AN APPROACH TO THE ASSESSMENT OF ARTEFACT DIMENSIONS
AS DESCRIPTORS OF SHAPE

M.N. Leese and P.L. Main

British Museum Research Laboratory
Great Russell Street, London WC1

1. Introduction

An attribute possessed by artefacts which is often of considerable interest in typological studies is that of shape. Equally, it is a 'problem' attribute. It is problematical because although it usually involves measuring a number of different dimensions and angles, e.g. height, height of maximum width, angle at base, and so on, the shape is still 'one thing' and one would like to treat it as such. In the words of David Clarke:

> "the problem feature in most archaeological studies is still the curious attribute of shape. Now the shape of an artefact is in a way a single unit entity - the shape - consequently if we seek to express a variety of shapes within a population as a series of multi-state ratios between key measurement proportions, then the shape will be overweighted and poorly expressed."
> (Clarke, 1968)

So one is faced with a choice between two approaches if one wishes to analyse shape quantitatively.

(i)   take a series of 'significant' measurements from the artefacts and use these to make comparisons between them,

(ii)   record the entire outline shape in an objective way and base inter-artefact comparisons on the complete outlines.

2. Measurements and Outline Shape

A disadvantage of the 'significant measurement' approach is that the archaeologist may be unsure of how many, and what, measurements ought to be taken to represent the shape accurately enough for typological purposes. This can result in measuring an excessive number, leading to inherent redundancy in the information recorded. The points between which to take the measurements can be:

(i)   'features' - i.e. points on the artefact with some archaeological significance, or more often something which 'hits the eye' such as a corner or point of local maximum in the curvature.

(ii)   arbitrarily defined points on the outline, e.g. width at half-height.

If the points chosen have true archaeological significance, then their use as they stand is of course justified, but this is unusual. Where arbitrarily defined points are used, they are only justifiable if they lead to a satisfactory representation of the shape, and preferably one without too much inherent redundancy.

The second approach to representing and comparing shape (using the complete outline shape) has the disadvantage that it requires the outline to be digitised, and this is laborious and may even be impossible for a large assemblage, unless very sophisticated equipment is available. Broken artefacts present a problem unless their original outlines can be completely reconstructed. Furthermore, it is usually a drawing that has to be digitised, and three-dimensional shape variation is difficult to represent adequately on paper.

It is often the case that even numerate archaeologists prefer the idea of taking measurements to that of digitising outlines, because the method of comparing outlines seems rather abstruse. They may also feel that such techniques are "too objective", and that artefact dimensions are more likely to reflect to some extent the intentions of the artisan who constructed them. Another advantage to the archaeologist is that dimensional measurements are relatively easy to take, and it is usually not even necessary to draw the artefact first. Moreover, a series of scores representing shape can be combined conveniently with other (non-shape) attributes, whereas this is difficult when dealing with complete outlines. Thus there are a number of advantages in the "significant measurement" approach which make it worth considering even when facilities for digitising are available.

3. The Aim of This Study

Even if one accepts the advantages of using artefact dimensions as shape descriptors, there is still the difficulty of deciding on the measurements to take. This difficulty is related to the fundamental problem that there is no benchmark against which any given set of attributes can be assessed as shape descriptors. If such a benchmark were available, however, and this would presumably involve digitising complete outlines, sets of dimensional attributes could then be compared with the benchmark on some representative subset of the complete assemblage. The most satisfactory set of measurements could then be used as shape descriptors for the whole assemblage with some degree of confidence, and without the need for digitising every artefact.

The aim of this study is to explore the adequacy of subjectively chosen dimensional measurements and descriptors of overall shape, using the tangent profile representation as benchmark. The tangent profile of a two-dimensional outline is derived from the digitised points by first passing a smooth curve through them and then converting this curve mathematically to a function of tangent angle versus arclength along the curve (measured from some fixed reference point). This

procedure, and the properties of tangent profiles are discussed in detail in Main (1981b). The tangent profile representation was chosen as a benchmark because it

(i)   allows mathematical representation of the original outline to any degree of accuracy (limited only by the density of digitising employed),

(ii)   provides a measure of comparison which treats the whole of each outline in a uniform manner,

(iii)   produces results comparable with those of human shape perception.

Main (1981a) describes well-developed software available at the British Museum for transforming cartesian outlines to tangent profiles and calculating a variety of distance measures (i.e. measures of shape dissimilarity) between pairs of outlines. The tangent profile representation satisfies (i) and (ii) above; see Main (1981b). Furthermore this combination of representation and distance measure has been shown in preliminary work to give good agreement with the archaeologist's own subjective groupings and hence satisfies (iii).

4. The Material

The data used for this investigation were taken from 546 Early Bronze Age axes collected in Southern Britain. The archaeologist concerned with the material had drawn all the axes in three sectional views and had taken 20 measurements from each axe. He had also recorded other non-shape attributes, and had used the ranges of variation within the attributes and measurements to divide the assemblage into five broad chronological classes.

From this data-set we chose 94 axes from two of the major classes (4 and 5) which were in a sufficiently good state of preservation to allow their complete outlines to be reconstructed. Only the 47 class 4 axes were considered initially and three outliers were omitted, leaving a set of 44. One of the sectional views is shown in figure (1), which illustrates 12 of the continuous measurements used in the study. The following variables were considered:

WE - Width of cutting edge; DE - Depth of cutting edge;

LB - Length of body; MO - Maximum Offset;

LC1 - Length between MO and tip; WB - Width of butt;

DB - Depth of butt; WS - Width at stop;

DEB - Depth of edge bevel to cutting edge; W1 - Width 0.25 length of body;
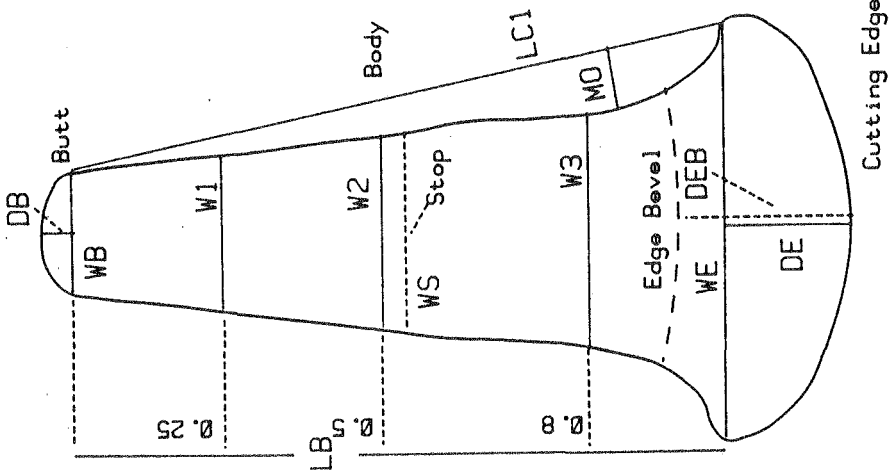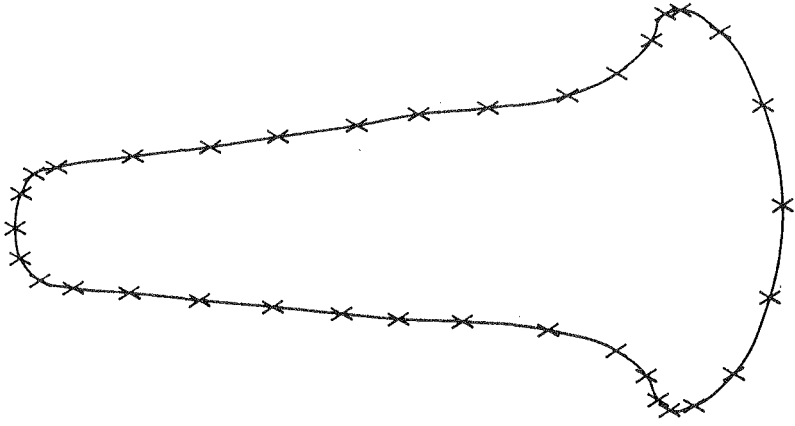
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Fig.(2)   Typical Axe Showing Digitising Layout

Fig.(1)   Typical Axe Showing Dimensional Measurements

W2 — Width 0.5 length of body; W3 — Width 0.8 length of body.

Before proceeding further, all these measurements were converted to proportions of the overall length L and were log-transformed. The reason for the using proportions of length was to obtain a set of measurements comparable with the outline shape representation, which was standardised to unit perimeter. Both representations are therefore independent of overall scale. There are some inherent intercorrelations among the variables; for example DE, LB and DB are defined to sum to 1; and a degree of correlation may be introduced by the standardisation to unit overall length. In order to simplify the problem it was decided to choose only one from pairs of variables that were highly correlated, the highest correlations among the basic variables being DE with DEB and LB; W1 with WB, W2 and WS; W2 with WS. On this basis it was decided to omit DE, W2 and WS because they were unlikely to add to the information provided by the other variables. The final set of measurements used was thus reduced to WE, LB, MO, LCl, WB, DEB, W1, W3.

In addition, the complete outline shape of each axe was digitised from the same plan view, as in the example in figure (2), where the crosses indicate the digitised points.

5. Distance Measures Dtp and Dmr

The measure of shape difference between pairs of outlines used was the unsigned area between the two tangent profiles, represented by the area between the graphs in figure (3). This distance measure was computed for all 946 pairs of axes and the square of each value, denoted Dtp, was stored as an element in a matrix.

In order to compare the performance of the dimensional measurements with the profile representation, we require a corresponding squared distance matrix based on measurements. Using one measurement (say WE) we can define the squared distance as $Dwe = (WE_i - WE_j)^2$ for axes i and j. For a number of measurements we either have to combine their basic values and then compute distances, or combine the squared distances to obtain a single overall squared distance Dmr. A straightforward approach to the latter is to fit a least squares regression of the form:

$$Dmr = A_0 + A_1 D_1 + A_2 D_2 + \dots A_n D_n$$

where D_k is the individual squared distance based on the kth measurement, n is the number of measurements and $A_0 \dots A_n$ are constants.

The presence of the constant A_0 means that Dmr (unlike Dtp) is not a strict distance measure. However the value of A_0 fitted is an indication of 'missing' information: if Dtp = 0 then D_1, D_2, ... D_n = 0, but not vice versa. We therefore included the term in these experi-

ments, since the magnitude of A_0 was of some interest in itself.

The least-squares program used to fit the values for A_0, A_1, ... A_n also computed the multiple correlation coefficient between Dmr and Dtp, and standardised partial regression coefficients for the individual contributors to Dmr. These were used as guides to the success of Dmr in approximating to Dtp, and the importance of the various contributors $D_1 ... D_n$. No statistical interpretation is put on the values obtained since we are not dealing with a random sample of axes from some larger population, and in any case most of the usual assumptions of regression analysis are obviously untenable. In particular the distances are dependent and, as Gower (1971) has pointed out, misleadingly high correlation can be observed in these circumstances. However, an increase in correlation can be expected to result from improvements in the approximation of Dmr to Dtp and hence it is reasonable to use it as a comparative measure of associa- tion. The following section describes the results of experimenting with various subsets of the eight possible contributors.

6. Correlation between Dtp and Dmr

The highest overall multiple correlation between Dtp and any linear combination of the eight possible measurement distances was 0.86, and this was achieved with the subset WE, LB, W3 and WB. These were subsequently used for the computation of Dmr. WE and LB together gave a multiple correlation coefficient of 0.83, the other variables contributing little extra. One interesting aspect is the small over- all effect of adding in MO, DEB, LC1 and W1 once the other variables are included. This is partly because of the intercorrelations, those greater than or equal to 0.2 being shown below in Table (1). The table also shows the correlations of the individual distance measures with Dtp. (The intercorrelations between the basic variables showed a similar pattern but were higher in magnitude than those between the distance measures.) It is worth noting that the intercorrelations between the dimensional distances were all much lower than that between a combination of them (Dmr) and Dtp. This suggests that the high cor- relation between Dmr and Dtp is not merely a spurious result of dependence within the distance matrices.

Table (1) Correlations Between Distance Matrices

	WE	LB	MO	WB	W3	DEB	W1	LC1	Dtp
WE*			.4		.2		.2		0.6
LB*			.3			.3			0.5
MO									0.5
WB*							.5		0.3
W3*						.3	.2	.4	0.2
DEB								.2	0.2
W1									0.0

*These give, in combination, a correlation of 0.86.

As a check on the plausibility of the Dmr distance matrix, it was used to produce a non-metric multidimensional scaling configuration in two dimensions and this gave reasonable agreement with one produced using the Dtp values, which had itself been in broad agreement with the archaeologist's grouping.

7. Discrepancies between Dtp and Dmr

We have proposed a distance measure for axe shapes which makes use of dimensional measurements traditionally used by archaeologists. We also have a representation of the whole outline shape in terms of a tangent profile, which can generate inter-axe distances. The discrepancies between the two distance measures are of interest, because they indicate improvements that might be made either in fitting Dmr to Dtp or in choosing the appropriate variables to include.

Discrepancies can arise from the use of an inappropriate combination of dimensional measurements: for example, the inclusion of an intercept term clearly contributes to an error in the estimates of distance when the axes are very similar in outline. A linear combination of individual measurement differences may not be appropriate. It will therefore be important to improve the model as far as possible so that any residual differences can be confidently attributed to missing measurements.

Leaving aside until future work the question of model inadequacies, it was decided to investigate pairs of axes where the dimensional measurement distances departed most radically from the outline shape distances. Among the programs being developed for the investigation of the relationship between the two distance measures is one which compares two distance matrices, ranks them and prints out an ordered list of the differences in rank. Those at the top of the list are cells in the matrix (that is, pairs of axes) that are close on distance measure 1, but far on distance measure 2. Those at the bottom of the list are far on distance measure 1 but close on measure 2.

An extreme example of a pair of axes differing in terms of outline shape but differing little in terms of measurement distance is shown in figure (3), which gives the outline shapes, their tangent profiles and the distance profile. The distance profile is the function $(T1-T2)^2$ where T1 and T2 are the tangent profiles being compared. It indicates two peaks at either end corresponding to the differences in curvature at either end of the cutting edge, and a smaller difference in the middle, corresponding to a difference of thickness in the body.

How does this compare with the individual measurement differences? Table (2) gives the logged standardised values for the measurements included in the distance measure, and the possible contributors that had been excluded.
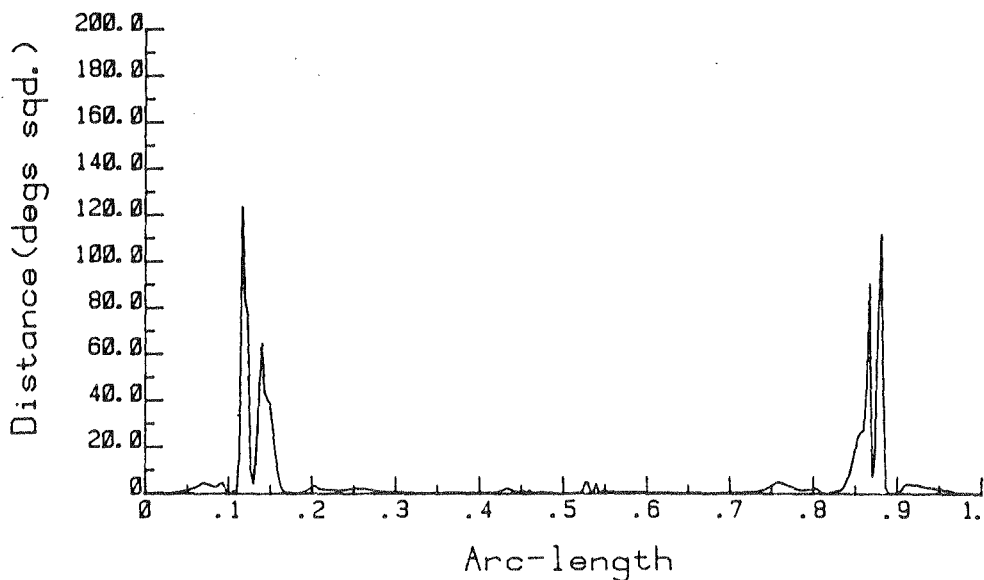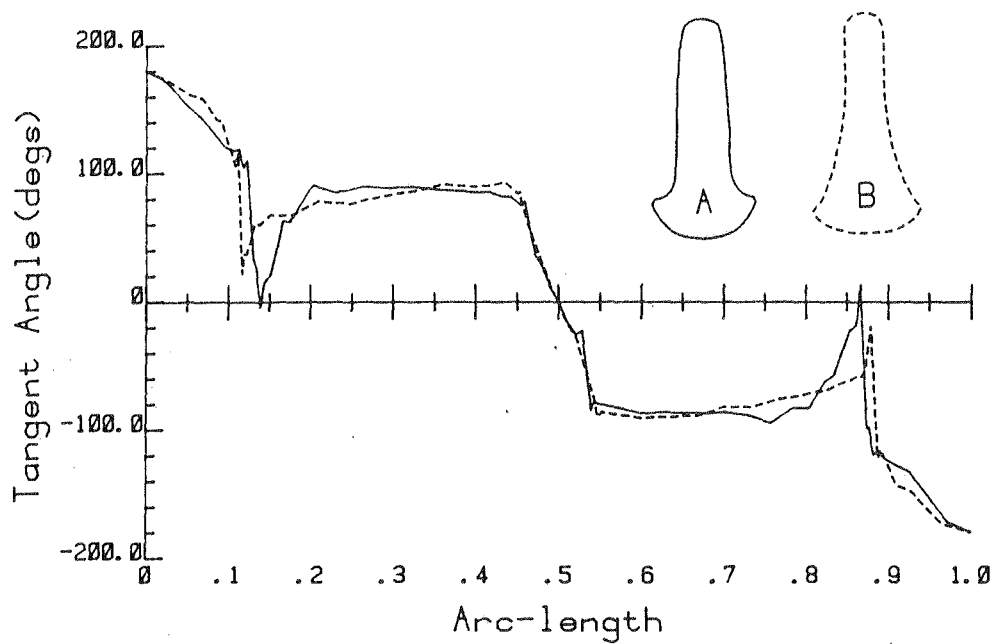
Fig.(3) Tangent- and Distance Profiles for Two Contrasting Axes

Table (2) Measurements for Axes A and B in Figure (3)

	Measurements included in Dmr				Measurements excluded from Dmr			
	LB	WE	WB	W3	MO	LC1	DEB	W1
A	-0.23	-0.73	-1.76	-1.36	-2.51	-0.37	-2.0	-1.50
B	-0.19	-0.70	-1.74	-1.17	-2.65	-0.72	-2.47	-1.72
$d = \lvert A - B \rvert$	0.04	0.03	0.02	0.19	0.14	0.35	0.49	0.22
S.d. (all axes)	0.04	0.11	0.13	0.16	0.30	0.15	0.67	0.13
Ratio (d/S.d.)	1·	⟨1	⟨1	1.2	⟨1	2.3	⟨1	⟨1.7

Clearly for this particular pair of axes the measurements LC1 and
W1 differ significantly (differences underlined above) but, because
these variables had been excluded from Dmr the two axes were not well
distinguished. In other cases where Dtp and Dmr diverged there was
usually a similar explanation in terms of omitted variables.

8. Summary

We have gathered together a number of techniques which it is hoped
will enable us to identify those measurements that are essential for
representing the outline shapes of EBA axes and those that are redund-
ant. At present they comprise the following computer programs with
which to investigate outline shapes in general and their representation
in terms of separate measurements:

(i)   Generation of tangent profiles and calculation of (squared)
        distances (Dtp) between them;

(ii)   Calculation of individual (squared) measurements distance measures,
        and weighted linear combinations of these (Dmr);

(iii)   Correlation between Dtp and Dmr, and identification of
        discrepancies between them.

The programs have been tested on a set of 44 axes, and the initial
experiments have revealed a need for a more careful consideration of
the combination of dimensional distances used to generate Dmr. In
particular the appropriate form of weighting and the initial elimination
tion of variables need to be investigated further and the intercept
term should be dropped. Nevertheless the simple linear fit as
described here explains a reasonably large proportion of the variation
in distances, and the discrepancies in the model could in the most
extreme cases be explained in terms of the omission of measurements

from Dmr.

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References

Clarke, D.   Analytical Archaeology, Methuen, London.
1968

Gower, J.C.   Statistical methods of comparing different multivariate
1971   analyses of the same data.   In: MATHEMATICS IN THE
               ARCHAEOLOGICAL AND HISTORICAL SCIENCES, 138-149, (Eds.
               Hodson, F.R., Kendall, D.G. and Tautu, P.) Edinburgh
               University Press.

Main, P.L.   SHU - an interactive graphics program for the storage,
1981a   retrieval and analysis of artefact shapes.   PROC.
               QUANTITATIVE METHODS IN ARCHAEOLOGY CONFERENCE (CAA 81),
               Institute of Archaeology, London, 1981.

Main, P.L.   A method for the computer storage and comparison of the
1981b   outline shapes of archaeological artefacts, PH.D. THESIS
               (CNAA), May 1981.