INFORMATION THEORY AND DENDROCHRONOLOGY; THE EFFECT OF PRE-WHITENING

R.R. Laxton and C.D. Litton

Department of Mathematics, University of Nottingham

Nottingham NG7 2RD

1. Introduction

As a practical method of dating oak structures, dendrochronology has been very successful. The results for the British Isles are contained in many articles and in two recently published books; Fletcher (1978) and Baillie (1982). As described in Fritts (1976), the weather is usually the main factor in determining the width of the annual ring of growth round the circumference of the trunk of the tree. Notwithstanding the details, this relationship between weather and ring-width makes the relative dating (ie cross-dating) of a piece of oak by comparing its sequence of successive indices of ring widths with other such sequences practical.

There has been much recent work analysing the relationship between the weather conditions and ring widths and this is important for dendrochronological theory and practice (see Fritts 1976, Hughes et al 1978 for further references). Equally important is the theory of cross-dating and it is to this aspect of the problem that this paper is devoted. In an earlier article, Laxton and Litton 1982, we showed how cross-dating could be interpreted as a decoding for a Gaussian channel. Here we take the programme a stage further by pre-whitening all sequences of indices before analysing the cross-dating process and as a result improve the process of cross-dating. In theory, and largely in practice too, pre-whitening removes the autocorrelation within the sequences so that they are better suited for interpretation by means of Gaussian channels.

In the following we will show that cross-dating within five (pre-whitened) dendrosystems from the East Midlands of England is well modelled by the decoding of 'good random' codes in appropriate Gaussian channels (Gallacher, 1968). In future this may provide a guide to the success or otherwise of a master dendrochronological sequence - especially if it is to be used to cross-date specimens from oak trees growing in a varied geographical and/or climatic region. The years of growths of the rings in the samples in these five dendrosystems are known since actual felling and boring dates are known.

We indicate the usefulness of this approach by applying it to five more dendrosystems from the East Midlands whose chronologies we have dated with a master chronology. For example, it is well known that 'short' sequences are difficult to cross-date with a site chronology. We estimate from characteristics of a given dendrosystem the least length for which the probability of error in cross-dating with the chronology is at most 0.01 (a 1% error bound). We show how the 1% error bound varies with the power to noise ratio and compare the theoretical bound with actual data. The agreement is quite good. Finally we make a further practical application to a collection of sequences from the tower of Hagworthingham Church, Lincolnshire.

Fritts in his book writes on the "...climatic and environmental information common to the sample trees" and also quotes Dean's remark that "cross-dating involves the recognition of identical messages". We show here that these ideas can be given quantitative expression in classical information theory.

2 Pre-whitening, chronology formation and dendrocodes

Let $\underline{x} = (x_1,\ldots,x_M)$ be a sequence of ring-widths, where as usual x_1 is the width of the innermost of these M rings. This sequence is first converted into a sequence $\underline{y} = (y_1,\ldots,y_{M-4})$ of indices by the transformation (Baillie and Pilcher 1973)

$$y'_{i-2} = \log_e(500x_i/(x_{i-2}+x_{i-1}+x_i+x_{i+1}+x_{i+2})), \text{ for } i = 3,\ldots,M-2.$$

Put $\bar{y}' = \frac{1}{(M-4)}\sum_{i=3}^{M-2} y'_{i-2}$ and $y_{i-2} = y'_{i-2} - \bar{y}'$ so that $\sum_{i=3}^{M-2} y_{i-2} = 0.$

Since the growth of a tree ring may be related to the growth of the rings in previous years, the terms of the sequence $\underline{x}$ may be autocorrelat Furthermore, the very form of the transformation from $\underline{x}$ to $\underline{y}$ may introduce more autocorrelation. It is well-known that cross-correlating two-autocorrelated but unrelated sequences (ie, perhaps from different time periods) will result in spurious matching (Box and Jenkins, 1970). Therefore in order to remove this autocorrelation, the sequence of indices $y = (y_1,\ldots,y_{M-4})$ is pre-whitened as follows.

Suppose the y_i form an auto-regression process of order R, then

$$y_i = \alpha_1 y_{i-1} + \alpha_2 y_{i-2} + \ldots + \alpha_R y_{i-R} + \varepsilon_i$$

where the α_j are the parameters of the process and the ε_i are independen and identically distributed Gaussian random variables with zero mean and constant variance σ_1^2. The α_j's can be estimated by using the sample autocorrelations and the Yule-Walker equations (ibid, 1970).

However, the order R of the process is unknown and so is determined using Akaike's Information Criterion (Akaike, 1973). Finally, the pre-whitened sequence $\underline{w}$ is calculated from

$$w_i = y_{i+R} - \hat{\alpha}_1 y_{i+R-1} - \hat{\alpha}_2 y_{i+R-2} - \ldots - \hat{\alpha}_R y_i$$

$i = 1,2,\ldots,M-4-R$. Thus we now have a sequence of length M-4-R of independent normal variables with mean 0 and constant variance.

If we start with K specimens, then the above process will lead to K sequences of ring-width indices $\underline{w}$ $,\ldots,\underline{w}_K$. (For simplicity only, all these sequences are of the same length $\bar{N}$.) We can write these in a form in which the K indices for a given year appear in one column together wi the average:

$$\begin{array}{c} w_{1,1},\ldots\ldots,w_{1,j},\ldots\ldots,w_{1,N} \\ \cdots\cdots \\ w_{k,1},\ldots\ldots,w_{k,j},\ldots\ldots,w_{k,N} \\ \cdots\cdots \\ w_{K,1},\ldots\ldots,w_{K,j},\ldots\ldots,w_{K,N} \\ \hat{\theta}_1,\ldots\ldots,\hat{\theta}_j,\ldots\ldots,\hat{\theta}_N . \end{array}$$

Here

$$\hat{\theta}_j = \frac{1}{K}\sum_{k=1}^{K} y_{k,j} \quad \text{for} \quad j = 1,\ldots,N$$

and $\hat{\theta}_1,\ldots,\hat{\theta}_N$ is the *site chronology* (for the period of N years in question). The whole; K sequences of indices and the site chronology is called a *dendrosystem*. The power of this average and the average of the sum of squares about the $\hat{\theta}_j$ is

$$\hat{P} = \frac{1}{N}\sum_{j=1}^{N}\hat{\theta}_j^2, \quad \text{and} \quad \hat{\sigma}^2 = \sum_{j=1}^{N}\left(\sum_{k=1}^{K}\left(w_{k,j}^2 - \frac{1}{K}\hat{\theta}_j^2\right)/(KN-1)\right), \text{ respectively.}$$

We will assume the existence of an underlying signal of ring width indices $\theta_1,\ldots,\theta_N$, called the *dendrosignal*, common to all trees growing under similar conditions. The sequence $w_1,\ldots,w_N$ of N consecutive indices of a particular specimen for the corresponding years is, then, composed of the dendrosignal plus a 'noise' factor due to the micro-environmental conditions of this specimen. That is, with each year i there is associated a random variable Z_i, called the *dendronoise*, with expected value 0 and $w_1,\ldots,w_N$ is an outcome of the sequence $W_1 = \theta_1 + Z_1,\ldots,W_N = \theta_N + Z_N$ of N random variables. That is, $w_1,\ldots,w_N$ is a 'noisy' version of the dendrosignal $\theta_1,\ldots,\theta_N$. If the Z_i are independent, identically distributed Gaussian random variables with variance σ^2, then we have described here a time-discrete *Gaussian channel* (Ash 1965, Gallacher 1968, Jones 1979) to model the dendronoise. The pre-whitening of the indices was introduced specifically for this purpose. The capacity of this channel is $\frac{1}{2}\log_e(1 + \hat{P}/\hat{\sigma}^2)$.

An actual dendrosystem can be simulated by first forming a random sequence of real numbers $\alpha_1,\ldots,\alpha_N$ generated from the Gaussian distribution with mean 0 and variance P together with a further K sequence $\gamma_{k,1},\ldots,\gamma_{k,N}$, $k = 1,\ldots,K$, where $\gamma_{k,i} = \alpha_i + \beta_{k,i}$ and where the K sequences $\beta_{k,1},\ldots,\beta_{k,N}$ are generated from the Gaussian distribution with mean 0 and variance $\hat{\sigma}^2$. The simulation can be carried out several times and various averages calculated and these can be used to compare an actual dendrosystem with its corresponding Gaussian channel. We will describe this now. The idea is not just to compare a specific sequence $x_1,\ldots,x_N$ with the *whole* dendrosignal $\theta_1,\ldots,\theta_N$ (or with the whole site chronology $\hat{\theta}_1,\ldots,\hat{\theta}_N$) but to obtain more understanding by comparing *subsequences* $x_i, x_{i+1},\ldots,x_{i+n-1}$ with subsequences $\theta_j,\ldots,\theta_{j+n-1}$ of the dendrosignal of the same length and to do this for various lengths n.

A *dendrocode* U_n of a dendrosystem consists of all the (N-n+1) subsequences $\hat{\theta}_i,\ldots,\hat{\theta}_{i+n-1}$ of length n in $\hat{\theta}_1,\ldots,\hat{\theta}_N$. Any of the K particular specimen subsequences $w_{k,i},\ldots,w_{k,i+n-1}$ is regarded as a 'noisy' version of $\hat{\theta}_i,\ldots,\hat{\theta}_{i+n-1}$ after passing through the Gaussian channel with dendronoise $\hat{\sigma}^2$. The power of U_n is $\hat{P}$ (which is reasonable for n large enough) and the rate is $R = \frac{1}{n}\log_e(N-n+1)$. The specimen subsequence $w_{k,i},\ldots,w_{k,i+n-1}$ is cross-dated with <u>all</u> (N-n+1) subsequences in U_n and is decoded as that subsequence $\theta_j,\ldots,\theta_{j+n-1}$ in U_n for which the t-value is maximum (cf. Fritts 1976, Baillie 1982). If $j = i$ the cross-dating is correct, otherwise an error is made. Cross-dating can be carried out in this manner for all $K(N-n+1)$ such specimen subsequences of length n and the proportion of errors $E(n)$ in cross-dating calculated. We determine $E(n)$ for various values of length n, at least until $E(n) = 0$, for each of the five (and later five more) dendrosystems.

We remark here that decoding by means of the maximum of the t-values (Baillie, 1982) is essentially equivalent to the more usual nearest neighbour technique used for decoding in information systems (Ash 1965, §8).

The above procedure can be carried out with data simulating the dendrosystem, as described above. For each simulation (ie, with the same power $\hat{P}$ and 'noise' variance σ^2) the proportion of errors is calculated and from, say, 100 such simulations an average proportion of errors $E_a(n)$ calculated (100 simulations were used in this work; also we took $K = 10$ in all cases). This is done for various values of n, at least until $E_a(n) = 0$. The average $E_a(n)$ is an estimate of the average probability of error in decoding a codeword over an ensemble of codes of codeword length n of given rate $\frac{1}{n}\log_e(N-n+1)$ and power at most $\hat{P}$ in a Gaussian channel with noise σ^2. Of interest are those n for which the rate is less than the capacity. Now $E_a(n)$ tends to zero as $n \to \infty$ and indeed in a manner accurately given by

$$E_a(n) = 0.5\, An \exp(-\ell An) \qquad (2.1)$$

where ℓ depends only on the estimated power to noise ratio $A = \hat{P}/\sigma^2$, at least for the range of rates considered here. This simple expression approximates more general ones given in Gallacher 1968, §7; we demonstrate its validity for the systems analysed here. An estimate for ℓ is obtained from the linearized form

$$\log_e(E_a(n)/An) = -\ell An + \log_e 0.5. \qquad (2.2)$$

Indeed we further show that ℓ is well approximated by

$$\ell = 0.27 - 0.053\, A \qquad (2.3)$$

over the range of A obtaining here, so that (2.1) reduces to

$$E_a(n) = 0.5\, An \exp((-0.27\, A + 0.053\, A^2)n) \qquad (2.4)$$

This formula for ℓ approximates general expressions given in Gallacher.

To return to the dendrosystems considered, we compare the proportions of errors $E(n)$ with the corresponding proportions $E_a(n)$ and show that they are approximately the same. Indeed we show that the $E(n)$ too satisfies an equation of the form

$$E(n) = c\, An \exp(-kAn) \qquad (2.5)$$

for suitable constants c and k and evaluate them with the linearized form

$$\log_e(E(n)/An) = -k\, An + \log_e c. \qquad (2.6)$$

For more direct comparison with the simulated average $E_a(n)$, we also fit the data to an equation of the form

$$E(n) = 0.5\, An \exp(-\hat{\ell} An) \qquad (2.7)$$

again via its linearized form

$$\log_e(E(n)/An) = \hat{\ell}\, An + \log_e 0.5 \qquad (2.8)$$

3 Analysis of pre-whitened data

The five dendrosystems here analyzed are: Thoresby 9 consisting of 9 specimens from different oaks which grew in or close to the Thoresby Estate, Nottinghamshire; Emily Wood 54 consisting of 2 cores from each of 27 oak trees which grow in Emily Wood, Suffolk (Carter 1981); Emily Wood 9 consisting of 9 specimens from this latter collection and representing a more homogeneous stand of 9 trees within the wood; BECT 12 consisting of 3 samples from different trees from each of four sites in the East of England and finally the Thoresby Tree 12 dendrosystems consisting of 12 radii round a disc from the trunk of one oak from the Thoresby Estate.

The details of the dendrosystems of these five collections are described in Table 1.

DENDROSYSTEM	NUMBER OF SPECIMENS K	LENGTH OF SITE CHRONOLOGY N	POWER P	NOISE	POWER TO NOISE RATIO A
BECT 12	12	163	0.0087	0.0241	0.3592
EMILY WOOD 54	2 × 27	137	0.0124	0.0281	0.4424
THORESBY 9	9	156	0.0163	0.0252	0.6445
EMILY WOOD 9	9	125	0.0298	0.0257	1.1594
THORESBY TREE 12	12 × 1	146	0.0314	0.0151	2.0820

Table 1 *Details of five (pre-whitened) dendrosystems analyzed (see also Laxton and Litton, 1982).*

In Figs.1 to 5 are plotted the proportions of errors $E(n)$ in cross dating all $K(N-n+1)$ particular specimen subsequences of length n of a dendrosystem with the $(N-n+1)$ subsequences of length n of the site chronology for various values of n and at least until $E(n)$ becomes 0. Also plotted in each case are the proportions of error $E_a(n)$ averaged over an ensemble of codes of codeword length n with the same power P and rate $R = \frac{1}{n}\log_e(N-n+1)$ in a time discrete Gaussian channel with noise variance σ^2.

n_0 is the minimum codeword length n for which the rate $R = \frac{1}{n}\log_e(N-n+1)$ $< C$, the channel capacity, and n_1 is the minimum codeword length n for which the proportion of errors $E(n) \leq 0.01$ (see Section 4).

Figs. 1 to 5 show that the proportions of errors $E(n)$ for the dendrosystems and the proportions $E_a(n)$ for the corresponding averages follow the same trend and are close to each other especially when $E(n)$ or $E_a(n)$ tends to 0. (Later we consider the least value of n for which $E(n) \leq 0.01$, ie, a 1% error in cross-matching.) For BECT12 the proportion $E(n)$ is less than the corresponding $E_a(n)$, for Thoresby Tree 12 it is the same, while for the other three it is greater than the corresponding average.

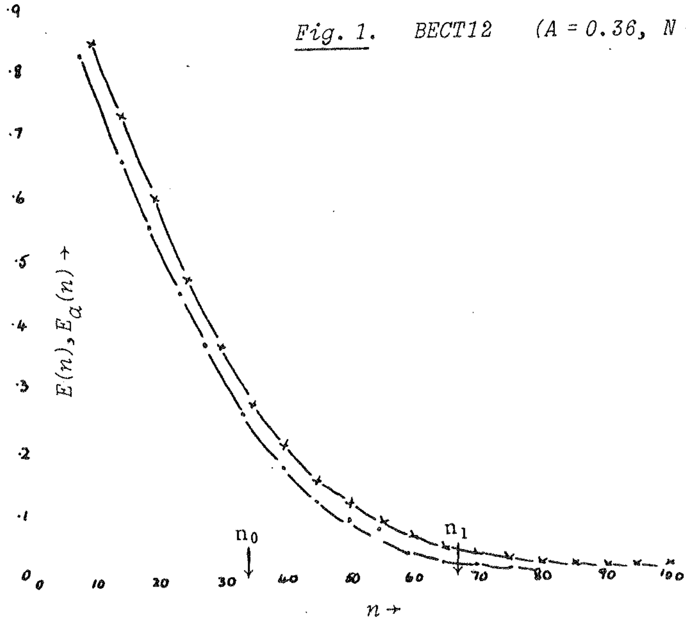


Fig. 1. BECT12 (A = 0.36, N = 163)

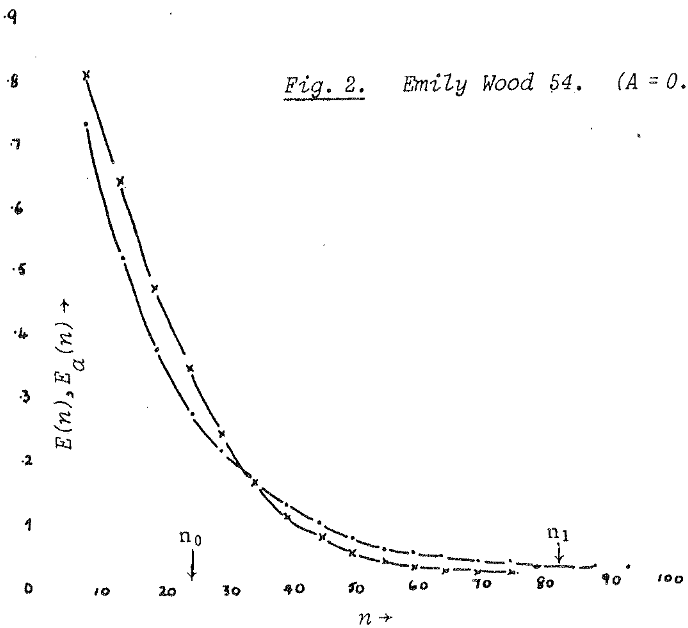
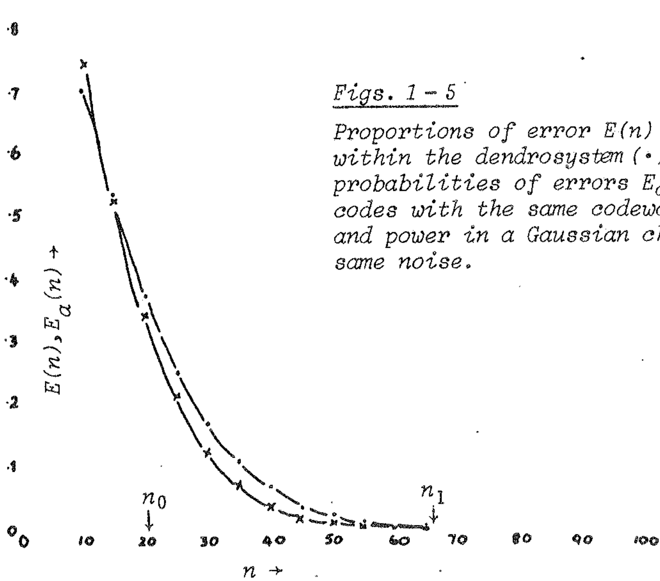


Fig. 2. Emily Wood 54. (A = 0.64, N = 156)

Fig. 3. Thoresby 9. $(A = 0.64,\ N = 156)$



Figs. 1 – 5

Proportions of error $E(n)$ in cross-matching within the dendrosystem (•) and the average probabilities of errors $E_{\alpha}(n)$ (x) in decoding codes with the same codeword length n, rate and power in a Gaussian channel with the same noise.

Fig. 4.

Emily Wood 9.
$(A = 1.16,\ N = 125)$

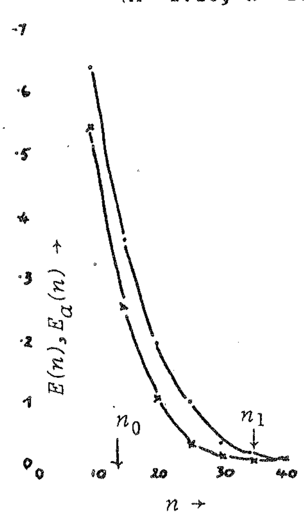
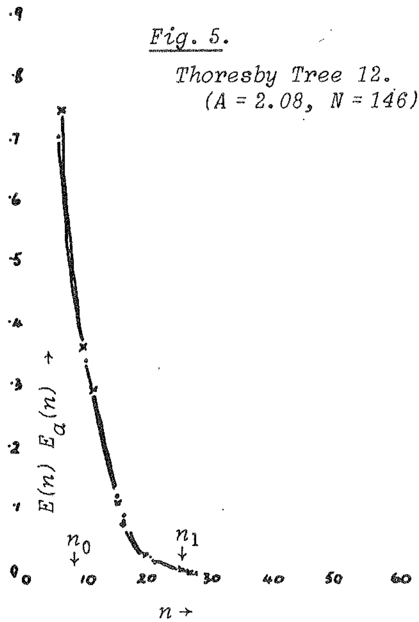


Fig. 5.

Thoresby Tree 12.
$(A = 2.08,\ N = 146)$



A more searching comparison can be made with the aid of the results listed in Table 2.

DENDROSYSTEM	A	c	k	d_1	$\hat{\ell}$	d_2	ℓ	d_3
	RESULTS FOR THE DENDROSYSTEMS						RESULTS FOR THE SIMULATED AVERAGE	
BECT12	0.36	0.99	0.31	0.97	0.27	0.95	0.25	0.99
EMILY WOOD 54	0.44	0.14	0.17	0.99	0.22	0.90	0.25	0.99
THORESBY 9	0.64	0.19	0.16	0.98	0.18	0.98	0.23	0.99
EMILY WOOD 9	0.16	0.72	0.19	0.99	0.19	0.94	0.20	0.99
THORESBY TREE	2.08	0.45	0.16	0.99	0.16	0.99	0.16	0.99

Table 2 . *Values obtained for the coefficients c, k,* $\hat{\ell}$ *and* ℓ *in equations (2.2), (2.6) and (2.8).* d_1, d_2 *and* d_3 *are the coefficients of determination in each case (Chatfield 1970). Least square fittings were obtained with a NAG Library routine (E04FDF; Numerised Algorithms Group, 1981).*

The values of the coefficients of determination d_3 and d_1 show that the fits to linear equations (2.2) and (2.6) are extremely good. Even $d_2 \geq 0.90$ in all cases, though the fits to equation (2.8) are not as good as to (2.6) - as is to be expected. The value of ℓ decreases with increasing value of A; an approximate expression for ℓ as a function of A is $\ell = 0.27 - 0.053A$. The values of k and ℓ also fall with increasing value of A with the exception that those for the Emily Wood 9 dendrosystem rise slightly. In all cases $|\ell - k| \geq |\ell - \hat{\ell}|$. These results show that the actual dendrocodes behave very much as 'good random' codes of the corresponding Gaussian Channel.

Finally, it is interesting to compare these results with those obtained when the sequences of indices remained unpre-whitened. In three cases, for BECT12, Emily Wood 9 and Thoresby Tree, the proportions of errors E(n) are decreased by pre-whitening, whilst those for the other two, Emily Wood 54 and Thoresby 9, remain about the same. The improvement is most marked for the BECT12 dendrosystem (Laxton and Litton, 1982).

4. An application; the 1% error bound

Here we consider a practical application of the method developed so far. We consider ten dendrosystems: the five of known date already considered together with five more, originally of unknown date, whose site chronologies had to be constructed first and then were subsequently dated by cross-dating with a master chronology (by the early summer of 1982). Now each of the former has at least 9 component subsequences and a site chronology of 125 to 163 indices (see Table 1). The five more chosen have roughly similar characteristics and are from the East Midlands (see Laxton et al, 1979 and 1983). The relevant data is included in Table 3; all ten sets are given for ease of comparison.

DENDROSYSTEM	K	N	A	c	k	d_1	$\hat{\ell}$	d_2	n_1
BECT12	12	163	0.36	0.99	0.31	0.97	0.27	0.95	68
EMILY WOOD 54	2 × 27	137	0.44	0.14	0.17	0.99	0.22	0.90	81
CROPWELL	2 × 14	230	0.51	0.63	0.23	0.98	0.22	0.98	60
THORESBY 9	9	146	0.64	0.19	0.16	0.98	0.18	0.98	65
ARNOLD	10	157	0.68	0.25	0.15	0.97	0.17	0.95	65
HAGWORTHINGHAM (25)	2 × 21	189	0.75	0.17	0.14	0.99	0.17	0.94	63
LINCOLN (55)	2 × 9	138	1.06	0.29	0.16	0.99	0.18	0.98	39
EMILY WOOD 9	2 × 9	125	1.16	0.72	0.19	0.99	0.19	0.94	35
SNEINTON	2 × 12	117	1.52	0.34	0.16	0.99	0.17	0.99	31
THORESBY TREE	9	146	2.08	0.45	0.16	0.99	0.16	0.99	23

Table 3 *Details of ten (pre-whitened)dendrosystems from the East Midlands together with the values obtained for the coefficients c, k and ℓ in equations (2.6) and (2.8). d_1 and d_2 are the coefficients of determination. n_1 is the minimum subsequence length n for which the proportion of errors $E(n) \leq 0.01$.*

Again it is apparent that the fits to the two linear equations are good and that with a few exceptions k and $\hat{\ell}$ are decreasing functions of the power to noise ratio A.

A clear impression of how the proportion of errors E(n) behaves relative to the average proportion of errors $E_a(n)$ for random codes in a Gaussian channel with the same power to noise ratio A can be gained from Fig.6. We have shown in the previous section that the relationship between $E_a(n)$ and n is well approximated by

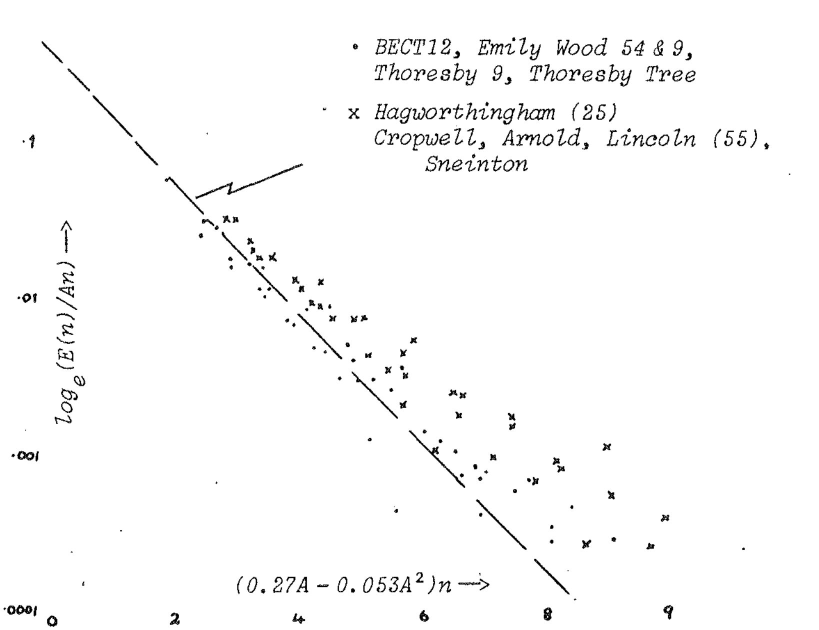
$$\log_e(E_a(n)/An) = -(0.27A - 0.053A^2)n + \log_e 0.5 \qquad (4.1)$$

In Fig.6 we have plotted E(n)/An (ie, for the dendrosystem itself) on a logarithmic scale against $(0.27A - 0.53A^2)n$ for all ten dendrosystems. It follows that if $E(n) = E_a(n)$ for all n the points would lie along a line with slope -1 and intercept 0.5 on the logarithmic axis. This line has been drawn in Fig.6. It is evident from this figure that while in general the five dendrosystems with samples of known felling and sampling dates, BECT12, Emily Wood 54 and 9, Thoresby 9 and the Thoresby Tree, have their data points lying along or close to this line those from the other five systems, Hagworthingham (25), Cropwell, Arnold, Lincoln (55) and Sneinton, tend to lie above it, especially for the larger lengths n. That is, E(n) tends to be greater than $E_a(n)$ in these cases. Presumably this imperfection is because one or two of the samples in each of these dendrosystems come from an environment somewhat different from the remaining ones.

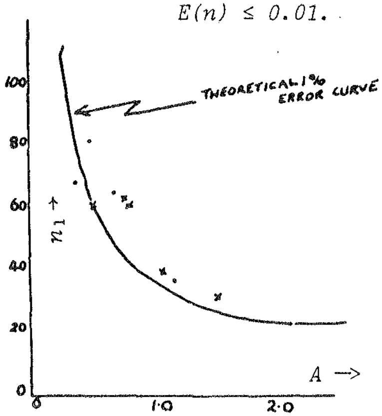
Generally, shorter sequences are more difficult to cross-date with a site chronology than longer ones. As a means of interpreting this quantitatively we have adopted the standard of a 1% error. In Table 3 are tabulated the values of n_1, the minimum subsequence length n for which $E(n) \leq 0.01$, for all ten dendrosystems (see also Figs.1 to 5). In general terms, Emily Wood 54 being the outstanding exception, n_1 declines steadily with increasing power to noise ratio. A theoretical estimate of a 1% error curve together with the ten values of n_1 are shown in Fig.7. The theoretical bound is obtained from equation (4.1) by putting $E_a(n) = 0.01$ to get

$$\log_e(0.01/An_1) = -(0.27A - 0.053A^2)n_1 + \log_e 0.5 \qquad (4.2)$$

<u>Fig.6</u> Plot of $E(n)/An$ on a logarithmic and $(0.25A-0.053A^2)n$ on a linear scale of data from the ten dendrosystems from the East Midlands.



<u>Fig.7</u> The theoretical 1% error curve and values of n_1, the minimum subsequence length n for which $E(n) \leq 0.01$.



The values of n_1 have been estimated from cross-dating the subsequences of *components* of the dendrosystems used in forming the site chronologies. But it can be used as an estimate for the probability of error in cross-dating *other* sequences of indices with a site chronology. To illustrate this consider the Hagworthingham (25) dendrosystem. The samples of this came from an oak frame inside the tower of Hagworthingham Church, Lincs. The tower was demolished in 1981. A full account of the dendrochronological analysis of this frame (and bell-stage at its top) will appear elsewhere. Suffice it to say here that each timber was sampled and measured twice (sequences A and B) and that 21 pairs of sequences formed a site chronology, HAGSEQ25, for the main frame. Its length is 189 indices, the maximum length of any (pre-whitened) constituent sequence is 149 and the minimum is 66 indices. 10 pairs of samples from the main frame were not incorporated in HAGSEQ25. These were then cross-dated with this site chronology. The success or otherwise of cross-dating with HAGSEQ25 was judged as follows. In each case sequence A from a timber was cross-dated with sequence B from the same timber and the relative position (offset) of one to the other obtained. Then sequences A and B were cross-dated with HAGSEQ25 and only if the maximum t-values for *both* were in the correct relative offset of A to B were the matches considered successful. Of these 10 pairs, 7 were successfully cross-dated with HAGSEQ25 and 3 were not. The shortest length of a pair A and B in each of these 3 cases is 68, 51 and 51, respectively. Thus each pair whose shortest length is greater than 68 has been successfully cross-dated with HAGSEQ25. Now n_1 for Hagworthingham (25) is 63 (see Table 3). In consequence, if we are satisfied with a 1% error in cross-dating, we should not consider these 3 pairs as not belonging to the same group of trees from which the other 28 samples came. It is quite possible they failed to match because of the nature of the dendrosystem - specifically, the value of its power to noise ratio A. The theoretical estimate for n_1 for $A = 0.75$ is 44, which is rather low.

5 Conclusions

We have recorded here a second attempt to discuss certain aspects of dendrochronology within the framework of information theory of Gaussian channels. In the first (Laxton and Litton, 1982) we used unpre-whitened whilst here we used pre-whitened sequences of indices of ring-widths. From the present work we conclude that it is a good approximation to the truth to say that pre-whitened dendrosystems tend to behave with respect to cross-dating between components and the site chronology as 'good random' codes of the same power and rate in a Gaussian channel with the same noise (the power and noise as estimated above). As a consequence behaviour in cross-dating can be predicted theoretically with reasonable accuracy and used in practical situations.

We pointed out in section 3 that as a consequence of pre-whitening there is an overall improvement in cross-dating. As Knonberg (1981) points out this is to be expected and, perhaps, indicates that all sequences of indices of ring widths should be pre-whitened in practice. Moreover pre-whitening may be especially significant for dendrosystems which cover a varied geographical and climatic region (ie, low power to noise ratio), for example BECT12.

The usefulness of the power to noise ratio has been underlined. It is the most important parameter which determines the behaviour of cross-dating with a site chronology. In particular, we have suggested the use of the 1% error curve as a quantitative measure of how short a sequence we can expect to successfully cross-date with a site chronology. (5% error curves could be used, perhaps, as profitably.) The theoretical estimate for n_1 is too low in practice for small values of A but is quite accurate for values of A greater than 1.

A consideration of the 5 dendrosystems Thoresby Tree, Emily Wood 9, Thoresby 9, Emily Wood 54 and BECT12 lead us to hope that the value of the power to noise ratio tells us something about the origin of the timbers from which the samples were taken (we mentioned this in the 1982 article also). Thus for pre-whitened data, a value of A of about 2 indicates that the samples come from one tree, a value of about 1 that they come from a smallish wood, a value of about 0.5 that they come from a largish but local estate, whilst a value below 0.4 indicates that the samples come from a wide area. In this manner we predict that the samples from Green's Mill, Sneinton come from but one or two trees, those from Lincoln (55) fror several but rather similar trees whilst those from Cropwell are from trees of a more varied nature or spread over a wider area.

Most of our samples have from 80 to 140 ring widths which after detrending and pre-whitening give from 70 to 130 indices. As a consequenc it may be impossible for us to form a site chronology with a power to nois ratio less than, say, 0.25 since, for example, the values of n_1 are greate than 130. This may provide a limit to the extent of the geographical and/ or climatic region that can be successfully covered in practice by a site chronology. Of course, other considerations besides straightforward cross dating may be used to overcome this difficulty. Fletcher, especially, use such methods (see, for example, Fletcher 1978).

Finally we mention that various other simulations were carried out to test the behaviour of the pre-whitened dendrosystems. These were mentione in the 1982 article by us and have not been described again in detail. Th results are substantially the same as those described here.

Acknowledgments

We thank Miss P J Whitley and Dr W G Simpson of the Nottingham University Tree-Ring Dating Laboratory. We are extremely grateful to Dr R Switzer of the Godwin Laboratory, University of Cambridge and Mr A Carter of Cambridgeshire Polytechnic who supplied us with the data for the specimens from Emily Wood. The calculations were carried out on the VAX 11/780 computer at the Cripps Computing Centre, University of Nottingham, and we must express our appreciation to Dr Hatfield of the Cripps Computing Centre for his help in implementing the programs.

References

Akaike, H. 1973 — Information theory and an extension of the maximum likelihood principle, 2ND INTERNATIONAL SYMPOSIUM ON INFORMATION THEORY, Akademia Kiado Budapest, pp 267-281.

Ash, R. 1965 — INFORMATION THEORY, Wiley and Sons, New York.

Baillie, M.G.L. & Pilcher J.R. 1973 — A simple cross-dating program for tree-ring research, TREE-RING BULLETIN 38, pp. 35.43.

Baillie, M.G.L. 1982 — TREE-RING DATING AND ARCHAEOLOGY, Croom Helm, London.

Box & Jenkins 1970 — TIME SERIES ANALYSIS, FORECASTING AND CONTROL, Holden-Day, San Francisco.

Carter, A. 1981 — A dendrochimatological Study of Emily's Wood, near Brandon, Suffolk, MA thesis, University of Cambridge.

Chatfield, C. 1970 — STATISTICS FOR TECHNOLOGY, Penguin Books, London.

Fletcher, J. 1978 — (ed) Dendrochronology in Europe, BAR INTERNATIONAL SERIES 51, Oxford.

Fletcher, J. 1980 — Tree-ring dating of Vernacular Buildings from Yorkshire VERNACULAR ARCHITECTURE 11, pp. 23-46.

Fritts, H.C. 1976 — TREE RINGS AND CLIMATE, Academic Press, New York.

Gallacher, R.G. 1968 — INFORMATION THEORY AND RELIABLE COMMUNICATION, Wiley, New York.

Graybill, D.A. 1979 — Revised Computer Programs for Tree-Ring Research, TREE-RING BULLETIN, 39, pp. 77-82.

Hughes, M.K., Gray, B., Pilcher, J., Baillie, M., Leggett, P. 1978 — Climatic signals in British Isles tree-ring chronologies. NATURE, 272, pp.605-606.

Jones, D.S. 1979 — ELEMENTARY INFORMATION THEORY, O.U.P., Oxford.

Kronberg, D. 1981 — Distribution of cross-correlations in two dimensional time series, with application to dendrochronology, Research Report No.72, Dept.of Theoretical Stats, Aarhus University.

Laxton, R.R., Litton, C.D., Simpson, W.G., Whitley, P.J. 1979. — Dendrochronology in the East Midlands, TRANS OF THE THOROTON SOC OF NOTTINGHAMSHIRE 93, pp.23-34.

Laxton, R.R., Litton, C.D. 1982 — Information theory and dendrochronology, SCIENCE AND ARCHAEOLOGY, 24, pp.9-24.

Laxton, R.R., Litton, C.D., Simpson, W.G., Whitley, P.G. 1983 — Tree-ring dated structures of the East Midlands, TRANS OF THE THOROTON SOC OF NOTTINGHAMSHIRE, to appear.