COMPUTER GENERATION OF KEY PATTERNS USED IN GRAECO-ROMAN MOSAICS

I.O. Angell

Department of Computer Science, Royal Holloway College, Egham, Surrey TW20 0EX

Although the title of this paper implies a concentration on the art of Greece and Rome, the geometrical art form I am considering is a fundamental part of cultures from all over the world. Key patterns, or lattice patterns as I prefer to call them, occur in the art of Islam, China, Burma, India, the Celts, the Mayas and many, many more cultures, as well as in the more well-known Graeco-Roman form.

I shall describe the pictorial results of a suite of computer programs written in FORTRAN IV and run on the University of London Computer Centre computers. There were two reasons for their production:

(i) I have been studying the geometrical art of ancient cultures for a number of years, in particular Celtic Art (Angell 1978a; 1978b). Writing programs to draw these designs has proved an invaluable aid in understanding the mathematical basis of this type of pattern, and realising its potential. Many authors who have written on this subject have, to my mind, over-complicated the problems (Romilly-Allen 1903; Bain 1951; Critchlow 1976). Working on the programs, and the practical implementation of such patterns, has simplified my understanding of the concept to a great extent.

(ii) Many archaeologists require diagrams of these patterns for books, papers and lectures (e.g. Blanchard et.al. 1973). Producing such designs by hand is time consuming and prone to error, and the resulting diagrams are often unsatisfactory. For this reason I intend to make microfilms of these patterns available to any archaeologist requiring them for valid academic reasons.

In some instances patterns can be used as pointers to a particular culture; I intend to point out the very real dangers of using lattice patterns for such a purpose. This type of pattern is universal, it is a direct consequence of the elementary and natural mathematical restrictions imposed by artists - namely some form of repitition of an initial pattern (a line sequence or shape), which I will call a CELL. The most obvious form of repetitive pattern is a frieze, where copies of the cell are drawn in a line, consecutive cells being the same distance apart. A style very popular as boundaries in all forms of classical and neo-classical art. I, however, will concentrate on the extension of this idea; now copies of the cell can be placed throughout two dimensional Euclidean Space. The natural mathematical device to use is the LATTICE.

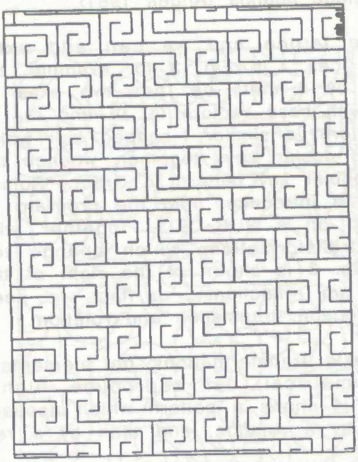
Any point in two-dimensional space is defined by x-y Cartesian co-ordinates. The lattice is defined by two base vectors:

$$\underline{v}_1 = (x_1 , y_1) \quad \text{and} \quad \underline{v}_2 = (x_2 , y_2)$$

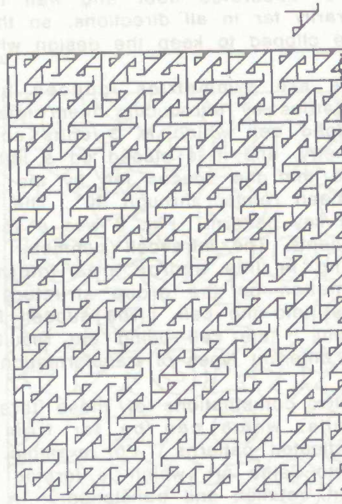
and it is formed by integer combinations of these vectors:

$$m\underline{v}_1 + n\underline{v}_2 = (mx_1 + nx_2 , my_1 + ny_2)$$

where m and n run independently through all the integers. Fig. 1 shows the lattice derived from the base vectors (5, 0) and (4, 4). A cell is a sequence of lines and/or areas placed relative to some reference point, and the reference point can be identified with a lattice point. This can be repeated at every lattice point to produce Fig. 2.



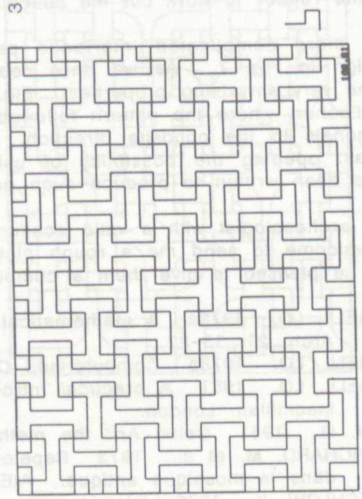
2



4

1

3



This simple idea has a straightforward practical implementation, namely the use of decorated floor and wall tiles. Obviously the lattice can extend arbitrarily far in all directions, so that such a potentially infinite pattern has to be clipped to keep the design within finite bounds (Angell 1981).

There are symmetries induced by the lattice and we can add extra symmetries by introducing them into the cell. In Fig. 3 for example, the I-shaped cell which is 3 units across by 5 units up, has two axes of symmetry, and it is placed on a lattice with base vectors (6, 0) and (3, 3). Note that the spaces left by the cell form a similar I-shaped cell which has been rotated through 90°. The whole design keys together, whence the name key pattern. Fig. 3 was a very popular key pattern in Graeco-Roman pavements. The intra-cell symmetry can be introduced into the program by reading in line and/or area information on a SUB-CELL, together with information on the group structure of transformations which change the subcell into the cell. The subcell for Fig. 3 is shown at the right of the diagram - the cell being the result of two reflections. In fact we need three different types of transformation: reflection, translation and rotation.

Complex combinations of these three basic transformations on elementary subcells, which can be far from symmetrical, can reproduce highly sophisticated patterns. For example, Fig. 4 shows a pattern which appears in Picto-Celtic art, which is formed first by a rotation; and then by a reflection, rotation and translation.

There is no need to use only vectors with integer co-ordinates, for example a lattice based on integer multiples of the vectors (0, 1) and ($\sqrt{3}/2$, 0) generates a hexagonal lattice, for example Fig. 5, used in many popular Islamic patterns, although it is not limited to this culture.

Naturally a subcell can contain curves as well as straight lines: as in Fig. 6, a Graeco-Roman design. It also contains a simple interlaced pattern which is not apparent from the subcell. In the remaining patterns it is left to the reader to work out the subcells.

Far more complicated interlaced patterns are possible, like the (Celtic?) design in Fig. 7. As we have seen, patterns produced in this way have some very surprising properties. Fig. 8 is in the Thai (?) style, popularised by Escher, where the pattern followed down a diagonal is repeated on offset diagonals in the opposite direction. The cells can be thought of as solid areas, opening the possibility of using colour graphics devices: programs have been written to produce such patterns.

Any archaeologist with a valid academic reason for producing these patterns is welcome to send me a rough picture or photograph of the design and I will be pleased to give them a computer-drawn pattern.

ANGELL, I.O. 1978a A mathematical appreciation of Celtic Art. Science & Arch. 21, 15-22.

ANGELL, I.O. 1978b Computerised Celtic Art. Comp. Appl. Arch. 6, 5-12.

ANGELL, I.O. 1981 A practical introduction to computer graphics. Macmillan, London.

BAIN, G. 1951 Celtic Art, the methods of construction. Constable, London.

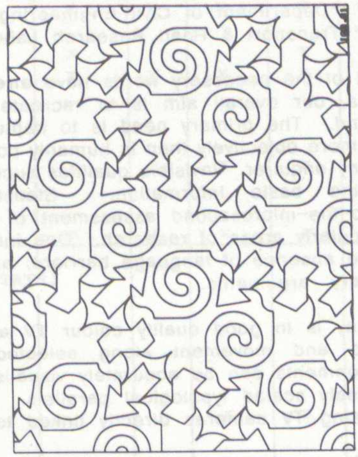
BLANCHARD, M. et al. 1973 Repetoire graphique du decor geometrique dans la mosaique antique. AIEMA, Paris.

CRITCHLOW, K. 1976 Islamic patterns. Schocken Books, New York.

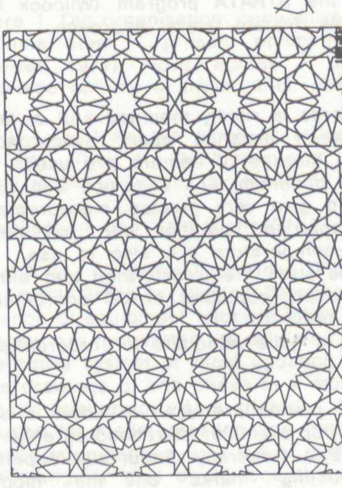
ROMILLY-ALLEN, J. 1903 The early Christian monuments of Scotland. Neill, Edinburgh.

6

8



5



7