

Partially Preserved Colonnades in Greek Architecture: the Probability of Matching Column Drums

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Abstract. The fragmentarily preserved colonnades of ancient Greek buildings are in many ways a challenge to architectural scholars; studying the column drums discovered at an archaeological site can give important information for reconstructing the building height and column shaft profile, but quite often the fieldwork is not conducted thoroughly enough or the collected data is not completely utilised. In this paper the formulae for calculating the mathematical probability of discovering complete columns or matching drums are presented. These formulae can be used to determine whether the expensive field documentation of the drums is likely to yield positive results for reconstruction purposes. Two case studies, the fourth-century BC temples of Athena Alea at Tegea and of Zeus at Labraunda, are also discussed in the paper.

Keywords: Greek architecture, probability, hypergeometric distribution, column drums.

1 Introduction

Monolithic column shafts were used in Greek architecture in some large temples in the Archaic period (c. 700-480 BC),¹ but later Greek monumental buildings were constructed with columns consisting of several drums (Fig. 1). Often none of the columns of a single building have endured standing the combined destructive forces of man and nature, and only part of the column drums can later be discovered scattered around the building. However, since the reuse of large drums as building material is less convenient than that of rectangular blocks, the proportion of drums preserved at archaeological sites is usually greater than that of e.g. cella wall blocks. Therefore, the column drums are often the most reliable guide to questions regarding the building height (the height of column drums often varies greatly and the shaft height cannot be simply calculated from the average drum height). If a sufficient number of blocks are preserved in good condition at the site, also the gently curving shaft profile, or entasis, can be reconstructed fairly accurately on the basis of drum measurements (Pakkanen 1998: 49-67; Pakkanen 1999).

The precision of the building reconstruction is greatly enhanced by detection of matching pairs of column drums; in an ideal case all the drums of a single shaft are discovered. Since this requires a great deal of fieldwork, calculating the mathematical probability of matching pairs will give some suggestions for what kind of priority is given to the time-consuming docu-

mentation of the blocks and what kinds of results might be expected with the excavated material.

However, it should be kept in mind that the preserved drums of a building never constitute a random sample of the original material:² the preservation of drums is not a random process, and if the site cannot be completely excavated, neither the selection of the excavation area can be considered as random.

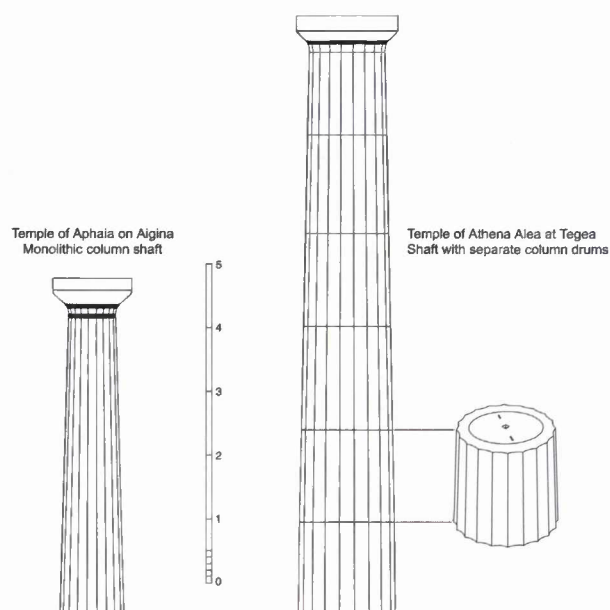


Fig. 1. Monolithic column shaft and a shaft consisting of separate drums

¹ The following Archaic buildings have at least part of the shafts monolithic: the temple of Hera at Olympia, the temples of Zeus Olympios and Apollo at Syracuse, the temple C at Selinous, the temple of Apollo at Corinth, and the temple of Aphaia on Aigina. On the buildings, see e.g. Dinsmoor 19503 (1985).

² Cf. Shennan 1997: 61: "It is obvious that no archaeological sample can be considered a random sample of what was once present." See also Edginton 1995: 6-8.

The effect of deviating from randomness may be in two directions:

- 1) if drums from a certain level of the column shaft are missing more often than others, the probability of discovering a complete shaft diminishes;
- 2) if complete columns are missing, the probability increases (on this point, see the last section and also Pakkanen 1998: 55-56).

2 Mathematical Probability

The following formulae are valid for calculating the probability of complete shafts and matching drums. Assume that the total number of columns in the building is n and the number of preserved drums on level k ($k = 1, 2, \dots, n$) is n_k . Let $P(k, h)$ be the probability that on level k there are h preserved complete columns. Then we have a recursive formula

$$P(k, h) = \sum_{j=h}^{n_{k-1}} \left[P(k-1, j) \frac{\binom{j}{h} \binom{n-j}{n_k-h}}{\binom{n}{n_k}} \right] \quad (1)$$

This formula is based on the observation that in order to have h 'complete' columns preserved on level k , the number of complete columns on the preceding level $k-1$ must be any of the numbers $j = h, h+1, \dots, n_{k-1}$. The probability of j complete columns on level $k-1$ is $P(k-1, j)$. The conditional probability that the number of complete columns drops from j to h when moving from level $k-1$ to level k is according to hypergeometric distribution

$$\frac{\binom{j}{h} \binom{n-j}{n_k-h}}{\binom{n}{n_k}} \quad (2)$$

Thus the final formula for $P(k, h)$ is obtained simply as the total probability. The numerical probabilities are calculated iteratively according to this formula by starting from initial conditions $P(1, n_1) = 1$, $P(1, h) = 0$ otherwise. The calculations presented in the paper were performed using editorial arithmetics in the statistical program Survo (Fig. 2).³

3 Case Studies

The probabilities of matching drums and complete columns are discussed in the following based on two case studies: the Doric temple of Athena Alea at Tegea (Peloponnesos, Greece) and the Ionic temple of Zeus at Labraunda (Karia, Asia Minor). Both of the buildings are from the fourth century BC.

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46 1 SURVO 98 Mon Apr 5 21:19:31 1999 D:\CA\99\ 300 100 0
3 *let P(k,h) be the probability that on level k there are h preserved
4 *complete column shafts.
5 *
6 *Examining separately level 1:
7 *P(k,h):=if(k=1)then(P1(h))else(P2(k,h))
8 *where the probability is 1 with value h=n(1) and otherwise 0, or
9 *P1(h):=if(h=n(1))then(1)else(0)
10 *
11 *In general, the following formula applies for P(k>1):
12 *P2(k,h):=for(j=h)to(n(k-1))sum(P(k-1,j)*C(j,h)*C(n-j,n(k)-h)/C(n,n(k)))
13 *where the Cs are binomial coefficients. The conditional distribution
14 *of h moving from level k-1 to level k is a hypergeometric distribution.
15 *
16 *The total number of columns is n=36 and the number of preserved drums
17 *on different levels n(k):=X(N,k)
18 N 7 12 10 7 7 6
19 *
20 *The probability of one or more complete columns being preserved at
21 *Tegea can be obtained by activating 1-P(6,0)=0.00480766
22 *
23 *P(2,0)=0.0414611 P(2,3)=0.28004428 P(2,6)=0.00265655
24 *P(2,1)=0.19348514 P(2,4)=0.12001898 P(2,7)=0.00009488
25 *P(2,2)=0.33605313 P(2,5)=0.02618596

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Fig. 2. The Survo 98 edit field for calculating the probability of matching pairs of column drums at Tegea. The last three lines of the edit field are used to produce the probability of matching first and second drums (cf. Table 1)

Table 1. Probability of matching pairs of column drums at Tegea

N	1st & 2nd	2nd & 3rd	3rd & 4th	4th & 5th	5th & 6th
0	0.04146	0.00772	0.07880	0.18697	0.24388
1	0.19349	0.06173	0.27580	0.39833	0.42678
2	0.33605	0.19097	0.35460	0.29875	0.25607
3	0.28004	0.29955	0.21491	0.09958	0.06566
4	0.12002	0.26211	0.06541	0.01532	0.00730
5	0.02619	0.13243	0.00981	0.00102	0.00031
6	0.00266	0.03863	0.00065	0.00002	0.00000
7	0.00009	0.00631	0.00001	0.00000	
8		0.00054			
9		0.00002			
10		0.00000			
1 or more	0.95854	0.99228	0.92120	0.81303	0.75612

3.1 Temple of Athena Alea at Tegea

At Tegea the 36 columns comprised each six drums, and 49 of the original 216 drums (23%) are well preserved (Pakkanen 1998: 11-12). At Labraunda there were 24 columns of five drums, and 33 of the 120 drums (28%) have been preserved (Hellström and Thieme 1982: 26).

The probability of one or more complete column shafts being preserved at Tegea can be calculated as $1 - P(6, 0) \approx 0.00408$. The numbers of preserved drums on each level are $n_1 = 7$, $n_2 = 12$, $n_3 = 10$, $n_4 = 7$, $n_5 = 7$, and $n_6 = 6$ (Pakkanen 1998: 11-12, A1-41). However, the chances of discovering separate pairs of matching drums is considerably higher (Table 1): e.g. the probability of exactly one existing pair of 3rd and 4th drums at Tegea is 27.6%, while the probability of at least one pair is as high as 92.2%.

The documentation of building blocks at the sanctuary has been carried out in 1993-1998 as part of the excavations at Tegea conducted by the Norwegian Institute at Athens (Østby 1994; Østby et al. 1994); two certainly matching pairs of drums were identified during the project (one pair of 3rd and 4th drums and one of 4th and 5th), and based on the supplementary data a new reconstruction of the column shaft has been recently proposed (Pakkanen 1998: 49-67). The reason for discovering only two joints is most probably due to

³ On Survo in general, see Mustonen 1992; the address for the web pages in English is www.survo.fi/english.

incompletely preserved drum surfaces and conditions at the site: for some drums the surfaces could not be adequately documented.

3.2 Temple of Zeus at Labraunda

At Labraunda the probability of a whole column shaft surviving is 2.8% (the numbers of preserved drums on each level are $n_1 = 4$, $n_2 = 5$, $n_3 = 7$, $n_4 = 11$, and $n_5 = 6$),⁴ and e.g. the probability of at least one existing pair of 3rd and 4th drums is 99.5% (Table 2). If it is reasonable to presume that the preserved drums are originally from fewer columns than the whole colonnade (at Labraunda most of the drums were excavated by the northwest corner and the north flank of the temple; Hellström and Thieme 1982: pl. 28), the possibility of a completely preserved column shaft becomes significantly higher: reducing the number of columns (n) from 24 to 12 in the calculation increases the probability of a complete surviving shaft from 2.8% to 39.3%. In conclusion, it is very probable that by conducting an extensive field documentation of the preserved drums of the temple of Zeus, a sufficient number of matching drums would be discovered for definitively solving the questions of the building height and shaft profile.

Table 2. Probability of matching pairs of column drums at Labraunda

n	1st & 2nd	2nd & 3rd	3rd & 4th	4th & 5th
0	0.36477	0.14559	0.00496	0.01275
1	0.45596	0.39196	0.05454	0.10518
2	0.16093	0.33597	0.20452	0.29217
3	0.01788	0.11199	0.34087	0.3506
4	0.00047	0.01400	0.27269	0.19124
5		0.00049	0.10412	0.04462
6			0.01735	0.00343
7			0.00095	
1 or more	0.63523	0.85441	0.99504	0.98725

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⁴ For the number of drums, see Hellström and Thieme 1982: 26; on the shaft height and profile, see Pakkanen 1999.