COMPUTERISED CELTIC ART

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I have always held the view that computers should be used for more than just completing the calculations at the end of an experiment or research project. They should be an integral part of any research, being aids to decision making, the means of instant information retrieval (in numerical or graphical form), and tools for the preparation and presentation of publications.

Recently I have been studying the interlacing techniques of Celtic Art, in particular those patterns found on Stone Crosses and Illustrated Manuscripts. In this paper I will describe a Fortran IV program I developed to aid my understanding of a subset of this Art Form, but before going into details I must introduce some underlying mathematical ideas.

I have shown that all interlaced art is equivalent to the manipulation of closed loops of strands by a small number of transformations (Angell,1978). Imagine a number of closed loops in two dimensions (i.e. no interlacing as yet!), each point of intersection (or crossover point) has two strand segments cutting at it. In order to change this diagram into an interlaced pattern we have to classify each crossover point by defining which strand segment goes over and which goes under (see diagram 1). The pattern is said to be consistently classified if, for every loop in the pattern, the types of classification alternate around the loop. I have proved that it is always possible to classify consistently loop patterns in the following (natural!) way. First choose one loop, take any crossover point on it and arbitrarily classify the point as over or under. Next continue around the loop alternating the classification of the crossover points as they occur. Then, repeatedly, choose another loop which contains at least one unclassified point, and another point in common with a classified loop. Starting at this classified point continue around the new loop alternating the typing of the crossover points. Eventually the whole diagram is completely classified and we never get a point classified with the same type on both its defining strand segments, and there is always an even number of points on a loop. For example in diagram 1, start at the point marked 1 on the strand going towards point 2, classify it as type over, i.e. $\overline{1}$, (in the diagram, pairs of short parallel lines are used in the natural way to distinguish between over and under). The point 2 is classified under, i.e. $\underline{2}$ and continuing around the loop we get:-

$\overline{1}$ $\underline{2}$ $\overline{3}$ $\underline{4}$ $\overline{5}$ $\underline{6}$ $\overline{2}$ $\underline{7}$ $\overline{8}$ $\underline{9}$ $\overline{1}$.

Note point 2 is met twice on the loop, once over and once under. Starting now at point 1, but going on its other strand (now classified $\underline{1}$) we get:-

$\underline{1}$ $\overline{10}$ $\underline{8}$ $\overline{11}$ $\underline{12}$ $\overline{4}$ $\underline{13}$ $\overline{6}$ $\underline{1}$.

Note that points in common between the first two loops have consistent opposite classifications. We finish with the third loop:-

$\underline{3}$ $\overline{14}$ $\underline{11}$ $\overline{9}$ $\underline{10}$ $\overline{7}$ $\underline{14}$ $\overline{12}$ $\underline{5}$ $\overline{13}$ $\underline{3}$.

Note point 14 is the only one that has not occurred before and thus it is classified twice on this final loop - consistently once over and once under.

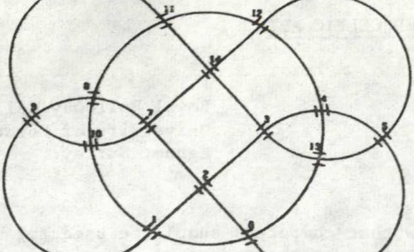


Figure 1.

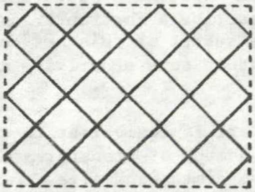


Figure 2.

Figure 3.

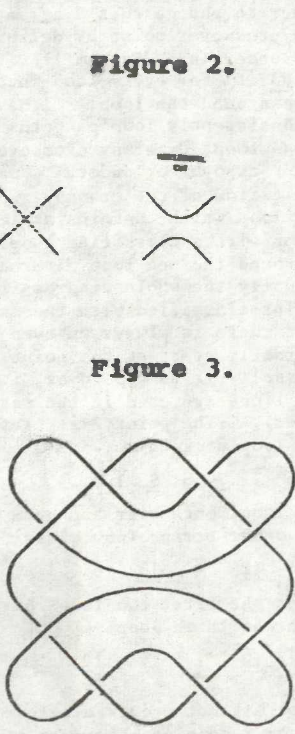


Figure 4.

The many complicated zoomorphic and phytomorphic designs are derived from such loop patterns by cutting the strands and rejoining them in groups. At the turn of the century, Romilly-Allen (1903) in his study of Celtic Art claimed that many designs were derived from a special subset of looped patterns viz. rectangular patterns. Consider a rectangle m units long and n units high filled with 'diamonds', each with vertical and horizontal diameters of 1 unit. First place m diamonds in a row at the base of the rectangle, then fit m-1 diamonds in the 'valleys' formed by them. Then another m diamonds above them, and so on until the rectangle is filled with 2n-1 layers (diagram 2 shows a 4×3 pattern). In practice an m×n pattern is completed by rounding the diamond edges in contact with the rectangle, and then suppressing the boundary. Another form of rectangular pattern ($m-\frac{1}{2}\times n$) is the skew pattern, where the rectangle is $m-\frac{1}{2}$ units by n units and there are 2n-1 layers each of m-1 diamonds - but this pattern can be derived from the previous type so we will ignore it for the time being. We may develop a coordinate system for such patterns:- the rectangle corners are (0,0), (0,n), (m,n), (m,0), and the crossover points are

$$\{(\frac{x}{2}, \frac{y}{2}) \mid 1 \le x \le 2m-1 \ ; \ 1 \le y \le 2n-1 \ ; \text{ only one of x and y is odd.}\}.$$

Now we may consider the number of loops in such a pattern.

Theorem In an m×n pattern there are hcf(m,n) closed loops (hcf(m,n) is the highest common factor of m and n; thus if m=4 and n=3 there is only one loop, if m=6 and n=9 there are 3 loops).

Romilly-Allen gave a number of transformations for changing rectangular patterns into the borders seen on Stone Crosses and in Manuscripts. These transformations are limited in scope and cannot be used to produce the more complicated designs, but we will consider four of his transformations - horizontal and vertical, single or double cuts. To make the single cuts two strand segments are cut at a crossover point and are rejoined in pairs (diagram 3). The double cuts are made by separating the strands at the four vertices of a diamond and rejoining as in diagram 4 (a horizontal double cut). With these limitations I wished to study the artistic potential of Romilly-Allen's method, and for mathematical interest to consider the question of how many loops are produced by making such cuts. One theorem I have proved is:-

Theorem The smallest number of single cuts required to change an m×n pattern into a single loop is hcf(m,n)-1.

Such a study would require either drawing hundreds of diagrams by hand and painstakingly counting the loops in each one, or resorting to a computer for the drawing and counting - the obvious choice!

Now for the program. The strands which go through a crossover point are one of two types - type 1 goes S.W. ⇄ N.E. and type 2 goes S.E. ⇄ N.W., and we have to classify every point on each of its defining strands. The program initially requests the size of the pattern, and then needs to know which strand type goes over point ($\frac{1}{2}$,1). Once this is defined then, if we wish to maintain consistency, every other point in the diagram is uniquely classified. The diagram is drawn on an IMLAC interactive graphics terminal which has a maximum of 96 superimposable frames, any frame may be suppressed, thus by drawing one loop per frame (hence a maximum of 96 loops are allowed by the program) we may consider parts of the pattern and analyse the loop relationships to understand the final effect.

To draw a loop the program finds a strand segment through a crossover point which has not been previously classified. From this point (say $(\frac{x_1}{2}, \frac{y_1}{2})$) the machine produces a sequence of points $(\frac{x_t}{2}, \frac{y_t}{2})$ t=1,2,... thus:-

$$x_{i+1} = x_i + XGRAD \quad ; \quad y_{i+1} = y_i + YGRAD$$

where the variables XGRAD, YGRAD are initialised

XGRAD=1 YGRAD=1 if the strand through $(\frac{x_1}{2}, \frac{y_1}{2})$ is type 1

or XGRAD= -1 YGRAD=1 if it is of type 2.

Obviously this sequence will keep going upwards (to N.E. if type 1, to N.W. if type 2) and so eventually it must hit the rectangle - where the values of XGRAD and/or YGRAD are changed:

XGRAD = -XGRAD if left or right boundary is met
YGRAD = -YGRAD if top or bottom boundary is met.

In this way the sequence of crossover points is constrained within the rectangle and we must return to the start point on the same strand as we started. The program simply draws lines joining up the points of the sequence; arcs are drawn when the boundary is met. Keeping track of the classification the line drawing stops short of an 'under' point and starts again just past it, thus giving an optical illusion of the interlacing effect (see diagram 5i).

The drawing of loops continues until every point is classified, and the information about each loop is stored on tape for later use. The information about the classification of points is stored using the array BEEN.

BEEN(I,J,K)=0 if the strand of type K through $(\frac{I}{2}, \frac{J}{2})$ has not been visited

=1 if it has.

Since half these values are never used (I and J cannot both be odd, or both even) we map the array into a linear vector by the transformation

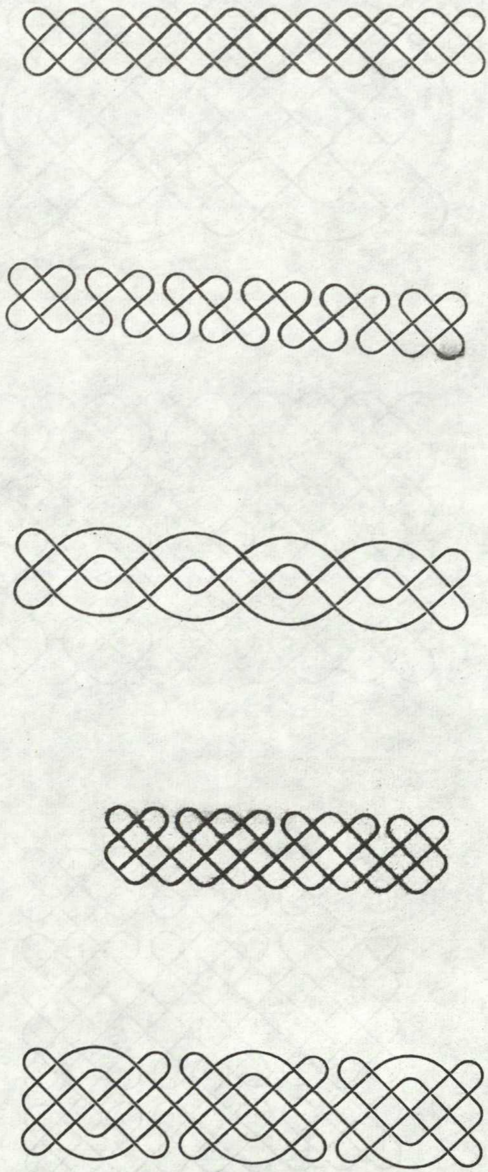
$$(I,J,K) \rightarrow ((\lfloor I-1 \rfloor /2)*2*N+J-1)*2+K .$$

Furthermore, because the value of each element in the array is 0 or 1 the linear vector is stored in groups of 48, one group per computer word (the C.D.C. 6400 on which the program was run has a 48 bit word).

This leads us to 'cuts'. Initially a complete m×n pattern is drawn as described above, and then the crossover points to be cut are specified by light pen. Next the type of cut is typed on the keyboard (H horizontal single cut, V vertical single, D horizontal double, E vertical double and R to restore any cut to its original form). The information on each cut (coordinate value and type) is stored in one computer word - and an array CUTS holds the information on all these cuts. The loop construction procedure is exactly as was described earlier, except now checks must also be made to see if a cutting point is reached (simply search array CUTS). If such a point is reached then the correct arc is drawn.

Now we consider some examples, in each case the pattern can be continued vertically up or down, but for ease of understanding I have 'rounded off' the patterns - the main design points are still readily seen.

5ii) is found on a stone from Sandbach, Cheshire, and is produced with horizontal single cuts on a 2×n pattern.



(i)

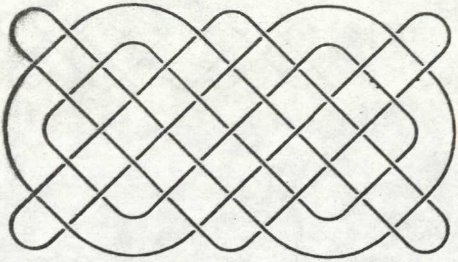
(ii)

(iii)

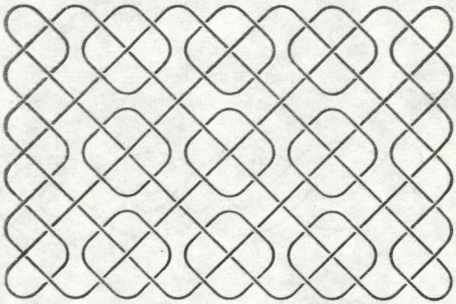
(iv)

(v)

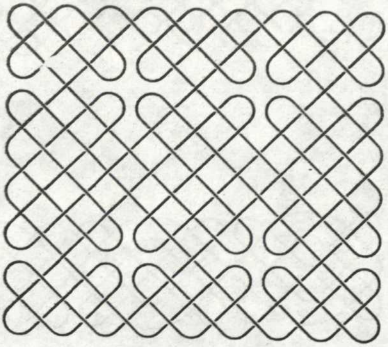
Figure 5.



(vi)



(vii)



(viii)

Figure 5.

5iii) from the Book of Kells, and the Tara Brooch, derived by vertical double cuts on a 2×n pattern.

5iv) from the Paris Gospels is a skew pattern ($2\frac{1}{2}$×n) produced by making vertical single cuts at all points ($2\frac{1}{2}$,y) y=1,...,n-1 on a 3×n pattern. There are also some horizontal single cuts.

5v) from Kells, derived by horizontal single cuts and vertical double cuts on a 3×n pattern.

5vi) from Stapleford, Notts. Derived by vertical and horizontal single cuts on a 6×n pattern.

5vii) from the Stockholm Gospels, vertical and horizontal double cuts on a 4×n pattern.

5viii) from Glamis, vertical and horizontal cuts forming subtle cruciform shapes in this 9×n pattern.

My final example (diagram 6i) is of a 12×12 pattern on a panel at the rear of the Cross of Eiudon, now in the National Museum of Wales. It was this pattern which started my detailed study of Celtic Art - I was intrigued since it is assymmetric and even with 13 single cuts it still contains three loops. Diagram 6ii) shows frame 1 of previous diagram; Diagram 6iii) shows frames 2 and 3. It should take only 11 single cuts to change a 12×12 pattern into one closed loop - however even replacing two of these cuts still will not give one loop. Was it meant to be this shape - or did the artist make a mistake? These two previous diagrams also give an insight into how the program works and how the optical illusion effect is produced.

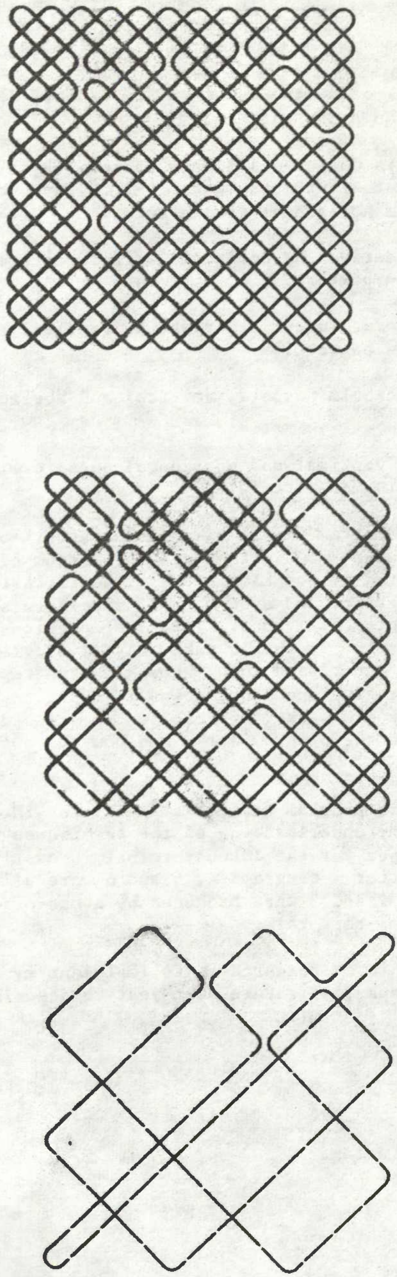
The use of this program gave me a 'feel' for interlacing and proved of great value in my understanding of the techniques involved. Furthermore the program developed for the IMLAC terminal is easily transformed for use by a microfilm plotter - diagrams 4, 5 and 6 were all produced in this way. Diagrams 1, 2 and 3 were produced by a one-off program written especially to give diagrams for this talk.

The next step in my research is to implement my own ideas on interlacing - perhaps I'll return next year to describe that program.

References

Angell, I. O. 1978 — 'A Mathematical Appreciation of Celtic Art' SCIENCE AND ARCHAEOLOGY (to be published)

Romilly-Allen, J. 1903 — THE EARLY CHRISTIAN MONUMENTS OF SCOTLAND Neill, Edinburgh.



(i)

(ii)

(iii)

Figure 6.