# Quantitative Measures of the Uniformity of Ceramics 

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#### Abstract

The introduction of computerized recording and measurement of archaeological ceramic vessels opens new channels of research, some of which we introduce and discuss in the present contribution. In particular, we show that the accurate measurements of wheel produced pottery provide information on the deviations from the ideal cylindrical symmetry which are due to faults in various stages of the production process. We present a systematic method to quantify two kinds of deviations from perfect symmetry: the uniformity of the profiles of cross-sections and the deformations of horizontal sections. We propose that they may be considered as indicators of the technological skill of the producers, or manufacture methods in a way which was not possible hitherto in archaeological research.


Keywords: pottery, uniformity, curvature analysis, Fourier transform

## 1. Introduction

Pottery is an important tool in comparative archaeological research. Traditionally, wheel produced ceramics (complete or fragmented) are represented by the hand drawn contours of their sections. This time consuming, and often inaccurate documentation suffers from further limitations: Often, vessels deviate from perfect symmetry, hence, a single section cannot represent the entire vessel, and a precise axis of symmetry cannot be defined. Hand drawn profiles provide (at best) an average profile of the vessel. The accuracy of this average profile is uncertain, and the information about the variability of the profile within the same vessel is completely lost. What is the effect of these uncertainties on shape (profile) typology which is based on the representation of vessels in terms of their average profiles? What information of archaeological significance can one deduce from measuring the variability and deviations from uniformity? In particular, can one use the non-uniformity to assess production skills, technological development or to characterize and distinguish between workshops and producers? The present research attempts to address these questions. A prerequisite to approach these issues is to accurately measure and record as many profiles of a single vessel as possible. We achieved this by using a computerized 'profilograph' (see
http://www.dolmazon.de/english.htm for a description of the profilograph). 3-dimentional optical scanning devices (for instance, Sablatnig and Menard 1996; Adler, Kampel et al. 2001; Leymarie, Cooper et al. 2001) are also well suited for this purpose. We measured and analyzed two assemblages of ceramics, which are described in the next section. We used two quantitative measures to characterize the deviation of the vessels from cylindrical symmetry. The variability of the various cross-sections of a vessel was determined from a
correlation analysis of profiles within a vessel. The deformation of selected horizontal sections, were deduced from the Fourier analysis of the section. Both quantities will be defined and discussed in section (3), and measured for the assemblages under study in section (4). Our conclusions and outlook are summarized in section (5).

## 2. Data Acquisition and Description

We analyzed two different assemblages of complete vessels. The first is a collection of 16 bowls from Tel Dor (Israel), which were picked at random from the Tel Dor collection. Their dates span the period between the Early Iron Age and the Hellenistic period, and therefore they do not represent a homogenous assemblage or a single morphological type. The second assemblage is composed of 11 flower pots, which were produced by a contemporary potter who uses the traditional kick wheel. This assemblage is as homogenous as hand-made, mass produced pottery can be. The two very different assemblages were chosen for methodological reasons which will become clear in the sequel.
For each vessel, six section profiles, as well as the horizontal contours of its rim and base were measured with an accuracy margin of 0.2 mm using a profilograph. (A useful method for its use is given in 'http://www.weizmann.ac.il/complex/ uzy/archaeomath/profilograph.html'). Typical sections which illustrate the deviations from perfect cylindrical symmetry are shown in Fig. 1. The deviations from symmetry can be as large as a few millimeters, well within the accuracy margin. They are the subject of the subsequent systematic study and comparison of sections belonging either to the same or to different vessels.


Fig. 1. Single profiles of two different bowls from Tell Dor, Israel $(\mathrm{a}, \mathrm{c})$, and the overlap of six profiles of the same bowls (b,d), respectively.

## 3. Mathematical Definitions

### 3.1 Correlation and Curvature Analysis

The comparison and correlations between the profiles of different sections was carried out using a recently developed method which was originally proposed for the classification of ceramics (Gilboa, Karasik et al. 2004). It is based on the idea that one may consider the profiles as curves in the plane which are represented by their curvature functions $k(s)$. The definition of the curvature function, its properties and its use in the archaeological context are explained in (Gilboa, Karasik et al. 2004). Fig. 2. shows a profile of a bowl (a) and its corresponding curvature function (b).


Fig. 2. A profile of a bowl (a) and its corresponding curvature (b), as a function of the normalized arc-length.

The correlation between two profiles is defined in terms of the scalar product of their corresponding curvature functions $k_{1}(s), k_{2}(s)$. (It is assumed throughout that profiles are scaled to a constant length). The correlation is given as
$C\left(k_{1}, k_{2}\right)=\frac{\int k_{1}(s) \cdot k_{2}(s) d s}{\sqrt{\int k_{1}(s) d s} \cdot \sqrt{\int k_{2}(s) d s}}$.
Notice that $-1 \leq \mathrm{C}\left(k_{1}, k_{2}\right) \leq 1$ and it assumes the highest value if and only if $k_{1}=k_{2}$. The smaller the correlation, the less similar are the compared profiles. A correlation value smaller then 0.5
usually indicates a significant difference between the compared profiles. Let $k_{\mathrm{n}}(\mathrm{s}) 1 \leq n \leq N$ with be the curvature functions of $N$ profiles of different sections of the same vessel. Using equation (1) we compute the correlations matrix $C_{i, j}=$ $C\left(k_{i}, k_{j}\right)=C_{j, i}$ for the profiles under study. The uniformity of the vessel is defined as
uniformity $=\frac{\sum_{i, j}^{N}\left(C_{i, j}\right)^{2}}{N^{2}}$
Notice that $1 / N \leq$. uniformity $\leq 1$. The uniformity reaches its maximum value when all the entries in the correlation matrix have the value 1 . That is, when all the profiles are identical. The minimum value is obtained when the only non-vanishing entries are on the diagonal, that is, every profile is similar only to itself and is very different from all the others. It is clear that the definition of the uniformity makes better sense when a larger number of sections are measured. Our results show that $\mathrm{N}=6$ is good enough.
The correlation as prescribed by equation (1) applies as well to profiles which belong to different vessels, and the uniformity of an assemblage of vessels can be quantified in terms of the uniformity defined in equation (2). We can thus compare two measures of uniformity, the intra-vessel and the inter-vessel uniformities, the former represents the quality of the production of single vessel, and the latter measures the reproducibility of the manufacturing process.

### 3.2 Deformation and Fourier Transform Analysis

Horizontal sections of wheel produced pottery are expected to be circular. Deviations may occur in various stages of the production, and their systematic occurrence may reflect production patterns or habits which can be used to identify a potter. The deviations from perfect symmetry are not expected to be large, and typically, the convex nature of the curve is maintained. In such cases, one can use the polar representation of the curve, which is defined in the following way: Choose a convenient interior point as the origin O. The points on the curve are specified by their direction $\varphi$ and their distance $r(\varphi)$ from O . A convenient choice of the origin is e.g., the center of gravity, as was done by previous authors who used the polar representation in the archaeological context. (Gero and Mazzullo 1984; Liming, Hongjie et al. 1989)). Expanding in Fourier series,
$r(\varphi)=R\left(1+\sum_{n=2}^{N} a(n) \cos (\varphi \cdot n)+\sum_{n=2}^{N} b(n) \sin (\varphi \cdot n)\right)$
the magnitudes of the Fourier coefficients provide the deviations from a perfect circle at different angular resolutions. The "elliptic" deformation are measured in terms of
$d_{2} \equiv \sqrt{[a(2)]^{2}+[b(2)]^{2}}$

Indeed, for small deformations, $d_{2}$ is the eccentricity of the ellipse. The coefficients with $n>2$ provide the distortions of
the vessel at higher angular scales. We lump them together to a single parameter

$$
\begin{equation*}
D \equiv \sum_{n=3}^{N} \sqrt{[a(n)]^{2}+[b(n)]^{2}} \tag{5}
\end{equation*}
$$

## 4. Results

We turn now to the determination of the uniformity and deformation of vessels in the two assemblages described above. The correlations matrix of the bowls from Tell Dor is shown in Figure 3. The columns and rows are arranged such that sections of each bowl appear consecutively (six per vessel, except bowl 8 which is represented by only 4 sections). The value of the correlation of any two profiles, $i$ and $j$, is given by the grey level of the pixel at the intersection of the $i$ 'th column and $j$ 'th row. The numerical equivalents of the grey levels are provided by the bar at the right of the figure.


Fig. 3. The correlations matrix of Tel Dor bowls. The index of the bowls is given along the axes.

The intra-vessel correlations are represented by the $166 \times 6$ square matrices on the diagonal. The values of these correlations are usually higher than the inter-vessel correlations, as can be seen by the lighter gray along the diagonal. Some high inter-vessel correlations do exist, for instance between the profiles of bowls 1 and 15 . On the other hand the intra-vessel correlation matrices are not uniform and darker pixels


Fig. 4. Summary of the inter/intra-vessel uniformity values for the Tel Dor bowls.
can be seen, for example in the intra-vessel matrix of bowl 7. We used the formulae of the previous section to compute the inter- and intravessel uniformity for the Tel-Dor assemblage. The results are summarized in figure 4.
The mean intra-vessel uniformity is about .75 , with bowl 7 being exceptionally non uniform. The mean of the inter-vessel uniformity is approximately 0.3 much lower than the typical individual uniformities. This difference is consistent with the fact that the bowls were chosen randomly and they do not represent a typologically homogenous assemblage. Different results are expected in the analysis of a uniform assemblage such as the flower pots discussed below.
A preliminary inspection of the flower pots immediately reveal that most of the pots are deformed - their base and rim planes are not parallel. (This might have been the result of the method by which the potter removed the still wet pots from the wheel (rope-cut)). Due to this deformation opposite profiles of the same pot have different lengths and therefore have lower correlations. Nevertheless, the pots look very similar in general. To give a quantitative description of the above observation, we compared the pot sections which were truncated at 5 levels which are shown in Fig 5. The first level corresponds to the upper part of the pot, and the fifth level is the entire profile including the base. The truncated sections can be considered as representing rim shards, and their correlations can be obtained by using the formulae of the previous section with minor modifications. The results of the


Fig. 5. Four truncation levels of a profile. The corresponding "fragments" were analyzed separately.


Fig. 6. The mean and the standard deviation of the inter/intra-vessel uniformities for each section size of the flower pots assemblage. The smaller the section the higher the correlations.
inter/intra-vessel uniformity as a function of the truncation level are shown in figure 6.
The most prominent trend in figure 6 is that the inter/intra vessels uniformities are rather high as long as the base is not included in the analysis. The numerical values of the inter vessel uniformity are almost as high as the intra vessel uniformity. The difference becomes larger as soon as the complete profiles are considered. Thus the quantitative analysis confirms the intuitive expectations concerning this assemblage. In contrast with the previous assemblage, the similar values of the inter and intra vessel uniformities show that we are dealing with a typologically homogeneous assemblage.
The quantitative analysis described above might lead to a potentially valuable application in the analyses of excavation assemblages. Most of the pottery found in excavations consists of small fragments, with whole pots being the exceptions. Two rim shards which are found at a small distance from each other, but whose fracture lines do not match, may or may not belong to the same vessel. The results of the previous sections indicate that a very high correlation may hint that the fragments belong to the same vessel. If e.g., two shards which cover approximately the same fraction as "section 1" of Fig. 5., have a correlation of approximately 0.9 , they might belong to the same vessel. On the other hand correlation around 0.7 and less may suggest that the two shards belong to different vessels. This line of thought needs further elaboration.
As for the deformation analysis, we computed the parameters $d_{2}$ ( $\sim$ eccentricity) and D (deformation) as defined in equations (4) and (5). We found that the base plane-views of the bowls have, on average, higher eccentricity and deformation than the rims plane-views. This may be explained by the fact that some of the bases are ring-bases, and they were attached manually to the body after they were manufactured on the wheel. Moreover, the rope-cut probably deforms the bowls which have flat bases. Due to lack of space we must defer further discussion to another publication.

## 5. Conclusion

The analysis presented above could be improved if a larger number of sections or plan views were available for each vessel. This amount of data could be provided at no additional effort had we used a 3D scanning camera to record our ceramics. In the coming season of excavation at Tell Dor (2004), we plan to employ a 3D scanner to digitize large quantities of pottery, which will add our research another important aspect.

Even at the limited level of resolution used here, we can safely conclude that the use of accurate, digitized recording of pottery contributes not only in providing a reliable, objective and efficient method for recording and sorting ceramic objects. The present study showed that such data can be used also in order to study in detail the deviations from uniformity of the vessels, and in this way to trace possible characteristic or systematic trends in the production of the corresponding vessels. Can these be indicative of technical developments or mark the production habits in specific workshops? This and many other questions await further studies involving a larger body of data, which is currently under way.

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