Least-Cost Kernel Density Estimation and Interpolation-Based Density Analysis Applied to Survey Data

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Several methods have been proposed for reconstructing find densities and delineating settlement areas. The computationally intensive approach by PETERSON et al. (2005) has already been applied several times in Latin American archaeology. But standard handbooks on GIS in archaeology recommend using kernel density estimation (KDE), which is readily available in some GIS packages. The two methods are compared both in theory and in a study of survey data published by DRENNAN (2006). The example survey data was recorded in Colombia in an area with steep slopes. As topography plays an important role in the study area, a least-cost KDE algorithm was developed which takes geographical features into account. This algorithm can also be extended to illustrate the differences in find distribution between two phases, and to create an overall accessibility map of the study area.

Keywords: Kernel density estimation, Least-cost calculations, Accessibility.

1. Introduction

Many studies aim to cluster archaeological observations into meaningful human interaction units on different scales, based on distribution maps of sites or survey data, either with mapped individual finds, site locations, or weighted data such as sherd counts. This has often been performed on an intuitive basis, which is fine if the groupings of the observations are easily discernible. But if the boundaries are not that obvious, approaches based on an explicit hypothesis are more appropriate.

In Latin American archaeology, a density calculation method based on inverse distance weighting (IDW) interpolation proposed by PETERSON *et al.* (2005) has already been applied several times. Standard handbooks on GIS in archaeology recommend using kernel density estimation (KDE; WHEATLEY *et al.*, 2002: 186–187; CONOLLY *et al.*, 2006: 175–177), which is readily available in some GIS packages. It is one aim of this study to compare the IDW-based method and the KDE approach both in theory and in analysing some survey data published by DRENNAN (2006). The example survey data was recorded in Colombia in an area with steep slopes. It did not seem appropriate to apply standard KDE methods to this data because they would disregard the topography. Instead a least-cost KDE algorithm was developed which takes geographical features into account.

2. Test data: Survey data from Colombia

DRENNAN (2006) presents the analysis of survey data recorded in the river valley of La Plata, Colombia. The aims of the fieldwork were to (i) document the ways in which the prehispanic inhabitants were distributed across the landscape at a regional scale; (ii) delineate social units; and (iii) monitor the change in these characteristics.

The data and part of the results were published on the web. This allowed us to compare our methods with the approach proposed by Drennan and his colleagues. Our study focuses on the most densely occupied survey zone, i.e. the western part of the study area (about 319 km²), where 4601 collection units were recorded with an average size of 0.75 ha. Some patches of dense forest with a total area of 28.7 km² were not surveyed. For each collection unit the sherd counts for several chronological periods were provided in a spreadsheet. This study selects both the *Formative 3* (ca. 300 BC–1 AD) and the *Regional Classic* period (ca. 1–900 AD) for comparative analysis. In the western study region, 2595 sherds of the Formative 3 and 34,967 sherds of the *Regional Classic* period were recorded.

Figure 1 shows the distribution of sherd densities for the *Regional Classic* period. For this data set it is difficult to delineate zones of high density based on the distribution map only.
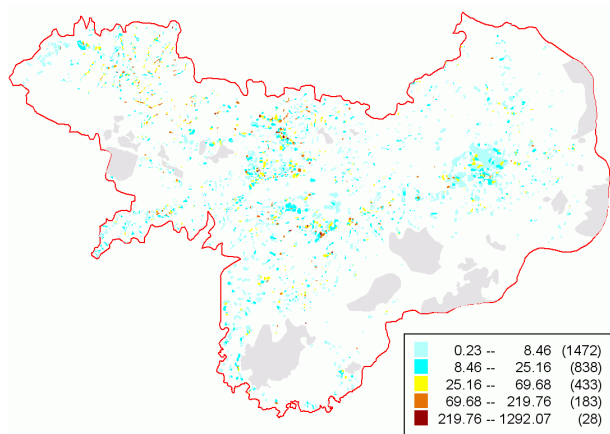
Figure 1: *Thematic map of Regional Classic sherd densities, unobserved areas are depicted in grey.*

According to DRENNAN (2006: 13), concentrations of ceramic and lithic artefacts are the best and most accessible indicator, at a regional scale, of ancient residential locations. This paper is based on Drennan's hypothesis (p. 61) that the amount of sherds found is proportional to the number of people who left them.

2.1. The digital elevation model

The Drennan team provides a classified digital elevation model (DEM) for the study area (30 classes). A DEM with higher accuracy was needed for our analysis, and therefore ASTER DEMs (ASTER GDEM is a product of METI and NASA) were downloaded. This elevation data is delivered as a raster grid with a resolution of ca. 33 m. Unfortunately, the quality of the data is poor in some areas.

Comparable gross errors could not be detected in the NASA Shuttle Radar Topographic Mission (SRTM) DEM (version 4.1), but due to its low resolution of about 90 m, most of the deeply incised gorges, often less than 100–150 m across, are smoothed away. For this reason, we tried to fix the higher resolution ASTER data in areas of large errors with the SRTM data (Figure 2). By comparing profiles of SRTM and ASTER elevation data an offset of 100 m along the north-south axis was identified. After correcting this offset, 75% of the elevation data differences between the two elevation models is in the range of –10.2 to 2.9 m.

All geospatial data was reprojected to the Bogota 1975 / UTM 18N datum. This task was difficult because Drennan's data was given in another projection, unknown at first, and some of the initial digits of the coordinates were not printed in the maps.

The range of the elevation values in the study area range from 1237–2568 m, the median being 1989 m. A slope map was calculated using the Vertical Mapper plugin of MapInfo. Only 25% of the slopes are below 14.5%, the median slope is 26.5%.
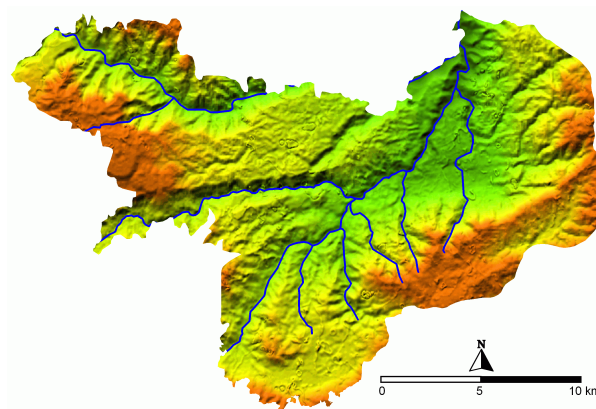

Figure 2: *Corrected ASTER DEM of the study area. Patches with ring structures indicate areas where incorrect ASTER elevations were replaced with SRTM data.*

According to DRENNAN (2006: 121–127), throughout the periods considered, a preference for more level land is prevalent, though non-trivial amounts of sherds were found in extremely sloping surroundings.

3. Interpolation-based density analysis

PETERSON *et al*. (2005) introduce a method based on the surface densities of sherd counts. In the field, collection units consisting of areas of 1 ha or less are defined according to field boundaries, streams, roads or other convenient features of the landscape. For each collection unit, the sherd count for different periods is recorded. To delineate the communities in a systematic way, first a regularly spaced grid of z-values at 100-m intervals is created and the surface densities of ceramics (sherds/m²) are assigned to each grid cell. Then contour maps are derived from this density grid and a cutoff contour is chosen intuitively that defines the areas of the clusters.

A refined variant of this method employs inverse distance interpolation (IDW) to smooth the density grid: The value for each 100-m cell is set to the weighted sum of all cell values, the weights are proportional to the inverse power of the cell distances. Lowering the power increases the smoothing effect. Power values in the range of 4 to .001 are considered by the authors, with 4 representing hardly any smoothing and .001 very strong smoothing. The analysis by Peterson and Drennan focuses on the IDW result with power 1 (see also DRENNAN, 2006: 145). According to Drennan, meaningful interaction communities at a larger scale become apparent as the level of smoothing is increased.

3.1. Application of IDW-based density analysis

The IDW-based density algorithm was applied by DRENNAN (2006, 139–153) to analyse the survey data recorded in the La Plata river valley. We tried to reconstruct the results using a different projection and different software (*gvSIG* and *MapInfo/Vertical Mapper* instead of *Surfer*). With *gvSIG*, each density calculation

took more than ten hours on a notebook with a 1.73 GHz processor and 512 MB RAM. The result with IDW exponent 1 for the *Regional Classic* sherds is combined with the boundaries suggested by Drennan (Figure 3), which are not convincing in some places.

The contour map presented by DRENNAN (2006: 153) based on the same IDW parameters is somewhat different, for example the closed contour lines in the eastern zone are not as elongated as those shown in Figure 3. It does not seem likely that these differences are only due to the change in projection. It is well-known that adding together a large number of values, the vast majority of which is very small, might cause numerical problems. Probably the two IDW implementations differ in the number of significant digits used to store and process floating point numbers.
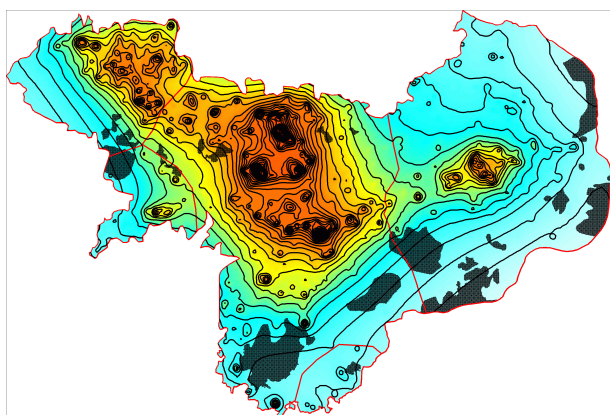

Figure 3: *Result obtained with the IDW-based algorithm for Regional Classic sherd counts using gvSIG. Boundaries suggested by Drennan and his colleagues are shown in red, unobserved areas are depicted in grey.*

So the disadvantages of this method are, on one hand, the heavy computational load, and on the other, probable numerical instabilities producing significant errors. Moreover, expanding the study area eastwards changes the density values in the west, because the calculation of the smoothed density estimate of one cell involves the initial densities of all the cells within the study area.

The method could be extended to take topographical features into account by replacing the distances in the IDW algorithm with the lengths of least-cost paths (LCPs). This requires calculating LCPs between each and every grid cell, which is still computationally prohibitive.

4. Kernel Density Estimation (KDE)

Kernel Density Estimation (KDE) is a well-established method in statistics (e.g. SCOTT, 1992) used for scatterplot smoothing. Some archaeological applications were presented by BAXTER *et al.* (1997). Standard handbooks on GIS in archaeology recommend using KDE for density analysis (WHEATLEY *et al.*, 2002: 186–187; CONOLLY *et al.*, 2006: 175–177). Recently,

MCMAHON (2007) applied this technique to discern broad trends in archaeological distribution maps. HERZOG (2007) presents simulation experiments that focus on the reconstruction of density distributions based on a sample, and concludes that KDE outperforms several other popular methods if the bandwidth parameter is chosen properly. Some GIS packages such as the ArcGIS Spatial Analyst extension and gvSIG support KDE calculations.

4.1. A short introduction to KDE

Kernel Density Estimation spreads the known quantity of some phenomenon across the landscape. Spreading is controlled by the bandwidth parameter, larger values produce a smoother, more generalized density map. The bandwidth is often referred to as the search radius in GIS software. A "bump" (i.e. a kernel) is placed over each point in turn, and these kernels are added together resulting in a density map.
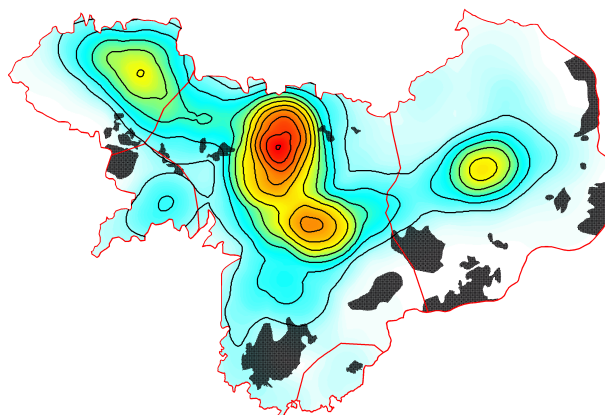

Figure 4: *Result obtained using standard KDE with a bandwidth of 2 km for Regional Classic sherd counts. Boundaries suggested by Drennan and his colleagues are shown in red.*

The size of the kernel depends on the weight of the point, the general shape of the kernel and the bandwidth. The weight determines the total height of the kernel. The shape of the bump could be a cone or the "bell" of the 2D normal distribution (i.e. a Gaussian kernel). The Epanechnikov kernel, which is a paraboloid of revolution, is considered optimal (for references see HERZOG, 2007). SCOTT (1992: 138–141) shows that the efficiency reductions due to non-optimal kernel shapes, even absurd ones, are fairly modest.

The choice of the bandwidth is far more important. To analyse the spatial distribution on a local scale, a small bandwidth should be chosen whereas a larger bandwidth is appropriate for a regional scale. Hence the optimal bandwidth often depends on the question to be answered. By setting the bandwidth, the user can control the radius of a sherd's impact on the cultural landscape. For most kernels, the impact is maximal at the sherd's location and decreases with increasing distance; outside the area delimited by the search radius the impact is nonexistent. Choosing a bandwidth is more intuitive than setting an exponent with the IDW-based method. A

more detailed introduction to KDE can be found in the references listed in the previous section.

Figure 4 shows a KDE result for the *Regional Classic* data. For all bandwidths up to 2 km the largest settlement area suggested by Drennan is divided roughly by the Loro river into two different density centres.

4.2. Least-cost KDE

DRENNAN (2006: 70–77) studies the garbage dispersal of ten modern houses in the eastern Andean mountain range where topography is generally similar to that of the study area. The aim was to obtain an estimate of the number of *Regional Classic* households per ha. Transects were walked in various directions, starting at the house. Drennan and his colleague Ana María Boada Riva delimit the garbage dispersal zones by ellipses of different sizes, ranging from 800 to 4830 m². However, some of the ellipses are not convincing because for one half or more of the area enclosed no garbage was recorded. According to Drennan and his colleague, the size of the garbage dispersal zones is mainly determined by the length of the occupation. In our view, this example shows that the dispersal of the sherds is influenced by topographic features.

For this reason, a least-cost KDE algorithm is proposed. This development bears some similarity to that in site catchment analysis, where the circular areas with a fixed radius of traditional site catchment analysis have been replaced by a more realistic alternative, i.e. the area that could be reached when expending costs up to a certain cost limit (CONOLLY *et al*., 2006: 224–225).

A model for a least-cost kernel could be constructed by adding slices of least-cost site catchment areas with increasing cost limits and with thicker slices for small cost limits. The smaller the slice thickness the closer the model approaches reality. For an area with constant costs of movement, the result is close to the standard KDE with the Epanechnikov kernel.

Most least-cost studies in archaeology focus on slope (CONOLLY *et al*., 2006: 218–221). In our view, both geology and the presence of rivers are also important in the La Plata area. However, reliable data on these cost factors are hard to obtain, and the combination of several cost factors is not trivial. Therefore the cost function applied for the La Plata data depends on slope only. The cost function is a sixth degree polynomial approximation to the energy expenditure values found by MINETTI *et al*. (2002) in physiological experiments. The minimum of the curve is at a downhill gradient of about 10.5%. The calculations are based on effective slope (CONOLLY *et al*., 2006: 217–218), use return paths and permit moves in up to 48 horizontal directions. Figure 5 shows the least-cost KDE result for the *Regional Classic* data obtained with a bandwidth set to the energy expenditure needed for walking 3 km on level ground.

In ecology, least-cost KDE with isotropic costs has been applied to model the dispersal of amphibians (COMPTON *et al*., 2007). However, we were not aware of this publication when we developed the least-cost KDE algorithm for archaeological data and implemented corresponding software. The program input is a CSV list of point coordinates with an additional weight attribute (the sherd density in the La Plata), and the output is a raster grid in ESRI's ArcInfo ASCII Grid format, which can be imported into most GIS.
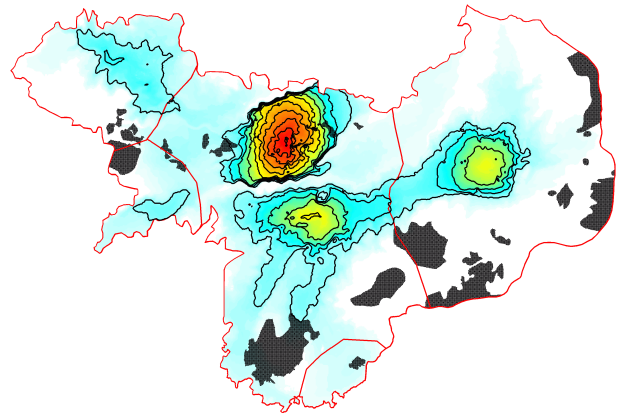

Figure 5: *Result obtained using least-cost KDE with a bandwidth of 3 km on level ground for Regional Classic sherds.*

With the least-cost KDE, the central cluster identified by Drennan and his team is divided by the Loro river into two different density centres. Only after increasing the bandwidth to the energy expenditure needed for walking 6 km on level ground, a small bridge is established between the two centres. If the costs of crossing the river were included in the calculations, the division might be even more pronounced.

On the other hand, the river valley provides a broad connection between the central area and the density centre in the east which are seen as two separate areas by Drennan and his co-authors. This connection is still maintained when the bandwidth is reduced to the energy expenditure required for walking 1.2 km on flat terrain.

The runtime of the program depends both on the number of grid points with positive weights and the bandwidth. The number of grid points to be considered in this case was 7316. The notebook with a 1.73 GHz processor and 512 MB RAM mentioned above required 43 minutes to compute the least-cost KDE result with a bandwidth set to 6 km walking on level ground, the runtime for the 3 km bandwidth was 3.5 minutes, and calculations with a bandwidth of 1.2 km took 49 seconds.
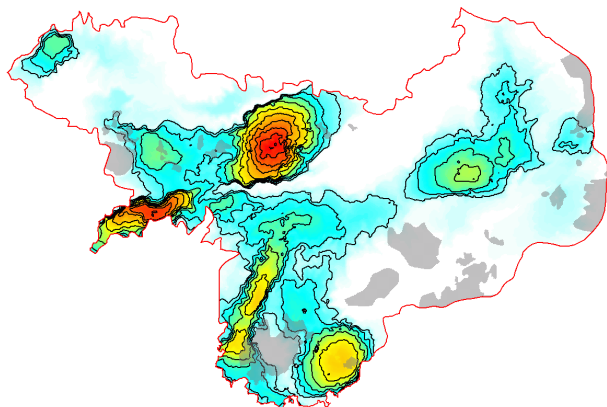
Figure 6: *Result obtained using least-cost KDE with a bandwidth of 3 km on level ground for Formative 3 sherd counts.*

4.3. Comparing sherd densities

The period before *Regional Classic* is called *Formative*, which is divided into three subperiods. It is the aim of this section to compare the sherd distributions of two successive time intervals, i.e. the distribution of 34,967 *Regional Classic* finds with that of 2595 sherds of *Formative 3* (300 BC–1 AD).

The comparison of Figures 5 and 6 shows that the distribution of the *Formative 3* sherds differs from that of the *Regional Classic* ceramics. Some changes are quite obvious: The density of *Formative 3* sherds is higher in the south of the survey zone. But is the sherd density in the area north of the river Loro lower compared to the *Regional Classic* period? DRENNAN (2006: 41) notes that the total amount of occupied area increases dramatically as *Formative 3* gives way to *Regional Classic*. This can hardly be discerned when comparing Figures 5 and 6, and therefore a method is needed to delineate areas of change.

A variant of the least-cost KDE may be applied to this purpose. The calculation is based on the two grids with a cell size of 100 m, which contain the surface densities of ceramics (sherds/m²). These grids were set up for the density calculations anyway. First the grids are rescaled so that each cell value stores the sherd density per century. The number of sherds per century for the *Formative 3* period is 865, whereas 3885 sherds per century were recorded for the *Regional Classic* period.

GIS programs allow subtracting the *Formative 3* from the *Regional Classic* grid and visualizing the result in a thematic map. But the thematic map shows areas of negative and positive values close to each other so that it is difficult to obtain an overall picture (Figure 7).

Some smoothing is required, and once again it seems plausible to take the topographic factors into account. The smoothing procedure was inspired by the weighted least-cost KDE approach. In general, only positive weights are allowed for a KDE, but for delimiting areas
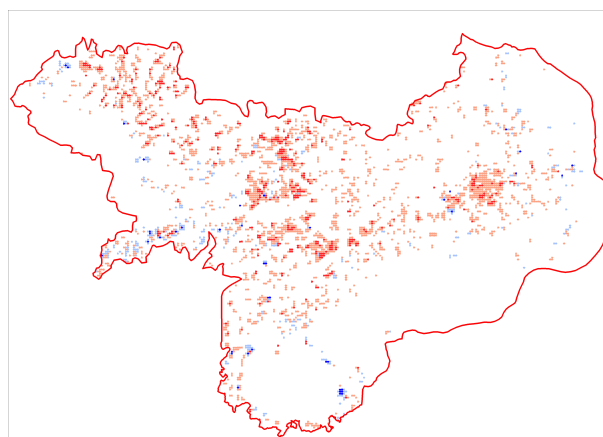

Figure 7: *Thematic map showing areas of positive (red) and negative (blue) differences.*

of positive and negative change, the method can be readily extended to include negative weights.

Of course, the resulting surface cannot be considered as an approximation to a probability density function because such functions are non-negative. For this reason, the term Kernel density estimation is not appropriate in this situation. Nevertheless, the least-cost KDE algorithm allowing negative weights creates an intuitive visualization of the differences between the two different sherd densities considered (Figure 8).
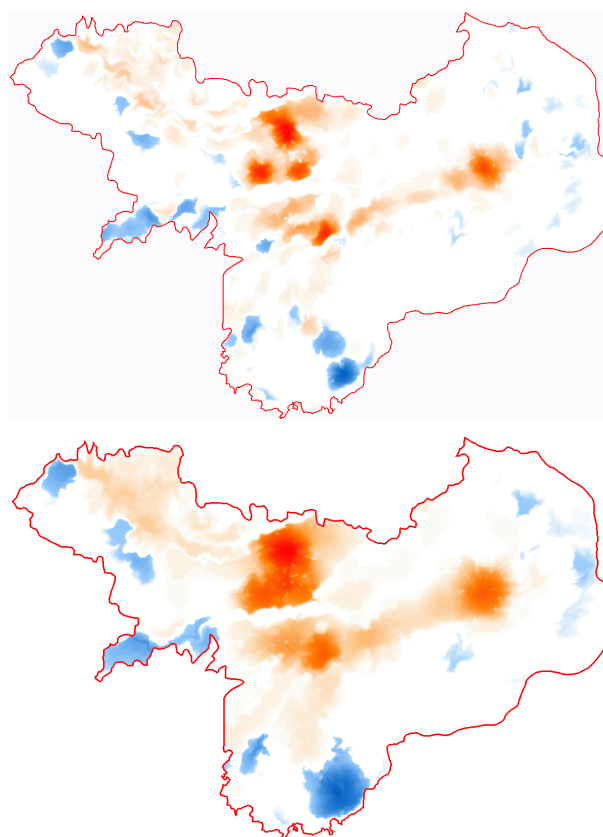

Figure 8: *Least-cost comparison of Regional Classic and Formative 3 sherd distributions using two different bandwidths.*

Figure 8 shows the results of this approach calculated with a bandwidth set to the energy expenditure needed for walking 1 and 2 km on level ground. Areas with a higher *Formative 3* sherd density (negative values) are depicted in blue, whereas areas with higher *Regional Classic* sherd density are shown in red. The higher the colour saturation the larger the difference. With the smaller bandwidth (image at the top), differences at a local scale are highlighted, whereas differences at a more global scale are shown in the least-cost KDE result based on a bandwidth of 2 km.

Both pictures illustrate that some areas at the east, west and south peripheries of the study area were more densely occupied during the *Formative 3* period, whereas some central areas with a high sherd density during *Formative 3* even saw an increased sherd accumulation in *Regional Classic* times.

4.4. Accessibility calculations

MILLER (1999) gives an overview of various methods for calculating accessibility: In general, accessibility measures consider *n* locations, each with a different opportunity size a_k. Opportunity sizes might be measured in terms of square metres, quality, and quantity of some desired substance which is available at the given location etc. Accessibility for a given location is a function of all pairs (a_k, d_k), where d_k is the distance between the location considered and all opportunities. Often, only the accessibility values of the opportunities are calculated. A typical opportunity in a prehistoric context might be a fresh water supply. The size in this case is the amount of water supplied.

In archaeological landscape analysis, accessibility often is considered as the ease with which a location may be reached from any other location in the area (LLOBERA, 2000), and the aim is to create a map showing the accessibility of all locations in a study area. It is assumed that an area was explored by the first settlers using routes of high accessibility. Llobera defines the accessibility of a cell as the reverse of the average path cost of moving from any cell within a certain radius to the selected cell, i.e. accessibility is defined on the basis of a cost-surface: For each cell, the costs of moving from this cell to any location within a certain radius from this cell are calculated and the negative sum (or the negative average) of these cost values is the accessibility index for the cell. An appropriate constant can be added (Llobera adds 100) to ensure that positive accessibility values result. The radius r can be set to a small value if accessibility is to be analysed on a local scale (r = 100 m), at a middle range (r = 1000 m) or to a larger value for a global scale (r = 6000 m). The radius values given in parentheses were suggested by Llobera. The disadvantage of Llobera's definition of accessibility is an inherent drawback of averaging: Some very high values create the same effect as many medium high values. This means for example that a gorge or an erratic boulder close to the origin of the cost-surface will

decrease the accessibility value considerably though accessibility is only low in one direction.

An alternative accessibility concept is based on the size of the area enclosed by the isoline corresponding to a certain radius of movement on flat terrain (CONOLLY *et al.*, 2006: 214–215). If this site catchment area is small, accessibility is low, whereas a large area corresponds to high accessibility. The radius can be varied as in Llobera's approach to account for different scales of accessibility.
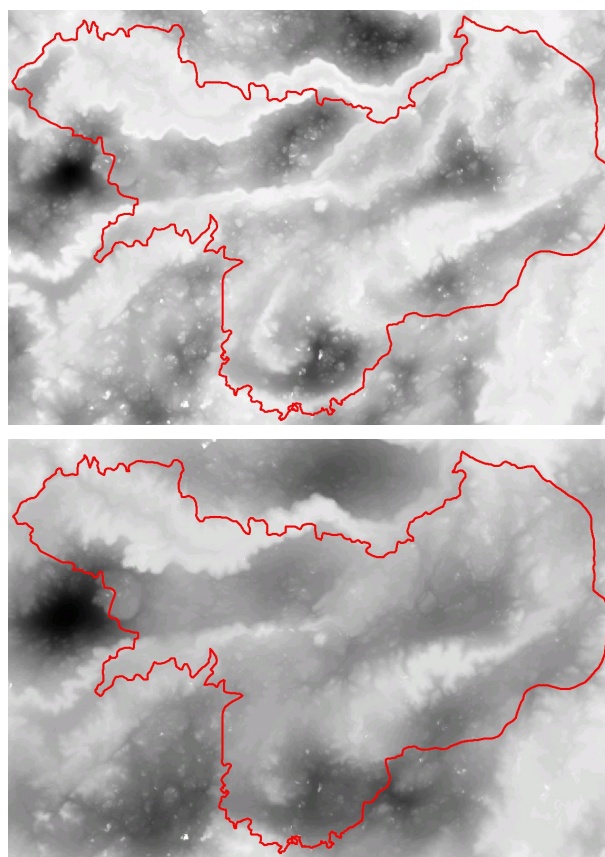

Figure 9: *Accessibility maps for the study area, with white representing low and black indicating high accessibility; top: bandwidth set to the energy expenditure needed for walking 2 km on level ground; bottom: bandwidth of 6 km.*

In section 4.2 a least-cost kernel was compared to adding slices of least-cost site catchment areas with increasing cost limits and with thicker slices for small cost limits. This model is appropriate for accessibility measures as well: Accessibility to near features is more important than to locations which are at the outer limit of a person's area of movement. Based on these conceptual considerations, the following procedure for calculating an accessibility map is proposed: (i) Create a dense regular grid of points for the study area. (ii) Calculate the least-cost KDE result for these points, with all weights set to 1. Varying the bandwidth of the least-cost KDE results in accessibility maps at different scales.

A similar accessibility concept was applied by COMPTON *et al*. (2007) in their connectivity study of

amphibian pools. They estimated for each vernal pool the accessibility to upland habitats.

This procedure was performed for the La Plata study area. The 200 m grid covering the area contains 18,860 centre points. For this grid, least-cost KDE results were obtained with bandwidths in the range of 2 to 6 km on level ground. The larger the bandwidth, the larger the computation times.

Figure 9 shows the results of the procedure described above using two different bandwidths depending on energy expenditure. In most river valleys slope-based accessibility is low. Within the La Plata survey zone the area with the highest accessibility values is located in the south, in the region which was occupied during the *Formative 3* period but was nearly deserted in *Regional Classic* times. In general, zones of high accessibility can be found in the east-west direction, whereas nearly all connections in the north-south direction have to pass zones of low slope-based accessibility. Compared to the result obtained with a least-cost bandwidth of 6 km, the picture remains roughly the same for smaller bandwidth values, though the east-western zones of high accessibility do not appear quite as uniformly accessible.

5. Discussion

The results presented in this study rely on a number of assumptions which may or may not be true (cf. DRENNAN, 2006: 15–19): The data consists of both surface collections and shovel probes. Shovel probes account for about 36% of the collection units and were taken where surface collections were not possible. They were approximately 40 by 40 cm and excavated to a depth of about 40 cm. The boundary of the area represented by the artefacts from a shovel probe was delimited fairly arbitrarily. According to Drennan, both collection types roughly show the same distribution patterns, but our statistic tests indicate significant differences between the two types.

Sherds of one type were regarded as an indicator of a period, based on fairly weak stratigraphic evidence. Issues of visibility, detectability, and sherd fragmentation during and after deposition were not taken into account. However, DRENNAN (2006: 15) states that conditions for surface collections in cultivated zones are generally good but the "density of the sherds found on the surface appears to depend very heavily on the nature of the present vegetation cover" (p. 59).

Nearly 9% of the study area remained unsurveyed, and the analysis is based on the assumption that the amounts of occupation within these areas is small enough so that no noticeable impact on the overall patterns is observable (DRENNAN, 2006: 33).

Furthermore, it was assumed that the number of sherds deposited per century remained constant throughout each period. Drennan's hypothesis that the amount of

sherds found is proportional to the number of people who left them, is especially problematic if two or more periods are considered. Drennan and Boada Riva are aware of this issue and note that the quantity of ceramics people used could have changed over time (p. 61).

The analysis of the sherd distributions using least-cost KDE was based on the grid of sherd densities constructed as proposed by PETERSON *et al*. (2005). This was due to the fact that the sizes of the collection unit areas vary considerably: The size of 25% of these units is below 0.38 hectare, the median is 0.57 ha (the average is 0.75 ha), and nearly 20% of the unit sizes exceed 1 ha. The grid approach is problematic insofar as isolated small collection units with high sherd density are treated differently, depending on their location with respect to the grid: If they fit in one grid cell, only one grid point is included in the calculations, whereas a location on the intersection of four grid cells results in four relevant grid points. Especially with more uniformly distributed collection unit sizes, it might be more appropriate to work with the collection unit centres.

The least-cost calculations are based on slope only, and the landscape might have changed substantially due to erosion and landslides (cf. DRENNAN, 2006: 91). Small-scale changes are to be expected if terraces are constructed on steep slopes so that houses can be built.

Different results are to be expected if swamps and rivers are taken into account. The inclusion of these cost components are desirable in a region with an annual precipitation of nearly 2000 mm (DRENNAN, 2006: 29).

So the results obtained for the La Plata study area with the LC-KDE approaches are only preliminary and may change dramatically if all the aspects mentioned above are included in the model. However, we are convinced, that the least-cost KDE approach is flexible enough to cope with changes in the model. In our view, the smoothed distribution maps created on the basis of unmodified sherd densities per survey unit and on the cost component slope, are only one step towards the goal of a comprehensive analysis of the settlement patterns in the La Plata region.

Conclusions

This study shows that the results of KDE are more intuitive than those of interpolation-based density estimation. Moreover, KDE can be readily adapted to take topographic features impeding progress into account.

The density calculations for the survey data from Colombia serve as an example of the fact that a least-cost method produces significantly different results as compared to an approach based on map distance only.

Furthermore, the least-cost KDE approach can be applied to visualize differences in sherd distribution and

to create accessibility maps, which reflect the ease with which a location may be reached from the surrounding area.

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