Realization of φ Josephson Junctions with a Ferromagnetic Interlayer

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Abstract

In this thesis, φ Josephson junctions based on 0-π junctions with a ferromagnetic interlayer are studied.

Josephson junctions (JJs) with a ferromagnetic interlayer can have a phase drop of 0 or π in the ground state, depending on the thickness of the ferromagnet (0 JJs or π JJs). Also, 0-π JJs can be realized, where one segment of the junction (if taken separately) is in the 0 state, while the other segment is in the π state. One can use these π Josephson junctions as a device in superconducting circuits, where it provides a constant phase shift, *i.e.*, it acts as a π phase battery.

A generalization of a π JJ is a φ JJ, which has the phase $\pm\varphi$ in the ground state. The value of φ can be chosen by design and tuned in the interval $0 < \varphi < \pi$. The φ JJs used in this experiment were fabricated as 0-π JJs with asymmetric current densities in the 0 and π facets. This system can be described by an effective current-phase relation which is tunable by an externally applied magnetic field.

The first experimental evidence of such a φ JJ is presented in this thesis. In particular it is demonstrated that (a) a φ JJ has two ground states $+\varphi$ and $-\varphi$, (b) the unknown state can be detected (read out) by measuring the critical current I_c (I_{c+} or I_{c-}), and (c) a particular state can be prepared by applying a magnetic field or a special bias sweep sequence. These properties of a φ JJ can be utilized, for example, as a memory cell (classical bit).

Furthermore, a φ Josephson junction can be used as a deterministic ratchet. This is due to the tunable asymmetry of the potential that can be changed by the external magnetic field. Rectification curves are observed for the overdamped and the underdamped case.

Moreover, experimental data of the retrapping process of the phase of a φ Josephson junction depending on the temperature is presented.

Kurzfassung

In dieser Arbeit werden φ-Josephson-Kontakte basierend auf 0-π-Kontakten mit einer ferromagnetischen Zwischenschicht studiert.

Josephson-Kontakte (JK) mit einer ferromagnetischen Zwischenschicht können abhängig von der Schichtdicke einen Phasensprung von 0 oder von π im Grundzustand aufweisen (0-JK oder π-JK). Es können auch 0-π-Josephson-Kontakte hergestellt werden, bei denen sich ein Teil des Kontakts (wenn er getrennt betrachtet wird) im 0-Zustand befindet, während der andere Teil im π-Zustand ist. In supraleitenden Schaltkreisen kann ein π-Kontakt als Bauteil benutzt werden, das eine konstante Phase liefert, d.h. als π-Phasen-Batterie.

Eine Verallgemeinerung eines π-Kontakts wird als φ-Kontakt bezeichnet und hat im Grundzustand die Phase $\pm\varphi$. Der Wert von φ kann durch das Probendesign gewählt werden und kann jeden Wert im Intervall $0 < \varphi < \pi$ annehmen. Die φ-Josephson-Kontakte, die in diesem Experiment verwendet wurden, wurden als 0-π-Josephson-Kontakte hergestellt und haben asymmetrische Stromdichten im 0- und π-Teil. Dieses System kann durch eine effektive Strom-Phasen-Beziehung beschrieben werden, die durch ein externes Magnetfeld verändert werden kann.

In dieser Arbeit wird der erste experimentelle Nachweis solch eines φ-Kontakts präsentiert. Im Besonderen wird gezeigt, dass (a) ein φ-Josephson-Kontakt zwei Grundzustände $+\varphi$ und $-\varphi$ hat, (b) der unbekannte Zustand der Phase ausgelesen werden kann, indem der kritische Strom I_c (I_{c+} oder I_{c-}) gemessen wird und (c) ein bestimmter Zustand präpariert werden kann, indem ein Magnetfeld oder eine spezielle Bias-Strom-Sequenz angelegt wird. Diese Eigenschaften können dazu benutzt werden, einen φ-Kontakt zum Beispiel als Speicherzelle (klassisches Bit) einzusetzen.

Außerdem kann ein φ-Josephson-Kontakt als deterministische Ratsche benutzt werden, da die Asymmetrie seines Potentials durch ein äußeres Magnetfeld verändert werden kann. Es werden Gleichrichtungskurven

für den überdämpften und den unterdämpften Fall gezeigt.

Darüber hinaus werden experimentelle Daten des Retrapping (Einfang)-Prozesses der Phase eines φ-Josephson-Kontakts in Abhängigkeit von der Temperatur präsentiert.

List of publications and contributions

This thesis is written cumulatively, based on the two publications listed below. A third and a fourth manuscript are in preparation. The manuscripts in preparation are presented in chapter 5 and chapter 6. The publications are attached at the end of the thesis.

List of publications

Publication I

H. Sickinger, A. Lipman, M. Weides, R. G. Mints, H. Kohlstedt, D. Koelle, R. Kleiner and E. Goldobin
Experimental Evidence of a φ Josephson Junction
Phys. Rev. Lett. **109**, 107002, (2012)

Publication II

E. Goldobin, **H. Sickinger**, M. Weides, N. Ruppelt, H. Kohlstedt, R. Kleiner and D. Koelle
Memory cell based on a φ Josephson Junction
Appl. Phys. Lett. **102**, 242602, (2013)

List of publications in preparation

Manuscript I

H. Sickinger, M. Weides, M. Knufinke, N. Ruppelt, H. Kohlstedt, D. Koelle, R. Kleiner and E. Goldobin
Deterministic ratchet based on a SIFS φ Josephson Junction

Manuscript II

H. Sickinger, M. Weides, N. Ruppelt, H. Kohlstedt, D. Koelle, R. Kleiner and E. Goldobin
Retrapping of the phase in a φ Josephson Junction

Publications not included in this thesis

Publication I

M. Kemmler, C. Gürlich, A. Sterck, **H. Pöhler**, M. Neuhaus, M. Siegel, R. Kleiner, and D. Koelle
Commensurability Effects in Superconducting Nb Films with Quasiperiodic Pinning Arrays
Phys. Rev. Lett. **97**, 147003, (2006)

Publication II

U. Kienzle, J. M. Meckbach, K. Buckenmaier, T. Gaber, **H. Sickinger**, Ch. Kaiser, K Ilin, M. Siegel, D. Koelle, R. Kleiner and E. Goldobin
Spectroscopy of a fractional Josephson vortex molecule
Phys. Rev. B **85**, 014521, (2012)

Publication III

N. Ruppelt, O. Vavra, **H. Sickinger**, E. Goldobin, D. Koelle, R. Kleiner, H. Kohlstedt
Combinatorial sputtering in planetary type systems for alloy libraries with perpendicular gradients of layer thickness and composition realised by a timing approach
Appl. Phys. A (2013), DOI 10.1007/s00339-013-8100-x

Contents

1 Introduction

Superconductivity is a macroscopic quantum phenomenon which can be observed in many materials below a critical temperature T_c. The value of T_c is specific for each material. Below this critical temperature T_c the electrical resistance vanishes. The superconducting charge carriers are correlated electron pairs, the Cooper pairs [BCS57]. In the simplest case, the two Cooper pair electrons have opposite momenta and opposite spin polarities (spin-singlet state). They can be described by the macroscopic wave function

$$\Psi = \Psi_0 e^{i\theta(r)}$$

where $\theta(r)$ denotes the phase and $|\Psi_0|^2$ is the Cooper pair density. Ψ can also be identified with the order parameter of the superconductor.

Another typical property of superconductivity is, that it expels a magnetic field B from its interior. This effect was experimentally discovered by W. Meissner and R. Ochsenfeld [MO33]. They found, that the field expulsion is irrespective of the application of the magnetic field before or after the transition to the zero resistive state. This means, that a superconductor is an ideal diamagnet ($\vec{B} = 0$). If the externally applied magnetic field exceeds a critical value $B_c(T)$, superconductivity is destroyed. On the other hand, the magnetic field tends to align the spins parallel and thus suppresses the singlet superconductivity.

In this sense, singlet superconductivity and ferromagnetism are antagonistic phenomena, the coexistence of both is possible in heterostructures within a thin region [Buz05]. The interplay of superconductivity (S) and weak ferromagnetism (F) is achieved, for example in SFS-multilayers. In such structures the order parameter is damped and oscillates spatially inside the ferromagnetic interlayer, similar to the Fulde-Ferrell-Larkin-Ovchinnikov state [FF64, LO65].

A heterostructure consisting of two superconductors separated by a thin barrier (*i.e.*, an insulator (I) or a ferromagnet) is called Josephson junction (JJ). A supercurrent can be passed through the barrier. This supercurrent depends on the phase difference $\phi = \theta_2 - \theta_1$ between the two superconducting electrodes, where θ_1 is the phase of one electrode and θ_2 corresponds to the phase of the second electrode. For a conventional Josephson junction the phase $\phi = 0$ corresponds to the ground state. This JJ is called 0 Josephson junction. In a SFS or SIFS JJ the order parameter is oscillating inside the F-layer and thus by properly choosing the thickness of the ferromagnetic interlayer a π JJ can be realized. For this JJ the ground state phase is $\phi = \pi$.

It would be even more interesting to have a Josephson junction with an arbitrary value φ of the phase in the ground state, rather than just 0 or π.

The motivation for the realization of a φ JJ is manifold, because in a φ JJ exceptionally interesting physics can be observed and there are various applications. For instance, a φ JJ can be used as a phase battery, which provides a constant phase, like a normal battery provides a constant voltage. This phase battery will never discharge and is dissipationless. The φ phase battery can be included into superconducting loops and due to that, bias lines can be removed. In addition, a φ Josephson junction can be utilized in Rapid Single Flux Quantum (RSFQ) circuits. Even for the improvement of the decoherence of Qubits (quantum bits) a phase battery proves beneficial. Furthermore, a φ JJ cannot only be utilized as a classical device, but also in the quantum mechanical regime. Due to the existence of a degenerate ground state, a φ JJ can be used as a quantum-mechanical two level system, which can be easily controlled.

For a long time, a controllable φ JJ existed only as a theoretical construct. The aim of this thesis was, first, to realize a φ Josephson junction experimentally. Second, the applications of a φ Josephson junction as a classical bit (memory cell) and as a ratchet were studied. Third, the retrapping process in a φ Josephson junction was investigated.

The thesis is organized as follows:
The **first chapter** deals with the basics of Josephson junctions. The physics of π Josephson junctions are reviewed and the concept of φ Josephson junctions is introduced. In the **second chapter**, the data of the first publication is presented. Therein the experimental evidence of a φ Josephson junction is discussed. In the **third chapter**, the first application of a φ Josephson junction as a memory cell (classical bit) is reported. The chapter is a summary of the second publication. In **chapter four**, a manuscript in preparation is presented. In this chapter the operation of a deterministic ratchet based on a φ Josephson junction is discussed. The asymmetry

of the ratchet is tunable by an external magnetic field. The overdamped and the underdamped regime of rectification are investigated. The **fifth chapter** deals with another manuscript in preparation. The topic is the retrapping process of the phase in a φ Josephson junction. The work is closing with a **Summary and Outlook**. The used abbreviations and symbols are described in the chapter **Acronyms and Symbols**. All the used references are given in the **Bibliography**. Reprints of the publications are attached at the very end of this thesis.

2 Basics

2.1 Josephson Junctions

A Josephson junction is a system consisting of two weakly coupled superconductors. In the simplest case the superconducting electrodes are separated by a thin (only few nanometers thick) insulating barrier (SIS junction). In the insulating barrier the order parameter of the two superconducting electrodes Ψ_1 and Ψ_2 is suppressed exponentially. If the barrier is thin enough, then the macroscopic wave functions of the two superconductors overlap, which leads to a weak link. The supercurrent I_s, which can be carried dissipationless through a conventional Josephson junction is described by the first Josephson equation [Jos62]

$$I_s = I_0 \sin(\phi) \tag{2.1}$$

The Josephson phase ϕ is the phase difference of the wave functions of the two superconductors

$$\phi = \theta_2 - \theta_1 - \frac{2e}{\hbar} \int \vec{A} \cdot d\vec{l} \tag{2.2}$$

The vector potential is integrated over the barrier from superconductor 1 to superconductor 2.

The current-phase relation (CPR) as described in Eq. (2.1) is the simplest form of the general relation

$$I_s = \sum_{n=0}^{\infty} I_{cn} \sin(n\phi) \tag{2.3}$$

with the coefficients I_{cn} [BK04], which depend on the coupling and the type of contact.

One can see from Eq. (2.3), that the CPR, for example, can have a second harmonic $\sim \sin(2\phi)$. The consequences of a second harmonic in the CPR will be discussed in section 2.3. For the discussion in this section only the CPR given by Eq. (2.1) is considered.

The maximum supercurrent that can be passed though the junction is the critical current I_0 [GKI04]. For $I > I_0$ the current cannot be carried by Cooper pairs alone anymore, but also by quasiparticles and a voltage drop is detected across the junction. A relation between the voltage V and the derivative of the phase is given by the second Josephson equation

$$\dot{\phi} \equiv \frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{2e}{\hbar}V = \frac{2\pi}{\Phi_0}V \tag{2.4}$$

where $\Phi_0 \approx 2.07 \cdot 10^{-15}$ Wb is the magnetic flux quantum. The Josephson potential in general is the integral of the CPR

$$U(\phi) = \frac{\Phi_0}{2\pi}\int I_s(\phi)\mathrm{d}\phi \tag{2.5}$$

and in this case for Eq. (2.1) the Josephson energy is given by

$$U(\phi) = E_J(1 - \cos\phi) \tag{2.6}$$

with the Josephson coupling energy $E_J = I_0\Phi_0/2\pi$ and the integration constant chosen so, that $U(0) = 0$.

As long as no current ($I = 0$) is flowing over the Josephson junction, the junction is in the ground state. The two superconducting wave functions of the electrodes behave coherently and establish a (Josephson) phase difference of $\phi = 0$. The ground state corresponds to the minimum of the Josephson energy (Eq. (2.6)). A Josephson junction with a CPR given by Eq. (2.1) has its minimum of energy at the Josephson phase $\phi = 0$ and for that reason the junction is called 0 Josephson junction (0 JJ), see Fig. 2.1.

The dynamics of a Josephson junction in the short junction limit can be described by a simple analog system, which has the same differential equation of motion as a Josephson junction. In this analogon the Josephson phase is represented as a particle in a washboard potential (compare Eq. (2.5), Fig. 2.1 and Fig. 2.2). Applying a bias current to the junction corresponds to a tilt of the washboard potential and exerts an external force to the particle. If the critical current I_0 is reached, the junctions jumps to the voltage state and the particle begins to roll down the potential.

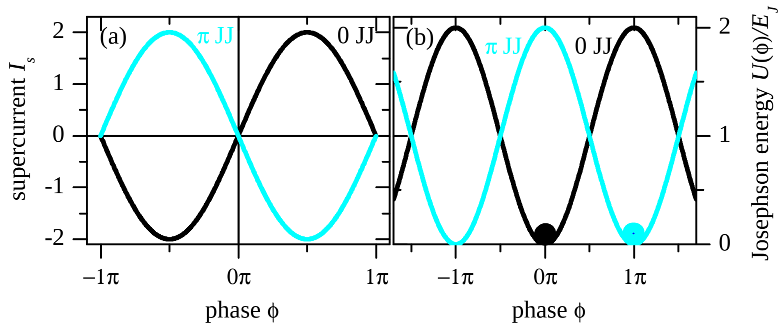

Figure 2.1: (a) Current-phase-relation $I_s(\phi)$ and (b) Josephson energy $U(\phi)/E_J$ for a 0 JJ and a π JJ. The sphere marks the ground state (minimum of energy) for a 0 and a π JJ.

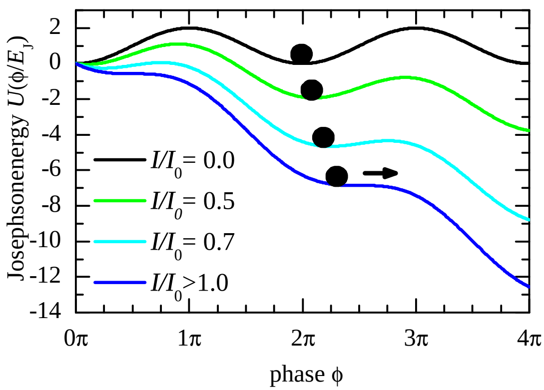

Figure 2.2: Washboard potential with the phase particle (sphere) for different tilts (bias currents).

2.2 π Josephson Junctions

Already in 1977, Bulaevskii *et al.* predicted [BKS77, BKSL78] that a negative critical current $I_0 < 0$ may occur in a Josephson junction with a barrier with magnetic impurities. The current-phase relation for this case then reads

$$I_s = -|I_0|\sin(\phi) = |I_0|\sin(\phi + \pi) \tag{2.7}$$

and the Josephson energy is

$$U(\phi) = E_J(1 + \cos\phi). \tag{2.8}$$

One can easily see that the minimum of the Josephson energy is at $\phi = \pi$ and that the ground state for such a Josephson junction is $\phi = \pi$, see Fig. 2.1(b). Accordingly, the junction is called π Josephson junction (π JJ).

2.2.1 Ferromagnetic π Josephson Junctions

The first experimental realization of a π JJ based on a heterostructure with a ferromagnetic barrier was demonstrated by Ryazanov *et al.* [ROR$^+$01].

A so called ferromagnetic Josephson junction is a Josephson junctions with a ferromagnetic interlayer, *e.g.*, **S**uperconductor–**F**erromagnet–**S**uperconductor (SFS) JJ [BTKP02, BBA$^+$04] or **S**uperconductor–**I**nsulator–**F**erromagnet–**S**uperconductor (SIFS) JJ [KAL$^+$02].

If one first considers the simple case of a superconductor in contact with a normal metal, the order parameter enters into the normal metal and is suppressed exponentially on the length scale of the coherence length ξ_N. This effect is called proximity-effect [Buz05] and is depicted in Fig. 2.3 (a).

Now, let us consider the case, when a superconductor is in contact with a ferromagnet, see Fig. 2.3 (b). In such a heterostructure the order parameter is also suppressed exponentially ($\sim \mathrm{e}^{(-z/\xi_{F1})}$) (due to the proximity effect) on the length scale ξ_{F1}, but in addition, it is oscillating on the length scale $2\pi\xi_{F2}$. This results in a order parameter that shows a damped oscillation in the ferromagnetic interlayer. The oscillating order parameter is a consequence of the exchange field H_{ex} of the ferromagnet. The Cooper pair electrons coming from the superconductor and entering into the ferromagnet experience the exchange field H_{ex}, which leads to a Zeeman splitting of the two spin polarities. The potential energy of the Cooper pair electrons changes by the exchange energy $E_{\mathrm{ex}} = \pm\mu_B\mu_0 H_{\mathrm{ex}}$, respectively for the spin direction ($\uparrow$ or $\downarrow$). However, the total energy of each electron must remain the same. Therefore the change in potential energy is accompanied by a change in kinetic energy. The electron with parallel spin direction decreases its potential energy and increases its

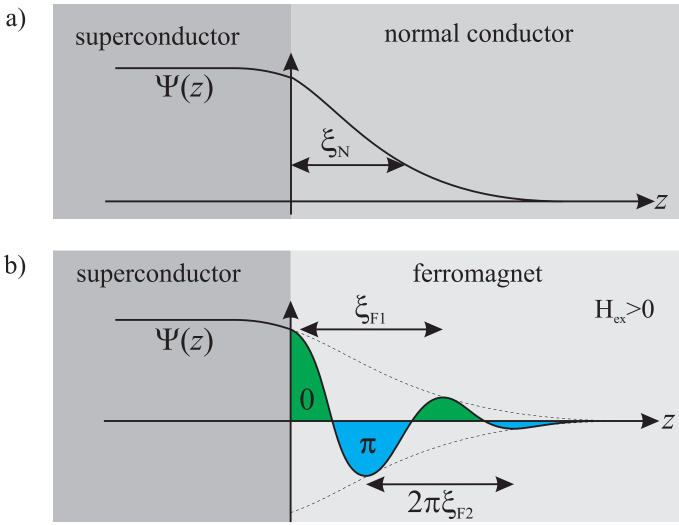

Figure 2.3: Spatial dependence of the superconducting order parameter $\Psi(z)$ at (a) the NS interface and (b) the SF interface with the corresponding decay lengths.

kinetic energy. For the electron with the antiparallel spin it is vice versa. In both cases the electrons gain a shift into the same direction in k-space. This leads to a shift of the center of mass momentum of the Cooper pair of $Q = 2E_{\mathrm{ex}}/v_F$, where v_F is the Fermi velocity in the ferromagnet. Since the spin wave function of a Cooper pair is antisymmetric, the orbital wave function is symmetric (for the spin-singlet state $(\uparrow\downarrow - \downarrow\uparrow)$) and given by: $\Psi \propto \mathrm{e}^{(iQz/\hbar)} + \mathrm{e}^{(-iQz/\hbar)} \propto \cos(Qz/\hbar)$. Additionally, by taking into account scattering effects of the Cooper pairs in the ferromagnet, the wave function will be damped on the length scale ξ_{F1}. This leads to a damped oscillating order parameter in the F-layer:

$$\Psi(z) \propto \mathrm{e}^{z/\xi_{F1}} \cdot \mathrm{e}^{-iz/\xi_{F2}} \tag{2.9}$$

[DAB97]. For a weak ferromagnet the coherence lengths $\xi_{F1,2}$ are given by

$$\xi_{F1,2} = \sqrt{\frac{\hbar D}{[E_{\mathrm{ex}}^2 + (\pi k_{\mathrm{B}} T)^2]^{1/2} \pm \pi k_{\mathrm{B}} T}} \tag{2.10}$$

where D is the diffusion constant of the electrons in the ferromagnet, k_{B} is the Boltzmann constant and T is the temperature [ROR⁺01].

If the exchange energy E_{ex} is of the same order of magnitude as $k_{\mathrm{B}}T$, the ratio of $\xi_{\mathrm{F1}}/\xi_{\mathrm{F2}}$ can be changed by changing the temperature T. For certain junction parameters a temperature induced transition between a 0 coupled and a π coupled junction can be achieved in a junction with a constant thickness of the ferromagnetic interlayer. This 0-π transition was observed experimentally [ROR⁺01, OBF⁺06, SBLC03, WKG⁺06].

Now, it is obvious that the ground state phase of a JJ depends on the thickness of the ferromagnetic barrier d_{F} of an S(I)FS JJ. By properly choosing the thickness of the ferromagnetic interlayer d_{F}, it is possible to realize either a 0 or a π JJ.

2.2.2 SIFS Josephson Junctions

To investigate dynamical properties of a JJ one needs underdamped (hysteretical) junctions. The damping of a Josephson junction in the framework of the resistively and capacitively shunted junction (RCSJ) model is described by the Stewart-McCumber parameter $\beta_c = (2\pi I_0 R^2 C)/\Phi_0$ with the capacitance C and the normal resistance R. To achieve π JJs with low damping it is necessary to have a JJ with high critical current I_0 and high resistance R. Unfortunately, the junction resistance in SFS JJs is quite low ($\sim$ 10μΩ). By introducing an additional insulating interlayer into the junction (SIFS JJ) the critical current is reduced, but the resistance R is enhanced. This results in a underdamped π JJ [KAL⁺02, BSH⁺06, WTK06].

The critical current in SIFS JJs as a function of the thickness of the ferromagnetic interlayer d_{F} is given by

$$I_0(d_{\mathrm{F}}) \propto \frac{1}{\gamma_{\mathrm{B2}}} \mathrm{e}^{\left(\frac{-d_{\mathrm{F}}}{\xi_{\mathrm{F1}}}\right)} \cos\left(\frac{d_{\mathrm{F}} - d_{\mathrm{F}}^{\mathrm{dead}}}{\xi_{\mathrm{F2}}}\right) \tag{2.11}$$

with the transparency factor γ_{B2} of the SIF-part and the thickness $d_{\mathrm{F}}^{\mathrm{dead}}$ of the non-magnetic part of the interlayer [WKG⁺06, Buz05].

2.2.3 Technology

There are many other possibilities to realize π JJs using different physical effects and technologies. For example [Gol12]:

1. **Josephson Junctions with spin flipping magnetic impurities in the insulating barrier**. In the pioneering work of Bulaevskii *et al.* [BKS77] it was predicted, that π JJs may be realized by such JJs. These π JJs with ferromagnetic insulating or semiconducting barriers were investigated theoretically by Barash *et al.* [BB02] and Fogelström [Fog00]. The proximity effect in this kind of JJs can be disregarded, because it is much weaker as in S(I)FS JJs. In this type of JJs the appearance of the π ground state is caused by quasiparticles scattering on a magnetically active interface [Fog00]. There may occur two different tunneling contributions to the critical current. The tunneling probability through the barrier without spin-flipping creates a positive current (0 JJ) and the tunneling with spin-flipping generates a negative current (π JJ). The two tunneling currents have opposite polarity and different temperature dependencies [VGG$^+$06]. Depending on which spin channel dominates, either a 0 JJ or a π JJ is formed. These SIS JJs with ferromagnetic impurities in the insulating barrier are called S(FI)S JJs. The first experimental realization of a π JJ based on a S(FI)S JJ was demonstrated by Vávra *et al.* [VGG$^+$06]. The transition from the 0 coupled JJ to the π coupled JJ was induced by the temperature.

2. **JJs based on unconventional superconductors.** The first experimental realization of a π JJ was based on a corner JJ, consisting of a d-wave (YBCO) and a s-wave (Pb) superconductor [vH95]. d-wave superconductors have an anisotropic order parameter, which depending on the direction changes the polarity. This sign change is utilized in combination with s-wave superconductors to create a π JJ or arrays of 0 and π JJs (zigzag JJs) [vH95, SAB$^+$02, HAS$^+$03].

With grain boundary JJs it is also possible to realize π JJs [TK00, CSG$^+$02].

With unconventional superconductors, it is not possible to fabricate a single π JJ, but a π JJ, that is in connection with a 0 JJ. This junction is called 0-π junction.

3. **Superconductor – Quantum dot – Superconductor Josephson Junctions**. The current transport through Superconductor – Quantum dot – Superconductor JJs is very similar to the one in JJs with spin flipping

magnetically impurities in the I-layer. But typically only one conduction channel is present. Often carbon nanotubes (CNTs) are used as quantum dots [KKR+99]. By applying a gate voltage the CNT JJs can be tuned from the 0 to the π state as proved by [CWB+06]. A reversal of the supercurrent (π state) through a quantum dot with a semiconductor nanowire was shown by van Dam *et al.* [vDNB+06].

4. **Superconductor – Normalmetal – Superconductor – Josephson Junctions (SNS JJs)** may be utilized to create π JJs. The energy distribution (double-step function) of the current-carrying density of states in the normal metal is influenced by two electron reservoirs. By applying a voltage the state can be changed between 0 or π [BMvWK99].

5. With **superfluid** ^3He a Josephson junction can be realized that can have a ground state phase of 0 or π [PLB+97, BPS+98, MSB+99].

2.2.4 Applications of π Josephson Junctions

As mentioned before a π JJ has the constant phase $\phi = \pi$ in the ground state. Due to that reason, the junction can be seen as a (never discharging) "phase battery", which provides a constant phase $\phi = \pi$, similar to a normal battery providing a constant voltage.

By integrating a π JJ into a superconducting loop one will observe a supercurrent though the superconducting wire. No energy is dissipated in this system and the phase battery does not discharge [Gol12]. A supercurrent is circulating in the loop, if $|I_0| > \Phi_0/(2\pi L)$ with the inductance L of the superconducting wire. If $|I_0| \gg \Phi_0/(2\pi L)$ then the provided phase of the phase battery is close or equal to π and otherwise smaller [Gol12]. The current in the loop may flow clock-wise or counter-clock-wise, depending on the phase shift being π or $-\pi$. The state is the degenerate ground state of the system and the current flow direction is spontaneous. The circulating current induces a magnetic flux of $\Phi_0/2$ or $-\Phi_0/2$ in the loop. This value is half of the magnetic flux quantum Φ_0 and due to that the created fractional vortex is called "Semifluxon".

This phase battery can be used to create a phase bias to several classical [OAM+06] and quantum mechanical [FOB+10] electronic circuits. In several superconducting electronic circuits (*e.g.*, RSFQ logic) the phase battery can be used for self biasing the JJs and due to that bias lines can be removed. In requirements related to Qubits and other quantum circuits the phase batteries can be used to decouple the system from the environment and to reduce the decoherence.

2.3 φ Josephson Junctions

For applications as phase battery it would be desirable to have a Josephson junction with a phase shift not only of 0 or π, but rather with an arbitrary phase shift. This requirement can be fulfilled by a φ Josephson junction. A φ Josephson junction is a generalization of a π JJ with an arbitrary phase $0 < \varphi < \pi$ in the ground state.

The first suggestions for a possible realization of a φ JJ was given by Mints in 1998 [Min98]. He considered a long chain of alternating short 0 and π JJs, a 0-π-0-π-0-...-array. Every facet of the array has a conventional current-phase-relation $j_s = \pm j_0 \sin\phi$, with j_s being the supercurrent density and j_0 the critical current density.

Further theoretical investigations on 0-π junction based φ JJs were performed by Mints *et al.* [MP00] and Buzdin *et al.* [BK03]. In their considerations the phase of the system (of the 0-π-0-π-0-...-array) for short junctions and small magnetic fields can be described by

$$\phi(x) = \psi + \xi(x) \sin\psi \tag{2.12}$$

where

$$\psi = \langle \phi(x) \rangle \tag{2.13}$$

is the constant spatially averaged phase ψ and $\xi(x)$ is a rapidly alternating function ($|\xi(x)\sin\psi| \ll 1$) on the order of the length of a single facet. $\xi(x)$ can also be seen as a deviation from the average phase ψ. For a simple illustration of the Josephson phases along one 0-π segment of the whole array see Fig. 2.4.

The effective CPR is given by [Min98, MPK$^+$02, BK03]

$$j_s = j_1 \sin(\psi) + j_2 \sin(2\psi) \tag{2.14}$$

where the spatially averaged critical current density is $j_1 = \langle j_c(x) \rangle$. The second harmonic term has the amplitude $j_2 < 0$, which depends on the length and the critical current densities of the facets.

The ground state of the system is doubly degenerate with $\psi = \pm\varphi$ for

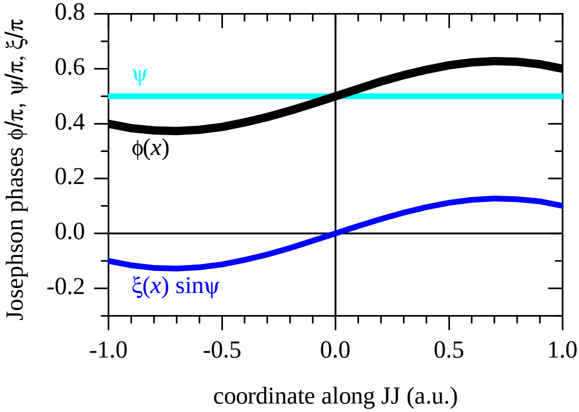

Figure 2.4: The total phase $\phi(x)$ of a short 0-π JJ can be described as a sum of the average phase ψ and a deviation $\xi(x)$ from the average value.

$j_2 < -j_1/2$ and

$$\varphi = \arccos[-j_1/(2j_2)] \tag{2.15}$$

where $0 < \varphi < \pi$ is the arbitrary phase.

This φ JJ has some interesting and remarkable physical properties, *e.g.*, Josephson vortices that are not quantized occur in such a JJ. These splintered vortices were predicted by Mints *et al.* [Min98] and could be observed experimentally at grain boundaries in $YBa_2Cu_3O_{7-\delta}$ [MPK$^+$02].

There are many proposals how a φ Josephson junction also can be realized [GS09, PGKK10, HPK$^+$13, Tur13]. Below I focus on Josephson junctions with a steplike ferromagnetic barrier (0-π SIFS JJs).

Let us now consider the smallest element of such a 0-π-0-π-0-...-chain, which is only one period of the chain and just a 0-π JJ, see Fig. 2.5. This is the simplest system for a potential realization of a φ JJ.

The physical lengths of the two segments L_0 and L_π are supposed to be short $L_0, L_\pi \lesssim \lambda_J$ and generally speaking asymmetric $L_0 \neq L_\pi$, where $\lambda_J \propto 1/\sqrt{j_0}$ is the Josephson penetration depth. $L = L_0 + L_\pi$ is the total length of the junction, see Fig. 2.6.

Nevertheless, it took a long time to solve the following contradiction concerning the dependence of the critical current in a magnetic field

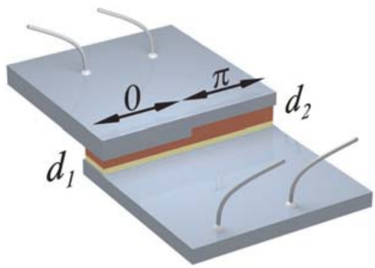

Figure 2.5: φ JJ based on a SIFS 0-π JJ with a step of thickness in the ferromagnetic interlayer.

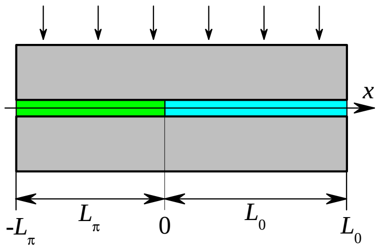

Figure 2.6: Schematic picture of an asymmetric 0-π JJ. Adapted from [GKKM11].

H ($I_c(H)$ pattern) of a φ JJ. On one side, by using the linear phase ansatz $\phi(x) = Hx + \phi_0$ and the usual CPR, this leads to a cusplike minimum in the $I_c(H)$ dependence at $H = 0$ in an asymmetric 0-π JJ. This $I_c(H)$ pattern is in agreement with the experiments on 0-π JJs [vH95, KWW+10, WPK+10]. On the other side, by using the second harmonics in the CPR (Eq.(2.14)) (like successfully used for the description of Josephson properties in $YBa_2Cu_3O_{7-\delta}$ grain boundary junctions [MPK+02]) and the linear phase for the average phase ψ, this results in a $I_c(H)$ pattern that looks like a Fraunhofer pattern and has a maximum at $H = 0$ [GKKB07].

This contradiction could be cleared by Goldobin *et al.* [GKKM11]. In this work an asymmetric 0-π JJ (see Fig. 2.6) is analyzed in an external magnetic field. The main idea is to use the ansatz $\phi(x) = \psi + \xi(x) \sin\psi$ (Eq. (2.12)) with the average phase ψ and substitute it into

$$\phi'' - \langle j_c \rangle \left[1 + g(x)\right] \sin\phi = -\gamma \tag{2.16}$$

with the normalized bias current density $\gamma = j/j_0$. This equation was derived from the static sine-Gordon equation

$$\phi''(x) - j_c(x) \sin[\phi(x)] = -\gamma \tag{2.17}$$

which has a step-function in the critical current density

$$j_c(x) = \begin{cases} -j_0, & \text{for} \quad -L_\pi < x < 0 \\ +j_0, & \text{for} \quad 0 < x < L_0. \end{cases} \tag{2.18}$$

The average critical current density is $\langle j_c \rangle$ and the deviation is

$$g(x) = \begin{cases} g_\pi = \frac{-2L_0}{L_0 - L_\pi}, & \text{for} \quad x < 0 \\ g_0 = \frac{+2L_\pi}{L_0 - L_\pi}, & \text{for} \quad x > 0 \end{cases} \tag{2.19}$$

of the average value $\langle g(x) \rangle = 0$.

The calculations of Goldobin *et al.* [GKKM11] result in the effective CPR for the current density

$$\gamma = \langle j_c \rangle \left[\sin\psi + \Gamma_h h \cos\psi + \frac{\Gamma_0}{2} \sin(2\psi)\right] \tag{2.20}$$

and accordingly, the experimentally very important effective CPR for the supercurrent

$$I_s = \langle I_c \rangle \left[\sin\psi + \Gamma_h h \cos\psi + \frac{\Gamma_0}{2} \sin(2\psi)\right] \tag{2.21}$$

with $\langle I_c \rangle = \langle j_c \rangle Lw$ and the width w of the JJ.

The normalized applied magnetic field is given by $h = 2\pi\lambda_J \Lambda H/\Phi_0$ and Λ is the effective magnetic thickness.

This effective CPR has the negative second harmonic $\sin(2\psi)$ like in earlier works [Min98, MPK$^+$02, BK03]. Furthermore there is an extra term $\propto h\cos\psi$ in the CPR induced by the magnetic field H. This means, that

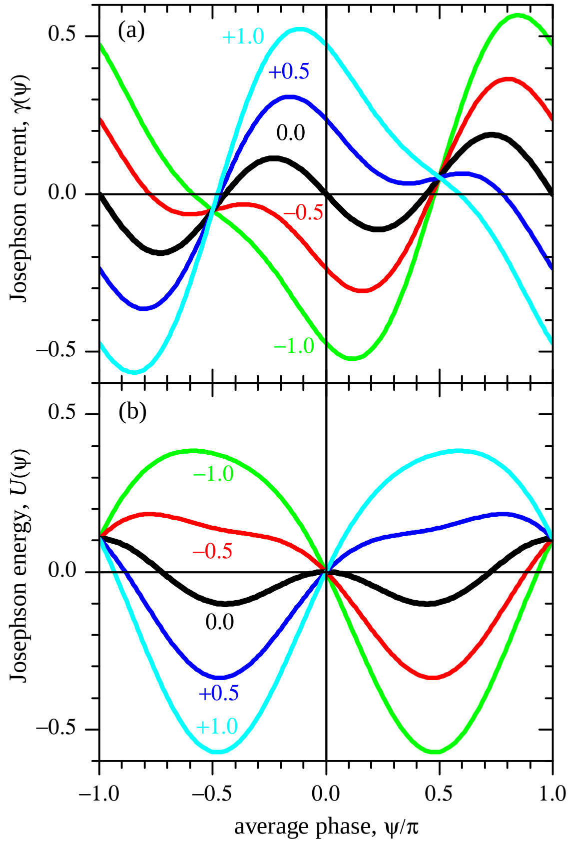

Figure 2.7: (a) Effective CPR $\gamma(\psi)$ and (b) effective Josephson energy $U(\psi)$ for different magnetic field values h for an asymmetric 0-π JJ with $L_0 = 1$ and $L_\pi = 0.9$. Adapted from [GKKM11].

the effective CPR is tunable by the applied magnetic field, see Fig. 2.7(a).

The Josephson energy is the integral of the CPR (Eq. (2.21)) and is given by

$$U(\psi) = \langle E_J \rangle \left[1 - \cos\psi + \Gamma_h h \sin\psi + \frac{\Gamma_0}{2} \sin^2\psi \right] \tag{2.22}$$

with the coupling energy $\langle E_J \rangle = \Phi_0 \langle I_c \rangle / 2\pi$. The coefficients are given by

$$\Gamma_0 = -\frac{4}{3} \frac{L_0^2 L_\pi^2}{L_0^2 - L_\pi^2} \tag{2.23}$$

$$\Gamma_h = \frac{L_0 L_\pi}{L_0 - L_\pi} \tag{2.24}$$

and depend only on the junction parameters.

Lipman *et al.* [LMK+12] extended the work of Goldobin *et al.* [GKKM11] to the more general case of arbitrary current densities $j_{c,0} \neq j_{c,\pi}$ of the 0 and the π part. This approach is more suitable for the 0-π SIFS JJs investigated in this thesis, because the length of the 0 and the π parts are equal, but the current densities $j_{c,0}$ and $j_{c,\pi}$ in both parts are different. The coefficients are then defined as

$$\Gamma_0 = -\frac{l_0^2 l_\pi^2}{3} \frac{(j_{c,0} - j_{c,\pi})^2}{(j_{c,0} l_0 + j_{c,\pi} l_\pi)^2} \tag{2.25}$$

$$\Gamma_h = \frac{l_0 l_\pi}{2} \frac{j_{c,0} - j_{c,\pi}}{j_{c,0} l_0 + j_{c,\pi} l_\pi} \tag{2.26}$$

with the lengths l_0 and l_π normalized to the Josephson length $\lambda_J(\langle j_c \rangle)$. Here the averaged critical current density is
$\langle j_c \rangle = (L_0 j_{c,0} + L_\pi j_{c,\pi})/(L_0 + L_\pi)$.

The effective CPR remains the same as in Eq. (2.20). For the case of equal 0 and π segments ($l_0 = l_\pi = l/2$), Eq. (2.25) and Eq. (2.26) reduce to

$$\Gamma_0 = -\frac{l^2}{12} \left(\frac{j_{c,0} - j_{c,\pi}}{j_{c,0} + j_{c,\pi}} \right)^2 \tag{2.27}$$

$$\Gamma_h = \frac{l}{4} \frac{j_{c,0} - j_{c,\pi}}{j_{c,0} + j_{c,\pi}} \tag{2.28}$$

In Fig. 2.7(b) the Josephson energy is depicted for the same values of the magnetic field h as the effective CPR in Fig. 2.7(a). In zero magnetic field, it is visible, that the cosine-like potential (of a conventional JJ) is transformed into a double well potential due to the existence of the second harmonic. The double well potential is symmetric and consequently has two degenerate ground states. For an applied magnetic field h the form of the potential $U(\psi)$ can be modified. One of the potential minima, depending on the polarity of the magnetic field, is getting deeper and the other one is getting more shallow, up to the point, where it totally disappears.

With the effective CPR (Eq. (2.25)) in zero field, it is possible to calculate the domain, where the φ state exists as a function of the segment lengths L_0 and L_π. An estimation for which length the degeneracy occurs can be done with

$$L_0 \geq L_\pi \sqrt{\frac{3}{4L_\pi^2 + 3}} \tag{2.29}$$

$$L_\pi \geq L_0 \sqrt{\frac{3}{4L_0^2 + 3}} \tag{2.30}$$

In Fig. 2.8 this approximation is shown as dashed line and compared with the exact results [BKSL78] indicated as solid line. It can be reasoned, that the approximation works very well. The areas of different colors indicate the validity of the approximation $|\xi(x)\sin\psi| \ll 1$. From green, with a well working approximation, to red, where the approximation is invalid.

To answer the question experimentally, whether one deals with a φ JJ or not, the $I_c(H)$ pattern at small magnetic fields h is observed.

In Fig. 2.9 the critical current dependence of the magnetic field $\gamma_c(h)$ for different lengths of the two facets ($L_0 \neq L_\pi$) is calculated and compared with numerical simulations (cyan) performed with STKJJ [Gol03]. The results are in good agreement. The length asymmetry used in Fig. 2.9(a) lies deep inside the φ region, compare Fig. 2.8. One finds a $\gamma_c(h)$ dependence with four possible critical currents for $|h| \geq 0.6$. The four critical currents correspond to the escape of the phase out of the two different potential wells in the two bias current directions. For higher magnetic fields $|h| \leq 0.6$ the second minimum is disappearing and due to that only two critical currents can be expected.

In Fig. 2.9(b) and Fig. 2.9(c), a 0-π JJ with larger length asymmetry is shown. The domain of existence of the four critical currents is getting more narrow and collapses with increasing asymmetry of the lengths. The disappearance of the crossing branches of the $\gamma_c(h)$ dependence

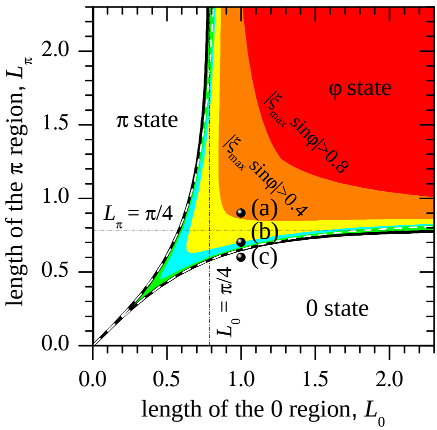

Figure 2.8: Domain of existence of the φ state at $h = 0$ as a function of the segment lengths L_0 and L_π. The color code shows how good the approximation $|\xi(x) \sin\psi| \ll 1$ works, from green (good) to red (invalid). The dots indicate the parameters that are used in Fig. 2.9. Adapted from [GKKM11].

corresponds to the crossing of the boundary of the φ state. Hence, a characteristic property of a φ JJ is the crossing of the branches at the main minimum of the $\gamma_c(h)$ pattern.

A further typical feature of a φ JJ is the shifted cusplike main minimum of the $\gamma_c(h)$ pattern. This shift depends on the length asymmetry and can be utilized to determine the value of φ.

To observe two critical currents $\gamma_{c\pm}$ for each bias current direction a Josephson junction with low damping is required. Otherwise, due to high damping, the phase is always trapped in the deeper minimum and only the higher critical current can be measured.

Due to the possibility to tune the Josephson potential asymmetry by applying a magnetic field, it is possible to use φ Josephson junctions as phase ratchets. Furthermore, the doubly degenerate ground state can be used as a macroscopic quantum two-level system. Additionally, the bistable system can be controlled by the magnetic field.

The experimental proof of a φ Josephson junction and its applications are discussed in the following chapters.

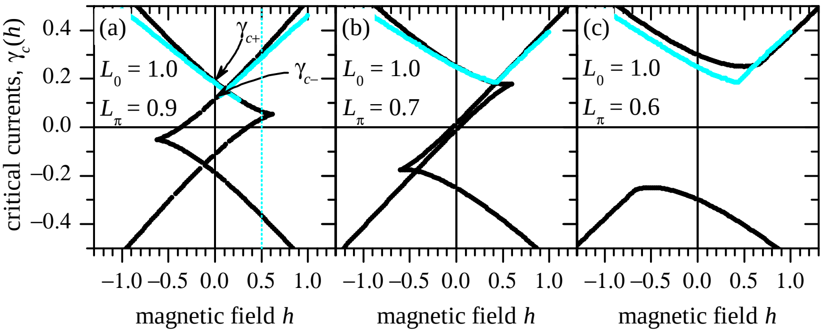

Figure 2.9: Normalized critical current versus normalized magnetic field, $\gamma_c(h)$, for different segment lengths of a 0-π JJ. Black lines are calculated analytically. In cyan the numerical simulations are shown. Adapted from [GKKM11].

3 Publication I: Experimental Evidence of a φ Josephson Junction

For a long time, a controllable φ Josephson junction existed only as a theoretical construct. Quite recently the first ferromagnetic φ JJ could be realized experimentally in our group. In this publication we report on the existence of such a φ Josephson junction.

The φ JJ we deal with is based on a 0-π JJ produced by SIFS-technology. The heterostructure is fabricated as a Nb$|$Al-$Al_2O_3|Ni_{0.6}Cu_{0.4}|$Nb multilayer. A step is etched into the ferromagnetic interlayer to realize a 0 and a π segment of the junction. The Josephson phase $\phi(x)$ of this 0-π JJ, due to the short normalized length, can be treated as a constant average phase $\psi = \pm\varphi$ along the whole junction. The value of φ lies in the interval $0 < \varphi < \pi$ and can be chosen by choosing the junction parameters. The 0 and the π facets of the 0-π JJ have almost the same length and an asymmetry in the current densities. The parameters of the junction are such, that the JJ lies well within the domain of existence of the φ state, see Fig. 3.1.

A typical property of a φ JJ is the doubly degenerate ground state $+\varphi$ and $-\varphi$ at zero magnetic field. The escape of the phase out of the $+\varphi$ or the $-\varphi$ well corresponds in experiment to the observation of two different critical currents I_{c+} and I_{c-}, respectively, for a positive bias current direction. For negative bias currents the situation is vice versa and in total four critical currents can be observed, see Fig. 3.2.

The larger critical current I_{c+} is observed at any temperature (at any damping) in the superconducting state. For the observation of the lower critical current I_{c-} $(-I_{c-})$ low damping is required, so that retrapping in the $+\varphi$ $(-\varphi)$ well is avoided.

In this experiment, $\pm I_{c+}$ and $\pm I_{c-}$ clearly are identified on the current-

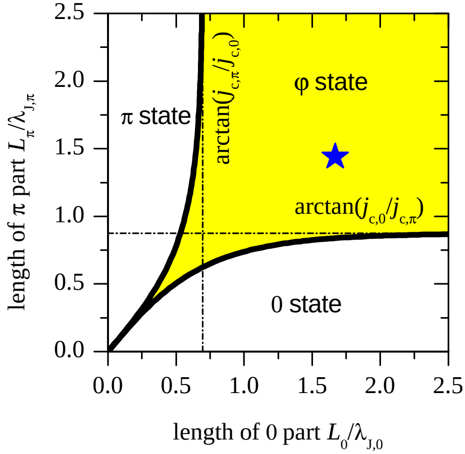

Figure 3.1: Domain of existence of the φ state. The star indicates the parameters of the investigated sample. Figure modified from the appended publication I. © 2012 by the American Physical Society.

voltage-characteristic in the temperature range between 3.5 K and 300 mK, see Fig. 3.2.

The two critical currents could not only be observed at zero field, but also at finite but small magnetic fields. This can be seen in the $I_c(H)$ dependence (see Fig. 3.3), where the crossing of two branches for each bias current direction is shown. At the crossing point, the cusplike main minimum is located and is shifted off from $H = 0$. The magnitude of the shift can be utilized to determine the value of φ.

With this measurement two typical features of a φ Josephson junction were demonstrated.

Another important property of a φ JJ is that the effective CPR and the effective Josephson potential are tunable by the magnetic field. By applying a sufficiently high magnetic field the second potential minimum can be removed. Consequently, the phase is trapped in the other potential well and the system is in a defined and well-known state. In the experiment the feasibility of this method of preparing the state by applying a magnetic field was demonstrated.

Another possibility to prepare the phase into a certain potential well

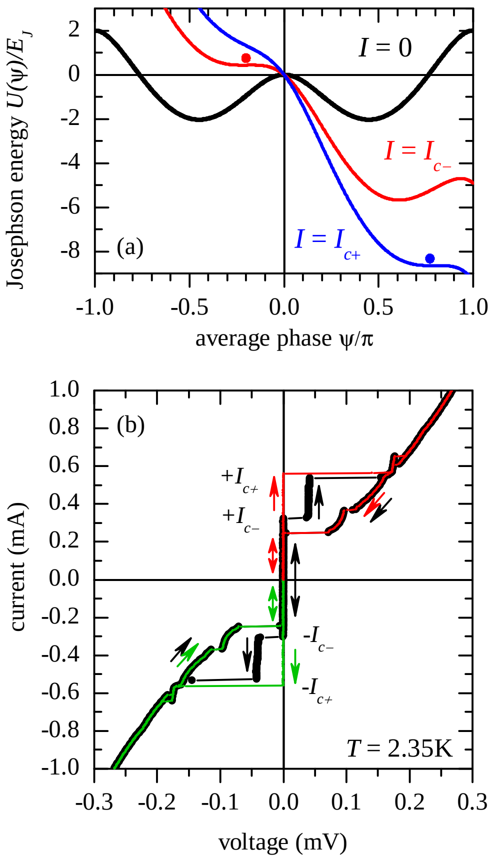

Figure 3.2: (a) Escape of the phase particle out of the potential well $-\varphi$ or $+\varphi$, tilted by the bias current I_{c-} or I_{c+}, respectively, at $h = 0$. (b)Experimentally measured current-voltage characteristics of the φ JJ at $T = 2.35\,\mathrm{K}$ and $H = 0$ with the two observed critical currents $\pm I_{c\pm}$ per bias current direction. Figure modified from the appended publication I. ©2012 by the American Physical Society.

is to apply a special bias sweep sequence. This method is based on the retrapping of the phase after an applied bias current is removed again. A certain damping is required to trap the phase in a particular

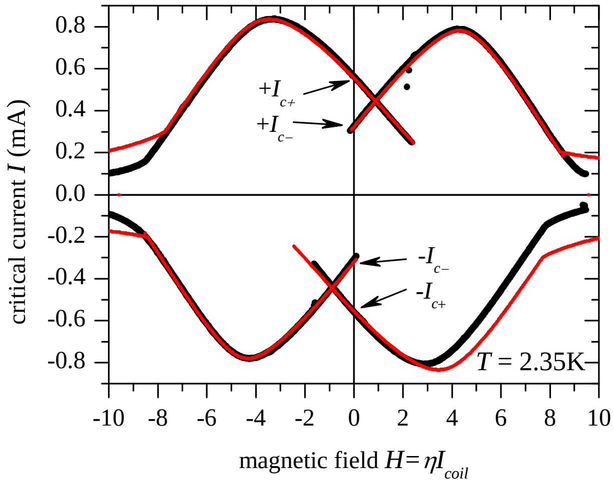

Figure 3.3: Experimentally observed $I_c(H)$ dependency (black) and numeric simulations (red). Figure modified from the appended publication I. ©2012 by the American Physical Society.

potential minimum. In the experiment this deterministic retrapping could be achieved at a temperature of $T \approx 2.35$ K.

The measured $I_c(H)$ pattern is compared with numerical simulations performed with the program STKJJ, which solves the full sine-Gordon equation. Additionally, some parameters of the simulations were extracted and used for further calculations, *e.g.*, the effective CPR.

For this φ JJ (because it lies deep in the φ state) the analytical equations of the effective CPR Eq. (2.21) and the effective Josephson potential Eq. (2.22) are not strictly applicable anymore. For this reason the CPR and the effective Josephson potential were calculated numerically. The results of numeric and analytic calculations are very similar and qualitatively the same. The numerically calculated CPR can be used to determine the value of $\varphi = 0.45\pi$ at $I = 0$ for the investigated φ JJ.

In conclusion, the first experimental evidence of a φ Josephson junction based on a 0-π SIFS JJ was demonstrated. In the $I_c(H)$ pattern the cusplike main minimum is shift off from $H = 0$. Furthermore, the existence of two critical currents $\pm I_{c\pm}$ was shown, which corresponds to the phase escape from the states $+\varphi$ and $-\varphi$. The phase can be prepared into a particular state $+\varphi$ or $-\varphi$ by (a) applying an external magnetic field, that removes the degeneracy or (b) applying a special bias current sweep sequence at a certain damping. To measure deterministic behavior a specific damping (temperature) is needed, otherwise the states $+\varphi$ and $-\varphi$ are observed randomly. The value of $\varphi = 0.45\pi$ at $I = 0$ was calculated for the investigated Josephson junction.

Own contributions to this publication

All publications were done in cooperation between our group in Tübingen and the group of H. Kohlstedt. The samples used in this publication were fabricated in the group of H. Kohlstedt. Until 2009 the group was located in FZ-Jülich and M. Weides did the sample preparation. Now, the group is situated at the Universität zu Kiel and N. Ruppelt is fabricating the samples.

My contribution was mainly in the experimental part of the work. From a very large number of samples I systematically searched and discovered the desired sample with the required properties for our measurements. On this sample I performed all the measurements at different temperatures and did the evaluation of the experimental data. Numerical simulations of the magnetic field dependence were done by myself with the software STKJJ (programmed by E. Goldobin). I wrote the first draft of the manuscript.

4 Publication II: Memory cell based on a φ Josephson Junction

Nowadays, when the prices of energy have increased and the discussion of saving energy is omnipresent, there is a big interest in energy efficient information and communication technology. Semiconductor microelectronics is still going strong, even though the processing of the digital information needs to be more efficient [LS91]. A good candidate for much lower energy dissipation are digital superconducting circuits based on Josephson junctions. For example, the Rapid Singe Flux Quantum (RSFQ) technology offers digital operations with very low power dissipation, extremely high speed and high sensitivity [Muk11]. Presently, in RSFQ technology the main challenge is to find a suitable superconducting memory, which has high capacity, is operating fast and is energy-efficient [VBB$^+$13].

In this publication we show, that a φ Josephson junction can be used as a memory cell (1 classical bit). The demonstration of this so-called "φ bit" is a proof of principle and the first step towards a memory consisting of φ Josephson junctions. The integration into a RSFQ logic is possible.

The doubly degenerate ground state of a φ JJ is utilized to store one bit of information. By applying a magnetic field one can remove the degeneracy and make the potential so asymmetric, that one of the potential wells disappears. This is how the desired state $+\varphi$ or $-\varphi$ can be written, see Fig. 4.1(a). After the writing process, one switches the magnetic field back to zero. The present state of the junctions will be stored infinitely long, as long as the Josephson junction is kept cold. For the readout of the state, the two different critical currents I_{c-} and I_{c+} are used, which correspond to the $-\varphi$ or $+\varphi$ states, Fig. 4.1(b). By applying a bias current (read out pulse) I_{RO}, which lays in between I_{c-} and I_{c+}, the state is detected. If a

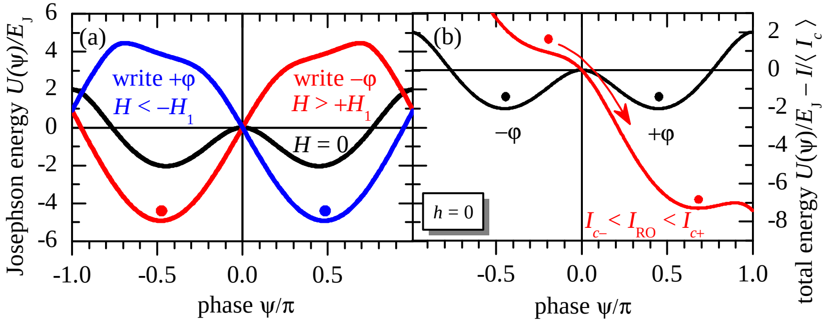

Figure 4.1: The principle of (a) writing the state by applying a magnetic field H and (b) reading the state by applying a bias current (read out pulse $I_{c-} < I_{\mathrm{RO}} < I_{c+}$). Figure modified from the appended publication II. ©2013 by the American Institute of Physics.

voltage V across the JJ can be measured, the initial state was $-\varphi$. If no voltage can be detected, the JJ was in the $+\varphi$ state.

In Fig. 4.2 the experimental operation of the φ bit is demonstrated. A sequence of the writing and reading process of different states is recorded. The series of the prepared states is $-\varphi$, $+\varphi$, $-\varphi$, $-\varphi$, $+\varphi$, $+\varphi$, $-\varphi$, $-\varphi$, $-\varphi$, $+\varphi$, $+\varphi$, $+\varphi$, see Fig. 4.2(a). In Fig. 4.2(b) the state is read out and in Fig. 4.2(c) on can see, that the φ bit works perfectly.

The advantage of this φ memory cell is, that it is easily controllable by the externally magnetic field. In the "store" state it does not dissipate any energy. Even the readout of one state ($-\varphi$) is not destructive and dissipationless. For RSFQ circuits the φ JJ is a ready-to-use flip-flop and can be integrated as on-chip memory. With regard to the speed of the operations, in principle, the characteristic frequency ω_c of the JJ is the only limiting factor (if an on-chip-coil is used). For this device, this results in $\omega_c \sim 75\,\mathrm{GHz}$, which is quite fast. Although the φ JJ used in this experiment is quite large ($L = 200\,\mu\mathrm{m}$), technologically it should be possible to realize a bit, which is only $L = 4\,\mu\mathrm{m}$ small.

In conclusion, the application of a φ JJ as a memory cell was demonstrated, which can be controlled easily by the external magnetic field and the bias current. No energy is dissipated during the information being stored.

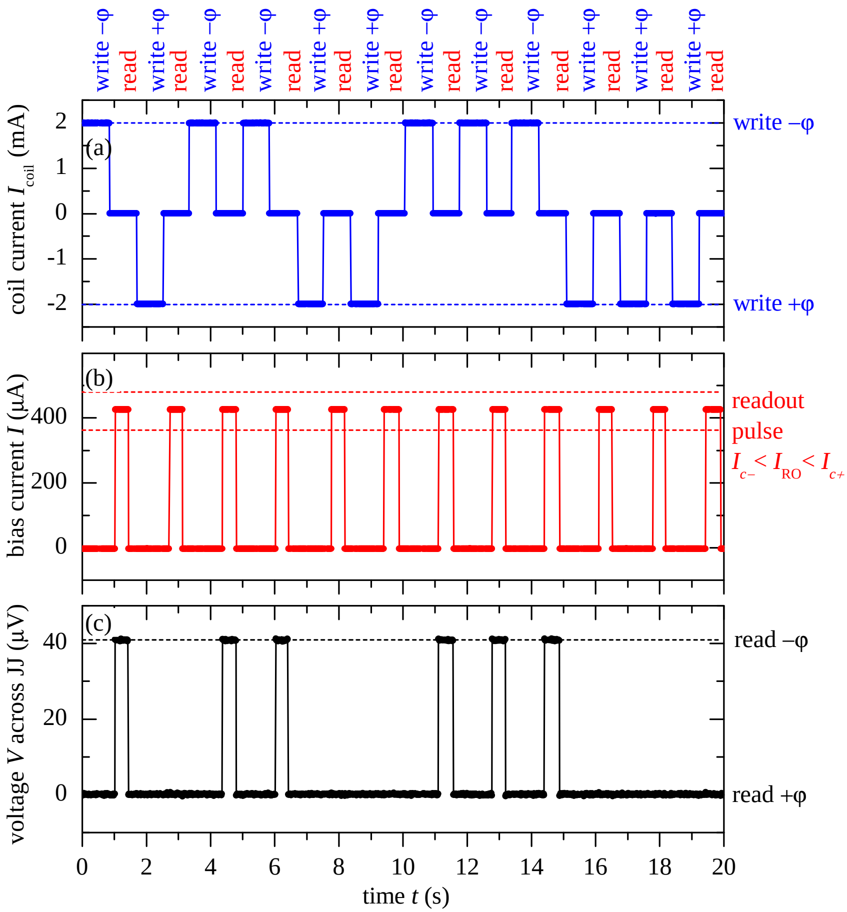

Figure 4.2: Demonstration of the operation of the φ JJ as memory cell. A sequence of (a) writing by externally applied magnetic field (b) reading out by the applied bias current and (c) detected voltage V. Figure modified from the appended publication II. © 2013 by the American Institute of Physics.

Own contributions to this publication

For this publication, I carried out all the measurements and did the data evaluation. I was involved in writing the manuscript and contributed the graphs with experimental data.

5 Manuscript in preparation I: Deterministic Ratchet based on a φ Josephson Junction

In this chapter, a summary of a manuscript in preparation is given.

H. Sickinger, M. Weides, M. Knufinke, N. Ruppelt, H. Kohlstedt, D. Koelle, R. Kleiner and E. Goldobin

Ratchets or Brownian motors attracted a lot of interest in the last decades. A so-called ratchet operation occurs, when a particle is moving in a potential and has a predominant direction even though the exerted external force vanishes in time average. A substantial requirement of a ratchet is a periodic potential with a broken reflection symmetry. Apart from answering some fundamental questions, they can be immediately employed for the extraction of work out of nonequilibrium thermal fluctuations, for rectification, or for particle separation [Lin02, HMN05, HM09]. Aside from the ratchets existing in nature [JAP97], there are plenty of artificial ratchet implementations, in particular, based on nanostructured superconductors: Abrikosov vortex ratchets [VSN+03, SN02], Josephson vortex ratchets [FMMC99, TMFO00, GSK01, Car01, CC01, CCM+02, Lee03, UCK+04, BGN+05, WZG+09, KIS+12], SQUID ratchets [WKM+00, SKK05, SKK09].

In particular, the Josephson junction (JJ) ratchets [FMMC99] have several advantages [BGN+05].

In a single point-like JJ the Josephson phase ϕ can be considered as a coordinate of a particle moving in a periodic potential (Josephson potential energy $U(\phi)$). However, the Josephson potential $U(\phi)$ in most types of JJ

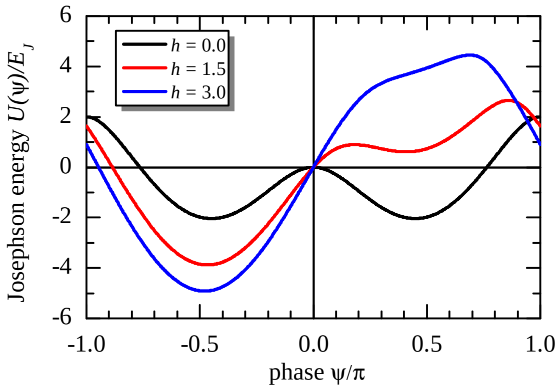

Figure 5.1: Josephson energy $U(\Psi)$ tuned by an applied magnetic field h. Note that at any h the $U(\Psi)$ profile remains 2π periodic. Figure modified from the appended publication I. ©2012 by the American Physical Society.

is reflection symmetric and its shape is hardly controllable.

Thus, during many years there was no possibility to create a simple Josephson junction ratchet. Researchers, however, were able to demonstrate more complex Josephson ratchets (with more than one JJ or with extended JJs), such as asymmetric SQUID ratchets [SKK05, SKK09] or Josephson vortex ratchets [TMFO00, CC01, BGN$^+$05, KIS$^+$12]. The physics of such devices is more complicated and they are not as reliable as the simple ratchet.

Recently, our group suggested [GKKM11] and demonstrated [SLW$^+$12] a φ JJ with a magnetic-field-tunable Josephson energy profile. For this kind of JJ, the following common behavior is observed. At zero magnetic field $h = 0$ the Josephson energy $U(\psi)$ is reflection symmetric. At $h \neq 0$ it becomes asymmetric (due to the presence of $\cos(2\psi)$ and $\sin(\psi)$ terms in Eq. 2.22). Thus, one is also able to tune its asymmetry during experiments by changing h, *e.g.*, switch it on, off, reverse its sign, etc., see Fig. 5.1. This is an extremely useful feature from a practical point of view as it allows to compare the transport with and without asymmetry and optimize the ratchet performance by tuning the asymmetry.

In this paper we demonstrate the operation of a deterministic ratchet based on a φ JJ with magnetic-field tunable asymmetry of $U(\psi)$.

The samples we used are superconductor-insulator-ferromagnet-superconductor (SIFS) 0-π Josephson junctions (JJs) as also investigated in our former experiments [SLW+12]. Such a JJ as a whole behaves [GKKM11, LMK+12] as a φ JJ with the average phase $\psi = \langle \phi(x) \rangle$, and the Josephson energy $U(\psi)$ is qualitatively similar to the one given by Eq. (2.22). In any case it is important that at bias current $I = 0$ and magnetic field $H = 0$, $U(\psi)$ is a 2π periodic double well potential with the wells at $\psi = \pm\varphi$. At $H = 0$ the wells are degenerate, while for $H \neq 0$ the degeneracy is removed [GKKM11, LMK+12, SLW+12]. We investigated two samples that show similar results. The samples we used for this experiment were also measured in earlier experiments [SLW+12].

As demonstrated in our previous works [GKKB07, SLW+12] a typical property of a φ JJ is to have two critical currents for each bias current direction at zero magnetic field $H = 0$, $|I_{c-}| < |I_{c+}|$. These two currents correspond to the escape of the phase out of the $-\varphi$ and the $+\varphi$ well of $U(\psi)$, respectively, for a positive bias current $I > 0$. In contrast to I_{c+}, the current I_{c-} can only be observed for low damping. In general the damping in SIFS JJs is strongly temperature-dependent and reduces for lower temperatures [PKK+08].

For this reason the measurements were performed in a ^{3}He cryostat at temperatures T down to 300 mK. This cryostat is equipped with a multilayer mu-metal shielding. All sample lines have been filtered electrically both at room temperature and at cryogenic temperatures. The magnetic field was applied by a coil with $H = \eta \cdot I_{\mathrm{coil}}$ with the coil factor $\eta \sim 5\,\frac{1}{\mu_0}\frac{\mu\mathrm{T}}{\mathrm{mA}}$. For temperatures well below 3.5 K the damping was low enough and both critical currents I_{c+} and I_{c-} were observed.

We performed our measurements at two different temperatures to investigate the regime of high damping (overdamped φ JJ) at $T = 3.5\,\mathrm{K}$ and of low damping (underdamped φ JJ) at $T = 2.35\,\mathrm{K}$. The current-voltage-characteristics (IVCs) are shown in Fig. 5.2.

The first step which appears on the IVC at $V \approx 39\,\mu\mathrm{V}$ in Fig. 5.2(a) for $T = 3.5\,\mathrm{K}$ is a half-integer zero field step (hi-ZFS). It can be treated as a Fiske step induced by the phase jump at the 0-π-boundary of the SIFS JJ [NSAN06, NSAN10]. The asymptotic voltage of this step is given by $V_{1/2}^{\mathrm{ZFS}} = V_1^{\mathrm{FS}} \propto 1/2L$. The same step can be identified in Fig. 5.2(b) for $T = 2.35\,\mathrm{K}$ with the voltage $V \approx 42\,\mu\mathrm{V}$ and the height of the step is $\pm I_{\mathrm{step}}$.

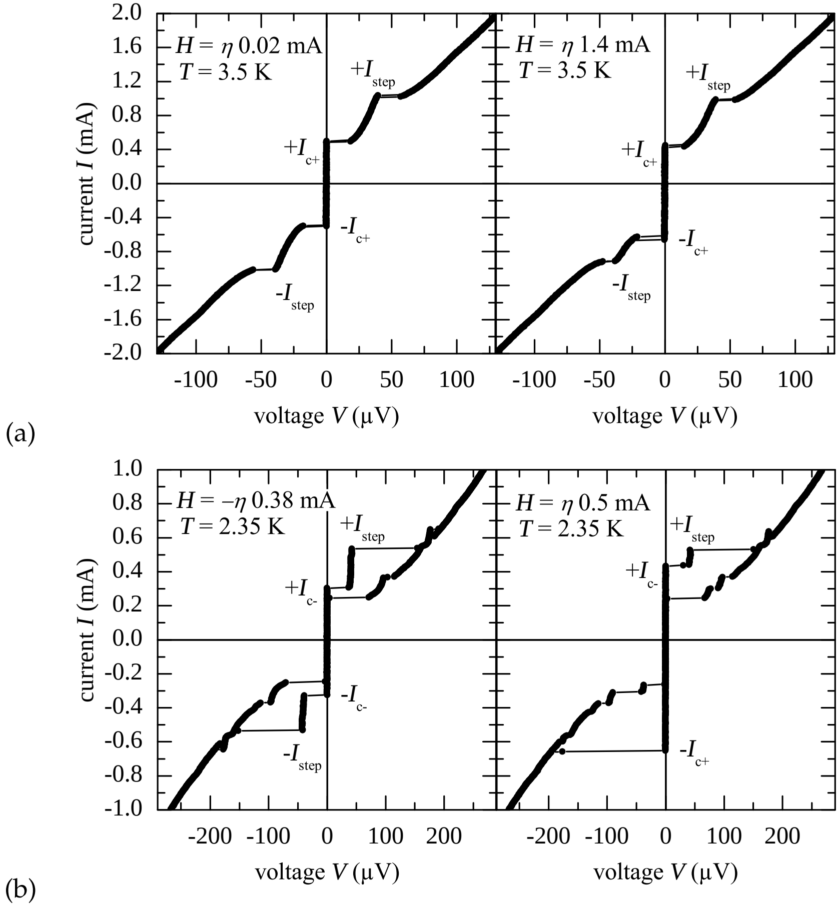

Figure 5.2: IVCs of the φ JJ in the (a) overdamped regime at $T = 3.5\,\mathrm{K}$ and (b) underdamped regime at $T = 2.35\,\mathrm{K}$ at different magnetic fields H.

The dependence of the critical current I_c on the externally applied magnetic field H is shown in Fig. 5.3. For $I_c(H)$ at $T = 2.35\,\mathrm{K}$ one can see, that we observed two critical currents I_{c-} and I_{c+} in zero-field.

By applying a magnetic field one can change the asymmetry between

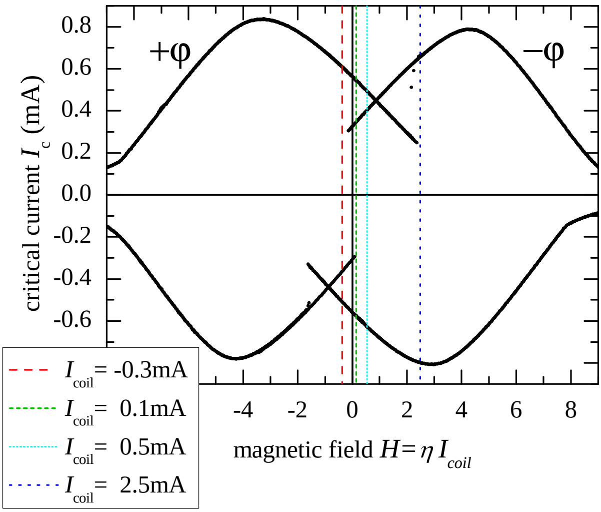

Figure 5.3: $I_c(H)$ at $T = 2.35\,\text{K}$. Different vertical lines (I_{coil}) correspond to different measured rectification curves in Fig. 5.4.

the wells of the Josephson potential energy $U(\psi)$ and create an asymmetric periodic potential required for a ratchet operation, *cf.*, Eq. (2.22). For large magnetic fields one of the wells can even disappear, *cf.* Fig. 5.1. Correspondingly, one of the critical currents disappears too.

At $T = 3.5\,\text{K}$ the damping is so high, that only I_{c+} was observed. $I_c(H)$ is not shown, but looks similar to the one shown in Fig. 5.3. However, the $I_c(H)$ pattern does not show a crossing near $H = 0$, *i.e.*, only one I_c is observed for each H.

In our experiment we measure the rectification curves $\overline{V}(I_{\text{ac}})$, *i.e.*, the average voltage *vs.* the amplitude of applied ac bias current. For this we apply a periodic bias current $I(t) = I_{\text{ac}} \sin(2\pi f t)$ with the frequency $f = 100\,\text{Hz}$ and the update rate of 100000 points/second (period $T = 10\,\text{ms}$) and we measure the voltage N times with the sampling rate $100N$ samples/second. At $T = 2.35\,\text{K}$ the value is $N = 100$, while at $T = 3.5\,\text{K}$ this results in $N = 1000$ measurements. In both cases the total sampling time is 10 ms. The collected samples are averaged to obtain $\overline{V}$ at

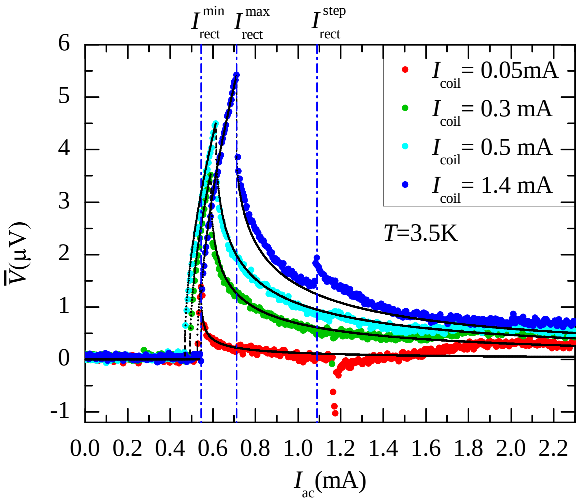

Figure 5.4: Rectification curve at $T = 3.5\,\mathrm{K}$ for different externally applied magnetic fields.

given I_{ac}. In Fig. 5.4 and Fig. 5.5 $\overline{V}(I_{\text{ac}})$ is shown for different values of the magnetic field in the range of $I_{\text{coil}} = -3.5\ldots 2.5\,\text{mA}$ at $T = 2.35\,\text{K}$ and $I_{\text{coil}} = -1.5\ldots 1.6\,\text{mA}$ at $T = 3.5\,\text{K}$, *i.e.*, for different asymmetries of the potential $U(\psi)$. Rectification was also observed for negative values of the field ($I_{\text{coil}} < 0$) and the results look very similar but with negative $\overline{V}(I_{\text{ac}})$ ($\overline{V} < 0$: at $T = 2.35\,\text{K}$ for $I_{\text{coil}} = -3.5\ldots - 0.5\,\text{mA}$ and at $T = 3.5\,\text{K}$ for $I_{\text{coil}} = -1.5\ldots - 0.05\,\text{mA}$).

For the overdamped case the rectification curves are shown in Fig. 5.4. For small I_{ac} the current $I(t)$ is so small, that it exceeds neither I_{c+} nor $-I_{c+}$, so that $\overline{V} = 0$. Note that due to the high damping only $\pm I_{c+}$ is visible on the IVC, see Fig. 5.2(a). If I_{ac} becomes larger, *i.e.*, $I_{c+} < I_{\text{ac}} < I_{\text{step}}$ ($I_{\text{rect}}^{\text{min}} < I_{ac} < I_{\text{rect}}^{\text{step}}$), the voltage becomes $\overline{V} \neq 0$, because for $I(t) > +I_{c+}$ the JJ jumps to the step. Again, due to the high damping the IVC does not show a hysteresis and, as a result, $\overline{V}(I_{\text{ac}})$ increases smoothly starting from zero.

In general, for zero magnetic fields $|H|$ the potential $U(\psi)$ is symmetric,

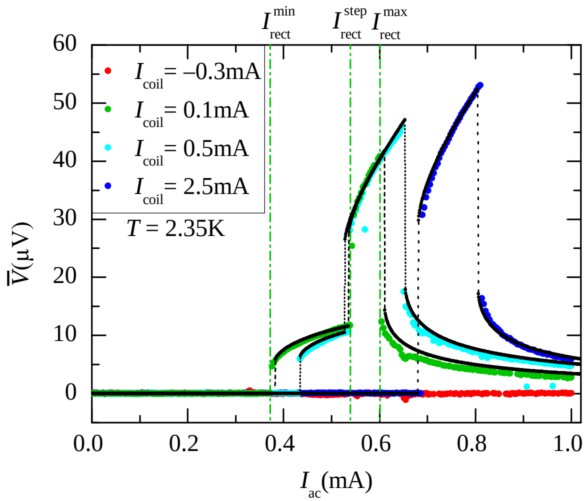

Figure 5.5: Rectification curve at $T = 2.35\,\mathrm{K}$ for different externally applied magnetic fields.

so that the IVC is symmetric, too, [see Fig. 5.2 (a) for $I_{\text{coil}} = 0.02\,\mathrm{mA}$], and rectification is absent (not shown in Fig. 5.4). For a small coil current $I_{\text{coil}} = 0.05\,\mathrm{mA}$, the rectification appears, see Fig. 5.4. The kink at $I_{\text{ac}} = 1.2\,\mathrm{mA}$ can be explained by a small asymmetry between the amplitudes of the steps in the positive ($+I_{\text{step}}$) and negative ($-I_{\text{step}}$) directions.

The higher the magnetic field, the larger is the rectification window, see Fig. 5.4. The height of the step on the IVC changes with H. In our φ JJ the heights of the steps $+I_{\text{step}}$ and $-I_{\text{step}}$ are different, which increases the asymmetry of the IVC (*cf.* Fig. 5.4 for $H = \eta 1.4\,\mathrm{mA}$). This leads to an additional rectification and is responsible for the kink (at $I_{\text{rect}}^{\text{step}}$) on the $\overline{V}(I_{ac})$ curve.

The rectification curves for the underdamped case are shown in Fig. 5.5. For a small applied current amplitude $I_{\text{ac}} < |\pm I_{c\pm}|$, no rectification can be observed and $\overline{V} = 0$. For a larger driving current amplitude $I_{c-} < I_{\text{ac}} < I_{\text{step}}$ ($I_{\text{rect}}^{\text{min}} < I_{ac} < I_{\text{rect}}^{\text{step}}$), the voltage $\overline{V}$ becomes positive, because the JJ jumps to the step. Due to the hysteresis on the IVC, see

Fig. 5.2(b), the voltage $\overline{V}$ jumps to a finite value in the beginning of the rectification region. Note, that near $H = 0$ both I_{c+} and I_{c-} are non-zero. Our sweep is such that we go alternatingly to the negative and positive direction so that upon return from the voltage state we always trap the phase in the state revealing $\pm I_{c-}$. For lower temperatures $T < 2.35\,\mathrm{mK}$ the phase can be retrapped in different minima of $U(\psi)$ or even show random behavior [GKKM13].

For a even larger ac current amplitude $I_{\mathrm{step}} < I_{\mathrm{ac}} < |-I_{c+}|$ ($I_{\mathrm{rect}}^{\max} > I_{ac} > I_{\mathrm{rect}}^{\mathrm{step}}$), the JJ reaches the resistive branch. For that reason there is yet another region with higher $\overline{V}$ on the $\overline{V}(I_{\mathrm{ac}})$ curve. For $I_{\mathrm{ac}} > |-I_{c+}|$ ($I_{\mathrm{ac}} > I_{\mathrm{rect}}^{\max}$) the voltage $\overline{V}$ decreases because the junction also picks up some negative voltage during the negative semiperiod.

For small magnetic fields, no rectification was observed, because the IVC is symmetric, see Fig. 5.5 for the IV at $I_{\mathrm{coil}} = -0.3\,\mathrm{mA}$. For higher magnetic fields ($H \geq \eta 0.07\,\mathrm{mA}$), $-I_{c-}$ does not appear, which results in an asymmetric IVC [see Fig. 5.2(b) for $H = \eta 0.5\,\mathrm{mA}$] and rectification. The higher the magnetic field, the bigger is the rectification window at higher voltage (related to the resistive branch) and the smaller is the rectification window at lower voltage (related to the step on the IVC). This tendency proceeds up to $H \approx \eta 1.2\,\mathrm{mA}$. For larger H the step on the IVC disappears and the first rectification region disappears too, see Fig. 5.5 for $I_{\mathrm{coil}} = 2.5\,\mathrm{mA}$.

For comparison with the measured rectification curves Fig. 5.4 and Fig. 5.5 also show analytically calculated curves as black lines for each value of the magnetic field. The model used for the calculations is a combination of the two hysteretic models – the constant voltage step model and the linear branch model [Gol13]. Those are, in fact, extensions of the nonhysteretic model [KIS+12] to the case with hysteresis.

This model is a first step to verify our experimental data theoretically. For a definite theoretical proof of this problem, more calculations based on the double-sine-Gordon equation need to be done. The implementation of this into our software and the calculations will be done in the near future.

Although there were many theoretical studies on ratchets where the particle moves in an asymmetric periodic potential, the practical implementation, especially in Josephson junction systems was missing, mainly, because the Josephson energy in conventional junctions is reflection symmetric. Here we have demonstrated that in φ Josephson junctions this symmetry is broken and one can obtain rectification and directed transport of the phase. The advantage of this system is that the asymmetry is tunable by a magnetic field H, so that one can clearly see a (dis)appearance of rectification as a function of H.

Own contributions to this manuscript

For this work, I carried out all the experiment and analyzed the data. I wrote the first draft of the manuscript.

6 Manuscript in preparation II: Retrapping of the Phase in a φ Josephson Junction

In this chapter, a summary of a manuscript in preparation is given.

H. Sickinger, M. Weides, N. Ruppelt, H. Kohlstedt, D. Koelle, R. Kleiner and E. Goldobin

The "butterfly effect" is a term of the chaos theory and used for complex nonlinear dynamical systems, that are extremely sensitive to the initial conditions. Only a small change (perturbation) of the initial conditions can result in a totally different final state. In long Josephson junctions the butterfly effect was studied numerically by Yugay *et al.* [YY06].

In this publication we consider a φ Josephson junction which has a Josephson energy profile equal to a 2π-periodic double-well potential [Min98, BK03, GKKM11, SLW$^+$12]. The ground state is doubly degenerate and has the phase $\psi = +\varphi$ or $\psi = -\varphi$. The application of a bias current to the junction corresponds to a tilt of the potential. If the tilt of the potential is large enough, the phase is not trapped in one of the two potential wells anymore and slides down the potential. This means for the Josephson junction, that it switches from the zero-voltage state to the state of finite voltage.

Now the question arises, what happens when the bias current is decreased? In which of the two wells $+\varphi$ or $-\varphi$ will the phase be retrapped?

In our earlier work [SLW$^+$12] we reported on the retrapping of the phase in a particular well by using a special bias sweep sequence. We showed, that when the junction is returning from the positive-voltage state to the zero-voltage state, the phase is always trapped in the $+\varphi$ state. Inversely, when the junction is returning from the state of negative voltage, the phase is retrapped in the $-\varphi$ state. We only observed this deterministic behavior at a certain temperature ($T = 2.35\,\mathrm{K}$), which corresponds to a certain and rather large damping. For lower temperatures $T < 2.35\,\mathrm{K}$, the damping is also lower and the destination well seemed to be random (at $T = 300\,\mathrm{mK}$) [SLW$^+$12] .

In this work we studied experimentally the temperature dependent retrapping process of the phase of a φ Josephson junction.

We focus on the switching of the Josephson junction from the voltage-state to the zero-voltage state. As long as no bias current ($I = 0$) is applied to the junction, the phase is trapped in one of the two potential wells, either in the $-\varphi$ well or in the $+\varphi$ well. By increasing the bias current, the potential is tilted and as soon as the corresponding potential well disappears, the phase is sliding down the potential. Thus, the junction is in the voltage-state. The required tilt for the escape of the phase out of the potential well depends on the initial state of the junction $-\varphi$ or $+\varphi$. For positive bias currents $I > 0$, the phase escape from the $-\varphi$ state occurs at a lower critical current I_{c-}. The phase escape of the $+\varphi$ well arises at a higher I_{c+}, see Fig. 6.1. We proofed the existence of the two critical currents $\pm I_{c\pm}$ per bias current direction in our former works [GKKM11, SLW$^+$12]. The possibility to use $I_{c\pm}$ for the read out of the state was demonstrated by Goldobin *et al.* [GKKM13].

Now, we are quasistatically decreasing the bias current and accordingly decreasing the tilt of the potential. The current at which the phase is retrapped in one of the potential wells is called the retrapping current I_r. This current depends on the damping (temperature) of the Josephson junction.

The measurement method we used is equivalent to the method we utilized to control the destination well in our previous work [SLW$^+$12]. For example, if the junction is in the positive-voltage state, the phase is sliding to the right. Then, the tilt is decreased and for a rather high damping (at $T = 2.35\,\mathrm{K}$) the phase is trapped in the right well ($+\varphi$ well). If we then read out the state by applying a bias current, we detect $+I_{c+}$ for the positive bias current direction $I > 0$ (tilt to the right) or $-I_{c-}$ for the negative bias current direction $I < 0$ (tilt to the left). And vice versa, if the junction was in the negative-voltage state, the phase will be retrapped in the $-\varphi$ state.

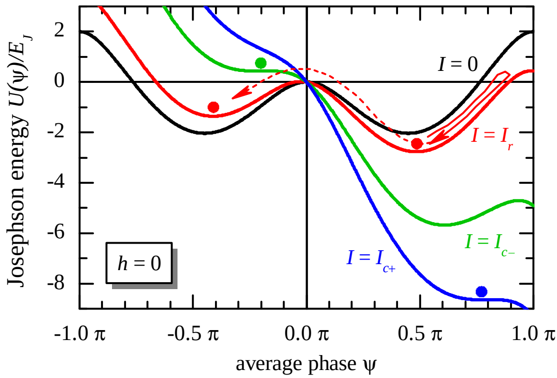

Figure 6.1: Josephson potential for the ground state $I = 0$, the retrapping current $I = I_r$, the escape of the phase from the $-\varphi$ well $I = I_{c-}$ and the escape from the $+\varphi$ well $I = I_{c+}$ (at magnetic field $h = 0$). The red solid line shows the retrapping of the phase in the $+\varphi$ state, when the tilt of the potential is decreased. The red dashed line shows, how the phase is not stopped in the right potential well, but passing the small potential maximum and being retrapped in the left $-\varphi$ state. Figure modified from the appended publication I. ©2012 by the American Physical Society.

If we decrease the damping by decreasing the temperature $T < 2.35\,\mathrm{K}$, then the retrapping process is not as simple as considered before. Let us consider a slightly lower damping ($T < 2.35\,\mathrm{K}$) and again the case of positive voltage. In this case, upon reducing I to the retrapping current I_r, the phase is not stopped in the right well but climbing up the right wall of the potential. Then it is drawn back by passing the smaller potential maximum and being retrapped in the left $-\varphi$ well, see Fig. 6.1. By decreasing the damping further, the phase particle first is passing the right well, then passing the left well and finally being trapped in the right well.

Theoretically the retrapping of the phase in a pointlike φ Josephson junction is discussed by Goldobin *et al.* [GKKM13]. They predict, that the destination well of the retrapping changes from $+\varphi$ to $-\varphi$ and back (and so on), if the damping of the system is decreased. For very low damping the initial conditions strongly influence the destination well. Only tiny fluctuations, *e.g.*, thermal or electronic noise, can change the destination well. Thus, one observes the butterfly effect.

The sample we used in our experiment is the same as already investigated in our previous work [SLW+12]. It is a φ Josephson junction based on a 0-π Josephson junction with a steplike ferromagnetic interlayer (SIFS JJ). The damping of SIFS Josephson junctions strongly depends on the temperature T.

We recorded the data by measuring the current-voltage-curve (IVC) several times ($N = 1000$) and used two special sweep sequences. The first sequence is starting at zero voltage (and zero current) and the current is swept to positive (negative) values, resulting in a positive (negative) voltage drop over the junction. Then the current is swept back to zero and the voltage jumps to zero, see Fig. 6.2. This sweep sequence is called "sequence 1 pos" for the positive bias current direction and "sequence 1 neg" for the negative bias current direction. The second sweep sequence is always starting from the finite-voltage state at positive (negative) currents. The current is swept to the negative (positive) direction up to resistive branch with negative (positive) voltage and back point-by-point. This sweep sequence we call "sequence 2", see Fig. 6.2.

In principle, sweep "sequence 1 pos" ("sequence 1 neg") and the negative (positive) part of "sequence 2" result in the retrapping of the phase in the same potential well. But the following escape of the phase takes place into the two different bias current directions. Thus, in the experiment, we have the possibility to record either $+I_{c+}$ ($-I_{c+}$) by using "sequence 1 pos" ("sequence 1 neg") or $-I_{c-}$ ($+I_{c-}$) by using "sequence 2" ("sequence 2") to observe the retrapping in the state $+\varphi$ ($-\varphi$).

In Fig. 6.3 we present experimental data of the retrapping of the phase in a certain potential well depending on the damping. Every data point gives the probability of the retrapping in a certain state. We utilized both sweep sequences in our experiment and the data is shown in Fig. 6.3 (a) and (b). Figure 6.3 (a) shows the probability to measure the $+\varphi$ (right) state by using the "sequence 1 neg" and detecting $-I_{c-}$. Additionally, the probability to detect the $+\varphi$ state $-I_{c+}$ by using "sequence 2" is shown. For high temperatures $T \gtrsim 2.3\,\mathrm{K}$ (high damping) the probability to detect the phase in the $+\varphi$ well for both sweep sequences is zero. The phase is always retrapped in the $-\varphi$ well. This behavior coincides with the deterministic behavior we reported earlier [SLW+12]. Correspondingly, in Fig. 6.3 (b) the probability for the retrapping in the $-\varphi$ state is shown by using "sequence 2" and measuring $-I_{c+}$.

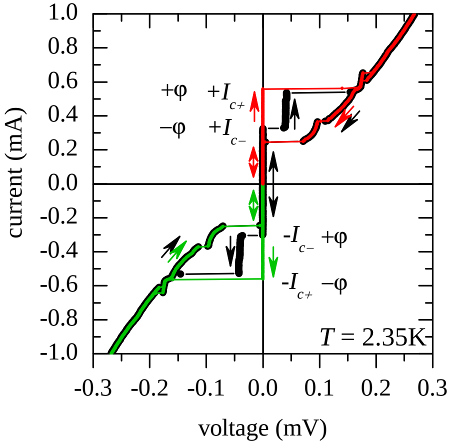

Figure 6.2: Experimentally measured current-voltage characteristics at $T = 2.35\,\mathrm{K}$ with the two observed critical currents $\pm I_{c\pm}$ per bias current direction. Red symbols and arrows show the "sequence 1 pos" and blue symbols correspond to "sequence 1 neg". The black curve and arrows show the sweep "sequence 2". Figure modified from the appended publication I. ©2012 by the American Physical Society.

For decreasing damping one would expect that the phase will be trapped alternatingly in one well and then in the other with a probability of 100% as predicted by Goldobin *et al.* [GKKM13]. However, due to thermal and electronic noise the probability distribution of the destination well is smeared.

In our experiment, for decreasing temperature, three peaks are visible in Fig. 6.3 (a) at $T = 1.5\,\mathrm{K}$, at $T = 1.97\,\mathrm{K}$ and at $T = 2.08\,\mathrm{K}$. One can see, that the two different sweep sequences coincide quite well. In Fig. 6.3 (b) two peaks can be observed as well, located at $T = 2.04\,\mathrm{K}$ and at $T = 2.22\,\mathrm{K}$. The peaks show an increase of the probability and give rise to the assumption that the phase is retrapped more often in the $+\varphi$ ($-\varphi$) well, as in the $-\varphi$ ($+\varphi$) well for Fig. 6.3 (a) (for Fig. 6.3 (b)). The measurements of the retrapping of the phase in the $+\varphi$ well (Fig. 6.3 (a)), in principle, should coincide with the retrapping in the $-\varphi$ well (Fig. 6.3 (b)) and the peaks are expected to be at the same values of the temperature

T. In our experiment, the peaks are shifted off a little bit from each other. A possible reason for this shift is, that the two potential wells of our φ JJ are not perfectly symmetric.

For a further decrease of the temperature $T \lesssim 1.4\,\mathrm{K}$ a plateau at $\sim 30\%$ is visible. It is not possible to predict the destination well and the critical currents $I_{c\pm}$ are measured in random order. Here the junction exhibits the butterfly effect as predicted theoretically by Goldobin *et al.* [GKKM13]. It is not clear yet, why the saturation value is not 50%, as expected. An asymmetry of the potential well due to an inhomogeneous magnetization is a possible explanation of this deviation.

In conclusion, we have experimentally investigated the retrapping process of a φ Josephson junction depending on the damping. We recorded the switching events of the junction depending on the temperature and evaluated the probability to find the phase trapped in a certain state. We observed an enhancement of the retrapping events at two different temperatures for two different sweep sequences. For very low temperatures we measured random behavior which can be explained by the butterfly effect.

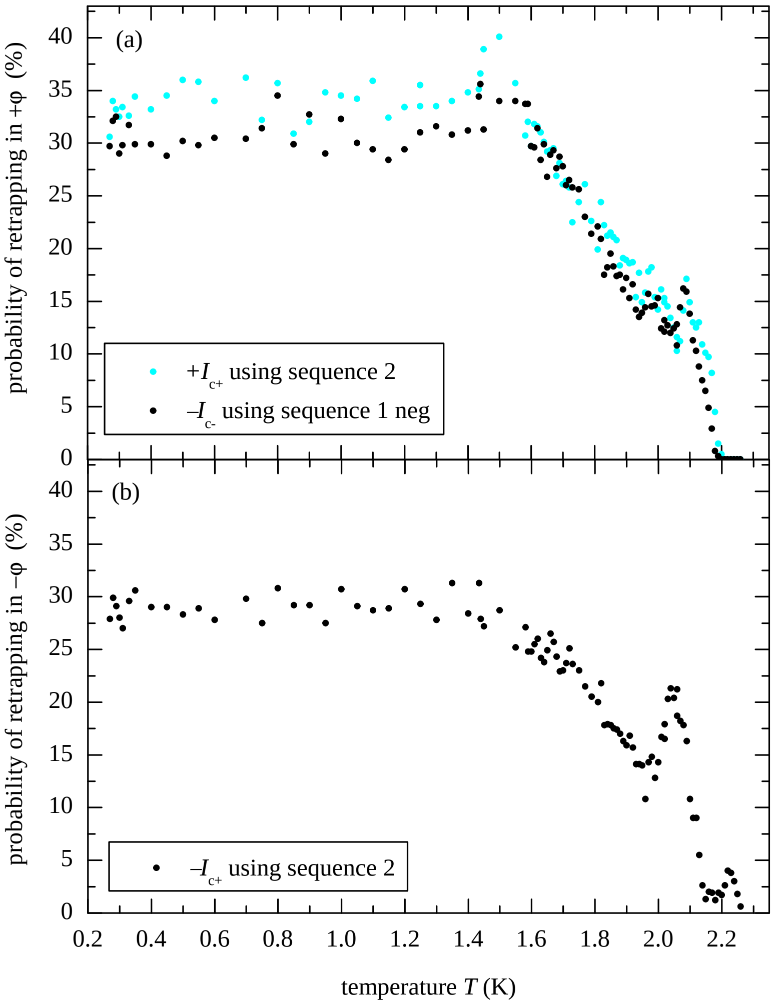

Figure 6.3: Probability (in percent) of the retrapping of the phase in a certain well, depending on the temperature. (a) shows the retrapping in the $+\varphi$ (right) well, while (b) shows the retrapping in the $-\varphi$ (left) state. The data is recorded by using two different sweep methods.

Own contributions to this manuscript

For this work, I carried out the experiment and evaluated the data. I wrote the first draft of the manuscript.

7 Summary and Outlook

In the framework of this thesis experimental studies of φ Josephson junctions (JJs) based on 0-π junctions with a ferromagnetic interlayer are presented.

Josephson junctions with a ground state phase of $\phi = 0$ or $\phi = \pi$ can be fabricated by using junctions with a ferromagnetic interlayer (SIFS JJs). By combining a 0 JJ and a π JJ, a φ junction can be realized which has an arbitrary ground state phase $\psi = \pm\varphi$ in the interval $0 < \varphi < \pi$. Thus, a φ JJ is a generalization of a π JJ. The value of φ can be chosen by design parameters. The Josephson potential of a φ JJ is a 2π-periodic double-well potential and can be tuned by an external magnetic field.

The motivation of this work was to demonstrate such a φ Josephson junction experimentally and to investigate the physical properties and possible applications.

In the first part of this work, the experimental evidence of a φ JJ is presented. It is demonstrated that a φ JJ has a doubly degenerate ground state $+\varphi$ and $-\varphi$. These two ground states can be detected (read out) by measuring the critical current I_{c+} or I_{c-}. It is shown that one can tune the potential by applying a magnetic field, which can be utilized to prepare the state. Furthermore, the state can be prepared by using a special sweep sequence at a certain temperature (damping). The value of φ can be extracted from the shift of the $I_c(H)$-pattern. The experiments agree with previous theoretical predictions.

In the second part, the application of a φ JJ as memory cell (classical bit) is discussed. One can make use of the above mentioned properties of reading out the state by an applied bias current and writing the state by an applied magnetic field. To store the information (state), no external power supply is needed, only the sample needs to be kept in the superconducting state. This "φ bit" is easy to control and operating very fast. Due to that, it is possible to integrate φ bits into rapid single flux quantum (RSFQ) logic.

In the third part, the experimental operation of a φ Josephson junction as deterministic ratchet is demonstrated. In conventional Josephson junctions the Josephson potential is reflection symmetric and due to that no ratchet operation is possible. In a φ JJ the Josephson potential can be made asymmetric by the applied magnetic field. Particularly favorable is, that during the experiment the rectification can be "switched" on and off and different asymmetries can be achieved by different magnetic field values. Additionally, by varying the temperature, the damping of the system can be changed. In this work rectification curves for the overdamped and the underdamped regime are shown, respectively for different asymmetries of the potential (magnetic field values).

In the fourth and final part of this thesis the process of retrapping of the phase of a φ Josephson junction is studied experimentally. The switching events of the junction from the state of finite voltage to the zero-voltage state, depending on the temperature (damping) are recorded. The probability to find the phase trapped in the $+\varphi$ state is measured and an oscillating behavior for a decrease of the damping ($T = 2.3...1.5\,\mathrm{K}$) is found. Consequently, the phase is trapped in the $-\varphi$ and the $+\varphi$ state alternatingly. For very low temperatures ($T \lesssim 1.5\,\mathrm{K}$) the probability to find the phase trapped in a certain state is nearly constant. The damping is so low, that even very small fluctuations (perturbations), *e.g.*, thermal or electronic noise can change the final state of the system. It is not possible to predict in which well the phase will be trapped. This behavior can be explained by the butterfly effect.

There are many other promising techniques to realize φ JJs. Gumann *et al.* [GS09] theoretically show, that with a constriction-type Josephson junction based on a *d*-wave superconductor thin film with an appropriate geometry a φ JJ may be realized. Another theoretical proposal for the realization of a φ Josephson junction was made by Pugach *et al.* [PGKK10]. They propose to realize a φ junction with an intrinsically nonsinusoidial current-phase relation based on a SFS JJ with a clean ferromagnet and alternating 0 and π segments (multifacet JJ). Furthermore a theoretical proposal was submitted by Heim *et al.* [HPK+13]. They propose the realization of a φ JJ based on a planar ferromagnetic structure. The advantage of this geometry is that there is no need of etching a step into the ferromagnetic interlayer (as for 0-π SIFS JJs), which makes the fabrication easier. Another possibility to realize a φ Josephson junction based on a 0-π ramp junction made of a *s*-wave and a *d*-wave superconductor, was investigated in our group [Tur13]. The benefit of this technology is, that the lengths of the two segments can be adjusted by using focused ion beam and thus the value of φ can be tuned.

φ Josephson junctions can not only be used in the classical regime as shown in this thesis. It is also possible to utilize φ JJs, *e.g.*, as quantum mechanical two-level systems. In the near future, it is planed to carry out experiments with φ Josephson junctions in the quantum mechanical regime in our group. For this purpose we prepared a special sample design, where conventional superconductor-insulator-superconductor JJs are equipped with extra current injectors [Ust02, GGS$^+$05]. This Josephson junctions with current injectors can be utilized to create φ JJs with a drastically higher critical current density compared to the φ JJs fabricated with SIFS-technology. This increase in current density leads to an enhancement of the crossover temperature to the quantum regime. This makes the quantum regime experimentally accessible for a φ JJ. Our group aims to investigate macroscopic quantum effects in this φ JJs, *e.g.*, manipulation and readout of the states.

8 Acronyms and Symbols

Acronym	Description
ac	Alternating current
CPR	Current-phase relation
hi-ZFS	Half-integer zero field step
IVC	Current-voltage curve
JJ	Josephson junction
Qubit	Quantum bit
RCSJ	Resistively and capacitively shunted junction
RSFQ	Rapid Single Flux Quantum
SIFS	Superconductor-Insulator-Ferromagnet-Superconductor
SIS	Superconductor-Insulator-Superconductor

Symbol	Physical quantity
$e = 1.602 \cdot 10^{-19}$ C	Elementary charge
$\Phi_0 = 2.07 \cdot 10^{-15}$ Tm^2	Magnetic flux quantum
$\hbar = 1.055 \cdot 10^{-34}$ Js	Reduced Planck's constant
$k_\mathrm{B} = 1.381 \cdot 10^{-23}$ J/K	Boltzmann constant
$\mu_0 = 4\pi \cdot 10^{-}$ Vs/Am	Vacuum permeability
$\mu_\mathrm{B} = 9.274 \cdot 10^{-24}$ J/T	Bohr magneton

Table 8.1: Acronyms and physical quantities in alphabetic order.

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9 Appended publications

Publication I

Experimental Evidence of a φ Josephson Junction

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We demonstrate experimentally the existence of Josephson junctions having a doubly degenerate ground state with an average Josephson phase $\psi = \pm\varphi$. The value of φ can be chosen by design in the interval $0 < \varphi < \pi$. The junctions used in our experiments are fabricated as 0-π Josephson junctions of moderate normalized length with asymmetric 0 and π regions. We show that (a) these φ Josephson junctions have two critical currents, corresponding to the escape of the phase ψ from $-\varphi$ and $+\varphi$ states, (b) the phase ψ can be set to a particular state by tuning an external magnetic field, or (c) by using a proper bias current sweep sequence. The experimental observations are in agreement with previous theoretical predictions.

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Josephson junctions (JJs) with a phase shift of π in the ground state [1] have attracted a lot of interest in recent years [2–7]. In particular, these JJs can be used as on-chip phase batteries for biasing various classical [8] and quantum [9] circuits, allowing for removing external bias lines and reducing decoherence. Currently, it is possible to fabricate simultaneously both 0 and π JJs using various technologies such as superconductor-ferromagnet heterostructures [10–14] or JJs based on d-wave superconductors [15–18].

It would be remarkable to have a phase battery providing an arbitrary phase shift φ, rather than just 0 or π. The simplest idea is to combine 0 and π JJs to obtain a φ JJ. However, this is not as straightforward as it may seem. The balance between 0 and π parts is complicated as shown in the pioneering work [19], where conditions for having a nontrivial φ-state were derived. Long artificial [20,21] and natural [22–24] arrays of ...-0-π-0-π-... JJs with short segments were analyzed in detail and suggested as systems, where a φ JJ could be realized. More recently, only one period of such an array, i.e., one 0-π JJ, was analyzed in an external magnetic field [25]. In these works, the Josephson phase $\phi(x)$ is considered as a sum of a constant (or slowly varying) phase ψ and a deviation $|\xi(x)| \ll 1$ from the average phase. Then, for the average phase ψ one obtains an effective current-phase relation (CPR) for the supercurrent [25]

$$I_s = \langle I_c\rangle\left[\sin(\psi) + \frac{\Gamma_0}{2}\sin(2\psi) + \Gamma_h h\cos(\psi)\right], \quad (1)$$

where the averaged value of the critical current $\langle I_c\rangle = \langle j_c\rangle Lw$. The CPR (1) exactly corresponds to a φ JJ at zero normalized magnetic field $h = 0$, if $\Gamma_0 < -1$, cf. Fig. 1(a). Here $L = L_0 + L_\pi$ is the total length of the JJ, while L_0 and L_π are the lengths of 0 and π parts, accordingly, w is the width of the JJ.

It is worth noting that the term "φ JJ," introduced in Ref. [20], refers to a JJ with a degenerate ground state phase $\psi = \pm\varphi$. In the particular case of Eq. (1) at $h = 0$, one has $\varphi = \arccos(-1/\Gamma_0)$. The coefficients Γ_0 and Γ_h are defined as [26]

$$\Gamma_0 = -\frac{l_0^2 l_\pi^2}{3}\frac{(j_{c,0} - j_{c,\pi})^2}{(j_{c,0}l_0 + j_{c,\pi}l_\pi)^2}, \quad (2a)$$

$$\Gamma_h = \frac{l_0 l_\pi}{2}\frac{j_{c,0} - j_{c,\pi}}{j_{c,0}l_0 + j_{c,\pi}l_\pi}, \quad (2b)$$

where l_0, l_π are the lengths normalized to the Josephson length $\lambda_J(\langle j_c\rangle)$ and $j_{c,0}$, $j_{c,\pi}$ are the critical current densities of 0 and π parts, respectively. Here λ_J is calculated using the average value of the critical current $\langle j_c\rangle = (L_0 j_{c,0} + L_\pi j_{c,\pi})/(L_0 + L_\pi)$.

The physics of φ JJs with a CPR given by Eq. (1) is quite unusual [27]. In particular, one should observe two critical currents [25,27] at $h = 0$, corresponding to the escape of the phase from the left ($-\varphi$) or the right ($+\varphi$) well of the double-well Josephson energy potential

$$U(\psi) = \langle E_J\rangle\left[1 - \cos(\psi) + \Gamma_h h\sin(\psi) + \frac{\Gamma_0}{2}\sin^2(\psi)\right], \quad (3)$$

where $\langle E_J\rangle = \Phi_0\langle I_c\rangle/(2\pi)$. The critical currents are different because the maximum slope [maximum supercurrent in Fig. 1(a)] on the right-hand side (positive bias) of the $-\varphi$ well is smaller than the maximum slope [maximum supercurrent in Fig. 1(a)] on the right-hand side of the $+\varphi$ well, see Fig. 1(b).

In this Letter we present experimental evidences of a φ JJ made of one 0 and one π segment, see Fig. 2(a).

The samples were fabricated as Nb|Al − Al_2O_3|$Ni_{0.6}Cu_{0.4}$|Nb heterostructures [11,28]. These superconductor-insulator-ferromagnet-superconductor (SIFS) JJs have an overlap geometry as shown in Fig. 2(b). Each junction consists of two parts, a conventional 0 segment and a π segment. It is well known [4,5,29,30] that the critical current in SFS or SIFS JJs strongly depends on the thickness d_F of the F layer and can become negative within

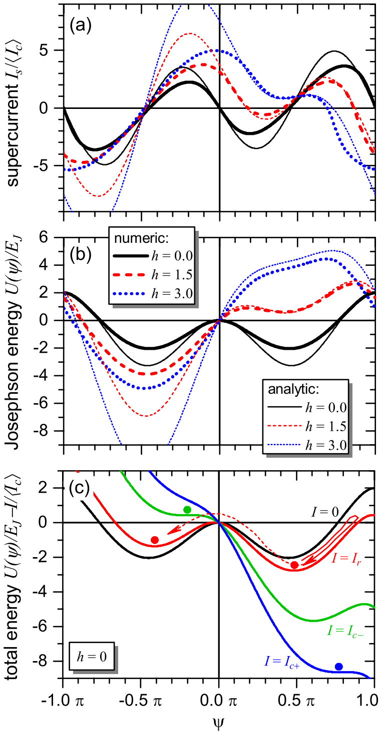

FIG. 1 (color online). (a) Effective CPR $j_s(\psi)$ and (b) effective Josephson energy $U(\psi)$ calculated numerically (thick lines) in comparison to those given by approximate analytical formulas (1) and (3) (thin lines). For $h < 0$ the $U(\psi)$ curves look mirror reflected with respect to the U axis, i.e., $U(\psi, h) = U(-\psi, -h)$. (c) Numerically calculated effective Josephson energy $U(\psi)$ at $h = 0$ tilted by an applied bias current $I > 0$ [energy supplied by the current source $-\Phi_0 I\psi/(2\pi)$]. Several important situations are shown: ground state $I = 0$ [same curve as in (b)], $I = I_r$ (retrapping), $I = I_{c-}$ (escape from $-\varphi$ well), and $I = I_{c+}$ (escape from $+\varphi$ well).

some range of d_F (π junction). Therefore, to produce the 0 and the π segments, the F layer has different thicknesses $d_{F,0}$ and $d_{F,\pi}$, as shown in Fig. 2(a). To achieve this, the F layer of thickness $d_{F,\pi}$, corresponding to $j_{c,\pi} \equiv j_c(d_{F,\pi}) < 0$ (π JJ) was fabricated first. Then the area indicated in Fig. 2(b) was etched down to $d_{F,0}$, corresponding to $j_{c,0} \equiv j_c(d_{F,0}) > 0$ (0 JJ). Usually one obtains asymmetric j_c values [31], i.e., $j_{c,0} \neq |j_{c,\pi}|$.

We have studied 3 samples. One of them has the very little j_c asymmetry of about 5%, and $L_0 = L_\pi = 25$ μm, which corresponds to $L_0/\lambda_J(j_{c,0}) \approx L_\pi/\lambda_J(j_{c,\pi}) \approx 0.37$. In this case even 5% of asymmetry brings the sample out of the φ domain, see Fig. 3 for a qualitative picture. The other two JJs have $L_0 = L_\pi = 100$ μm and are deep inside the φ domain in parameter space. Both samples show similar results. Here we present the results obtained on one of them. Its parameters are summarized in Table I and its position within the φ domain is indicated in Fig. 3. The values of $j_{c,0}$ and $j_{c,\pi}$ cannot be measured directly. In our case we have measured the $I_c(H)$ dependence (see below) and then simulated it numerically using $j_{c,0}$ and $j_{c,\pi}$ as fitting parameters. The best fitting was obtained for the values specified in Table I.

According to theoretical predictions [25,27] for a φ JJ at zero magnetic field $H = 0$ one expects two critical currents $|I_{c-}| < |I_{c+}|$ (for each bias direction), corresponding to the escape of the phase from $-\varphi$ and $+\varphi$ wells of $U(\psi)$, respectively, for bias current $I > 0$ and vice versa for $I < 0$. The current I_{c+} is always observed. To observe I_{c-} one has to have low damping so that retrapping in

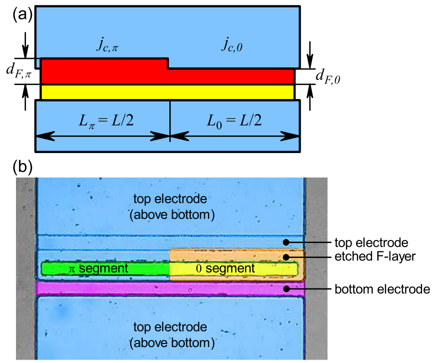

FIG. 2 (color online). (a) Sketch (cross section) of the investigated SIFS 0-π JJ with a step in the F-layer thickness. (b) Optical image (top view, colored manually) of an investigated sample having overlap geometry. The junction area (0 segment and π segment) is 200×10 μm^2. The "etched F-layer" area shows where the thickness of the F layer was reduced from $d_{F,\pi}$ to $d_{F,0}$ to produce a 0-π JJ.

TABLE I. Junction parameters at $T = 2.35$ K. The values of $j_{c,0}$, $j_{c,\pi}$ and the normalized length $L/\lambda_J(\langle j_c(x)\rangle)$ are obtained from the fits.

Parameter	0 part	π part	Whole JJ
Physical length	100 μm	100 μm	200 μm
$j_{c,0}$, $j_{c,\pi}$, $\langle j_c(x)\rangle$ (A/cm^2)	+62.9	−47.9	+7.5
Normalized lengths	0.68	0.68	1.36

the $+\varphi$ well is avoided (for positive I). The I-V characteristic (IVC) of the investigated 0-π JJ at $H = 0$ measured at $T \sim 4.2$ K shows only one critical current. Therefore the experiments were performed in a ^{3}He cryostat at T down to 300 mK where the damping in SIFS JJs reduces drastically [14]. In the temperature range from 3.5 K down to 300 mK both I_{c+} and I_{c-} are clearly visible in the IVCs as shown in Fig. 4.

Earlier [27] we proposed a technique that allows us to choose which critical current one traces in the IVC, i.e., from which well the phase escapes. The control is done by choosing a proper bias sweep sequence. For example, if the junction is returning from the positive voltage state, the potential $U(\psi)$ is tilted so that the phase slides to the right. When the tilt becomes small enough, then the phase will be trapped, presumably in the right $+\varphi$ well, cf. Fig. 1(c). However, this natural assumption is not always true (see below). Then, if the phase is trapped at $+\varphi$, we can sweep the bias (a) in the positive direction and will observe escape from $+\varphi$ (to the right) at I_{c+} or (b) in the negative direction to observe escape from $+\varphi$ (to the left) at $-I_{c-}$.

In experiment, at $T \lesssim 2.3$ K, when the damping is very low, the currents $\pm I_{c+}$ and $\pm I_{c-}$ are traced in random order. Recording one IVC after the other, we were able to obtain IVCs with all 4 possible combinations: (a) $(-I_{c-}, +I_{c-})$, (b) $(-I_{c-}, +I_{c+})$, (c) $(-I_{c+}, +I_{c-})$, (d) $(-I_{c+}, +I_{c+})$. Choosing a specific sweep sequence as described above does not make the outcome (I_{c-} or I_{c+}) predictable. We believe that in this temperature range the damping is so low that, upon returning from the positive voltage state, the phase does not simply stop in the $+\varphi$ well, but can also reflect from the barrier and find itself in a $-\varphi$ well, cf. Fig. 1(c). The absence of determinism suggests that most probably we are dealing with a system exhibiting chaotic dynamics. This issue will be investigated elsewhere. Nevertheless, at $T \approx 2.35$ K we managed to achieve deterministic behavior as described above, see Fig. 4.

Another fingerprint of a φ JJ is its $I_c(H)$ dependence, which (a) should have the main cusplike minima shifted off from $H = 0$ point symmetrically with respect to the origin, see Fig. 4 of Ref. [25] or Fig. 5 discussed later, and (b) should show up to four branches in total (for both sweep polarities) at low magnetic field [25]. In essence, the latter feature results from the escape of the phase from two different energy minima in two different directions and is an extension of the two-critical-currents story to the case of nonzero magnetic field. Instead, the feature (a) alone cannot serve as a proof of a φ JJ as it is a common feature of every asymmetric 0-π JJ even if its ground state is 0 or π [25].

The experimentally obtained $\pm I_c(H)$ dependence at $T = 2.35$ K is shown in Fig. 5. First, in the whole temperature range 0.3–4.2 K we observe the main minima shifted

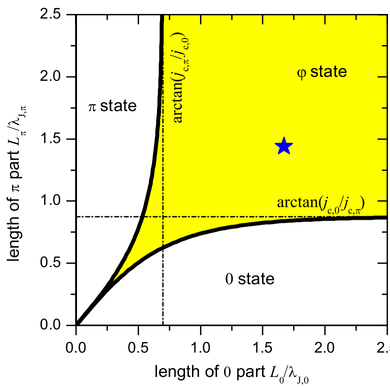

FIG. 3 (color online). Domain of existence of φ state. The ⋆ shows the position of the investigated JJ at $T = 2.35$ K.

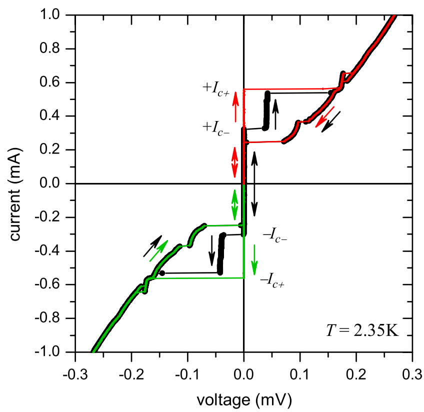

FIG. 4 (color online). Current-voltage characteristics showing lower $\pm I_{c-}$ and higher $\pm I_{c+}$ critical currents measured at $T \approx 2.35$ K. At this temperature the behavior is deterministic: if one sweeps I as $-I_{\max} \to +I_{\max} \to -I_{\max}$ one always observes the critical currents $\pm I_{c-}$; if one sweeps I as $+I_{\max} \to 0 \to +I_{\max}$, one always observes $+I_{c+}$; finally, if one sweeps I as $-I_{\max} \to 0 \to -I_{\max}$, one always observes $-I_{c+}$.

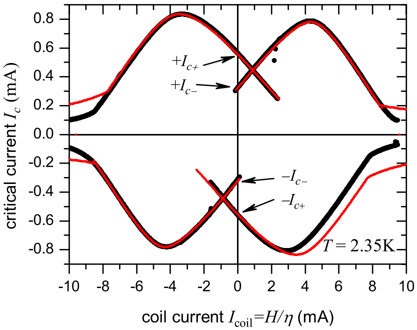

FIG. 5 (color online). Experimentally measured dependence $I_c(H)$ (black symbols) and numerically calculated curve (red [gray] smaller symbols) that provides the best fit.

point symmetrically to a finite field, which is $H_{min} = \eta I_{coil} = \eta \times (\pm 879\ \mu A)$. Second, at $T \lesssim 3.5$ K one observes the crossing of two branches and two critical currents for each bias current polarity. The left-hand branches correspond to the escape of the phase out of the $+\varphi$ well, while the right branch corresponds to the escape from the $-\varphi$ well. Note, that at $T = 4.2$ K the crossing of branches is not visible. One just observes a cusplike minimum. In this case, both $I_{c\pm}$ cannot be seen together and the existence of two states, as should be present for a φ-JJ, cannot be proven.

To trace the intersection of the branches better we applied a special value H_{reset} of the magnetic field during the reset from the voltage $V > 0$ state back to $V = 0$ state, to "prepare" a specific state ($+\varphi$ or $-\varphi$) of the system. Then the field was set back to the "current" value H and the bias was ramped up to trace $I_c(H)$. By doing this for different values of H_{reset}, one is able to trace the intersection of the branches much better than without this technique, see Fig. 5. Still, in experiment we were not able to trace the branches for positive and negative I_c up to the point where they meet probably because of retrapping.

To extract some parameters of our JJ from the $I_c(H)$ curve, we performed numerical simulations, by solving the full sine-Gordon equation for a 0-π JJ, using the normalized length l and the critical currents $j_{c,0}$ and $j_{c,\pi}$ as fitting parameters. The simulations were performed using STKJJ [32]. The objective was to obtain the best fit close to the origin, especially the point where the branches of $I_c(H)$ cross. The best fit is shown in Fig. 5 and was obtained for $l = 3.7$ and the values $j_{c,0}$ and $j_{c,\pi}$ from Table I. Measured data in Fig. 5 are shifted along I_{coil} axis by 530 μA to compensate for average remanent magnetization of the F layer. To obtain an almost perfect fit we have also assumed a difference in the constant remanent magnetizations in the 0 and in the π parts of $\Delta\langle M\rangle = \eta 1.5$ mA, similar to earlier studies [28,31]. One can also see that one of the experimental branches after the main maximum runs parallel to the simulated curve [the experimental $I_c(H)$ is not point symmetric]. This shift stays the same even if we cycle the sample through T_c of Nb, but changes if we cycle through the Curie temperature T_C of the F layer. We conclude that this is related to the nonuniform remanent magnetization of the F layer.

The parameters obtained from the fit allow us to see the location of our JJ on the (l_0, l_π) plane, see Fig. 3. One sees that the JJ is not really short and lays quite deep in the φ domain. In this region the analytic results [25,26] and, in particular, Eqs. (1) and (3) are valid only qualitatively. Therefore we calculated the CPR of our JJ numerically. We started with a 0-π JJ with parameters l_0, l_π, $j_{c,0}$ and $j_{c,\pi}$ obtained from the fit, assumed some applied bias current I and found *all* static solutions $\phi(x)$ numerically. Then for each of these solutions we calculated $\psi \equiv \langle\phi(x)\rangle$ and plotted all those ψ on a $\psi(I)$ plot. By repeating this procedure for different I, we obtain an effective CPR $\psi(I)$ shown in Fig. 1(a) together with the curve produced by the analytical formula (1). One can see that the exact effective CPR calculated numerically and the approximate CPR calculated analytically are qualitatively similar. From the numerical CPR the value of $\varphi = 0.45\pi$ for our particular φ JJ is extracted.

In conclusion, we have demonstrated experimentally the realization of a φ Josephson junction based on a 0-π SIFS junction. We have observed experimentally a shift of the $I_c(H)$ minimum according to the effective CPR (1), as predicted [25], as well as two critical currents $\pm I_{c\pm}$ close to zero magnetic field for each bias current polarity. We showed that one can choose between the $\pm\varphi$ states of the system using an externally applied magnetic field, which removes their degeneracy. We also showed that one can bring the system into one of the two states $\pm\varphi$ by properly tilting the potential using the bias current. Depending on the damping (temperature) one can achieve deterministic behavior (when damping is small enough to see the lower critical current, but large enough to trap the phase in a particular well) as well as random behavior at very low damping. The obtained JJ has $\varphi = 0.45\pi$ at $I = 0$.

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Publication II

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Memory cell based on a φ Josephson junction

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The φ Josephson junction has a doubly degenerate ground state with the Josephson phases $\pm\varphi$. We demonstrate the use of such a φ Josephson junction as a memory cell (classical bit), where writing is done by applying a magnetic field and reading by applying a bias current. In the "store" state, the junction does not require any bias or magnetic field, but just needs to stay cooled for permanent storage of the logical bit. Straightforward integration with rapid single flux quantum logic is possible. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4811752]

Superconducting digital circuits, in particular, based on Josephson junctions (JJs), potentially offer much lower energy dissipation per logical operation than the existing semiconducting devices.[1] However, any practical computer as well as many digital devices need compatible superconducting random access memory (RAM). The largest superconducting RAM was demonstrated more than a decade ago and had a capacity of 256 kwords (1 word $\equiv$ 16 bits).[2] Its operation frequency of $\sim$1 GHz is already similar to the one of modern computers, and a huge feed current of about 2 A was necessary for operation.

Since the beginning of this century, the JJ technology progressed significantly. For example, it became possible to obtain JJs with a phase difference of π in the ground state[3–7]—the π junctions. φ JJs[8] consisting of 0 and π parts connected together were proposed[9] and demonstrated experimentally[10] recently. Such a JJ has a doubly degenerate ground state with the Josephson phase $\psi = \pm\varphi$, where the value of $0 < \varphi < \pi$ depends on design parameters. This results from the peculiar Josephson energy profile $U_J(\psi)$, which has a form of a 2π periodic double well potential. One can use these two intrinsic states of the φ JJ to store one bit of information.

In this paper, we demonstrate the operation of a memory cell (1 bit) based on a φ JJ—the so-called "φ bit," which is the first step towards a φ Josephson memory.

For the demonstration of the memory cell, we have used the same samples as in Ref. 10. Namely, the sample is a Nb|Al-AlO$_x$|Ni$_{0.6}$Cu$_{0.4}$|Nb multilayered heterostructure, with a step-like thickness of the ferromagnetic layer to obtain a 0-π JJ of moderate length. The sample fabrication is published elsewhere.[11,12] For this work, we have chosen a 0-π JJ with an in-plane length $L = 200\,\mu$m and the width $W = 10\,\mu$m. By treating the structure as a whole (like a black box with two electrodes), one deals with a φ JJ.[9,10,13]

The current-voltage characteristic $V(I)$ as well as the dependence of the critical current I_c on applied in-plane magnetic field H are shown in Fig. 1. Already here one can see that we observe two different critical currents, I_{c-} or I_{c+}, depending on the initial state of the JJ, $\psi = -\varphi$ or $\psi = +\varphi$, as demonstrated in the original work.[10] By applying a magnetic field, one can remove this degeneracy and observe only one branch on both $V(I)$ (not shown) and on $I_c(H)$. A point-symmetric $I_c(H)$ with shifted cross-like minimum around zero field, as in Fig. 1(b), is the signature of a φ JJ.

For writing the state, we use the fact that an applied magnetic field changes the asymmetry between the two wells in the Josephson energy $U(\psi)$ up to the point that one of the wells disappears,[9,10] see Fig. 2(a). Hence one can set (write in) the desired state: $-\varphi$ for $H > H_1$ and $+\varphi$ for $H < -H_1$, where $\pm H_1$ is the field, at which one minimum in $U(\psi)$ disappears. After the state is written, one can return the magnetic field back to zero, and the state of the junction will be stored for infinitely long time provided the energy barrier is considerably larger than the energy of thermal or quantum fluctuations.

For reading out the state, we use the fact that the critical current of the φ JJ is equal to I_{c+} or I_{c-} depending on the initial state of the JJ, $+\varphi$ or $-\varphi$, see Fig. 2(b) as well as $V(I)$ characteristics shown in Fig. 1(a). In our case, $I_{c+} \approx 480\,\mu$A and $I_{c-} \approx 370\,\mu$A. Therefore, for the readout, we apply a bias current $I_{RO} = 426\,\mu$A shown in Fig. 1(a), which lays in between I_{c+} and I_{c-}. Then, if the φ JJ switches to a voltage state, its critical current was $I_{c-} < I_{RO}$, i.e., the JJ was in the $-\varphi$ state. If, on the other hand, after the application of I_{RO} the JJ stays in a zero voltage state, then its critical current is $I_{c+} > I_{RO}$, i.e., its is in the $+\varphi$ state, see Fig. 2(b).

Note, that this readout technique is destructive, at least when we detect the $-\varphi$ state. After switching to a finite-voltage state, resetting the readout bias current back to zero, the phase may be retrapped either in the $-\varphi$ or in the $+\varphi$ state, depending on the JJ parameters such as on the damping. This retrapping of the phase is investigated in detail elsewhere.[14]

In Figs. 2(c)–2(e), we demonstrate an experimentally recorded sequence of writing and reading $\pm\varphi$ states in different order. We start from an unknown state by applying a

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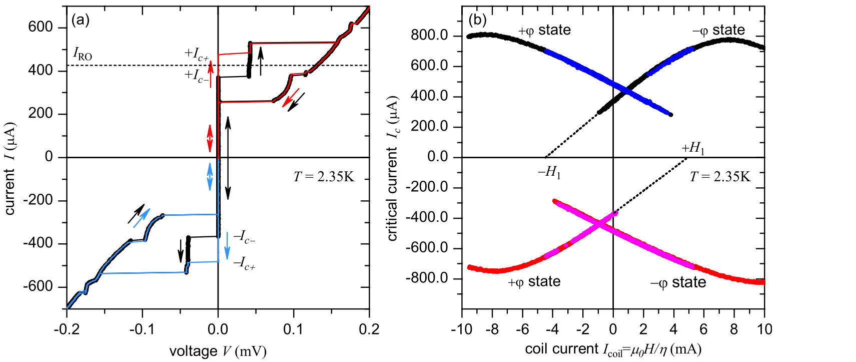

FIG. 1. (a) $V(I)$ characteristic at zero field and (b) $I_c(H)$ curve of the investigated φ JJ measured at $T = 2.35$ K (overlap of several measurements with different initial conditions to trace out different branches).

positive magnetic field pulse, see Fig. 2(c), which removes the $+\varphi$ well, see Fig. 2(a), so that the phase relaxes into the $-\varphi$ well. After such a "writing" operation, the magnetic field is reset to zero, Fig. 2(c). Then, we apply a readout pulse, see Fig. 2(d) and observe a non-zero voltage across the junction, see Fig. 2(e), indicating a readout of the $-\varphi$ state. During the next write-read cycle, we apply the negative magnetic field $H < 0$, see Fig. 2(c), to change the asymmetry in the opposite direction, see Fig. 2(a), and write the $+\varphi$ state. After application of the readout pulse, see Fig. 2(d), we detect no voltage across the junction, see Fig. 2(e), indicating a $+\varphi$ state indeed. Further, the write-read cycles are repeated for the states $-\varphi, -\varphi, +\varphi, +\varphi, -\varphi, -\varphi, -\varphi, +\varphi, +\varphi, +\varphi$. One can see that the φ bit performs perfectly, reproducing the written sequence. Different longer sequences were played out as well without problems.

We have also investigated the margins of operation of our φ bit. The readout current I_{RO} should be well in between I_{c-} and I_{c+}, i.e., $|I_{RO} - I_{c\pm}| \gg I_{noise}$—the background noise (electronic or thermal) in the bias current circuitry. In our case, $I_{noise} \sim 3\mu$A, so we have very large margins. The margins in terms of the magnetic field were determined experimentally. We have started seeing faults in operation when the writing magnetic field was decreased down to $|\mu_0 H_{write}^{min}| \approx 1.9\,\text{mA} \times \eta$ or increased up to $|\mu_0 H_{write}^{max}| \approx 2.1\,\text{mA} \times \eta$, where $\eta \sim 5\,\mu\text{T/mA}$ is the coil factor.

We have demonstrated the use of a φ Josephson junction as a memory cell (classical bit). The advantage of such a memory cell is that it is easily controllable by an applied magnetic field. The destructive readout can be performed as described here. However in real circuits, e.g., Rapid Single Flux Quantum (RSFQ) logic, one can construct novel cells that will perform readout with an output in a form of a single RSFQ pulse (or no pulse). In this case, the temperature limitation $T < 2.4$ K necessary in the presented readout scheme to avoid the retrapping of the phase in a $+\varphi$ well upon escape from the $-\varphi$ well, see Fig. 2(b), is not relevant. By increasing the characteristic voltage V_c, one reduces the damping and facilitates operation at $T = 4.2$ K. This can be done, e.g., by using a superconductor with the larger energy gap such as NbN. Besides this, λ_L for NbN is larger than for Nb, i.e., λ_J will be smaller. In fact, if used in RSFQ circuits, a φ JJ is a ready-to-use flip-flop.

In terms of the speed of the operation, the measurements presented in Fig. 2(c)–2(e) are performed on a 10 ms scale (delay time between setting the field/current and reading the voltage). In our case, the field is supplied by a coil, which is very slow and limits the writing time by $\sim$5 ms. Applying the field by an on-chip control line will substantially accelerate the writing cycle. The most effective can be a control line passing through the upper or lower electrode of the JJ. Note also that the writing time is not limited by the remagnetization (domain flipping or rotation) of the ferromagnetic layer, like in some alternative superconducting memory proposals,[15] but rather by a characteristic frequency ω_c (damping) of the φ JJ. For the device presented here, $V_c \sim 150\,\mu$V, i.e., $\omega_c/2\pi \sim 75$ GHz. In addition, we have a zero-field step at the voltage $V_1 \approx 50\,\mu\text{V} \propto 1/L$, which appears only sometimes. If one takes shorter JJs, the step will appear at much larger voltages or will disappear, i.e., become irrelevant for $V_1 > V_c$.

In terms of miniaturization, the φ JJ used in this proof-of-principle experiment is rather large ($L = 200\,\mu$m). However, already for existing technology, one can easily reduce its length down to 40–50 μm. Further decrease of the size becomes difficult because the margins for the realization of a φ junction become very narrow. To realize a φ JJ on the basis of a 0-π JJ, one has to keep a certain relation between the lengths and critical currents of the 0 and π parts.[9,13,16,17] If these parts have a normalized length $\sim\lambda_J$ and larger, then these requirements are quite relaxed, i.e., even 30% of asymmetry still results in a φ JJ. On the other hand, if the length of each part is shorter than $\sim\lambda_J$ then these requirements are rather strict (asymmetry should be controlled on a level better than 3%) and difficult to meet technologically. The way out of this problem is the further development of Superconductor-Insulator-Ferromagnet-Superconductor (SIFS) 0-π junction technology with the aim to increase $j_{c,\pi}$ from the present value of 40 A/cm^2 up to, say 4 kA/cm^2 (2 orders of magnitude). This can be done either by using a clean F layer or by using a

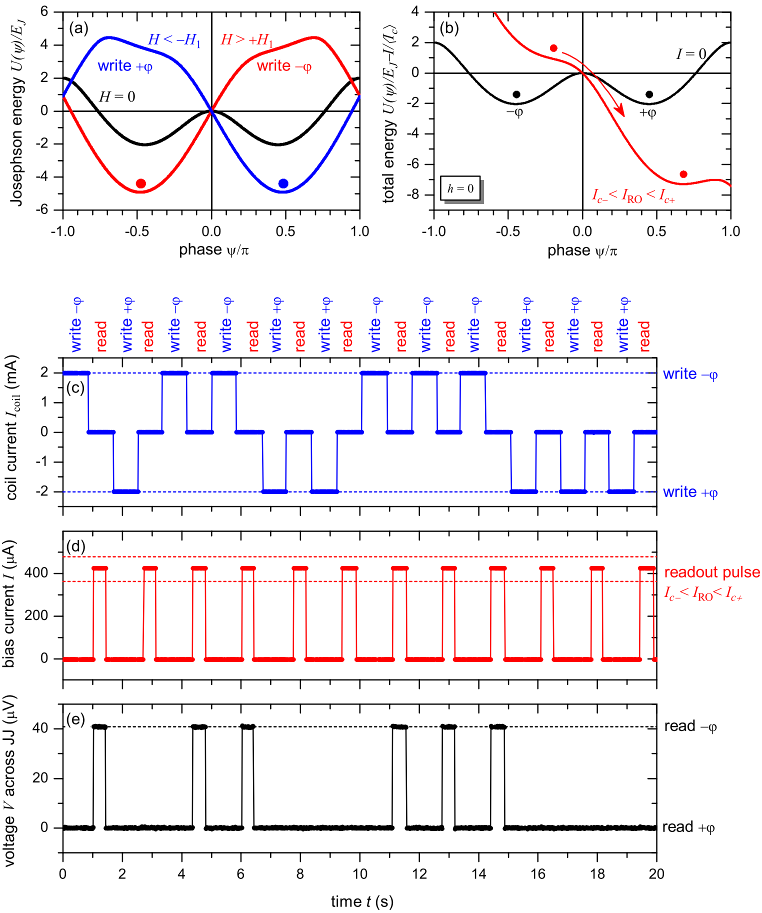

FIG. 2. (a) The principle of writing the state $+\varphi$ or $-\varphi$ by applying a magnetic field. (b) The principle of reading the state by applying the bias current $I_{c-} < I_{RO} < I_{c+}$. (c)–(e) The sequence of the external magnetic field (c), readout by the bias current (d) and the measured voltage (e), demonstrating the operation of the φ JJ as a memory cell. The speed of operation is limited by our dc setup (coil) and electronics.

SISIF structure.[18] In this case, $\lambda_J \propto 1/\sqrt{j_c}$ will become 10 times smaller, so that one can have a 4 μm large bit. Another possibility to reduce λ_J is by increasing the specific inductance of the JJ electrodes

$$\mu_0 d' \approx \lambda_L \coth\left(\frac{d_1}{\lambda_L}\right) + \lambda_L \coth\left(\frac{d_2}{\lambda_L}\right). \quad (1)$$

This can be done (a) by increasing λ_L, e.g., by using NbN electrodes; or (b) by decreasing the thicknesses d_1 and d_2 of the superconducting electrodes.

Alternative φ JJ technologies,[19–21] especially those providing intrinsic φ JJs[22] may, at the end, lead to an extremely high-density memory.

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