Optimization of Sustainable Groundwater Management in a Rapidly Developing Urban Setting

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Abstract

The growing population of the city of Lucknow, India has resulted in an increased water demand which is met by enormous groundwater withdrawal, with the consequences of fast depletion of water resources. The current study aims at implementing a sustainable pumping rate (SPR) concept to calculate groundwater extractions at different wells across the city and the pre-assessment of any artificial groundwater recharge scheme to be implemented in the city.

The SPR concept used in previous studies are used in different ways without actually addressing the principal components of water budget changes, due to groundwater extraction. In this study the groundwater extraction rates were calculated by maximizing the total extraction at all wells, subject to constraints with regards to socially and environmentally relevant measures which were similar or same while accounting for the water budget terms. The constraints were varied to find different groundwater extractions, by developing a groundwater model and coupling it with an optimization code. The analysis of resulting different groundwater extractions illustrates the wells to be preferred and avoided for groundwater extraction. An analysis of the sum of the groundwater extraction rates and corresponding values of constraints showed that the higher values of either of the constraints have resulted in the same amount of total groundwater extraction, and such groundwater extraction should be avoided by the water manager. The analysis also showed that on increasing the value of either of the constraint, the resulting groundwater extraction usually favored extraction at well locations spatially distant from the particular constraint. Hence, this study aims at helping a water manager to estimate groundwater extractions, based on water demand at locations which are spatially close to either of the constraints and in providing the deficit amount of water that needs to be supplied in parts of the city, where the application of a SPR concept has resulted in a smaller amount of groundwater extraction.

The artificial groundwater recharge systems are installed as a solution to the water problems in order to augment the groundwater reservoir for later use during the dry period. The artificial groundwater recharge should aim at the social and economic benefits of the investments

involved, keeping in mind other space and geological constraints while applying such systems. Hence, the pre-assessment of the applications of such systems can be advantageous in order to achieve these objectives. The groundwater model was used in the second part of this thesis, which gives a pre-assessment of the socio-economic benefits, along with a planned artificial groundwater recharge system. The social and economic benefits of the artificial groundwater recharge system are compared with a scenario where no artificial groundwater recharge system was implemented by optimizing the objective functions associated. The social and economic benefits of the artificial groundwater recharge systems were shown using the unit groundwater extraction prices. Also, the amount of risk associated with the application of such systems can be reduced if the investments to such systems are made carefully. Although the total costs of planned and unplanned artificial groundwater recharge systems were almost the same, it was found out that a planned artificial groundwater recharge system lowers the investment costs implying less risks associated.

Zusammenfassung

Die stetig steigende Bevölkerungszahl der Metropole Lucknow, Indien führt zu einem gleichfalls steigenden Bedarf an Trinkwasser. Diesem Bedarf wurde mit einer Erhöhung der Grundwasserentnahme entgegengetreten, der jedoch zu einer schnellen Reduzierung der Wasserreserven führt. Die vorliegende Arbeit setzt sich zum Einen zum Ziel ein Konzept anzuwenden, das eine Pumprate berechnet, die eine nachhaltige Grundwasserentnahme an verschiedenen Brunnen im Stadtgebiet gewährleistet (SPR-Konzept). Des Weiteren soll eine frühzeitige Bewertung des Einflusses jeglicher künstlicher Grundwassererneuerung im Gebiet von Lucknow erfolgen.

Die SPR-Konzepte, die auf verschiedene Art und Weise in vorherigen Studien angewandt wurden, haben sich bisher nicht den Komponenten gewidmet, die entscheidend sind, wenn der Grundwasserhaushalt durch Grundwasserentnahme beeinflusst wird. Diese Grundwasserentnahmeraten wurden berechnet, indem der Gesamtbetrag der Entnahme unter Einhaltung bestimmter Grenzwerte bezüglich Grundwassersabsenkung maximiert wurde. Um verschiedene Grundwasserentnahmeraten ermitteln zu knnen, wurden diese Grenzwerte variiert, indem ein Grundwassermodell erstellt und dieses mit einem Optimierungscode gekoppelt wurde. Eine Untersuchung der verschiedenen Grundwasserentnahmeraten verdeutlicht, welche Brunnen für die Grundwasserentnahme bevorzugt verwendet und welche vermieden werden sollten.

Bei Betrachtung der Summe der Grundwasserentnahmeraten und der zugehörigen Grenzwerte des SPR-Konzepts zeigte sich, dass bestimmte Brunnenkombinationen bei gleicher Grundwasserentnahme zu stärkerer Absenkung führen. Diese Brunnenkombinationen sollten von Wasserwirtschaftlern vermieden werden. Es zeigte sich ebenfalls, dass bei Erhöhung des Grenzwertes der Grundwasserabsenkung an einer bestimmten Stelle, die Grundwasserentnahme an Brunnen stattfinden sollte, die sich räumlich deutlich abseits dieser Stelle befinden. Die vorliegende Studie eignet sich daher als unterstützendes Werkzeug, das es einem Wasserwirtschaftler erlaubt, Grundwasserentnahmeraten sowohl aufgrund des Wasserbedarfs als auch aufgrund der Grenzwerte bezüglich Grundwasserabsenkung an dicht beieinander liegenden Lokationen abzuschätzen. Weiterhin liefert diese Studie Informationen, welche es dem Wasserwirtschaftler erlauben eventuelle Defizite in der Wasserversorgung in bestimmten Bereichen des Stadtgebietes, in denen die Anwendung des SPR-Konzeptes zu einer Reduzierung der Grundwasserentnahme geführt hat, zu kompensieren.

Die künstlichen Grundwassererneuerungssysteme wurden installiert, um vorhandene Wasserreserven für zukünftige Trockenperioden zu erweitern. Die künstliche Grundwasserneubildung sollte auf sozialen und ökonomischen Nutzen der investierten Ausgaben ausgelegt sein, wobei auch immer die vorhandenen räumlichen und geologischen Faktoren berücksichtigt werden müssen. Aus diesem Grund kann eine frühzeitige Bewertung des sozio-ökonomische Nutzens einer Anwendung solcher Systeme von Vorteil sein. Im zweiten Teil der vorliegenden Arbeit wurde das Grundwassermodell zu Zweck verwendet, der frühzeitigen Bewertung des sozio-ökonomischen Nutzens eines geplanten künstlichen Grundwasseranreicherungssystems. Um den sozialen und ökonomischen Nutzen von künstlichen Grundwasseranreicherungssystems hervorzuheben, wurde ein solches Szenario mit einem anderen Szenario verglichen, in dem kein solches System eingebaut wurde. Dies geschah über die Optimierung der dazugehörigen Zielfunktionen. Als Maß für den soziale und ökonomischen Nutzen dient der Preis pro entnommener Grundwassermenge. Das Risiko bei der Installation solcher Systeme kann reduziert werden, wenn die Investitionen sorgfältig geplant werden.

Obwohl die Gesamtkosten sowohl geplanter als auch ungeplanter Grundwasseranreicherungssystems etwa gleich sind, zeigte es sich, dass die geplanten Systeme die Investitionskosten durch eine Senkung der damit verbundenen Risiken herabsetzen.

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Introduction

1.1 Motivation

The population of the world has grown several folds in the last few decades. India is the second most populated country in the world with a population of over 1.2 billion in the year 2010. The continuous increase in population and economy of India has resulted in an imminent increase of water demand. Groundwater is the only source of water in many parts of the world. Since some regions are becoming overly dependent on groundwater resources, the groundwater depletion rates become larger than the natural replenishment rates. There is evidence that this happens in North India, but there has been no regional assessment of the rate of groundwater depletion (CGW, 2009). The GRACE satellite data during the study period of August 2002 to October 2008 estimated the groundwater depletion equivalent to a loss of 109 km^3 of water due to groundwater extraction, which is twice as much as the capacity of Indias largest surface-water reservoir (Rodell et al., 2009). The analysis of satellite data showed that the annual meteorological conditions were almost constant and the future groundwater conditions can lay stress on the socio-economic prospects of the region. The two main reasons of groundwater depletion are irrigation and urban usage in rural and urban areas, respectively. In the current study, the groundwater depletion in the city of Lucknow, India is examined and hence only the groundwater depletion due to urban usage is considered.

The depletion of near-surface unconfined aquifers is observed over the last few decades in the urban parts of northern alluvial plains in India due to overexploitation (CGW, 2009; Chatterjee and Purohit, 2009a; Chatterjee et al., 2009b; Foster and Choudhary N., 2009; Livingston, 2009; Lorenzen et al., 2010). Lucknow, the capital city of the state of Uttar Pradesh in the north of India may be assessed as a representative case. Situated in the Sai-Gomti basin, the Lucknow district faces a threat to its natural groundwater resources: Out of the city′s total water demand of $4.9\times10^5 m^3$/day, $2.5\times10^5 m^3$/day of the demand is met by the river Gomti after treatment, while government tubewells supply $2.4\times10^5 m^3$/day from the aquifers to the city (CGW, 2009). However, 30% of the water supply is lost due to leakage in the water supply pipes. This supply deficit has led to the installation of over 10,000 private tubewells in the city, with few tubewells extracting groundwater up to 8,000 m^3/day from the unconfined aquifer. Also, the increasing population of the city (over 100,000 people every year) further stresses the groundwater resources (Foster and Choudhary N., 2009; Livingston, 2009). Large scale groundwater extraction rate of $2.5\times10^5 m^3$/day from the unconfined aquifer across the city since 2001 has resulted in an observed average annual decrease of the groundwater levels by 0.7 m (Foster and Choudhary N., 2009; Livingston, 2009).

The river Gomti and its surrounding alluvial aquifers are the only sources of water in the region. Although the city is underlain by a rich alluvial aquifer system with multiple productive layers, the groundwater overexploitation occurs usually in the uppermost unconfined aquifer due to easy accessibility and low drilling and operational costs. Most of these groundwater resources are characterized as semi-critical or over-exploited since numerous groundwater level gauges across the city show a continuous decline of groundwater levels over the last few years. The river Gomti is fed by groundwater discharge from the Sai-Gomti basin which infiltrates water to the groundwater system only during the short monsoon period (45 days a year) and is fed by groundwater during the rest of the year (Bhatnagar, 1966; Foster and Choudhary N., 2009). However, recent world bank reports state a loss in river-aquifer connection due to groundwater overexploitation within the city (Foster and Choudhary N., 2009; Livingston, 2009). The overexploitation of groundwater resources has created drawdowns, spatially extending to the peri-urban and rural areas outside of the city (Livingston, 2009). Although these places are less densely populated they too face depletion of water resources.

Addressing the aforementioned groundwater problem requires a better groundwater extraction strategy based on the concept of sustainability and an applicable artificial groundwater recharge method. Also, the drawdowns observed in the peri-urban and rural areas surrounding the city, create socio-economic disparity. In case such drawdowns continue to occur due to groundwater extractions in the city, proper compensation should be provided to the population staying in the outside areas. Another option of performing groundwater recharge in the city should be considered in order to reach sustainability goals. In such cases, an artificial groundwater recharge measure is required in the city which utilizes the economic, social and geological feasibilities.

The central groundwater board (CGWB) manages the groundwater resources of the city by estimating the groundwater resource in the first step, followed by a groundwater management strategy. The water table fluctuation method is used to estimate the groundwater resource, as described in Healy and Cook (2002) which has been applied in water management studies, for instance in Chatterjee et al. (2009b). The water table fluctuation method is individually applied to a group of piezoemeters within a city block, where the recharge is estimated by the product of the groundwater level difference before and after the monsoon (averaged over 3-5 years), the area of the block and the estimated specific yield of the block. In the next step of the groundwater resources management, a small amount of the estimated recharge is reserved for water supply. However, there exist procedural drawbacks, related to 1) the consideration of a block as a hydrogeological unit, 2) neglecting spatial variation of groundwater levels in a block 3) uncertain specific yield in a block and 4) unaccounted groundwater inflow/outflow of the block. As the assessment is performed for each block, the effects of groundwater extraction in neighboring blocks are not considered, thus lacking the groundwater rights of the neighboring communities in the presence of a developing cone of depression. Hence, this approach is not feasible for the case of Lucknow city. Due to the lack of spatial and temporal coverage of groundwater level and hydrogeological data, this method was widely applied in India in almost all groundwater management related studies since 1984. The groundwater monitoring has increased considerably since 2003, but the method is still widely used. In the recent past, immense emphasis has been laid on the sustainability of water resources and special attention is required when there is a threat to the quality or quantity of the water resources. In the current study only the threat to quantity is concerned and the use of the term water sustainability only addresses the issue of quantity. The indicators present for the requirement of water sustainability are fast declining groundwater levels or/and reduction/loss of groundwater discharge to the river Gomti. Since both indicators are present in the city of Lucknow 1) the sustainability of the groundwater resources, 2) the future groundwater extraction plans to meet the growing water demand and 3) water management require utmost attention.

The authors working in the field of groundwater management have approached the sustainability problem using either the concept of sustainable development of groundwater resources or the concept of sustainable pumping rates (SPR). The SPR concept was defined by Devlin and Sophocleous (2005) as the maximum groundwater extraction that can be maintained indefinitely without dewatering or mining an aquifer while the concept of sustainable development of groundwater resources is a broader term going beyond the SPR concept and also aims at ecology, water-quality and socio-economic welfare of the society. It is important to state that the groundwater system holds a hydrological balance between the surface water, inflows/outflows and the aquifer. Hence, the SPR concept of 'not dewatering the aquifer' is a constraint which applies to the surface water and inflows/outflows.

However, the authors working on sustainability problems of groundwater resources developed concepts based on the requirements of a specific scenario (Liu et al., 2008; Romano and Preziosi, 2010). For instance, a study on groundwater sustainability focused on the calculation of groundwater extraction rates in the North China plain considered two scenarios (Liu et al., 2008). One scenario aimed at maximizing total groundwater extraction under the constraints of minimum and maximum groundwater extraction at individual wells and the groundwater heads at the extraction wells. The second scenario focused on the optimization of costs involved for extraction under the same constraints as the first scenario and the additional constraint of (total) minimum groundwater extraction. In order to achieve the aims, a transient groundwater model was developed and coupled to an evolutionary algorithm for optimization. Although the modeled domain comprised of a river, the river drainage term was left unconsidered during the study and the author concluded that the constraint should be based upon heads along with the consideration of the river discharge (referred as ecological water requirement) in the model domain.

A similar study performed in Central Italy (Romano and Preziosi, 2010) showed the importance of a transient groundwater model to achieve sustainability but continuously focused on the unachievable objectives of SPR concept due to the use of the term 'indefinitely', which refers to a steady-state condition of the aquifer and is usually not found in nature. Although the studies performed so far for e.g. Liu et al. (2008); Munoz et al. (2003); Romano and Preziosi (2010) used constraints to achieve sustainability, it stayed either focused on cost optimization or declined to consider the SPR concept, and the issue of sustainability remained unaddressed.

In order to augment the storage of groundwater reservoir, various artificial groundwater recharge measures are taken in India, which have been funded by foreign institutions such as the British Geological Society (Gale et al., 2006). The two major focal points of the study Gale et al. (2006) were that (1) there should be a source of water and storage space for artificial groundwater recharge and (2) it should be usable in a certain period of time. The other key issues addressed in this report were (1) the sustainable use of the water (including groundwater) resources, i.e., a balance between recharge, discharge and extraction is needed, either annually or over a few years, (2) impact of the artificial groundwater recharge on the downstream users, (3) the significance of the quantity of the artificially recharged water in relation to natural recharge, (4) distribution of recharged water in social (community with population including varying economic status and interest) and physical strata, (5) and the extent to which local geological, meteorological, economic, social and institutional conditions need to be taken into account. But as stated by the authors of the report (Gale et al., 2006), there exists no panacea to solve water problems in all parts of India. Hence, an artificial groundwater recharge measure should be site-specific.

The study (Gale et al., 2006) compared three artificial groundwater recharge studies performed in three different places in India and concluded that the socio-economic factor was a major hindrance in redistributing of the recharged groundwater. Although a modeling study was performed in the same report to see the effect of water mounds generated due to artificial groundwater recharge, it was never applied to the respective case studies. Hence, the major assessment of the amount of recharged water usable after a certain time was not calculated which is a drawback of the study (Gale et al., 2006). Another drawback of the study was that it did not assess the time span for the recharged water to stay in the subsurface under natural flow conditions, which is required in order to know the maximum time allowed for reusing it.

The amount of investment required in a project such as an artificial groundwater recharge project which yields economic benefits can be assessed beforehand. In case of equal benefits a project involving smaller investment on artificial groundwater recharge holds less risk during failure.

The goal of this thesis is to develop a framework for the groundwater management problem of Lucknow, which can address the aforementioned drawbacks of the water table fluctuation method and calculate groundwater extraction rates based on the SPR concept. This study also aims to overcome the aforementioned drawbacks in the report of Gale et al. (2006) and addresses the socio-economic factor concerned with the peri-urban and rural areas of the city of Lucknow. The problems related to groundwater management are solved by the aforementioned authors using a tool that can quantify the effect of groundwater extractions on drawdowns.

The tools used to address the groundwater management problem in the city are empirical models and mathematical models, which were applied in the previous studies such as Kallioras et al. (2010); Liu et al. (2008); Lûaiciga and Leipnik (2001); Romano and Preziosi (2010). The empirical models are based on the observed relationship between concurrent input and output datasets, which are assessed using the statistical properties of the dataset and fail if input datasets of varying statistical properties are used. For example, sometimes the empirical models obtain results which are physically impossible such as negative groundwater discharge. Also, empirical models are not based upon a theory of first principles i.e. the conservation of mass, momentum and energy.

Mathematical models indirectly simulate groundwater flow by means of governing equations based on first principles and are thought to represent the groundwater processes as understood by a conceptual model of the groundwater system together with equations that describe heads or flows along the boundaries of the model (boundary conditions). These mathematical models in the field of groundwater modeling can either be, analytical models or numerical groundwater models. In case that the assumptions regarding the conceptual model of the groundwater system are not simple, a numerical groundwater model should be used (Anderson and Woessner, 1992).

The objective of the groundwater management in a logical sense aims to compute groundwater extractions using the SPR concept. However, the calculation of maximum groundwater extraction under constraints adds more relevance to the groundwater management using the SPR concept and therefore in this thesis the optimization tools are coupled with the groundwater model. In this study, a numerical groundwater model is developed in order to solve the problem of groundwater management for the city of Lucknow by simulating the observed groundwater heads, which is then coupled with the optimization codes (as similar to previous studies) to obtain maximum groundwater extraction, while meeting constraints of the SPR concept.
These numerical groundwater models also calculate the lost groundwater due to extraction, which can be multiplied with the water prices in the peri-urban and rural areas in order to assess the socio-economic effect. The effect of artificial groundwater recharge measures was therefore known in the terms of (1) storage and movement of recharged water in subsurface and (2) the usability of the stored water considering socio-economic effects as discussed before by Sanford (2002). The institutional conditions are based upon the agencies implementing the required artificial groundwater recharge measure, which will not be addressed in the study. Hence, in the next step the groundwater model is used for assessing the effect of artificial groundwater recharge subject to geological and meteorological conditions, while the economic and social conditions in the peri-urban and rural areas are met with the help of optimization procedures.

1.2 Research questions and objectives

The principal question investigated in this thesis are:

1. *What relevance does the SPR concept have in terms of applicability to Lucknow city? Can the water manager trade off between the amount of groundwater extractions at respective locations and still follow the SPR concept?*
   This question is divided into two main parts.

   How is the SPR concept defined in the context of Lucknow and what should be the approach towards finding 'groundwater extraction that can be maintained indefinitely (steady state)' in a basin where the groundwater flow is never steady? What are the assumptions for such an approach?

   How do we develop a methodology applicable to achieve the trade-off following the SPR concept?

2. *What spatial groundwater extraction patterns can be identified amongst the groundwater extractions estimated following the SPR concept as a function of constraints? Which general rules should the groundwater manager follow?*

3. *Is it economically beneficial to implement rooftop rainwater harvesting?*

In order to answer these research questions a model framework is prepared utilizing the groundwater extraction data and hydrogeological and meteorological data for the city of Lucknow. It should be kept in mind that in a developing nation like India the amount of data available for a scientific study is much smaller than for any developed country. Hence, the thesis also aims towards understanding the groundwater flow regimes under limited data conditions.

Since the groundwater extractions are regulated by the water managers, the objective has been formulated and used in a manner which is acceptable and easy to follow for the water managers. Also, the formulation of the groundwater extraction scenario is performed in a flexible manner so that it can be altered to the trade off required by the water manager. Also an emphasis is laid upon the uncertainty in the scientific study and the thesis tries to give a scientific outline to the effects of action versus inaction of artificial groundwater recharge.

1.3 Outline

The details about the SPR concept and its relation to the groundwater management are provided in chapter 2. The chapter also discusses the constraints involved in the SPR concept due to the hydrological balance between the surface water, inflows/outflows and the aquifer. Furthermore, this chapter formulates the requirements of the numerical groundwater model and the optimization processes involved.

The components of the hydrological balance are described in chapter 3 along with further details about the groundwater system. The equations used for the numerical assessment of various components of the hydrological cycle are also provided. The groundwater flow equations are also stated in the chapter along with the description of different initial and boundary conditions required for solving such equations.

In chapter 4, the purpose of optimization is stated which will be coupled to the numerical groundwater model in the coming chapters. This chapter also provides some detail about the optimization techniques, along with some simple derivations, which is required in the current work for maximizing groundwater extraction and economic benefits.

In chapter 5, the procedure of extracting valuable information from a set of results is explained (also called data mining). The methods involved are stated in ample detail to make the reader aware of the application of such procedures used in extracting important information from the optimization results and are used later in the study do identify the spatial patterns when the constraints of the SPR concept are enforced.

In chapter 6, various artificial groundwater recharging methods are briefly stated which are relevant to an urban setting in India. The chapter also covers the feasibility of rooftop rainwater harvesting method subject to geological and spatial constraints in an urban setting such as Lucknow. The chapter also covers the economic considerations before the implementation of rooftop rainwater harvesting and provides a background of the required formulation.

Chapter 7 provides a protocol which is widely used in the field of groundwater modeling. This chapter also provides logical explanation for each part of the protocol, including the sequence it follows with the help of a flowchart. The protocol laid down in this chapter was followed while developing a groundwater model for the city of Lucknow.

Chapter 8 widely follows the groundwater model protocol laid down in the previous chapter 7. The chapter discusses the development of a conceptual model of the city of Lucknow in view of the available data and the objective of the study i.e. estimating groundwater extractions following the SPR concept. The initial and boundary conditions for the conceptual model are also stated in this chapter. Sensitivity analysis and calibration is performed to estimate the parameters which are used in the groundwater model in order to simulate the groundwater heads as close as possible to the groundwater observations. The groundwater model is validated in the next step and its performance is judged on the basis of various statistical measures.

In chapter 9, the groundwater extractions following the SPR concept is estimated by combining the optimization formulation developed in chapter 2 and the developed groundwater model. In the first step, the simulated groundwater heads are proven to be linear with respect to the groundwater extraction rates to save time and computational effort. Later, the optimization code is coupled with the linear formulation instead of the developed groundwater model. The results are analyzed using the methods provided in chapter 5 to advise a water manager about the groundwater extraction wells to be preferred or avoided for groundwater extraction. Also, a methodology of meeting location based demands using the results of SPR implementation is provided.

In chapter 10, the economic feasibility of rooftop rainwater harvesting is performed on the basis of the water prices in the different peri-urban and rural areas. The different blocks in the city are shown where the rooftop rainwater harvesting measures can be taken, followed by the assessment of the economic feasibility in a scenario where the rooftop rainwater harvesting measure is implemented in all blocks of

the city, using optimization algorithms. As similar to the previous chapter, the optimization protocol is followed and a linear formulation is used for estimating the state variables of the groundwater model which is required for the economic consideration. In another scenario, the blocks to be selected for rooftop rainwater harvesting are a part of the optimization procedure and the economic efficiency of these two scenarios are compared with a scenario where no rooftop rainwater harvesting takes place.
In chapter 11, the conclusions on the basis of the results are widely discussed, covering more aspects of the water scenario in the city. The answers to the aforementioned research questions are given on the basis of the results obtained in the previous chapters. A broader perspective of applying this study in other SPR concept based groundwater management is provided. It is also stated that a framework for managing the urban water is performed but should continue to address other key issues which were left unattended during the current research work.

2

Sustainable pumping rates

In this chapter the term 'water budget' for the groundwater flow system of Lucknow is explained before discussing other terms such as the SPR concept and the 'water budget myth', which is found in almost all of the publications related to groundwater management. It is followed by the significance of the SPR concept, as followed in this study, to a water manager of the city.

2.1 Water budget

A control volume representative for the groundwater flow system of Lucknow is shown in figure 2.1. The figure shows the scenarios before and after groundwater extraction, and the terms used in the figure and in the description below are adopted from previous studies (Bredehoeft, 1997, 2002). However, the meaning of the induced recharge term, ΔR_0 used in Bredehoeft (1997, 2002) refers to changes in external conditions such as rainfall, vegetation cover and soil permeability, while in this study the term refers to the recharge from areas outside the city limits. The recharge and discharge terms in which the virgin amount of recharge[1] is represented using R_0 and $R_{neighbor}$ due to precipitation and inflow from neighboring areas, respectively, balances the virgin discharge D_0 [2] as shown in figure 2.1.
The control volume has inflows from the city boundary (from the rural and peri-urban areas) as $R_{neighbor}$, outflow to the river D_0, recharge due to precipitation R_0 before groundwater extraction. The water budget of the groundwater system of the city of Lucknow can be computed using the following equation:

$$R_0 + R_{neighbor} = D_0 \qquad (2.1)$$

Considering a long term fixed groundwater extraction P, the extracted water comes initially from the

[1] Virgin recharge rate is the natural recharge to the aquifer in absence of any pumping
[2] Virgin discharge rate is the natural discharge to the river in absence of any pumping

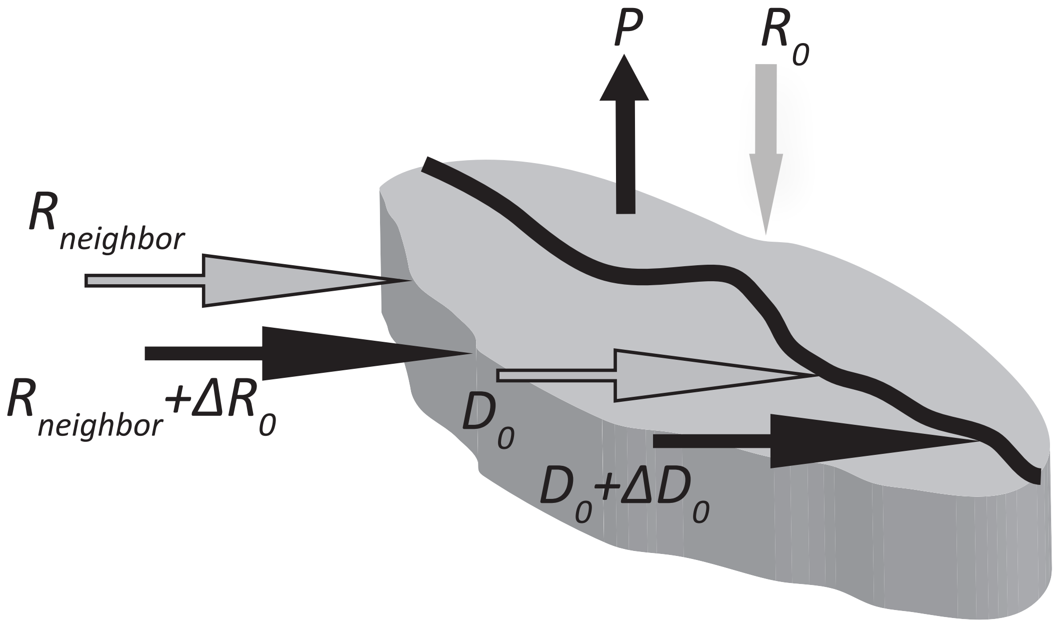


Figure 2.1: The city of Lucknow shown as a control volume for which SPR concept is defined. The terms shown in the figure are flow from different boundaries.

vicinity of the well but later from the neighboring rural or peri-urban area as $R_{neighbor} + \Delta R_0$ or from the river in the city as $D_0 + \Delta D_0$ due to the development of a cone of depression up to the boundaries of the control volume and the river (Theis, 1940), changing previous water balance equation to the following:

$$R_0 + R_{neighbor} + \Delta R_0 = D_0 + \Delta D_0 + P - \frac{\partial V}{\partial t} \tag{2.2}$$

where $\partial V/\partial t$ is the rate of change of storage, ΔR_0 is the change in the virgin rate of recharge and ΔD_0 is the change in the virgin rate of discharge due to extraction, these terms are called induced recharge and discharge, respectively. The term induced is used since it appears due to pumping. The cone of depression reaches a steady state for long term pumping P, when the extraction rates equal the lateral flow rates from the neighboring areas and/or from the surface water bodies (see (Theis, 1940)).

2.2 SPR concept

Sustainable pumping rates (SPR) is a common term used often in the groundwater management literature described as the maximum permissible groundwater extraction which can continue for a long term without disturbing too much the other terms of the groundwater system, mainly recharge and discharge, where recharge is anything which enters the aquifer and discharge is anything which leaves the aquifer. ”Too much” is a quantitative term which has to be decided by the water manager considering the social and economic aspects of the aquifer concerned. Devlin and Sophocleous (2005) has defined the SPR concept as the maximum exploitation that can be maintained indefinitely without ”dewatering” or mining an aquifer. The SPR concept is one way to address the sustainable management of water resources in a groundwater system (Bredehoeft, 2002; Devlin and Sophocleous, 2005; Kalf and Woolley, 2005; Maimone, 2004; Munoz et al., 2003; Romano and Preziosi, 2010; Stanghellini and Collentine, 2008), though other terms such as safe yield and sustainable yield have also been mentioned rarely e.g. Bredehoeft (1997); Maimone (2004); van der Gun and Lipponen (2010), but are often called to be abandoned due to ambiguity (Alley and Leake, 2004; Kalf and Woolley, 2005; van der Gun and Lipponen, 2010). For sustainable development, the steady-state case must be considered since it represents the ”indefinite” term used in the SPR concept. At steady state, the change in storage, $\partial V/\partial t$ is zero so that the water is no longer being removed from the storage which transforms the equation 2.2 in:

$$P = \Delta R_0 - \Delta D_0 \tag{2.3}$$

on substituting equation 2.1 in 2.2. Hence, similar to Devlin and Sophocleous (2005) it is found that P is a function of induced recharge ΔR_0 and induced discharge ΔD_0.
The SPR concept has been debated for the last decades among hydrogeologists, especially over the importance of estimating the virgin recharge rate. It is called the ”water budget myth”, stating that the calculation of virgin recharge rate R_0 was important to know the groundwater extractions P for aiming towards sustainable extraction, which was refuted in all studies related to the calculation of groundwater extractions based on the SPR concept e.g. Bredehoeft (2002); Devlin and Sophocleous (2005); Kalf and Woolley (2005); Maimone (2004); Munoz et al. (2003); Romano and Preziosi (2010); Stanghellini and Collentine (2008) and can also be seen in the above equation.

Considering the same long term fixed extraction P in a groundwater basin, a cone of depression will be formed, where the extracted water comes from the neighboring areas outside the city limits as ΔR_0 (call it a positive number r) and groundwater discharge to the surface water body ΔD_0 (call it q_{crit}, since the discharge is decreased due to pumping) resulting in $P = r + q_{crit}$ (Devlin and Sophocleous, 2005). Hence, the estimation of P by a water manager depends upon r and q_{crit}, which serve as constraint. In a case, when there are groundwater extraction wells across the city, total pumping P equals the sum of all groundwater extraction rates in the city i.e. $\vec{1} \cdot \mathbf{q_{spr}}$.
In the previous study performed in Central Italy (Romano and Preziosi, 2010), a groundwater model

was developed for both, steady and transient simulation, and the main emphasis of the study was that the calculation of P based on the SPR concept should be performed using transient simulations, which allows for the necessary time spans of aquifer depletion and recovery. Nevertheless, no P or $\mathbf{q_{spr}}$ was calculated, rather it was concluded similar to other studies like Bredehoeft (2002); Devlin and Sophocleous (2005); Kalf and Woolley (2005); Maimone (2004); Munoz et al. (2003); Romano and Preziosi (2010); Stanghellini and Collentine (2008) that this calculation should be based on long term effect of temporally varying P upon q_{crit}. Although many examples of estimating groundwater extractions based on the SPR concept can be found in the literature cited above, the application of the concept to determine P differed from study to study, although a widely accepted definition of SPR in terms of r and q_{crit} exists. The studies did not consider r and q_{crit} as constraints, though these are the only two factors the SPR depends upon. Instead, the studies mainly focused towards the drawbacks of the SPR concept and its application (Bredehoeft, 1997, 2002) due to its long-term average extraction value.

In the current study for the city of Lucknow, the SPR concept defined by Bredehoeft (1997, 2002) is used, by fixing the values of r indirectly and q_{crit} directly as constraints. The indirect and direct terms are used due to the fact, that the value of drainage to the river can directly be measured using q_{crit} whereas the value of r corresponds indirectly to the groundwater drawdowns at the city limits as d_{crit}, implying that $P(\mathbf{q_{spr}})=f(d_{crit},q_{crit})$, where $\mathbf{q_{spr}}$ is the groundwater extraction estimated using the SPR concept. The transient nature of P could have been considered in a case that some data on the seasonal urban demand was available. Only a steady-state pumping case is considered in this study and P is a constant value as mentioned in the beginning of the chapter.

The study performed by Romano and Preziosi (2010) and other authors concluded that the SPR concept can only be used for a steady state case which can also be seen in the above derivation where change in storage is zero. Hence, a dynamic water balance approach is used, where the groundwater heads and storage terms at a particular time instance becomes constant, after long simulation times with the same groundwater extraction P (Anderson and Woessner, 1992). The problem of finding the SPR adopts the strategy of developing a transient groundwater flow model similar to the studies performed so far for e.g. Liu et al. (2008); Romano and Preziosi (2010). Also the formulation of the optimization problem is similar to Liu et al. (2008) in the sense that the sum of groundwater extraction rates is maximized for a fixed duration of extraction.

This approach mainly helps a water manager to estimate $P(d_{crit},q_{crit})$ based on the SPR concept by varying constraint values. The characterization of different $\mathbf{q_{spr}}$ provides general suggestions about the wells to be preferred or avoided for groundwater extraction. The study also helps in explaining the water manager the $\mathbf{q_{spr}}$ to be avoided. Also, in a case the water manager does not care about one of the constraint d_{crit} or q_{crit}, a $\mathbf{q_{spr}}$ can be chosen based upon the total extraction P.

This study is significant in the field of groundwater extraction plans in urban and non-urban regions. It illustrates a combination of groundwater modeling, linearization and optimization to evaluate groundwater extraction rates in the city of Lucknow, based on the SPR concept. The study can be applied to groundwater extraction plans in other cities in India or other countries, in order to find the SPR as a function of the same or larger number of numerical constraints. In all such cases, considering the trade-off between two $\mathbf{q_{spr}}$ helps the water manager to improve the groundwater extraction strategy, in terms of meeting water demands. Although the groundwater extractions based on the SPR concept may be insufficient to meet the water demands in the city, the study holds importance in the sense that the proposed groundwater extraction meets the groundwater sustainability criteria. Also, the study can provide the water manager with the water deficit amount in the wake of current and future demands.

In order to effectively manage the groundwater resources of the city, most of the groundwater balance estimates have been done only using WTF (water table fluctuation Healy and Cook (2002)) and empirical methods as described in Chatterjee et al. (2009b) and CGW (2009), whereas the question of sustainability has not been addressed yet. The water managers have put emphasis on artificial groundwater recharge using rooftop rainwater harvesting to save water for groundwater extraction during dry periods in the city. However, the impact and economic benefit of such artificial groundwater recharge on change in groundwater flow system and groundwater extraction respectively has to be known before its application.

In the current study a quantitative investigation on the amount of groundwater extraction under sustainable conditions was performed. This study is also performed to the see the effect of the groundwater extraction during same recharge regime and to evaluate the effect of artificial groundwater recharge on the groundwater extraction regime.

3

Groundwater Model

A groundwater model represents groundwater flow in a natural or artificial groundwater basin. These models can be a laboratory setup or a set of mathematical equations in order to understand or predict a specific groundwater basin. A groundwater basin is an underground reservoir of water which may take the form of a single aquifer or a group of linked aquifers (Anderson and Woessner, 1992; Perez et al., 2011).

A groundwater model based on a set of governing mathematical equations is used to verify the conceptual understanding or to predict the effect of hydrological changes of a groundwater basin. The mathematical equations represent the physical processes occurring in the ground- water flow system along with equations describing heads or normal fluxes along the boundaries of the basin. The mathematical groundwater model can be solved analytically or numerically. In case that the analytical solution is judged to be insufficient for the particular system, a numerical model may be selected. In case that the theoretical understanding of the groundwater basin on the basis of governing equations is verified by the observations in nature, it is conjectured that the mathematical equations and the boundary conditions well represent the groundwater processes occurring in the basin. The groundwater models can then be used to simulate the effect of hydrological changes on the basin.

3.1 Components of a groundwater flow model

In order to model a natural or an artificial groundwater basin, the basic components of a groundwater flow have to be considered. They can be assessed by simple consideration of the water budget equation of a control volume, which states that the change in storage must equal the difference between inflow and outflow, which is a formulation of the first principle (Cirpka, 2011) i.e.

$$\Delta S = I - O \tag{3.1}$$

In equation 3.1, I represents the sum of all inflows, i.e. precipitation and groundwater inflows while O represents the sum of all outflows, i.e. evapotranspiration, groundwater outflows, groundwater extraction or gaining streams. These different terms of the inflows and outflows are measured directly or indirectly in the field as follows.

3.1.1 Precipitation

Precipitation can be defined as any product of the condensation of atmospheric water vapor that falls under gravity. Usually, the amount of precipitation is measured using a vessel open to the air at the point of observation, which periodically (pluviometer) or continuously (pluviograph) records the collected quantity of water. Recorded point values are used to estimate regional precipitation (Anderson and Woessner, 1992; Geer, 1996; Glickman and Zenk, 2000; Perez et al., 2011). Precipitation can have spatial variations, in larger basins, where Thyssen polygons, simple arithmetic means or interpolation methods (e.g. Kriging, Spline or Inverse Distance Weighting) are employed in order to estimate the spatial distribution of precipitation. The spatial and temporal nature of precipitation controls the response of the groundwater basin in terms of groundwater heads or discharge to the surface water bodies. Especially in cases of groundwater basins, where the objective of the study requires precipitation data of higher spatial or temporal resolution or the groundwater basin has high topographic variability, such data is required. Usually, the spatial and temporal variability of the precipitation data is unavailable in large catchments and the basin is studied with limited amount of data. In these cases a decision is required if the observation can represent at least an average value of precipitation in the groundwater basin and does not affect the objective of the study.

After entering the land surface, the precipitation partially infiltrates which can flow into the saturated via the unsaturated zone of the basin and the rest is lost in evapotranspiration and/or flows on the surface. The net precipitation is calculated by deducting this amount from the precipitation. In case it enters the unsaturated zone, the amount of uninfiltrated water must be deducted from the total precipitation to estimate groundwater recharge. This is done in cases where the groundwater basin only considers the saturated zone.

Due to the limited amount of precipitation data from a single meteorological station in the city, it is assumed that the amount of the precipitation entering the saturated zone is a product of the factor defined for respective zones in the city based upon the data, *RCH*, and the net precipitation. The net precipitation is calculated by deducting the evapotranspiration from the precipitation and surface runoff is neglected.

3.1.2 Evapotranspiration

Evapotranspiration is a component of the hydrological cycle used to describe the total sum of water lost due to evaporation from the Earth's land surface, surface water bodies and plant transpiration to the atmosphere. In groundwater basin studies, it is hard to separate evaporation from transpiration. Also, the groundwater studies aim to calculate total water loss in order to assess the net recharge, and not the individual components (Cirpka, 2011; Davie, 2008):
Evapotranspiration can be measured directly using the following methods.

- A weighing lysimeter can be used to measure directly the amount of evapotranspiration by continuously weighing the soil column and calculating the change in storage of water in the soil by measuring the change in weight of the lysimeter. The change in weight is converted to units of length based on the surface area of the weighing lysimeter and the unit weight of water. Evapotranspiration is computed as the change in weight plus rainfall minus percolation.

- The most direct method of measuring evapotranspiration is using the eddy covariance technique in which fast fluctuations of vertical wind speed are correlated with fast fluctuations in atmospheric water vapor density. This directly estimates the transfer of water vapor (evapotranspiration) from the land surface (or canopy) to the atmosphere.

Since the direct measurement of evapotranspiration is expensive, methods were developed that estimate it on the basis of direct measurements of meteorological parameters that are available in high spatio-temopral resolution and routinely surveyed at meteorological stations (Cirpka, 2011; Davie, 2008; Monteith, 1965).
The models used to calculate evapotranspiration are adopted from atmospheric physics, which are directly based upon energy and mass conservation. A free-water evaporation theoretical concept was introduced initially in order to group general methods for estimating evapotranspiration, neglecting advection and heat storage changes. Different approaches have been used thereafter to calculate evaporation from free water: water-balance, mass transfer, Eddy-correlation, energy balance and the Penman equation, which combines the mass transfer concept and energy balance approaches (Cirpka, 2011; Davie, 2008; Monteith, 1965).
The model used to calculate evapotranspiration in this thesis are adopted from Penman (Monteith, 1965)

and is stated in the coming paragraphs. The Penman equation estimates the evaporation rate from a (water) saturated surface using meteorological data measured in a reference height of 2 *m* above ground, based on the assumption that water supply is not a limiting factor. Hence, it is termed potential evapotranspiration (ET_P). The conceptual idea behind this method considers that a weighted net radiation on the surface of water and the mass transfer due to convective heat transfer are the driving forces for ET_P i.e.

$$ET_P \propto \frac{\Delta \times net\ radiation + \gamma \times mass\ transfer}{\Delta + \gamma} \tag{3.2}$$

The weighting criteria is specified by the psychrometric constant, γ, and Δ which is the derivative of the saturation pressure e_{sat} curve with respect to air temperature $T_a[^oC]$:

$$\Delta = \left.\frac{\partial e_{sat}}{\partial T}\right|_{T_a} \tag{3.3}$$

and the *net radiation* is a result of the following energy balance:

$$net\ radiation = \frac{K + L - G + A_w - \Delta Q/\Delta t}{\rho_w \lambda_v} \tag{3.4}$$

implying that the net radiation is expressed as the volume of water transferred per unit time and area. $K[Wm^{-2}s^{-1}]$ is the net shortwave radiation which depends on the solar radiation and the reflection properties of the water surface (albedo); $L[Wm^{-2}s^{-1}]$ is the net long-wave radiation; $G[Wm^{-2}s^{-1}]$ is the net heat conduction to the ground; $A_w[Wm^{-2}s^{-1}]$ is the net water advected energy (inflows and outflows); and $\Delta Q/\Delta t[Wm^{-2}s^{-1}]$ is the change in the amount of heat stored in the body during the time Δt. $\rho_w[kgm^{-3}]$ is the mass density of water, and $\lambda_v[Wm^{-1}]$ is the latent heat of evaporation.
In the combined approach, the mass transfer relation is assumed to depend on the difference between the actual vapor pressure $e_a[kgm^2]$ and the saturation vapor pressure at the air temperature $e_{sat}(T_a)[kgm^2]$. Thus, the mass transfer can be written as:

$$mass\ transfer = f(v_a)(e_{sat}(T_a) - e_a) \tag{3.5}$$

where $f(v_a)[m^3kg^{-1}T^{-1}]$ is a function of the wind speed $v_a[ms^{-1}]$ that represents the aerodynamic conductance (inverse of the aerodynamic resistance) of the transport of water vapor from the surface to a reference height.
The relative humidity, $W_a[-]$, which can be directly measured at a representative location is expressed as:

$$W_a = \frac{e_a}{e_{sat}(T_a)} \tag{3.6}$$

Hence, the equation 3.5 can be written as:

$$mass\ transfer = f(v_a)e_{sat}(T_a)(1-W_a) \tag{3.7}$$

By substituting equation 3.7 and 3.4 in equation 3.2 and neglecting the water advected energy A_w, the change in heat storage ($\Delta Q/\Delta t$), and the net conduction to the ground G, the generalized Penman equation is obtained:

$$ET_P = \frac{\Delta(K+L-G)+\gamma\rho_w\lambda_v[f(v_a)e_{sat}(T_a)(1-W_a)]}{\rho_w\lambda_v(\Delta+\gamma)} \tag{3.8}$$

In certain cases, the dew point is used to calculate the relative humidity due to the unavailability of direct humidity data. The dew point is defined as the temperature where the water vapor in a volume of humid air at a constant barometric pressure will condense into liquid water. Condensed water is called dew when it forms on a solid surface. The dew point is a water-to-air saturation temperature. The dew point is associated with relative humidity. A high relative humidity indicates that the dew point is closer to the current air temperature. Relative humidity of 100 % indicates the dew point is equal to the current temperature and that the air is maximally saturated with water. The relationship between relative humidity and dew point is defined using the following equation

$$W_a = 100\ \ exp\left(a\left[\frac{T_d}{b+T_d} - \frac{T_a}{b+T_a}\right]\right) \tag{3.9}$$

where a and b are constants with values 17.271 and 237.7 °C, respectively, while T_d is the dew point. $e_{sat}(T_a)$ is calculated by the formulation provided by National Oceanic and Atmospheric Administration (NOAA) as

$$e_{sat} = 611.2\ \ exp\frac{17.0543\ T_a}{(241.15+T_a)} \tag{3.10}$$

3.1.3 River drainage

River drainage is an important part of the groundwater basin, where the discharge quantity is determined by the surface runoff processes and the groundwater flow in the region. Depending on the losing or gaining nature of the river, it either provides water to the groundwater or extracts water from the groundwater, respectively.

Surface runoff (or overland flow) is a sheet flow defined as the precipitation excess that moves over the land surface to the river. There are two distinct surface-runoff generation processes Hortonian and Dunne overland flow (Dunne, 1983; Mehta et al., 2003). These processes are modeled by coupling the Saint-Venant equations (Strelkoff, 1970) and the subsurface exchange flux. However, solving these coupled equation requires a considerable computational effort which is worthwhile only when the mean river discharge depends upon surface runoff processes. Hence, a modeling effort is meaningless in a case in which the mean annual river discharge depends only upon the groundwater flow.

A losing or gaining river result in exchange with the groundwater flux, which can be modeled by a simpler approach, using a function of the groundwater heads in the basin, river levels or river bed elevations and riverbed conductances. Furthermore, such a modeling procedure is applied in large groundwater basins where the major amount of discharge is a result of groundwater interaction.

In the thesis, the gaining nature of the river is known beforehand from the groundwater reports and hence the groundwater discharge to the river for large basins was computed using a drainage boundary condition

$$q_{riv} = \begin{cases} RIVP \cdot (h - h_{drainage}(\mathbf{x}_{dr})) & \text{if } h \geq h_{drainage} \\ 0 & \text{if } h < h_{drainage} \end{cases} \tag{3.11}$$

as stated in Harbaugh (2005), where $q_{riv}[ms^{-1}]$ is the specific groundwater discharge to the river (computed by using the product of q_{riv} and the cross sectional area of the river), $RIVP[s^{-1}]$ is the riverbed conductance, $h[m]$ are the groundwater heads and $h_{drainage}(x_{dr})[m]$ is the riverbed elevations at the river location $x_{dr}[m]$.

3.1.4 Groundwater extraction

Groundwater extraction is an anthropogenic process in the groundwater basin, being part of the outflow in the groundwater equation 3.1, and is performed to cater to the agricultural, industrial and household water demand. Groundwater extraction may exhibit spatial and temporal variation and can lead to considerable drawdowns laterally, depending upon the magnitude and duration of groundwater extraction and hydrogeological setting of the groundwater basin (Bredehoeft, 2002; Theis, 1940). In a groundwater basin, the spatial extension of the cone of depression due to groundwater extraction holds key importance in discharge of the gaining stream and serves as a controlling factor in order to maintain a specific discharge in the stream. Furthermore, the cone of depression can reach the boundaries of the city, which affect the groundwater rights of the users in these areas.

3.2 Modeling groundwater flow

3.2.1 Groundwater flow equation

In groundwater flow, *Darcy's* law determines the specific discharge, or *Darcy* velocity, $\mathbf{q}[ms^1]$ as a linear function of the hydraulic gradient ∇h:

$$\mathbf{q} = -\mathbf{K}\nabla h \tag{3.12}$$

in which $\mathbf{K}$ $[ms^{-1}]$ is the hydraulic-conductivity tensor. The specific discharge is the total discharge per bulk cross-sectional area perpendicular to the direction of flow (no discrimination between area occupied by pores and grains). The hydraulic head $h[m]$ consists of the hydrostatic pressure $p[kgm^{-1}s^{-2}]$, expressed in the height of a water column, and the geodetic height $z[m]$ is the elevation of the aquifer bottom:

$$h = \frac{p}{\rho_w g} + z \tag{3.13}$$

in which ρ_w $[kgm^{-3}]$ is the mass density of water and g $[ms^{-2}]$ is the acceleration due to gravity. The negative sign in equation 3.12 signifies the flow direction of water to be in the negative direction of the hydraulic gradient, from higher to lower heads. The hydraulic conductivity controls the groundwater flow, and depends on the size and shape of the pores, hence it changes for different types of soils. $\mathbf{K}$ can be measured in the field via pumping tests or can be estimated using a grain-size analysis. Estimates of $\mathbf{K}$ from soil texture can also be found in the literature (Carsel and Parrish, 1988). Hydraulic conductivity of porous media may also depend on the the direction of flow (anisotropy), and it may vary in space (heterogeneity) (Cirpka, 2011; Zhang, 2002).

The groundwater flow equation is derived using the equation 3.12 and for isotropic porous media as considered in this study is commonly expressed as:

$$S_s \frac{\partial h}{\partial t} - \nabla \cdot (\mathbf{K}\nabla h) = q_{in} - q_{out}$$

in which q_{in} and q_{out} are the volumetric sources and sinks, $S_s[m^{-1}]$ is the specific storage made of a term relating to the compressibility of the fluid and a term expressing the compressibility of the porous matrix:

$$S_s = \frac{n_e}{\rho_w}\frac{\partial \rho_w}{\partial h} + \frac{\partial n_e}{\partial h}$$

in which n_e is the effective porosity of the porous medium.

In regional groundwater studies, flow is often considered to be horizontal. In such cases, integration over the thickness of the groundwater body yields:

$$S\frac{\partial h}{\partial t} - \nabla \cdot (\mathbf{T}\nabla h) = q_{in/out}$$

in which, gradients are evaluated only in the horizontal directions. In the above equation, S is the storage coefficient [-] and $\mathbf{T}$ is the transmissivity (conductivity times thickness of aquifer) obtained by integrating over the depth and $q_{in/out}[ms^{-1}]$ is the source/sink term representing the volumetric flux per unit area. In phreatic or unconfined aquifers the groundwater head is the top of the groundwater body and the unconfined groundwater flow equation used in the thesis can be written as:

$$S_y\frac{\partial h}{\partial t} - \nabla \cdot (\mathbf{K}(h-z)\nabla h) = q_{in/out} \tag{3.14}$$

where S_y is the specific yield [-] and $q_{in/out}[ms^{-1}]$ defined as

$$q_{in/out} = RCH \cdot max(0, P - E_{PT}) - \delta(x_w)Q_{ext} - q_{riv} \tag{3.15}$$

where $RCH[-]$ is a recharge flux factor, $P[ms^{-1}]$ is the amount of precipitation, $E_{PT}[ms^{-1}]$ is the calculated evapotranspiration (using equation 3.8), $Q_{ext}[m^3s^{-1}]$ represents groundwater extraction and x_w represents its location.

The function associated with a point source or sink such as a well involves Dirac delta functions. For practical purposes, we may define the Dirac delta function as a function that has a nonzero value only over an interval $[x_w - a, x_w + a]$, is restricted to having an integral equal to 1, and is defined by taking the limit as a $\rightarrow 0$. This function is represented symbolically as $\delta(x_w)[m^{-2}]$.

In equation 3.15, precipitation, evapotranspiration and groundwater extraction are directly or indirectly measured or are already known while groundwater discharge to the river is estimated using groundwater heads. The groundwater heads are the state variables which are calculated using equation 3.14.

3.2.2 Boundary and initial conditions

The groundwater flow equation stated above is a partial differential equation, which requires particular conditions to be defined on all boundaries of the domain. The boundary and initial conditions along with the partial differential equation 3.1 is called the structure of model in the current study. The boundary and initial conditions are a necessity for solving the partial differential equation 3.1. The importance of right boundary conditions is essential for making prediction using the model and has been discussed in details by Franke et al. (1987) and Reilly et al. (1987).

In transient applications, where the state variable are calculated as a function of time, initial conditions

are required. Initial conditions represent the groundwater heads at the beginning of a transient simulation, hence serving as a boundary condition in time for the transient head response of a groundwater model solution (Cirpka, 2011; Reilly et al., 2004).
There are mainly four kinds of boundary conditions and these are typically used in the context of the groundwater head defined in Cirpka (2011) as:

- **Dirichlet boundary conditions**, or boundary conditions of the first type, are set at boundaries where the primary unknown (here groundwater head h) are known:

$$h(x) = h_{fix}(x) \quad at \quad \Gamma_D \tag{3.16}$$

  in which h_{fix} is the fixed distribution of hydraulic head along the Dirichlet boundary Γ_D. In practice, this type of boundary condition is assumed if an aquifer is considered to be in perfect hydraulic contact to a river or lake with known water stage.

- **Neumann boundary conditions**, or boundary conditions of the second type, are set at boundaries where the normal flux crossing the boundary is known:

$$\mathbf{n} \cdot (-\mathbf{K}\nabla h) = q_{fix}(x) \quad at \quad \Gamma_N \tag{3.17}$$

  in which $\mathbf{n}$ is the normal unit vector pointing outwards, and $q_{fix}(x)$ is the known flux perpendicular to the Neumann boundary Γ_N. A particular case of the Neumann boundary condition is the no-flux condition, in which q_{fix} = 0. The latter is a typical assumption at the base of an aquifer or at lateral boundaries to bedrock. While Dirichlet boundaries conditions make statements about the unknown itself, Neumann boundary conditions refer to gradients of the unknown.

- **Cauchy boundary conditions**, or boundary conditions of the third type, include combinations of the unknown and its derivative normal to the boundary. An example is the so-called leaky boundary:

$$\mathbf{n} \cdot (-\mathbf{K}\nabla h) = \frac{K_{leak}}{\Delta z_{leak}}(h - h_{ref}) \quad at \quad \Gamma_C \tag{3.18}$$

  in which K_{leak} is the hydraulic conductivity of the leaky layer, Δz_{leak} is its thickness, and h_{ref} is a given reference hydraulic head at the other side of the leaky layer. Γ_C denotes the Cauchy boundary. This type of boundary condition is frequently used for aquifers connected to surface-water bodies via a semi-pervious (mud) layer.

- **Free-exit boundary conditions** determine a situation in which the normal flux component is considered to be constant:

$$\mathbf{n}\cdot\nabla(\mathbf{n}\cdot(-\mathbf{K}\nabla h)) = 0 \quad at \quad \Gamma_F \tag{3.19}$$

  implying that the second derivative of h with respect to the spatial coordinate in the normal direction must be zero if $\mathbf{K}$ is uniform. Γ_F denotes the free-exit boundary. The free-exit boundary condition is popular in the modeling of unsaturated flow if the distance to groundwater is much larger than the zone of interest.

The different boundaries should not intersect each other and all the boundaries together, $\Gamma = \Gamma_D \cup \Gamma_N \cup \Gamma_C \cup \Gamma_F$ should be the total boundary of the domain Γ. For groundwater modeling applications their selection is critical to the development of an accurate model (Franke et al., 1987). It should also be noted that sometimes the same hydrological feature can be represented by more than one boundary condition and hence the modeler should be careful in choosing boundary conditions (Reilly et al., 2004). As a rule of thumb, the aforementioned boundaries should either correspond to hydrogeological boundaries such as rivers, valleys and groundwater divides or lie far from the area of interest (Cirpka, 2011).

The changes in head in a transient groundwater model due to any applied stress is affected by the change in stress on the system and any adjustments in heads as a result of errors in the initial head configuration (the initial conditions). Adjustments in the heads resulting from errors in the initial head configuration do not reflect changes that would occur in the actual system, but would rather represent heads, due to the mismatch between the heads specified as the initial condition and the structure of the groundwater model. Hence, it is usually preferred, to chose initial groundwater head distribution from a steady-state solution of the model, which is a valid solution regarding the structure of the groundwater model (Cirpka, 2011; Reilly et al., 2004).

However, it is important to mention that sometimes it is not possible to use the solution of a steady-state groundwater model as initial condition of a transient groundwater model. This situation arises when the groundwater model intends to simulate seasonal or other cyclic conditions where the system is never at steady state, or in instances when there is a period of unknown stress that cannot be reproduced accurately, or when it is not feasible to simulate the entire period of record from a time of steady state because of time and computational constraints. Since, it is still important that the initial conditions used do not bias the results for the period of interest, some rules of thumb for the evaluation of the appropriateness of the initial conditions in these non-ideal situations are used. Such an approach is called dynamic water balance and is used to calculate initial conditions. It makes use of the fact that groundwater flow systems do not depend upon the initial conditions after long simulation times, i.e. the impact of initial values of hydraulic heads and flow velocities decreases with time, while the impact of the structure of the model does not. The idea behind the dynamic water balance is to reduce the influence of initial values by extending the simulation period. Measured data and model results are only compared during the final part of the simulation period when it is justified to assume that the impact of errors in initial values has disappeared. The simulation period is extended by repeating the boundary conditions which are valid during the period of comparison, e.g. groundwater recharge values are projected backward in time.

4

Optimization

Optimization is an important tool in decision making which are usually applied in the field of finance and engineering. In order to make use of this tool an objective is identified which is a quantitative measure of the performance of the system under study. This objective could be profit, time, potential energy, or any quantity or combination of quantities that can be represented by some numerical measure. Solving groundwater management problems like the maximization of total groundwater extraction under constraints, as in the current study, requires optimization techniques, while the problem itself is termed as optimization problem. An optimization problem can be represented in the following way:
Given: a function $F : A \to \mathbb{R}$ from some set A to the real numbers.
Sought: an element x_0 in A such that

1. $G(x) \geq d$ or $G(x) \leq d$.

2. $F(x_0) \leq F(x)\ \forall x \in A$ ("global minimization") or such that $F(x_0) \geq F(x)\ \forall x \in A$ ("global maximization").

F is the objective function, $G(x)$ defines additional constraints and the $x \varepsilon A$ is a vector of decision variables. Depending upon whether F and G are linear or non-linear a linear or non-linear optimization method has to be applied.
Furthermore, there can be also multi-objective problems where instead of just one $F(x)$ we have $F1(x)$, $F2(x) \ldots Fn(x)$, requiring a multi-objective optimization. Only linear and multi-objective type of optimization problems and solutions are used in this study.

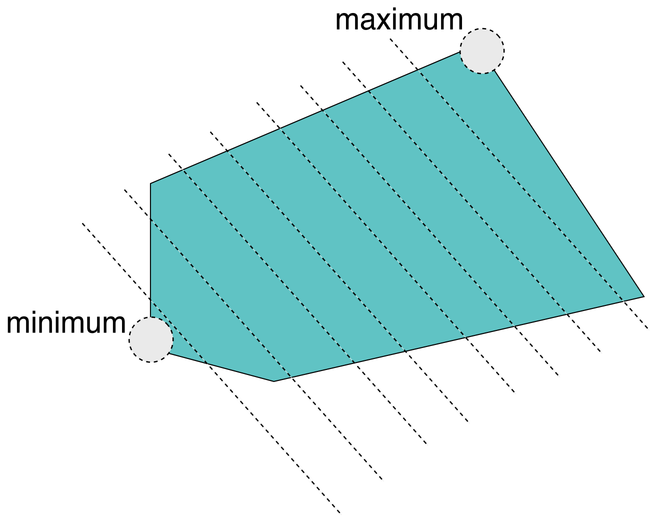


Figure 4.1: The feasible region of the solution of a linear problem with two variables and five inequalities leads to a solution space of two dimensions and five edges, respectively, and is shown in green. The contour lines of the function $F(x)$ is shown in dashed lines.

4.1 Linear optimization

Linear optimization problems can be expressed as

$$\begin{aligned} F(\mathbf{x}) &= \mathbf{c}^T\mathbf{x} \\ \mathbf{G}\mathbf{x} &\leq \mathbf{b} \end{aligned}$$

where **c** and **b** are *known* vectors while **G** is a *known* matrix. The inequality in the above problem provides a convex polytope (as shown in figure 4.1) shape over which the objective function has to be optimized and also represents the solution space. The solution to such problem always ensure a global solution and is always a vertex of the convex polytope shown in figure 4.1 (Nocedal and Wright, 2006). The pioneering contributions to the solution of linear optimization was made by John von Neumanns *theory of duality* in the year 1945 (Neumann, 1945) and George Dantzigs *simplex method* in the year 1951 (Dantzig, 1951). The *theory of duality* is used as simple formulation of the linear problem adopted from the solution of convex and non-smooth non-linear problems which used basic calculus to find the solution, while the simplex method aimed at finding the convex polytope (or a simple geometric shape) of the linear problem and selecting the solution based on the value of the objective function at vertices of the polytope (Nocedal and Wright, 2006). Also, in the 1980s it was discovered that many large linear programs could be solved efficiently by using formulations and algorithms from nonlinear programming and nonlinear equations (Nocedal and Wright, 2006; Zhang, 1998). One characteristic of these methods was that they required all iterates to strictly satisfy the inequality constraints in the problem, so they became known as interior-point methods. By the early 1990s, a subclass of interior-point methods known as primal-dual methods had distinguished itself as the most efficient practical approaches, and proved to be strong competitor to the simplex method on large problems (Nocedal and Wright, 2006). A variant of the primal-dual and interior point method as stated in (Zhang, 1998) and programmed in the MATLAB optimization function `linprog` is used in the thesis. Details of the algorithm are stated by Zhang (1998) and are not described further as this would be beyond the scope of this thesis.

4.2 Multi-objective optimization

Multi-objective optimization is the process of simultaneously optimizing two or more conflicting objectives usually subjected to certain constraints depending upon the nature of the problem. A multi-objective optimization problem can be stated in the following way:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & [F_1(\mathbf{x}), F_2(\mathbf{x}), F_3(\mathbf{x}), \ldots, F_M(\mathbf{x})] \\ \text{subject to} \quad & G(\mathbf{x}) \leq \mathbf{b} \\ & \mathbf{x_l} \leq \mathbf{x} \leq \mathbf{x_u} \end{aligned} \tag{4.1}$$

where $F_i(\mathbf{x})$ is the i^{th} objective function, M is the number of objective functions, $G(\mathbf{x}) \leq b$ is the inequality constraint, $\mathbf{x}$ is a vector of decision variables and $\mathbf{x_l}$ and $\mathbf{x_u}$ are the lower and upper bounds of the decision variable, respectively.

For example, in figure 4.2 the solution set of minimizing two linear objective functions of two variables with five linear inequality constraints is shown. The solution to this problem is a set of points called *Pareto points*, since a single value of $\mathbf{x}$ does not optimize both objective functions. Just for the sake of explanation let's call the resulting set of objective functions as objective values. Then, a set of *Pareto efficient* or non-dominated points are a result of choosing those points amongst the *Pareto points* which result in a set of objective values where none of the objective values can be minimized further without increasing at least one individual objective value. For the above cited example there are two points, the minimum of $F_1(x)$ and the minimum of $F_2(x)$, which are the *Pareto points*. The front obtained by joining such non-dominated points is called the *Pareto front* and other points are called the *Pareto inefficient* or the dominated points.

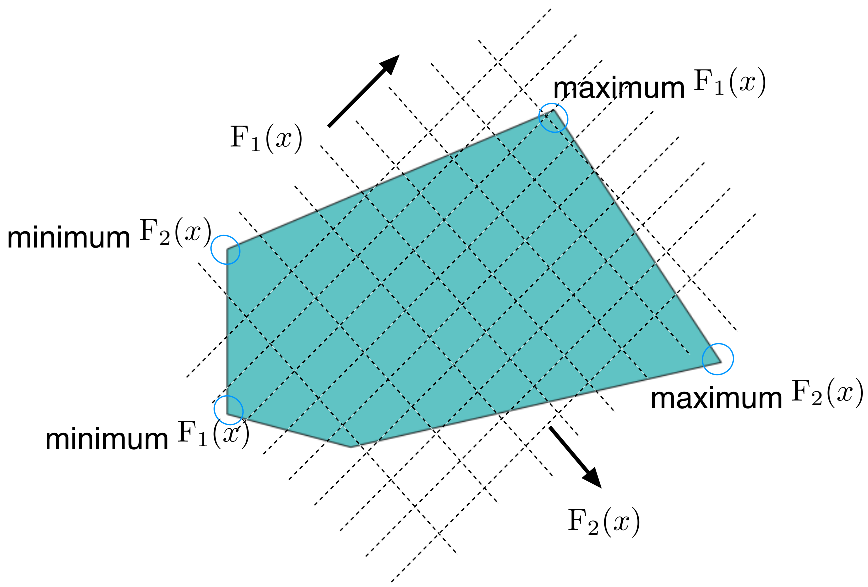


Figure 4.2: The feasible region of the solution of two linear objective problems with two variables and five inequalities leads to a solution space of two dimensions and five edges.

4.2.1 Aggregate objective function

An aggregate objective function is formed by combining the objective functions using weights followed by varying weights and the optimization as a single objective function. Such multi objective optimization has been performed in the last decade but later proved to be futile in certain cases when it was unable to capture points in the non-convex parts of the aggregate objective function (Das and Dennis, 1997). The weighted sum method, like any method of selecting a single solution as preferable to all others, is essentially subjective, in that a decision manager needs to supply the weights. In such cases it is always preferable to use other approaches in which no-prior information regarding weights is required. Although, only convex functions are optimized in the current study and the linear combination of such functions are always convex, the weights still have to be scaled so that the functions have almost the similar magnitude for optimization (Das and Dennis, 1997). An evolutionary algorithm is used in this study to overcome the aforementioned weights and scaling related problem and the multi-objective problem involved objective functions which required a binary solution.

4.2.2 Evolutionary algorithms

As mentioned before, *Pareto points* have to be selected from a large set of points resulting from multi-objective optimization. This demands a multi objective method to find as many as possible values of objective functions, resulting from a set of points x_0. The classical multi objective optimization approach such as aggregate objective function where all the objective functions are combined into a single objective function using weights, have been used in many applications and the weights have been continuously changed, hoping for new solutions, per simulation run (Das and Dennis, 1997; Deb et al., 2002). Hence, nowadays it is suggested by many authors such as Deb et al. (2002); Goldberg (1989); Srinivas and Deb (1994) to use multi-objective evolutionary algorithms based on heuristic (improving the solution by measure of quality of the current solution) approaches for solving such problems due to their ability to find multiple Pareto-optimal solutions in a single simulation run. The evolutionary optimization procedures include non-dominated sorting in genetic algorithm, strength pareto evolutionary algorithm, multi-objective evolutionary optimization vector (Das and Dennis, 1997; Goldberg, 1989; Srinivas and Deb, 1994) etc. while other methods like Particle-Swarm optimization (Kennedy and Eberhart, 1995; Venter, 2002), simulated annealing (Aarts and Laarhoven, 1989; Kirkpatrick et al., 1983) have also been suggested for solving multi-objective optimization problems. Genetic algorithms are explained in the following paragraph followed by a brief description of the non-dominated search genetic algorithm, which is a modified approach of the original genetic algorithm.

4.2.3 Genetic Algorithm

The genetic algorithms or GA are search heuristics that mimic the process of natural evolution and belong to the larger class of the evolutionary algorithms. These algorithms search for an optimal solution based upon the theory of ”survival of the fittest” which is an evolutionary selection approach.

There exists numerous algorithms based on evolutionary algorithms using heuristic optimization techniques, inspired by biological evolution mechanisms i.e. the *operators*; reproduction, mutation, selection and crossover. The solutions to the optimization problem play the role of individuals in a population, and the fitness function determines the environment within which the solutions ”live”. Evolution of the population then takes place after the repeated application of the above mentioned *operators* as described below.

Describing briefly how the GA works, a set of points (population) **x** are randomly selected and the value of the objective function is calculated. Depending upon the type of optimization problem, maximization or minimization, the randomly selected **x** are ranked on the basis of the values of the objective function, and the members of **x** corresponding to lower ranks are removed. The members of **x** corresponding to the best rank in the current generation is called the ”King”, and is sometimes represented once more, for the mating pool in the crossover, ensuring that the next population is close to the best solution, which is also called the elitist selection. The crossover performed in the next step imitates the reproduction

process observed in nature, and is performed by representing **x** in a binary number system. In the next step, mutation of a small percentage (usually up to 5 %) is performed on the obtained set of **x**, in order to preserve ”good chromosomes” lost during crossovers and to preserve genetic diversity. A high mutation rate can lead to loss of the ”good chromosomes” unless an elitist selection is followed.
Although the concept of GA is easily explained, the implementation procedures vary and a large number of genetic algorithms are available today. The aforementioned example of optimizing a single objective function can be applied similarly to the case of multi-objective optimization as defined below.

4.2.4 Non-dominated sorting in genetic algorithm (NSGA)

The non-dominated sorting in genetic algorithms is a popular approach of solving multi-objective problems. In this section the NSGA2 algorithm is directly explained, without explaining the previous strategy of its predecessor NSGA. NSGA2 implemented the usage of an elitist approach which was previously lacking in NSGA and decreased the computational complexity from $O(MN^3)$ to $O(MN^2)$, where N is the population size and M is the number of objectives (Deb et al., 2002). A brief description of the NSGA2 algorithm is adopted from the work of Seshadri (2006).
The population is initialized using a random population as described before in section 4.2.3. The population is sorted based on non-domination into each Pareto front, after the population is initialized. Successively more than one Pareto front are formed, the first one being a population of the non-dominated points, the second one being the next set of non-dominated points formed after removing the non-dominated points of the first pareto front, the third one being the next set of non-dominated points formed after removing the non-dominated points of the first and second pareto front and so on. It is obvious that the first Pareto front is completely non-dominant in the current population and the second front being dominated by the individuals in the first front only and the front goes so on. Each individual in each front are assigned rank (fitness) values based on the front in which they belong to. Individuals in the first front are given a fitness value of 1 and individuals in the second are assigned a fitness value as 2 and so on. In addition to the fitness value a new parameter called crowding distance is calculated for each individual. The crowding distance is a measure of how close an individual is to its neighbors. Large average crowding distance results in better diversity in the population. The population to be selected for crossover for the next generation is selected from the current population by using binary tournament selection based on the rank and crowding distance. The binary tournament selection performs a ”tournament” between randomly selected population and the winner is selected for the crossover. An individual is selected when the rank is lower than the other or if the crowding distance is greater than the other. The selected population reproduce from crossover and mutation operators. The population comprising

of the current population and the population after crossover is sorted again based on non-domination and only the best N individuals are selected. The selection is based on the rank and the on crowding distance on the last front. The whole procedure is repeated for the total number of generation specified.

5

Multivariate data analysis

Multivariate analysis is a term used in this study for analyzing more than one statistical variable at a time, which can be in a single or multiple dimensions. The importance of multivariate analysis holds in providing a summary and analyzing the most important dimensions in the multiple dimensional dataset of the statistical variable. In order to achieve this aim similar statistical variables in the dataset are grouped together using cluster analysis in the first step, which is followed by a principal component analysis of each group to identify patterns in the dataset of the statistical variable. Cluster analysis and principal component analysis are frequently used tools for multivariate data analysis and are therefore described in the following paragraphs.

In this thesis, the statistical variable is the vector of groundwater extraction rates estimated based on the SPR concept as a function of the constraints, as explained earlier in chapter 2. The number of dimensions equals the number of groundwater extraction wells in the city. The aim of the multivariate analysis is to find the most important dimensions or the groundwater extraction wells in each grouped dataset.

5.1 Cluster analysis

Cluster analysis is a class of statistical techniques that can be applied to data in order to analyze similar groupings. Cluster analysis sorts through the raw data and groups them into clusters. A cluster is a group of relatively homogeneous cases or observations. Objects in a cluster are similar to each other. They are also dissimilar to objects outside the cluster, particularly objects in other clusters (Abdi and Williams, 2010; Jolliffe and MyiLibrary, 2002).

The following steps should be taken during cluster analysis:

1. Formulate the problem by selecting the dataset and the variable to which the clustering technique has to be applied.

2. Select a distance measure using following ways of computing distance:

   - Euclidean distance or 2-norm - the sum of the squared differences in value for each variable and is calculated as $\sqrt{\sum(|a-b|^2)}$
   - Manhattan distance or 1-norm - the sum of the absolute differences in value for any variable and is calculated as $\sum|a-b|$
   - Chebyshev distance or infinity-norm - the maximum absolute difference in values for any variable and is calculated as $\max|a-b|$
   - Mahalanobis (or correlation) distance - this measure uses the correlation coefficients between the observations and uses that as a measure to cluster them. This is an important measure since it is unit invariant (can figuratively compare sets including apples and oranges). It is calculated as $\sqrt{\sum(|a-b|^2 C_{ab}^{-1})}$
     where a and b is a pair selected from the dataset and C_{ab} is the covariance between a and b.

3. Select a clustering procedure (see below)

4. Decide on the number of clusters

5. Interpret clusters

There are many possible methods of cluster analysis, and several books have appeared on the subject, for example in Abdi and Williams (2010); Blashfield and Aldenderfer (1978); Everitt et al. (2001); Johnson and Wichern (2002); Jolliffe and MyiLibrary (2002).
Clustering techniques are mainly classified as non-hierarchical and hierarchical clustering (Johnson and Wichern, 2002). The non-hierarchical clustering or k-means clustering (used in this thesis), determines a cluster center in the first step, followed by grouping objects within a center distance.

5.2 Principal component analysis

The central idea of principal component analysis (PCA) is to reduce the dimensionality of a dataset consisting of a large number of interrelated variables, while retaining as much variation present in the dataset as possible. This is achieved by transforming to a new set of variables, the principal components (PCs), which are uncorrelated, and ordered such that the first few retain most of the variation present in all original variables (Abdi and Williams, 2010).

PCA is mathematically defined as an orthogonal linear transformation that transforms the dataset to a new coordinate system, such that the greatest variance by any projection of the data comes to lie on the first coordinate (called the first principal component), the second greatest variance on the second coordinate, and so on.

In this thesis, PCA is done by eigenvalue decomposition of the grouped dataset (using cluster analysis) covariance (or correlation) matrix or singular value decomposition of the grouped dataset matrix, after mean centering the grouped dataset for each dimension, using singular value decomposition as explained in the next section. The results of a PCA are discussed in terms of component scores, sometimes called factor scores (the transformed variable values corresponding to a particular data point), and loadings (the weight by which each standardized original variable should be multiplied to get the component score see Abdi and Williams (2010); Jolliffe and MyiLibrary (2002); Shaw (2003)). The scores give information about the most important dimensions of the grouped dataset in the corresponding principal component. The loadings are correlation coefficients between the principal component scores and the original variables and are also a measure of the importance of each variable in accounting for the variability in the principal component.

5.2.1 Singular value decomposition

Singular value decomposition (SVD) is used to find the principal component by finding a linear combination of dimensions in the dataset matrix, which results in maximum covariance and orthogonal vector of loadings (Abdi and Williams, 2010; Jolliffe and MyiLibrary, 2002). The next principal components are formed in a decreasing order of these covariances while preserving orthogonality between loadings. The singular value decomposition of an $m \times n$ matrix X is calculated by factorization as follows:

$$\mathbf{X} = \mathbf{W}\Sigma\mathbf{V}^{\mathbf{T}} \tag{5.1}$$

where $\mathbf{X}$ is the grouped dataset, $\mathbf{W}$ is an $m \times m$ orthogonal matrix, Σ is an $m \times n$ rectangular diagonal matrix with nonnegative numbers on the diagonal, and $\mathbf{V}^{\mathbf{T}}$ (the transpose of $\mathbf{V}$) is an $n \times n$ orthogonal unitary matrix. The diagonal entries $\sum$ are known as the singular values of $\mathbf{X}$. The m columns of $\mathbf{W}$ and the n columns of $\mathbf{V}$ are called the left singular vectors and right singular vectors of $\mathbf{X}$, respectively.

The singular value decomposition and the eigendecomposition are closely related in which the left singular vectors and right singular vectors are eigenvectors of $\mathbf{X}\mathbf{X}^{\mathbf{T}}$ and $\mathbf{X}^{\mathbf{T}}\mathbf{X}$, respectively. For example, considering the grouped dataset matrix $\mathbf{X}^{\mathbf{T}}$, with zero empirical mean (the empirical (sample) mean of the statistical variable has been subtracted from the dataset), where each of the n rows is a statistical variable representing a different repetition of a similar problem, and each of the m columns gives a dimension of the statistical variable. It should be noted that $\mathbf{X^T}$ is defined here and not $\mathbf{X}$ itself, and what we are calling $\mathbf{X^T}$ is often alternatively denoted as $\mathbf{X}$ itself. The scores and loadings are calculated by performing PCA transformation of each grouped dataset, such that the dimensionality is preserved (i.e. gives the same number of principal components as original variables)

$$\begin{aligned} \mathbf{Y^T} &= \mathbf{X^T W} \\ &= \mathbf{V \Sigma^T W^T W} \\ &= \mathbf{V \Sigma^T} \end{aligned} \tag{5.2}$$

The first column of $\mathbf{Y^T}$ is made up of the scores of the cases with respect to the principal component, the next column has the scores with respect to the second principal component, and so on while $\mathbf{W\Sigma}$ is the loading matrix.

6

Artificial groundwater recharge

Artificial recharge to groundwater aims at augmentating an aquifer by modifying the natural movement of surface water, utilizing suitable techniques. Artificial recharge techniques addresses the following issues:

1. To raise the water levels in areas where overexploitation has depleted the aquifer.
2. To conserve and store excess surface water for future requirements, since these requirement often changes within a season or a period.
3. To improve the quality of existing ground water through dilution.
4. To remove bacteriological and other impurities from sewage and waste water so that water is suitable for re-use.

Since the study focuses on groundwater quantity issues, only the first two points are considered. The application of the concept requires considering geological, social and economical constraints. Considering geological constraints, it is important to specify that a proper groundwater recharge measure is taken depending upon the surface and subsurface storage capacity. Artificial recharge methods can be classified into two broad groups (i) direct methods, and (ii) indirect methods (REC, 2000; Bhattacharya, 2010).

6.1 Direct artificial recharge

The direct recharge measures include surface and subsurface recharge methods. The surface groundwater recharge methods include surface spreading techniques such as

1. Flooding: It is performed by spreading the surplus surface-water from canals / streams over a large area for a sufficiently long period so that it recharges to the groundwater body. This technique can be used for gently sloping land with a slope of up to 1 to 3 % without gullies and ridges.

2. Ditches and Furrows: In areas with irregular topography, shallow, flat-bottomed and closely spaced ditches and furrows provide maximum water contact area for recharging water from the source stream or canal. This technique requires less soil preparation than the recharge basin technique and is less sensitive to silting.

3. Recharge Basins: Artificial recharge basins are either excavated or enclosed by levees. They are commonly built parallel to stream channels. The water contact area in this method is quite high which typically ranges from 75 to 90 % of the total recharge area. In this method, efficient use of space is made and the shape of basins can be adjusted to suite the terrain and the available space.

4. Run-off Conservation Structures: In areas receiving low to moderate rainfall, mostly during a single monsoon season, and not having access to water transferred from other areas, the entire effort of water conservation is required to be related to the available in-situ precipitation.

5. Gully plugs: These are the smallest run-off conservation structures built across small gullies and streams rushing down the hill slopes carrying drainage of tiny catchments during the rainy season. Usually, the barrier is constructed by using local stones, earth and weathered rock, brushwood, and other local materials. Sloping lands with surface gradients up to 8 % having adequate soil cover can be leveled through bench terracing for bringing under cultivation. It helps in soil conservation and holding run-off water on terraced area for longer durations giving rise to increased infiltration recharge.

6. Contour barriers: This method involves a watershed management practice so as to build up soil moisture storages. This technique is generally adopted in areas receiving low rainfall. In this method, the monsoon run-off is impounded by putting barriers on the sloping ground all along contours of equal elevation. Contour barriers are taken up on lands with moderate slopes without involving terracing. In areas where uncultivated land is available in and around the stream-channel section, and sufficiently high hydraulic conductivity exists for sub-surface percolation, small tanks are created by making stop dams of low elevation across the stream. The tanks can also be located adjacent to the stream by excavation and connecting them to the stream through delivery canals.

7. Stream-channel Modification: The natural drainage channel can be modified to increase the infiltration by detaining stream flow and increasing the stream-bed area in contact with water. This method can be employed in areas having influent streams (stream-bed above water table) which are mostly located in piedmont regions and areas with deep water table (semiarid, arid region and valley filled with impervious deposits). Stream-channel modification methods are generally applied in alluvial areas.

6.2 Indirect artificial recharge

In case impervious layers overlie deeper aquifers, infiltration from the surface cannot recharge the sub-surface aquifer under natural conditions and special sub-surface recharge techniques need to be applied (REC, 2000; Bhattacharya, 2010).

1. Injection Wells: Injection wells are structures similar to groundwater extraction wells but with the purpose of augmenting the groundwater storage of an aquifer by actively injecting treated surface water. These injection wells are used for replenishment of the overexploited aquifers where declining trends of water levels have already set in. Similar injection well recharge measures are also taken in coastal regions to stop seawater intrusion and to combat the problems of land subsidence in areas where confined aquifers are heavily pumped. Due to higher well losses caused by clogging, injection wells display lower efficiency (40 to 60 %) as compared to a pumping well of similar design in the same situation. The source water and the water in the aquifer should be compatible to avoid precipitation, causing clogging of the well. Injection together with pumping wells are more efficient because the well can be cleaned during pumping operation.

2. Gravity-Head Recharge Wells: As an alternate to specially designed injection wells, ordinary bore wells and dug wells used for pumping may also be used as recharge wells, whenever source water becomes available. In certain situations, such wells may also be constructed for effecting recharge by gravity inflow. In areas where water levels are currently declining due to over-development, using available structures for inducing recharge may be both, an immediate and economically available option.

3. Connector Wells: Connector wells are a special type of recharge wells where, due to difference in potentiometer head in different aquifers, water can be made to flow from one aquifer to another without any pumping (syphon principle). The aquifer horizons having higher heads start recharging the aquifer having lower heads.

4. Recharge pits: Recharge pits are structures that overcome the difficulty of artificial recharge of phreatic aquifer from surface-water sources. Recharge pits are excavations of variable dimensions that are sufficiently deep to penetrate less permeable surficial strata. A canal trench is a special case of a recharge pit dug across a canal bed. An ideal site for a canal trench is the influent stretch of a stream that appears as a dry patch. One variation of the recharge pit is a contour trench extending over long distances across the slope and following topographical contour. This measure is more suitable in piedmont regions and in areas with higher surface gradients. As in case of other water spreading methods, the source water used should be free of silt and turbidity. In case of hard rock terrain, a canal bed section crossing permeable strata of weathered fractured rock or the canal section coinciding with a prominent lineament or an intersection of two lineaments, form ideal sites for canal trench.

5. Recharge Shafts: In case, poorly permeable strata overlie the water table aquifer located deep below land surface, a shaft is used for causing artificial recharge. A recharge shaft is similar to a recharge pit but much smaller in cross-section.

The depleted aquifers are technically feasible alternatives for storing surplus monsoon run off, which can store a substantial quantity of water. These aquifers may be considered as ”warehouse” for storing water that come from sources located on the land surface. Besides suitable lithological condition, other considerations for creating sub-surface storages are favorable geological structures and physiographic units, whose dimensions and shape will allow a retention of substantial volume of water in permeable formations (REC, 2000; Bhattacharya, 2010).

6.3 Feasible recharge measures

The sub-surface storage has the advantage of being free from the adverse effects like inundation of large surface area, loss of cultivable land, displacement of local population, substantial evaporation losses and sensitivity to earthquakes. No gigantic structures are needed to store water. The underground storage of water would also have beneficial influence on the existing ground water regime. The deeper water levels in many parts of India, either of natural occurrence or due to excessive ground water development, may be substantially raised, resulting in reduction in lifting costs and energy saving. The structures required for subsurface groundwater recharge are of small dimensions and cost effective, such as check dams, percolation tanks, surface spreading basins, pits, subsurface dykes etc.
The indirect methods of artificial groundwater recharge include methods such as raising the groundwater

level in order to fill the surface reservoirs, aquifer modification involving techniques to modify aquifer properties by bore blasting or hydro-fracturing, and creating groundwater dams to retard the groundwater flow.
Out of the above mentioned direct recharge methods, the surface recharge measures can hardly be considered in an urban setting due to space constraints. In the case of India with sub-tropical climate, stagnant water in surface reservoirs holds risk of water borne diseases such as malaria. Also the high potential evapotranspiration rates in Lucknow, prove such measures to be futile. Additionally, most of the above techniques are not cost-efficient or require a considerate amount of capital investment by the local, state or federal authorities. Considering sub-surface recharge measures, the percolation tanks can be used to collect the rainwater to use for recharge. This method is called rooftop rainwater harvesting (RRWH) and is considered geologically feasible due to the mostly alluvial subsurface geology in the city of Lucknow (CGW, 2009). Furthermore, RRWH has been already implemented in Chennai, India, where the method was socially accepted.

6.4 Economic importance

In the current study, a hypothetical economic-equality based water pricing for the city of Lucknow is aimed at. The industrial and residential water demand in the city is comparatively higher than its peri-urban and rural counterparts. The water prices are therefore higher in the city as compared to the peri-urban and rural areas. Neglecting the necessity of water in peri-urban and rural areas can lead to social issues such as unemployment and poverty. An economic feasibility assessment of the RRWH scheme based on the economic-equality based water pricing informs in advance about the likelihood of a benefit. The feasibility depends upon the investment and drawdown costs and is explained in the following paragraphs.
By the definition of Theis (Theis, 1940), long term groundwater extraction results in spatially extending groundwater drawdowns. Considering the city of Lucknow, the groundwater extractions have led to drawdowns extending in the peri-urban and rural areas outside the city of Lucknow. This can be converted to the amount of groundwater withdrawn from these areas, which can be estimated using the groundwater levels before and after extraction. Hence groundwater extraction should be furnished with a cost in order to compensate the peri-urban and rural population and are called drawdown costs. Similarly another compensation cost should be attached to groundwater extraction in the city to pay for expenses related to artificial groundwater recharge methods if deployed in the city. These are called investment costs hereafter.

For example, the investment and drawdown costs depend upon the number of houses and investment per RRWH installation, and the spatial costs of water in the peri-urban and rural areas, respectively. Hence, if RRWH is implemented, the cost of the drawdowns (in peri-urban and rural areas) due to extraction is added to the investment costs i.e.

1. RRWH implementation costs = *investment costs(number of houses, unit investment per RRWH installation) + drawdown costs(location, unit price of drawdown, groundwater extraction, RRWH)*
2. Without RRWH implementation costs = *drawdown costs(location, unit price of drawdown, groundwater extraction).*

Economic benefit can be estimated by dividing total costs in RRWH by total groundwater extraction as

$$\text{cost benefit} = \frac{\text{RRWH implementation costs}}{\text{Without RRWH implementation costs}} \tag{6.1}$$

where a cost benefit greater than 1 indicates that the RRWH implementation is beneficial. Also, the minimum efficiency of the RRWH can be calculated as $\frac{1}{\text{cost benefit}}$ for the economic feasibility of such measure.

An inverse procedure can be used to estimate the areas yielding maximum economic benefit. In this study, the areas in such case are defined on the basis of local administration blocks and in the absence of detailed data of the urban structures in individual areas, the number of houses in each area can be assumed to be directly proportional to the area.

7

Regional groundwater flow model

During the introduction of the thesis it was shown that a numerical groundwater model was necessary to overcome the drawbacks of the water table fluctuation methods and for the application of the SPR concept. The required groundwater model can simulate the groundwater heads up to a desired level of accuracy, which should be taken into account while assessing the model results.

The groundwater flow equations used in the groundwater model can simulate the groundwater heads, in a case the exact values of the parameters used in the equations are exactly known. Using parameter values from different sources, the state variables can be predicted on the basis of physical interactions in the model. The calculated state variables, namely the groundwater heads and groundwater discharge to the river depend on the parameter values. However, these calculations do not give exact predictions, due to error in the model structure and parameter uncertainties.

In order to achieve the aforementioned objective, it is necessary to develop a regional (due to the size of the city) groundwater flow model for the city of Lucknow which requires the steps of code selection and verification, appropriate model structure, sensitivity analysis, model calibration and model validation. These requirements are listed stepwise in the coming section and are shown in the form of flowchart in the figure 7.1. They serve as a foundation block in supporting the site-specific groundwater model of producing meaningful results (Anderson and Woessner, 1992) and will be used in the coming chapters for the development of the regional groundwater flow model for the city of Lucknow.

7.1 Establish the purpose of the model

The purpose of developing the regional groundwater model has to be established first, in order to proceed with the further steps in the model protocol. The model purpose will lead to a proper selection of

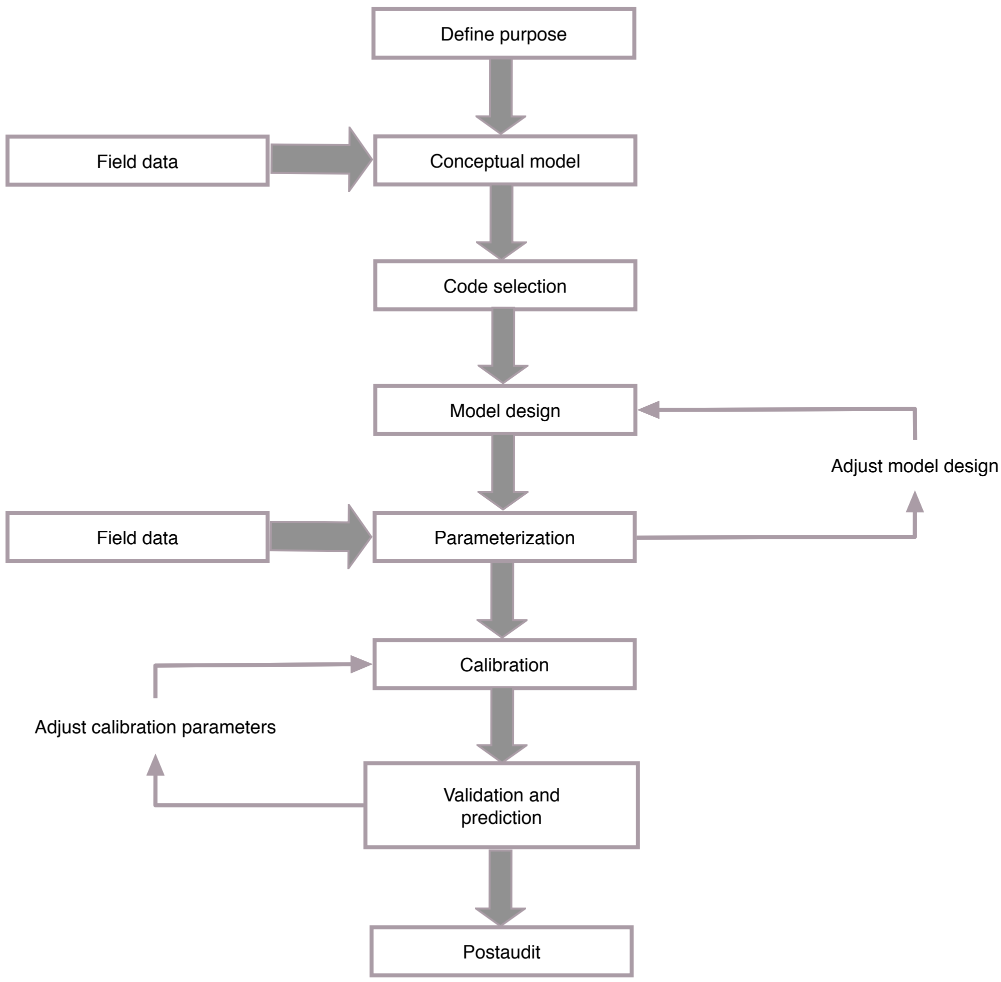


Figure 7.1: Steps for developing regional groundwater model adopted fromAnderson and Woessner (1992)

equations and thereafter the numerical code. It also decides the data necessary for using such a code.
As established in the introduction, the SPR concept aiming for sustainability requires the estimation of groundwater heads and groundwater drainage to the river, for values of d_{crit} and q_{crit}. These state variables can be calculated using the groundwater equations 3.14 and 3.15.

7.2 Develop a conceptual model of the system

A conceptual model of the system is a pictorial representation of the groundwater flow system, usually in the form of a block diagram or a cross-section, required for defining the hydrostratigraphic units and system boundaries. The conceptual model is simplified in order to remove unimportant data requirements and additional equations to build the regional groundwater flow model. Theoretically stating, the closer the conceptual model is to the reality, the better is the numerical model. However, in practice parsimony is always desired, to simplify the model but still retain enough complexity to correctly produce the state variables. The conceptual model should represent correctly the important hydrogeological conditions, since the failure of numerical models to represent the reality is often due to wrong conceptual models (Anderson and Woessner, 1992).
The important features required of a conceptual model are; defining the area of interest, identifying the boundaries of the model, defining the groundwater flow system, and defining hydrostratigraphic units.

7.3 Selecting the computer code

Selection of the computer code mainly depends upon the groundwater equations which represent the groundwater flow system in the area of interest. The selected computer code is based on an algorithm which solves a set of groundwater equations numerically. In the case that the groundwater equations are based upon differential equation, a finite difference or finite element scheme can be applied for numerical solution.
The verification process is performed at two levels: the computer code and the governing equations. The verification of the computer code ensures that the algorithm can successfully solve the equations, and can be checked by comparing solutions to analytical solutions, while the verification of the governing equations establishes some confidence by comparing results with laboratory experiments or field data.
The verification of the governing equations, i.e. the groundwater flow equations, can be ensured by comparing the observed and simulated groundwater heads, which is ensured by executing the following stated steps of the modeling protocol.
The groundwater equations 3.14 and 3.15 are assumed to represent the groundwater flow system in

Lucknow. In order to solve these groundwater equations a variety of commercial (such as FEFLOW (Diersch, 2005)) and non-commercial (such as MODFLOW (Harbaugh, 2005)) codes are available.

7.4 Model design

The conceptual model is numerically defined during the step of model design, aiming towards grid discretization, time steps, boundary and initial conditions.

In a numerical model, the continuous problem domain is replaced by a discretized domain, consisting of an array of nodes and associated finite difference cells or finite element (Anderson and Woessner, 1992). The first step of model design is to discretize the domain on the basis of the type of numerical scheme selected to solve the differential equations, which define the groundwater flow system. It is important to state that the number of finite difference cells or finite elements and the number of groundwater flow equations, determine the computational requirements of the numerical model, while the size of the cells or the elements determine the extent to which hydraulic properties and stresses can vary throughout the model. Hence, the higher the number of cells, the easier it becomes to represent the lateral variation of hydraulic properties and stresses in the model. Again, following the principle of parsimony, a decision has to be made between the cell size and acceptable computational processing and time demands associated. In few cases, a variable cell size (also known as variable discretization) is used to reduce the computational demand.

The finite difference method solves the groundwater equation using a suitable numerical solution algorithm in computational domains discretized by a rectangular grid. The finite difference solution algorithm obtains primarily the unknown hydraulic heads at the nodes of rectangular elements in the model. The grid can be discretized in two ways, block-centered and mesh-centered. In the block centered approach, flux boundaries are always located at the edge of the block while in the vertex-centered grid the boundary coincides with a node (Anderson and Woessner, 1992).

The finite element method divides the domain into a finite number of elements, characterized by discrete nodes, generally allowing for more flexibility while creating grid, and the groundwater equation at each node is solved using a suitable numerical scheme. One-dimensional elements can be one-dimensional line or curve elements; two-dimensional elements can be either triangular, rectangles or quadrilaterals; and three-dimensional elements can be prisms, pyramids or bricks. The key of the finite element method is a minimization principle, that allows to solve the nodal values so that the numerical error is minimized on average over the domain.

Temporal discretization is required in a transient groundwater model, deciding the effect of stress on the state variables over time. These time steps are required to simulate the distribution of state variables over time, similar to the need of more cells or nodes in a groundwater model. Boundary conditions are required for simulating a groundwater model since the numerical scheme solves one or more differential equations that require particular conditions to be defined along all boundaries of the model domain (Cirpka, 2010) and represent any flow or head constraint within the groundwater flow domain (Franke et al., 1987). These boundary conditions are not only mathematical constraints but also represent the sources and sinks within the model domain. For example, in a groundwater model, the recharge due to precipitation is a source, represented using a Neumann boundary condition while a gaining river in a groundwater model is a sink represented using Cauchy boundaries (as per definition of the boundaries see section 3.2.2).

Similarly, while addressing transient solutions of groundwater flow equations represented using the differential equation, the solution requires an initial value. The initial value of the transient model has already been discussed in the section 3.2.2 and an appropriate initial condition can be chosen.

7.5 Parameterization

During the process of making a relevant regional groundwater flow model, producing meaningful results, a set of parameters are required which simulate groundwater heads as close as possible to the observations. This process is called calibration. Before parameter estimation is performed using calibration, the process of parameterization identifies the aspects of the simulated system that are to be represented by the estimated parameters (Hill and Tiedeman, 2005). Hence, before performing calibration it is important to identify the parameters that are to be used in model calibration. These parameters are usually the hydraulic parameters used in the groundwater flow equations, while sometimes parameters of other components of the hydrological cycle, termed as non-hydraulic parameters, are used.

Same hydraulic or non-hydraulic parameter can be spatially distributed in the model, representing the spatial distribution as given in the field tests and geological reports. In most circumstances, it is useful to begin with simple models. Complexity can gradually be increased as warranted by the complexity of the system, the inability of the model to match observed values, and the importance of the complexities to the predictions of interest.

Furthermore, the parameterization process includes the range of acceptable values for the hydraulic and non-hydraulic parameters. The range of acceptable parameter values determines; the valid search space in mathematical terms which determines the computational effort and gives an idea of the right initial values for finding the calibrated parameters.

7.6 Sensitivity Analysis and Calibration

It is usually assumed that the groundwater flow equations as well as the boundary and initial conditions represent the groundwater flow system and the groundwater observations. The model with parameters used from various sources cannot predict exactly the groundwater observations. Although the value ranges of the parameters are usually known through literature or field tests, parameter estimation is always required either due to incomplete quantitative information of these parameters or due to the difference in scales at which the field test are performed and the scale at which they are used in the model. Also, there can be uncertainties in the conceptual model related to the boundary conditions. Despite of our knowledge that the conceptual uncertainties are the most important cause of error, only the parameter uncertainties are determined during calibration. In order to match the observed and simulated groundwater heads, the parameters **p** established during the process of parameterization are estimated, termed as parameter estimation.

Sensitivity analysis and calibration can be performed automatically using computer codes (such as UCODE (Hill and Tiedeman, 2005)) or manually. A high computational effort for running the forward simulation may be required, depending upon the number of equations being solved, grid discretization and time steps, as discussed above. These factors govern the decision of choosing an automated or manual calibration (Cirpka, 2010). However, an automated calibration is always preferred over manual calibration due to its adequate search strategies for finding values of calibration parameters. Also, manual calibrations are prone to inconclusive decision of the person performing calibration.

Sensitivity analysis is the evaluation of model parameters in terms of their importance to observations. Sensitivity analysis holds importance, since most data sets only support the estimation of relatively few parameters, which need to be identified in order to reduce the computational effort during model calibration (Foglia et al., 2009; Hill and Tiedeman, 2005). Such information may be gathered using composite scaled sensitivities (CSS). The advantage of composite scaled sensitivities over standard sensitivities is that these are dimensionless and allows a direct comparison of the relative importance of the parameters with different dimensions. The composite scaled sensitivity of each parameter j is calculated using:

$$Css_j = \left(\frac{1}{n_{obs}} \sum \left(\left. \frac{\partial f(\mathbf{x},t,\mathbf{p})}{\partial p_j} \right|_{\mathbf{p}} p_j C_n^{-\frac{1}{2}} \right)^2 \right)^{\frac{1}{2}}$$

where $\partial f(\mathbf{x},t,\mathbf{p})/\partial p_j$ is the sensitivity of the simulated state variable $f(\mathbf{x},t,\mathbf{p})$ with respect to the j^{th} parameter, n_{obs} denotes the number of observations, C_n is the measurement error of the n^{th} observation $o(\mathbf{x},t)$. $\mathbf{x}$ and t are the location of the observation and time, respectively. In case that the number of parameters is n_p, $n_p + 1$ simulations are required for evaluating composite scaled sensitivities. n_p simulations are required for incrementing each parameter in the vector **p** while keeping all other parameters the same while one simulation is required using unchanged parameters.

The correlation coefficient between two parameters j and k is calculated to identify whether co-ordinated change between these two parameters results in same model fit. It is calculated by:

$$PCC_{jk} = \frac{C_{p_j,p_k}}{\sigma_{p_j}\sigma_{p_k}} \tag{7.1}$$

where C_{p_j,p_k} and σ_{p_j} are calculated by:

$$\begin{aligned} \sigma^2_{p_j} &= \sum \left(\frac{\partial p_j}{\partial f(\mathbf{x},t,\mathbf{p})} \right)^2 C_n \\ C_{p_j,p_k} &= \sum \left(\frac{\partial p_j}{\partial f(\mathbf{x},t,\mathbf{p})} \right) \left(\frac{\partial p_k}{\partial f(\mathbf{x},t,\mathbf{p})} \right) C_n \end{aligned}$$

A PCC value above 0.85 is considered to be high linear correlation (Foglia et al., 2009; Hill and Tiedeman, 2005). The assessment of composite scaled sensitivities and parameter correlation aid in identification of the parameters that can be estimated during model calibration. However, the set of parameter values which result in minimum difference between the simulated and observed groundwater heads stays unknown.

The transient state calibration of the model is performed to estimate model parameter values by minimizing the objective function:

$$\chi^2 = \varepsilon^T \mathbf{C}^{-1} \varepsilon \tag{7.2}$$

where ε is a vector of model errors consisting of n_{obs} entries $\varepsilon_i = o(\mathbf{x},t) - f(\mathbf{x},t,\mathbf{p})$. The error covariance matrix **C** is diagonal under the assumption that the errors are uncorrelated. The errors represented in the matrix **C** correspond to the measurement errors and ε reflect the model errors related to the uncertainty in the structure of model and boundary conditions. The measurement errors accounts for inaccuracy of the water level measuring device and location of the groundwater observation point (Anderson and Woessner, 1992).

Depending upon the number of parameters n_p, greater than, less than or equal to the number of observations n_{obs}, the calibration problem can be described as, underdetermined, overdetermined and well determined, respectively. However, in practice, overdetermined problems are often found due to high number of groundwater observations as compared to the number of parameters used in the model.

χ^2 can be minimized using proper search techniques such as Gauss-Newton, Levenberg-Marquardt etc..

Describing briefly, a Gauss-Newton method iterates to the final solution by evaluating χ^2 using an initial guess and then updating the current value of **p** using the Jacobian of χ^2. The Levenberg-Marquardt method is a modified Gauss-Newton method in which the current value of **p** is modified using the Jacobian and a scaling factor.

Removing correlation during sensitivity analysis ensures that the calibration process is not caught in iterations with successive iterations producing the same model fit and increases the chance of the solution being unique. However, the search technique cannot ensure if the solution is unique. Hence, different initial value of the parameters should be used while performing a calibration.

7.7 Validation and prediction

The step of model validation and prediction in the model protocol determines the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model (Anderson and Woessner, 1992; Foglia et al., 2009; Hill and Tiedeman, 2005; Klemeš, 1986; Perez et al., 2011). This information is used to decide whether or not the model has resulted in an acceptable agreement with the groundwater observations. The question of whether or not the model is adequate for its intended use is usually broader and has to be decided by the modeler based on his preference. The decision of acceptable agreement focuses only on the level of agreement between groundwater observations and simulation results, the criteria for selecting the observations are specified as part of the conceptual model. If the value of χ^2 is high, the model and/or the groundwater observations can be revised. Model revision is the process of changing the basic assumptions, structure, parameter estimates, boundary values, or initial conditions of a model to improve agreement with groundwater heads. Revising groundwater observations is the process of determining the errors in measurements or human error. Whether the model or the groundwater observations (or both) are revised depends upon the judgment of the model developer (Anderson and Woessner, 1992).

However, the decision of the modeler who accepts the agreement between observed and simulated groundwater heads, has to defend the end results using statistical criteria, and provide quantitative information about the amount of error existing in the results of the groundwater model during prediction. The four statistical criteria employed to judge quantitatively the goodness of fit between observed and simulated groundwater heads are, the maximum error (ME), the root mean square error (RM), coefficient of residual mass (CR) and Nash-Sutcliffe (NS) (Nash and Sutcliffe, 1970). These statistics have been used previously in estimating residuals and systematic characterization of underestimation and overestimation (e.g. Loague and Kyriakidis (1997), Jones et al. (2008) and Foglia et al. (2009); Perez et al. (2011)). *RMSE* represents an aggregated error of model precision; negative and positive values of *CR* is the aggregated measure of the over or underestimation of the simulated values, respectively; the Nash-Sutcliffe index compares the explained variation in the observations to a model consisting of mean observations. The ideal value for *ME*, *RMSE*, *CR* and *NS* are 0, 0, 0 and 1, respectively, implying that the observed and simulated groundwater heads are identical.

$$ME = max|o(\mathbf{x},t) - f(\mathbf{x},t,\mathbf{p})| \tag{7.3}$$

$$CR = \frac{\sum(o(\mathbf{x},t) - f(\mathbf{x},t,\mathbf{p}))^{\mathbf{2}}}{\sum o(\mathbf{x},t)} \tag{7.4}$$

$$RMSE = \left[\frac{1}{n_{obs}}\sum(o(\mathbf{x},t) - f(\mathbf{x},t,\mathbf{p}))^{\mathbf{2}}\right]^{\frac{1}{2}} \tag{7.5}$$

$$NS = 1 - \frac{\sum(o(\mathbf{x},t) - f(\mathbf{x},t,\mathbf{p}))^{\mathbf{2}}}{\sum\left(o(\mathbf{x},t) - \frac{1}{n_{obs}}\sum o(\mathbf{x},t)\right)^2} \tag{7.6}$$

There exists no strict protocols for validation and prediction. However Klemeš (1986) gave a hierarchical scheme for systematic testing of such models.

1. Split-sample test: calibration based on one time period and validation on another period.

2. Differential split-sample test: calibration on periods with certain conditions (climatic or land-use) and validation on periods with different conditions.

3. Proxy basin test: calibration of a model on data from one or several catchments and validation in another, but similar, catchment. Adjustment of parameter values according to catchment properties but no calibration is allowed.

4. Proxy-basin differential split-sample test: calibration of a model on data from one or several catchments and validation in another catchment with different characteristics. Adjustment of parameter values according to catchment properties but no calibration is allowed.

In general, the scheme proposed by Klemeš (1986) describes tests on how well a model can be represented temporally and spatially. These tests are possible for state variables calculated using the groundwater model. The importance of this scheme can not be neglected, but as mentioned by Klemeš (1986), it is a minimum standard rather than a complete catalogue of possible tests.

7.8 Post-audit

Model calibration and verification demonstrates that the model can mimic past behavior. As explained in the model protocol by Anderson and Woessner (1992), post-audit is a predictive validation requiring new field data which is collected several years after the modeling study was completed to determine whether the prediction came true. During the process of post-audit, the simulated heads are compared with the new field data (Anderson and Woessner, 1992). The work of Anderson and Woessner (1992) showed that the validation of the post-audit should be performed long enough after the prediction is performed, in order to ensure that the groundwater model was given enough time for significant changes (due to stress) to occur. A post-audit performed too soon after the initial calibration may lead to a conclusion that the prediction came close to estimating the observed values, when in fact not enough time elapsed to allow the system to move sufficiently far from the calibrated solution (Anderson and Woessner, 1992). However, it should be noted that it is not common for groundwater modelers to perform a post-audit as can be seen in the recent works by Foglia et al. (2009); Perez et al. (2011); Romano and Preziosi (2010) etc.. Also Anderson and Woessner (1992) states that post-audit may not be necessary, if the purpose of the model is to analyze current steady-state behavior or to make short-term term predictions while it may be necessary when the model is used to make predictions on the order of tens or hundreds of years. Hence, it is necessary that the modeling process involves post-audit in a case in which it is necessary followed by subjective judgements about the magnitude of acceptable error, which includes measurement error in the field data and modeling error represented by the difference between simulated and observed values.

8

Lucknow groundwater model

8.1 Data inventory

Lucknow is situated in the Sai-Gomti basin, which is a part of the Central Ganga basin of India. Its geographical location is shown in figure 8.1. The hydro-meteorological data collected for this study is shown in table 8.1. River discharge and groundwater head data were collected from Ministry of Water Resources and Central Groundwater Board in India, while the meteorological data was obtained from the World Data Center of Meteorology. The availability of time series was the deciding factor for choosing appropriate calibration and validation periods, while following the model protocol.
The city of Lucknow stretches across both banks of the Gomti river which is a tributary of the river Ganges. The river Gomti originates near the Madho Tanda in the Pilibhit district, India. The total length of the river is about 900 km and drains a catchment area of about 30,437 km^2. It is situated in the interfluvial region of the rivers Ganges and Ghaghara. The river forms an elongated basin trending in northwest-southeast direction. The northern part of the basin occupies the piedmont zone of the Himalyan foothills. The tributaries of the river Gomti are Kukrail, Loni, Betha and Reth. After traversing for approximately 240 km from its source, river Gomti enters the city of Lucknow through which it meanders for about 12km (Verma et al.). Except during monsoon, the river is fed by groundwater (CGW, 2009).
The climate in the region is subtropical with three distinct seasons, summer, monsoon and winter with an average rainfall of 1,110 mm/yr, out of which 1000 mm/yr occurs during the 45 monsoon days between June and August (Foster and Choudhary N., 2009; Livingston, 2009). The average temperature of the region lies between 35° C to 40° C. The average relative humidity varies between 25% in the morning to 68% in the evening. The district of Lucknow approximately covers an area of 2528 km^2

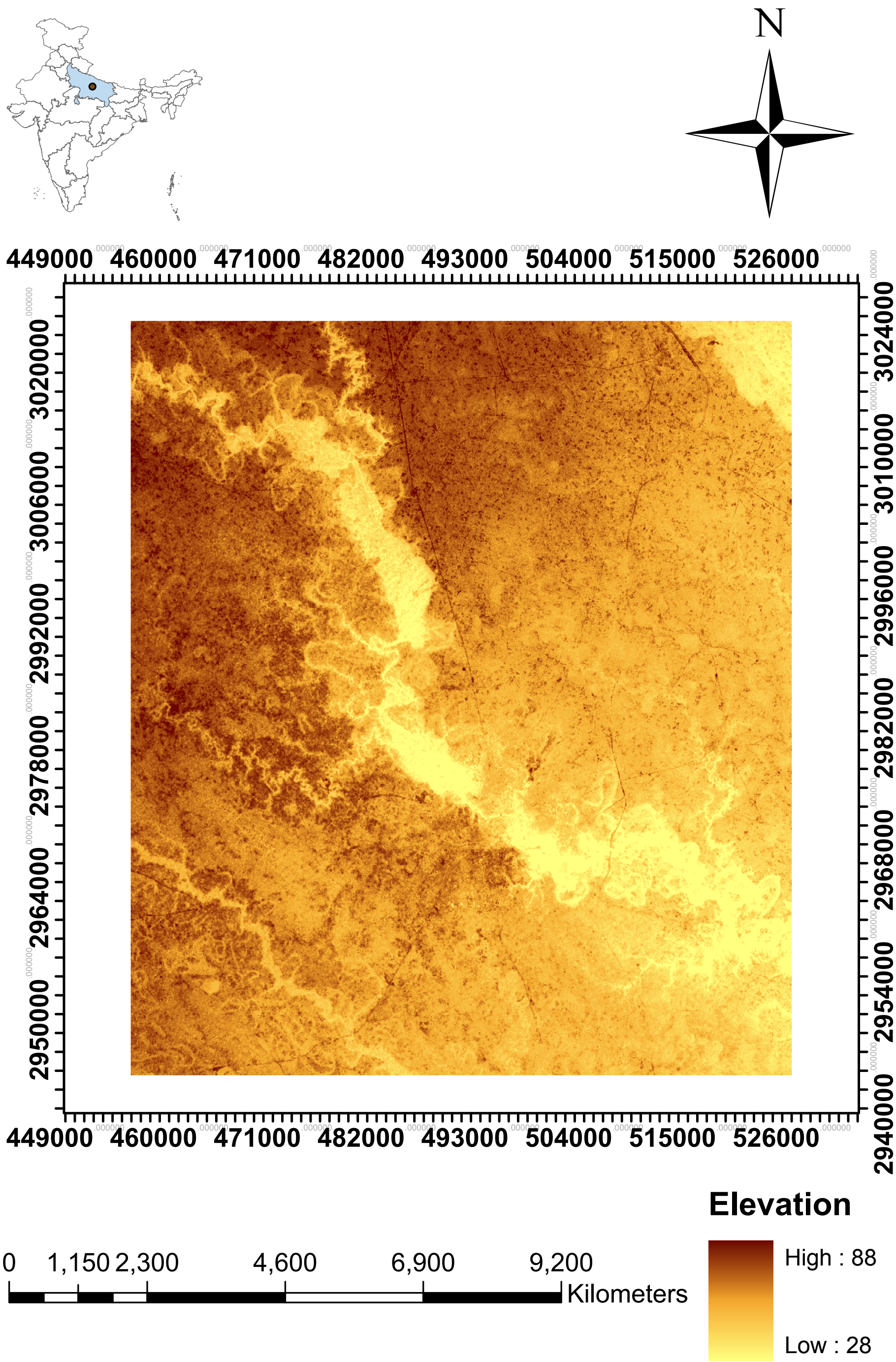


Figure 8.1: Digital elevation model for the city of Lucknow, India. The color scheme represents the elevation of the city above mean sea level.

and lies inside the coordinates (449313, 3022830) and (530520,2927331) in zone 44N, according to the UTM co-ordinate system. The hydrogeological setting of the city and the Sai-Gomti sub-basin is

Table 8.1: Data available for Lucknow groundwater model

Data	Time span	Temporal availability
River discharge	January 1990 − December 2001	10^{th} day
Groundwater heads	September 2003 − December 2011	monthly
Meteorological data	June 2003 −February 2007	daily

described in CGW (2009); Bhatnagar (1966). Further, a soil map from the state soil testing department (see figure 8.2), published and unpublished borehole data from geological survey of India (Bhatnagar, 1966) and various private drilling companies of the city, respectively, was acquired in order to interpret the hydrogeological situation. The direction of groundwater flow in the region is from north-west to south-east.

CGW (2009); Bhatnagar (1966) provide hydrogeological information about the underlying aquifers in the city. The acquired borehole data shows a top unit of sand and silt with intercalations of clay. The soil map of the city shows mainly silty-sand and clay layers and a slight variation from these soil textures was seen in the whole region. Geological cross-sections found in Bhatnagar (1966) are shown in figures 8.3 and 8.4 provide information regarding the plausible ranges of riverbed conductances of river Gomti in various parts of the city. The river in the north-west region of the city has a thin sandy-clay riverbed with two layers of sand and sandy-gravel of varying thickness underneath. The south eastern part of the riverbed consists of a thick layer of sand and gravel. The south western part of the river flows over a thick layer of clay while there was no information of the riverbed in the north east region of the city. It can be seen in figures 8.3 and 8.4 that a good connection exists between the river and the uppermost phreatic aquifer due to the underlying sand and sandy-gravel in the riverbed. Since most of licensed/unlicensed groundwater extraction occurs within the topmost unconfined aquifer, only one aquifer layer is considered in this study. The general direction of the groundwater flow is from the north-west to the south-east part of the basin (CGW, 2009; Bhatnagar, 1966; Foster and Choudhary N., 2009). The current groundwater extraction ($\mathbf{q_{ext}}$) in all parts of the city is estimated on the basis of population and groundwater extraction data provided in Bhatnagar (1966); Foster and Choudhary N. (2009); Livingston (2009).

The natural and anthropogenic factors affecting the groundwater levels in the city are recharge through precipitation and water exchange with the river, and the urban extraction in various parts of the city, respectively. Since short duration peak flows occurs rarely in the river, (CGW, 2009) and Bhatnagar

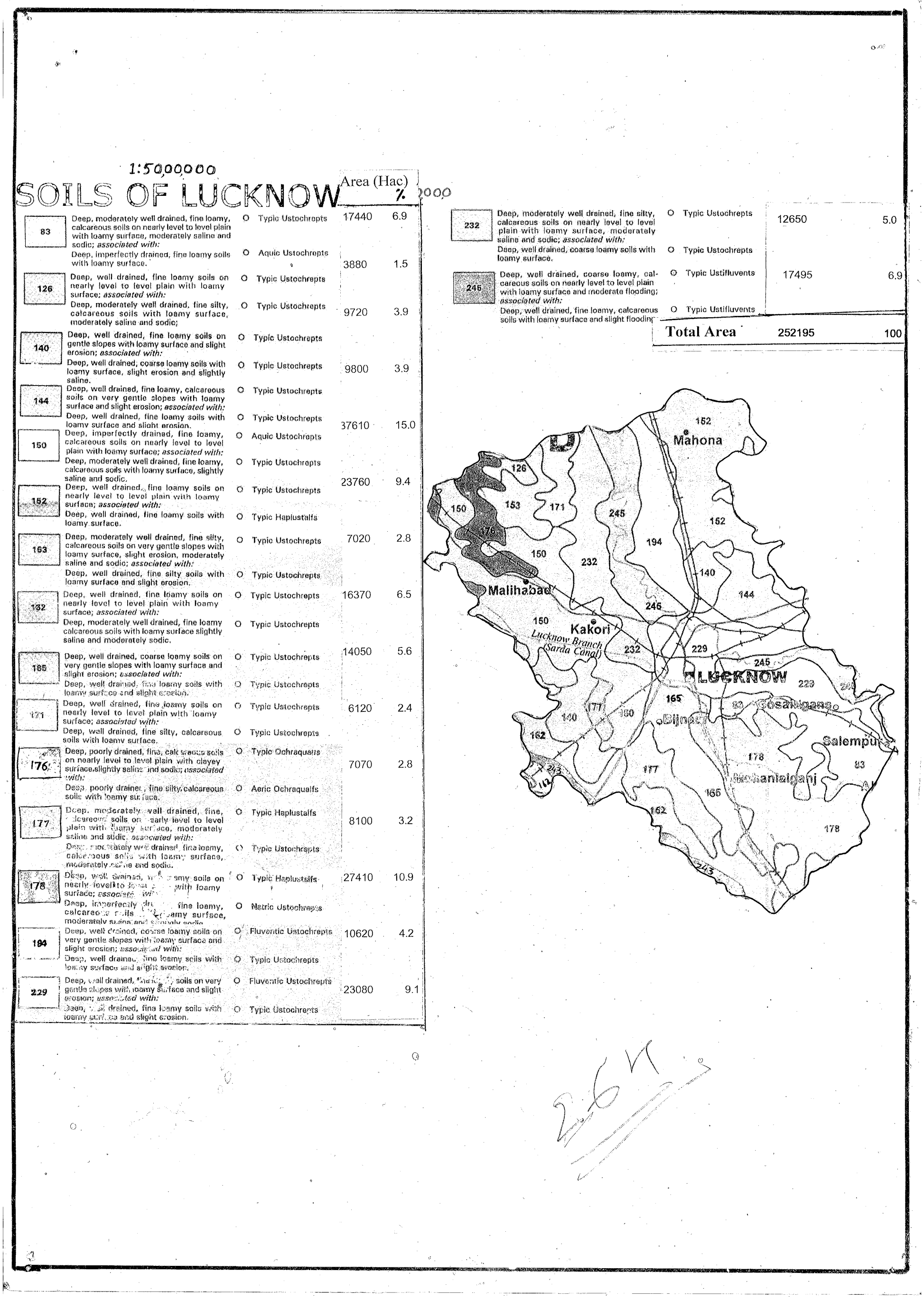


Figure 8.2: Soil map of the Lucknow city

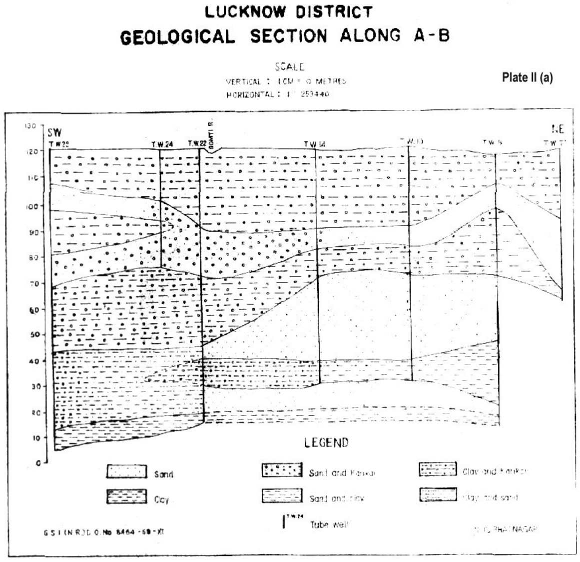


Figure 8.3: Geological section of the city in the north-east to south west direction

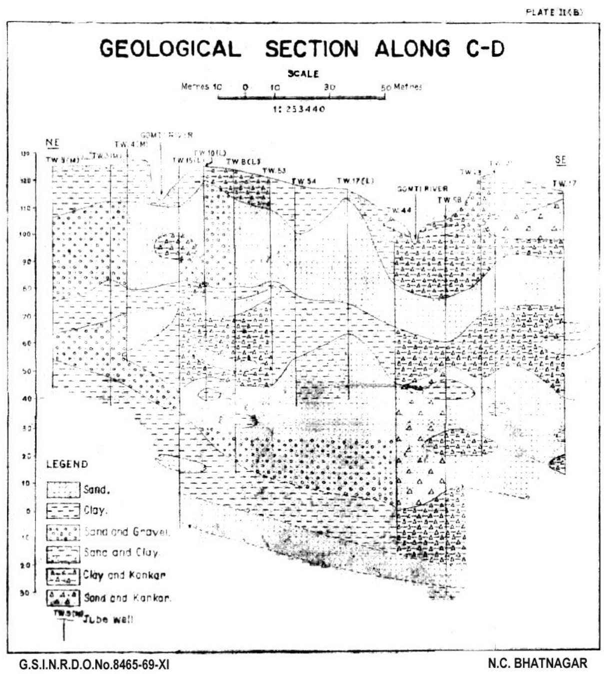


Figure 8.4: Geological section of the city in the north-west to south east direction

(1966) mention only the gaining nature of the river and the study aims at studying the long term behavior of overexploitation, only the gaining nature of river Gomti is considered here.

966 groundwater observations, $d(\mathbf{x},t)$ (depth to the water table from the surface) from 23 locations were acquired from the Central Groundwater Department, Lucknow, where $\mathbf{x}$ refers to the location of the observation at time t. The groundwater observations are recorded monthly for each location, spanning a period from September 2003 to February 2007. The groundwater elevation $y(\mathbf{x},t)$ was calculated using $y(\mathbf{x},t) = h_{elev}(\mathbf{x}) - d(\mathbf{x},t)$, where $h_{elev}(\mathbf{x})$ is the elevation of the location $\mathbf{x}$ acquired from the SRTM (Shuttle radar topographic mission) data, which was provided by DLR (the German aerospace center Taubenboeck et al. (2009)). It should be noted that there exists an error in the elevation data due to the presence of tall urban features and hydrological features such as lakes and rivers (Bhang et al., 2007; Gamba et al., 2002), and h_{elev} measurements at such locations had large error. These locations were either too close to the hydrological feature, i.e. river Gomti, or existed in the newly developed residential areas of the city. At such biased locations (observation wells marked in green in figure 8.7), the error of upto 6m was deducted from $y(\mathbf{x},t)$ using the available topographical maps, for correction.

In figure 8.5 the locations of the groundwater extraction wells and of constraints are provided. The

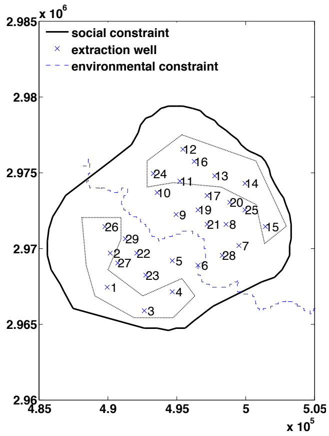


Figure 8.5: Location of the groundwater extraction locations, social constraints (city limit) and environmental constraints (river) in the city. Inside the dotted lines are well locations close to social constraints.

constraints considered are:

1. a minimum groundwater discharge into river Gomti, hereafter referred to as environmental constraint acting along the river course, and

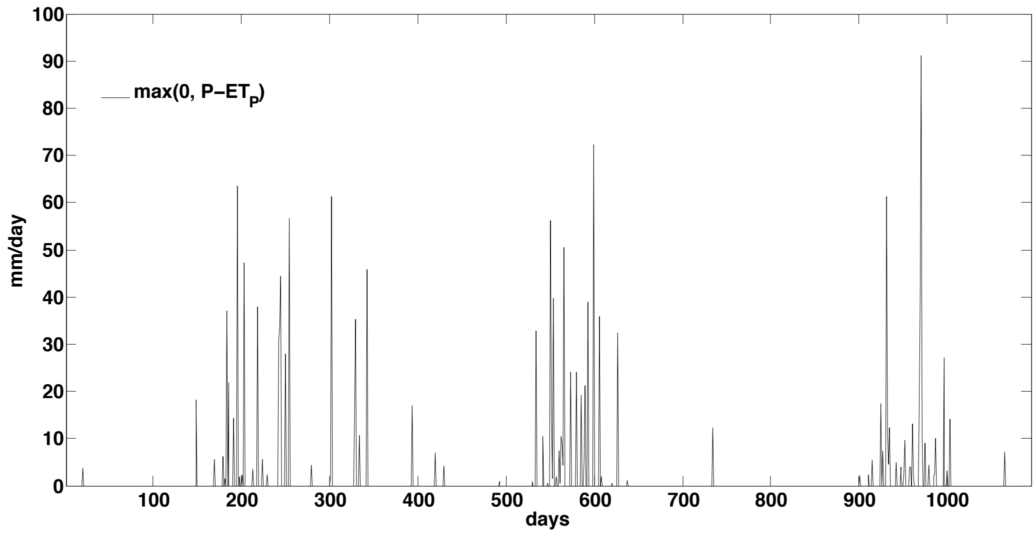


Figure 8.6: Recharge boundary condition is spatially distributed and equals Recharge flux factor . max(0,(P-ET_P)) where P is the precipitation and ET_P is evapotranspiration

2. a maximum groundwater drawdown in the peri-urban and rural areas, denoted as social constraint in the following, which acts along the city boundary

The meteorological data used for estimation of recharge was measured at the meteorological station in the city and was acquired from the World Data Center for Meteorology. The data obtained included the air temperature T_a, net shortwave radiation K, downward long wave radiative flux L, earth skin temperature T_{skin} (used for calculating net outward radiation G using black body radiation relationship σT^4) and the dew point T_d. The mean potential skin evapotranspiration is estimated to be 500 mm/yr based on equations 3.8, 3.9 and 3.10, which is similar to the formulation of Penman equation, as described in Monteith (1965) and the transient values of precipitation minus potential evapotranspiration is shown in the figure 8.6.

8.2 Purpose of the model

The purpose of the model was mentioned in chapter 1, focusing towards calculating the maximum groundwater extraction rates based on the SPR concept and the feasibility of an appropriate groundwater recharge method. The necessity of the regional groundwater model arises from the required calculation of the spatial groundwater heads in the city area and the groundwater drainage to the river as a function of groundwater extraction rates.

This objective helps in the calculation of the aforementioned constraints in order to know in advance the effect of the groundwater extraction rates. The ongoing groundwater extraction in the city of Lucknow leads to reduction in groundwater discharge to the river and spatially spreading groundwater drawdowns, requiring such estimates. Though the SPR concept is not explained explicitly by the hydrogeologist for each and every case study, a change in the groundwater budget in terms of the local water balance acts as an indicator in such cases, and should be used to estimate groundwater extraction rates through a regional groundwater model.

The SPR concept needs to be applied in the current research by comparing the effect of groundwater extraction to virgin conditions as explained in chapter 2. Since the groundwater contours are not available for a virgin scenario, it has to be estimated using no groundwater extraction for twelve years. Also, the groundwater extraction during the same 12 years of extraction should estimate the effect on constraints as compared to the virgin case.

It should be noted that the virgin conditions which are simulated using 12 years with no groundwater extraction is only an approximation because virgin conditions have not existed in the time since beginning of recording of groundwater levels. However, due to computational limits the simulation period more than 12 years was inefficient.

The next purpose of the model is to simulate the effect of artificial groundwater recharge on the groundwater heads. It is applied using extra recharge over all administrative blocks in the city uniformly, shown in figure 10.1. The drawdown prices are estimated due to groundwater extractions using a spatial map of the water prices in different parts outside the city. It is used to judge the economic feasibility of rooftop rainwater harvesting application in the city of Lucknow. Also the minimum efficiency of the rooftop rainwater harvesting application is estimated in order to ensure the economic feasibility.

In order to estimate the administrative blocks producing maximum economic efficiency, the groundwater model is used in the final step. Hence, it can be learned from the aforementioned objectives that a regional groundwater model is necessary to simulate groundwater heads and groundwater discharge to the river, due to varying groundwater extractions, more recharge in the city and varying recharge in different administrative blocks of the city.

8.3 Conceptual model

The conceptual model of the basin is shown in figure 8.7. It is based on the hydrogeology of the city and the key processes affecting the groundwater in the region such as dynamic meteorological forcing, the gaining stream and the groundwater extraction in the city. The pictorial representation of the conceptual model also includes the groundwater observations. As mentioned before, the conceptual model is simplified by removing unimportant data. It should be noted that the river discharge observation location is not shown since it will not be used, due to different time span at which it was available. Since major groundwater extraction takes place from the first unconfined aquifer in the area, only one layer is considered throughout the model and a two-dimensional representation is considered to be sufficient. The important features of the groundwater flow system for the city of Lucknow have been defined in the section 7.2.

The conceptual model chosen is a compromise between simplifying the model and keeping all important aspects of the groundwater flow in the region. The model boundaries could not be extended more due to computational limits.

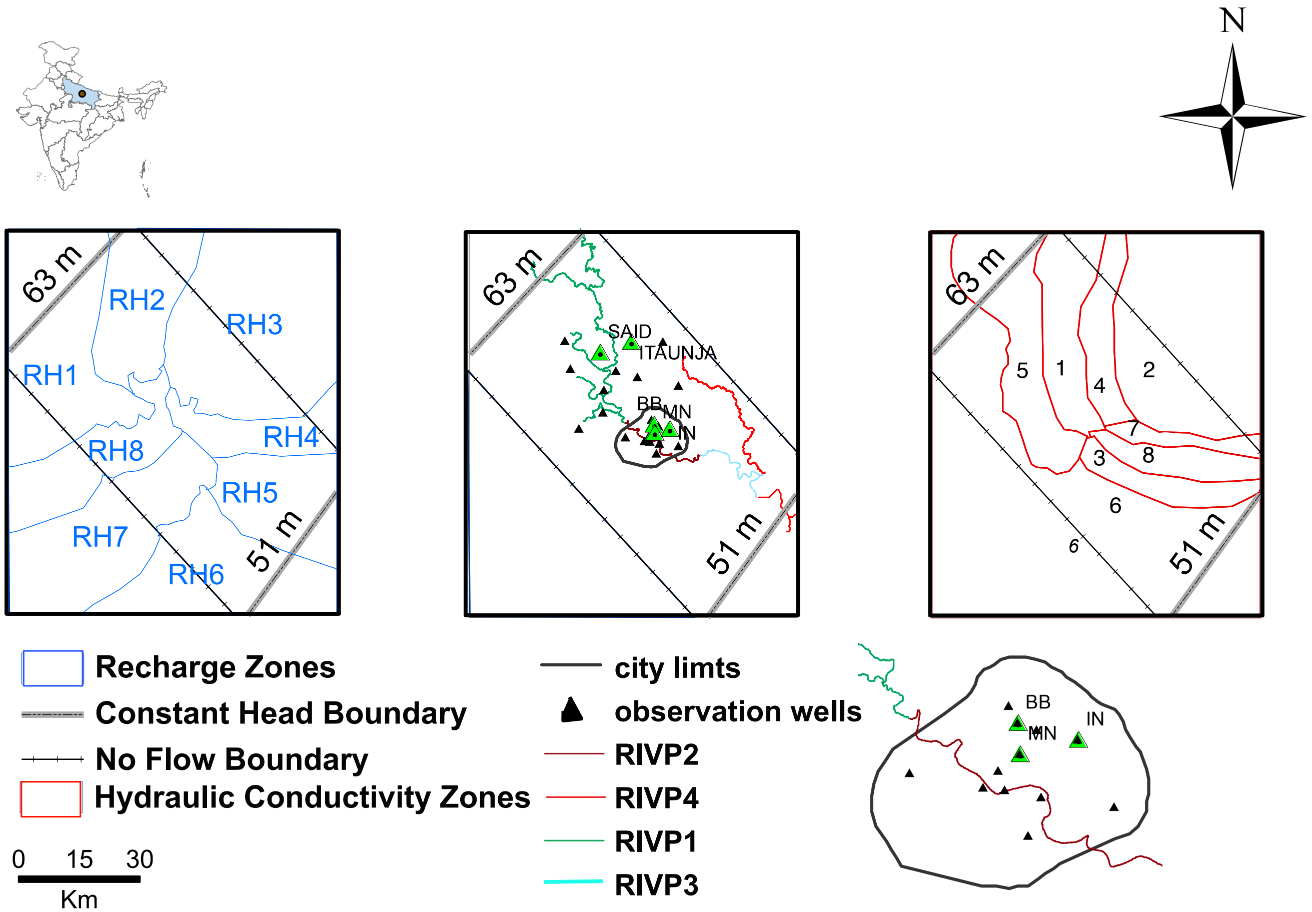


Figure 8.7: Conceptual Model: The left figure shows hydraulic conductivity zones, the middle figure shows the riverbed conductances and spatial locations of head observations and the figure in the right shows the recharge zones.

8.4 Selecting the computer code

As discussed in section 7.3 a simple code is required which has been validated earlier. Hence, MODFLOW (Harbaugh, 2005) is used in order to represent the conceptual model numerically. MODFLOW is a finite difference based code which solves the two-dimensional groundwater equation 3.1 and has been used extensively to solve such groundwater problems. The choice was also based upon easy accessibility of the code and intensive documentation provided. ModelMuse (Winston, 2009) is a graphical user interface which was used during the study to produce the input files required by MODFLOW.

8.5 Model design

The following groundwater equation is used to represent the groundwater flow conditions in Lucknow city which is the same as mentioned before (see equation 3.14). The different parameters used in the equation are defined spatially (see figure 8.7) and will be described later in the parameterization section.

$$S_y \frac{\partial h}{\partial t} - \nabla \cdot (\mathbf{K} h \nabla h) = q_{in/out} \tag{8.1}$$

$$\begin{aligned} q_{in/out} = RCH \cdot max(0, P - E_{PT}) - \mathbf{q}_{ext} \\ - RIVP \cdot (h - h_{drainage}(\mathbf{x}_{dr})) \end{aligned} \tag{8.2}$$

The model domain is discretized in 354 rows and 362 columns, encompassing a total area of $\approx$ 7755 km^2. A variable discretization was used for the model domain to reduce the computational costs. The city area in the model domain was finely discretized (since it is the major area of interest) as compared to the regions outside the city. The discretization dimensions in X and Y dimensions varied between 135 to 400 m. All aforementioned parameters were defined for each cell of the finite difference grid.

The boundary of the city outlines mainly the groundwater extraction region in the city. A time varying Neumann boundary condition was used for simulating the recharge due to precipitation. A sink term was also used for simulating the constant groundwater extraction in the region. The representation of streams in the conceptual model complies with the position of streams in topographic maps. A third type boundary condition, was used in the model to represent the groundwater discharge to the river, which depends on the riverbed conductances and the difference between the riverbed elevations and the general water table.

The common practice of choosing model boundaries by a natural boundary such as a river, the limit of a valley or groundwater divide, could not be applied, which resulted in opting spatially distant ($>$

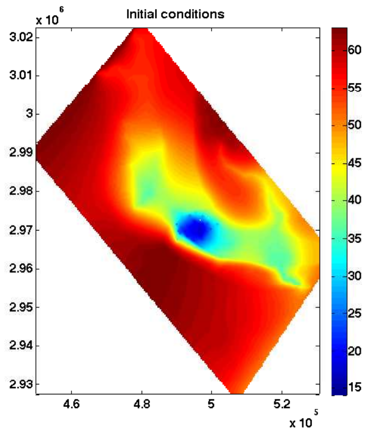


Figure 8.8: Initial condition of groundwater heads calculated using steady state model

13km from the city) model boundaries due to inaccuracy and consideration of regional groundwater flow. Boundary conditions in the groundwater model are represented using the constant head boundary (Dirichlet) and no flow boundary (Neumann), to ensure the regional groundwater flow from north-west direction to south-east direction of the conceptual model (see figure 8.7).

An initial condition of groundwater heads was not readily available for the numerical model. Hence, the initial conditions of the transient groundwater model on 1^{st} of September 2003 were generated using a manual calibration of the steady state model. The steady-state model had the same model setup as shown in figure 8.7 with a recharge of $5{\cdot}10^{-4}$m/day. The simulation of initial heads using the groundwater model also resulted in generating initial conditions where the groundwater head were consistent with the numerical groundwater model in terms of model structure and boundary conditions (Anderson and Woessner, 1992). The initial conditions generated of the model is shown in figure 8.8. The steady state groundwater model matched well the annual averaged groundwater head for the year 2003 as well as the average baseflow of the river of 15 m^3s^{-1}.

However, the parameters used to reach the initial heads might not simulate the minimum difference between simulated and observed groundwater heads during transient runs of the groundwater model, thus requiring an automated calibration for the transient groundwater model.

8.6 Parameterization

A map constructed from hydraulic conductivity values found in the literature (Bhatnagar, 1966; Foster and Choudhary N., 2009; Srivastava et al., 2003) and the above mentioned geological data helped in interpreting the eight hydraulic conductivity zones in the basin as shown in figure 8.7. In total, four different riverbed conductance zones were considered for river Gomti, which were adopted from the geological cross sections mentioned in the report of Bhatnagar (1966), shown in the figures 8.3 and 8.4. The data from only one meteorological station was available from the city of Lucknow, which cannot be an accurate representation of the recharge system for the whole model area. Due to the limitations of the spatial meteorological data, a recharge potential map (obtained from the central groundwater board, CGWB (CGW, 2009)) was used to estimate the recharge for different blocks of the city. The parameters $S_y, \mathbf{K}, RCH$ and $RIVP$ are spatially distributed (shown in figure 8.7) and constitute the parameter vector $\mathbf{p}$. The spatial zonation of the specific yield was assumed to be exactly the same as for the hydraulic conductivity and hence is not shown in figure 8.7.

8.7 Sensitivity Analysis

In order to match the observed $o(\mathbf{x},t)$ and simulated groundwater heads $f(\mathbf{x},t,\mathbf{p},\mathbf{q_{ext}})$ between a period of September 2003 to August 2006, the parameter vector $\mathbf{p}$ is estimated. Sensitivity analysis is performed to aid calibration and removing non-uniqueness of parameters, which is widely discussed in groundwater modeling literature. The composite scaled sensitivities (CSS) values and parameter correlations were estimated using UCODE (Hill and Tiedeman, 2005) for this purpose. The computational time required for a single simulation was 12 minutes. For faster estimation of the sensitivities, UCODE was used in parallel with the help of seven 2.83 GHz-Xeon-Quadcore-CPU (total 28 cores) at the high performance computing center, University of Tübingen.
The composite scaled sensitivities of each parameter of the vector $\mathbf{p}$ for 828 observations (n_{obs}) is calculated using equation 7.6, as mentioned in the last chapter and shown in figure 8.9. The parameter correlation coefficient between all possible pair of different parameters are calculated using equation 7.1 and the correlations higher than 0.85 are shown in table 8.2. The parameters RH1, RIVP3, RH5 and RH6 were removed during model calibration due to high correlation (more than 0.85) and lower values of composite scaled sensitivity (less than 5×10^{-3} times the highest composite scaled sensitivity).

8.8 Calibration and Validation

The assessment of composite scaled sensitivities and parameter correlation aid in identification of the parameters that can be estimated during model calibration. However, the set of parameter values, which results in minimum difference between the simulated and observed groundwater heads stays unknown.

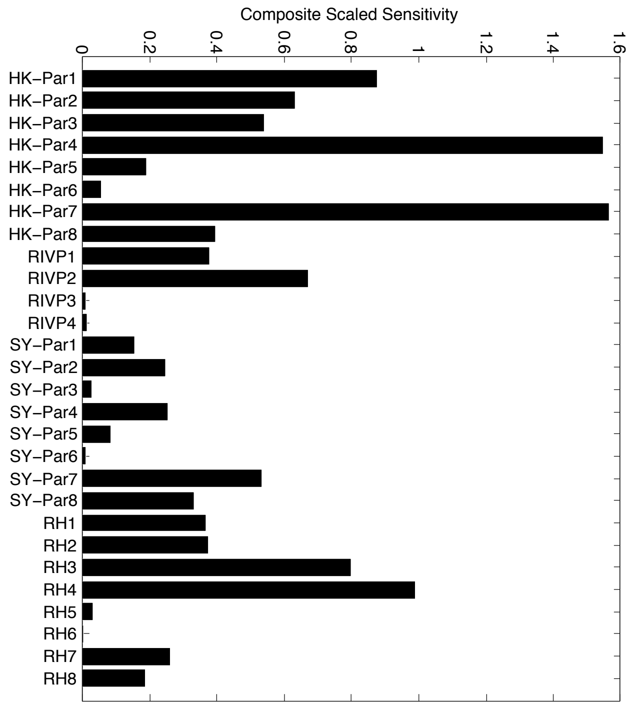


Figure 8.9: Composite sensitivity analysis of the model parameters.

Calibration of the groundwater model addresses this issue and estimates the parameter values due to: 1) incomplete information of the numerical values of drainage bed conductances, specific yield and hydraulic conductivity, which relies upon the geological cross- sections in CGW (2009); Bhatnagar (1966) 2) non quantitative information on the amount of recharge in different zones which are adopted from the recharge potential map obtained from the central groundwater board report (CGW, 2009). The process of validation establishes greater confidence in the groundwater model by simulating the groundwater heads under different stresses than the one existing. In the current study a split-sample test is performed during which the first 36 months of observations are used for calibration, while the latter 6 months of observations are used for validation.

The transient state calibration of the model is performed to estimate model parameter values by minimizing the objective function 7.2.

The four statistical criteria employed to judge quantitatively the goodness of fit between monthly observed and simulated groundwater heads are the maximum error (ME), the root mean square error ($RMSE$), coefficient of residual mass (CR) and Nash-Sutcliffe (NS) (Nash and Sutcliffe, 1970) has been stated in the previous chapter and are calculated using the same set of equations (7.3-7.6).

Table 8.2: Parameter correlations greater than 0.85 of the parameters shown in the figure 8.7

Parameter Names	Correlation
RH1,RH4	-0.96
HK-Par6,SY-Par6	0.92
RH1,RH3	-0.94
RH2,RH8	0.90
RIVP3,RH5	0.86
RH1,RH2	-0.86
RH1,RH7	-0.89
RH2,RH3	0.86

The groundwater model was calibrated using monthly observed groundwater head measurements at 23 locations, between a period of September 2003 to August 2006. Automated calibration of the model was preformed to minimize the value of χ^2 using UCODE (Hill and Tiedeman, 2005), assuming the same measurement error of 0.5 m for all observations in the covariance matrix **C**. The parameter values estimated were as follows: the eight different hydraulic conductivity and specific yield and riverbed conductance estimated were within a realistic range of 6 m/day to 170 m/day, 0.05 to 0.3, and 285 to 864 day^{-1} respectively; the recharge flux factor were estimated within a range of 0.06 - 0.3. All the hydrogeological parameters lie well within the range of the pumping based measurements and are in agreement with an alluvial aquifer with a lateral variation in the clay content.

The observed and simulated groundwater heads for the calibration period (first 36 months) are shown in figure 8.10. The simulated and observed groundwater heads match considerably well. It can be seen that occasionally the simulated heads are lower than the observed heads. This occurred during the small monsoon period during which the river Gomti infiltrates. The groundwater model can well represent the seasonality and the declining groundwater levels in the city.

In order to validate the groundwater model, it was simulated using the meteorological forcing for the next six months with the model structure and the set of parameters obtained by calibration. The observed and simulated groundwater heads for the next six months are plotted in the same figure and show that the groundwater model can simulate the heads well.

8.9 Performance Measures

The model performance is judged on the basis of calculated statistical criteria, as shown in table 8.3. The maximum error ME of 5.9 m occurred due to the proximity of the observation location to the model

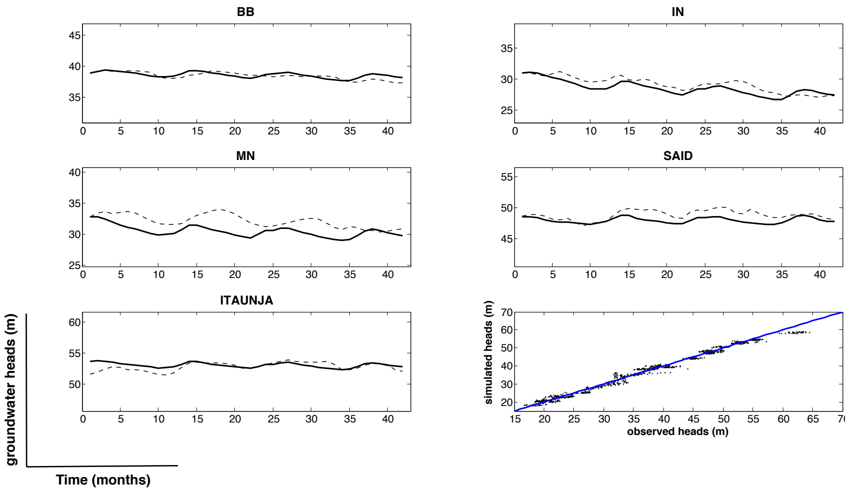


Figure 8.10: Calibration and validation results. Dashed lines are observations while continuous lines represent simulated values. First 36 months are for calibration and the rest for validation.

boundary, which can be ignored, considering that the area of interest lies inside the city limits, spatially distant from the model boundaries. The *ME* at other locations inside the city remained below 1.6 *m* during calibration and validation. The value of RMSE was observed to be 1.5 *m* and 1.63 *m* for calibration and validation, respectively. However, inside the city the *RMSE* was estimated to be 1.2 *m*. The maximum difference between highest and lowest position of groundwater head was 10 *m*, which existed in the groundwater observation at one location (GN observation, see figure 12.2 in the Appendix), leading to the conclusion of a maximum error of 12% and 15% inside and outside of the city, respectively. The values of CR during calibration and validation were estimated to be 0.012 and -0.012, which is a measure of slight underestimation and overestimation of the groundwater heads. The underestimation of heads occurred due to unrepresented influent nature of the river during the monsoon. The slight overestimation of heads during validation might have occurred due to increased extraction in the city of Lucknow, which was also unrepresented in the groundwater model. A further detailed analysis of overestimation can be performed but is unimportant for the scope of the thesis and its low value of -0.012 m. The values of *NS* for calibration and validation as 0.94 and 0.93, respectively, demonstrating the suitability of the model to reproduce groundwater heads subject to meteorological forcings and groundwater extraction.

The slight mismatch between the simulated and observed groundwater heads might have been overcome if the river discharge data and detailed hydrostratigraphic units were available. In such a case the representation of the influent nature of the river and coupled 3D groundwater model of the city could have even further improved the model performance. However, due to unavailability of the aforementioned data, the model complexity could not be increased. A root means square error of 1.2 *m* (RMSE) in areas inside the city was calculated at locations inside the city.

As mentioned before in chapter 7.1, the post audit of such a model could have further increased the level of confidence in the groundwater model, but could not be performed due to lack of data.

Table 8.3: Model performance statistics for the calibration period September 2003 to August 2006 and the validation period (September 2006 to February 2007). ME: maximum error, RMSE: root mean square error; CR: coefficient of residual mass; and Nr: the NashSutcliffe coefficient.

Run	ME(m)	RM(m)	CR(-)	NS(-)
Calibration	5.9	1.5	0.01	0.94
Validation	4.2	1.6	-0.01	0.93

9

Application of the SPR concept

The SPR concept stated in chapter 2 was explained in terms of the water budget terms of a control volume. The social and environmental constraints explained so far are not yet applied to estimate groundwater extractions based on this concept. However, the groundwater model developed during the previous sections forms the basis of simulating the induced stress upon the groundwater heads and the groundwater discharge to the river inside the city limits denoted as the social and environmental constraints, due to different groundwater extraction scenarios. This chapter focuses on estimating P as per the SPR concept, which aims towards maximizing the total groundwater extraction subject to a set of constraints on a future date of 1^{st} May 2024 (t_1) as explained in section 8.2.

The model simulated the period between September 2006 and August 2012 using the same model structure, parameter vector $\mathbf{p}$, groundwater extraction $\mathbf{q_{ext}}$ and repeating the meteorological forcing calculated between September 2003 and August 2006. The simulated groundwater heads at the end of August serve as initial conditions for the groundwater model simulated from 1^{st} September 2012 to t_1.

It is assumed that on keeping other factors unchanged as mentioned above, except the ground- water extraction rates, $\mathbf{q}$, implemented from 1^{st} September 2012 to t_1 (12 years) results in dynamic-steady state conditions of groundwater heads and groundwater discharge to the river at time t_1. The time t_1 of any year had the lowest groundwater heads in all the temporal groundwater observations and the minimum river discharge in the river Gomti, making it the most suitable time instant for assessing the severe consequence of groundwater extraction.

9.1 SPR problem formulation

The SPR problem, $P(\mathbf{q_{spr}}) = f(d_{crit}, q_{crit})$ is formulated as follows (similar to the previous works of Ahlfeld and Baro-Montes (2008); Liu et al. (2008); Shen et al. (2004)), where d_{crit} (social constraint) and q_{crit} (environmental constraint) are the reduction in heads at locations at the city limits and reduction in the river discharge, respectively, at time t_1. $0m < d_{crit} \leq 3m$ and $0m^3/s < q_{crit} \leq 1.35m^3/s$ values were used in order to estimate P with a step size of 0.01.

$$\max_{\mathbf{q_{spr}}=\mathbf{q}} \; P(\mathbf{q}) = \sum_{j=1}^{29} q_j \tag{9.1}$$

subject to the following constraints

$$\begin{gathered} h(\mathbf{x}_1, t_1, \mathbf{p}, \mathbf{q}) \geq h(\mathbf{x}_1, t_1, \mathbf{p}, \vec{0}) - d_{crit} \\ \sum Q_{sim}(\mathbf{x}_{dr1}, t_1, \mathbf{p}, \mathbf{q}) \geq \sum Q_{sim}(\mathbf{x}_{dr1}, t_1, \mathbf{p}, \vec{0}) - q_{crit} \\ \vec{0} \leq \mathbf{q} \leq \mathbf{q_{ext}} \end{gathered}$$

in which $\mathbf{x}_1$ and $\mathbf{x}_{dr1}$ refers to the vector of locations at the city limit and drainage constraints, respectively (as shown in figure 8.5). The first two constraints are the social and environmental constraint, respectively, while the third constraint sets an upper limit $\mathbf{q_{ext}}$ to the individual groundwater extraction rates $\mathbf{q}$, since the values above $\mathbf{q_{ext}}$ resulted in dry cells in the groundwater model.

If the constraints depend linearly on the groundwater extractions, a linear optimization procedure can be used to solve equation 9.2, which requires less iterations. Also less computation time and effort is required since the CPU time for calculating constraints at time t_1 using a linear procedure is negligible as compared to a single transient groundwater flow simulation of 12 years requiring ~ 50 minutes. In order to prove the constraints to be linear with respect to $\mathbf{q}$ a procedure is followed similar to the methodology adopted by Ahlfeld et al. (2005) and Ahlfeld and Baro-Montes (2008).

The social constraints are linear with respect to groundwater extraction if

$$||h(\mathbf{x_1}, t_1, \mathbf{p}, \mathbf{q_{ext}}) - h(\mathbf{x_1}, t_1, \mathbf{p}, \vec{0}) - \mathbf{J}(\mathbf{q_{ext}} - \vec{0})|| < \varepsilon \tag{9.2}$$

with the Jacobian or response matrix as

$$J_{ij} = \frac{h(x_{1,i}, t_1, \mathbf{p}, \mathbf{q_{ext,j}} + \delta_j \cdot \mathbf{q_{ext,j}}) - h(x_{1,i}, t_1, \mathbf{p}, \mathbf{q_{ext,j}})}{\delta_j \cdot \mathbf{q_{ext,j}}} \tag{9.3}$$

where δ_j represents the amount by which the groundwater extraction rate is perturbed. $\mathbf{q_{ext,j}}$ contains the unperturbed value of the groundwater extraction vector $\mathbf{q_{ext,j}}$ at position j while ε is the tolerance criterion for a valid linear assumption.

The aforementioned linear model is appropriate for calculating the constraints, for values of groundwater extraction in the range of $\vec{0}$ to $\mathbf{q_{ext}}$ for small values of ε.

9.2 SPR Problem Solution

In order to use a linear optimization technique to estimate $\mathbf{q_{spr}}$, as stated in equation 9.1, the linearity check of the groundwater model was performed using equation 9.2. The response matrices (equation 9.3) were calculated for 29 extraction wells and by calculating heads at 63 social constraints. A similar formulation was used for the environmental constraint. The values of ε for the social and environmental constraints evaluated using equation 9.2 had a maximum value of 0.3 m and 0.02 m^3/s, respectively, which is two orders of magnitude lower than the minimum groundwater depth and minimum groundwater drainage to the river in the model ($20m$ and $1.7m^3/s$). $P(\mathbf{q_{spr}})$ is obtained by using the `linprog` function implemented in MATLAB (Zhang, 1998), in order to solve equation 9.1 subjected to the constraints.

The constraints used in the equation 9.1 are slightly modified to estimate the constraint values by combining (first two) the social and environmental constraints in a single equation as following:

$$\begin{pmatrix} \mathbf{J_{social}} \\ \mathbf{J_{env}} \end{pmatrix} (\vec{0} - \mathbf{q}) = \begin{pmatrix} \vec{1} \cdot d_{crit} \\ q_{crit} \end{pmatrix} \tag{9.4}$$

where $\vec{1}$ is a vector of ones consisting of i rows and $\vec{0}$ is a vector of zeros consisting of j rows. The social and environmental response matrices, $\mathbf{J_{social}}$ and $\mathbf{J_{env}}$, obtained from equation 9.3 are substituted in equation 9.1 to combine the value of the first two constraints. The third constraint

$$\vec{0} \leq \mathbf{q} \leq \mathbf{q_{ext}} \tag{9.5}$$

stays the same and sets a constraint for groundwater extractions up to which the linear approximation is valid.

Using the SPR concept, $P(\mathbf{q_{spr}})$ is estimated as a function of constraints, d_{crit} and q_{crit}, and a contour plot is shown in figure 9.1. The contour plot forms the basis for the decision of a water manager to implement a set of groundwater extractions across the city. This decision does not only depend upon the total groundwater extraction demand but also upon the total amount of groundwater extraction that can be provided based on the SPR concept. For example, it can be seen that higher P demands higher values of both social and environmental constraints.

Figure 9.1 shows a contour plot of the P (sum of all groundwater extractions obtained using the SPR concept) as a function of the constraint d_{crit} and q_{crit} revealing important features which should be known by a water manager before undertaking any decision to implement $\mathbf{q_{spr}}$. The first feature revealed by figure 9.1 is that the same amount of P $(= \vec{1} \cdot \mathbf{q_{spr}})$ can be achieved by higher d_{crit} and q_{crit} values, which is unadvisable for implementation. Hence, the $\mathbf{q_{spr}}$ corresponding to $3m^2/s > |\partial q_{crit}/\partial d_{crit}| >$

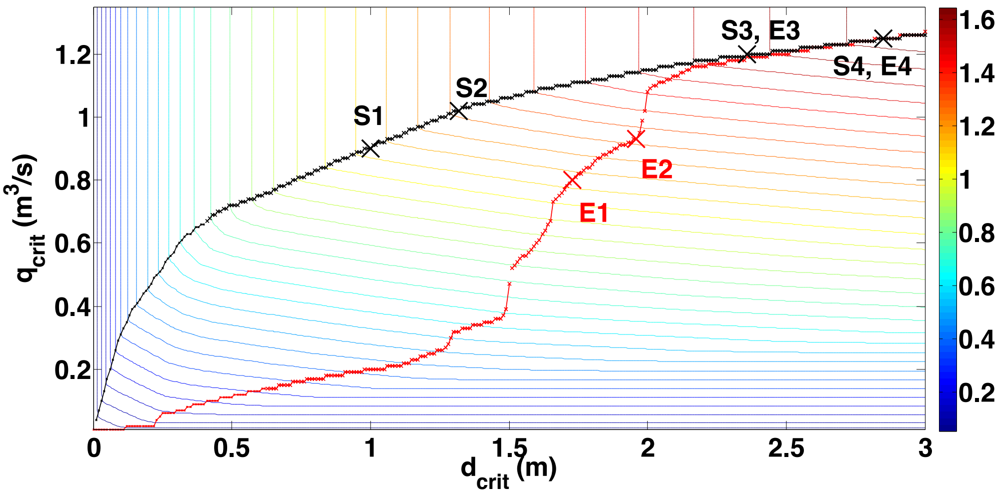


Figure 9.1: A contour plot of SPR. The two curves refer to minimum effect on each constraint. Four clusters for social curve (S1-S4) and environmental curve (E1-E4) are shown in different color. red (cluster 1), blue (cluster 2), black (cluster 3) and green (cluster 4).

$0.1m^2/s$ in each contour should only be considered for implementation. The values 3 and 0.1 are chosen to initiate a process to exclude points lying almost horizontally or vertically on each contour. The points lying vertically on each contour can be easily depicted as compared to the ones lying horizontally. However, the points lying on the horizontal part ($|\partial q_{crit}/\partial d_{crit}| > 0.1m^2/s$) of the contour are still excluded from further analysis due to the fact that the decrease of q_{crit} is much lower than the increase in d_{crit}. On excluding these points, a group of points are left along each contour, having two group of points on either side of each contour which prove that the process initiated to remove unadvisable $\mathbf{q_{spr}}$ was successful.

These points are grouped together to form two separate curves i.e. social curve (S1-S4) and environmental curve (E1-E4) corresponding to minimum value of d_{crit} and q_{crit}, respectively.

The second feature revealed is that on increasing the P (the points S3, E3 and S4, E4 in figure 9.1) either of the constraints on the social and environmental curves have the same value (see table 9.2), which indicate that the higher groundwater extractions based on the SPR concept aim at the same value of constraints.

Without performing any further data analysis, few general observations useful for the water manager can be made in figure 9.1. The groundwater extractions, $\mathbf{q_{spr}}$ along the social curve or the environmental curve can be followed by the water manager without any detailed analysis, by using the sum total of required groundwater extraction P, if he does not care about one of the constraints. For example, in case that the water manager gives no importance to the reduced groundwater discharge to the river, then the groundwater extraction along the social curve can be followed.

A visual analysis of the results has been performed so far. In the next step a multivariate analysis of $\mathbf{q_{spr}}$ along the social and environmental curves is performed to state important facts related to groundwater extraction locations.

9.3 Multivariate Analysis

The multivariate analysis is performed due to the huge amount of $\mathbf{q_{spr}}$ data obtained by varying d_{crit} and q_{crit}. It helps to identify important groundwater extraction wells which play a key role in the optimization results. It should be kept in mind that three wells 18, 19, 20 are taken out of the analysis, since well 18 lies spatially far from the location of the constraints and wells 19, 20 dried up during the MODFLOW simulations between September 2006 and August 2009.

The following analysis is performed in order to assist the water manager in selection of $\mathbf{q_{spr}}$, giving general ideas about the wells to be preferred or avoided for extraction and trade-off implementation between $\mathbf{q_{spr}}$ points on the social and environmental curves.

The multivariate analysis investigates the groundwater extraction wells where groundwater extraction has to be avoided or preferred in view of either of the constraints, by dividing the environmental and social curve into clusters and finding the principal component of each groundwater extraction well in the cluster.

In order to achieve these objectives two separate groups consisting of $\mathbf{q_{spr}}$ along the social and environmental curves are formed in the first step. In the next step, a cluster analysis on each of the two groups is performed and therefore $\mathbf{q_{spr}}$ of each curve is divided into four clusters. Now these $\mathbf{q_{spr}}$ are similar in each cluster as compared to other clusters.

In the final step, a principal component analysis of each of the four clusters of the social and environmental curves (which include similar $\mathbf{q_{spr}}$) is then performed. Explaining briefly, in case the principal component of a well from each cluster in the environmental and social curve shows higher or lower score indicates its preference and rejection, respectively for groundwater extraction. Such information stays hidden in figure 9.1 and cannot be explored without the analysis of $\mathbf{q_{spr}}$ of the social and environmental curves.

9.3.1 Cluster Analysis

Cluster analysis is performed on two group of points comprising of $\mathbf{q_{spr}}$ for each curve and each group is divided into four clusters. It is achieved by performing a centroid based clustering using the k-means algorithm implemented in MATLAB. $k-means$ uses a two-phase iterative algorithm to minimize the sum of squared distances from point to centroid, summed over all $\mathbf{q_{spr}}$. In $k-means$ method, the number of clusters are specified in advance and is widely used for large datasets. Current work only applies the method and further description of the analysis lies beyond the scope of paper. For details of the algorithm refer to the works of Seber (1984) and Spath (1985).

9.3.2 Principal Component Analysis

In the current study, principal component analysis is applied using singular value decomposition of each cluster of both curves, which identifies the most important wells, favored or avoided for groundwater extraction during optimization (see equation 9.2). This identification is based upon the score of a well in the principal component and singular values (directly proportional to the variance) of the $\mathbf{q_{spr}}$ as stated in the chapter 5.

Table 9.1: Results of cluster analysis and principal component analysis

Curve	Cluster 1	Cluster 2	Cluster 3	Cluster 4	parameter
environmental	0.032	0.82	0.04	0.23	sum of squared distance
	63	88	78	102	number of $\mathbf{q}_{spr}$ in a single group
	0.36	0.41	0.41	0.36	maximum singular value
	0.28	0.21	0.21	0.31	$\vert$maximum score$\vert$
	1, 2, 7, 11, 15, 16, 24, 26	1, 2, 7, 10, 11, 15, 16, 21, 26, 27	4, 5, 6, 8, 12, 15, 16, 17, 22, 28, 29	3, 4, 9, 12, 13, 15, 16, 17, 23, 25	wells with $\vert$score$\vert$ $>$0.0001
social	0.137	0.233	0.14	0.19	sum of squared distance
	40	72	79	109	number of $\mathbf{q}_{spr}$ in a single group
	0.42	0.38	0.34	0.35	maximum singular value
	0.21	0.19	0.26	0.30	$\vert$maximum score$\vert$
	2, 5, 6, 7, 9, 10, 21, 22, 24, 26, 28, 29	1, 2, 4, 7, 8, 11, 15, 23, 24, 27	1, 4, 11, 12, 15, 16, 17	1, 3, 9, 12, 13, 15, 16, 17, 25	wells with $\vert$score$\vert$ $>$0.0001

9.3.3 Analysis of the Results and Plausibility Check

The results of the cluster and principal component analysis are shown in table 9.1. In order to check whether the SPR concept and the characterization of $\mathbf{q_{spr}}$ show similar results, the groundwater extraction rates $\mathbf{q_{spr}}$ for the marked points in figure 9.1 were used to calculate the drawdown, $h(\mathbf{x}_1, t_1, \mathbf{p}, \vec{0}) - h(\mathbf{x}_1, t_I, \mathbf{p}, \mathbf{q})$ using MODFLOW and are shown in figure 9.4. The corresponding values of d_{crit} and q_{crit} and P is given in table 9.2.

The higher sum of squared distances of a cluster inhibits the property of wider range of well extraction rates. It can be seen in table 9.2 that the ranges of extraction rates in clusters of the social curve were almost similar (0.13-0.23) as compared to the clusters of environmental curve (0.032-0.82). This property is also reflected in figure 9.1 where the social curve does not have a sudden peak as compared to the second cluster of the environmental curve. However, since the groundwater extraction rates are of a similar scale and the maximum singular value of the principal components (directly proportional to the explained variance) in clusters of both the curves are almost similar, it can be concluded that the group of $\mathbf{q_{spr}}$ in each cluster is similar.

Analysis of the wells with |score|>0.0001 for the environmental curve from the first cluster to the fourth cluster, showed that the well locations initially favored for extraction, lie spatially close to the city limits. With the same principal component analysis of the social curve a vice versa behavior was observed, favoring the extraction close to the river. However, in the contour plot in figure 9.1 it can be observed that for higher P, the wells for groundwater extraction become independent of the locations where the constraints act, which hold for both curves. Simulating the drawdown showed similar results to the $\mathbf{q_{spr}}$ characterization, as it can be seen in the plots (see figure 9.4) that E1 and E2 showed a higher stretch of spatial drawdown outside the city due to higher extraction close to the city limit constraint, and a vice versa effect for plots S1 and S2.

As mentioned in chapter 5, the magnitude of score gives information about the most important dimension in the dataset, which is groundwater extractions in this case. Hence, the wells 3, 9, 13, 14, 23, 25 and 27 having lower score, i.e. |score|<0.0001, in the first three clusters of both curves should not be preferred by a water manager during groundwater extraction. Two wells marked as 13 and 14 in figure 8.5 always had a |score|<0.0001 and should not be preferred for extraction.

Until now, general information regarding the wells to be preferred or avoided for extraction is provided to the water manager. Different wells are preferred for extraction in cluster 1 on the environmental and social curves (comparing the |scores| >0.0001 stated in table 9.1). Hence for a given amount of total extraction P a trade-off between $\mathbf{q_{spr}}$ on the environmental and social curves should be performed. This feature can also be seen in figure 9.2.

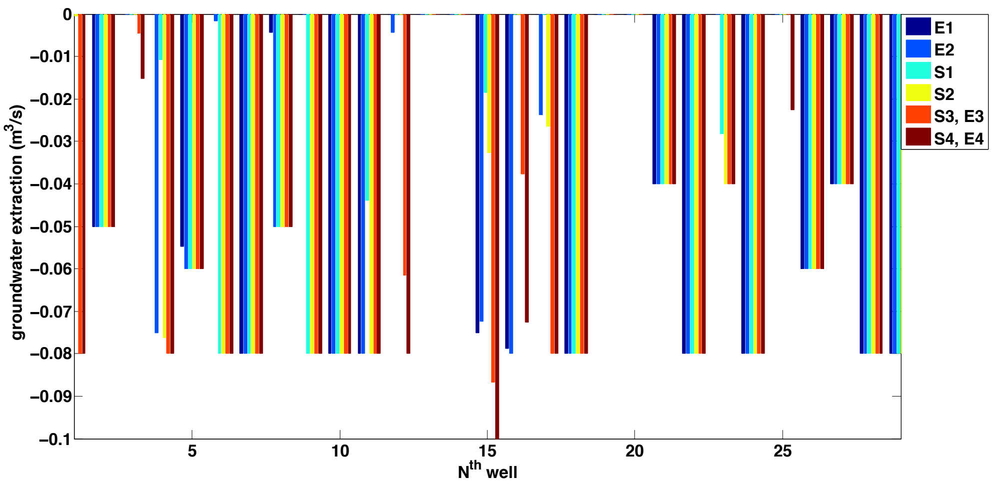


Figure 9.2: Individual groundwater extraction rates at each of the well

Table 9.2: List of points used for performance analysis (shown in the figure 9.1)

	$d_{crit}(m)$	$q_{crit}(m^3/s)$	$P(m^3/s)$
E1	1.73	0.80	1.12
E2	1.96	0.93	1.27
S1	1.00	0.90	1.12
S2	1.32	1.02	1.27
S3, E3	2.36	1.19	1.57
S4, E4	2.85	1.25	1.67

9.4 Linear trade off

For a known P, one point from each of the social and environmental curves can be picked, consisting of different $\mathbf{q_{spr}}$ with known constraint values. As these $\mathbf{q_{spr}}$ lie on either of the curves, either of them favors a higher value of one of the constraints and consequently favoring higher groundwater extraction at certain locations. The constraints and $\mathbf{q_{spr}}$ from the two curves can be traded off to the ratio of water demands, using a methodology defined as follows. In case of implementing trade-off, firstly the P has to be selected, followed by the trading-off between the two $\mathbf{q_{spr}}$ on the social and environmental curve. The trade-off has to be based on the ratio of demands at locations close to the locations where the constraint d_{crit} and q_{crit} act. In case that the ratio of demands is $w1 : w2$, where $w1$ is the total demand at well locations close to the city limit constraint as shown in figure 8.5 and $w2$ is the total demand at

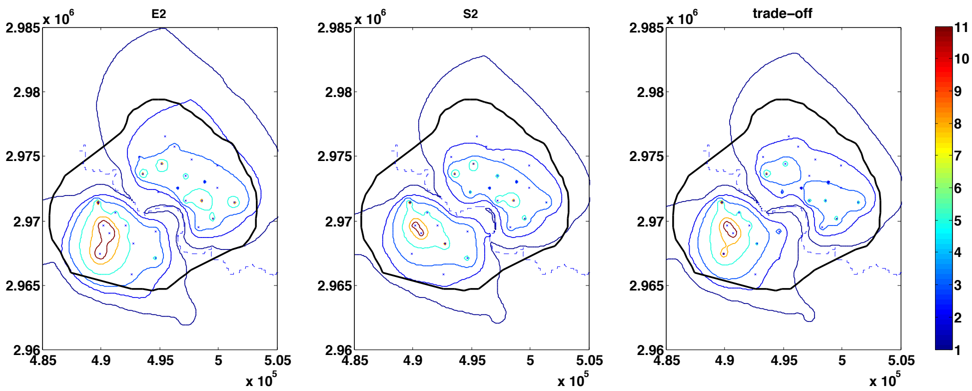


Figure 9.3: Groundwater drawdowns [m] after implementing trade off in E2 and S2.

othere wells, $w1 + w2 = 1$ and $w1 > 0, w2 > 0$, a vector $\mathbf{q}^{\mathbf{tr}}_{\mathbf{ext}}$ is determined by dividing the $\mathbf{q_{spr}}$ of the two points in the ratio $w1 : w2$.

For example consider these two points to be E2 (point E2 groundwater extraction vector $\mathbf{q_{spr}}^{E2}$) and S2 (groundwater extraction vector $\mathbf{q_{spr}}^{S2}$) which have the same P=1.27 m^3/s. The groundwater extraction vectors $\mathbf{q_{spr}}^{E2}$ and $\mathbf{q_{spr}}^{S2}$ are divided in the ratio of demands $w1 : w2$ i.e. $\mathbf{q_{spr}}^{tr} = \mathbf{q_{spr}}^{E2} \cdot w1 + \mathbf{q_{spr}}^{S2} \cdot w2$. In case the demands are identical, $w1 = w2$, the constraints are also traded-off in the same ratio of 1:1 due to linear nature of the problem. The groundwater drawdowns due to the three groundwater extractions $\mathbf{q_{spr}}^{E2}$, $\mathbf{q_{spr}}^{S2}$ and $\mathbf{q_{spr}}^{tr}$ are shown in figure 9.3. The lowest drawdowns in the trade-off plot can be seen in the middle of the lowest drawdowns in E2 and S2 in areas close to the environmental constraint or social constraint. The implementation of trade offs provided the trade off to d_{crit} as 1.64 m and q_{crit} as 0.9750 m^3/s. Hence, the ratio of water demands trades off the constraints in the same ratio of 0.5. The trade off implementation also resulted in more groundwater extraction of 100% at well locations 4, 6, 9, 17, 23 and less groundwater extraction of 43% at 1, 12, 15, 16 as compared to E2 and vice versa as compared to S2.

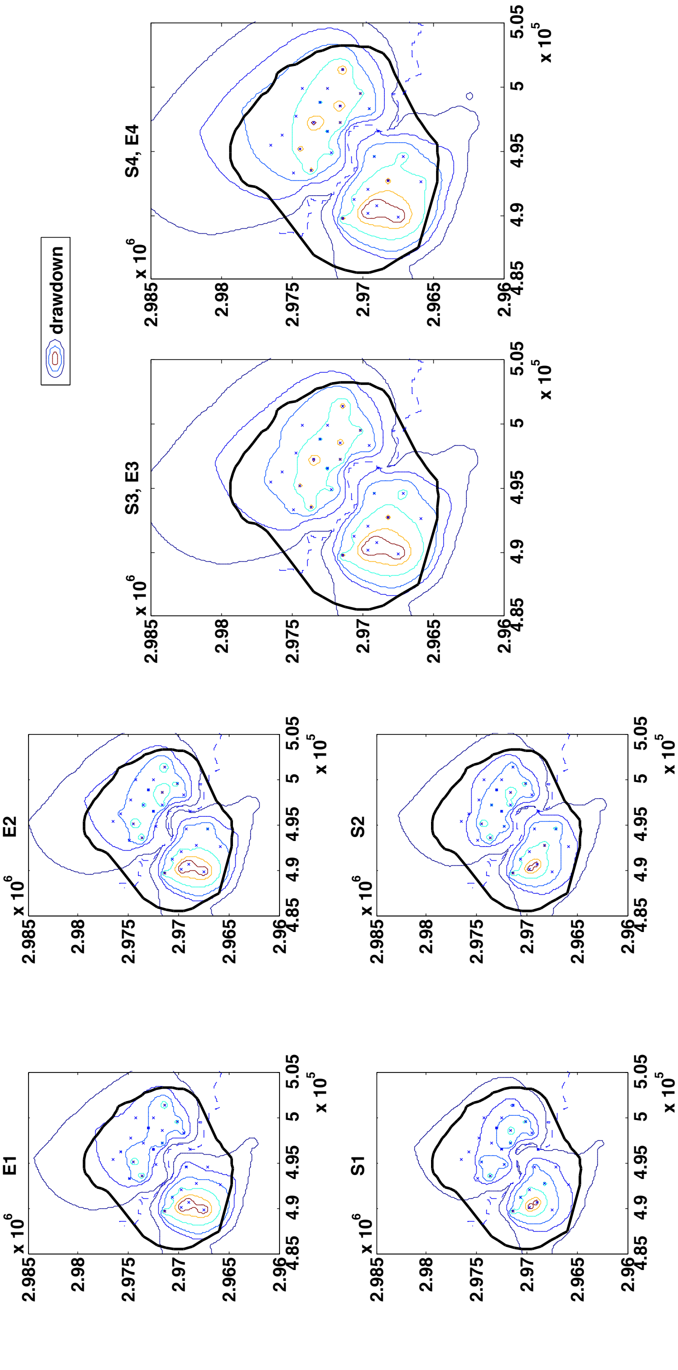


Figure 9.4: Head drawdowns [m] for time t_1 for various groundwater extraction rates (given in table 9.2), as compared to no groundwater extraction

9.5 Discussion

The SPR concept, $P = \Delta R_0 - \Delta D_0$ transformed to $P(\mathbf{q_{spr}}) = f(d_{crit}, q_{crit})$ was implemented in the groundwater sustainability study of Lucknow. It should be noted that the d_{crit} and q_{crit} values are significant not only from a sustainability point of view but also covers a wider area of social equality and river ecology. Although the $P(\mathbf{q_{spr}})$ covers the water supply issue based on the SPR concept, higher values of the constraint does not always permit higher extraction rates (as some wells dried out during simulations) i.e. the hydrogeological setting of the groundwater basin is the actual constraint for the SPR concept which defines the range of the selected d_{crit} and q_{crit} values.

As per definition, since the SPR concept focused only on steady-state simulations, the groundwater model was simulated for 12 years to achieve a dynamic water balance, under the same groundwater extraction. The SPR concept was successfully implemented using the groundwater model, the response matrix and the optimization tool. The groundwater model could simulate the heads at a satisfactory level inside the city limits while the response matrix could simulate the heads at time t_1 with less computational time and effort. The plausibility check showed that the optimization was successful as the groundwater drawdowns stretched and confined close to the city limits when $\mathbf{q_{spr}}$ for the environmental and social curve was used, respectively. The multivariate analysis of the computed $\mathbf{q_{spr}}$ using cluster analysis and principal component analysis helped in guiding the water manager to know the wells to be avoided or preferred for groundwater extraction based on |scores|. The trade-off helps the water manager to evaluate $\mathbf{q_{spr}}$ based upon the water demands spatially close to the location where the constraints act.

In a case that the SPR concept was unable to support groundwater extraction at certain extraction wells due to constraints and the water demand was higher than the total extraction, alternative water supply measures should be taken to meet the deficit by importing water. The supply deficit can be calculated using the developed methodology.

The social and environmental curves converge (points E3 or S3), since for larger P, only one feasible set of (d_{crit}, q_{crit}) exists, the other sets have larger value of either d_{crit} or q_{crit} but not P. It suggests that the implementation of the trade-off is also restricted by constraints and $\mathbf{q_{spr}}$. Also, since the error of the MODFLOW groundwater model is $\sim 1.2m$ inside the city limits (section 8.1), the SPR results where $d_{crit} > 1.2m$ can only be considered.

The SPR concept can be transformed to a constraint based function, similar to this study, when different number of constraints exist. Similar characterization of $\mathbf{q_{spr}}$ can be used to help a water manager find to wells to be preferred or avoided for extraction.

However, the developed methodology is not applicable in a case where the sustainable results are required before or after the aforementioned date. In such a case the procedure listed from previous sections has to be performed again. In a case the constraints hold a non-linear relationship with respect to the groundwater extraction rates, the developed groundwater model should be used to calculate constraints, under acceptable computational time and effort. Otherwise another methodology of simulating the constraints within time limits should be developed. Also in such a case the trade-offs can still be implemented by calculating the $\mathbf{q_{spr}}$ in a similar fashion.

10

Rooftop rainwater harvesting

The groundwater extraction in the city of Lucknow has resulted in decreasing groundwater discharge to the river and groundwater drawdowns, spatially extending up to the peri-urban and rural areas as shown in several reports (REC, 2000; CGW, 2009; Foster and Choudhary N., 2009; Jain et al., 2007) and confirmed by the current model runs. Artificial groundwater recharge measures can counter these spatially extending drawdowns, but the space, geological and economic constraints related to surface or sub-surface storage (see chapter 6 for details) play a major role in selecting the type of measure to be adopted.

The space constraints due to urban features and water borne diseases do not allow any surface groundwater recharge methods, while the investment costs for injection wells are not foreseeable due to the lack of data. Also, an experimental study performed at Mehsana Area And Coastal Saurashtra - Gujarat resulted in clogging of the injection well, decreasing the injection rate from 12 liters per second to 3 liters per second (REC, 2000). The clogging occurred due to lacking removal of the silt in the recharged water.

Hence, the only artificial groundwater recharge method considered feasible for the city of Lucknow is rooftop rainwater harvesting (RRWH). RRWH is a common practice in cities worldwide, mainly depending upon the decision of law-enforcing institutions (Gale et al., 2006). Suspended matter can be filtered out from the recharged water by a gravel sand filter in the RRWH installation. An RRWH scheme has been applied already in the metropolitan city of Chennai, India, facing similar groundwater problems as Lucknow (Coelho and Reddy, 2004; Sethuraman and Shukla, 2000). The success of RRWH in the city of Chennai, is a motivation to study RRWH in the city of Lucknow. Both cities are similar with respect to socio-economic conditions and the type of law-enforcing agencies.

10.1 Data Inventory

The city of Lucknow has been divided into different blocks by the administrative authorities as shown in figure 10.1. The area of the corresponding city blocks is given in table 10.1. These blocks are the residential areas in the city of Lucknow. Sarojininagar is a newly developed residential area of the city. All blocks listed in table 10.1 can be considered as urban areas, except few parts of Sarojininagar, which is usually considered to be peri-urban. However, due to the growth of the city, some areas which were previously considered to be peri-urban have changed to urban in the past years. Hence, the urban limits in the figure 10.1 cross a significant portion of the Sarojininagar area. Currently, there are approximately 350,000 households (total number of households including all in a single building or house) in the city of Lucknow and 100,000 houses. This data has been estimated by rough figures provided by different newspaper reports and have been used in the study due to unavailability of any other specific geo-informatics based data. Also, the number of structures in each administrative block is considered to be directly proportional to the size of the area. The amount of investment required for a single rooftop rainwater installation is $\sim$ ₹5000 ($http://CGWB.gov.in/CR/rain\_water\_har.html$ accessed on 23/05/2012).

In order to consider the prices to be paid as compensation to the population staying in the peri-urban

Table 10.1: Data available for Lucknow groundwater model

Name of block	Name in figure 10.1	Area (m^2)
Gomti nagar	RH9_1	7.2221e+06
Indira nagar	RH9_2	1.0020e+07
Mahanagar and Aliganj	RH9_3	1.3130e+07
Vikas nagar	RH9_4	1.0207e+07
Gulistan colony	RH9_5	5.0585e+06
Aminabad	RH9_6	1.2186e+07
Chowk	RH9_7	5.6237e+06
Rajajipuram and Aishbagh	RH9_8	1.3645e+07
Sarojininagar	RH9_9	1.6731e+07
Cantonment	RH9_10	2.7199e+06

and rural areas, a spatial map of the water prices in such areas was adopted from the water board of Lucknow and world bank reports (Sethuraman and Shukla, 2000). These prices decrease with distance from the urban limits as shown in the figure 10.2.

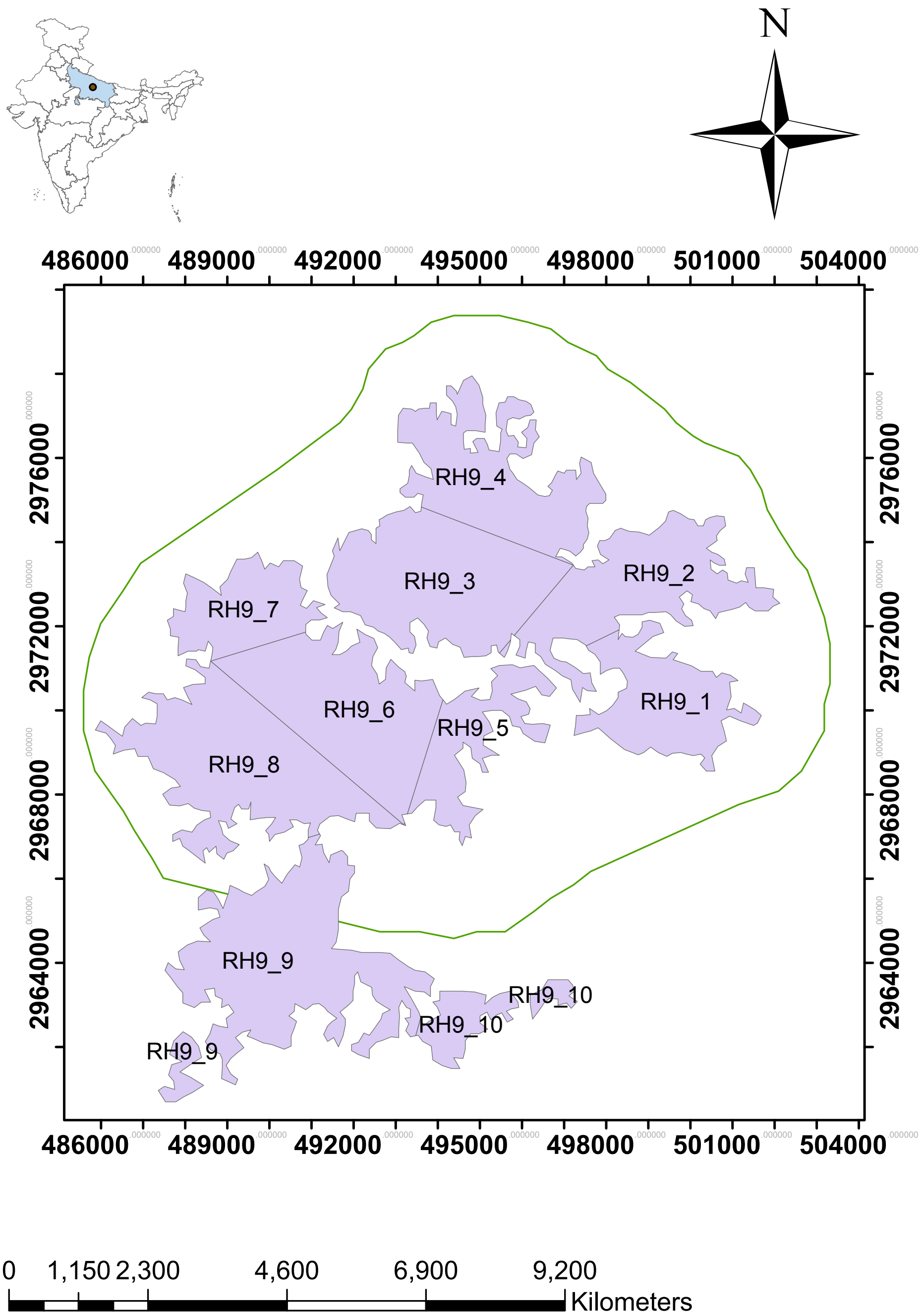


Figure 10.1: Administrative blocks in the city of Lucknow, India. The green line shows the considered urban limit of the city.

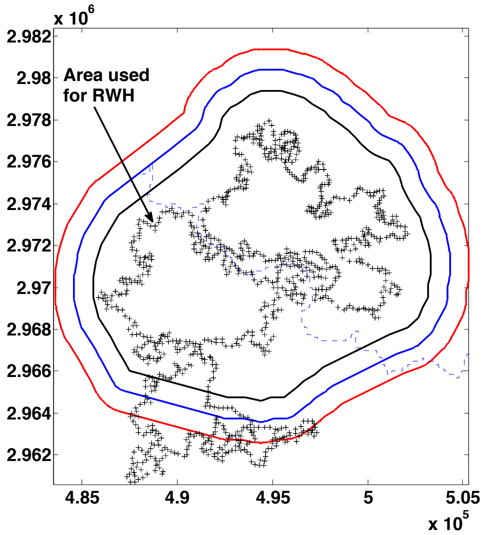


Figure 10.2: The groundwater prices between black and blue, blue and red and outside red boundary is ₹$2/m^3$, ₹$1/m^3$, ₹$0.50/m^3$ respectively. No price of water inside the black boundary (city limits).

10.2 Purpose of Optimization

The purpose of optimization in this study is to predict the socio-economic feasibility of the artificial groundwater recharge system by minimizing the total costs involved in such a project. However, the other optimization functions maximizing the groundwater discharge to the river and total groundwater extractions, cannot be excluded while maximizing the economic benefit, which are therefore included in the optimization problem. Furthermore, the total costs are the only objective, which has not been numerically described so far. Hence, in this section the assumptions behind the total cost is described, followed by the qualitative judgement of the total costs. In the next section, the quantitative description of total costs is provided.

The following evaluation of the economic feasibility is completely performed on the hypothesis that the groundwater extraction users in the city should pay for any RRWH scheme that is applied along with the price for lost amount of groundwater in peri-urban and rural areas depending upon the water prices in the respective regions, as shown in figure 10.2. The purpose of optimization is to evaluate the economic feasibility of RRWH, based upon the comparison of compensation to be provided with and without rainwater harvesting to the urban, peri-urban and rural residents, inside or outside of the city limits, in terms of:

- *drawdown*, advisable to be paid for social equality in areas outside the city, depending upon the water price and extracted groundwater in the respective area due to groundwater extraction in the city, named as *drawdown* costs. (*drawdown* costs = unit price of water at the location outside the city times water lost due to extraction)
- *investment* for installation costs of the rainwater harvesting system inside the city, named as *investment* costs. (*investment* costs = number of houses times amount spent on each installation)

where the unit price of water is based upon a spatial map (see figure 10.2) and the water lost due to extraction is calculated using the product of area, specific yield and drawdown difference between q extraction rates and no extraction between September 2012 and t_1. The price of water is inversely proportional to the distance from the city. The city limit shown in figure 10.2 corresponds to the limits of groundwater extraction users. The cost of water in the city equals ₹$5/m^3$, which is left unconsidered while estimating *drawdown* compensation, since the groundwater extraction wells benefits the users inside the city. The *drawdown* compensation in two peri-urban (up to 2 km distant from the city) and rural areas are based on the assumption of lower water prices than the ones in the city (see figure 10.2). The boundary marked in black is the current city limits, while the boundaries marked in blue and red are at a distance of 1 km and 2 km, respectively.

In the scenario where RRWH is applied throughout the city without spatial considerations, it is assumed that it is applied in all households, regardless whether they lie in the urban, peri-urban or rural areas, as seen in the urban sprawl of the city (Scenario 2). In such case, the total investment costs are fixed and equals the product of number of buildings and the costs associated with each RRWH installation. In another scenario, the total investment costs depend upon the respective areas where the RRWH is applied (Scenario 3). These two scenarios, along with a scenario without any RRWH (Scenario 1) are compared to assess the economic efficiency of the RRWH scheme. Furthermore, each scenario is divided into two parts (a) and (b), the only difference being that the former maximizes the average discharge in the river during last 90 days before time t1 and the latter maximizes the minimum discharge at time t_1.

These six scenarios (see figure 10.3) are used to evaluate the economic feasibility based upon the unit groundwater extraction price comparison[1]. An optimization function for each scenario aims at minimizing the total costs, minimizing the reduction in groundwater discharge to the river and maximizing the total groundwater extraction under constraint (see table 10.1), using respective Jacobian matrices as mentioned in table 10.2. The unit groundwater extraction prices are mainly compared by dividing the total costs by total groundwater extraction in 12 years, i.e. $\sum q \times 86400 \times 12 \times 365$.

[1] Readers should not confuse with the term used earlier unit price of water which applies to the price paid for water in different parts of the city.

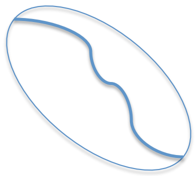
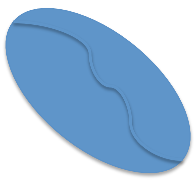
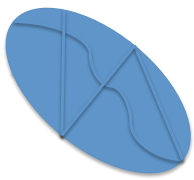
	**Scenario 1** **No artificial groundwater recharge** **a) Minimize average reduction in discharge over last 90 days before 1st May 2024** **b) Minimize reduction in discharge on 1st May 2024**
	**Scenario 2** **Complete artificial groundwater recharge in the city** **a) Minimize average reduction in discharge over last 90 days before 1st May 2024** **b) Minimize reduction in discharge on 1st May 2024**
	**Scenario 3** **Selective artificial groundwater recharge in the city** **a) Minimize average reduction in discharge over last 90 days before 1st May 2024** **b) Minimize reduction in discharge on 1st May 2024**

Figure 10.3: Scenarios used for evaluating economic feasibility

Table 10.2: List of Scenarios where J stands for corresponding Jacobian matrix used for calculation

		Scenario		
		1(a), 1(b)	2(a), 2(b)	3(a), 3(b)
investment costs (₹)	Δ r	**0**, **0**	**1**·0.7, **1**·0.7	evaluated,evaluated
	b	**0**, **0**	**1**, **1**	evaluated,evaluated
drawdown costs (₹)	RRWH(M1)		✓(**J1**, **J1**)	✓(**J1**, **J1**)
	extraction(M2)	✓(**J2**, **J2**)	✓(**J2**, **J2**)	✓(**J2**, **J2**)
drawdown constraints	RRWH(N1)		✓(**K1**, **k1**)	✓(**K1**, **k1**)
	extraction(N2)	✓(**K2**, **k2**)	✓(**K2**, **k2**)	✓(**K2**, **k2**)

$$
\begin{aligned}
J1_{ij} &= \frac{h(x_2, t_1, \mathbf{p}, \mathbf{q_{ext}}, \Delta\mathbf{r_{,j}}) - h(x_2, t_1, \mathbf{p}, \mathbf{q_{ext}}, \mathbf{0})}{\Delta\mathbf{r_{,j}}} \\
J2_{ij} &= \frac{h(x_1, t_1, \mathbf{p}, \mathbf{q_{ext,j}}, \mathbf{0}) - h(x_2, t_1, \mathbf{p}, \mathbf{q_{ext,j}}, \mathbf{0})}{\Delta\mathbf{q_{ext,j}}} \\
K1_{ij} &= \frac{\sum Q(x_{dr}, \mathbf{t_{1,i}}, \mathbf{p}, \mathbf{q_{ext}}, \Delta\mathbf{r_{,j}}) - \sum Q(x_{dr}, \mathbf{t_{1,i}}, \mathbf{p}, \mathbf{q_{ext}}, \mathbf{0})}{n_t \Delta\mathbf{r_{,j}}} \\
k1_{j} &= \frac{\sum Q(x_{dr}, t_1, \mathbf{p}, \mathbf{q_{ext}}, \Delta\mathbf{r_{,j}}) - \sum Q(x_{dr}, t_1, \mathbf{p}, \mathbf{q_{ext}}, \mathbf{0})}{\Delta\mathbf{r_{,j}}} \\
K2_{ij} &= \frac{\sum Q(x_{dr}, \mathbf{t_{1,i}}, \mathbf{p}, \mathbf{q_{ext,j}}, \delta_j \cdot \mathbf{0}) - \sum Q(x_{dr}, \mathbf{t_{1,i}}, \mathbf{p}, \mathbf{q_{ext}}, \mathbf{0})}{n_t \cdot \delta_j \cdot \mathbf{q_{ext,j}}} \\
k2_{j} &= \frac{\sum Q(x_{dr}, t_1, \mathbf{p}, \mathbf{q_{ext,j}}, \mathbf{0}) - \sum Q(x_{dr}, t_1, \mathbf{p}, \mathbf{q_{ext,j}}, \mathbf{0})}{\delta_j \cdot \mathbf{q_{ext,j}}}
\end{aligned}
$$

where x_2 is the location of the cells outside the city limits, $\Delta\mathbf{r}$ are column vector of size 10×1 which represent the same amount of recharge due to RRWH in the respective urban blocks, $\mathbf{b}$ is also a column vector of the same size comprising of only binary elements 1 or 0 to denote if RRWH is applied in the corresponding area or not, respectively, and $\mathbf{t_1}$ is a vector of size 91, consisting of a daily time step for the last 90 days (n_t) before t_1. A recharge flux factor of 0.3 was estimated in areas inside the city using calibration. During RRWH it is assumed that the rest of the recharge infiltrates into the first unconfined aquifer. Hence, a factor of 0.7 is chosen to represent RRWH.

10.3 Cost evaluation

For each scenario the Jacobians are used to evaluate the drawdown costs, investment costs and reduction in groundwater discharge to the river. It was based on the assumption that the groundwater drawdowns and the reduction in groundwater discharge to the river is linear with respect to the groundwater extractions and $\Delta\mathbf{r}$, which is valid in a case if the value of ε in equation 10.1 is negligible. Similar formulation for reduction in groundwater discharge has to be verified as well.

$$
||h(x_2, t_1, \mathbf{p}, \mathbf{q_{ext}}, \Delta\mathbf{r_{,j}}) - h(x_2, t_1, \mathbf{p}, \mathbf{q_{ext}}, \mathbf{0}) - \mathbf{M1} \cdot \Delta\mathbf{r} \cdot \mathbf{b} - \mathbf{M2} \cdot (\mathbf{q_{ext}})|| < \varepsilon \tag{10.1}
$$

and a similar formulation for reduction in groundwater. The costs are evaluated for each scenario by substituting the respective Jacobian matrices M1, M2 as stated in table 10.2 in the following equation.

1. Drawdown costs

The drawdown costs are computed as follows:

$$\text{drawdown costs (₹)} = \mathbf{pr}(\mathbf{x_2}) \cdot \mathbf{a}(\mathbf{x_2}) \cdot \mathbf{s_y}(\mathbf{x_2}) \cdot (\mathbf{M1}\Delta\mathbf{r} \cdot \mathbf{b} + \mathbf{M2} \cdot \mathbf{q}) \tag{10.2}$$

where $\mathbf{pr}(\mathbf{x_2}), \mathbf{a}(\mathbf{x_2}), \mathbf{s_y}(\mathbf{x_2})$ are the price of water (₹/m^3), area of the cell and specific yield, respectively, at location x_2.

2. Investment costs

   The investment costs are computed as follows:

$$\text{investment costs (₹)} = \begin{cases} 0 & \text{if No RRWH} \\ 5 \cdot 10^8 & \text{if RRWH without spatial considerations} \\ 5 \cdot 10^8 \cdot \sum \frac{Area}{\sum Area} \mathbf{b} & \text{if RRWH with spatial considerations} \end{cases}$$

10.4 Optimization

The optimization function is formulated numerically in the first step for cost evaluation as follows. The total costs and groundwater drainage to the river for each scenario given in table 10.2 are computed using a set of Jacobian matrices, and the optimization function is formulated as follows:

$$\min_{\mathbf{q},\mathbf{b}} \quad \text{total costs}(\mathbf{q},\mathbf{b}) = \text{drawdown costs + investment costs} \tag{10.3}$$

$$\min \quad (\vec{1}\mathbf{N1}\Delta\mathbf{r} \cdot \mathbf{b} + \mathbf{N2} \cdot \mathbf{q}) \tag{10.4}$$

$$\max \quad \sum_{j=1}^{29} q_j \tag{10.5}$$

subjected to constraints

$$\vec{0} \leq \mathbf{q} \leq \mathbf{q_{ext}} \tag{10.6}$$

The equations 10.3 and 10.4 calculates the total costs and the reduction in groundwater discharge to the river, respectively, and have to be minimized while equation 10.5 is maximized to obtain maximum groundwater extractions. Alltogether it is a multi-objective optimization. All equations are subject to constraints of maximum groundwater extraction as given in equation 10.6. The matrices used in these equations from 10.3-10.5 are calculated using the respective Jacobians for each scenario as stated in table 10.2.

The total costs and groundwater drainage to the river are linear with respect to RRWH, which is represented using $\Delta\mathbf{r}$ and the groundwater extraction rates $\mathbf{q}$. The maximum value of ε in equation 10.1, using respective Jacobian matrices as stated in table 10.2, was calculated to be 0.286 m and 0.0650 m^3/s (using similar formulation for reduction in discharge), which is negligible in comparison to the minimum groundwater depth in the model and the lowest discharge. Hence, the assumption that the total costs and reduction in groundwater discharge are linear with respect to the groundwater extractions and $\Delta\mathbf{r}$ in a case $\mathbf{0} < \mathbf{q} < \mathbf{q_{ext}}$ and $\mathbf{0} < \Delta\mathbf{r} < \vec{1} \cdot 0.7$ is valid.

10.5 Results

The multi-objective problems for scenarios 1-3 are formulated by substituting the respective Jacobian matrices as mentioned in table 10.2 and **b** as a constant column vector of ones for scenarios 1 and 2, in equation 10.3. The solution to the multi-objective problem provides the groundwater extractions and reduction in groundwater discharge for scenario 1-3, and the binary vector **b** (scenario 3).

The multi-objective function aims at finding the maximum number of non-dominated points (pareto points), in order to compare the benefits of scenarios 1-3 against each other. An aggregate objective function using weights for each individual objective function was combined to form a single objective function. However, it was not used further since the weights had to be transformed accordingly to the magnitude of the optimization functions, which were unable to produce a number of non-dominated points. Hence, the groundwater extractions and the binary vectors are evaluated by performing multi-objective optimization. The NSGA2 algorithm was applied for solving the problem, using a population size and maximum generation of 200 each, which was chosen based on preliminary runs and seen on the graphical animated plot. The population generated after every 10^{th} generation was produced as results. In total 4100 results are analyzed which are compared in the next paragraph.

Three plots for all scenarios (as given in table 10.2) are shown in figures 10.4-10.6. The explanation of the results is presented in three parts. The first part covers the comparison of the reduction in groundwater discharge to the river and the total costs for scenarios 1-3. The second part compares the unit groundwater extraction prices, based upon the total costs and total groundwater extraction for scenarios 1-3. The third part emphasizes the differences in the binary vector and total investment costs, based upon the favorable RRWH recharge areas for scenarios 3(a) and 3(b).

The reduction in groundwater discharge to the river (d_{crit}) and total costs decreases in 2(a) and 3(a), as compared to 1(a) and exactly the same behavior was observed in 1(b) to 3(b), for the same amount of total groundwater extraction. It clearly indicates that RRWH is an efficient method in terms of increasing groundwater discharge to the river considering an optimized groundwater extraction scenario. However, it can be seen in figures 10.4 and 10.5 that 2(a) and 2(b) had either slightly lower or same values of d_{crit} and total costs, as compared to 3(a) and 3(b), in figures 10.4 and 10.5, respectively. This is due to the fact that scenario 2 used a complete recharge of all areas in the city, as compared to scenario 3 where the recharge areas were selected on the basis of the optimization code NSGA2. Hence, in case the optimization of the recharge area resulted in recharge for all blocks of the city, equal reduction in groundwater discharge to the river and total costs are observed. On comparing (a) and (b) part of the same scenario (for e.g. 1(a) and 1(b)), it can be seen that the total costs are smaller in (b), as compared to (a) for an optimized groundwater extraction for all scenarios 1-3 (figure 10.6). This occurs due to the different optimized groundwater extraction scenario as a result of the NSGA2 algorithm. Further investigations on this topic are outside the scope of the study, since it focuses only on the economic feasibility of RRWH and choosing scenario (a) or (b) is a management decision. In the first plot of figure 10.4 (total groundwater extraction d_{crit}) a linear relationship can be seen for scenario 1(a) and 2(a), since a linear relationship for evaluating d_{crit} was used. Also it can be seen that the NSGA2 algorithm was unable to produce extractions of 2 m^3/s (except in scenario 2(b)), although the costs for the highest extraction can directly be computed for scenarios 1 and 2 without any optimization procedure, which followed the above mentioned trend.

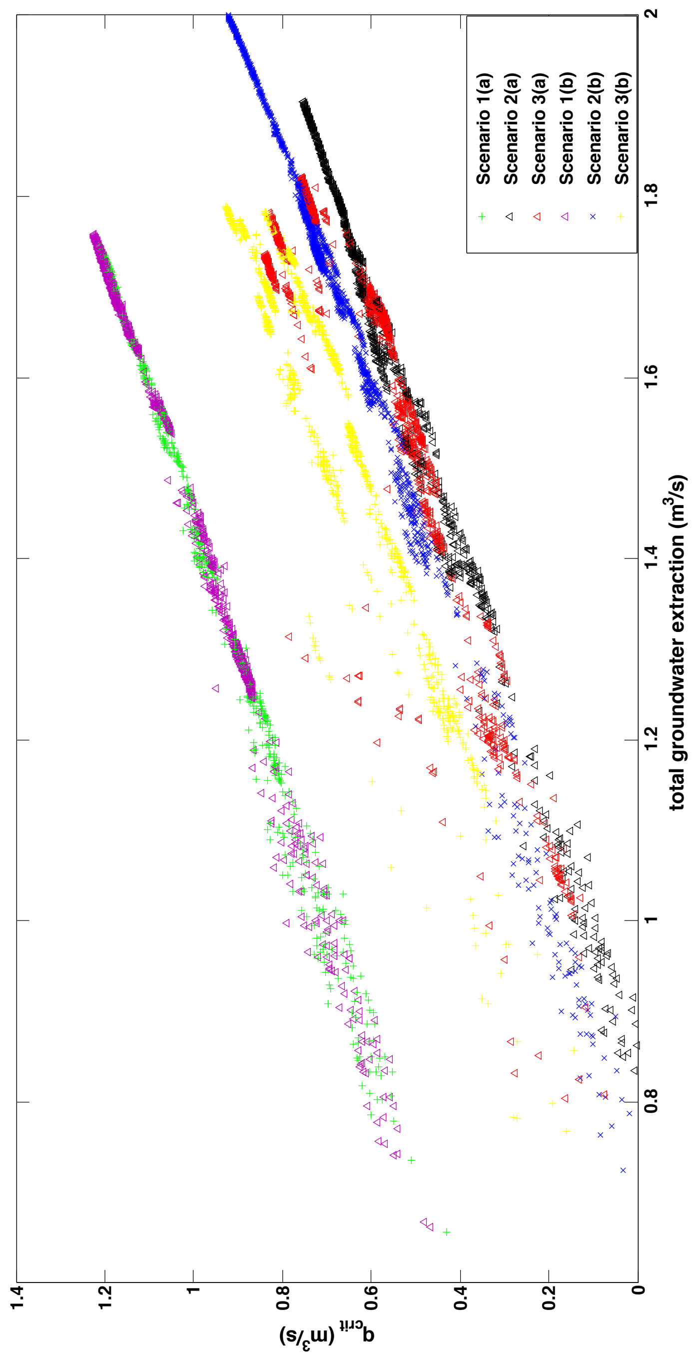


Figure 10.4: Optimization results of scenarios 1-3. q_{crit} is the reduction in groundwater discharge to the river.

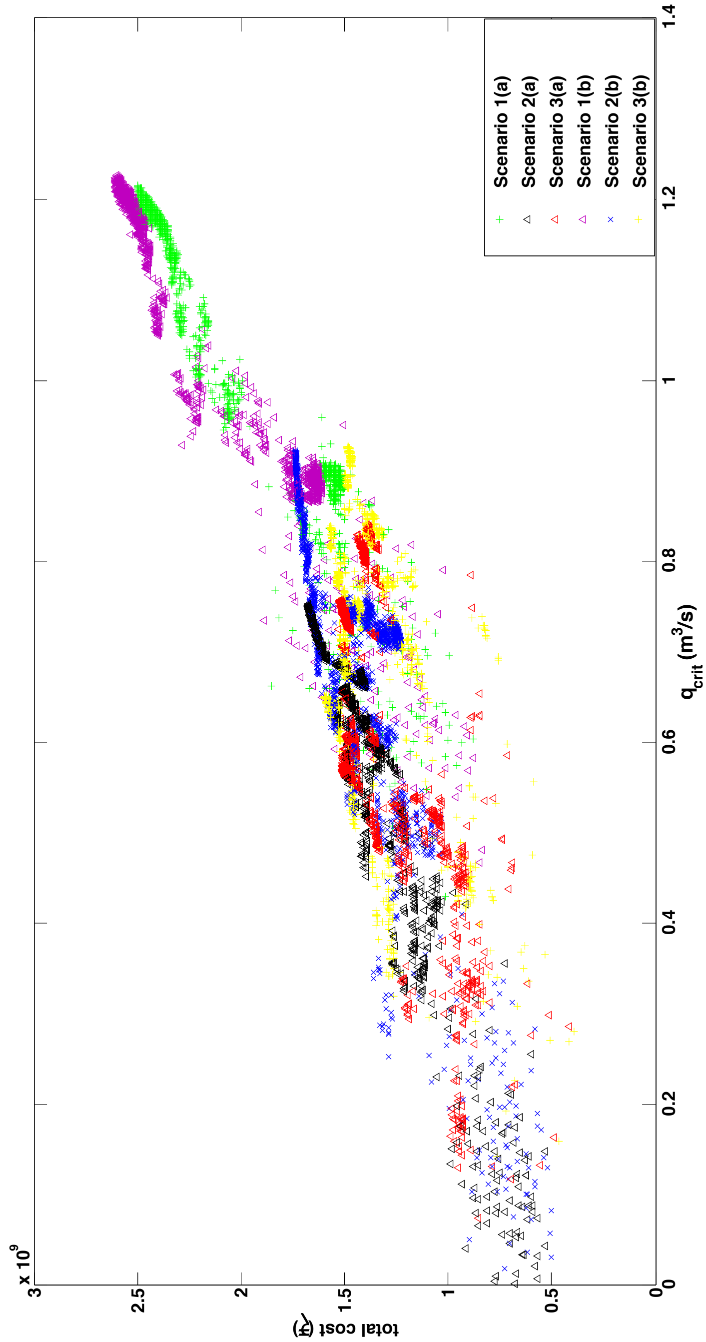


Figure 10.5: Optimization results of scenarios 1-3. q_{crit} is the reduction in groundwater discharge to the river.

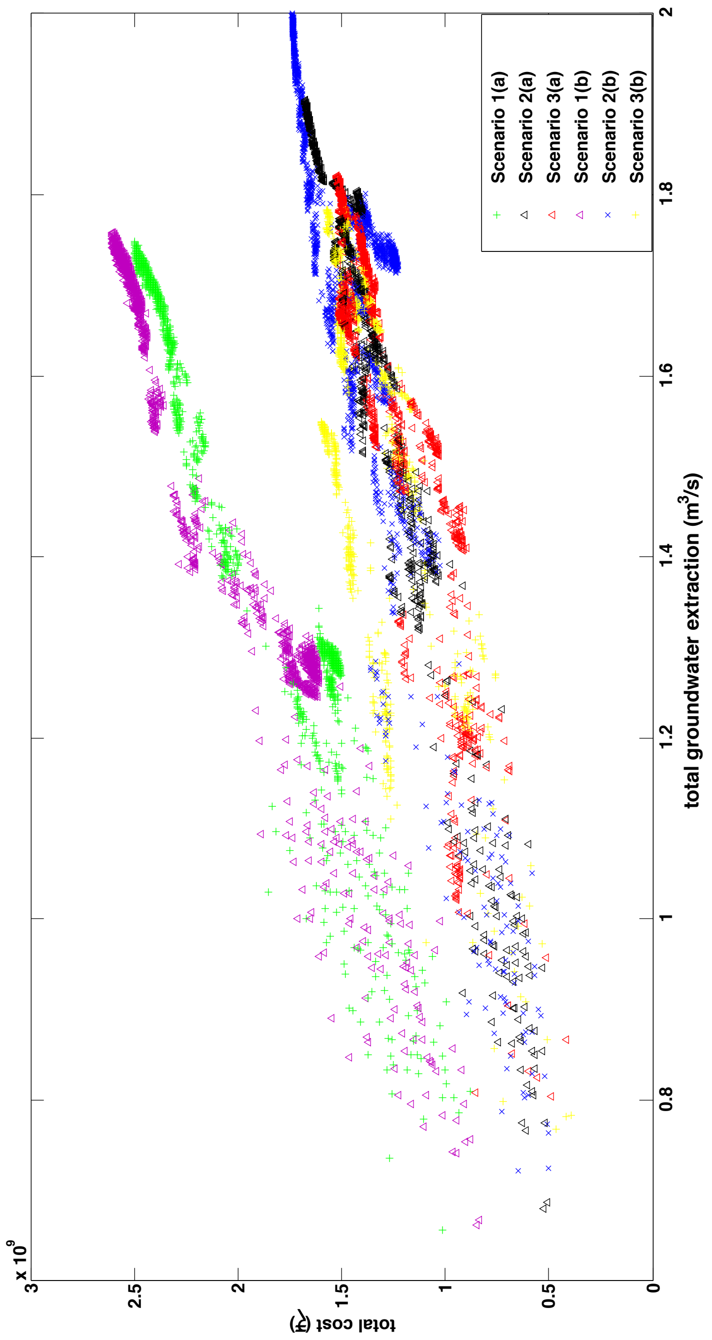


Figure 10.6: Optimization results of scenarios 1-3.

It is important to evaluate the unit groundwater extraction prices, which is an exact indicator of the savings groundwater extraction users inhibit in a case where RRWH is implemented in the city. As mentioned before, the unit groundwater extraction prices, evaluated by dividing the total costs by the total groundwater extraction over 12 years, i.e. $\sum q \times 86400 \times 12 \times 365$, are shown in the figure 10.7. In the plot for each scenario, there exists for each total groundwater extraction a higher and a lower price (₹$/m^3$), which correspond to lower and higher reduction in groundwater discharge to the river Gomti, respectively. It can be seen in figure 10.7 that the prices of groundwater extraction are much lower when RRWH is implemented in the city. The NSGA2 algorithm used a population of 200 in each of the 200 generations for scenario 1-3, but was unable to optimize groundwater extraction up to the maximum extraction of $\mathbf{q_{ext}}$, except in scenario 2(b). However the price benefit of up to ₹1/m^3 is obtained if prices of total groundwater extraction is considered, in a case in which RRWH is implemented, which proves the economic feasibility of its implementation. The unit groundwater extraction prices in scenario 2 and 3 are almost similar. In few cases the unit groundwater extraction price in scenario 3 is slightly smaller than in scenario 2, due to the optimization of the RRWH area required for recharge, which is explained in the following paragraphs.

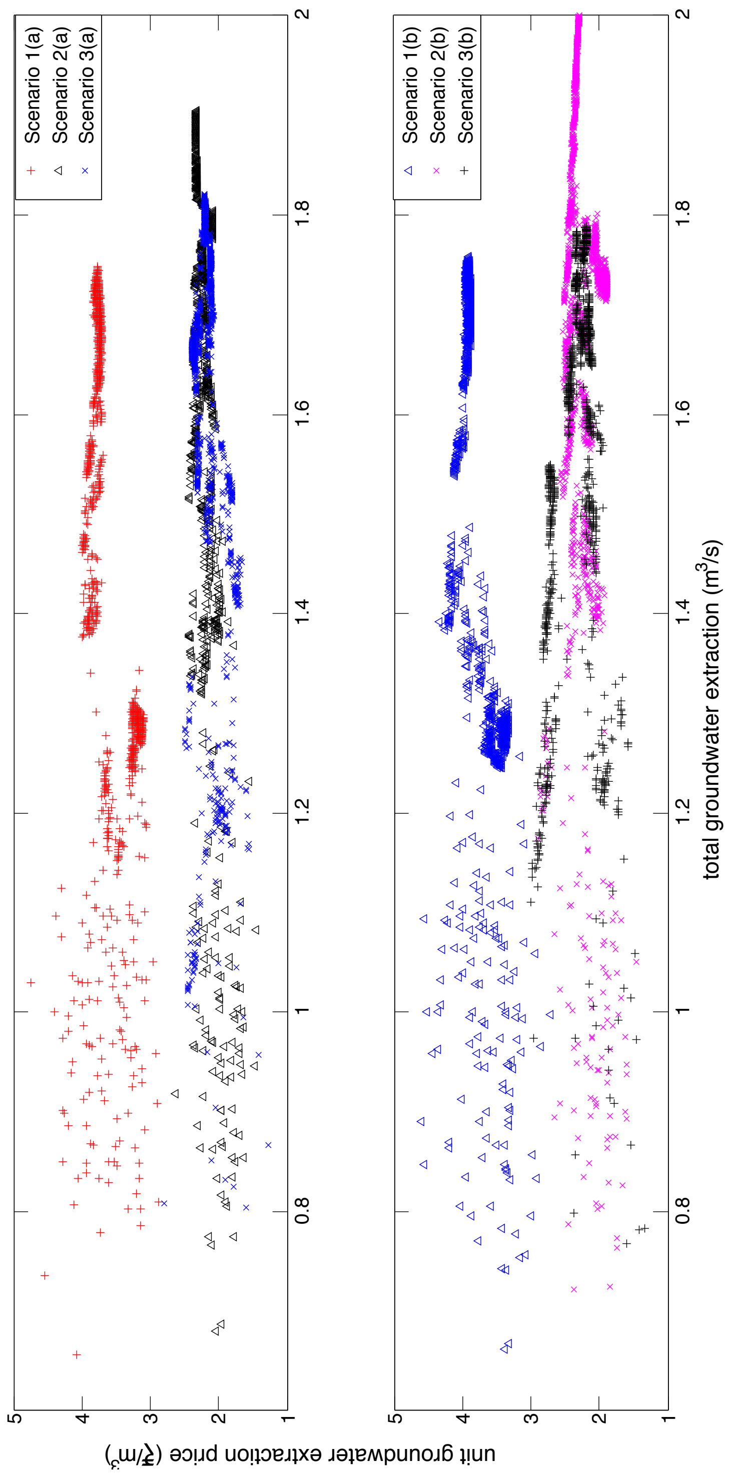


Figure 10.7: Unit groundwater extraction evaluated in different scenarios

The NSGA2 algorithm is also used for the assessment of the best blocks in the city for RRWH. The results of the optimization are shown in figure 10.8. It can be seen that the optimization results iterated initially, using different choices for the binary vector **b** in the beginning, but showed similar solutions in the end, which was also seen during restart of the optimization. Hence, the populations of **b** converge to solutions in both scenarios, 3(a) and 3(b). The city blocks RH9_4, RH9_5 and RH9_7, and RH9_1, RH9_5, RH9_6, RH9_7 and RH9_10 should be avoided for RRWH implementation, considering scenario 3(a) and 3(b), respectively. It can be seen that city blocks RH9_5 and RH9_7 were avoided for RRWH implementation in both scenarios 3(a) and 3(b), considering the spatial distance from the urban limits and small areas. The city blocks RH9_2, RH9_3, RH9_8, and RH9_9 were always preferred and should be considered by the water manager for RRWH implementation. However, it can be seen that both scenarios favored different city blocks for RRWH implementation.

Although it can be seen that the blocks were selected for implementing RRWH during optimization, the total costs of Scenario 3(a) are almost similar to the total costs in scenario 2(a) or were slightly lower. This is due to the fact that the total investment costs are an order of magnitude lower than the drawdown costs. However, the implementation of NSGA2 could efficiently find the blocks where RRWH should be implemented. In order to highlight the investment difference between scenario 3(a) and 2(a), another plot of total groundwater extraction and investment costs is shown in figure 10.9. The costs of investment in scenario 2(a) and 2(b) are always the same of ₹500 million, based on the formulation of the problem. Furthermore, it can be seen that the investment costs can be decreased by ₹100 million for the same benefits in terms of groundwater extraction or d_{crit}. Hence, the RRWH implementation in different blocks of the city as calculated by the NSGA2 algorithm shows that the same amount of groundwater extraction can be achieved using smaller investment costs. Also, smaller amount of investment costs for a new project to be implemented amounts to smaller risk as compared to larger investments. Hence, considering this, the RRWH scheme in the city should be started from the city blocks RH9_2, RH9_3, RH9_8 and RH9_9, and depending upon the benefits gained in terms of total costs, should be extended to other areas of the city.

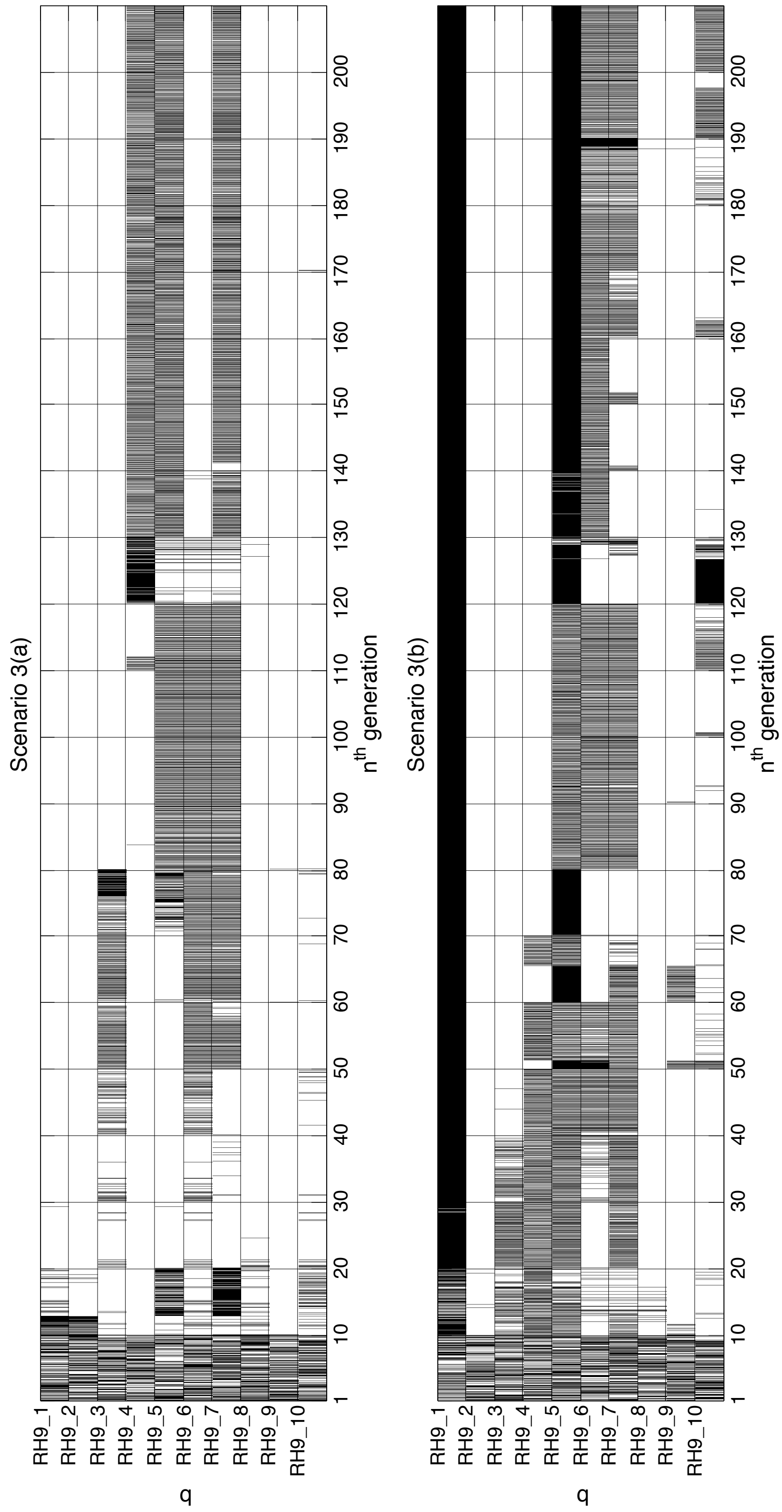


Figure 10.8: Optimized area for recharging where b is the binary vector estimated using the NSGA2 algorithm. The color black signifies no recharge while white signifies recharge. A population of 200 for every 10^{th} generation is plotted in the figure.

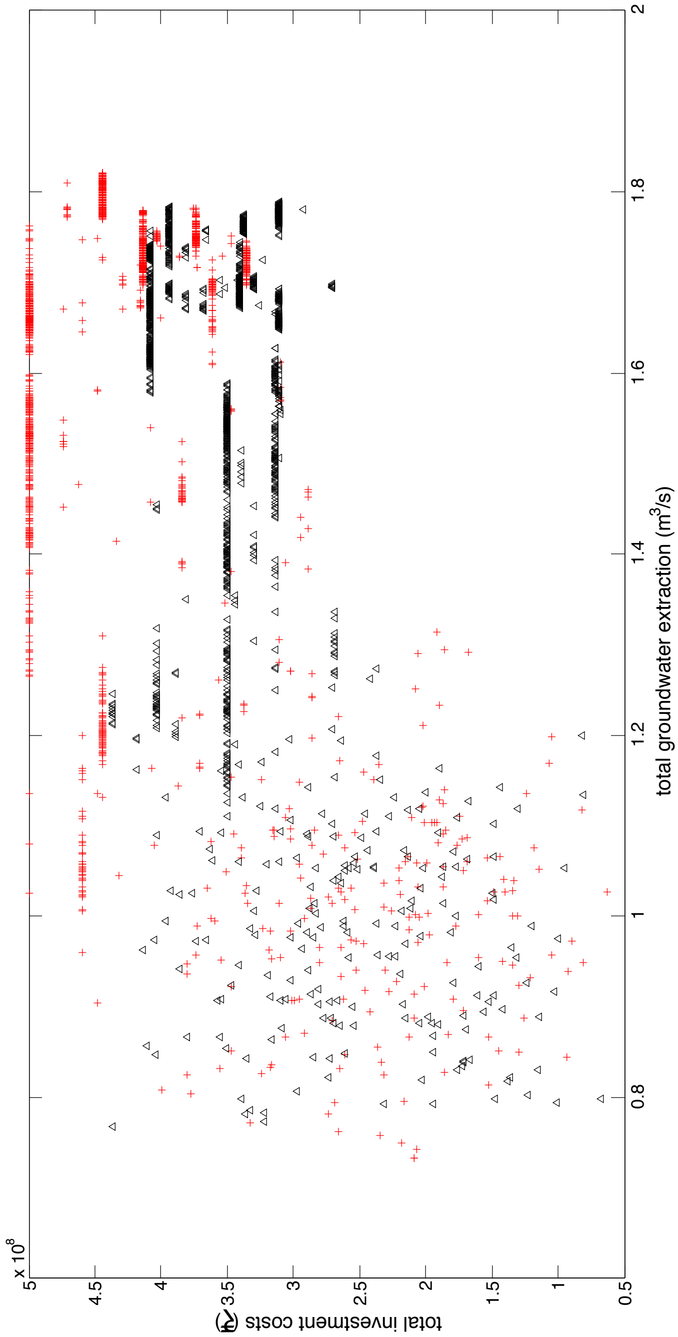


Figure 10.9: Optimized area for recharging. The color black signifies no recharge while white signifies recharge.

It can be noted that none of the scenarios considered the cost of groundwater discharge to the river, while drawdown costs are considered. It can be understood in terms of the spatial cost distribution of the unit price of water, as compared to only one unit price of groundwater discharge to the river. Since the second function in equation 10.3 needs only to be multiplied with a constant factor, the results of multi-objective function will not be affected. This is due to the fact that the multi-objective optimization used in this study minimizes the functions independently, without considering the weight of individual functions. Also, in case that the water prices in the peri-urban and rural areas increase by a constant factor the results of the multi-objective optimization will not change. The only change will be in the scale of the axis of total water costs, but multi-objective optimization will provide all information needed.

Efficiency evaluation

In order to compute the minimum value of $\Delta\mathbf{r}$ in a way that the RRWH is economically feasible, the drawdown costs of groundwater extraction $\mathbf{q}$ with RRWH should be lower than without RRWH i.e.

$$\begin{aligned}\mathbf{pr}(\mathbf{x_2})\mathbf{a}(\mathbf{x_2})\mathbf{s_y}(\mathbf{x_2})(\mathbf{J1}\Delta\mathbf{r}\cdot\mathbf{b}+\mathbf{J2q}) &- \mathbf{pr}(\mathbf{x_2})\mathbf{a}(\mathbf{x_2})\mathbf{s_y}(\mathbf{x_2})(\mathbf{J2q}) \\ &\geq \sum\frac{Area}{\sum Area}\cdot\mathbf{b}\cdot 5\cdot 10^8\end{aligned}$$

Considering that $\mathbf{b}$ is a constant vector of value 1 for Scenario 2, i.e. RRWH is applied in all parts of the city, the solution of the above equation reveals that $\Delta\mathbf{r}$ is independent of $\mathbf{q}$. Also for the economic feasibility of RRWH, all elements of the vector $\Delta\mathbf{r}$ should be greater than 0.2. Hence, an efficiency of RRWH system less than 0.2/0.7 results in RRWH not yielding economic benefits. In a case in which the RRWH scheme is applied, the price (₹/m^3) will start yielding profits of ₹315 / year for each RRWH installation after t_1. Solving the same equation for scenario 3 follows another procedure than the one used for scenario 2, since $\mathbf{b}$ is no longer a constant vector of value 1. Hence, a value of $\Delta\mathbf{r}$ could not be determined.

11

Conclusions

This thesis has provided a framework for managing the groundwater resources in the city of Lucknow by calculating the sustainable extraction in the city and studying the feasibility of an artificial groundwater recharge scheme using a numerical groundwater model and optimization algorithms. The results of the thesis have provided a water manager with the pre-assessment of the application of optimized groundwater extractions and artificial groundwater recharge.
In this chapter the answers to the research question stated in the introduction of the thesis are provided, followed by the importance of the study in similar urban groundwater extraction scenarios and future work that can enhance the performance of the sustainable groundwater management of Lucknow city.

What relevance does SPR concept in terms of applicability to the Lucknow city? Can the water manager have a trade off between the amount of groundwater extractions at respective locations and still follow the SPR concept?

The approach towards groundwater sustainability can be applied using the SPR concept. Although the concept provides a general definition of the SPR based on the groundwater budget, it was never applied in terms of induced recharge and discharge, which can be represented as constraints while considering a groundwater extraction scenario. This study emphasizes upon including only those induced recharge and discharge terms, which should be comparable to the amount of groundwater extraction required. For example, in this study, both these terms were comparable to the amount of groundwater extraction estimated using the SPR concept. The classical SPR concept assumed the water balance at a steady state which was again cited as a drawback of the concept rather than making the definition applicable.
All these limitations or drawbacks of the SPR concept could be overcome using a numerical ground-

water model. The steady state drawback was overcome applying the concept of a dynamic-steady state which yields transient, but time periodic results. Although the time required to reach this dynamic-steady state was chosen to be 12 years, it can be further extended by the water manager as needed.

In this thesis, the SPR concept itself formed the basis of building the conceptual model by assuming the urban setting of Lucknow as the control volume of the SPR concept. Also, the numerical representation of the conceptual model assisted in quantitatively defining the social and environmental constraints as the amount of water entering from the peri-urban and rural areas and the groundwater discharge to the river, respectively. The implementation of the SPR concept using the groundwater model is the first effort of achieving sustainable water management for the city of Lucknow and can be similarly applied to such scenarios where groundwater management is required.

In addition to the existing SPR concept subject to constraints, the total groundwater extractions across the city were maximized. The total groundwater extraction estimated as a function of the constraints had the information of groundwater extractions to be implemented, but lacked the requirement of water manager in meeting location based demands. Hence, a separation of curves which favored either of the constraints was done, followed by explaining a methodology to be performed for meeting location based demands.

What spatial groundwater extraction patterns can be identified amongst the groundwater extractions estimated following the SPR concept as a function of constraints? Which general rules should the groundwater manager follow?

The groundwater extraction rates were calculated as a function of constraints. However, a contour plot of the sum of groundwater extraction rates against the constraints revealed some important features. These features lay down a foundation behind advising the water manager for implementing optimized groundwater extraction rates, obtained by the SPR concept. The features revealed were analyzed both visually and by performing a data analysis.

In order to extract these features a multivariate analysis of the groundwater extraction rates was performed. The first feature revealed in the plot was the same amount of total groundwater extraction obtained by higher values of either of the constraints. The second feature revealed in the plot was the same amount of total groundwater extraction, represented using a minimum value of either of the constraints during optimization. These two features are explained in two separate paragraphs as follows.

The first feature implies that, based on the SPR concept, the higher value of either induced recharge or discharge values can result in the same amount of total groundwater extraction, which should be avoided by a water manager. The total groundwater extraction represented a sum of groundwater extractions across the city and although it resulted in higher values of constraints, it benefitted certain groundwater extraction locations. However, in order to obtain maximum total groundwater extractions across the city, such groundwater extractions should be avoided.

The second feature implies that for a minimum value of either of the constraints, a required total groundwater extraction can be achieved. The respective groundwater extraction could be followed in a case, in which the water manager wants to minimize either of the induced recharge or discharge assessed in the SPR concept, which favors the users spatially distant from the respective social or economic constraint. The groundwater extractions between these two points should be the area of focus for the water manager, since these groundwater extractions neither resulted in a minimum value of either of the constraints, nor resulted in a higher value of constraint for the same amount of groundwater extraction.

The first feature recommended the water manager about the groundwater extractions that should be avoided completely, while the second feature suggested the water manager to obtain groundwater extractions that minimize either of the constraints favoring respective users. In this study, a tool was developed, which could calculate groundwater extractions according to the demand interpretation of the water manager. The demands assessed by the water manager close to either of the locations where constraints act, using a linear interpolation on the respective groundwater extractions (corresponding to minimum value of each constraint). It lays down a better groundwater extraction strategy in meeting location specific water demands.

Hence, in this study the water manager was advised to chose an optimized groundwater extraction strategy based upon the SPR concept and his preferences, based on the water demand. Also, a multivariate analysis was performed during the study to find the groundwater extraction locations which should be preferred and avoided in general. This study therefore suggests the relevance of result analysis using multi-variate techniques, which gives more insight into the results obtained and in helping the water manager in meeting demands. Also, the work shows that it is important to develop techniques which meet the water manager's objectives rather than strictly implementing the results obtained. In the current study, the tool consisted of forming two separate groups and using multi-variate analysis. The common groundwater extraction wells in both of the groups were considered to be the most important groundwater extraction locations, according to the SPR concept. These wells can be selected by the water manager for groundwater extraction if the linear interpolation procedure is incomprehensible, due to the technological background involved for understanding such schemes.

Is it economically beneficial to implement rooftop rainwater harvesting?

Artificial groundwater recharge methods should not be implemented in the city without investigating the feasibility of such projects. The procedure specified in this study for pre-assessing benefits of such projects should be followed which mainly comprises of finding the direct or indirect recharge method to be applied followed by economic assessment. Various artificial groundwater recharge measures were stated in the study and the feasible methods were discussed. The feasibility should be assessed similar to this study in terms of geological details of the site, and other social and economical considerations. Considering the city of Lucknow with high population density, only subsurface recharge measures were feasible (1) considering the space constraints, (2) considering the water borne diseases due to stagnant water and (3) considering high evapotranspiration rates. Hence, only indirect measures of recharging the aquifer were considered. However, if the water manager thinks one or more artificial groundwater recharge measures to be feasible, the economic benefit of each recharge project should be done separately or in combination of others.

For the city of Lucknow, the geological feasibility of storing the water in the subsurface was assessed in this study. The high specific yield of the alluvial aquifer of the city, obtained during the calibration of the groundwater model allowed a subsurface storage of water using direct methods. Hence, a groundwater model availability ensures higher confidence in making a decision of direct or indirect recharge. Out of the two methods that could have been implemented for direct storage of water in the subsurface, rooftop rainwater harvesting (RRWH) and injection wells, only the former was considered, since no power and investment costs from the government is involved. Also during personal discussions, the water manager of the city agreed to the idea of RRWH. RRWH was also considered to be socially acceptable as it has been fully implemented in another city of India (Chennai). In a case both methods are feasible, a combined approach for artificial groundwater recharge should be considered.

However, the social and economic feasibility in terms of drawdown and investment costs, are needed to be assessed in advance in order to know the benefits of an RRWH project. The drawdown costs were based upon the product of the amount of water lost in peri-urban and rural areas due to groundwater extraction in the city and the respective water prices in the outside areas. The investment costs were based upon the product of costs per single RRWH installation and the respective number of houses in each block of the city, where RRWH is implemented. Although the comparison of economic benefits is performed for different scenario of RRWH implementation, the current study aims at comparing the possible set of artificial groundwater recharge measure. This is of special relevance both for developing and developed countries which can focus towards specific recharge measures. The implementation of evolutionary algorithm to solve multi-objective problem holds importance in such study where weights

or the direct amount per installation becomes unimportant. Also, the study has always compared the unit prices of groundwater extraction instead of the whole amount.
The main component of the cost in this study aimed at either proving water to the rural and peri-urban population or providing proper compensation for the under privileged. This study provides efficient application of modeling and optimization techniques to include socio-economic parameters during policy decisions.
A better strategy had to be chosen for RRWH amongst the total recharge and selective recharge implementation. Although the total recharge scenario benefits slightly more in terms of groundwater discharge to the river, it resulted in slightly higher groundwater extraction prices, as compared to the ”selective recharge” scenario. Hence, according to me it is always preferable to use the scenario ”selective recharge”, since it involves less financial risk, if the project does not hold required efficiency. The relevance of this part of the thesis was that the lucrative measure of implementing a recharge method throughout the investigated ares should be thoroughly investigated since a significant amount can be saved while giving the same socio-economic benefits.
The economic and social feasibility of the RRWH measures has been assessed in this study. The groundwater model developed in this study was used to quantitatively assess the total costs of RRWH implementation versus no RRWH implementation. The unit groundwater extraction prices and total costs were found to be lower in a case in which RRWH was implemented. The selective recharge scenario of implementing RRWH results in a lower risk due to failure or inefficiency of such methods.

11.1 Remarks

This study contributes mainly in the manner the SPR concept can be defined and applied numerically for the urban setting of Lucknow. The study depends on the developed groundwater model and should be considered for making the first step towards implementing the SPR concept. A transient groundwater model calibration is computationally expensive and hence the principal of parsimony should be cautiously chosen. Nevertheless, the importance of such groundwater models cannot be neglected.
However, it should be emphasized that the study makes a pathway for implementing SPR concept for a different urban setting which will be based upon the same physical quantities of the SPR concept, induced drainage and recharge terms but may differ in the number of constraints i.e. an urban setting having other discharge terms comprising of lakes and other smaller streams, may have more constraints. Although in this study the SPR concept is defined only for the constraints in the Lucknow urban setting, it mainly emphasizes to maximize the total groundwater extraction, which has not been included yet

in the SPR concept. The visual interpretation of the data may become cumbersome in this case and should be completely replaced by numerical analysis. The multivariate analysis can be performed in a similar way as in this study i.e. the curves will have to be separated for each constraint followed by a similar methodology. The study provides a contribution for implementing trade-offs with respect to each constraint for including location based water demands, based upon different curves extracted for each constraint as stated above.

The artificial groundwater recharge method proposed in this study is RRWH. A different urban setting may lead to another favorable artificial groundwater recharge method depending on the geology, the socio-economic setting and law-enforcing agencies. This study provides a framework of performing the pre-assessment of a planned artificial groundwater recharge application. For example, if injection wells are used, the wells providing a maximum benefit and posing least financial risk can be assessed. Also, the long term benefit of such artificial groundwater recharge method can be assessed similar to this study.

11.2 Future work

This study is a first step towards providing a framework for groundwater management plans in the city of Lucknow. The number of groundwater observations should be increased in the city along with the permanent monitoring of the groundwater extractions in the peri-urban and rural areas outside the city. In the next step, this data should be incorporated along with the meteorological data into the current groundwater model, as a part of post-audit. If the post-audit of the model did not provide the expected groundwater heads, the model should be reviewed and changed, if possible by following all steps of the model protocol. After a successful post-audit of the model, the groundwater model should be extended further to model the effect of groundwater extractions from the confined aquifer in the city.

The urban sprawl is spatially increasing due to the increasing population. This leads to increasing water demands and hence the possibility of considering more groundwater extraction wells outside an investigated limits should also be considered. The whole SPR concept should be implemented again with optimization of the spatial location of the new groundwater extraction wells after having the new groundwater extraction wells. Another option could be importing water from nearby perennial streams. In order to add more relevance to the existing SPR concept, which includes only the quantity, a term related to quality should be added for example usable water, defined as the amount of potable water. Hence the amount of potable water a city has, should be used for taking policy decisions for groundwater extraction. The SPR concept should be used to find groundwater extractions in a transient manner

subject to the social and environmental constraints. Also the economic benefit of extraction and injection of the excess water in high precipitation period, should be assessed.
The consideration of factors like land use and climate change should also be included into the groundwater model. While considering the artificial groundwater recharge measures, more modeling studies are needed to know the possible extent of pollution. The implementation of RRWH should be followed by regular monitoring of the groundwater quality.

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Appendix

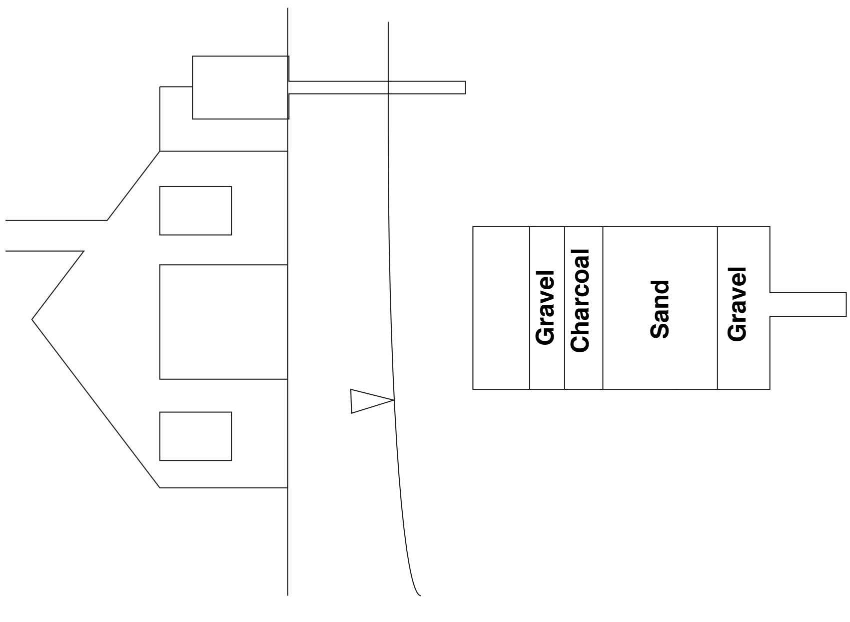

Figure 12.1: Rooftop rainwater harvesting installation

Table 12.1: Groundwater extraction rates in various parts of the city

extraction location	groundwater extraction rates
1	-8.000000000000E-002
2	-5.000000000000E-002
3	-8.000000000000E-002
4	-8.000000000000E-002
5	-6.000000000000E-002
6	-8.000000000000E-002
7	-8.000000000000E-002
8	-5.000000000000E-002
9	-8.000000000000E-002
10	-8.000000000000E-002
11	-8.000000000000E-002
12	-8.000000000000E-002
13	-1.000000000000E-001
14	-1.000000000000E-001
15	-1.000000000000E-001
16	-8.000000000000E-002
17	-8.000000000000E-002
18	-8.000000000000E-002
19	-6.000000000000E-002
20	-8.000000000000E-002
21	-4.000000000000E-002
22	-8.000000000000E-002
23	-4.000000000000E-002
24	-8.000000000000E-002
25	-8.000000000000E-002
26	-6.000000000000E-002
27	-4.000000000000E-002
28	-8.000000000000E-002
29	-8.000000000000E-002

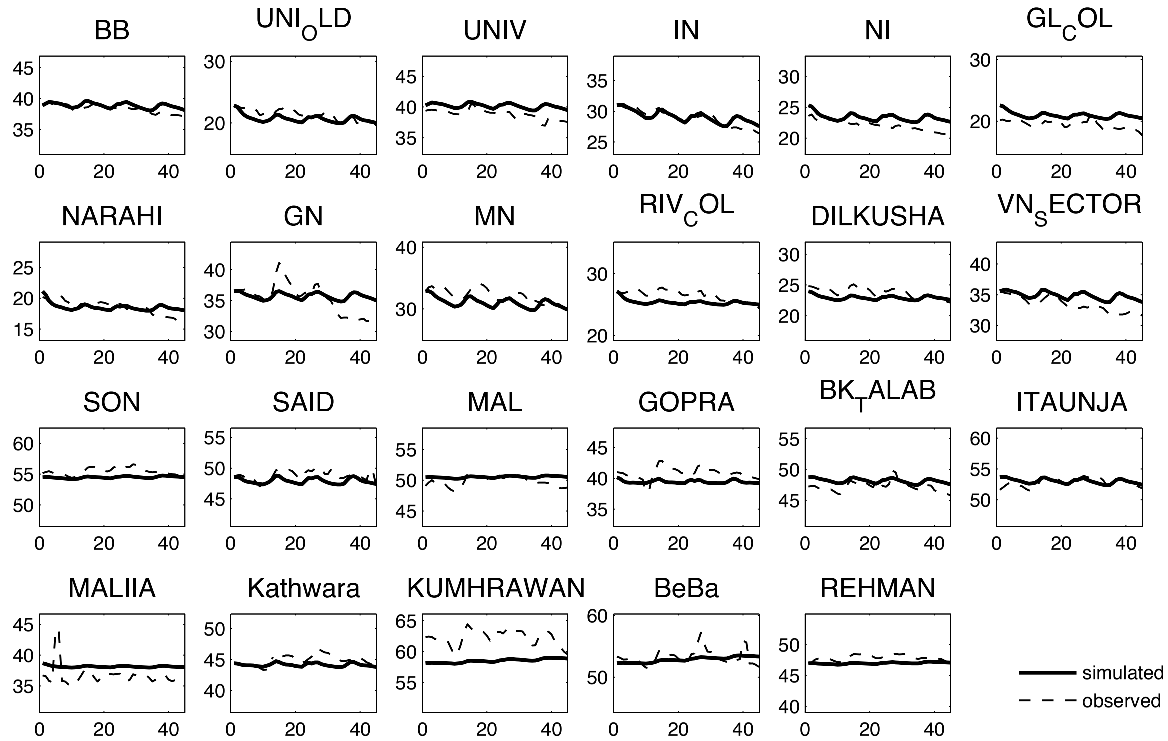

Figure 12.2: Observed and simulated heads at all 23 locations inside the model domain. X axis represents the time in months and Y axis represents the groundwater head in m

Declaration

I herewith declare that I have produced this thesis without the prohibited assistance of third parties and without making use of aids other than those specified; notions taken over directly or indirectly from other sources have been identified as such. This thesis has not previously been presented in identical or similar form to any other German or foreign examination board.

The thesis work was conducted from January 2009 to March 2013 under the supervision of Prof. Dr.-Ing. Olaf A. Cirpka and Dr. Claudius Bürger at the University of Tübingen.

Tübingen