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Abstract

In his basic model of debt renegotiation, BESTER [1994] argues that collateral is more effective if high risk projects are financed. This result, however, crucially depends on the definition of risk. Using the second-order stochastic dominance criterion introduced by ROTHSCHILD AND STIGLITZ [1970], we show that it is not a project's high risk, induced by a high probability of default, that makes collateral more effective. Instead it turns out that, given the expected return, the probability of default has no impact on the collateral's effectiveness. Moreover, a higher risk of the project caused by a higher loss given default makes the use of collateral even less effective.

Keywords: Debt renegotiation; Collateral; Risk; Stochastic dominance

JEL Classification: D81, D82, G21, G32

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1 Introduction

The optimal design of a loan contract is one of the topics most intensively analyzed in institutional economics. Particularly, the importance of collateral in mitigating problems of asymmetric information due to credit risk is pointed out in a large theoretical and empirical literature. Ever since the mid 70s, information asymmetries have been highly emphasized in economic theory. On the one hand, moral hazard provides risk incentive arising when a borrower protected by limited liability has the choice between different levels of risk (cf. JENSEN AND MECKLING [1976]). On the other hand, within an adverse selection setting, less risky projects may be crowded out. As STIGLITZ AND WEISS [1981] have pointed out, the lender cannot be compensated for increased risk by an additional risk premium because under both regimes, moral hazard and adverse selection, higher contractual interest rates imply even worse incentives. Therefore, a rising interest rate may lead to a decline in the lender's expected return. This, in turn, implies the possibility of equilibrium credit rationing. Within the literature relying on the Stiglitz and Weiss-model, higher risk is typically associated with a lower expected return since otherwise risk incentive caused by moral hazard or crowding out caused by adverse selection are irrelevant from a social point of view. However, as already noted by KÜRSTEN [1995], these results do no longer hold in general if increasing risk is associated with a constant expected return. We will take up this point in the main section of our paper.

Apart from a higher interest rate, the introduction of collateral to loan contracts increases the lender's expected return, and improves the borrower's incentives. Bester [1985], [1987] has shown that under adverse selection or moral hazard, collateral reduces agency costs associated with debt financing. In both cases, collateral should be used in financing less risky projects while debt contracts specified for riskier projects should not include collateral. Under adverse selection, borrowers endowed with a less risky project provide collateral in order to send a signal which is too costly for borrowers with a riskier project to imitate. Under moral hazard, the use of costly collateral only pays off if the borrower cannot commit himself to a less risky project otherwise. If he cannot commit himself to that less risky project even using costly collateral, he will certainly refrain from doing so. Therefore, both types of debt models imply that collateral should be associated with low-risk projects.

To our best knowledge, there is just one model leading to mixed predictions regarding the relation between risk and the use of collateral. BOOT, THAKOR AND UDELL [1991] use a two-stage set up with two types of borrowers. The first step consists of a pure effort-incentive problem. By assumption, only for the bad type it is first best to exert any effort. The second-best solution calls for the bad type's contract stipulating some collateral while the good type does not have to provide collateral. The second step integrates adverse selection into the model implying that any loan contract contains some collateral. If debt contracts are not fully

secured, the contract for the bad type uses less collateral than the good type's one.[1]

From an empirical point of view, the relationship between project risk and the use of collateral is ambiguous. Unlike the theoretical predictions of the models cited above, according to practitioner's wisdom, the optimal loan is an unsecured credit because it has been profitably paid out even without collateral. As a corollary, collateral is used in the case of a substantial default risk. Econometric studies reveal mixed evidence. While, e.g., BERGER AND UDELL [1990] and BOOTH AND BOOTH [2006] report a positive relation between project risk and collateralization, BERGER AND UDELL [1995], MACHAUER AND WEBER [1998], or ELSAS AND KRAHNEN [2002] do not find any significant correlation. LEHMANN AND NEUBERGER [2001] even report a negative correlation between risk and the use of collateral. Quite remarkably, the studies indicating an ambiguous or a negative correlation focus on relationship lending which itself has no clear implication on the creditor's risk.

The model of BESTER [1994] provides an explanation for this negative correlation by accounting for the role of collateral on debt renegotiation. Therefore, his model implies comparative static results which are in clear contrast to the findings of the Stiglitz and Weiss-type models. Bester uses the setting of a costly state-verification as introduced by TOWNSEND [1978] and as applied to debt contracts by GALE AND HELLWIG [1985]. Within this type of model, project risk is exogenously given and commonly known. Instead, the financial outcome is observable only for the borrower while the lender has to incur some costs to monitor the project's success. Bester extends this setting by allowing for mixed strategies on part of both, the borrower and the lender. If the project is successful, the borrower has the choice between meeting his contractual obligations or cheating and opting for a strategic default, i.e. not paying back the loan even if he is capable to do so. The latter strategy might be individually superior because the lender cannot distinguish between a strategic default and a liquidity default which occurs if the project fails. (In the following, we use the term "default" exclusively for the liquidity default or project failure, whereas we refer to "cheating" in case of a strategic default.) If a borrower has not paid back his loan because of one reason or the other, a lender may either choose to take over the firm via a bankruptcy procedure or to opt for a renegotiation resulting in a reduced repayment obligation. Games including this kind of costly monitoring are typically solved by mixed-strategy equilibria.

In Bester's model, collateral may be used to alter the borrower's incentive to cheat and the lender's incentive to choose a bankruptcy strategy towards a less costly solution, i.e. a lower probability of both cheating and choosing bankruptcy. Collateral turns out to be most effective if the probability of default is high. Therefore, Bester's model stands in clear contrast to any alternative model of debt contracts – except for the BOOT, THAKOR AND UDELL [1991] paper pointing on monetary theory – in linking collateral to high-risk projects, thereby aligning

[1] It should be noted, however, that the Boot et al. paper focuses on monetary policy rather than on financial contracting.

theoretical rigor with practitioner's wisdom and some part of the empirical literature.

In the present paper, we address the question if the relation between increasing risk and the profitability of collateral continues to hold if a varying risk is modeled as a shift in pure risk, i.e. holding constant the project's expected return. We do this employing the ROTHSCHILD AND STIGLITZ [1970] notion of increasing risk. The question is tackled within the framework of the BESTER [1994] model – i.e. assuming just costly state-verification and neglecting any other problem of information asymmetry like moral hazard or adverse selection – because it is precisely this framework leading to a comparative static result contradicting alternative models on the use of collateral. For the same reason we do not refer to inside collateral which might be of interest if multiple lending is taken into account.

In fact, we show that Bester's result crucially relies on defining risk by the project's probability of default, holding all other parameters constant. Obviously, in this interpretation, increasing risk implies a decreasing expected return. Holding constant the project's expected return, increasing risk is shown either to have no influence on the benefit of collateral or to reduce its benefit, depending on whether risk is measured by the probability of default or by the loss given default.

The rest of the paper is organized as follows: In section 2, we briefly present the basic structure of an extended model of debt renegotiation. In Section 3, we analyze the influence of the probability of default and the loss given default as mean-preserving risk measures. Section 4 concludes.

2 The model

Following BESTER [1994], we consider a game of debt contract design and renegotiation where the borrower B is a risk-neutral entrepreneur who needs to raise capital for a risky investment project and where the lender L is a risk-neutral bank. Without loss of generality, the fixed investment is normalized to one and yields the random return

$$x = \begin{cases} 1+r & \text{with prob } 1-PD \\ 1-LGD & \text{with prob } PD, \end{cases}$$

where r is the project's rate of return in case of success, $PD \in (0;1)$ is the probability of default, and $LGD \in (0;1)$ is the loss given default. The project's expected rate of return

$$\begin{aligned} \mu &= (1-PD)(1+r) + PD(1-LGD) - 1 \\ &= r - PD(r+LGD) \end{aligned} \tag{1}$$

is assumed to be positive so that the project is ex ante profitable if the riskless rate of return is normalized to zero.

The borrower observes the realization of the return without any cost. The lender, however, receives this information only if taking over the project. Monitoring and liquidating the project is costly for him. Since return realizations

are not verifiable to third parties, the borrower's repayment obligation cannot be conditioned on the project's outcome.

While the borrower has no liquid funds, he owns some amount w of collateralizable wealth. This wealth cannot be used to finance investment directly, but the lender may use it as collateral $c \leq w$ for the loan. Taking possession of and liquidating c typically involves transaction costs $(1-\beta)$ with $\beta \in (0,1)$, so that the lender's net valuation of c equals βc. Alternatively, one may think of specific assets which are of a higher value to the borrower than to the lender. We further assume $w < LGD$ so that the loan cannot be fully secured even if the total wealth is made use of.

The figure below describes the sequential moves in the renegotiation game.

((Insert figure about here.))

In stage one, the project succeeds with probability $(1-PD)$ and fails otherwise. This is observed by the borrower while the lender remains uninformed.

In stage two, a successful borrower has the choice between meeting his obligations and paying $(1+i)$ where $i(<r)$ is the contractual interest rate or cheating, i.e. pretending the project has failed and just paying $(1-LGD)$. It is common knowledge that the project's outcome cannot be lower than $(1-LGD)$. Therefore, the borrower can be forced to pay out the respective amount so that he will never pay less than $(1-LGD)$. He chooses to pay the low amount of $(1-LGD)$ with probability h. A borrower whose project has failed cannot pay more than $(1-LGD)$ and defaults anyway. Whenever the borrower does not pay back the loan completely, the amount of collateral specified by the contract is transferred to the lender.

In stage three, having received a payment of $(1-LGD)$, the lender has to choose between initiating a bankruptcy procedure and taking over the project or to renegotiate the loan. We denote the probability of a bankruptcy procedure by b. In the bankruptcy case, the lender takes over the firm which causes bankruptcy costs of $(1-\alpha)$, $\alpha \in (0;1)$, times the assets of the firm, thus increasing the lender's wealth by $\alpha(1-LGD)$ or $\alpha(1+r)$, respectively. In case of renegotiation, the lender reduces the borrower's obligation to $(1-LGD)$ so that the contract is settled by the borrower's prior payment. Any reduction to an amount exceeding $(1-LGD)$ turns out to be useless because the borrower could still claim a liquidity default has happened. Again, the lender would have to induce bankruptcy in order to verify whether the borrower has been cheating or not.

If the bankruptcy procedure reveals the borrower has been cheating, the lender may, depending on parameter values, receive a net payment exceeding the borrower's obligation. In fact, there may be a legal environment limiting the lender's net receivable to the borrower's obligation, thus forbidding contractual sanctions exceeding the effective damage. As a robustness check we analyzed this setting as well. The results depend on the parameter values. In case of sufficiently high bankruptcy costs $(1-\alpha)$, the modified model completely corresponds to the basic model presented below. If instead bankruptcy costs are low, the use of

collateral turns out to be generally suboptimal because it is cheaper to transfer the firm's assets to the borrower rather than to incur the costs associated with collateral. In what follows, we continue to assume that the lender completely takes over the in case of bankruptcy.

Then, the expected profit of the lender is

$$\begin{aligned} \mathrm{E}\{\pi_L(b)\} &= (1-PD)[(1-h)(1+i)+hb(\alpha(1+r)+\beta c)+h(1-b)(1-LGD+\beta c)] \\ &+ PD[b(\alpha(1-LGD)+\beta c)+(1-b)(1-LGD+\beta c)]-1. \end{aligned} \tag{2}$$

The borrower yields the expected profit

$$\mathrm{E}\{\pi_B(h)\} = (1-PD)[(1-h)(r-i)-hbc+h(1-b)(r+LGD-c)]-PDc\,. \tag{3}$$

In order to solve the renegotiation game, we have to determine the equilibrium mixed strategies. Due to equation (2), the lender is indifferent between choosing a bankruptcy procedure or not, $\partial \mathrm{E}\{\pi_L\}/\partial b = 0$, if the borrower chooses to cheat with probability

$$h^* = \frac{PD(1-\alpha)(1-LGD)}{(1-PD)(\alpha(1+r)-(1-LGD))}\,. \tag{4}$$

Substituting r from equation (1) shows that $(1-LGD) < \alpha(1+\mu)$ is a necessary and sufficient condition for mixed strategies $(h^* < 1)$. Otherwise, it is strictly dominant for the lender to never induce bankruptcy $(b^* = 0)$, implying that the borrower always prefers to cheat $(h^* = 1)$. Intuitively, bankruptcy cannot be optimal for the lender if the loss given default or the bankruptcy costs are too high because there is either too little to be gained or only at a too high cost. Given the restrictions on collateral, the lender cannot earn non-negative profits in this case so that he will never sign a contract. Therefore, a pure strategy on part of the borrower must be off the equilibrium path in the contracting game.

If the borrower does not pay back the loan, the lender has to decide between inducing a bankruptcy procedure or not. Due to equation (3), the borrower is indifferent between cheating and paying back, $\partial \mathrm{E}\{\pi_B\}/\partial h = 0$, if the lender chooses bankruptcy with probability

$$b^* = \frac{i+LGD-c}{r+LGD}\,. \tag{5}$$

Given our assumptions, it is obvious that the lender will never choose pure strategies so that $0 < b^* < 1$.

Potential lenders compete by offering contracts of the form $\Gamma = ((1+i),c)$. If the borrower finds none of the contracts acceptable, the game is over and all players realize zero profits. Otherwise the borrower undertakes the investment by accepting the offer of one of the lenders. The equilibrium contract maximizes the borrower's expected profit (equation (3), having inserted (4) and (5) for the mixed strategy variables) subject to the lender's participation constraint (equation (2), accounting for the reservation profit level). We allow for imperfect competition between lenders who require a reservation profit of $\bar{\pi} \geq 0$, indicating the lender's

market or bargaining power. Most models stipulate perfect competition between lenders, i.e. $\bar{\pi} = 0$. Given $\bar{\pi}$, the lender's participation constraint implies

$$i = \frac{(h^* + PD(1-h^*))(LGD - \beta c) + \bar{\pi}}{(1-PD)(1-h^*)}. \tag{6}$$

In the mixed strategy equilibrium, the borrower is indifferent between all cheating strategies. Hence, we may set $h = 0$ in equation (3), substitute for i using equation (6), and substitute for r using equation (1) to obtain for the borrower's expected profit in equation (3)

$$\mathrm{E}\{\pi_B\} = \mu - \frac{h^*}{1-h^*} LGD + \frac{h^* - (PD + (1-PD)h^*)(1-\beta)}{1-h^*} c - \frac{\bar{\pi}}{1-h^*}.$$

Note that due to equation (4), h^* is not affected by the level of collateral. Thus we find

$$\frac{\partial \mathrm{E}\{\pi_B\}}{\partial c} > 0 \Leftrightarrow$$

$$1 - \beta < \frac{h^*}{PD + (1-PD)h^*} = \frac{(1-\alpha)(1-LGD)}{(1-PD)\alpha(r+LGD)} \equiv \kappa, \tag{7}$$

where κ is the critical upper cost level up to which the use of collateral increases the borrower's wealth. Therefore,

$$c = \begin{cases} w & \text{if} \quad 1-\beta \le \kappa \\ 0 & \text{if} \quad 1-\beta > \kappa \end{cases},$$

implying that κ is the key variable indicating the role of collateral in debt contracting. As it becomes obvious from (7), $\partial\kappa / \partial PD > 0$, i.e., the riskier the project, measured by the probability of default, the more probable is the use of collateral. Up to here, our results confirm the main findings of BESTER [1994] (in particular proposition 4) and add the additional insight that the relation between project risk and the use of collateral is not influenced by the degree of competition between lenders.

Equation (7) further implies $\partial\kappa / \partial LGD < 0$ so that the comparative statics are reversed if risk is measured by *LGD* instead of *PD*. As both variables contribute to the lender's expected loss, this observation raises first doubts on the robustness of Bester's results.

3. Restatement of the model using mean-preserving risk measures

Raising the probability of default *PD* while holding constant any other explanatory factor does not only imply a certain reasoning of "increasing risk" but at the same time a decline in the expected return. Therefore, it is not clear whether the implications are due to an increasing risk or to a decreasing expected return.

In order to get clear cut results with respect to the relation between project risk and the use of collateral we argue that the propositions of ROTHSCHILD AND

STIGLITZ [1970] on increasing risk should be taken into account. Rothschild and Stiglitz propose three different definitions of increasing risk which (in case of continuously distributed random variables) turn out to be equivalent holding constant the expected value: the addition of noise, the mean-preserving spread, and second-order stochastic dominance $(SD2)$. Within the two-point distribution in our model, the last version is the most useful one to analyze increasing risk. In fact, using the $SD2$ criterion is the only alternative which allows for staying in the two-point setting.

According to equation (1), there are in principle two sources of increasing risk, either an increasing probability of default PD or an increasing loss given default LGD while adjusting the respective other parameter and r in order to ensure a constant expected rate of return μ. These possibilities are analyzed successively.

3.1 Increasing risk by an increased probability of default

We start with the analysis of an increase in the probability of default PD to compare the results to the BESTER [1994] model. If increasing risk is introduced by an increase in PD, the invariance of μ can be insured either by an increase in r or by a decrease in LGD or by a combination of these two variations. As can easily be seen, any variation of a two-point distribution going along with a decrease in LGD cannot be compatible with the original distribution being stochastically dominant because the support of the distribution is shifted to the right. Therefore, the new distribution holds the property of being more risky according to $SD2$ than the original distribution if and only if the increase in PD is compensated solely by an increase in r, the rate of return in case of the project's success. Substituting r from (1) in condition (7) leads to

$$\kappa = \frac{(1-\alpha)(1-LGD)}{\alpha(\mu + LGD)}. \tag{8}$$

Obviously, if the increase in PD is compensated for, there is no further influence on the critical cost level κ. This unambiguously implies that an increase in PD has no impact on κ:

PROPOSITION 1: *Holding constant the project's expected return, increasing risk induced by a higher probability of default does not have any impact on the use of collateral.*

Proposition 1 stands in clear contrast to the result of BESTER [1994] who argues that collateral is more effective if high risk projects are financed. We show that Bester's argumentation is only true if the project's expected return is not constant thereby violating the $SD2$ criterion of the Rothschild and Stiglitz definition of increasing risk.

3.2 Increasing risk by an increased loss given default

The second way to impose an increased default risk on the lender is to increase the loss given default LGD. If increasing risk is introduced by an increase in LGD,

the logic as above applies: the invariance of μ can be insured either by an increase in r or by a decrease in PD or by a combination of these two variations. Here, all alternatives may be compatible with the new distribution being dominated according to the $SD2$ criterion.

Again, substituting either r or PD from equation (1) into equation (7) yields equation (8). Therefore, no matter which way of compensation for the increased LGD is used, holding constant the project's expected rate of return, the critical cost level κ does not depend on PD but only on LGD with the property $\partial\kappa / \partial LGD < 0$. Therefore, we state

PROPOSITION 2: *Holding constant the project's expected return, increasing risk measured by the loss given default leads to a decrease in the use of collateral.*

Proposition 2 not only clarifies the conditions under which the results of BESTER [1994] hold but also gives an explanation for the ambiguous empirical evidence presented above.

4. Conclusion

In his predecessor model of debt renegotiation, BESTER [1994] argues that collateral is more effective if high risk projects are financed. This result, however, crucially depends on the definition of risk. Using the second-order stochastic dominance criterion, we have shown that it is not a project's high risk, induced by a high probability of default, that makes collateral more effective. Under conditions implying a constant expected return, the probability of default has no impact on the effectiveness of collateral. Moreover, an increasing risk of the project caused by an increasing loss given default makes the use of collateral even less effective. These results stand in sharp contrast to the main conclusion in Bester's model. Varying only a project's probability of default, it is not the project's risk that determines the influence of collateral but its expected return. As our analysis has shown, if the project's expected return is assumed to be exogenously given, increasing risk either does not have any impact on collateral at all or makes the use of collateral even less probable. Therefore, our model seems to be appropriate to bridge the gap to the alternative models of debt contracts as cited in the introduction as well as to the empirical evidence in this important area of research.

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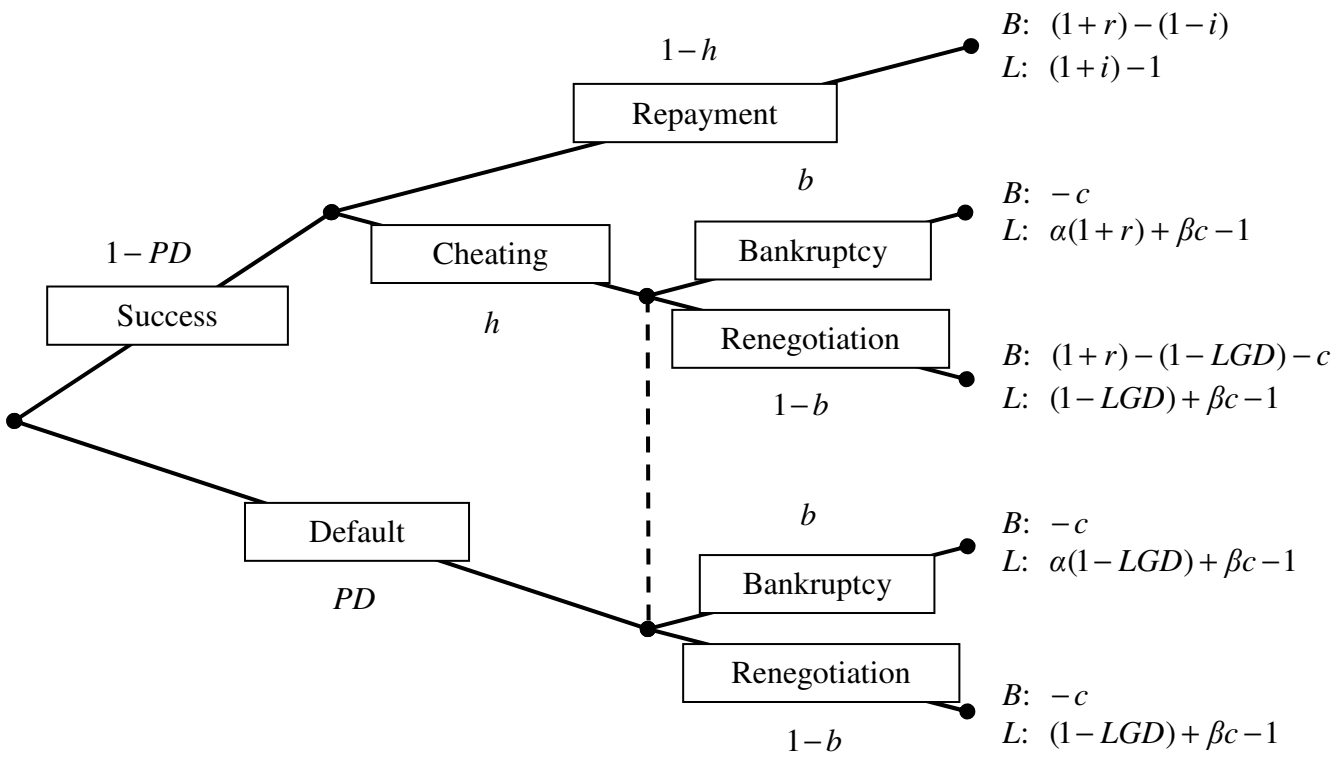
Figure: Renegotiation game



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