RANK-ORDER TOURNAMENTS AS SELECTION MECHANISMS:

APPLICATIONS IN PROJECT FINANCE

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Chapter 1

Introduction

Tournaments are a fact of life. They are used to rank athletes in sports events, workers competing for a promotion, and offers for government procurement contracts. The realm of applications for tournaments is virtually limitless, given that in almost every aspect of life, ranking decisions of some sort or another have to be made.

Part of what makes rank-order tournaments or contests so appealing is their relatively low informational requirement. In a world of complete and perfect information, rewards for achievements or for certain characteristics could be distributed on the basis of absolute input or output measures. However, if such cardinal ranking is costly or even impossible because it calls for costly monitoring or screening activities, or because certain traits simply cannot be compared on a cardinal scale (as, for example, in beauty contests), rank-order tournaments can prove to be superior selection or incentive mechanisms in the face of incomplete or imperfect information. Building on the well-known principal-agent paradigm, which is also concerned with the provision of incentives and selection mechanisms in the face of imperfect or incomplete information, tournaments are useful mechanisms in multiple-agent settings. In that,

they are closely related to auctions. Indeed, a specific type of auction, the first-price all-pay auction, is often used to model tournament applications.

While the theoretical tournament literature has long focused on analyzing incentive effects for homogeneous contestants, or for heterogeneous contestants with complete information, this thesis explores the potential of tournaments as selection devices in the context of heterogeneous competitors and incomplete information. It develops two applications in the field of finance, where managers/entrepreneurs compete over the allocation of scarce resources. In the first application, two division managers compete over limited funds in a conglomerate's internal capital market. In the second application, two entrepreneurs compete in a business plan contest, sponsored by a venture capitalist.

The main focus of this thesis is on "selection efficiency", defined as the probability with which the best contestant is picked as the tournament winner. In particular, both applications restrict contestants' actions to pure signaling activities, thus avoiding a mix of incentive and selection considerations. When analyzing the effect of exogenous parameters on the equilibrium level of information generation, changes in the quality composition of the contestant pool are considered only insofar as they influence contestants' strategic decisions. This is in contrast to a related contribution by Hvide and Kristiansen (2003), which defines selection efficiency in a broader sense, as the probability with which a high-quality contestant is picked as the winner. Their definition thus extends the concept from a purely relative to an absolute standard,

which is not only influenced by the specific tournament rules and equilibrium strategies, but also by the purely statistical impact of changes in the composition of the contestant pool.

In both presented applications, the winner's prize is endogenized, by allowing for a post-tournament auction between different competing investors. In the internal capital market application, two divisions of a conglomerate compete in an internal tournament, in order to receive financing from corporate headquarters. Afterwards, the division which loses the internal tournament is divested in a corporate auction in which two outside investors compete for financing the division. Before the auction, however, one of the outside investors acquires inside information from the division manager, thus gaining an information advantage over the other outside investor. In the business plan contest application, two entrepreneurs compete for winning the tournament sponsored by a venture capitalist.[1] After the contest, the winning entrepreneur is announced, thus eliciting interest in outside investors who will then compete with the inside venture capitalist in an implicit auction for financing the winning project. Hence, in both applications the bids made in the post-tournament auctions determine the expected rewards of the respective tournament winners and losers.

Such a tournament-cum-auction design approximates many real-life tournaments better than those with a pre-specified fixed prize. In corporate finance, what counts

[1]While business plan contests are often sponsored by universities or public institutions, they are usually co-sponsored by venture capitalists, and the latter typically also play an important role in the jury of such contests. For simplicity, the sponsor of the contest is therefore called venture capitalist.

more than a nominal sum of prize-money is the actual funding of a project, allowing the winning manager/entrepreneur to reap control benefits or project-specific monetary payoffs. In the two presented applications, the post-tournament auction is modelled as a common value auction with asymmetric information, as it is assumed that one investor has superior inside information. This specific form of auction, as well as the assumption of heterogeneous tournament participants and the focus on selection efficiency, set this tournament-cum-auction framework apart from a related contribution by Fullerton and others (2002). In their setup, a research tournament is combined with a subsequent private value auction, where contestants have to make offers to the tournament sponsor, which consist in a combination of the achieved innovation quality and price.

In addition to the question of selection efficiency, the tournament-cum-auction framework allows to address a second research question, which is the interaction between the contestants' signaling efforts in the tournament and the equilibrium strategies and payoffs in the subsequent auction, where contestants' prizes are endogenously determined. In addition, the effects of changes in external parameters, such as the probability of a high project quality or the effectiveness of signaling efforts, on equilibrium effort levels (and, hence, information generation) are analyzed, as well as the effects of such changes on equilibrium strategies and payoffs in the post-tournament auction. Since changes in both the probability of a high-quality project/division and the effectiveness of signaling inherently affect the degree of information asymmetry, conclusions can be formulated in terms of the impact of information asymmetry on strategies and expected payoffs.

The main findings of the two applications concerning selection efficiency can be summarized as follows:

(i) In the context of the internal capital market cum corporate auction, selection efficiency is higher in more highly valued conglomerates, and in more traditional and well-understood markets. It also rises with an increase in managers' nonmonetary control benefits. As argued in chapter 4, this is owing to the fact that division managers' incentives to exert information-generating effort rises with (a) an increase in the *a priori* probability that any division is of high quality (and, thus, of high value), which in turn is more likely in a more highly valued conglomerate; (b) an increase in the efficiency of information-generating efforts, which in turn is higher in more traditional and well-understood markets; and (c) an increase in nonmonetary control benefits that a manager derives from being in charge of his division.

(ii) In the context of the business plan contest cum financing competition, selection efficiency is generally higher in more competitive and highly reputed business plan contests, but it falls when competitiveness increases further from an already very high level. Selection efficiency also tends to be higher in more industry-specific contests where the venture capitalist has special expertise, as well as in contests that focus on higher value-added industries. As argued in chapter 5, this is owing to the fact that entrepreneurs' incentives to exert information-generating effort rises with (a) an increase in the *a priori* probability that any project is of high quality (and, thus, of high value), which in turn is more likely in more competitive and highly reputed business plan contests. When competitiveness is very high, however, a further increase causes a decline in effort, as the value of additional project-specific quality information is

considered less important against the backdrop of the general expectation of a high-quality contestant pool. In addition, incentives to exert information-generating effort rise with (b) an increase in the efficiency of information-generating efforts, which in turn is higher in industries for which the venture capitalist has a special expertise; and (c) an increase in the expected returns of a high-quality project, which in turn are generally higher in higher value-added industries.

These findings add to the literature in two ways. Firstly, they allow for different degrees of selection efficiency. This contrasts with earlier contributions that focus merely on the distinction between efficiency and inefficiency, in a quest for mechanisms that would induce complete selection efficiency. Secondly, in assessing the exogenous factors that affect selection efficiency, the present setup includes more factors than just the quality of the contestant pool, as in Hvide and Kristiansen (2003). Also, while these authors' result on the effects of changes in contestants' quality is in line with the results of the model of the business plan contest (where the quality composition of the contestant pool measures the degree of competitiveness), this is not the case for the model of the internal capital market (where the quality composition of the divisions translates into the valuation of the conglomerate). Their result of a nonlinearity in the effect of an increase in the contestants' quality pool on selection efficiency is thus not robust to changes in the contestants' reward structure.

The main findings concerning the impact of information asymmetry on agents' expected payoffs can be summarized as follows:

(i) In the context of the internal capital market cum corporate auction, an increase in

the probability for a high-quality division and a decrease in the effectiveness of signaling efforts both lead to a decrease in information asymmetry between the informed outside investor and the uninformed outside investor (since the losing division is auctioned off).[2] This, in turn, decreases the expected payoff of the informed investor, while increasing the expected payoff of the conglomerate's headquarters. As argued in chapter 4, this is owing to the fact that (a) an increase in the *a priori* probability that any division is of high quality induces the uninformed outside investor to increase his bid for the divested division, thus also driving up the equilibrium bid of an informed investor who knows that the division is indeed of high-quality, and reducing her information rent; and (b) a decrease in the effectiveness of signaling efforts lowers the probability that a divested division is indeed of lower quality, inducing the uninformed outside investor to bid more, thus forcing an informed investor who knows that the division is of high quality to also increase her bid, and reducing her information rent.

(ii) In the context of the business plan contest cum financing competition, an increase in the probability for a high-quality division as well as an increase in the effectiveness of signaling efforts both lead to a decrease in information asymmetry between the outside investor and the venture capitalist (since they compete for financing the winning project). This, in turn, decreases the expected payoff of the venture capitalist and increases the expected payoff of an entrepreneur with a high-quality project. As argued in chapter 5, this is owing to the fact that (a) as above, an increase in the *a*

[2]If the inside investor receives a high-quality signal for the divested division, her informational advantage is reduced by a reduction in signal precision, since the outside investor is more likely to assume that a high-quality project might be mistakenly divested if signal precision is low. This in turn increases the expected value of the divested division for the outside investor, bringing his expectations closer in line with those of the informed investor. For a more detailed discussion, see section 4.3.3.

priori probability that any project is of high quality induces the uninformed outside investor to increase his bid for the winning project, thus also driving up the equilibrium bid of a venture capitalist who knows that the division is indeed of high-quality, and reducing her information rent; and (b) an increase in the effectiveness of signaling efforts also increases the probability that the higher-quality project will win the contest, inducing the outside investor to raise his bid, thus forcing the informed venture capitalist to also increase her bid for a high-quality winning project, and reducing her information rent.

While these findings are in line with well-known results of the principal-agent literature, they also feed back into the equilibrium signaling efforts, allowing to assess the connection between information asymmetries in the post-tournament auction and selection efficiency in the tournament. In the model of the business plan contest, a decrease in information asymmetry is always associated with an increase in selection efficiency, with the exception of highly competitive tournaments (if the decrease in information asymmetry in such highly competitive tournaments is triggered by a further increase in competitiveness). In the model of the internal capital market, on the other hand, a decrease in information asymmetry is associated with an increase in selection efficiency only if it is triggered by an increase in the conglomerate's valuation. If it is triggered by a decrease in the effectiveness of signaling efforts, as captured by less traditional and well-understood markets, it is associated with a decrease in selection efficiency.

The basic model setup is the same for both applications: Both in the internal capital market setting and in the business plan contest, two division managers/entrepreneurs compete for limited resources by signaling their respective qualities (expected future returns). There are two types of contestants, one with a high-quality division/project and one with a low-quality division/project. The quality signal of the high-quality division/project can be distorted, and become identical to the signal of a low-quality division/project with a positive probability. High-quality contestants can exert unobservable effort in order to enhance their quality signal, and the tournament sponsor (corporate headquarters or venture capitalist, respectively) then picks the division/project with the higher quality signal as the tournament winner.

The two applications differ in the second phase, however, when all agents' payoffs are determined in a post-tournament auction. In the internal capital market setting, it is in corporate headquarters' best interest to divest the losing division and to retain the tournament winner within the conglomerate. Two outside financiers then compete for acquiring the divested division in a corporate auction. Before the auction, however, one of the outside investors teams up with the division manager and acquires insider information about the division's quality signal. This enables her to realize positive expected payoffs - her information rent - from the corporate auction, while the other outside investors make zero expected profits. A division manager reaps nonmonetary benefits if he remains in control of his division - which he will in case he wins the internal tournament, and also in case his division is taken over by the informed investor.

In the business plan contest setting, the venture capitalist has an interest in financing the contest winner. After winning the contest, however, the winner can also seek outside finance, accepting the best financing offer from either the venture capitalist - who sponsored the tournament and therefore has inside information about the project's quality signal - or from an uninformed outside investor. Again, the informed investor (the venture capitalist) realizes an information rent, while the outside investor makes zero expected profits. The winning entrepreneur receives the bid of the highest bidder in return for the residual claims on his project's eventual returns.

As suggested above in the discussion of the main findings, the crucial difference in the two applications lies in the object of the post-tournament auction. While it is the loser of the internal capital market tournament who will be divested and thus put up for a corporate auction, it is the winner of the business plan contest who finds himself at the center of the subsequent bidding game between competing investors. Since in each case, the investors know wether they are competing for the winner or for the loser of the tournament, this affects their beliefs about the quality of the object they are bidding for, and their reactions to changes in external parameters. This is particularly true for the respective uninformed investors whose only information is whether a division/project has lost/won the pre-auction tournament. The different setups also affect the expected payoffs and hence the strategic decisions in the tournament phase, through the different bidding strategies in the auction phase, but also through the different payoff structures for the division manager/entrepreneur.

Despite the long historical tradition of use of rank-order tournaments, rigorous

theoretical models were only possible based on previous work in the realm of principal-agent theory, and the first theoretical paper was published in 1981, by Lazear and Rosen ("Rank-Order Tournaments as Optimum Labor Contracts"). In their basic tournament setup, these authors model the interaction between two homogeneous, risk-neutral agents and one risk-neutral principal (tournament sponsor) who sets the spread between first and second prize such as to optimize her expected payoff from the agents' combined output, when effort levels are unobservable, and output is influenced both by effort and by an unobservable luck component. One of their main results is that under risk neutrality, rank-order tournaments induce first-best effort levels, resulting in the same optimal resource allocation as would other efficient mechanisms, such as piece rates.

In the aftermath, a large body of research has evolved, both theoretical and empirical, analyzing the workings of tournaments under different assumptions about participants' preferences and institutional settings, and assessing under which circumstances tournaments are preferred to other compensation schemes. The theoretical literature has focused on analyzing existence and properties of equilibria, in the presence of risk aversion, heterogeneous contestants, multiple contestants and prizes, different information structures, and assumptions about common or idiosyncratic shocks. Fields of application range from the traditional setting of internal labor markets (as first introduced by Lazear and Rosen, 1981) to rent-seeking in the political process, research tournaments, internal capital markets, evolutionary processes, and others. While the larger part of the literature takes a specific tournament mechanism as given, some authors also analyze the tournament structure itself, deriving the optimal number

of prizes/contestants, and suggesting additional performance measures and selection mechanisms.

In addition to the literature on rank-order tournaments, this thesis also draws on auction theory, and particularly on first-price common value auctions with asymmetric information. This type of auction was first fully characterized by Engelbrecht-Wiggans and others (1983), and chapters 4 and 5 draw on their main arguments and results for modeling the post-tournament auctions. Furthermore, the two applications are placed within the context of the literature on internal capital markets, spin-off decisions, and venture capital, as laid out in the introductions of the respective chapters.

The remainder of this thesis is organized as follows: Chapter 2 gives an overview of the literature on rank-order tournaments, beginning with the basic model as based on Lazear and Rosen (1981), and followed by a discussion of further theoretical developments and empirical applications. The chapter ends with the presentation of a model that focuses on selection properties of tournaments. Chapter 3 discusses the model by Engelbrecht-Wiggans and others (1983) on common value auctions with asymmetric information, thereby laying the groundwork for modeling the post-tournament auctions in the subsequent chapters. Chapter 4 presents an application of the tournament-cum-auction framework in the context of an internal capital market with subsequent spin-off decision, and chapter 5 presents an application of this framework in the context of a business plan contest with subsequent financing decision. Chapter 6 concludes, summarizing and discussing the main findings of the two applications, and assessing the broader implications of the presented framework.

Chapter 2

Rank-order tournaments

Rank-order tournaments or contests are used in a wide range of contexts, whenever a principal wants to choose the best performer among multiple agents. While imposing small information requirements on the principal, a tournament allows to tackle moral hazard as well as adverse selection problems. Without the need to know agents' effort levels or the necessity to rank results on a cardinal scale, it suffices to rank results on an ordinal scale to pick the winner. In addition, a public tournament allows the principal to credibly commit to paying a prespecified prize to the winner, thereby mitigating possible hold-up problems as analyzed in the incomplete contracts literature.

The most obvious application of tournaments is in sports, where a winner is chosen based on his superior performance relative to his competitors. Sometimes, as in car races, a cardinal ranking (based on individual timing) would also be possible, but in other cases, like in a tennis tournament, results are purely relative. Thus, while additional information is sometimes available, it is not used in a pure rank-order tournament. It is, however, possible to include additional information in order to refine

tournament results.[1]

Beginning with the seminal work by Lazear and Rosen (1981), the analysis of rank-order tournaments has evolved into a vast body of literature, both theoretical and empirical. While it has been acknowledged early on that a tournament can serve for both the provision of incentives and for the selection of the most qualified contestant[2], the bulk of this literature has focused on incentive effects. Only recently has there been increased interest in analyzing the selection properties of tournaments with heterogeneous contestants, private information, and strategic actions. In this vein, both Clark and Riis (2001) and Hvide and Kristiansen (2003) analyze selection efficiency properties of tournaments.

Lazear and Rosen (1981), as well as a big part of the subsequent literature, focus on the tournament properties of internal labour markets, where a promotion among peers is used as the winner's prize for higher work effort. In addition, other strands of the literature have analyzed tournaments in a variety of contexts. The most prominent among those are research and development tournaments (beginning with Taylor, 1995), electoral competition and rent-seeking (starting with Tullock, 1980), as well as internal capital markets (the first to introduce the notion of "winner picking" in the allocation of funds between projects was Stein, 1997). The field of competitive sports has also inspired a fair amount of research, particularly in the empirical realm (for an early analysis of golf tournaments, see Ehrenberg and Bognanno, 1990a, b).

[1]For example, Clark and Riis (2001) suggest the inclusion of additional performance standards to improve selection efficiency (defined as the probability with which the best contestant wins the tournament).

[2]See for example Rosen (1986).

The remainder of this chapter is organized as follows: Section 2.1 presents a basic tournament model, based on Lazear and Rosen (1981), which focuses on the provision of incentives. Section 2.2 deals with subsequent analytical work focusing on the existence and properties of equilibria in tournaments. This is followed by a discussion of different fields of application, and a brief overview of the empirical literature in section 2.3. Finally, section 2.4 focuses on the selection properties of tournaments, presenting a basic model by Hvide and Kristiansen (2003).

2.1 The basic tournament model

The most basic tournament setup consists of one principal (sponsor) and two homogeneous agents (contestants) A and B, all of whom are assumed to be risk-neutral. The sponsor *ex ante* announces two distinct prizes, v_W for the winner of the tournament, and v_L for the loser (with v_L possibly equal to zero). As suggested in Lazear and Rosen (1981), the contestants can exert unobservable effort x_i, with $i = A, B$, to produce output $q_i = x_i + \epsilon_i$, where ϵ_i is a random or luck component, which is drawn from a known distribution with zero mean and variance σ^2. Effort can be exerted only at a cost $c(x_i)$, which is equal for both contestants, thus inducing identical equilibrium behavior. The form of the cost function is assumed to satisfy the standard requirement $c'(x_i), c''(x_i) > 0$.

The expected payoff of contestant i is then defined by

$$\begin{aligned}&\Pr\{win\}[v_W - c(x_i)] + (1 - \Pr\{win\})[v_L - c(x_i)] = \\ &\Pr\{win\}v_W + (1 - \Pr\{win\})v_L - c(x_i)\,,\end{aligned} \tag{2.1.1}$$

with the success function $\Pr\{win\}$ giving the probability that contestant i wins the tournament. As the realization of output is the only observable variable for the tournament sponsor, she awards the winner's prize to the contestant with the higher output, such that $\Pr\{win\} = \Pr\{q_i > q_j\} = \Pr\{x_i - x_j > \epsilon_j - \epsilon_i\}$.[3] Setting $\epsilon_j - \epsilon_i = \varepsilon$, with $\varepsilon \sim g(\varepsilon)$, the cumulative distribution function becomes $G(\varepsilon)$, with $E[\,\varepsilon] = 0$, and $E[\,\varepsilon^2] = 2\sigma^2$ (since ϵ_i and ϵ_j are i.i.d.). Plugging in for this relationship results in

$$\Pr\{win\} = \Pr\{x_i - x_j > \varepsilon\} = G(x_i - x_j)\,. \tag{2.1.2}$$

Each contestant chooses x_i to maximize equation (2.1.1). Assuming interior solutions, the first order condition becomes

$$(v_W - v_L)\frac{\partial \Pr\{win\}}{\partial x_i} - c'(x_i) \stackrel{!}{=} 0\,, \tag{2.1.3}$$

and the second order condition becomes

$$(v_W - v_L)\frac{\partial^2 \Pr\{win\}}{\partial x_i^2} - c''(x_i) \stackrel{!}{<} 0\,. \tag{2.1.4}$$

Since contestant i takes contestant j's effort decision as given, it follows from equation (2.1.2) that $\frac{\partial \Pr\{win\}}{\partial x_i} = \frac{\partial G(x_i - x_j)}{\partial x_i} = g(x_i - x_j)$. Plugging this back into the

[3] In the tournament literature, different forms of success functions are being used. These can be broadly divided into two classes, "perfectly discriminating" and "not perfectly discriminating" success functions. In the former, the superior contestant always wins, while the latter includes an element of luck, allowing the inferior contestant to win with positive probability. Lazear and Rosen's (1981) success function would be perfectly discriminating only for $\epsilon_j = \epsilon_i$.

first order condition (2.1.3) yields contestant i's reaction function:

$$(v_W - v_L)g(x_i - x_j) - c'(x_i) = 0\,. \tag{2.1.5}$$

Assuming that a Nash equilibrium exists, symmetry implies that both contestants choose the same equilibrium strategy, resulting in the same equilibrium effort levels $x_A = x_B = x^*$.[4] The winning probability for each contestant therefore becomes $Pr\{win\} = G(0) = \frac{1}{2}$, and the outcome in equilibrium becomes purely random, depending only on the realizations of the idiosyncratic shocks ϵ_i. Substituting x^* for x_i and x_j in equation (2.1.5) gives

$$c'(x_i) = (v_W - v_L)g(0)\,,\ i = A, B\,. \tag{2.1.6}$$

Equation (2.1.6) reveals that a contestant's equilibrium effort decision only depends on the spread between winning and losing prizes, rather than on their absolute levels. The spread $\Delta v = v_W - v_L$ is set by the sponsor, who maximizes her own expected payoff, which is determined by the expected total output $E[\,q_A + q_B]$, the price p of the product per unit, and the tournament prizes v_W and v_L. In a competitive industry, the principal must offer competitive remuneration levels, to the point where her expected receipts are equal to the total prize money offered. Hence, her expected payoff can be expressed as $p \cdot E[\,q_A + q_B] - (v_W + v_L) = 0$, which, in equilibrium and with $x_A = x_B = x^*$, reduces to

$$p \cdot E[\,q_A + q_B] = v_W + v_L \Leftrightarrow p \cdot x^* = \frac{v_W + v_L}{2}\,. \tag{2.1.7}$$

[4] As Lazear and Rosen (1981) point out, an equilibrium does not necessarily exist. However, it can be shown that the objective function will be concave in the relevant parameter range if the variance σ^2 of the exogenous shock is large enough, thus ensuring that an equilibrium exists. Thus, "Contests are feasible only when chance is a significant factor." (Lazear and Rosen, 1981, p. 845)

In her optimization problem, the principal is not only restrained by her own zero expected profit condition, she must also take into account the contestants' participation conditions and their incentive compatibility constraints. It can be assumed that the participation conditions are satisfied, simply by setting the contestants' reservation utilities to zero. The incentive compatibility constraints in turn must be respected by choosing Δv so as to maximize the contestants' expected utility in equilibrium. Substituting equation (2.1.7) into contestant i's utility function (2.1.1) and setting $\Pr\{win\} = \frac{1}{2}$ gives a contestant's expected utility in equilibrium as

$$p \cdot x^* - c(x^*) \; . \tag{2.1.8}$$

Maximizing equation (2.1.8) over the prize spread Δv yields the first order condition

$$[\, p - c'(x^*)] \, \frac{\partial x^*}{\partial \Delta v} \stackrel{!}{=} 0 \, . \tag{2.1.9}$$

Thus, the marginal cost of effort $c'(x^*)$ in equilibrium equals its marginal social return p. This implies that, under risk neutrality, a competitive tournament is an efficient incentive mechanism that induces first best effort levels, resulting in the same resource allocation as would other efficient mechanisms, like piece rates.

To complete the results, insert equation (2.1.9) into equation (2.1.6) to obtain the optimal prize spread $\Delta v = \frac{p}{g(0)}$. Together with equation (2.1.7), this allows to compute the optimal prize levels

$$\begin{aligned} v_W &= p \cdot x^* + \frac{p}{2g(0)} \\ \text{and} \quad v_L &= p \cdot x^* - \frac{p}{2g(0)} \quad . \end{aligned} \tag{2.1.10}$$

As Lazear and Rosen (1981) point out, it is possible to interpret $\frac{p}{2g(0)}$ as an entrance fee which each contestant has to pay up front. The winning and losing prizes then

pay off the value of the expected output of one contestant, plus or minus the entrance fee. "That is, the players receive their expected product combined with a fair winner-take-all gamble over the total entrance fees or bonds. The appropriate social [effort] incentives are given by each contestant's attempt to win the gamble." (Lazear and Rosen, 1981, p.846)

2.2 Existence and properties of equilibria in tournaments

Starting from the basic tournament model, as set out by Lazear and Rosen (1981), extensive research has been done on the existence and theoretical properties of equilibria under a varying set of assumptions. In particular, the effects of risk aversion, of heterogeneity among contestants, and of different numbers of contestants and prizes, as well as different informational structures have been analyzed. This section provides a selective overview of this literature and its main results.

2.2.1 Risk aversion

This subsection shows that tournaments can be second-best incentive contracts when contestants are risk averse. Starting with Lazear and Rosen (1981), it explores under which circumstances tournaments are superior incentive contracts when compared to simple piece-rate schemes. While these authors identify several such circumstances, the subsequent literature focuses mainly on the existence of a common exogenous shock that affect all contestants' outputs and that gets cancelled out in the purely ordinal ranking of a tournament, thereby insulating individual contestants' payoffs from risk.

In their seminal article, Lazear and Rosen (1981) consider several extensions to the basic setup, allowing for risk aversion and for heterogeneous contestants. In the case of risk averse contestants, the authors show that under certain conditions, contestants prefer a tournament setting to a piece rate scheme that is based on individual output levels. This contrasts with the result under risk neutrality, where both compensation schemes induce efficient effort levels.

Under risk aversion, every compensation scheme faces a trade-off between setting incentives and providing insurance, an effect which is well-known from the principal-agent literature. Therefore, the first-best solution cannot be achieved, and it remains to be seen which compensation scheme can induce a second-best solution. In addition to complicating the formal analysis substantially, the introduction of risk aversion leads to inconclusive results, which depend on parameter values as well as on the specification of contestants' utility functions. Hence, Lazear and Rosen (1981) cannot present a complete characterization of the conditions under which a rank-order tournament dominates a piece rate scheme and vice versa. Instead, they provide examples which allow a characterization of factors that favor one scheme over the other.

For further analysis, Lazear and Rosen (1981) focus on a utility function of the form $U_i = \alpha y_i^{\alpha}$, ($y_i$ being contestant i's expected payoff) with constant relative, but declining absolute risk aversion. Results for different parameter specifications suggest that contestants with a lower absolute risk aversion and a higher exogenous, non-labor income will generally prefer contests, while contestants with higher absolute risk aversion and a lower level of exogenous income are more likely to prefer

piece rates. This preference structure can be attributed to the effects that each compensation scheme has on the income distribution: While piece rates concentrate the mass of the distribution near the mean, a rank-order tournament results in a binomial distribution where 50 per cent of the weight is significantly above the mean (for the winner) and 50 per cent significantly below (for the loser). This tournament-specific feature of "winner takes all" is relatively unattractive for more risk averse contestants.

Another feature that can make rank-order tournaments more attractive than piece rate schemes for risk averse contestants is a positive correlation between the random error terms ϵ_i and ϵ_j. The intuition is that common noise is cancelled out in rank-order tournaments, as they only evaluate contestants' relative performances. In contrast, a (positive or negative) common shock affects both contestants' outputs and compensations in individualistic piece rate schemes - increasing the standard deviation of payoffs while maintaining the same expected payoffs as without the shock. Thus, in the presence of sufficiently large common noise, tournaments can be more efficient compensation mechanisms than piece rate schemes for risk averse contestants, by reducing the volatility of payoffs. In the subsequent literature, the argument of a common noise term or common shock gained popularity as the main explanatory factor for the superiority of tournaments over other compensation schemes in the presence of risk aversion.[5]

In an early survey of tournament models, McLaughlin (1988) explicitly analyzes the effect of risk aversion on the optimal prize spread and on contestants' optimal

[5]The concept of a common shock is, among others, applied in Nalebuff and Stiglitz (1983), Green and Stokey (1983), and - in the context of internal capital markets - in Stein (1997).

effort levels. He points out that in the Lazear and Rosen (1981) setup, the optimal prize spread decreases with an increase in risk aversion. This observation matches the above result that more risk averse contestants tend to prefer compensation schemes with less spread around the mean. In addition, and as expected, the trade-off between incentive provision and insurance motive implies that optimal effort decreases as contestants' risk aversion increases: As an increased need for insurance reduces the optimal price spread, optimal effort levels decline.

In alternative model specifications, the described fundamental relation between risk aversion, prize spread and effort levels remains valid. However, depending on the specific form of the output equation, additional factors can have an influence. Unlike the Lazear and Rosen (1981) model analyzed in section (2.1), where output is additive in effort and disturbances, the model by Nalebuff and Stiglitz (1983) features a multiplicative (common) disturbance term. As a result, the variance of the common shock enters the equations of optimal prize spread and effort levels. Both variables increase with an increase in the variance of the common error term. As optimal effort levels increase with a larger variance of the common error term, contestants' expected utility also increases. This result is in line with the finding of Lazear and Rosen (1981) that tournaments become more efficient as common noise increases. For sufficiently large common error variance, the optimal effort levels even approach first-best in the Nalebuff and Stiglitz (1983) setup.

2.2.2 Heterogeneity of contestants

This subsection introduces heterogeneity to the contestant pool, within different types of tournaments - with or without the inclusion of a luck component -, and also allowing for different informational setups. While the literature on incentive provision has traditionally focused on complete information setups, where all contestants' know their own and each other's characteristics, as well as their strategy spaces and possible payoffs, the literature on selection properties of tournaments has naturally focused on the incomplete information setups, including the case of private information. Both types of informational setups, and their implications for equilibrium strategies, are analyzed in this subsection in the context of the two different types of tournament.

While the analysis of risk aversion was an early feature of the tournament literature - inspired by the principal-agent theory, which served as starting point and benchmark - the analysis of heterogeneous contestants was long neglected. Although Lazear and Rosen introduced heterogeneous contestants in another extension of their 1981 article (assuming risk neutrality), they limited the analysis to showing that in the presence of private information, heterogeneous workers do not self-sort into groups of equal skill levels. As a result, a separating equilibrium can only be achieved at additional cost - a finding which is in line with the literature on adverse selection. When the contestants' types are common knowledge, however, a handicap system can be used to produce an efficient outcome. In spite of this and several subsequent attempts, a complete characterization of tournament equilibria with heterogeneous contestants and private information was only provided more than a decade later, by

Amann and Leininger (1996).[6]

Heterogeneous agents in perfectly discriminating contests

Unlike the basic tournament model described above, the setup used by Amann and Leininger (1996) does not feature a luck component. Instead, the authors analyze a first-price all-pay auction in which the bidder with the highest bid wins with certainty. This type of auction is frequently used in the contest literature, as it captures the main feature that every contestant has to bear the cost of his own effort while only one contestant wins the prize. In addition, as Baye and others (1996) point out and Che and Gale (2000) show, a wide array of not perfectly discriminating contests (including the one analyzed in Lazear and Rosen, 1981) converges to the perfectly discriminating first-price all-pay auction as the element of luck is reduced to zero. Another advantage of this setup is that it is well understood for the cases of homogeneous contestants, and for heterogeneous contestants with complete information (see Baye and others, 1996).

In their 1996 article, Baye and others present a complete characterization of equilibria for first-price all-pay auctions with $n \geq 2$ potentially heterogeneous bidders and complete information. The authors show that unlike suggested by earlier research, for $n > 2$, there exists a large set of equilibria, which depend critically on the configuration of the players' types. Their main results can be summarized as follows: (a) While

[6]Rosen (1986) considers the case of heterogeneous contestants without private information, when the contestants themselves don't know their own type (incomplete information). This contribution is discussed at some length in subsection 2.2.3, where it is shown that the case of incomplete information without private information generates similar outcomes as the case of homogeneous contestants.

there exists a symmetric and unique Nash equilibrium in the case of two homogeneous players, a continuum of asymmetric equilibria arises (in addition to the unique symmetric equilibrium) if the number of homogeneous bidders is larger than two. This holds true whether or not there are additional players with (possibly heterogeneous) lower types than the group of homogeneous players at the top. In any equilibrium, the expected payoff for each player is zero, and all equilibria are revenue equivalent (i.e. the expected sum of all bids is equal). (b) When one player of a higher type competes against several weaker, but equal, players, revenue equivalence no longer holds. There exists a continuum of Nash equilibria, in which the strongest player earns a positive expected payoff, while all other players earn an expected payoff of zero, and the expected sum of the bids varies across the range of equilibria (it is maximized when only one of the weaker players bids with positive probability). (c) Lastly, when at least the three top players are of unequal strength, only the first two players bid with positive probability; and the resulting Nash equilibrium is unique. Again, the strongest player earns a positive expected payoff, while expected payoffs for all other players are zero.[7]

In their detailed analysis of asymmetric all-pay auctions with incomplete and private information, Amann and Leininger (1996) focus on the two-contestant case, thereby again narrowing the range of potential equilibria. While they analyze both first- and second-price all-pay auctions[8], the discussion of their results here is restricted to the first-price all-pay auction used to model tournaments. As their main result, Amann and Leininger (1996) prove existence and uniqueness of a Bayesian

[7]This last result is a known feature first described by Hillman and Riley (1989).

[8]The second-price all-pay auction is also of particular interest, as it is generally used to model the "war of attrition".

Nash equilibrium for the first-price all-pay auction with two asymmetric bidders and incomplete information.

Amann and Leininger (1996) consider two players with different valuations of an indivisible object to be auctioned off, where each player's valuation is private information, and both have independent priors about the distribution of the other's valuation. A player's optimal bidding strategy must then maximize his expected payoff against the backdrop of his opponent's expected type distribution and employed strategy. The authors derive a set of characteristics for the resulting bid distributions, which are in line with common findings in the auction theory literature: (a) The bid distributions of the two players have common support; (b) they are continuous, and monotonically increasing in the valuation of the object; (c) both distributions have full support, meaning that two players with the highest valuation (according to their respective type distribution) must submit the same bid; (d) at most one player can have an atom at zero in his bid distribution.

Based on these characteristics of the bid distributions, the authors show that for independently distributed valuations, whose distributions have densities that are continuously differentiable and positive on $(0, 1)$, there exists a unique Bayesian Nash equilibrium of the first-price all-pay auction. As a corollary, Amann and Leininger (1996) prove that in the special case of common priors about the respective distributions of contestants' valuations, the earlier conjecture by Weber (1985) holds, namely that "...the unique symmetric equilibrium found by [him] in this case is the only equilibrium." (Amann and Leininger, 1996, p. 9). In addition, they show that with a

continuously diminishing degree of uncertainty about the other player's valuation, the equilibrium bid distributions (both with and without common priors) under certain conditions converge to the equilibrium bid distributions of the complete information case.

Heterogeneous agents in not perfectly discriminating contests

As the main focus of this thesis lies on not perfectly discriminating contests, it is closer to more recent work, like that of Hvide and Kristiansen (2003), who analyze properties of not perfectly discriminating contests with heterogeneous contestants and incomplete and private information. These authors also explicitly consider the selection properties of tournaments, which are largely neglected by the bulk of the earlier literature. However, due to the very specific model-setup, their contribution does not provide a general characterization of contest equilibria, and will be discussed in more depth in section 2.4, which focuses on selection properties.

An earlier article by Baik (1998) is closer to the literature reviewed above, in that it focuses on the provision of incentives. But while it provides a complete characterization of equilibria in a not perfectly discriminating contest (i.e. a contest with a luck component) with two heterogeneous contestants, it does so only for the case of complete information (i.e. all contestants' types, as well as strategy and payoff spaces are common knowledge).[9] In his model setup, Baik (1998) allows for differences between the contestants' valuations for the prize and between their respective

[9]See Baik (1998), p. 686 for different ways to model not perfectly discriminating contests. In his own setup, Baik uses a so-called "difference form" success function, where one contestant's probability of winning is a function of the difference between his own and his contestant's bid.

skill levels (different relative abilities to convert effort into probability of winning). As both players strive to maximize their expected payoffs given their respective opponent's type and strategy, their optimal effort choices are described by their reaction correspondences. One of the main features of the reaction correspondences is that given an opponent's effort level, a player who decides to participate in the contest will always exert such a degree of effort that the difference between the two players' "effective effort levels" (adjusted for skill levels) remains constant. As a consequence, each participating player's probability of winning also remains constant along his reaction correspondence.

In his equilibrium analysis, Baik (1998) first considers the case when both contestants choose their effort levels simultaneously, and then analyzes the case when both contestants choose effort levels sequentially. In the simultaneous case, he finds that there is no pure-strategy Nash equilibrium in which both players participate in the contest, i.e. exert effort. Only when one player has a much higher composite strength (as a combination of skill level and valuation of the prize) than the other, this stronger player will participate and exert effort, while the weaker player will not exert effort. If both players have low valuations for the prize and their marginal probabilities of winning at effort levels of zero are sufficiently small, none of them will exert effort.[10] In the sequential case, the author also finds that there is no subgame-perfect equilibrium (in pure strategies), in which both players exert effort. In addition, there exists a first-mover advantage which causes player 2 to only exert effort if his composite strength by far outweighs that of player 1. Again, if both players' valuations

[10]This last result is idiosyncratic for difference-form contests and does not hold for other types of not perfectly discriminating contests.

are relatively low, and marginal winning probabilities at zero are small, neither player will exert effort.

When considering a not perfectly discriminating contest between heterogeneous contestants with *incomplete information*, the question of selection efficiency arises almost naturally. It therefore does not come as a surprise that much of the pertaining literature focuses on this aspect and moves away from the analysis of incentives and effort levels. The article by Bhattacharya and Guasch (1988) stands out in that it includes both selection and incentive effects. In the tradition of Lazear and Rosen (1981), the authors consider the possibility of ($n \geq 2$) contestants' self-selection into homogeneous "leagues", which would allow to induce first-best effort levels by holding separate contests for each "league". But in contrast to Lazear and Rosen (1981), they argue that self-selection is indeed possible, when introducing ordinal performance comparisons across self-selected cohorts. In addition, they allow for a continuum of contestants' types, whereas Lazear and Rosen only analyze the case of two different types. To allow for intra-cohort ranking, however, the authors have to resort to the use of absolute test standards, rewarding every contestant who passes the standard. This violates the tournament-specific "winner-take-all" condition by granting the same prize to all contestants who achieve the chosen standard.

Clark and Riis (2001), who do not treat the moral hazard issue, claim that their solution to the self-selection problem is easier than the one suggested by Bhattacharya and Guasch (1988), and at the same time "...closer to the spirit of tournaments in that it does not involve arranging separate tournaments for different types." (Clark

and Riis, 2001, p. 171). The authors use a two-player setup to show how a standard single-prize tournament fails to ensure that the principal chooses the most able contestant with certainty. They then go on to introduce a test standard, but unlike in Bhattacharya and Guasch (1988), a prize for passing the standard is only awarded to the winner of the tournament. This mechanism allows the authors to guarantee selection efficiency while at the same time preserving the winner-take-all aspect of tournaments. However, they use a perfectly discriminating contest in which the contestant with the higher effort wins with certainty. In addition, in order to implement their test standard, the authors must assume that effort levels can be precisely measured by some outside observer, who then only transmits ordinal information and information about the achievement of the test standard to the tournament sponsor.

2.2.3 Multiple contestants and prizes

Of the papers discussed in the previous subsection, some focus on the case of two contestants, while others choose a more general approach by admitting $n \geq 2$ contestants. None of them, however, explicitly considers setups with more than one winning prize (except for Bhattacharya and Guasch, 1988, who suggest rewarding every contestant who passes a given test standard). This subsection first presents a perspective on tournaments with multiple prizes, before turning to the question of the optimal number of contestants.

Multiple prizes

The earliest work to explicitly consider multiple prizes is the article by Rosen (1986) on elimination tournaments in labor markets.[11] The author's main objective is to analyze the effect of increasing rewards in sequential elimination tournaments on effort decisions and selection efficiency. His model setup therefore allows for $n \geq 2$ possibly heterogeneous risk neutral contestants, with possibly incomplete information (i.e. contestants' types are either common knowledge, or are unknown even to the contestants themselves), who engage in a multi-stage sequential elimination tournament with interim prizes. This framework approximates the observed career and promotion patterns in internal labor markets. As Rosen puts it, "A career trajectory is, in part, the outcome of competition among peers to attain higher ranking and more remunerative positions over the life cycle." (Rosen 1986, p. 701). The most salient feature of organizations' internal labor markets is a marked concentration of rewards in the top ranks. The author shows how this phenomenon is caused by the survival and incentives aspects of the elimination tournament. The case of risk aversion is treated as an extension of the risk neutral case.

Rosen (1986) models the tournament as a tennis-ladder type paired elimination tournament, which begins with 2^N players and proceeds sequentially through N stages. The winners of each round proceed to the next, while the losers will remain at their attained level and are not allowed to participate in any subsequent rounds. Hence, while the loser's prize is fixed, the winner's prize includes the option

[11] Another early contribution is by Green and Stokey (1983), who extend Lazear and Rosen's (1981) framework to allow for $n \geq 2$ contestants. Their analysis is, however, limited by very restrictive assumptions (such as homogeneous and risk averse contestants, and a common external shock), and does not explicitly discuss the role of the number and size of prizes.

of continuing to compete in future rounds. As the last stage approaches, the number of remaining stages diminishes, and the value of this option decreases accordingly. At each stage, the two paired contestants can exert non-observable effort which, together with both contestants' skill levels, positively influences their respective winning probabilities. The tournament sponsor is assumed to be interested in maintaining effort levels constant at each stage of the tournament. Only pure-strategy equilibria are considered.

In the basic setup with risk neutrality and homogeneous contestants, each contestant knows that he will meet an opponent of equal skill at every level. Solving the game by recursion, taking into account contestants' maximizing behavior and the sponsor's objective to hold effort levels constant, gives a unique prize structure for any desired effort level (or, put differently, for a given overall prize budget, there is one unique constant effort level which can be maintained). Rosen shows that the optimal spread between prizes at each stage is a constant, with the exception of the last stage. At that point, the continuation value is zero, and the prize spread must be larger in order to maintain effort incentives. By introducing this jump in the otherwise linearly increasing prize structure, it is possible to convert the difference between the loser's prize and the continuation value at each stage into a "...perpetuity of constant value at all stages." (Rosen 1986, p. 706). If the sponsor's aim were to elicit higher effort levels at each successive stage, she could do so by concentrating even more of the overall prize money on the top, thereby increasing the continuation value at higher stages.

Introducing risk aversion does not qualitatively alter the optimal prize structure.

When the absolute values of prizes are replaced by a monetary utility equivalent, then - independently of the utility function's form - the optimal spread between utility equivalents of each prize is again linearly increasing until the second-to-last stage, and jumps up at the final stage.

When contestants have heterogeneous but known abilities (in a risk-neutral setting), a player's continuation value at each stage depends on his own ability and on that of the other contestants. While it rises at any given stage with an increase in own ability (all else being equal), it decreases with an increase in opponents' abilities, which, owing to the elimination character of the tournament, are likely to be higher in the later stages.[12] Due to the asymmetries in the continuation values for heterogeneous contestants, it is impossible for the tournament sponsor to maintain identical effort levels across stages and agent types. However, by choosing the appropriate prize structure, she can maintain constant type-specific effort levels throughout all stages. The exact form of the required prize structure is less straightforward than in the case with homogeneous contestants, as it depends on the specific assumptions on the functional form of effort costs, on the success function (determining a contestant's winning probability based on his effort level), as well as on the total sum of prize money. However, the need for a larger prize spread at the final stage continues to hold as a general result.

In the case of heterogeneous contestants with unknown abilities (but without private information), Rosen introduces a new objective function for the principal. In

[12]In this vein, the matching process (random or seeded) by which contestants are selected also influences effort levels.

addition to trying to maintain constant effort levels at all stages, the sponsor is assumed to be trying to select the best contestant as the tournament's overall winner. As pointed out earlier, the question of selection properties arises naturally in the context of unknown abilities.[13] While these are discussed in more detail in section 2.4 below, it is worth pointing out at this point that there exists a unique symmetric equilibrium. In this equilibrium, incentives for effort may in fact decrease with an increasing prize spread in the final stages, but only for highly heterogeneous contestants. If heterogeneity is small, the incentive-diluting effects of uncertainty are less relevant, and the earlier results on optimal incentive-maintaining spreads continue to hold.

While Rosen (1986) discusses optimal prize spreads in a multi-stage contest, he takes the form of the tournament as given and does not explicitly analyze whether a multiplicity of prizes leads to outcomes superior to those of a single-prize tournament. Such optimal design questions are addressed by Clark and Riis (1998) and Barut and Kovenock (1998), who analyze optimal reward structures with homogeneous prizes and heterogeneous prizes, respectively.

In their contribution, Clark and Riis (1998) provide an extension of the analysis of complete-information all-pay auctions, by allowing for n heterogeneous contestants and $m \geq 1$ homogeneous prizes. They show that known differences between

[13] By assuming symmetric information, Rosen (1986) avoids the type of adverse selection problems that arise in Lazear and Rosen (1981), and that other authors have subsequently tried to resolve (see section 2.4).

contestants' abilities lead to an equilibrium in which only the $m + 1$ strongest contestants participate, while weaker players bid zero with probability one. This result holds both in the case of simultaneous and sequential awarding of prizes. However, these two variants induce different bidding behavior, leading to different overall sums of contestants' bids (i.e., they are not revenue equivalent for the principal). Thus, in an internal labor market setting, it depends on the distribution of workers' abilities which of the two tournament forms leads to a higher overall effort level and is therefore preferred by the principal. For the case of $m = 2$ prizes, the authors show that a sequential tournament generates higher overall effort if the two strongest workers have similar skill-levels, while the simultaneous setup generates higher overall effort if the strongest player has significantly higher skill-levels than the two runners-up.

While Clark and Riis (1998) discuss the optimal tournament design in terms of sequencing, like Rosen (1986), they take the number of prizes as given. Unlike these authors, Barut and Kovenock (1998) allow for any number of prizes less or equal to the number of homogeneous contestants. In their setup, multiple prizes are valued in weakly decreasing order, but identically among contestants. The authors provide a complete characterization of Nash equilibria and expected revenue, showing that expected revenue is maximized by setting the lowest prize as low as possible (zero in their case), while it remains unaffected by any further variations in prize structure. If the lowest prize is unique (i.e. lower than all other prizes), a unique and symmetric equilibrium arises; if there are more then one equally valued lowest prizes, a continuum of Nash equilibria arises. In both cases, expected utility of every player equals the utility of the lowest prize. Competition for higher valued prizes hence completely

dissipates the expected rents generated by these prizes.

In a more recent paper, Moldovanu and Sela (2001) allow for heterogeneous contestants with private information in a multiple prize all-pay auction, with (observable) effort levels as contestants' "bids". In turn, the principal's objective function is to maximize overall effort. One of the authors' main results is that it is optimal for the principal to allocate the entire prize money to one single prize at the top, when contestants' cost functions of exerted effort are linear or concave. For the case of convex cost functions, the authors specify conditions under which two or more prizes are optimal.

Other papers considering multiple prizes include that by Gradstein and Nitzan (1989), who study a setting in which players can allocate resources to different rent-seeking games simultaneously. They have to decide over how many prizes to compete and how much to bid for each prize. Glazer and Hassin (1988) analyze properties of a symmetric equilibrium in a similar setting to that used by Barut and Kovenock (1998), but without providing a complete characterization of equilibria. Sunde (2003) is an exponent of the empirical tournament literature, who examines professional tennis data in a two-stage setting (semi-finals and finals) with interim prizes and heterogeneous contestants.

Optimal number of contestants

In their seminal work, Lazear and Rosen (1981) focused on the case of two contestants, claiming, however, that all major results could readily be generalized to

more contestants. The ensuing literature covers both cases, assuming either two or more contestants. For the most part, however, the number of contestants is assumed to be exogenous, thereby foregoing the possibility of endogenously determining the optimal number of contestants. In the context of internal labor markets, this is consistent with the fact that promotions to a higher position are open to a given pool of workers at the next lower level. In the case of research tournaments, however, the sponsor has the freedom to invite potential contestants, possibly limiting the overall number of contestants in order to improve the expected outcome of the tournament. It is in this vein that Taylor (1995) and Fullerton and McAfee (1999) consider mechanisms such as entry fees and tournament entry auctions, respectively.

Taylor (1995) was the first to model research tournaments as all-pay auctions, veering from the standard innovation-race (based on Loury, 1979, and Lee and Wilde, 1980) used to model R&D competitions. In his setup, $n \geq 2$ homogeneous research firms, after paying an entry fee $E \geq 0$, compete for a fixed monetary prize for achieving the best innovation. Research takes place over a pre-determined number of rounds, costing a fixed amount c per round and each resulting in a random realization of innovation quality. Each contestant can decide to try and improve on attained quality levels by conducting further research in the following rounds. For simplicity, innovation quality is randomly drawn in each round and is statistically independent across time and among firms. During the contest, research decisions in each round as well as attained innovation qualities are private knowledge to each firm. A firm's optimal research strategy in the resulting unique subgame-perfect Nash equilibrium is characterized by the decision whether to pay the fixed fee to enter the tournament and by

the decision when to stop research - Thus settling for the attained level of innovation quality and foregoing further costly research.

After determining firms' equilibrium strategies, Taylor (1995) turns to the question of optimal tournament design. The principal, who wants to maximize the expected value of the winning innovation, can decide about how many firms to invite, the amount of the entry fee, and the size of the winning prize. In a first step, the author shows that free and unlimited entry generally induces a suboptimal amount of research, given that in this situation, the pure strategy equilibrium provides only for a limited number of firms to undertake research activities, and those that do, do so only in one round. Even less research is induced by (symmetric) mixed strategy equilibria as the active participants randomize over their research decisions. In a second step, Taylor (1995) derives the optimal number of invited participants as well as the optimal entry fee, which depend on the tournament's length (number of rounds), the cost of R&D, and the distribution function of the random innovations. He also shows that while the optimal tournament *ex ante* induces an expected overall research level equal to the first-best solution, it causes either an overshooting or a shortfall in research activities *ex post*.

While Taylor (1995) shows that it is generally optimal to limit the number of invited contestants and to charge a positive tournament entry fee, Fullerton and McAfee (1999) expand on his work and show that under a broad range of assumptions, the optimal number of contestants in a research tournament is two. Their setup differs from Taylor (1995) in that it allows for heterogeneous contestants and assumes

fixed costs in addition to variable costs of R&D. They model the tournament as a two-stage game. In the first stage, potential contestants decide whether to incur (symmetric) fixed costs to enter the tournament, and in the second stage, they decide on their effort levels, after the set of participants and their respective cost functions have become common knowledge. The authors analyze the resulting unique efficient subgame-perfect equilibrium[14], finding that it is efficient for a unique number k of the lowest-cost firms to enter the tournament.

In the case of homogeneous contestants, it is straightforward for the the tournament sponsor to maximize her profit by setting an entry fee E and choosing the prize level v, thereby determining the number of contestants k and total effort level $\sum x$. The entry fee also allows her to extract expected profits from the contestants. Owing to the fixed cost element of the contestants' R&D cost, the total cost of procurement is minimized (and, hence, expected profits are maximized) by limiting the number of contestants to two. In the case of heterogeneous contestants (with different marginal costs of R&D), the optimal number of contestants is also two - even in the absence of fixed costs - as long as a technical condition is satisfied.[15] As the authors show, this condition is satisfied for a large variety of cost functions, ensuring that the optimal number of contestants is two for a broad class of tournaments.

[14]If two contestants' cost functions are very similar, it is possible that both would realize negative profits if they both entered the tournament, while either one of them would realize positive profits if the other one were not to enter, thus allowing for the possibility that the one with slightly higher marginal costs enters and precludes the more efficient competitor from entering as well. This type of inefficient equilibrium does not arise when the fixed costs of R&D are such that they would allow only for the more cost-efficient contestant but not for the (slightly) less cost-efficient one. In what follows, only this "efficient" case is considered.

[15]This technical condition calls for the highest marginal cost of research of k participating firms to be increasing relative to the average cost of research of the k firms.

Like Taylor (1995), Fullerton and McAfee (1999) consider the possibility of using entrance fees to limit the number of contestants. However, as they point out, setting the optimal entrance fee requires information about would-be contestants' cost functions. Since an auction has less informational requirements, an entry auction can be a viable alternative in case of incomplete information. The authors show that an all-pay auction with uniform interim prizes for those contestants who enter the research tournament can serve as an efficient selection mechanism for the lowest-cost contestants (if their types are independent).

2.3 Fields of application and empirical findings

2.3.1 Fields of application

The models presented above analyze tournament design and resulting equilibria in different fields of application, such as internal labor markets, sports, and research and development. Beyond this, however, rank-order tournaments have been used to describe compensation schemes in multiple different contexts. This subsection presents an overview over the main fields of application as treated in the literature. As McLaughlin puts it: "...the principal features of tournaments apply to any compensation scheme which bases pay on relative performance." (McLaughlin, 1988, p. 225)

Internal labor markets

Following the seminal article by Lazear and Rosen (1981), a large part of the tournament literature analyzes internal labor markets. While the traditional principal-agent theory focuses on the provision of incentives and on contract design in the single-agent case, basing rewards on absolute performance, the tournament structure allows for a multiple-agent setting, in which incentives or selection criteria are based on relative performance. This seems to be a more adequate concept to model reward structures within firms, as promotions to higher positions - which are associated with a fixed higher pay and social standing - have all characteristics of a fixed prize, set to elicit effort from a pool of workers/contestants. In addition, a rank-order tournament has lower informational requirements as it does not require absolute performance measures, but relies only on an ordinal ranking of all competitors. This is in line with empirical characteristics of internal labor markets, where exact monitoring is costly, but information on ordinal differences may be readily available. In addition, the literature shows that tournaments can outperform individual incentive contracts in the presence of common shocks affecting all workers, as they are cancelled out by the relative evaluation mechanism.

Beyond the basic framework of risk neutrality and homogeneous contestants, contributions to this strand of literature include analyses of heterogeneous contestants (for example Lazear and Rosen, 1981, Rosen, 1986, Bhattacharya and Guasch, 1988, Clark and Riis, 2001), risk aversion (for example Lazear and Rosen, 1981, Nalebuff and Stiglitz, 1983, Rosen, 1986), common shocks (for example Lazear and Rosen, 1981 Nalebuff and Stiglitz, 1983), multiple contestants and prizes (for example Rosen, 1986,

Bhattacharya and Guasch, 1988, Clark and Riis, 1998), and selection efficiency (for example Rosen, 1986, Glazer and Hassin, 1988, Münster, 2006).

While most of the models describing labor market tournaments are based on not perfectly discriminating success functions (i.e. there always remains a random component in determining the winner), there are contributions that use the perfectly discriminating all-pay auction for modeling internal labor markets and rank-order tournaments in general (for example, Clark and Riis, 1998).

Rent-seeking

The concept of rent-seeking as a wasteful economic activity was first described by Tullock (1967), who discussed the welfare costs of tariff seeking, monopoly seeking, and theft. The term "rent-seeking" was, however, only coined later, by Anne O. Krueger (1974), who analyzed the phenomenon in the context of the allocation of import licenses, and provided empirical estimates of associated welfare losses. Tullock (1980) formalized the rent-seeking mechanism in a tournament model, in which agents (lobbyists) employ resources (favors, votes, bribes) to gain a prize (certain economic or political benefits) from a principal (politician). While it is in the principal's interest to exert as much lobbying resources from the contestants as possible in return for the privilege she can grant the winner, the traditional focus of the analysis has been on "rent dissipation", i.e. the relationship between total resource outlays and the value of the contested privilege. In the basic framework with $n \geq 2$ homogeneous risk neutral contestants and a (not perfectly discriminating) Tullock contest success function, the degree of rent dissipation is increasing in the number of contestants.

Beyond the basic framework, contributions to this strand of literature include analyses of risk aversion (for example Hillman and Katz, 1984; Hillman and Samet, 1987), heterogeneous rent seekers (for example Hillman and Riley, 1989), multiple contestants and rents (for example Gradstein and Nitzan, 1989), and rent-seeking by groups (for example Farrell and Lander, 1989).[16]

While Tullock (1980) used a not perfectly discriminating success function, much of the subsequent work is based on the perfectly discriminating framework of the standard first-price all-pay auction. This seems appropriate in the rent-seeking contest, as contestants' performances are likely to be less distorted than in an internal labor market: It should not be a problem for a principal to determine with certainty the one contestant who paid the largest bribe. It is worth noting that, in the case of homogeneous risk neutral contestants, the perfectly discriminating framework induces complete rent dissipation (see for example Hillman and Samet, 1987). In a more recent article, Che and Gale (2000) analyze equilibrium properties under a continuum of success functions. As distortions diminish and approach zero, the success function converges to the standard first-price all-pay auction. The authors also show that for a large range of parameters, the main qualitative equilibrium features of the first-price all-pay auction persist.

[16]See Nitzan, 1994, for a comprehensive review of the rent-seeking literature up to the mid-1990's.

Research tournaments

As discussed above, Taylor (1995) was the first to model a research tournament as an all-pay auction. In contrast to a standard innovation-race - where the first competitor to achieve a pre-specified innovation receives a reward - the research tournament ends after a pre-specified time period, and the competitor with the best innovation receives the reward. Examples for innovation races are numerous, and have inspired historical achievements such as Charles Lindbergh's crossing of the Atlantic in 1927, the mathematical proof of Fermat's last theorem by Andrew Wiles circa 1995, and, in 2004, the flight of the first private manned spacecraft, SpaceShipOne, into space, exceeding an altitude of 100 km twice within two weeks.[17]

One obvious problem with innovation-races is the potentially large amount of time that can elapse before the specified innovation is realized. A second problem is the exact specification of the innovation itself. Since an innovation is by definition a new and hitherto unknown object or technique, it may be difficult to specify *ex ante*. Hence, a research tournament can be a better alternative when it comes to generating innovations for productive application within a restricted time frame. In addition, verifiability problems are mitigated, as no absolute standard has to be matched and the prize is awarded for the best innovation on a relative basis. Typical examples for research tournaments are government procurement of innovative military equipment systems and of other research intensive technologies, but privately sponsored R&D contests have also contributed to boost innovative efforts in multiple fields.[18]

[17]Lindbergh won the Orteig Prize, which was established in 1919 originally for a period of five years, but subsequently extended. Wiles won the Wolfskehl Prize, which was set in 1906, and the builders of SpaceShipOne won the Ansari X-prize, set in 1996.

[18]See Fullerton and others (2002) for some examples.

Since the nature of the innovative process is characterized by an element of luck in addition to the deliberate R&D effort, the success functions generally used in research tournament models are not perfectly discriminating. This feature, in the presence of possibly heterogeneous contestants, adds to the importance of appropriate contestant selection - in addition to the incentive function of the contest, which is shown to be generally stronger for a smaller number of contestants. Different authors approach the problem of efficient contestant selection from different angles. As discussed above, Taylor (1995) suggests entrance fees for homogeneous contestants, while Fullerton and McAfee (1999), who allow for contestant heterogeneity, advocate a pre-contest selection auction.

Fullerton and others (2002) introduce yet another variant. In contrast to the aforementioned contributions, their model does not rely on a pre-contest selection of the (homogeneous) participants, but instead lets them compete against each other in a post-contest auction. Instead of granting the contest winner a fixed prize, the tournament sponsor elicits bids from all participants combining their achieved innovation quality and a price. The selection of the research tournament winner is then based on the innovation quality/price combination that generates the largest surplus for the sponsor. The authors show that while the post-contest auction greatly reduces the information burden for calculating the optimal fixed prize before the tournament and/or the optimal entry fee, it can still generate efficient research level efforts. In addition, by introducing a second dimension (price) to the competition, the authors

allow for greater flexibility for the sponsor after the tournament. Even mediocre innovation results can become profitable alternatives when competitively priced.

The idea of the post-tournament auction in Fullerton and others (2002) is the closest a tournament literature model comes to the two stage selection models presented in the following chapters. In their model, as in the models that will be presented in chapters 4 and 5, the tournament serves to determine the quality of a product or project, while the following auction serves to determine its price. As the price depends on the (real or perceived) quality of the product/project, the auction's equilibrium strategies and outcome determine effort choices in the tournament. A contestant with only medium perceived project quality can still be successful, although it will generate lower expected bids in the post-tournament auction. Another common result is the fact that when a contestant expects to achieve higher prices in the post-tournament auction, he will exert more effort during the tournament.

In spite of the structural similarities with the Fullerton and others (2002) model, the models presented in chapters 4 and 5 differ in several important aspects. First, the models presented in those two chapters allow for potentially heterogeneous contestants. Second, effort levels in the tournament enhance the precision of quality assessment, but do not in themselves improve project quality. Third, the post-tournament auction does not entail tournament contestants bidding against each other, but has potential investors (the tournament sponsor and/or outside financiers) compete against each other over financing either the losing project (after a spin-off decision, chapter 4) or the winning project (after a business plan contest, chapter

5). In addition, while the main focus of Fullerton and others (2002) is on generating effort incentives, the main focus of chapters 4 and 5 is on selection properties.

Internal capital markets

While the tournament literature has traditionally analyzed internal labor markets, the application of tournament models to internal capital markets is a more recent phenomenon. The role of internal capital markets in allocating finance across divisions, in combination with their inherent informational advantage over external financing, has long been stressed in the analysis of large conglomerate firms.[19] Alchian (1969), Williamson (1975) and Donaldson (1984) all emphasize the importance of this "smarter money" effect, allowing the CEO to engage in active winner picking, by reallocating scarce resources from less efficient divisions to more efficient ones. In this respect, a CEO is superior to the external market essentially because of her total and unconditional control rights, which increase her incentives for information acquisition (via monitoring of each division). Stein (1997) was the first to formalize these ideas in a tournament framework, where divisions compete against each other for the allocation of limited internal funding.

In his 1997 paper, Stein analyzes efficient internal capital markets by combining the idea of winner picking with that of relative performance evaluation. As corporate headquarters is interested in distributing a fixed amount of resources between projects, absolute performance errors are irrelevant as long as they are correlated across projects. In line with the reasoning of Lazear and Rosen (1981), the existence

[19]For a comprehensive review of the literature on internal capital markets, see Stein (2003).

of a common error term enhances the relative efficiency of the internal capital market compared to external financing for each division. In the light of this result, Stein (1997) suggests that internal capital allocation has a stronger value-enhancing effect within a focused conglomerate than it has in a conglomerate with unrelated divisions, which are not subject to common shocks.

Given Stein's (1997) assumption that a division's outcome depends solely on the level of investment and on the observable state of nature, headquarters determines the winning division on the basis of a perfectly discriminating success function. There are no agency problems between headquarters and division managers, as the latter cannot influence either project profitability or information generation. Later works analyze different aspects of such agency problems and their adverse effects on the efficiency of internal capital markets. While sticking to the basic framework of winner picking, Stein (2002), De Motta (2003), and Brusco and Panunzi (2005) investigate how *ex post* efficient resource allocation influences *ex ante* incentives for information generation (Stein, 2002) and for output-enhancing effort (De Motta, 2003, Brusco and Panunzi, 2005). Others, such as Rajan, Servaes, and Zingales (2000), and Scharfstein and Stein (2000), focus on wasteful influence activities (rent-seeking) by division managers to increase resource allocation to their division.

Building on the literature on internal capital markets, Nanda and Narayanan (1999) analyze corporate divestiture decisions. Within the literature on divestitures and corporate spin-offs, those contributions that focus on the role of internal capital markets mostly point to their potential inefficiencies, citing them as a possible cause

for an empirically observed "conglomerate discount" (i.e. capital markets valuing a conglomerate firm at a discount compared to the sum of their divisions' value).[20] In contrast, Nanda and Narayanan (1999) analyze the workings of an efficient internal capital market in the presence of a financing constraint. With insufficient resources to finance both of two indivisible divisions, headquarters has to decide how to raise additional funds - either through a secondary offering or via a spin-off of one of the divisions. Given the information asymmetry between headquarters and the outside capital market, any decision will be interpreted by the market as a signal for the value of the two divisions and hence the conglomerate firm. In equilibrium, headquarters will always choose the divestiture (spin-off) of a division over a secondary offering, and it will always divest the worse-performing division.

The creation of superior information through an internal capital market is at the center of analysis both in Stein (2002) and Nanda and Narayanan (1999), although it is explicitly modelled only by the former. The idea that information creation in internal capital markets plays a central role in winner picking and in spin-off decisions will be further pursued in chapter 4. However, unlike Stein (2002), who places the burden of information generation on headquarters, the analysis in chapter 4 places it on the division managers.

Other applications

In addition to the presented fields of application, authors have used tournament theory to explain behavior in such diverse contexts as evolutionary processes and portfolio

[20] See for example Dittmar and Shivdasani (2003) and Schlingemann and others (1999).

management. The literature also covers a range of contestants' strategic variables, starting with the choice of effort levels, and including the production of information (as discussed above), as well as risk taking.

Risk taking is analyzed for example in the evolutionary model by Dekel and Scotchmer (1999), where the attitude towards risk in the male members of a population determines their chances to win a winner-take-all competition over the right to mate with the females in the group. As a result, the winner's attitude towards risk will be passed on to his offspring. Thus, while it is not a strategic variable *per se*, the attitude towards risk becomes an endogenous variable after a sufficient number of rounds.

Hvide (2002), in contrast, treats risk taking as a strategic variable by adding it to a standard Lazear-and-Rosen-(1981)-type setup with risk neutrality to analyze CEO compensation mechanisms. In allowing for interaction between the two strategic variables effort choice and risk taking, Hvide (2002) is in line with the work of Palomino and Prat (2003), who study delegated portfolio management. These authors, however, do not focus on tournaments, but more generally on optimal contracts - which, in their case, they find to be a simple bonus contract.

Other contributions which focus on the role of risk taking in tournaments are Cabral (2003), who analyzes risk taking decisions in an R&D context, and Hvide and Kristiansen (2003), who analyze the selection properties of a rank-order tournament with risk taking as the strategic variable. The latter contribution will be analyzed in greater detail in section 2.4 on selection properties of tournaments.

2.3.2 Empirical findings

Competitive sports constitute a natural testing ground for tournament theory, as champions are usually determined by rank-order tournaments of one form or another. It is therefore not surprising that a large part of the empirical literature on tournaments focuses on professional sports where, in addition, data is quite readily available.[21]

Sports tournaments have been used to test different hypotheses emanating from the theoretical literature as discussed above. Several authors find strong empirical support for the hypothesis that higher prizes induce higher effort levels. Sunde (2003) claims to find this result in professional tennis data from the Association of Tennis Professionals (ATP), by showing that the number of games played in a match rises with rising prize money, implying higher effort levels by both players (with his reasoning being that higher effort implies riskier play, producing more mistakes on each side, which leads to more games being lost/won by each side). In a similar vein, Becker and Huselid (1992) show that professional NASCAR drivers tend to drive faster - and riskier - when the prize money is higher (the use of individual racing times as the effort variable makes measurement of absolute effort levels easier than in tennis). Analyzing professional golf data, Ehrenberg and Bognanno (1990a, b) show that golfers on the European circuit tend to have lower scores when they compete for a higher prize.[22]

[21] See Prendergast (1999) for a brief overview of selected empirical literature.

[22] However, this result was later challenged by Orszag (1994), who showed that it did not hold for comparable data from a different season.

A second hypothesis analyzed by Sunde (2003) is that a higher degree of heterogeneity (if contestants' different skill levels are common knowledge *ex ante*) reduces effort levels. In line with earlier studies on political campaigning[23], he finds support for the hypothesis, based on observations of games won by higher-ranked and lower-ranked players, respectively.

Another hypothesis derived in the theoretical literature claims that in a single-prize tournament, the prize for winning should increase in the number of competitors. As quoted by Prendergast (1999), support for this hypothesis is provided by Main and others (1993), Eriksson (1999), and Conyon and Peck (2001), who study executive compensation - one of the few examples of empirical applications of tournaments outside the sports realm - and find that "...the return to becoming CEO is increasing in the number of individuals competing at the next rank below." (Prendergast, 1999, p. 35).

2.4 Selection properties of tournaments

The literature presented so far has mainly focused on the provision of incentives in a moral hazard setting. As pointed out earlier, however, there is a small but growing strand of literature that focuses on the selection properties of tournaments. The following subsection gives a brief overview of the work that has been done along these lines. Subsection 2.4.2 below presents a basic tournament model with selection properties in greater detail, and subsection 2.4.3 contributes some further considerations.

[23]Snyder (1989) and Levitt (1994) find that campaigning is more intense in constituencies where the outcome is expected to be closer (i.e. contestants are less heterogeneous).

2.4.1 Selection with and without private information

After the seminal paper by Lazear and Rosen (1981), which is pessimistic about the selection efficiency of tournaments - showing that workers of different ability types will not self-select into different groups -, O'Keeffe and others (1984) and Rosen (1986) analyze the topic in more depth and find that under certain conditions, tournaments can combine selection properties with efficient incentive provision.

Rosen (1986) analyzes a multiple stage elimination tournament in which prizes are rewarded at every stage.[24] At any stage, the loser receives a guaranteed prize and is then eliminated from the tournament, while the winner's reward is the option value of competing for higher prizes in subsequent stages. As discussed above, in the case of homogenous competitors, prizes must increase linearly in every stage of the game in order to maintain competitors' effort levels as the last stage approaches. The increase in prizes between the second-to-last and the last stage must be larger in order to dominate the end-of-game effect.

When considering heterogeneous contestants, Rosen (1986) allows for two informational setups - one with complete information (in which the competitors' abilities are common knowledge) and one with incomplete information in which their abilities are unknown even to themselves. Rosen (1986) does not treat the private information case in which only the contestants know their own types.

[24] See subsection 2.2.3, where the model is discussed in the context of multiple prizes.

In the case of unknown abilities, and with Bayesian updating of beliefs, equilibria at each stage are symmetric (as all contestants have the same priors about their own and their competitors' abilities, and share the same track record, as only the winners move on to the next stage). However, the degree of contestant heterogeneity affects effort levels at each stage, in addition to the continuation value. This is because an increase in uncertainty about one's own and the competitors' abilities reduces the marginal effect of effort on the probability to win at any given stage. As the elimination process proceeds, the degree of uncertainty is reduced, since weaker contestants are likely to drop out at earlier stages. The continuation value and hence the prize spread gain importance in determining effort levels, and the spread between the second-to-last and the last prize must again exceed earlier spreads.

Concerning selection efficiency, Rosen shows that while contestants make strategic effort decisions based on their own abilities and those of their contestants, there is a strong tendency for survival of the fittest. This is a general result for the case of heterogeneous contestants, given that the winning probability at each stage increases in a contestant's ability. However, as the success function is not perfectly discriminating, there remains an element of luck in the determination of the winner, and the selection process is not fully efficient. This result is not further discussed by Rosen (1986), as he is more interested in the provision of incentives than in selection efficiency.

O'Keeffe and others (1984) are also more interested in the provision of incentives than in selection efficiency. In their setup, as in Lazear and Rosen (1981), the authors analyze a two-contestant one-stage tournament (which they show can be generalized

to $n \geq 2$ contestants) where the spread between the winner's prize and the loser's prize is the strategic variable of the tournament sponsor. In addition, O'Keeffe and others assume that the sponsor can control the monitoring precision, i.e. the degree of luck in the (not perfectly discriminating) contest success function.[25] With homogeneous contestants, efficient effort levels can be achieved through different combinations of prize spreads and monitoring precision. An increase in the prize gap (which leads to increased individual effort levels) can be compensated by a decrease in monitoring precision (which leads to decreased individual effort levels) and vice versa.

When considering heterogeneous contestants, O'Keeffe and others (1984) allow for three types of informational setups. While considering both the case of known and of unknown abilities as in Rosen (1986), they also analyze the case of private information, where contestants' abilities are known only to themselves. As in Rosen (1986), the case of unknown abilities is similar to the case with homogeneous contestants, as no asymmetries arise. Hence, an appropriate combination of a large prize spread and precise monitoring (making the success function perfectly discriminating) can provide effort incentives as well as ensure that the most able contestant is selected as the winner. With known abilities, the authors show that handicapping of the abler contestants can generate efficient effort incentives even in asymmetric matches.

For the case of private information, the authors show that precise monitoring will in general lead to inefficient incentives, since the individual equilibrium effort function is then determined by the distribution of abilities, while the optimal effort function

[25] While O'Keeffe and others (1984) further distinguish between fair and unfair tournaments, the discussion here is limited to the case of fair tournaments.

is independent of the distribution. In particular, equilibrium effort levels of more able contestants will be below the social optimum. Hence, while precise monitoring can guarantee selection efficiency, it cannot guarantee efficient effort levels. Imprecise monitoring can attenuate the incentive problem by inducing abler workers to exert more effort, but the authors show that it induces efficient effort levels only for a very limited set of ability distributions. The trade-off between incentive provision and selection efficiency therefore remains.

Bhattacharya and Guasch (1988) also consider the interplay of incentive provision and selection properties in the presence of private information (across a continuum of different ability types). In contrast to O'Keeffe and others (1984), however, these authors are explicitly concerned with achieving selection efficiency. Building on Lazear and Rosen (1981), and as discussed above in subsection 2.2.2, they find that the introduction of additional performance standards can induce contestants of different ability types to self-select into different groups. Unlike Lazear and Rosen (1981), who analyze tournaments within cohorts, Bhattacharya and Guasch (1988) assume that tournaments are held across cohorts, by comparing each agent's performance on an ordinal scale with that of a random member of the lowest ability group. Based on this comparison, each contestant is then awarded a prize (or wage) depending on the comparison's outcome as well as on agents' types and type-dependent expected output and efficient effort levels. The authors show that for certain distributions of error terms in the contest success function, this cross-cohort comparison mechanism can indeed provide efficient effort incentives while also guaranteeing selection efficiency.

More recent work on on the selection properties of tournaments has been done by Fullerton and McAfee (1999), Clark and Riis (2001), and Hvide and Kristiansen (2003), among others. As discussed above, Fullerton and McAfee (1999) show that a contestant selection auction before the tournament can be an efficient mechanism to restrict the number of contestants while selecting only the best to compete. Clark and Riis (2001), like Bhattacharya and Guasch (1988), introduce additional standards to achieve selection efficiency in labor tournaments, but they do so without having to recur to multiple prize/wage contracts in order to induce self-selection. The work by Hvide and Kristiansen (2003) focuses on selection efficiency when the strategic variable is risk taking, and will be discussed in more detail in subsection 2.4.2 below. Another contribution by Münster (2006) analyzes the effects of sabotage among co-workers on selection efficiency.

Clark and Riis (2001), like the authors before them, show that the basic single-prize tournament cannot guarantee selection efficiency in the presence of heterogeneous contestants and private information. This is the case although Clark and Riis (2001) use a perfectly discriminating success function. In their setup, contestants are, however, better informed about their rivals' ability distribution than the tournament sponsor, causing abler contestants to exert less effort when they expect to meet a less able opponent. The result is an inherent "bias" in favor of the contestants with expected lower ability. To solve this problem, the authors introduce additional performance standards against which the winner's performance is evaluated (on a purely ordinal scale). The actual winner's prize is then made contingent on which performance standard has been passed. By using appropriate performance standards

and bonus payments, the tournament sponsor can indeed generate selection efficiency.

2.4.2 A tournament model with selection properties and private information

This section presents the tournament model by Hvide and Kristiansen (2003), which is related to the models in chapters 4 and 5 in that it analyzes the effects of both the contestants' type distribution and their strategic decisions on the tournament's selection property. Unlike most other tournaments discussed above, the strategic variable is not productive in the way that it unambiguously contributes to a better outcome. Instead, contestants can (costlessly) choose between safe and risky strategies, enhancing their prospective outcomes in case of a successful gamble, and reducing it in case of failure. Within this setup, the authors analyze the effect of changes in the external variables (number of contestants, and quality of contestant pool) on selection efficiency, which they define as the probability that a good type wins the tournament.

As one of their main results, the authors show that, counter to simple intuition, selection efficiency may indeed decrease with an increase in the quality of the contestant pool.[26] Since the authors assume only two discrete types of contestants, "good" and "bad", the pool's quality is determined by the share α of good types in the pool. For simplicity of analysis, Hvide and Kristiansen (2003) focus on the case of only two contestants, reinterpreting α as the probability of a contestant being good. This implies that a contestant is bad with counterprobability $1 - \alpha$. An increase in α

[26]The author's other main result, which is the ambiguous effect of an increase in the number of contestants on selection efficiency, will not be discussed here.

can thus be interpreted as an increase in the contestant pool's quality, causing two competing effects: The purely statistical effect raises the probability of a good type winning, while the strategic effect increases the incidence of risk-taking in equilibrium, possibly decreasing selection efficiency.

The model

Although Hvide and Kristiansen (2003) allow for more than two contestants, the analysis presented here will be limited to two - for ease of exposition and in line with the models that will be presented in chapters 4 and 5. The two contestants A and B are assumed to be risk neutral, and to compete for a single prize v, which is normalized to 1. They have private information about their type, which can be either good (g) or bad (b). Each contestant can choose between two different strategies, safe (s) or risky (r). Playing safe yields a sure output of q_2 for a bad contestant, and q_3 for the good. The risky strategy results in a high output of q_4 with probability γ for a good contestant and with probability δ for the bad. It results in a low output of q_1 with probability $1-\gamma$ for the good contestant and with probability $1-\delta$ for the bad. These probabilities are public knowledge, with $\gamma > \delta$, as are the possible output levels $q_4 > q_3 > q_2 > q_1$. Another piece of public knowledge is the probability α that a contestant is of the good type.

The contestant with the highest output wins the prize $v = 1$, while the other receives a payoff of zero. In case of a tie, each contestant wins with equal probability. Contestant i's expected payoff is thus equal to his winning probability $\Pi_i = \Pr\{q_i > q_j\} + 1/2 \Pr\{q_i = q_j\}$, with $i = g, b$, and it depends on his own type,

the expected type of his opponent j, as well as on both types' risk taking decisions. While only considering pure strategies explicitly, Hvide and Kristiansen (2003) point out that mixed strategies are not excluded, ensuring the usual continuity properties. Focusing on symmetric Bayesian Nash-equilibria (where every contestant of the same type plays the same strategy) in pure strategies, the set of possible equilibria is defined as $S := \{(s,s),(s,r),(r,s),(r,r)\}$, with the bad type's action written first. The probability of a good type winning is denoted by P, and depends on the equilibrium strategies and, therefore, on the contestant pool's quality.

Equilibrium bidding strategies

Analyzing the four possible equilibria in terms of their payoff structures for good and bad types and juxtaposing individual's deviation payoffs reveals that there exist parameter ranges (α, γ, δ) such that each strategy pair can indeed be an equilibrium. It also reveals that in the corresponding parameter range, each of these equilibria is unique.[27] In keeping with the above notation that the bad type's action is written first, and remembering that the winner's prize is normalized to 1, expected equilibrium

[27] The authors do not claim, however, existence of an equilibrium for all possible parameter ranges. There are, indeed, parameter ranges for which no stable equilibrium exists in pure strategies.

payoffs for a type g player are as follows:

$$\begin{aligned}
\Pi_g(r,r) &= \alpha \cdot [\gamma(1-\gamma) + \frac{1}{2}\gamma^2 + \frac{1}{2}(1-\gamma)^2] \\
&+ (1-\alpha) \cdot [\gamma(1-\delta) + \frac{1}{2}\gamma\delta + \frac{1}{2}(1-\gamma)(1-\delta)] \\
&= \frac{1}{2}[1 + (1-\alpha)(\gamma - \delta)] && \text{(2.4.1.a)} \\
\Pi_g(s,r) &= \alpha \cdot \frac{1}{2} + (1-\alpha) \cdot \gamma && \text{(2.4.1.b)} \\
\Pi_g(r,s) &= \alpha \cdot \frac{1}{2} + (1-\alpha) \cdot (1-\delta) = 1 - \frac{1}{2}\alpha - \delta + \alpha\delta && \text{(2.4.1.c)} \\
\Pi_g(s,s) &= \alpha \cdot \frac{1}{2} + (1-\alpha) = 1 - \frac{1}{2}\alpha && \text{(2.4.1.d)}
\end{aligned}$$

Expected equilibrium payoffs for a type b player are:

$$\begin{aligned}
\Pi_b(r,r) &= (1-\alpha) \cdot \frac{1}{2} + \alpha \cdot [\delta(1-\gamma) + \frac{1}{2}\gamma\delta + \frac{1}{2}(1-\gamma)(1-\delta)] \\
&= \frac{1}{2}[1 + \alpha\delta - \alpha\gamma] && \text{(2.4.1.e)} \\
\Pi_b(s,r) &= (1-\alpha) \cdot \frac{1}{2} + \alpha(1-\gamma) = \frac{1}{2}(1+\alpha) - \alpha\gamma && \text{(2.4.1.f)} \\
\Pi_b(r,s) &= (1-\alpha) \cdot \frac{1}{2} + \alpha\delta && \text{(2.4.1.g)} \\
\Pi_b(s,s) &= (1-\alpha) \cdot \frac{1}{2} && \text{(2.4.1.h)}
\end{aligned}$$

Expected payoffs from individual deviation from equilibrium are denoted by Π_i', with $i = g, b$, and are computed under the assumption that the deviant's opponent

sticks to his own type's equilibrium strategy (again, the bad type's equilibrium strategy is written first):[28]

$$\Pi'_g(r,r) = \alpha(1-\gamma) + (1-\alpha)(1-\delta) \quad (2.4.2.a)$$

$$\Pi'_g(s,r) = \alpha(1-\gamma) + (1-\alpha) \quad (2.4.2.b)$$

$$\Pi'_g(r,s) = \alpha\gamma + (1-\alpha)\cdot[\gamma(1-\delta) + \frac{1}{2}(1-\gamma)(1-\delta) + \frac{1}{2}\gamma\delta] \quad (2.4.2.c)$$

$$\Pi'_g(s,s) = \gamma \quad (2.4.2.d)$$

$$\Pi'_b(r,r) = \alpha(1-\gamma) + (1-\alpha)(1-\delta) \quad (2.4.2.e)$$

$$\Pi'_b(s,r) = \alpha\big[\delta(1-\gamma) + \frac{1}{2}\delta\gamma + \frac{1}{2}(1-\delta)(1-\gamma)\big] + (1-\alpha)\delta \quad (2.4.2.f)$$

$$\Pi'_b(r,s) = (1-\alpha)(1-\delta) \quad (2.4.2.g)$$

$$\Pi'_b(s,s) = \delta \quad (2.4.2.h)$$

Given both types' equilibrium and deviation payoffs, it is possible to determine the parameter ranges (regions) for which each equilibrium is stable. This exercise will also show uniqueness of equilibrium, as it will turn out that there is no overlap between parameter regions.

For the pure strategy symmetric Bayesian Nash equilibrium (r,r) to be a stable equilibrium, neither a type g player nor a type b player must have an incentive to deviate. This is the case iff

$$\Pi_g(r,r) > \Pi'_g(r,r) \quad \text{and} \quad \Pi_b(r,r) > \Pi'_b(r,r)\,. \quad (2.4.3)$$

[28] For example, in equation (2.4.2.a), both the bad type's and the good type's equilibrium strategies are risky (r). If a good type deviates, this implies that he will play safe (s). Since the opponent is assumed to stick to his respective equilibrium strategy, the deviant player will meet another good player who plays risky (r) with probability α, and win with probability $(1-\gamma)$, and he will meet a bad player who plays risky (r) with probability $(1-\alpha)$ and win with probability $(1-\delta)$.

Plugging in for (2.4.1.a, 2.4.2.a, 2.4.1.e, and 2.4.2.e) gives the conditions

$$\delta > \frac{1-\alpha\gamma}{2-\alpha} \quad \text{and} \quad \gamma > \frac{1-\delta+\alpha\delta}{1+\alpha}. \tag{2.4.4}$$

Analogously, conditions for the equilibrium (s,r) can be derived as $\gamma > 1 - \frac{1}{2}\alpha$ and $\delta < \frac{\alpha\gamma-1}{\alpha-2}$. For the equilibrium (r,s), one gets the conditions $\gamma < \frac{1-\delta+\alpha\delta}{1+\alpha}$ and $\delta > \frac{1}{2}(1-\alpha)$. Conditions for the equilibrium (s,s) are $\gamma < 1 - \frac{1}{2}\alpha$ and $\delta < \frac{1}{2}(1-\alpha)$. A close look at these conditions reveals that there is no overlap of parameter regions, implying that the analyzed equilibria are indeed unique.[29]

Variation in the quality of the contestant pool

Having derived the equilibrium structure, it is now possible to analyze how a change of quality of the contestant pool affects selection efficiency, i.e. the probability P that a good type wins the tournament. It will be shown that for high levels of quality α, an increase in the quality will always increase selection efficiency. The same holds true for low levels of quality, while for intermediate levels, it may be the case that an increase in α causes a decrease in selection efficiency.

Within each given equilibrium, the probability P of a good type winning always

[29]The conditions for the equilibrium (r,s) given here differ from those in Hvide and Kristiansen (2003), owing to a mix-up in their analysis between $\Pi_g(r,s)$ and $\Pi_g(r,r)$.

increases in α, as can be easily verified:[30]

$$P(r,r) = \alpha^2 + 2\alpha(1-\alpha)\underbrace{[\frac{1}{2}\gamma\delta + \frac{1}{2}(1-\gamma)(1-\delta) + \gamma(1-\delta)]}_{=[\cdot],\text{ with } 0\leq[\cdot]\leq 1} \quad (2.4.5.a)$$

$$\Leftrightarrow \frac{\partial P(r,r)}{\partial \alpha} = 2\alpha + 2(1-\alpha)[\cdot] - 2\alpha[\cdot] > 0$$

$$P(s,r) = \alpha^2 + 2\alpha(1-\alpha)(1-\gamma) \quad (2.4.5.b)$$

$$\Leftrightarrow \frac{\partial P(s,r)}{\partial \alpha} = 2\alpha\gamma + 2(1-\alpha)(1-\gamma) > 0$$

$$P(r,s) = \alpha^2 + 2\alpha(1-\alpha)\delta \quad (2.4.5.c)$$

$$\Leftrightarrow \frac{\partial P(r,s)}{\partial \alpha} = 2\alpha(1-\delta) + 2(1-\alpha)\delta > 0$$

$$P(s,s) = \alpha^2 + 2\alpha(1-\alpha) \quad (2.4.5.d)$$

$$\Leftrightarrow \frac{\partial P(s,s)}{\partial \alpha} = 2(1-\alpha) > 0$$

From the conditions above, it can be derived that (r,r) is a unique equilibrium if $\alpha > \frac{1-2\delta}{\gamma-\delta}$, i.e. if the contestant pool is of high quality. Once this equilibrium is reached, every further increase in α will always increase the probability of a good type winning. In the case of a low quality contestant pool, i.e. for $\alpha < min[1-2\delta, 2-2\gamma]$, the unique equilibrium is (s,s). Within this equilibrium, an increase in α also causes P to increase. Once α increases so far as to reach intermediate levels, however, a further increase will trigger a switch from (s,s) to a different equilibrium, either (s,r) or (r,s). Meanwhile, it follows from equations (2.4.5.a) to (2.4.5.d) that the probability of a good type winning is higher in equilibrium (s,s), than in any other equilibrium. If a small increase in α leads to a switch from (s,s) to a different equilibrium, it will therefore reduce that probability. This is the case for example for $\alpha = 1-2\delta < 2-2\gamma$,

[30] For example, in equation (2.4.5.a), the probability P of a good type winning is derived as the sum of the probability with which both contestants are good (α^2), and the probability that exactly one of the two contestants is good ($2\alpha(1-\alpha)$) multiplied with the probability that the good contestant wins, given that both play a risky strategy ($[\frac{1}{2}\gamma\delta + \frac{1}{2}(1-\gamma)(1-\delta) + \gamma(1-\delta)]$).

where a small increase in α would cause a switch to equilibrium (r, s) - which is unique for the parameter range $1 - 2\delta < \alpha < \frac{1-\delta-\gamma}{\gamma-\delta}$. This example demonstrates how for intermediate levels of contestant pool quality, a small increase in that quality can cause a decrease in selection efficiency. This is owing to the strategic effect which leads to a switch from an equilibrium where both types of contestants play it safe, to an equilibrium where either the good or the bad type plays a risky strategy.

2.4.3 Some further considerations on selection efficiency

The findings of Hvide and Kristiansen (2003) on the effects of an increase in the quality of the contestant pool on selection efficiency are noteworthy in that they allow to form hypotheses about empirical circumstances under which tournaments can be more or less efficient selection mechanisms. This is similar in spirit to the analysis in chapters 4 and 5, where the comparative statics effects of changes in external parameters on selection efficiency are assessed.[31]

The basic model setup in the subsequent chapters of this thesis, however, allows for a richer analysis than Hvide and Kristiansen (2003), in that it considers additional exogenous variables, such as the effectiveness of contestant's costly efforts. Also, it should be pointed out that both the type of strategic variable and the outcome of contestants' actions differ. While Hvide and Kristiansen (2003) model a costless strategic risk-taking decision which influences the potential output of each contestant, the models in chapters 4 and 5 focus solely on the selection aspect of the tournament.

[31] As outlined in the introduction, the concept of selection efficiency is defined more narrowly in those chapters than in Hvide and Kristiansen (2003), where an increase in the quality of the contestant pool in itself already contributes to higher selection efficiency.

This is achieved by assuming that contestants' efforts cannot influence the expected outcome of their divisions/projects, but are transformed solely into enhanced signal precision concerning their true quality. Also, by analyzing two setups with different, endogenized reward structures, it is shown that the authors' non-linearity result in the comparative statics analysis is not robust to the model specification. While a similar non-linearity indeed arises in chapter 5, this is not the case in chapter 4. In the latter, selection efficiency always rises with an increase in the quality of the contestant pool.

The reward structures in the subsequent chapters are endogenized through the introduction of a post-tournament auction, which determines among other things the payoffs of the contestants. As argued above in subsection 2.3.1, this approach is most similar in spirit to the post-tournament auction introduced in the research tournament model by Fullerton and others (2002). While the main differences between these authors' approach and the one chosen in chapters 4 and 5 are discussed above, it is worth highlighting again that the type of post-tournament auction is different, according to the problem at hand. To allow for the specific informational structure that arises both at the end of the tournament in the internal capital market and from the business plan contest, the post-tournament auction in the two chapters is modelled as a common value auction with asymmetric information. The specifics of this type of auction are discussed in detail in chapter 3.

Chapter 3

Common value auction with asymmetric information

In most of the tournament literature, the analysis ends when the winner is selected and awarded a - usually exogenous - prize. In most real life contexts, however, the true value of winning a tournament does not lie in some form of static, exogenous payment.

Although the ex ante fixed prizes in Lazear and Rosen's (1981) labour tournaments can be interpreted as a form of bonus payment contingent on relative performance criteria, a multi-stage game with several rounds of promotion as suggested by Rosen (1986) may be the more realistic scenario. In that setting, there is an endogenous element to expected benefits if the contestants have heterogeneous types: The more qualified a contestant is, the higher is his probability to win consecutive rounds of the tournament, and the higher is therefore his expected benefit.[1]

For other types of tournaments, the endogenous character of the winner's prize is even more obvious. Research tournaments are a case in point. As discussed above,

[1]The same is true for all tournaments among heterogeneous agents, when their type influences their winning probability.

Taylor (1995) showed that the principal can achieve optimal research efforts by choosing appropriate fixed prizes, limiting the number of competitors, and charging an entry fee to the tournament. Different authors have added to this strand of literature, by introducing pre-tournament selection auctions, such as Fullerton and McAfee (1999), Rob (1986), and Goel (1999). In the latter two models, only the winner of the selection auction subsequently enters into R&D activities. Fullerton and McAfee (1999) go one step further and combine the selection auction with a genuine research tournament between two contestants - after showing that two is the optimal number for a large class of tournaments. In their 1999 paper, however, Fullerton and McAfee still assume that the winner of the research tournament is awarded a fixed price. It is only Fullerton and others (2002) who endogenize the winner's prize by placing a first-price auction after a Taylor-style research tournament, thereby allowing competitors to compete for a contract on the basis of quality and price.

By placing a first-price auction after the tournament, Fullerton and others (2002) come closest to the models presented in chapters 4 and 5, where the prize for tournament winners is also endogenized by a subsequent first-price auction. Yet, the bidders and the information sets analyzed in those chapters differ substantially from their assumptions. While Fullerton and others (2002) assume that the contestants themselves enter a bidding process after the research tournament, this thesis focuses on tournaments with a subsequent competitive bidding between competing principals. While they use the framework of a standard first-price, independent-private value auction to derive optimal bids, this thesis uses a common value framework, assuming that the value of the auctioned project is independent of the financier's identity.

A general feature of common value auctions with symmetric bidders - who receive independently and identically distributed private signals about the value of the indivisible object that is to be auctioned off - is the phenomenon of the winner's curse. In a symmetric equilibrium, the bidder with the highest estimate of the object's value will make the highest bid and win the object. The fact that he has the highest estimate, however, implies that he is likely to have overestimated the object's true value, and will on average pay more than it is worth. Rational bidders avoid the winner's curse by shading their bids downwards, thus realizing zero expected profits.[2] Against this backdrop, each bidder has an incentive to acquire private information about the object's value, in order to create an informational rent and increase his expected payoff.

The analysis in chapters 4 and 5 allows for different information sets in the after-tournament auction. The tournament sponsor (and, potentially, an informed outside investor) holds information about the project's quality which is superior to the information of an outside investor. This turns the subsequent bidding process into a first-price common value auction with asymmetric information.

The first author to analyze the effect of information asymmetries on bidding strategies in common value auctions was Wilson (1967), based on a case study on the auctioning of oil drilling rights by Woods (1965). Hughart (1975) and Weverbergh (1979)

[2]The first to characterize the symmetric Nash equilibrium of the common value auction was Wilson (1977), and Milgrom and Weber (1982) later generalized the setup to include both private- and common-value elements.

formalized the analysis, but it was only Engelbrecht-Wiggans, Milgrom and Weber (1983), who fully characterized the first-price common value auction with asymmetric information - including equilibrium bids by informed and uninformed bidders and expected payoffs for all parties.

The results of Engelbrecht-Wiggans and others (1983) will be used in chapters 4 and 5 to analyze the bidding process that takes place after the tournament. Since the information that is revealed during the tournament influences the subsequent bidding process and its outcomes, the expected payoffs feed back into the effort decisions during the tournament.[3] The post-tournament auction is modelled along the lines of the Engelbrecht-Wiggans and others (1983) model, albeit in a slightly modified version, which is similar to the one presented by Hendricks and Porter (1988). The Engelbrecht-Wiggans and others (1983) setup, however, is more general, as it does not impose any restrictions on the joint distribution of the unknown common value of the object being auctioned off and the private information of the better-informed bidder. The following section presents an in-depth discussion of this setup, laying the groundwork for the analysis of the tournament-cum-auction selection problems that will be discussed in the later chapters.

[3]As mentioned before, effort decisions are assumed to influence the precision of information generation, rather than output itself.

3.1 The Engelbrecht-Wiggans and others (1983) model

3.1.1 The model

The authors analyze the sale of a single indivisible object of fixed, but unknown value. In a first-price sealed bid auction, the object is sold off to the highest bidder who pays the amount of his own bid, while the losers pay nothing. The bidders are asymmetrically informed. Only one bidder is assumed to have private information on the object's value, while all other bidders have only public information. The informed bidder thus has a double advantage: she has a more precise assessment of the object's true value, and she knows exactly what information her competitors rely on.

The joint distribution of the random pair $(\mathbf{Z}, \mathbf{X})$ is common knowledge for all $n+1$ bidders, where $\mathbf{Z}$ is the unknown true value of the object being sold, and $\mathbf{X}$ is the private information of the informed bidder. Only the informed bidder knows the realization of her private information variable $\mathbf{X}$. While $\mathbf{Z}$ takes on values in $\mathbb{R}_+$ and has finite expectation, the values of $\mathbf{X}$ are not restricted to real values, provided that they lie in any sort of measurable space.

Once the informed bidder has observed the realization of her information variable $\mathbf{X} = x$, she must choose a bid β to maximize her expected profit:

$$\Pr\{\beta \text{ wins}\} \cdot (E[\mathbf{Z}|\, \mathbf{X} = x] - \beta) \tag{3.1.1}$$

Where $\Pr\{\beta \text{ wins}\}$ is the winning probability, and $E[\mathbf{Z}|\, \mathbf{X} = x]$ is the expected value

of the object, given the realization x of the private information variable $\mathbf{X}$. Her private information affects her expected profit only through its impact on the expected value of the object, which is captured by the real-valued variable $\mathbf{H} = E[\mathbf{Z}|\,\mathbf{X}]$. Consequently, it can be assumed without loss of generality that the informed bidder only observes $\mathbf{H}$, rather than the possibly more complicated variable $\mathbf{X}$.

The optimal bid β is non-decreasing in the expected value of the object. This means that, if β is an optimal bid when the realization of $\mathbf{H}$ is h, then no lower bid β' can be optimal when the realization is any larger value $h' > h$. To see why this is the case, consider that for any $\beta' < \beta$, it must be true that $\Pr\{\beta \text{ wins}\} > \Pr\{\beta' \text{ wins}\}$. Otherwise, β would not be optimal when $\mathbf{H} = h$, since the informed bidder could bid less and win with a higher probability, which would raise her expected payoff. Therefore,

$$\begin{aligned} \Pr\{\beta \text{ wins}\}\overbrace{(h' - \beta)}^{>(h-\beta)} &- \Pr\{\beta' \text{ wins}\}\overbrace{(h' - \beta')}^{<(h-\beta')} \\ > \Pr\{\beta \text{ wins}\}(h - \beta) &- \Pr\{\beta' \text{ wins}\}(h - \beta') \geq 0 \end{aligned} \tag{3.1.2}$$

The term in the second line must be ≥ 0, since β is the optimal bid if $\mathbf{H} = h$, and therefore no other bid β' can generate a higher expected payoff. Also, the term in the first line is greater than the term in the second line, as can be easily seen by subtracting the latter from the former, while taking into account that $\Pr\{\beta \text{ wins}\} > \Pr\{\beta' \text{ wins}\}$. The term in the first line must therefore also be greater than zero. Hence, for $\mathbf{H} = h'$, any bid $\beta' < \beta$ leads to a lower expected payoff than the bid β and can therefore not be optimal. This concludes the proof that the optimal bid must be non-decreasing in the expected value of the project.

If no *ex ante* assumptions are to be made about the joint distribution of $(\mathbf{Z}, \mathbf{X})$, and, therefore, the distribution of $\mathbf{H}$, the informed bidder must be allowed to bid according to both pure and mixed strategies. This can be achieved by letting her observe a random variable $\mathbf{U}$, which is independent of $(\mathbf{Z}, \mathbf{X})$, and has an atomless distribution on $[0, 1]$. She can then use $\mathbf{U}$ whenever she needs to randomize her bids. A mixed strategy β then is a function $\mathbb{R}^2 \to \mathbb{R}_+$, where $\beta(h, u)$ is the amount bid when $\mathbf{H} = h$ and $\mathbf{U} = u$. Also, assume without loss of generality that $\beta(h, u)$ is non-decreasing in u for every fixed value of h.

3.1.2 Equilibrium bidding strategies

A mixed strategy for uninformed bidder i is a distribution G_i on $\mathbb{R}_+$, where $G_i(\beta)$ is the probability that he tenders a bid not exceeding β. Let $G(\beta) = G_1(\beta) \cdot ... \cdot G_n(\beta)$ denote the distribution of the maximum of the bids made by n uninformed bidders. Put differently, $G(\beta)$ represents the probability that none of their bids exceeds β. Proposition 1 then describes the equilibria of the bidding game:

Proposition 1: *The* $(n+1)$*-tuple* $(\beta, G_1, ..., G_n)$ *is an equilibrium point if and only if*

$$\beta(h, u) = E[\mathbf{H} | \, \mathbf{H} < h, \text{ or } \mathbf{H} = h \text{ and } \mathbf{U} < u], \tag{3.1.3}$$

and

$$G(\beta) = P(\beta(\mathbf{H}, \mathbf{U}) \leq \beta(h, u)). \tag{3.1.4}$$

For the extreme cases of $u = 0$ and $u = 1$, the informed bidder's bid becomes $\beta(h, 0) = E[\mathbf{H} | \, \mathbf{H} < h]$, and $\beta(h, 1) = E[\mathbf{H} | \, \mathbf{H} \leq h]$, respectively. If $\mathbf{H}$ has no atom at h, these two expressions are equal. When $\mathbf{H}$ is atomless, β hence describes a pure strategy. Note that the informed bidder does not bid an amount equal to the observed realization h, but only the expected value $E[\mathbf{H} | \, \mathbf{H} \leq h]$, with h as an upper

bound. This means that the informed bidder bids less than her expected value, thus effectively shading her bid downward, enabling her to extract an information rent and realize a positive expected payoff.

For the proof of Proposition 1, Engelbrecht-Wiggans and others (1983) use the notion of "distributional type", a concept first introduced by Milgrom and Weber (1985). By transforming the conditional expected value $\mathbf{H}$ of the object into the informed bidder's distributional type $\mathbf{T}$, each of the (at most countably many) atoms of $\mathbf{H}$ are "opened up" into an interval, which allows for the analysis to proceed as if $\mathbf{H}$ had originally been atomless.

The informed bidder's distributional type is defined as $\mathbf{T} = T(\mathbf{H}, \mathbf{U})$ and is uniformly distributed. The realization $t = T(h, u)$ denotes the probability of the event that $\{\mathbf{H} < h, \text{ or } \mathbf{H} = h \text{ and } \mathbf{U} < u\}$. Letting $H(t) = \inf\{h|\ \Pr\{\mathbf{H} \leq h\} > t\}$, leads to $\mathbf{H} = H(\mathbf{T})$ almost surely. This implies that the distributional type $\mathbf{T}$ carries all relevant information for the informed bidder to make an optimal bid.

Using the distributional type, the informed bidder's optimal bid β can be expressed in its distributional form. When she observes $\mathbf{T} = t$ (i.e. $(\mathbf{H}, \mathbf{U}) = (h, u)$), she bids:[4]

$$\beta(t) = E[H(\mathbf{T})|\, \mathbf{T} \leq t] = \int_0^t H(s)\frac{1}{t}\ ds = H(t) - \frac{1}{t}\int_0^t s\, dH(s)\,. \tag{3.1.5}$$

In this form, β is continuous and non-decreasing in t, with $\beta(0) = H(0)$ and $\beta(1) = E[\mathbf{H}]$.

[4]While the third term of equation (3.1.5) simply rewrites the expected value by taking into account that $\mathbf{T}$ is uniformly distributed on $[0, 1]$, the fourth term follows from integration by parts after factoring out $\frac{1}{t}$.

After this transformation, the first step of the proof of Proposition 1 consists in showing that the range of the informed bidder's bid β, $[H(0), E[\mathbf{H}]]$, is also the support of an uninformed bidders' bid distribution G_i. Suppose the informed bidder has learned that $\mathbf{T} = t$. A bid $b < H(0)$ by an uninformed bidder would surely lose and result in a payoff of zero. While any bid $b > E[\mathbf{H}]$ would win with certainty, a bid of precisely $b = E[\mathbf{H}]$ would be strictly preferred, since it would also win with certainty and at a lower cost. Any optimal bid b must therefore lie within the range of β.

The second step of the proof consists in showing that β is indeed an optimal strategy for the informed bidder. First, note that a bid $\beta(\tau)$ wins with probability τ: Consider that $\beta(\tau)$ wins if and only if it exceeds every non-informed bidders's bid b. Given their optimal bidding strategies, this happens with probability $G(\beta(\tau)) = \Pr\{\beta(\mathbf{T}) \leq \beta(\tau)\}$. Since $\mathbf{T}$ is uniformly distributed on $[0, 1]$, and β is continuous and non-decreasing in t, this probability is equal to τ.[5] A bid of $\beta(\tau)$ therefore yields an expected payoff of

$$\begin{aligned}[H(t) - \beta(\tau)] \cdot \tau &= H(t) \cdot \tau - \left[\frac{1}{\tau}\int_0^\tau H(s)ds\right] \cdot \tau = \int_0^\tau H(t)ds - \int_0^\tau H(s)ds \\ &= \int_0^\tau (H(t) - H(s))ds \ . \end{aligned} \tag{3.1.6}$$

The first derivative of the expected payoff with respect to τ is $H(t) - H(\tau)$, which is non-negative for $\tau < t$ and non-positive for $\tau > t$. Therefore, $\beta(t)$ must be an optimal bid when $\mathbf{T} = t$.

[5] This is true as long as $\mathbf{H}$ has no atom at $H(0)$. If this were the case, G would also have an atom at $H(0)$ and it would be optimal for the informed bidder to bid $\beta(\tau)$ if and only if $H(t) = H(0)$. In what follows, this case will be omitted.

In a third step, G is shown to indeed describe optimal strategies for the uninformed bidders. As argued above, the support of any G_i is identical to the range of β. Hence, for every possible bid b, there exists some t such that $\beta(t) = b$. An uninformed bidder's (say, bidder 1's) payoff of any bid $b = \beta(t)$ is therefore

$$E[\mathbf{Z} - \beta(t) | \, \mathbf{T} \leq t] \cdot t \cdot G_2(\overbrace{b}^{=\beta(t)}) \cdot ... \cdot G_n(\overbrace{b}^{=\beta(t)}) \, , \tag{3.1.7}$$

where t is the probability that $\mathbf{T} \leq t$ and therefore the informed bidder's unknown bid $\beta(\mathbf{T}) \leq \beta(t) = b$.

Using the fact that $H(\mathbf{T}) = \mathbf{H} = E[\mathbf{Z} | \, \mathbf{X}]$, one gets

$$E[\mathbf{Z} - \beta(t) | \, \mathbf{T} \leq t] = E[\mathbf{Z} | \, \mathbf{T} \leq t] - \beta(t) = E[H(\mathbf{T}) | \, \mathbf{T} \leq t] - \beta(t) = 0, \tag{3.1.8}$$

and, therefore, an uninformed bidder's expected payoff is always equal to zero, showing that there are only optimal bids in the support of the distribution G_1.

Note that neither the common value assumption, nor the bidders' risk-neutrality, nor the fact that all uninformed bidders have identical information sets, is essential for the outcome that any uninformed bidder makes zero expected profits in equilibrium. It is a general result in the literature that a bidder cannot profit from a sealed-bid auction if his information is completely known to a competitor, and if his risk-adjusted valuation for the object is no greater than that of this competitor.[6]

After having established that the bidding strategies described in Proposition 1 are indeed equilibrium strategies (the "if" part of the argument), what is left is to

[6]See Milgrom (1979), among others, for a detailed proof of this result.

prove is the "only if" part, i.e. the uniqueness of the equilibrium. In order to do this, Engelbrecht-Wiggans and others (1983) draw on a result by Griesmer and others (1967), who show that in equilibrium each bidder's distribution of bids is atomless, except possibly at its lower bound. While the following argument is based on atomless bids, it could be easily modified to cover this extreme case as well.

With atomless bid distributions, an uninformed bidder realizes a conditional expected payoff of $E[\mathbf{Z} - b|\, \beta(\mathbf{H}, \mathbf{U}) < b]$ if his bid b wins. Recall that β represents the non-decreasing strategy of the informed bidder. In addition, the range of β is convex, as can be shown with the arguments of Griesmer and others (1967). From equation (3.1.8), it follows that since the uninformed bidder can make any bid $b = \beta(h, u)$, one gets $\beta(h, u) = E[\mathbf{Z}|\, (\mathbf{H}, \mathbf{U}) < (h, u)]$ if β is atomless. Also, since $\beta(h, u)$ is optimal, it must maximize the informed bidder's expected payoff, solving $max_{\beta}(h - \beta) \cdot G(\beta)$. The first order condition for this problem is $0 = -G(\beta) + (h - \beta)G'(\beta)$, a first order linear differential equation in G that must hold for all (h, β)-pairs for which β is in the range $[H(0), E[\mathbf{H}]]$. Since no uninformed bidder will bid higher than $E[\mathbf{H}]$ in equilibrium, the resulting boundary condition for G is $G(E[\mathbf{H}]) = 1$. Given the convexity of β, only one function G can satisfy both the differential equation and the boundary condition. Therefore, the distribution G as characterized in Proposition 1 is the unique equilibrium distribution for the uniformed bidders' maximum bid.[7] This completes the proof of Proposition 1.

[7]Note that this proof requires that the equilibrium distribution G be differentiable everywhere. Dubra (2006) shows that this is not necessarily the case. However, he provides an alternative proof of uniqueness which holds even if G is not differentiable.

After establishing the equilibrium bidding strategies, Engelbrecht-Wiggans and others (1983) characterize the equilibrium further by establishing a continuity result (i.e. the equilibrium distribution of bids G varies continuously with the assumed distribution of $\mathbf{H}$), and by determining the expected payoffs of the seller and the informed bidder. Since these refinements do not contribute to the analysis of the tournament-cum-auction selection problems that are the focus of this thesis, they are omitted here. It is worth pointing out, however, that Engelbrecht-Wiggans and others (1983) assume that the seller does not have private information about the value of the object. The seller's expected payoff is therefore the difference between the object's expected value $E[\mathbf{Z}]$ and the informed bidder's expected payoff before learning the realization of $\mathbf{T}$.

3.2 Applications of the Engelbrecht-Wiggans and others (1983) model

The results of the Engelbrecht-Wiggans and others (1983) analysis have been used in a number of subsequent papers dealing with first-price auctions with asymmetric information. Two of them are discussed here, highlighting both the the empirical validity of the results and their application possibilities in different contexts.

3.2.1 Empirical findings

In an empirical application, Hendricks and Porter (1988) analyze federal auctions for oil and gas drainage leases. Their findings suggest that firms owning adjacent tracts to the tract that is auctioned off possess superior information and bid accordingly. While the application to drilling rights remains within the realm of the earlier work on asymmetric information auctions, it contributes to the literature by formulating a

testable hypothesis and showing that the model closely matches empirical findings.

The data that the authors analyze covers first-price sealed bid auctions of drainage and wildcat tracts off the coasts of Louisiana and Texas during the period 1954 to 1969. Drainage tracts are adjacent to tracts on which deposits have already been discovered, while a wildcat sale consist of tracts in areas that have not been drilled and where firms are permitted to acquire only seismic information. Since the drilling results on adjacent tracts are an indicator for expected returns from a new tract, neighboring firms have superior private information about the expected common value of the tract. In the case of a wildcat sale, it is reasonable to assume that all bidders have symmetric information.

In line with the Engelbrecht-Wiggans and others (1983) model, the findings show very different bidding behaviors in the two cases: The average value of drainage tracts in the analyzed sample turned out to be more than twice the average value of wildcat tracts. Nevertheless, bidding was less competitive, and, as a consequence, profit was about four times higher than on wildcat tracts. Also, while the government captured 77 per cent of the value of wildcat tracts, it was only able to capture 66 per cent of the value of drainage tracts. If it is true that neighboring firms in drainage tract auctions have private information, their bidding behavior matches the model's optimal bidding strategy of shading bids downwards, thus bidding less aggressively and allowing for information rents. The additional finding that non-neighboring firms in drainage tract auctions make zero average profits is also in line with the model's predictions.

Hendricks and Porter (1988) modify the theoretical setup in several ways. Firstly, they explicitly model a public information variable which influences the uninformed bidders' optimal strategy. The informed bidder's private information is a sufficient statistic for the public information, which means that the public information variable enters the informed bidder's optimization only indirectly, through the expectations of the uninformed bidder. Secondly, they introduce a reservation price R, and thirdly, they allow for differences in valuations of drainage tracts.[8]

While the explicit modeling of the public information variable and the introduction of valuation differences do not qualitatively affect the equilibrium strategies, the - realistic - assumption of a reserve price or minimum bid R leads to a truncation of bidding functions: neighboring firms will not bid for tracts with expected values $< R$, and non-neighboring firms will adapt their bidding strategies accordingly (recall that the range of the informed bidder's bid is identical with the support of the uninformed bidders' bid distribution).[9]

3.2.2 An application to finance

Rajan (1992) applies the model to a financing setup, in which an entrepreneur can choose between different financing options for a two-period project with stochastic

[8]Such differences in valuations are likely to result from negative external effects - neighboring tracts generally tap into the same pool of resources, such that the new tract's profitability is affected by the neighboring tract's production. While this reduces the tract's value for a non-neighboring firm, a neighboring firm can internalize these effects if it acquires the second tract. To capture this effect, Hendricks and Porter (1988) reduce a non-neighboring firm's valuation by a constant term.

[9]In two follow-up papers (Hendricks, Porter, and Spady, 1989, and Hendricks, Porter, and Wilson, 1994), the authors further explore the characteristics and equilibrium implications of the reservation price, claiming that - from the point of view of the bidders - it behaves like an unknown random variable, correlated with the bid of the informed bidder.

payoff. One of the options he analyzes is a short-term bank credit which has to be rolled over at the end of the first period. By assuming that a bank that gives a loan in the first period acquires insider information about the project's quality, Rajan can use the Engelbrecht-Wiggans and others (1983) results to derive a lock-in effect which enables the bank to make a more profitable continuation decision at the beginning of the second period.

In Rajan's (1992) setup, the entrepreneur has to make an effort decision at the beginning of the first period, which influences the probability of the project being of good or bad quality. For simplicity, the author assumes that only a good quality project generates a positive return at the end of the second period. Also, if the project is financed by a short term bank loan, this must be rolled over at the beginning of the second period. At this point, the bank that provided the first period loan has an informational advantage over any potential outside financier, owing to the information it has been able to acquire during the first financing round. This allows Rajan to model the competition between one insider and one outsider bank at the beginning of the second period as a first-price sealed-bid auction with asymmetric information as analyzed by Engelbrecht-Wiggans and others (1983).

In equilibrium, the informed bank only offers a new loan if the project is of good quality (since the payoff of a bad quality project is zero with certainty). The outside bank randomizes its bid, allowing it to win a financing contract for either a bad quality project or for a good quality project, both with positive probability. As a result, and in line with the Engelbrecht-Wiggans and others (1983) model, the outside bank

(the uninformed bidder) makes zero expected profits, while the informed bank uses its informational advantage to realize positive expected profits. Since the quality and hence the expected value of the project depends on the entrepreneur's effort decision, Rajan's setup also allows for a broader set of implications: The fact that one bank acquires inside information allows for early termination of a low quality project after the first period. However, it also reduces the entrepreneur's incentives to exert effort at the beginning of the first period, since the bank can appropriate part of the expected payoff.

Rajan (1992) thus adds to the literature on asymmetric information auctions in two ways: by widening the field of application to include financing decisions, and by placing the auction itself in a broader model setting where the expected payoffs from the auction influence the entrepreneur's effort decision and ultimately his financing decision.[10] Also, by assuming specific project payoffs and distribution functions, the author can derive specific optimal bids and expected payoffs in lieu of a general description of equilibrium properties.

Remaining in the realm of corporate finance, the following two chapters use a common value auction with asymmetric information for modeling the second step of analysis, after the object of the auction has been determined either through an internal capital market tournament or through a business plan contest. The formal arguments in in the respective sections draw on the analysis in Engelbrecht-Wiggans and others (1983), but also on the contributions made by Hendricks and Porter (1988)

[10]In addition to short-term bank lending, Rajan also analyzes a long-term arm's length credit and a mix of arm's length and bank credit.

and Rajan (1992).

Chapter 4

Internal capital markets, information revelation and corporate auctions

This chapter uses the tournament-cum-auction setup to analyze the process of information revelation in a competition between two divisions for scarce resources in an internal capital market and its consequence for a subsequent spin-off through a corporate auction.

Conglomerates' divestiture or spin-off decisions have been at the center of a growing literature since the mid-1990's. This academic interest was triggered by the wave of corporate focusing during that decade, which followed a trend towards diversification in the 1970's, and a number of spectacular hostile take-overs in the 1980's. Both theoretical and empirical papers have suggested different rationales for such restructuring, among them the notion that some divisions would be better managed outside of the conglomerate, a lack of sufficient financing within the conglomerate, as well as inefficiencies of internal capital markets.[1] While there there are divergent findings

[1]For a brief introduction to this branch of literature and its main arguments, see for example

about the efficiency or inefficiency of resource allocation in internal capital markets, it is a common result that liquidity constraints within the conglomerate are a main determinant of spin-off decisions. Specifically, and as discussed in subsection 2.3.1, Nanda and Narayanan (1999) find that corporate headquarters has an incentive to divest the worse performing of two divisions, in order to raise capital for the better performing division.

This chapter adds to the literature by combining the two elements of the liquidity constraint with an analysis of the efficiency of the internal capital market. By explicitly modeling both the workings of the internal capital market and the spin-off procedure (in a corporate auction), it analyzes the incentives for information generation and, hence, the degree of efficiency of the internal capital market, while also supporting Nanda and Narayanan's (1999) finding that the worse-performing division will be divested.

In this vein, the tournament-cum-auction setup allows to shed light on central issues in the corporate restructuring process and on its implications for selection efficiency, as well as for the bidding process during the corporate auction. The analysis specifically focuses on two main questions. In the first phase of this multi-stage game, it examines the incentives for information-generation during the internal capital market tournament.[2] In the second phase, it focuses on how the quality signals produced in the first phase influence the bidding process in the corporate auction, when outside

Dittmar and Shivdasani (2003), Nanda and Narayanan (1999), and Schlingemann and others (1999).

[2]As discussed in the introduction, "selection efficiency" is defined as the probability with which the best contestant is picked as the tournament winner. This probability rises with an increase in information generation.

financiers compete to acquire the division that has lost the tournament. In turn, the bidding strategies in the corporate auction affect the equilibrium level of information generation in the internal capital market, highlighting how the spin-off decision itself feeds back into the efficiency of the internal capital market.

Most of the literature on internal capital markets stresses the incentives of headquarters to gather information on the prospects of individual divisions (see for example Alchian, 1969, Gertner and others, 1994 and Stein, 1997). The present chapter extends this "smarter-money effect" (Stein, 2003, p.138) by arguing that the threat of a spin-off - which implies the loss of control benefits to the manager - acts as an incentive for the manager to engage in costly information-generation efforts. In addition, it examines how this information generation not only enhances corporate headquarters' efficiency in winner-picking, but also affects bidding strategies and expected payoffs in the post-tournament corporate auction. Similarly to Hvide and Kristiansen (2003), the analysis in this chapter also covers the effects of changes in exogenous parameters on information-generation and, hence, on selection efficiency.

The present analysis combines two strands of literature. On the one hand, it is closely related to studies on internal capital markets. This literature (see Stein, 2003 for a recent overview) concentrates mainly on a comparison of internal and external capital markets and on circumstances under which the former adds most value. In contrast, in this chapter the existence of an internal capital market is taken as given, and the focus is on the processes within this particular framework and the incentives for division managers to provide information-generating effort. In a spirit similar to

this analysis, Brusco and Panunzi (2005) take a closer look at management incentives in internal capital markets. They show that diversity of projects enhances managerial incentives. However, they only consider productive efforts, while the focus here is exclusively on information production. This is in line with the work of Stein (2002) and De Motta (2003). However, while their contributions focus on the incentives of headquarters to invest in information production, this chapter stresses the possibility of information-generation and transmission by divisional managers.

One key ingredient of the analysis is the existence of liquidity constraints of the conglomerate. Under the assumption that external financing for a division is more costly than internal financing, both divisions would ideally be financed by corporate headquarters. If the conglomerate is liquidity-constrained, however, this is impossible, and one division has to be divested in order to finance investment in the other. Following similar arguments as Lang and others (1995), and Nanda and Narayanan (1999), this assumption allows for an analysis of the divestiture decision and the subsequent corporate auction without entering a discussion of the advantages and disadvantages of internal capital markets.

On the other hand, the present analysis contributes to the literature on corporate auctions (see for example Boone and Mulherin, 2002, Bulow and others, 1999, and Hansen, 2001). It assesses the consequence of the asymmetries of information between an outside investor (another conglomerate) and a well-informed inside investor whose interests are modelled to be in line with those of the manager of the

division. In doing so, the analysis draws heavily on the theory of auctions, especially the work which is centered around the asymmetry of information of bidders in common-value auctions. In particular, it uses the Engelbrecht-Wiggans and others (1983) framework, as presented above. One extension is added, however, allowing for some information revelation to the otherwise uninformed outsider[3] - as argued below, any outside investor can deduce from the mere fact of a divestiture that the division which is auctioned off must be the loser of the internal capital market tournament.

The interaction of the internal tournament for scarce resources and the subsequent auction process is modelled as a multi-stage game. In a first step, the competition between two divisions is investigated, which can invest in unproductive signals that are used by headquarters to pick the winner. Among other things, it is shown that headquarters indeed has an incentive to pick the division with the higher expected payoff as the winner, rather than strategically distorting its selection and investment policy (see De Motta, 2003 on this conjecture). In a second step, the auction process is analyzed, between an uninformed outsider (for example another conglomerate) and an inside investor who receives her information from the management of the spin-off (for example a buy-out specialist). If the former wins the auction, the manager of the spin-off loses his control benefits entirely, while in the reverse case, management stays in command.

As argued below in subsection 4.1.3, the assumption of an asymmetric information framework for the corporate auction does justice to the incentive for information

[3]This is in line with Hendricks and Porter (1988), who introduce a public information variable, as discussed in subsection 3.2.1.

acquisition in such a common (but unknown) value auction. On the other hand, it is in the interest of the manager of the divested division to remain in control of his division in order to retain his control benefits. Hence, while lacking sufficient funds of his own, it is in his best interest to ally himself and share information with one of the outside investors in order to finance a management buy-out (MBO). To simplify matters, it is assumed that the buy-out specialist receives the same quality signal from the division manager as corporate headquarters before her.[4] In addition, she is assumed to become the residual claimant of the division in return for financing the MBO, thus leaving the manager only with his nonmonetary control benefits.

As in Engelbrecht-Wiggans and others (1983), the outside investor relies on a mixed bidding strategy and experiences zero expected profits. The inside investor, on the other hand, is able to realize positive expected profits, as her informational advantage allows her to realize information rents by shading her bid downwards, thus taking some of the rents away from selling headquarters.

The comparative statics exercises reveal that headquarters' expected payoff increases with both an increase in the expected return of a high-quality division and with an increase in the *a priori* probability that a division is indeed of high quality, which are both related to a higher overall valuation of the conglomerate. In this context, it is worth noting that an increase in the *a priori* probability that a division is of high quality also reduces information asymmetry between the two outside investors. This is owing to the fact that it induces the uninformed investor to raise his

[4]If this quality signal exists in the form of market studies and revenue projections, it could easily be shared with an outside investor, once it has been produced for the internal tournament.

bid for the divested division, as he attributes a higher probability to the event that it will be of high quality. Thus, his beliefs are more aligned with those of an informed outside investor who has received a good quality signal, and knows for sure that the divested division is of high quality.[5] Accordingly, the informed investor will also have to raise her equilibrium bid, which in turn lowers her informational rent. The higher equilibrium bids from both outside investors in turn increase headquarters' expected payoff.

On the other hand, headquarters' expected payoff falls with an increase in the effectiveness of the division managers' signaling efforts. This is owing to the related increase in information asymmetry between the uninformed and the informed outside investor, as the uninformed investor is less likely to assume that a high-quality project is mistakenly divested when signal effectiveness is higher. He will therefore attribute a lower expected value to the divested division, and reduce his equilibrium bid. In contrast, an informed investor who has received a good quality signal knows for sure that the divested division is of high quality, but she can lower her bid in response the to outsider's lower bid, thus extracting a higher information rent. The lower equilibrium bids from both outside investors in turn reduce headquarters' expected payoff. For low levels of signaling effectiveness, however, the result of an increase may actually lead to an increase in headquarters' expected payoff, as it reduces the likelihood of the inside investor winning the corporate auction. Since the inside investor's equilibrium bid is generally lower than the outside investor's expected bid,

[5]Note that the informed outside investor only realizes positive expected payoffs when she receives a high-quality signal. In contrast, as will be shown in subsection 4.2.2, she receives zero expected profits in the case of a low quality signal.

the reduction in that likelihood increases headquarters' expected payoff.

In addition to conclusions about different agents' expected profits and information asymmetries, the model allows for a direct assessment of selection efficiency under different market conditions. Managers' incentives for information generation and, hence, overall selection efficiency in equilibrium is higher (a) in more highly valued conglomerates, with a higher *a priori* probability that a division is of high quality; (b) in more traditional and well-understood markets, where the effectiveness of signal production is higher; and (c) when managers' nonmonetary control benefits are higher.

The next section outlines the model. The following two sections analyze equilibrium bids and payoffs, and the effects of changes in exogenous parameters. The last section provides a brief summary and discussion of the main results.

4.1 The model

Consider a four-stage bidding game with no discounting. The first two stages of the game represent the internal tournament, which is used to rank two divisions within a corporate firm according to their expected profitability. At the first stage, the division managers have to decide how much costly effort to exert in order to generate a meaningful quality signal. At the second stage, quality signals are realized and the firm suffers a negative liquidity shock. Such a shock makes it impossible for headquarters to finance both divisions and, accordingly, one of them has to be divested. Headquarters has to decide which division to spin off in order to maximize

the conglomerate's expected payoff.[6] This decision can only be based on the quality signals of the two divisions which were generated in the internal tournament. At the third stage, potential outside investors can bid for the division in a corporate auction. However, before the (sealed) bids are made, one of the investors can become an inside investor, acquiring the same information as headquarters. This inside financier could be thought of as a buy-out specialist, assisting the current manager in taking over his division. After the bids are realized, the division is sold to the highest bidder and payoffs are realized.

4.1.1 Agents

There are three types of risk neutral agents: (a) division managers, (b) corporate headquarters, and (c) outside investors.

Two division managers have to report to headquarters about the expected future profitability of their divisions. They can exert costly effort to raise the precision of their reports. It is based on these quality reports that the two divisions get ranked according to perceived quality. Division managers derive personal utility (control benefit) v from being in charge of the division. When a division is sold to an uninformed outside investor, management gets replaced, reducing the division manager's utility to zero. When it is sold to an informed outside investor, in the form of a management buy-out, the manager retains his control benefit v.

Corporate headquarters ranks the two divisions in the conglomerate according to

[6] In this setup, it is assumed that the liquidity shock occurs with certainty. It could also be modelled to occur with a probability less than one, but that case is not treated here as it complicates the analysis without generating additional insights.

their perceived quality. If there were enough slack liquidity within the conglomerate, both divisions would be worth financing, as they are both assumed to have a positive expected net present value. However, in the presence of a negative liquidity shock, only one division can be financed, and headquarters has to decide which one to divest. Headquarters' incentives are assumed to be in line with the company's shareholders', such that the former maximizes expected total payoffs. In equilibrium, this leads to the division with the lower quality signal being divested. If the quality signals happen to be identical, there is a 50:50 chance that either one of the divisions will be divested.

Two outside investors have the liquidity that is needed to finance the division that is to be divested. Given that even the losing division has a positive expected net present value, they are willing to bid a positive amount for it. Whoever bids the highest amount in a first-price sealed bid auction, pays their bid to headquarters and becomes the division's new residual claimant. As stated above, there exists an incentive for one of the outside investors to acquire inside information.

4.1.2 Technology and information

The conglomerate consists of two divisions with an expected net present value larger than or equal to zero. The division managers A and B have private information about the expected future returns μ_i, with $i = A, B$, of their divisions, which can be either high ($\mu_i = \mu_g$) or low ($\mu_i = 0$). In order to get the necessary resources from headquarters to run the division, they have to signal their respective divisions' quality to headquarters. The signals S_i will, however, be distorted: For a low-quality division, which has $\mu_i = 0$, the signal will be $S_i = 0$ with probability one. However, a high-quality division generates a high signal $S_i = \mu_g$ only with probability γ, with

$0 \leq \gamma \leq 1$. With probability $1 - \gamma$, a high-quality division generates a low signal $S_i = 0$, and could be mistaken for a low-quality one. Against this backdrop, the manager of a high-quality division can exert costly effort in order to increase the signal's precision (i.e. the probability that it reflects the true quality level). This effort x_i, $i = A, B$, can take the form of conducting a study on technical feasibility, investing into test-runs for a prototype, or preparing a market analysis, or other ways to convert information about costs and earning prospects into more verifiable hard facts. By exerting effort x_i, the manager of a high-quality division can raise the probability of producing a quality signal $S_i = \mu_g$ to $\gamma(1+x_i)$. With the counterprobability $1 - \gamma(1 + x_i)$, the signal still takes on the value $S_i = 0$. As can be easily seen, x_i must be restricted to $0 \leq x_i \leq \frac{1-\gamma}{\gamma}$. While it is natural to assume the effort level to be positive, it has to be less than $\frac{1-\gamma}{\gamma}$ for purely technical reasons, so as not to allow for probabilities greater than one.

The following information is public knowledge: The expected values μ_i of the two divisions are independently and identically distributed and take on the strictly positive value μ_g with probability α and zero with probability $1 - \alpha$. The probability functions for the signals $S_i = 0$ and $S_i = \mu_g$ are also common knowledge, while the realized effort level, x_i, is private knowledge of each division manager.

After observing the signals S_A and S_B, headquarters can rank the divisions. Due to a liquidity shock, it becomes impossible for headquarters to finance both divisions, and one of them has to be divested in order to finance the other. The outside investors do not know the quality signals S_i, but they can draw conclusions about a division's

quality ranking from the fact that it is divested. Given the two possible realizations of the quality signal, there are three possibilities: Both divisions have a low quality signal, $S_A = S_B = 0$, both divisions have a high quality signal, $S_A = S_B = \mu_g$, or one can have a high signal while the other has a low signal, $S_A = \mu_g \wedge S_B = 0$ or $S_A = 0 \wedge S_B = \mu_g$. Only in the latter case, there is a meaningful spin-off decision to be made: Headquarters could divest the division which did worse in the internal ranking and keep the division which performed better, or vice versa. In equilibrium, the division with the lower quality signal will be divested, which the outside investors will anticipate and condition their bids upon.

When both the informed and the uninformed outside investors have made their bids β and b, respectively, the division is sold to the highest bidder, who then becomes the residual claimant of the division.

Figure 4.1 summarizes the four stages of the model.

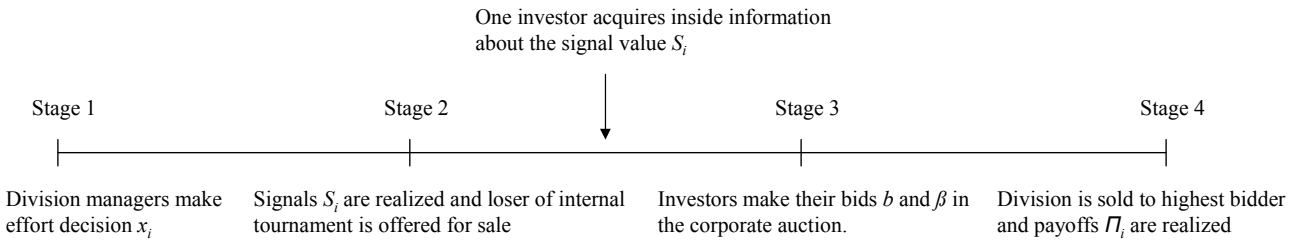

Figure 4.1: Internal capital market: stages

4.1.3 Common value auction and information acquisition

Since the corporate auction for the divested division takes the form of an unknown common value auction, the outside investors are prone to the winner's curse, meaning that the highest bidder is likely to overestimate the true value of the object which is auctioned off. Rational bidders will therefore shade their bids downwards. Also, any private information that a bidder can generate in addition to the public information on the value of the object gives her an advantage over the other bidders. As discussed before, the resulting asymmetric common value auction results in a positive expected payoff for the better informed bidder, and in a zero expected payoff for the less informed bidder.

But not only do the bidders have an incentive to obtain private information on the object's value - in this case, the quality of the division - according to the setup of the model, the division manager also has an incentive to ally himself with one of the bidders: By allying himself with one of the outside investors and providing her with inside information, he can establish a long-term relationship and make sure he remains the manager of the stand alone firm his division will eventually be turned into (as in a management buy-out). The advantage of this is immediately clear for a division manager with a high quality signal. However, a division manager with a low-quality signal can still benefit from an alliance with an outside investor. Although the investor will bid less for the division, there is still a chance that she could win the auction, which in turn would allow the manager to stay in charge. Given these incentives to acquire and to reveal private information, it is assumed that the division manager can

and does transmit the quality signal S_i of his division to one of the outside investors.[7]

4.1.4 Equilibrium analysis: why the division with the lower quality signal will be divested

As a starting point for all further computations and economical intuition, it is necessary to establish that when confronted with a liquidity shock, headquarters will always spin off the division which loses the tournament for internal financing. To get an intuition of why this is the case, consider the following argument:

Assume on the contrary that it is more profitable for headquarters to spin off the winner of the contest. Possible outside investors will foresee this behavior and bid accordingly. One inside investor acquires the same knowledge as headquarters, and knows the quality signal of the division to be spun off. The winner of the contest can either have a high quality signal or a low quality signal. The latter can happen if both divisions generate a low signal - the winner of the tournament is then determined by a 50:50 lottery.

If the inside investor observes a high quality signal, she will not bid up to the expected net value of the division but will instead shade her bid downwards such that

[7]While there are a plethora of potential contract forms that could ensure efficient information revelation, it can simply be assumed that one investor makes a lump-sum investment to learn the realization of the signal S_i, which would in turn suffice to prevent other outside investors from trying to obtain the same costly information. This is due to the fact that such an investment is only worthwhile if it results in an informational advantage, which would be nullified as soon as more than one outside investor acquired the same information.

she will earn positive expected returns from bidding.[8] The uninformed outside investor will randomize his bids between zero and the expected value of a division with a good signal, placing relatively more probability mass on the higher bids - assuming that he is bidding for a winning division. Given these two bidding strategies, a division with a high quality signal will on average be sold off for less than its expected value.

On the other hand, if the inside investor observes a low quality signal, she will bid up to the expected net value of the division. There is no bid shading and she will earn zero expected returns from bidding. The outside investor will again randomize between zero and the expected value of a division with a good signal. Given that the only information he has is that he is bidding for a winning division, he will again place relatively more probability mass on the higher bids. This bidding behavior causes a division with a low quality signal to be on average sold off for more than its expected value.

If a division with a good quality signal is on average sold for less than its expected value, and a division with a bad quality signal is sold at a price above its expected value, it will be in headquarters best interest to spin off the division with a low quality signal and to realize the high-quality division within the conglomerate (remember that a high quality signal implies that the division is of high quality with certainty). This is, however, inconsistent with the above assumption that it is more profitable for headquarters to spin off the winning division. If both divisions have a high quality

[8]This typical bidding behavior was discussed above in section 3.1 on the Engelbrecht-Wiggans and others (1983) model, and will be derived for this specific model setup in subsection 4.2.1.

signal or a low quality signal, then the winner is determined by a lottery and it does not matter for headquarters which division is divested. However, if one division has a high quality signal and one has a low quality signal, then the former will win the tournament and it will be more profitable for headquarters to keep it and to spin off the losing division with the low quality signal.

If one assumes that headquarters will always spin off the losing division, no such inconsistency arises: The inside investor will bid as before, given that she knows the quality signal of the division she bids for. The outside investor will still randomize between zero and the expected value of a division with a good signal, since it is still possible that the spun-off division has a high quality signal.[9] However, knowing that he is bidding for a losing division, he will put relatively less probability mass on the higher bids. This increases the amount by which a division with a high quality signal will be undervalued, and reduces the amount by which a division with a low quality signal will be overvalued. Qualitatively, though, the net effect remains the same, and it will always be more profitable for headquarters to spin off the division with the low quality signal. Whenever the two divisions generate different signals, this will be the losing division. If they both generate the same signals (high or low quality), then headquarters is indifferent as to which division will be spun off.

[9]This happens when both divisions have high quality signals and the winner is determined by a 50:50 lottery.

4.2 Analysis

In order to determine the division managers' expected payoffs, and to analyze the effects of a liquidity shock on their incentives to generate information, the multiple stage game is solved by backward induction. In a first step, the bidding behavior of the two outside investors will be analyzed, after a liquidity shock has occurred and it has been announced that a division will be divested. Following the above argument, it is clear that the division to be sold off is the one with the weaker quality signal. In a second step, the effort decisions of the division managers will be examined, during the tournament in which they have to provide headquarters with information on the quality of their divisions. In a last step, the results of changes in exogenous parameters on the effort decisions will be discussed, as well as those on bids and expected payoffs.

4.2.1 Equilibrium strategies in the corporate auction

During the corporate auction, two outside investors find themselves in a common but unknown value auction for one of the conglomerate's divisions. Given the discussed incentives for information acquisition and revelation, one of the two investors will have acquired inside information about the division's quality signal, while the other investor only has public information on the *a priori* probabilities for high- and low-quality divisions, on the probability of signal distortion and on the fact that the division with the lower quality signal is the one which will be auctioned off. This asymmetric bidding contest is similar to the one analyzed in Engelbrecht-Wiggans and others (1983), and the formal arguments presented in this subsection follow their line of arguments. In addition, some use is made of the contributions of Hendricks and Porter (1988) and Rajan (1992) to this type of model.

The equilibrium bidding strategies by the informed and the uninformed investors are determined by the maximization of their respective expected payoffs, given the strategy of the other party. The informed investor maximizes

$$\Pi_I^e = \Pr\{\text{ informed investor wins }\} \cdot [\, E[\mu|\, S_l] - \beta(S_l)\,]\ . \tag{4.2.1}$$

Equation (4.2.1) combines the winning probability with the expected net payoff from becoming residual claimant of the division. The latter is the difference between the expected net present value of the division, given its observed quality signal S_l[10], less the bid β, which also depends on the realization of the signal. The fact that the division generated the lower of the two quality signals is only of indirect interest to the informed investor, in that it influences the beliefs of the uniformed investor about the division's value. For the informed investor, the exact realization of S_l is a more precise information, thus constituting a sufficient statistic for the fact that a division has the lower signal.

Given that the division's quality signal can only take on two different values (0 or μ_g), it is possible to specify the informed investor's expected profit for these two cases:

For $S_l = 0$*:*

$$\Pi_I^e(S_l = 0) = \Pr\{\text{ informed investor wins}|\, S_l = 0\,\}\cdot[\, E[\mu|\, S_l = 0] - \beta(S_l = 0)\,] \tag{4.2.2.a}$$

For $S_l = \mu_g$*:*

[10] S_l denotes the quality signal of the division that has lost the internal tournament and is therefore divested. This signal could take on both a high or a low value, as even a division with a high quality signal could be divested in case that the other division also generates a high quality signal.

$$\Pi_I^e(S_l = \mu_g) = \Pr\{\text{ informed investor wins}|\ S_l = \mu_g\ \} \cdot [\ E[\mu|\ S_l = \mu_g] - \beta(S_l = \mu_g)\ ] \tag{4.2.2.b}$$

Simultaneously, the uninformed investor maximizes

$$\Pi_U^e = \Pr\{\text{ uninf. investor wins}\} \cdot [\ E[\mu|\ \text{lower S; uninf. investor wins}\ ] - b\ ]\ . \tag{4.2.3}$$

Like equation (4.2.1), equation (4.2.3) combines the winning probability with the expected net payoff from becoming residual claimant of the division. However, in this case, the expected net payoff is conditioned on two pieces of information: In contrast to the informed investor, the uninformed investor can only observe the fact that the division in question is being divested. From this fact, he can deduce that the quality signal must be lower or at best equal to the other division's signal (which in equation (4.2.3) is indicated by the expression "lower S"). This assumption about the quality signal influences the beliefs of the uninformed investor concerning the value of the division. The second piece of information which the uninformed investor can draw on is the fact that in order for him to win, his bid must be higher than the informed investor's. Since the bid of the latter depends on the observed quality signal, this also conveys information about the division's expected value. The resulting equilibrium is characterized in

Proposition 2: *(i) No equilibrium exists in pure strategies. (ii) There is an equilibrium in mixed strategies where: (a) The informed investor bids according to a pure strategy and conditions her bid on the observed Signal S_l. She makes positive expected profits. (b) The uninformed investor bids according to a mixed strategy independent of the state. He makes zero expected profits.*

Proof: See appendix.

Intuitively, it is clear that the uninformed outside investor cannot bid according to a pure strategy. Not knowing the true (common) value of the auctioned division, he is, however, aware that the other investor has superior information. Therefore, if he were to bid according to a pure strategy, and bid an amount less than the expected value of a high-quality division, he would have a low probability of winning a high-quality division - since the better-informed investor would always outbid him if she observed a good quality signal. He would win a high-quality division with positive probability only when the division generates a low quality signal, in which case the informed investor bids according to the expected value of the division, conditional on observing a low quality signal. In order to ever win a high-quality division, the uninformed investor would therefore always have to bid higher than this amount, thereby generating sure losses in the case the division is of low quality, which would more than offset his expected payoff from getting lucky and winning a high-quality division. Similarly, were he to bid a fixed amount equal to or higher than the expected value of a high-quality division, he would always realize zero or negative payoffs.[11]

The informed investor, on the other hand, can bid according to a pure strategy, conditioning her bid on the observed quality signal of the division. Knowing the randomizing strategy of the uninformed investor, she can shade her bid downwards when she observes a good quality signal, thereby generating a positive information rent, and when she observes a low quality signal, she bids according to the expected

[11]For a more detailed discussion of the uninformed investor's bidding strategy, see the appendix.

value of the division, conditional on observing this signal.

The equilibrium bidding strategies of both outside investors are described in

Corollary 1: *(a) The equilibrium bidding strategy of the informed investor is*

$$\beta(S_l = 0) = \mu_g \cdot \frac{\xi}{\hat{t}} = E[\mu | S_l = 0] \tag{4.2.4}$$

if she observes $S_l = 0$*, and*

$$\beta(S_l = \mu_g) = \mu_g \cdot (1 + \xi - \hat{t}) = E[\mu | S_l = \mu_g] \cdot (1 + \xi - \hat{t}) \tag{4.2.5}$$

if she observes $S_l = \mu_g$.[12]

(b) The equilibrium bidding strategy of the outside investor is a distribution G *on* $\Re_+$*, where* $G(\beta)$ *is the probability that he tenders a bid not exceeding* β*, and* u *is the realization of a random variable* U *which is independent of* (μ, S) *and has an atomless distribution on* $[0, 1]$*:*

$$G(\beta(S_l = 0)) = u \cdot \hat{t} \tag{4.2.6}$$

and

$$G(\beta(S_l = \mu_g)) = u\,. \tag{4.2.7}$$

Proof: See appendix.

[12] With $\xi = \Pr\{S_l = 0 \cap \mu_l = \mu_g\} = [\alpha - \frac{1}{2}(2 + x_A + x_B)(1 - \alpha)\alpha\gamma - (1 + x_A)(1 + x_B)\alpha^2\gamma^2]$ and $\hat{t} = \Pr\{S_l = 0\} = 1 - \alpha^2\gamma^2(1 + x_A)\,(1 + x_B)$. From this definition, it follows that $\hat{t} \geq \xi$ and therefore $0 \leq \frac{\xi}{\hat{t}} \leq 1$ and $0 \leq (1 + \xi - \hat{t}) \leq 1$.

The winning probability of the informed investor, conditional on the observation of signal S_l, follows directly from Corollary 1 (b). If the informed investor observes $S_l = 0$, her bid is higher than the uninformed investor's with probability $G(\beta(S_l = 0)) = u \cdot \hat{t}$. If she observes $S_l = \mu_g$, her bid is higher with probability $G(\beta(S_l = \mu_g)) = u$. In both cases, the winning probability of the uninformed investor is the respective counterprobability.

4.2.2 Equilibrium expected payoffs after the corporate auction

Having determined the equilibrium bidding strategies of the informed and the uninformed investor as well as their respective winning probabilities, it is now possible to determine the expected payoffs of all agents. In a first step, the expected payoffs of the division managers are assessed, which in turn influence their respective effort levels in the internal capital market tournament.

To simplify matters, only the expected payoff of division manager A is examined. The argument for manager B follows the same pattern and is therefore omitted. More specifically, the analysis is confined to the case where manager A knows that the expected net present value of his division is $\mu_A = \mu_g$. By construction, and as laid out in subsection 4.1.2, it is only in this case that the manager can influence the precision of the signal by exerting costly effort.

As discussed, division manager A derives a privat nonmonetary benefit v from managing the division. As long as he stays in control, it does not matter to him whether the division remains inside the conglomerate or whether it is divested via a

management buy-out to become a stand-alone firm.[13] Only if the division is acquired by an outside investor who replaces the manager, the latter loses his control benefits and his utility is reduced to zero. Since he has to make his effort-decision in stage one, he maximizes his expected payoff before his quality signal is realized:

$$\begin{aligned}\Pi_A^e(\mu_A = \mu_g) &= Pr\{\text{win}\} \cdot v + Pr\{\text{lose}\} \cdot Pr\{\text{MBO}\} \cdot v \\ &\quad + Pr\{\text{lose}\} \cdot \big(1 - Pr\{\text{MBO}\}\big) \cdot 0 - c(x_A)\,. \end{aligned} \tag{4.2.8}$$

Entrepreneur A's effort enters his expected payoff in two ways: On the one hand, it enhances the probability that the quality signal is high, on the other hand, it causes him disutility, which is modelled by the cost function $c(x_A)$. This cost function $c(x_i)$ is assumed to be symmetric for both entrepreneurs, with $c'(x_i) > 0$ and $c''(x_i) \geq 0$. $Pr\{win\}$ is the probability that the division wins the internal ranking competition. It loses the competition with probability $Pr\{lose\}$, which is equal to $1 - Pr\{win\}$. $Pr\{\text{MBO}\}$ is the probability that the informed outside investor wins the corporate auction and the manager remains in charge of the division.

Regrouping of equation (4.2.8) results in

$$\Pi_A^e(\mu_A = \mu_g) = v\left[Pr\{\text{win}\} + \big(1 - Pr\{\text{win}\}\big) \cdot Pr\{\text{MBO}\}\right] - c(x_A)\,. \tag{4.2.9}$$

Plugging in for $Pr\{\text{win}\}$ and $Pr\{\text{MBO}\}$ - both of which are derived in the appendix - and simplifying yields

$$\begin{aligned}&\Pi_A^e(\mu_A = \mu_g) = \\ &v\left[1 + \frac{1}{4}\left(2 - \hat{t} - (1 - \hat{t})\,(1 + x_A)\,\gamma\right)\left(-1 + (1 + x_A - (1 + x_B)\,\alpha)\,\gamma\right)\right] - c(x_A). \end{aligned} \tag{4.2.10}$$

[13] Remember that by assumption, the buy-out specialist becomes residual claimant of the project, in return for helping the manager stay in control of his division.

The expected payoff functions of the outside investors competing in the corporate auction are as stated in equations (4.2.1) and (4.2.3). Now that the optimal bidding strategies are established, these can be plugged in to explicitly determine the expected payoffs.

As established above, a standard result in the literature on the winner's curse problem is the fact that the uninformed outside investor makes zero expected profits. Since he has inferior knowledge of the division's expected value, he pursues a mixed bidding strategy which generates positive expected profits in the case he gets to finance a division with a good signal ($S_w = \mu_g$) and negative expected profits in the case he wins the corporate auction for a division with a bad signal ($S_w = 0$). Weighted with the probabilities of winning, this results in an overall expected payoff of zero for the uninformed investor.

Also in line with the literature, and as stated in Proposition 2, the informed investor makes positive expected profits. These are specified in

Corollary 2: *If the informed investor observes a losing signal $S_l = 0$, she will not shade her bid in equilibrium, but she will bid up to the expected gross profits from financing such a division. Therefore, her expected net payoff is $\Pi_I^e(S_l = 0) = 0$. In the case of $S_l = \mu_g$, she will shade her bid downwards such that she can profit from her information advantage. The informed investor's expected payoff conditional on observing $S_l = \mu_g$ is positive and equal to $\Pi_I^e(S_l = \mu_g) = E[u] \cdot E[\mu | \, S_l = \mu_g](\hat{t} - \xi) = \frac{1}{2}\,\mu_g\,(\hat{t} - \xi)$.*

Proof: Corollary 2 immediately follows from Corollary 1.

It should be noted, however, that the informed investor's expected payoff is reduced by the *ex ante* lump-sum cost of information acquisition. This is not a problem as long as the lump-sum cost is lower than the expected payoff.

The expected payoff for headquarters in stage one is somewhat less straightforward. It depends on the quality signals that headquarters observes in the internal tournament and on which it bases the spin-off decision. It also depends on the bidding strategies of the outside investors and on who of them wins the corporate auction. Equation (4.2.11) below gives a detailed description of the components of headquarters' expected payoff Π^e_{HQ}. The term in the first line is the expected payoff if both divisions generate a high quality signal. This implies that both are of good quality. The division that remains with headquarters will therefore provide an expected payoff of μ_g, while the expected payoff from the spin-off depends on the outcome of the corporate auction. The second line is the expected payoff if both divisions generate a low quality signal. The division that remains with headquarters provides an expected payoff of $E[\mu_w \mid S_w = 0]$, while the corporate auction determines the outcome for the spin-off. The third line describes the expected payoff for the case that the divisions generate diverging signals, resulting in the spin-off of the division with the bad signal.

$$\begin{aligned}\Pi^e_{HQ} = \; & Pr\{S_A{=}S_B{=}\mu_g\}\cdot\big[\mu_g + Pr\{\text{MBO} \mid S_l{=}\mu_g\}\cdot\beta(S_l{=}\mu_g) + (1 - Pr\{\text{MBO} \mid S_l{=}\mu_g\})\cdot E[b \mid b{>}\beta(S_l{=}\mu_g)]\big] \\ & + Pr\{S_A{=}S_B{=}0\}\cdot\big[E[\mu_w \mid S_w{=}0] + Pr\{\text{MBO} \mid S_l{=}0\}\cdot\beta(S_l{=}0) + (1 - Pr\{\text{MBO} \mid S_l{=}0\})\cdot E[b \mid b{>}\beta(S_l{=}0)]\big] \\ & + Pr\{S_A{\neq}S_B\}\cdot\big[\mu_g + Pr\{\text{MBO} \mid S_l{=}0\}\cdot\beta(S_l{=}0) + (1 - Pr\{\text{MBO} \mid S_l{=}0\})\cdot E[b \mid b{>}\beta(S_l{=}0)]\big] \qquad (4.2.11)\end{aligned}$$

$Pr\{S_A = S_B = \mu_g\}$ and $Pr\{S_A = S_B = 0\}$ represent the probabilities that the

quality signals of both divisions are good respectively bad. In both cases, headquarters will have to randomly decide which division to keep and which to auction off. With probability $Pr\{S_A \neq S_B\}$, one division generates a good quality signal and the other produces a bad quality signal. In this case, headquarters will always keep the division with the good signal and spin off the other one. Headquarters' payoff from the corporate auction depends on $Pr\{\text{MBO} \mid S_l\}$, the probability of a management buy-out, conditional on the quality signal of the divested division. This is derived in the appendix, as is the expected bid of the uninformed investor conditional on winning the corporate auction, $E[b \mid b > \beta(S_l)]$.

The tournament probability tree (figure A.2 in the appendix) helps to determine

$$Pr\{S_A = S_B = \mu_g\} = \alpha^2(\gamma + \gamma x_A)(\gamma + \gamma x_B)\,, \tag{4.2.12.a}$$

$$Pr\{S_A = S_B = 0\} = (-1 + (1 + x_A)\alpha\gamma)(-1 + (1 + x_B)\alpha\gamma) \quad \text{and} \tag{4.2.12.b}$$

$$Pr\{S_A \neq S_B\} = \alpha\gamma(2 + x_A + x_B - 2(1 + x_A)(1 + x_B)\alpha\gamma)\,. \tag{4.2.12.c}$$

The expected bid of the uninformed investor, conditional on his winning the corporate auction, can be derived from his bid distribution as described in Corollary 1 (b), equations (4.2.6) and (4.2.7):[14]

$$E[\,b \mid b > \beta(S_l = \mu_g)] = \frac{1}{2}\left(\mu_g + \beta(S_l = \mu_g)\right) \tag{4.2.12.d}$$

$$E[\,b \mid b > \beta(S_l = 0)] = \frac{(1 - \hat{t}\,)\left(\beta(S_l = \mu_g) + \beta(S_l = 0)\right) + \left(\mu_g + \beta(S_l = \mu_g)\right)}{4\left(1 - \frac{1}{2}\hat{t}\,\right)} \tag{4.2.12.e}$$

[14] See appendix for details.

Plugging $\beta(S_l)$, $Pr\{\text{MBO} \mid S_l\}$, and equations (4.2.12.a)-(4.2.12.e) into equation (4.2.11) and simplifying (while taking into account that $E[u] = \frac{1}{2}$) results in:

$$\Pi^e_{HQ} = \frac{1}{4\hat{t}}\big[\mu_g\big(\hat{t}\,[3 - 3\hat{t} + \hat{t}^2 + 4(2 + x_A + x_B)\alpha\gamma - (3 + \hat{t})(1 + x_A)(1 + x_B)\alpha^2\gamma^2] + 5\xi$$
$$+ [-(-3 + \hat{t}^2)\hat{t} - 4(2 + x_A + x_B)\alpha\gamma + (3 + \hat{t}^2)(1 + x_A)(1 + x_B)\alpha^2\gamma^2]\,\xi\big)\big] \quad (4.2.13)$$

This completes the solution of the corporate auction.

4.2.3 Equilibrium in the internal capital market

Having determined the equilibrium bids and expected payoffs of all parties in the corporate auction, it is now possible to solve for the optimal strategies in the internal capital market tournament. In this stage of the game, the two division managers have to decide how much effort to invest in the precision of their quality signals. A manager's effort raises the probability that his division's quality signal is high[15], thereby raising his probability to win the internal tournament. Winning the tournament means that the division is guaranteed to receive financing from the internal capital market. Internal financing in turn implies that the division manager will stay in control, reaping control benefits v. If the division manager loses the tournament, his division is divested. In this case, the equilibrium effort decision still influences outside investors' expectations and therefore their bidding behavior. As a result, a manager's effort also affects the probability that a management buy-out succeeds in case a division has to be divested. On the other hand, effort is costly. A division manager therefore has to weigh the benefits of his effort against its cost to maximize his expected payoff. While doing so, he also has to bear in mind the expected effort decision of his opponent, the other division manager.

[15]Remember that only a manager with a high-quality division can exert effort.

As can be seen from equation (4.2.10), the expected utility of a manager with a high-quality division depends on a set of variables, in addition to his own effort and the effort of his competitor: First, and most obviously, the higher a manager's control benefits v, the higher his expected utility. Furthermore, the manager's expected utility depends on the value of α, which is the *a priori* probability of a high-quality division, and on γ, which influences the probability of having a high quality signal if the division's quality is high. Both variables influence expectations and therefore payoffs in a number of ways.

From the point of view of a manager with a high-quality division, a high α means that there is a high probability that the other manager also has a good division. It therefore reduces the probability to win the internal tournament. On the other hand, it also drives up the bids of both the informed and the uninformed investor in the corporate auction, since it increases their expected payoffs from the spun-off division (even if the winning division has a bad quality signal, the informed investor will bid higher in the face of a higher α, since this raises the probability that a division with a bad signal is nonetheless of high quality).[16] Since it drives up both external investors' bids, the effect of a higher α on the probability of a management buy-out is not clear cut.

For γ, the case is similarly complex: A high γ means that the probability of having a good signal ($S_i = \mu_g$) is high, even without exerting costly effort. However, it also

[16]For a more detailed discussion of the effects of parameter changes on outside investors' bids, see subsection 4.3.2.

makes effort more worthwhile since it works as a weighting factor for the amount of effort x_i. On the other hand, it has the same effects on any competing manager who also has a high-quality division. From the informed investors' point of view, a high γ means a high probability of a good division generating a good signal, which therefore leads to a lower expected bid for divisions with a bad signal, which, taken by itself, would lower the probability of a management buy-out. This effect is counteracted by the expectations of the uninformed investor: Since a higher γ - all else being equal - implies a higher degree of signal precision, he will also bid less in the corporate auction.

To determine the equilibrium effort decision of a manager with a high-quality division, the standard procedure is to compute both manager's respective reaction functions in order to identify the Bayesian Nash equilibrium. Manager A's implicit reaction function is given by the first order condition for a maximum expected payoff:

$$\begin{aligned}\frac{\partial \Pi_A^e(\mu_A = \mu_g)}{\partial x_A} &= v\,\gamma\,\frac{1}{4}\big[1 + (1+x_B)\alpha^2\gamma\,\big(-1 + (4 + 4x_A - (1+x_B)\alpha)\gamma \\ &\quad - (1+x_A)(3 + 3x_A - 2(1+x_B)\alpha)\gamma^2\big)\big] - c'(x_A) \stackrel{!}{=} 0 \quad (4.2.14)\end{aligned}$$

Due to the symmetry assumption, manager B's reaction function is identical (switching only the indices). Manager A's reaction function $r_A = x_A(x_B)$ is depicted for $\alpha = \frac{1}{2}$, $\gamma = \frac{1}{2}$, $v = 2$ and $c(x_i) = \frac{1}{2}\,x_i^2$ in figure 4.2.

The reaction function is the curve which is formed by all (x_B, x_A)-combinations for which manager A's first order condition is satisfied, i.e. for which $\frac{\partial \Pi_A^e(\mu_A = \mu_g)}{\partial x_A} = 0$. The shaded area above the curve indicates the zone in which this derivative is less than zero, while it is greater than zero in the area below the curve. From the fact that the value of the first derivative of A's expected profit switches from positive to

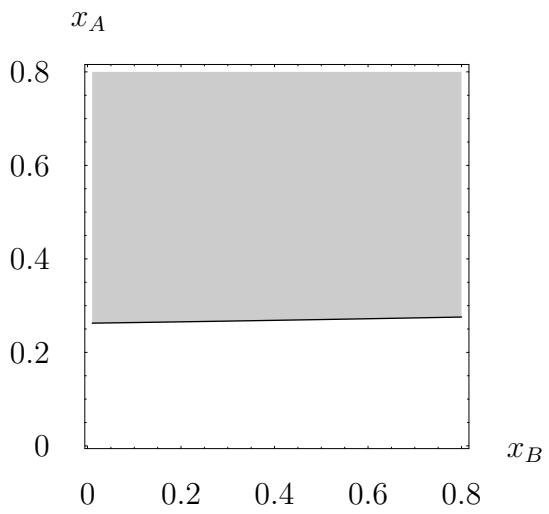

Figure 4.2: Manager A's reaction function $r_A = x_A(x_B)$

negative as x_A surpasses the critical value $x_A(x_B)$, it follows that the slope of the first derivative must be negative at this point, which means that the second derivative is negative and, therefore, second order conditions are satisfied.[17]

The symmetric Bayesian Nash equilibrium lies at the intersection of the two reaction functions in the (x_A, x_B)-space. It is visualized in figure 4.3 for the same parameter values that were used in figure 4.2. As before, only manager A's reaction function is plotted - the symmetric Bayesian Nash equilibrium lies at the intersection with the dashed 45°-line.

For $\gamma = \frac{1}{2}$, the optimal effort levels $x_A^* = x_B^* = x^*$ lie in the interval $[0, \frac{1-\gamma}{\gamma}]$ and are therefore valid solutions to the effort decision problem. These unique symmetric equilibrium values depend on the specifications of the exogenous parameters α, γ

[17]For a formal discussion of the second order condition, see the appendix.

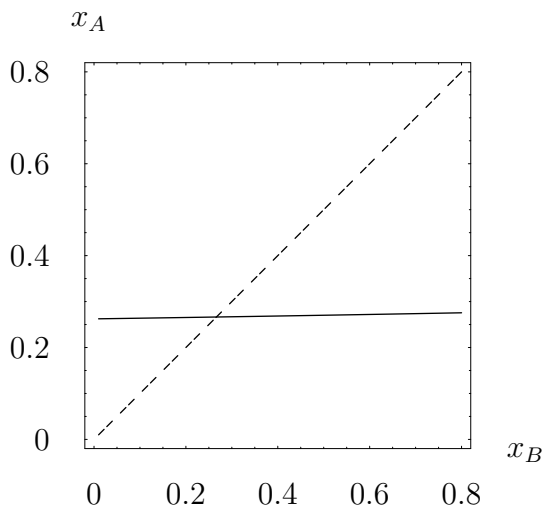

Figure 4.3: Equilibrium effort levels $x_A^* = x_B^* = x^*$

and v. As part of the comparative statics analysis in the next section, the effects of changes in these parameters on equilibrium effort levels will be analyzed.

4.3 Comparative statics

This section analyzes the effects of changes in exogenous parameters on all agents' optimal decisions and payoffs. This allows to characterize specific market conditions which are more or less favorable to information production and conducive to profitable spin-off decisions. Since parameter changes affect expected returns, they also affect effort decisions in the internal tournament, agents' beliefs, and bids in the corporate auction. The first step of the comparative statics analysis is to assess the effects on managers' effort decisions of changes in α (the *a priori* probability that a division is of high quality), γ (the probability with which a high-quality division generates a good signal if no effort is exerted), and v (a manager's nonmonetary control benefit). Once these effects are established, the impact of parameter changes on equilibrium

bids in the corporate auction and on agents' expected payoffs can also be assessed.

4.3.1 Effects of parameter changes on equilibrium effort levels

Figure 4.4 visualizes the effects of changes in the *a priori* probability α for a division to be of high quality (i.e. to have an expected return of μ_g). The values on the ordinate give the resulting equilibrium effort levels x^* for an entrepreneur with a high-quality division. Different values of v are used as location parameters. The dotted graph results from $v = 1$, the solid graph from $v = 2$, and the dashed one from $v = 3$. The dot-dashed graph results from $v = 4$. The value of γ is fixed at $\gamma = \frac{1}{2}$. Note that the maximum level of x^* must be lower than one, to ensure that the condition $0 \leq x \leq \frac{1-\gamma}{\gamma}$ is satisfied.

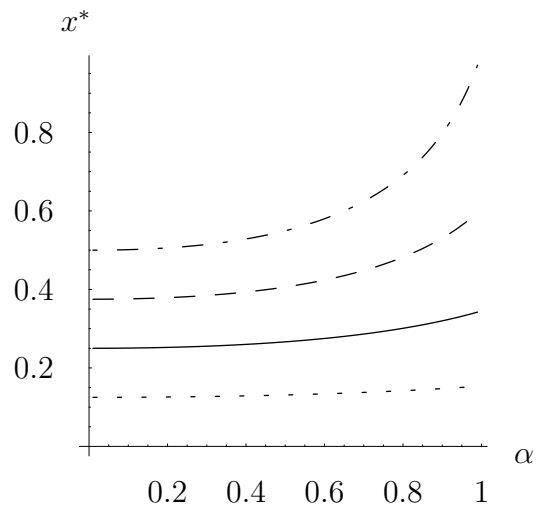

Figure 4.4: Equilibrium effort levels for different α and v

The effect of variations in the location parameter v on effort levels is straightforward: The higher the control benefits, the more effort a division manager is willing to exert in order to improve signal precision. All else being equal, a higher effort level raises the chances of winning the internal tournament as well as the chances of an MBO in case the division gets spun off. This in turn improves his chance to remain in control of his division and reap the higher control benefits. Obviously, the cost of effort provides an upper bound, such that the equilibrium effort level does not grow without bound as v rises.

A variation of α influences the equilibrium effort levels in two ways: On the one hand, if a manager of a high-quality division observes a low α, he knows that the probability that the competing division is of high quality is low. This means that he has good chances to win the internal tournament even without exerting much costly effort. With rising α, however, the chance that the competing division is of high quality rises. It therefore becomes more worthwhile for the manager to exert effort in order to raise his chance of generating a high quality signal and thereby winning the tournament. In addition, equation (A.2.6) implies that in case a division gets spun off, a rising α lowers the probability of a management buy-out, and thus the chance of retaining the control benefits. This second effect reinforces the first one: While the first effect makes it more important to invest effort into signal precision in order to win the tournament, the second effect makes it more important to win the tournament as it raises the danger of losing control over the division in case of a spin-off.

A higher v also raises the elasticity of the optimal effort level in response to a given increase in α: The higher the control benefits, the more worthwhile it becomes to generate a high quality signal at every given increase of α in order to win the internal tournament, thereby reducing the imminent danger of losing these higher benefits in a spin-off.

To determine the effects that changes in γ - the probability with which a high-quality division generates a good signal if no effort is exerted - have on the equilibrium effort level x^*, consider figure 4.5. The same values of v as above are used as location parameters, and the value of α is fixed at $\alpha = \frac{1}{2}$. Values above $\gamma = 0.65$ are not considered, since for higher values of the location parameter v, the resulting x^* would easily violate the condition $0 \leq x \leq \frac{1-\gamma}{\gamma}$.

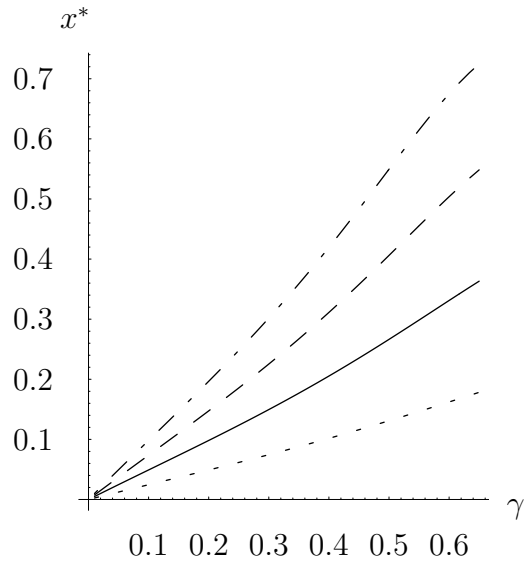

Figure 4.5: Equilibrium effort levels for different γ and v

As before, a higher v leads to higher equilibrium effort levels x^*. The intuition is the same as above and needs not be repeated.

Like the variation in α above, the variation in γ affects equilibrium effort levels through two channels: By influencing the probability to win the internal tournament and by influencing the probability of a management buy-out in case of a spin-off. In the internal tournament, an increase in γ has two countervailing effects. On the one hand, a higher γ implies a higher probability of a good signal, even without exerting costly effort, thus in fact reducing effort incentives. On the other hand, a high γ makes costly effort more worthwhile, since it raises its effectiveness (remember that the probability of a high-quality division generating a good signal is defined as $\gamma + \gamma x_i$). As for its influence on the probability of a management buy-out, equation (A.2.6) shows that for low values of γ, a rise in γ leads to a fall in the probability of a management buy-out, making it more important to avoid losing the internal tournament, and hence increasing effort incentives. In contrast, for high realizations of γ, a further rise leads to a rise in the probability of an MBO, causing a decrease in effort incentives.[18] Taken together, however, the positive effects outweigh the negative ones, and a rise in γ leads to a rise in the equilibrium effort level x^*.

The relationship between v and x^* is described in figure 4.6. In line with figures 4.4 and 4.5, the relation is unambiguous: Higher control benefits make it more worthwhile for division managers to spend effort on signal precision (i.e. information generation) in equilibrium.

[18] As shown in the appendix, the first derivative of $Pr\{MBO\}$ with respect to γ is negative for small γ and turns positive as γ approaches its upper bound, $\gamma \to \frac{1}{1+x}$.

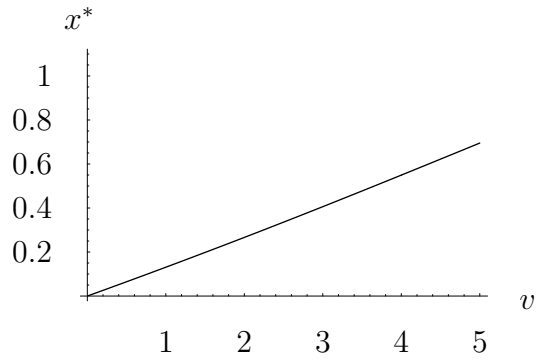

Figure 4.6: Equilibrium effort levels for different v

Summarizing the results of the above comparative statics analysis, one can derive the following hypotheses on information generation and, hence, selection efficiency in the internal capital market: (a) The higher the probability of a good quality division is, the more effort is spent on enhancing signal precision. This leads to the hypothesis that information generation is higher in more highly valued conglomerates - which are the ones where the *a priori* probability for high-quality divisions would be highest. (b) The higher the probability of generating a good signal is, and the more effective the effort to enhance signal precision is, the more effort is exerted. This leads to the hypothesis that divisions operating in traditional and well-understood markets and techniques are superior in terms of information generation. (c) The higher the control benefits are, the higher is the effort spent on enhancing signal precision. The corresponding hypothesis would be that the more nonmonetary benefits a division manager can extract from his position, the more information will be generated.

4.3.2 Effects of parameter changes on equilibrium bids in the corporate auction

After having determined the effects that parameter changes have on equilibrium effort levels, it is now possible to determine the effects of such changes on equilibrium bids and payoffs in the corporate auction. In the next step, the analysis will turn to the bids by both the informed and uninformed investor; afterwards, the comparative statics analysis will be concluded by assessing the effects on all agents' expected payoffs.

As shown above, the informed investor's optimal bid β depends on the quality signal that she observes. If she observes a low quality signal ($S_l = 0$), she will bid exactly the amount of the conditional expected value of the division, $E[\mu|\, S_l = 0] \;=\; \mu_g \cdot \frac{\xi}{\hat{t}}$, with ξ and $\hat{t}$ as defined in equation (4.2.4). Obviously, the joint probability ξ that the divested division is of high quality and has a low quality signal, depends on the parameters α and γ, as does the probability $\hat{t}$ that the divested division has a low quality signal.

When γ (the probability that a high-quality division generates a high quality signal, if no effort is exerted) rises, both ξ and $\hat{t}$ must fall. Since ξ falls faster than $\hat{t}$, the informed investor's bid falls with rising γ if she observes a low quality signal.[19] Intuitively, this is clear: As γ rises, it becomes ever less likely that a high-quality division generates a low quality signal. Therefore, the conditional probability for the division to be of high quality when it has a low quality signal, $\frac{\xi}{\hat{t}}$, falls. This causes the expected value $E[\mu|\, S_l = 0]$, and therefore the bid $\beta(S_l = 0)$, to fall.

[19] A formal discussion of the derivative of $\beta(S_l = 0)$ with respect to γ can be found in the appendix.

When α rises, the probability $\hat{t}$ that the divested division has a low quality signal falls, as both divisions are more likely to be of high quality. The effect of a rise in α on the joint probability ξ is somewhat more ambiguous. On the one hand, an increase in α raises the probability that any division is of high quality. On the other hand, it also increases equilibrium effort levels, making it more likely that a high-quality division generates a high signal. This in turn increases its chances to win the internal contest, and as a consequence, there is a higher chance that the division to be divested is of low quality. Also, if a high-quality division loses, the increase in effort levels implies a higher probability that it has a good quality signal. For small values of γ, the effect of a higher effort level is not very strong and therefore, the positive first effect dominates and ξ rises with a rise in α. Only for very high values of γ and low levels of v, the negative effect could dominate. For most parameter ranges, however, a rise in α leads to a rise in the joint probability ξ that the divested division is of high quality and has a low quality signal. In sum, given that $\hat{t}$ falls and ξ rises with a rise in α (or falls more slowly than $\hat{t}$), the expected value $E[\mu|\, S_l = 0]$, will rise and so will the bid $\beta(S_l = 0)$.[20]

If the informed investor observes a high quality signal ($S_l = \mu_g$), she will shade her bid downward and bid less than the conditional expected value of the division: $\beta(S_l = \mu_g) = E[\mu|\, S_l = \mu_g] \cdot (1 + \xi - \hat{t})$, as stated in equation (4.2.5). Changes in α and γ affect $\hat{t}$ and ξ in the same way as discussed above. Therefore, the informed investor's bid $\beta(S_l = \mu_g)$ falls if γ rises and rises if α rises.[21]

[20] A formal discussion of the derivative of $\beta(S_l = 0)$ with respect to α can be found in the appendix.

[21] A formal discussion of the derivatives of $\beta(S_l = \mu_g)$ with respect to γ and α can be found in

This means that there is more room for bid-shading as γ (the probability with which a high-quality division generates a good signal if no effort is exerted) rises. On the other hand, there is less room for bid-shading as α (the *a priori* probability for a division to be of high quality) rises. To see why this is the case, it is now time to take a closer look at the bidding strategy of the uninformed outside investor.

As discussed in the appendix, the uninformed outside investor always randomizes his bid between zero and μ_g. The only way in which parameter changes affect his bidding behavior is by inducing him to reallocate probability mass between these two extremes. As argued in the appendix, the bidding distribution of the uninformed investor is piecewise uniform on three different intervals. The boundaries of these intervals are determined by the equilibrium bidding strategy of the informed investor: Between zero and $\beta(S_l = 0)$, the uninformed investor bids according to density function $g_1(b)$; between $\beta(S_l = 0)$ and $\beta(S_l = \mu_g)$, he bids according to density function $g_2(b)$; and between $\beta(S_l = \mu_g)$ and μ_g, he bids according to $g_3(b)$.

As γ rises, so does the the probability that a high-quality division generates a good signal. This makes it more likely that the division that loses the internal ranking tournament and is therefore divested, is of low quality (as an increase in signal precision reduces the chance that a good division will be mistakenly divested). Therefore, an increase in γ reduces the uninformed investor's valuation of the division and induces him to lower his expected bid by placing more probability mass in the lower

the appendix.

intervals of his distribution function. In technical terms, this shift in the probability mass takes place via two different channels: Firstly, the slope of the distribution function in the third interval, $g_3(b)$, declines with a rise in γ (which implies that the slope must rise in the first two intervals), and secondly, the lower boundaries of the intervals shift downwards, given that the informed investor's bid decreases with an increase in γ (for both good and bad signals).[22] In more intuitive terms, it should also be noted that it is this decrease in the uninformed investor's expected bid which in turn allows the informed investor to reduce her bid for higher values of γ, thereby reducing her costs in case of winning, without reducing her probability of winning.

As α rises, it becomes more likely that both divisions are of high quality, which in turn makes it more likely that the divested division is of high quality. Therefore, the uninformed investor values the auctioned division more highly and raises his expected bid by placing more probability mass in the higher intervals of his distribution function. In technical terms, this implies a decrease in the slope of the distribution function in the first interval, $g_1(b)$, and an increase in the third interval. Also, the upper boundary of the first interval, $\beta(S_l = 0)$, increases, as does the upper boundary of the second interval, $\beta(S_l = \mu_g)$. In more intuitive terms, it should be noted that is is this increase in the uninformed investor's bid which in turn induces the informed investor to raise her equilibrium bid for higher values of α, thereby raising her costs, but maintaining her chances of winning the corporate auction.

This completes the analysis of the effects of changes in the exogenous variables

[22] A formal discussion of the effects of changes in α and γ on the uninformed investor's bidding strategy can be found in the appendix.

α and γ on the bidding strategies in the corporate auction. It was shown that a rise in α induces the informed investor to make higher equilibrium bids both if she observes a good signal and if she observes a bad signal. The uninformed investor in turn responds to a rise in α by placing more probability mass on higher bids. A rise in γ reduces the informed investor's equilibrium bids, and it induces the uninformed investor to also place more probability mass on lower bids.

4.3.3 Effects of parameter changes on expected payoffs

This subsection explores the effects of parameter changes on all agents' expected payoffs. A first step examines the expected utility of a manager with a high-quality division. The second step analyzes the changes in expected payoffs of both informed and uninformed outside investors, and the last step assesses the effects on headquarters' expected payoff.

As can be seen from equation (4.2.8), the expected utility of a manager with a high-quality division depends on the probability with which he wins the internal tournament and on the probability of a management buy-out in case his division loses the tournament and is auctioned off. Clearly, it also depends on his control benefits v and on the cost function $c(x)$. In what follows, it is assumed that the marginal cost of effort, $c'(x)$, is not prohibitively high, so it will not dominate the comparative statics analysis.

Intuitively, it is clear that a rise in v should lead to a higher expected payoff for the division manager. The partial derivative of $\Pi_A^e(\mu_A = \mu_g)$ according to equation

(4.2.10) for the equilibrium effort level $x^* = x_A = x_B$ is:[23]

$$\begin{aligned}\frac{\partial \Pi_A^e(\mu_A = \mu_g)}{\partial v} = \frac{1}{4}\big[&4 + (x^*(-1+\alpha)+\alpha)\,\gamma\,(-1+(1+x^*)^2\,\alpha^2\,\gamma^2\,(-1+\gamma+x^*\gamma)) \\ &- 4\,c'(x^*)\,\frac{\partial x^*}{\partial v} + v\gamma\big(1+\alpha[-1+(1+x^*)\alpha\gamma^2(1+3x^*-3\alpha-3x^*\alpha \\ &+(1+x^*)(-1-4x^*+4(1+x^*)\alpha)\gamma)]\big)\cdot\frac{\partial x^*}{\partial v}\big]\,. \qquad (4.3.1)\end{aligned}$$

It is straightforward to verify that this term is generally positive. It turns negative only for high values of $c'(x^*)$, as a prohibitively high marginal cost of effort would render the whole analysis meaningless. This can, however, be precluded since it was shown that the first order condition for an optimal effort decision holds, and x^* is an internal solution. It then follows from equation (4.3.1) that a rise in control benefits v leads to a rise in the division manager's expected utility.

A rise in α leads to an increase in the probability that the competing division is also of high quality, and it therefore reduces the probability of winning the internal ranking tournament. As discussed above, it also reduces the probability of a management buy-out in case the division gets spun off. In addition, a higher α induces the manager to exert more costly effort in order to keep up with the stronger competition. Therefore, and from equation (4.2.8), it is intuitively clear that the expected utility of a manager of a high-quality division falls with a rise in α. The derivative of the manager's

[23] As before, the discussion focuses on the expected payoff of division manager A, who is assumed to have a high-quality division.

expected utility (as given by equation (4.2.10)) with respect to α is less than zero:[24]

$$\begin{aligned}\frac{\partial \Pi_A^e(\mu_A = \mu_g)}{\partial \alpha} =& - c'(x^*)\,\frac{\partial x^*}{\partial \alpha} + \frac{1}{4}\,v\,\big[\gamma[-1 + (1+x^*)^2\alpha^2\gamma^2(-1+\gamma+x^*\gamma)] \\ & \cdot(1 + x^* + (-1+\alpha)\frac{\partial x^*}{\partial \alpha}) + (1+x^*)\alpha(x^*(-1+\alpha)+\alpha)\gamma^3 \\ & \cdot[2(1+x^*)(-1+\gamma+x^*\gamma) + \alpha(-2+3(1+x^*)\gamma)\frac{\partial x^*}{\partial \alpha}]\big] < 0\,. \quad (4.3.2)\end{aligned}$$

The effect of a change in γ is less clear cut. A higher γ makes it more likely that a high-quality division generates a good signal and therefore raises its chances of winning the internal tournament. However, for low values of γ, a rise in γ reduces the probability of a management buy-out in the case of a spin-off. For small values of α, the first effect is strong, since the competing division is unlikely to benefit from a rise in γ. The first effect therefore dominates the second and the expected utility rises with a rise in γ. However, for large α, the first effect becomes weaker as it becomes more likely that the competing division is also of high quality and could generate a high quality signal. The second effect dominates and the expected utility falls with a rise in γ. Also, the level of γ itself has an influence on the effect: For high values of γ, the probability of a management buy-out rises with a rise in γ, and the second effect also becomes positive, causing expected utility to rise with a rise in γ. Again, the rise in γ causes a rise in costly effort. The derivative with respect to γ is given

[24] To verify this result, also take the partial derivative of equation (4.2.9), and plug in for $\frac{\partial Pr\{\text{win}\}}{\partial \alpha} < 0$, $\frac{\partial Pr\{\text{MBO}\}}{\partial \alpha} < 0$, $c'(x^*) > 0$, and $\frac{\partial x^*}{\partial \alpha} > 0$.

as:[25]

$$\frac{\partial \Pi_A^e(\mu_A = \mu_g)}{\partial \gamma} = -c'(x^*)\,\frac{\partial x^*}{\partial \gamma} + \frac{1}{4}\,v\,\big[(x^*(-1+\alpha)+\alpha)[-1+(1+x^*)^2\alpha^2\gamma^2(-1+\gamma+x^*\gamma)] + (-1+\alpha)\gamma[-1+(1+x^*)^2\alpha^2\gamma^2(-1+\gamma+x^*\gamma)]\frac{\partial x^*}{\partial \gamma} + (1+x^*)\alpha^2(x^*(-1+\alpha)+\alpha)\gamma^2(-2+3(1+x^*)\gamma)(1+x^*+\gamma\frac{\partial x^*}{\partial \gamma})\big]\ . \quad (4.3.3)$$

In sum, the expected payoff of a division manager with a high-quality division thus rises with an increase in his nonmonetary control benefits v, and it falls with an increase in the *a priori* probability α for a division to be of high quality. The effect of an increase in γ is ambiguous and depends on the levels of α and γ.

The expected payoffs of the outside investors bidding in the corporate auction depend only in part on exogenous parameters. As stated in Proposition 2, the uninformed investor makes zero expected profits. This result is independent of the exogenous parameter values and therefore remains unchanged.

As stated in Corollary 2, the informed investor makes positive expected profits. Her expected payoff conditional on observing a good quality signal is given as $\Pi_I^e(S_l = \mu_g) = \frac{1}{2}\,\mu_g\,(\hat{t}-\xi)$, varying with the exogenous parameters μ_g, v, α, and γ.[26] Plugging in for $\hat{t}$ and ξ while taking into account that in equilibrium $x_A = x_B = x^*$, and simplifying yields

$$\Pi_I^e(S_l = \mu_g) = \frac{1}{2}\,\mu_g(1-\alpha)[1+\alpha\gamma(1+x^*)]\,. \quad (4.3.4)$$

[25]For the parameter values used in the analysis above ($\alpha = \gamma = \frac{1}{2}$ and $v = 2$), $\frac{\partial \Pi_A^e(\mu_A=\mu_g)}{\partial \gamma}$ is negative.

[26]Remember that her expected payoff conditional on observing a low quality signal is zero.

Clearly, a rise in the division's future returns μ_g leads to a rise in the expected payoff for the informed investor. The same is true for the control benefit v of the division manager. A rise in v influences the investor's expected payoff only indirectly, however, by raising the equilibrium effort level and thus the precision of the quality signal, thus increasing her informational advantage over the uninformed outside investor.

As shown above, a rise in the *a priori* probability α for a division to be of high quality induces the uninformed investor's expected bid to rise and forces the informed investor to also raise her equilibrium bid, allowing for less bid shading. This phenomenon can also be explained by analyzing the effect of an increase in α on the degree of information asymmetry between the informed and the uninformed outside investors. This in turn also allows to draw conclusions on the informed investor's expected payoff. Since an increase in the *a priori* probability for a division to be of high quality raises the probability that any divested division is of high quality, the outside investor raises his valuation of the division and increases his expected bid. The increase in his valuation also brings the uninformed investor's belief closer in line with that of an informed investor who observes a good quality signal, and therefore knows for sure that the division is of high quality. This approximation of beliefs reduces the degree of information asymmetry between the two investors, and reduces the information rent - the expected payoff - of the informed investor.

In more technical terms, the need to raise her bid increases the informed investor's cost of acquiring the division, while its expected value is not affected by a change in

α (given that it has a high quality signal). Therefore, a rise in α reduces the informed investor's profits in case she wins the corporate auction. The probability for winning the corporate auction, i.e. the probability of a management buy-out, conditional on observing a good quality signal, remains unchanged. As a result, the effect is unambiguous: A rise in the *a priori* probability α for a division to be of high quality leads to a decrease in the informed investor's expected payoff. The partial derivative of equation (4.3.4) with respect to α supports this reasoning:

$$\frac{\partial \Pi_I^e(S_l = \mu_g)}{\partial \alpha} = -\frac{1}{2}\,\mu_g[1 + (1 + x^*)(-1 + 2\alpha)\gamma + (-1 + \alpha)\alpha\gamma\frac{\partial x^*}{\partial \alpha}] < 0\ . \quad (4.3.5)$$

A rise in γ (the probability with which a higher quality division generates a good signal if no effort is exerted) has the opposite effect on bids and payoffs: As it reduces the uninformed investor's expected bid, it also reduces the informed investor's equilibrium bid, allowing for more bid shading. In terms of information asymmetries, a rise in γ increases the informational advantage of the informed investor.[27] The higher γ is, the more informational rent can be appropriated by the informed investor. Again, the division's expected future returns and the probability of winning the corporate auction (conditional on observing a good quality signal) remain unchanged, such that the informed investor's expected payoff unambiguously rises with an increase in γ. The derivative with respect to γ is given as:

$$\frac{\partial \Pi_I^e(S_l = \mu_g)}{\partial \gamma} = -\frac{1}{2}\,(-1 + \alpha)\alpha\mu_g(1 + x^* + \gamma\frac{\partial x^*}{\partial \gamma}) > 0\ . \quad (4.3.6)$$

In sum, the expected payoff of the inside investor thus rises with increases in μ_g,

[27] With higher γ, the probability that a high-quality division is mistakenly divested falls (as the probability of generating a high quality signal increases). As a consequence, the uninformed outside investor will lower his valuation of the division and decrease his bid accordingly. This causes an increase in information asymmetry when the informed investor observes a high quality signal for the divested division.

v, and γ, and it falls with an increase in α.

Last but not least, the expected payoff of corporate headquarters is also affected by changes in the exogenous parameters. As defined in equation (4.2.13), headquarters' expected payoff depends on the exogenous variables μ_g, v, α, and γ.

Clearly, a higher division return μ_g results in a higher expected payoff for headquarters, as both the expected value of the retained division and the bids for the divested division rise. The control benefits v only have an indirect effect, by influencing division managers' effort levels and therefore signal precision. For small values of γ and α, a rise in v results in a higher expected payoff for headquarters, while it results in lower expected payoffs for medium and large values of α and γ. For the latter parameter ranges, it is the informed investor who can appropriate the benefits of the increase in signal precision, to the detriment of corporate headquarters. A closer look at the derivative of Π^e_{HQ} with respect to v allows to specify the parameter ranges for which it is positive and negative, respectively:

$$\frac{\partial \Pi^e_{HQ}}{\partial v} = -\frac{1}{4}\,(-1+\alpha)\alpha\gamma(-1+(1+x^*)\alpha\gamma)(1+(1+x^*)\alpha\gamma)^2\,\underbrace{(-1+5(1+x^*)\alpha\gamma)}_{sign}\,\mu_g\frac{\partial x^*}{\partial v})\,. \tag{4.3.7}$$

This derivative is positive if the term *sign* is negative, i.e. if $\alpha\gamma < \frac{1}{5(1+x^*)}$. For larger α, γ, the derivative turns negative.

A rise in α raises the probability for both divisions to be of good quality. It also

leads to higher signal precision by inducing higher equilibrium effort levels, and reduces the information rent of the informed investor, as both outside investors' bids rise. It therefore results in an increase in headquarters' expected payoffs, as is supported by the derivative:

$$\begin{aligned}\frac{\partial \Pi^e_{HQ}}{\partial \alpha} = -\frac{1}{4}\mu_g\big\{ &-7+(1+x^*)\gamma\big[-1+\alpha\big(2+(1+x^*)\gamma\{4+\alpha[-6+(1+x^*)\gamma(6\\ &+\alpha(-8+(1+x^*)\gamma(-4+5\alpha+(1+x^*)\alpha(-5+6\alpha)\gamma)))]\}\big)\big]\\ &+(-1+\alpha)\alpha\gamma(-1+(1+x^*)\alpha\gamma)(1+(1+x^*)\alpha\gamma)^2(-1+5(1+x^*)\alpha\gamma)\frac{\partial x^*}{\partial \alpha}\big\} > 0\,. \end{aligned} \tag{4.3.8}$$

The effects of a rise in γ are more ambiguous: It leads to higher signal precision, but allows for more bid shading and a higher information rent for the informed investor. For small values of α and γ, a rise in γ results in a rise in headquarters' expected payoff, while for medium and high values, it leads to a decline. Again, a closer look at the derivative allows to identify the relevant parameter ranges:

$$\begin{aligned}\frac{\partial \Pi^e_{HQ}}{\partial \gamma} &= -\frac{1}{4}(-1+\alpha)\alpha(-1+(1+x^*)\alpha\gamma)(1+(1+x^*)\alpha\gamma)^2\\ &\quad\cdot\underbrace{(-1+5(1+x^*)\alpha\gamma)}_{sign}\mu_g(1+x^*+\gamma\frac{\partial x^*}{\partial \gamma}) \end{aligned} \tag{4.3.9}$$

This derivative is positive if the term *sign* is negative, i.e. if $\alpha\gamma < \frac{1}{5(1+x^*)}$. For larger α, γ, the derivative turns negative.

On a more intuitive level, an increase in γ from very low levels leads to an increase in headquarters' expected payoff, as it reduces the - unconditional - probability of a management buy-out.[28] This increases the probability that the uninformed outside

[28] See appendix for the derivation of $\frac{\partial Pr\{\text{MBO}\}}{\partial \gamma}$.

investor wins the corporate auction. Since the expected bid of the uninformed investor is higher than the informed investor's expected bid, this implies a higher expected payoff for corporate headquarters.

In sum, the expected payoff of headquarters rises with increases in μ_g and α. The effects of increases in v and γ are ambiguous and depend on the levels of α and γ.

All results of the comparative statics analysis are summarized in figure (4.7).[29]

	$v \uparrow$	$\mu_g \uparrow$	$\alpha \uparrow$	$\gamma \uparrow$
x*	$\uparrow$	-	$\uparrow$	$\uparrow$
$\beta(S_l = 0)$	-	$\uparrow$	$\uparrow$	$\downarrow$
$\beta(S_l = \mu_g)$	-	$\uparrow$	$\uparrow$	$\downarrow$
$E[b]$	-	$\uparrow$	$\uparrow$	$\downarrow$
$Pr\{\text{MBO}\}$	-	-	$\downarrow$; $\uparrow$ (for large γ)	$\downarrow$; $\uparrow$ (for large γ)
$\Pi^e_A(\mu_A = \mu_g)$	$\uparrow$	-	$\downarrow$	$\downarrow$; $\uparrow$ (for small α and large γ)
$\Pi^e_I(S_l = \mu_g)$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$
Π^e_{HQ}	$\downarrow$; $\uparrow$ (for small α and γ)	$\uparrow$	$\uparrow$	$\downarrow$; $\uparrow$ (for small α and γ)

Figure 4.7: Internal capital market: comparative statics

4.4 Conclusion

The model that was laid out and discussed in this chapter combines two strands of literature, by analyzing the workings of an internal capital market, and by assessing the mechanisms of a subsequent corporate auction.

[29] In the figure, "-" means there is no direct effect of the exogenous parameter on the variable in question. While it may have indirect effects through its effect on equilibrium effort levels x^*, these are not reported, as they are only second-order effects.

In the internal capital market, which is modelled as a rank-order tournament, two divisions within one liquidity-constrained conglomerate compete for internal financing. The analysis focuses here on the incentives for managers to generate meaningful information on their respective divisions' quality. While this information is unproductive in the sense that it does not in itself enhance a division's expected payoff, it helps to convey private information about its expected future returns (i.e. its quality) to corporate headquarters. The latter has to make a financing decision based on the received quality signals, and it is shown that it is always in headquarters best interest to divest the division with the lower quality signal.

In the analysis of the corporate auction, equilibrium bidding strategies and expected payoffs are established - conditional on managers' equilibrium effort levels in the internal capital market. The corporate auction is modelled as a first-price sealed-bid common value auction with asymmetric information. It is assumed that before the auction, one of the outside investors acquires inside information from the division's management, thus gaining an informational advantage over the uninformed investor. It is shown that no equilibrium exists in pure strategies, but that there exists a unique Bayesian Nash equilibrium where the inside investor bids according to a pure strategy (conditional on her inside information), making positive expected profits, and the uninformed investor randomizes his bid, making zero expected profits.

In a second step of the analysis, managers' equilibrium effort decisions for information generation in the internal capital market are established. It is shown that

in equilibrium, the manager of a high-quality division exerts an intermediate effort level which does not guarantee full information revelation, but balances the costs of effort with the increased probability for winning the internal tournament and reaping control benefits associated with remaining in control of the division.

In a third step, comparative statics highlight the impact of changes in exogenous parameters on the previous results. The effects on equilibrium effort levels are at the center of this assessment, since on the one hand, they affect all agent's expected payoffs, and on the other hand, they allow a direct assessment of selection efficiency in the internal capital market. This is owing to the fact that a higher effort level leads to higher signal precision, and the chance component in winner picking is therefore reduced (as soon as at least one division generates a high-quality signal, a high-quality division is with certainty picked as the winner of the internal tournament).

It is shown that equilibrium effort levels, and, hence, selection efficiency, increase with an increase in all relevant exogenous parameters. This allows to derive hypotheses about selection efficiency under different market conditions: (a) Selection efficiency tends to be higher in more highly valued conglomerates, where the probability for a division to be of high quality is higher, since an increase in the *a priori* probability that a division is of high quality increases the managers' incentives to exert costly signaling efforts; (b) it also tends to be higher in more traditional and well-understood markets, where the probability of generating a good signal is higher, since the increased probability of creating a good quality signal makes the signaling effort more worthwhile; and (c) it tends to be higher when managers earn higher

nonmonetary control benefits, since the potential cost of losing the internal capital market tournament, and losing control of the division, increases with higher control benefits.

It is further shown that headquarters' equilibrium payoff from the spin-off increases with an increase in the expected return of a high-quality division, as well as in the probability that a division is indeed of high quality. This is not surprising, as it coincides with a higher overall valuation of the conglomerate. More interestingly, headquarters expected payoff generally decreases with an increase in the probability of generating a good signal.

When taking into account information asymmetries and the distribution of rents between corporate headquarters and the informed outside investor, it is also intuitively clear that an increase in the *a priori* probability that a division is of high quality increases headquarters's expected payoff. This is owing to the fact that it reduces the information asymmetry between the uninformed and informed outside investors, as the former's beliefs about the quality of the divested division become better aligned with the latter's (who after observing a good quality signal knows for sure that the division is of high quality). This reduction of her informational advantage leads to a decrease in the informed investor's expected payoffs, and drives up the expected payoff of corporate headquarters.

For an increase in the probability of generating a good quality signal (and the effectiveness of signaling efforts), it was shown that an increase leads to an increase

in information asymmetry between the uninformed outside investor and an inside investor who observes a good quality signal. This is owing to the fact that the uninformed investor reduces his valuation of the divested division with an increase in signal precision, as it reduces the chance that a high-quality division is mistakenly divested. The related increase in the informed investor's information rent generally leads to a decrease in headquarters' expected payoff. Only for very low probabilities for generating a good signal, does an increase in this probability lead to an increase in headquarters' expected payoff. This is owing to the effects of changes in this parameter on the likelihood of a management buy-out. For low levels of this probability, the chance of an MBO falls with an increase in the probability of generating a good signal. This translates into a reduction in the likelihood that the informed investor wins the corporate auction. This, in turn, results in a higher expected payoff for corporate headquarters, as the uninformed investor's expected bid is always higher than the informed investor's bid, conditional on winning the corporate auction. Thus, while all bids are reduced by an increase in signaling effectiveness, the increased likelihood of the uninformed investor winning drives up headquarters' expected payoff for low levels of this parameter.

In line with these results, the expected payoff of the informed outside investor decreases with an increase in the *a priori* probability that a division is of high quality, as it decreases her informational advantage over the uninformed investor. On the other hand, an increase in the probability of generating a good quality signal (and the effectiveness of signaling efforts) increases the expected payoff of an informed investor who observes a good quality signal, as it increases her information rent. In

addition, and not surprisingly, her expected payoff also increases with an increase in the expected return of a high-quality division.

As a result of the above, corporate headquarters could be expected to have a higher interest in a corporate auction when the overall valuation of the conglomerate is higher, suggesting a higher *a priori* probability for its divisions to be of high quality. On the other hand, headquarters could be expected to be less eager to meet their financing needs through the divestment of one division, the more traditional and well-understood the markets in which it operates are, as this lowers outside investors' valuations of the divested division. In turn, it would be in this situation that outside investors would be most interested in such corporate auctions, and in acquiring inside information, as it is in this setting that their informational rent would be highest. Thus, in this setting, an MBO would be the most likely result if corporate headquarters had to make a spin-off decision. The empirical verification of these hypotheses as well as those on selection efficiency is left for future research.

Chapter 5

Business plan contests, information revelation and financing decisions

This chapter uses the tournament-cum auction setup to analyze the process of information revelation in a business plan contest, sponsored by a venture capitalist (VC), where two entrepreneurs compete for the financing of their respective projects.[1]

Business plan contests are still a fairly recent phenomenon in the realm of financing for start-up companies. They were first introduced in the 1980's, in order to give young entrepreneurs a chance to present their projects and potentially receive seed financing for their business ideas. Such a contest typically consists of several rounds, in which increasingly detailed business plans are judged, and competitors are successively eliminated. Over the years, business plan contests have become increasingly popular, with growing numbers of institutionalized contests worldwide, and competing projects rising sharply both in numbers and in quality of the competing projects.

While business plan sponsors can act out of a multitude of different motivations,

[1]While the terms "contest" and "tournament" are synonyms, "contest" will be preferred throughout this chapter, since the term "business plan contest" is a fixed expression.

some of which are discussed further below, one advantage for the sponsor is that it shifts the burden of information generation to the entrepreneur, who has to invest effort in writing a detailed business plan, in order to signal the quality of his project. While this may be considered only a slight difference to a routine due diligence where both the venture capitalist and the entrepreneur have to exert effort in order to generate and transmit meaningful information, the business plan contest also offers a second informational advantage, in that it requires only an ordinal ranking of different projects in order to determine the best project (as discussed above, a low information requirement is one of the distinguishing features of rank-order tournaments).

Apart from anecdotal evidence, little research has been done on business plan contests, both in the theoretical and empirical realm. One recent exception is the work by Elitzur and Gavious (2006), who present a theoretical model of a business plan contest in which several entrepreneurs compete for financing from one venture capitalist. In this, their work is related to the model presented in this chapter, where two entrepreneurs compete in a business plan contest sponsored by a venture capitalist. In doing so, it relies on a series of simplifications, which, nonetheless, are in line with stylized facts in the industry. As mentioned above, it is assumed that the entire cost of the information production lies with the two competing entrepreneurs. In addition, it is assumed that the contest sponsor is a venture capitalist who is interested in financing the winning project. While this does not coincide with the most commonly observed types of business plan contests, it captures the spirit of such contests, where VCs are involved in secondary roles (such as jury members) because they are eager to acquire insider information about promising new business projects.

After the contest, the winning entrepreneur can choose from competing financing offers for his project - both from the VC who sponsored the contest, and from an outside financier. While the winner of a business plan contest usually receives a predetermined prize (in line with traditional tournament rules), the present analysis stresses the role of the improved access to finance for the winning project. This is clearly the more important aspect for an aspiring entrepreneur, since it allows him to realize the expected returns of his project. The fact that winning the contest acts as a quality seal makes the project an interesting investment object for outside financiers, who then compete with the venture capitalist for the chance of financing it. As a result, the prize for the contest winner is determined endogenously by an auction-type bidding competition between prospective financiers. To account for information asymmetries between the VC and outside investors, the post-contest financing game is modelled as a common value auction with asymmetric information.

Applying the tournament-cum-auction setup to a business plan contest allows to shed light on the workings of the contest and its implications for information generation, as well as for the financing decision that takes place after the contest. The analysis focuses on the same two main questions as chapter 4. In the first phase of the game, it examines the incentives for information generation during the business plan contest.[2] In the second phase, it focuses on how the quality signals produced in the first phase influence the bidding process after the contest, when the VC and the

[2]As discussed in the introduction, "selection efficiency" is defined as the probability with which the best contestant is picked as the tournament winner. This probability rises with an increase in information generation.

outside financier compete for financing the winning project.

While the basic setup is similar to the one in chapter 4, it differs in two important aspects. Firstly, the object of the post-tournament auction is different - in chapter 4, outside investors compete for financing the loser of the internal tournament, while in the current setup, the two potential investors (the VC and one uninformed outside investor) compete for financing the winner of the business plan contest. Since in both cases it is common knowledge that the auction is over the loser/winner of the preceding tournament/contest, this affects agents' beliefs about the value of the division/project, and therefore also their equilibrium bidding strategies. Secondly, the winning bid in the post-contest auction is paid to the winning entrepreneur. Therefore, his expected payoff depends more directly on the investors' equilibrium bidding strategies than that of a division manager in the model in chapter 4, where the corporate auction only determines the probability with which a manager can retain his fixed nonmonetary control benefit. This direct connection, in turn, feeds back into the entrepreneurs' strategic effort decision in the business plan contest.

The venture capital industry is still a comparatively young phenomenon in the realm of financial intermediation. It emerged only after World War II, when new businesses sprang up around innovations and technologies originally developed for military use. The first venture capital firm was thus established in the United States in 1946, to be followed by only a few others during the period up to the 1960's. It was not until the late 1970's and early 1980's that the venture industry reached significant scale, both in terms of fundraising and amounts disbursed. The academic literature

on the role and characteristics of venture capital followed that pattern, with an early empirical assessment by Noone and Rubel (1970), and the first theoretical paper in the early 1980's (Chan, 1983).[3]

The academic literature on venture capital has been exploding since the early 1990's, both in the empirical and theoretical realm. All aspects of the venture capital cycle have been subject of analysis, from the initial fundraising to the eventual exit of the VC from a financed company. Special emphasis has been placed on the typical VC's organization and compensation patterns, on the staging and monitoring of investments, as well as on syndication between several VCs who invest in the same company. It has also been tried to analyze the connection between venture capital financing and the rate of innovation, and - inspired by the apparent correlation - whether governments can successfully act as venture capitalists. Considering the eventual exit decision, the roles of market conditions, of reputation effects, and of share-distribution have been analyzed, among other things.[4]

Chan (1983) already pointed out the importance of a venture capitalist's ability to screen projects before investing in order to reduce information asymmetries - thereby not only benefiting the individual VC, but also improving economy-wide resource allocation. In spite of this, comparatively little analytical research has been done on the selection mechanisms used by VCs to determine in which projects to invest. One empirical study by Kaplan and Strömberg (2000) focuses on the pre-investment

[3]For an overview of the evolution of venture capital, as well as the related empirical literature, see for example Gompers (2006).

[4]For an exhaustive research summary, see Gompers and Lerner (2004).

"due diligence" process. Among other things, these authors show that venture capitalists typically spend a lot of effort on assessing opportunities and risks prior to their investment decision, and they also find a significant correlation between this initial assessment and the company's subsequent performance in case an investment is undertaken.

Concerning the theoretical literature, however, efficient information generation is usually assumed rather than explicitly modelled (see, for example, Ueda, 2004). The cited work by Elitzur and Gavious (2006) is an exception, as it also models a business plan contest in which several entrepreneurs compete for financing from one venture capitalist. Unlike the model that is presented in this chapter, however, they analyze productive effort by entrepreneurs that increases the expected return of their respective projects, from which the VC will ultimately benefit. This feature brings their model closer to the realm of the literature on research tournaments, and in particular Fullerton and McAfee (1999), who allow for heterogeneous contestants.[5] Also, Elitzur and Gavious (2006) model the business plan contest as a one-stage game, where the contest winner is automatically financed at a predetermined rate by the VC who sponsors the competition. In contrast, the model presented in this chapter allows to assess selection efficiency in the business plan contest in a first step, and in a second step it explicitly models the competition between different investors for financing the winning project. Accordingly, this setup allows to draw conclusions about how the subsequent financing game influences the equilibrium effort levels in the business plan contest.

[5]See chapter 2 for a discussion of research tournaments.

As pointed out above, the screening process (due diligence) can require considerable effort from the VC if every project is to be precisely and independently evaluated. As discussed in chapter 2, one advantage of rank-order tournaments is the relatively low information requirement that they impose on the tournament sponsor in order to create an ordinal ranking. A business plan contest is thus an ideal screening instrument for a venture capitalist who is resource-constrained and wants to maximize her expected returns by financing only the most promising projects.

However, the first business plan contests were held only in the early 1980's, as university-based projects that were meant as a learning activity as much as a way to provide start-up financing for students' projects.[6] The number of such contests worldwide has since dramatically increased, and while most of them are still university-based, some venture capitalists have taken to sponsoring their own events. In addition, in view of the presumed positive externalities of innovative start-ups on the regional economy, public institutions have also started to sponsor business plan contests, often by teaming up with local universities and/or venture capitalists.[7] Even where venture capitalists are not the main sponsors of a competition, they are present as judges, responsible for analyzing business plans and picking the winners - thereby gaining inside information about the projects' expected returns. Prize money typically ranges from a few thousand dollars to more than $125.000, but the most important incentive for young entrepreneurs is the chance to gain access to additional

[6]The first such contest was Texas University's "Moot Corp." in 1984, which was extended to a US-wide competition in 1989 and went international in 1990.

[7]For an overview of some better known business plan contests, see Small Business Notes (http://www.smallbusinessnotes.com/planning/competitions.html).

financing for their projects either from an insider venture capitalist or from an outside financier.

In the remainder of this chapter, the interaction of the business plan contest and the subsequent bidding process is modelled as a multi-stage game. In a first step, the competition between two entrepreneurs is investigated, who can invest in the precision of their quality signals that are used by the venture capitalist to pick the perceived higher quality project as the winner.[8] In a second step, the financing decision of the winning entrepreneur is analyzed. After winning the contest, he can choose between an offer from the venture capitalist who sponsored the tournament - and therefore has inside information about the the entrepreneur's expected payoff - and a competing offer from an uninformed outside financier. Both offers are modelled in the simplest possible way, as buy-out offers that allow the entrepreneur to cash in on the expected value of his project, while the winning financier becomes the residual claimant. This assumption is arguably not the most realistic rendition of real-life venture capital financing contracts. It allows, however, to distinguish between the financiers' different conditional valuations for the winning project and the implications for the entrepreneur's payoff, while avoiding the intricacies of modeling the financing contract and the payoff structure over the project's life span. Optimal contracts and control as well as repayment considerations have been at the center of the bulk of the venture capital literature, as discussed above.

[8]The effort that entrepreneurs invest in signal precision can be interpreted as the effort needed to write a detailed business plan. It does not in itself enhance the expected returns of the project, but it is needed to transmit more precise information about its quality to the contest sponsor.

As in Engelbrecht-Wiggans and others (1983), the uninformed outside investor relies on a mixed bidding strategy and experiences zero expected profits. The VC, however, is able to realize positive expected profits, by taking some of the rent away from the winning entrepreneur. The comparative statics exercises reveal that the venture capitalist's expected payoff increases with an increase in the expected return of a high-quality project, and it decreases with an increase both in the probability that a project is of high quality and in the probability for generating a good quality signal. The negative effect of an increase of these two probabilities is owing to the corresponding reduction in information asymmetry between the VC and the outside financier, which drives up the VC's equilibrium bids in the post-contest financing game. This in turn reduces the VC's informational rent, and allows the winning entrepreneur to appropriate a larger chunk of his project's expected return. In line with this result, it is shown that the expected payoff of an entrepreneur with a high-quality project increases with an increase in these probabilities, as well as with an increase in the expected return of a high-quality project.

In addition to conclusions about different agents' expected profits, the model allows for a direct assessment of selection efficiency in different institutional settings. Entrepreneurs' incentives for information generation and, hence, overall selection efficiency in equilibrium is higher in (a) more competitive and highly reputed business plan contests with a higher *a priori* probability that a project is of high quality;[9] (b) in more industry-specific contests where the venture capitalist has special expertise,

[9]It is also shown, however, that this effect is reversed in an extremely competitive setting, where projects are of high quality almost certainly, and a further increase in competitiveness reduces entrepreneurs' effort incentives.

and where signaling effectiveness is therefore higher. It also tends to be higher (c) in contests that focus on higher value-added industries.

The next section outlines the model. The following two sections analyze equilibrium bids and payoffs, and the effects of changes in exogenous parameters. The last section provides a brief summary and discussion of the main results.

5.1 The model

Consider a four-stage bidding game with no discounting. The first two stages of the game represent the business plan contest, which is sponsored by a venture capitalist and used to rank two entrepreneurs' business projects according to their expected profitability. At the first stage, both entrepreneurs have to decide how much costly effort to exert in order to produce a quality signal. At the second stage, the quality signals are realized and the winner of the business plan contest is announced. Since a project's true quality is private information, and effort levels cannot be observed, the selection of the winner can only be based on the projects' respective quality signals. At the third stage, both the venture capitalist and an outside financier make financing offers for the winning project. After the offers are submitted, the entrepreneur who won the contest chooses the higher offer to cash in on his project, and all parties' payoffs are realized. To keep the analysis simple, it is assumed that the losing entrepreneur will not be financed, meaning that he will not receive a payoff at the end of the contest.

5.1.1 Agents

There are three types of risk neutral agents: (a) two entrepreneurs, (b) a venture capitalist, (c) an outside financier.

The two *entrepreneurs* both seek financing for their respective projects. In the business plan contest they compete with each other by signaling the quality/expected payoff of their projects to the venture capitalist who sponsors the contest. They can exert costly effort to enhance their business plans, thereby raising the precision of their quality signals. Based on the presented business plans, the projects get ranked according to their perceived quality. After the contest, only the winner gets a chance to realize his project, receiving financing either from the venture capitalist or from an outside financier. His payoff depends on the financing offers he receives and on the effort costs he incurs during the contest. For simplicity it is assumed that the losing entrepreneur does not receive financing offers, thus only incurring effort costs.

The *venture capitalist* uses the business plan contest to rank the two entrepreneurs according to their quality signals. Her own objective is to generate the highest possible return by financing one of the two entrepreneurs. It is assumed that she cannot finance both, due to a budget constraint that results from her own refinancing structure. Therefore, she is interested in financing the entrepreneur whose project has the higher expected net present value. She uses the quality signals that are generated in the course of the business plan contest to pick the entrepreneur whom she expects to have the higher value project, and declares him the winner. In the case of identical signals, the VC will have to make a random pick (resulting in a 50:50 winning chance

for each entrepreneur). After the winner of the business plan contest is determined, the VC has to compete with the outside financier in a sealed bid first-price auction for the chance to finance the winning project.

The *outside financier* can observe which entrepreneur wins the business plan contest and then he competes with the VC for financing that entrepreneur's project. Whoever makes the highest financing proposal wins and becomes the project's new residual claimant.

5.1.2 Technology and information

Both entrepreneurs have projects with an expected net present value larger than or equal to zero. The entrepreneurs A and B have private information about the expected future returns μ_i, with $i = A, B$, of their projects, which can either be high ($\mu_i = \mu_g$) or low ($\mu_i = 0$). To compete in the business plan contest, they have to signal their respective projects' quality to the venture capitalist. It is assumed that the mere description of the project conveys useful information to the VC - who can rely on her experience in financing start-ups to make an informed guess about any project's net present value. This guess is, however, not always correct. In fact, it will be distorted by misperceptions on the side of the VC, which may be due to expectations associated with a specific industry that do not hold for the individual entrepreneur. To offset this distortion, an entrepreneur can influence the precision of his quality signal S_i, with $i = A, B$, by exerting effort (i.e. by writing a detailed business plan).

A low-quality project with $\mu_i = 0$ will generate a low signal $S_i = 0$ with probability

one. The signal distortion is assumed to arise only in the case of a high-quality project, which is underestimated with positive probability. Thus, a high-quality project generates a high signal $S_i = \mu_g$ only with probability γ, with $0 \leq \gamma \leq 1$. With counterprobability $1 - \gamma$, a high-quality project generates a low signal $S_i = 0$ and could be mistaken for a low-quality one. Against this backdrop, an entrepreneur with a high-quality project can exert costly effort x_i, $i = A, B$, and write a detailed business plan in order to increase the signal's precision (i.e. the probability that it reflects the true quality level). The probability of generating a high quality signal is then raised to $\gamma(1 + x_i)$. With probability $1 - \gamma(1 + x_i)$, the signal still takes on the value $S_i = 0$. As can be easily seen, x_i must be restricted to $0 \leq x_i \leq \frac{1-\gamma}{\gamma}$. While it is natural to assume the effort level to be positive, it has to be less than $\frac{1-\gamma}{\gamma}$ for purely technical reasons, so as not to allow for probabilities greater than one.

After the winner of the business plan contest is announced, the VC and the outside financier make their financing offers in the forms of sealed bids, β and b, respectively. The investor with the higher bid wins and pays the amount of his bid to the entrepreneur in return for becoming the residual claimant of the project.

The following information is public knowledge: The expected values μ_i of the two projects are independently and identically distributed and take on the strictly positive value μ_g with probability α and zero with probability $1 - \alpha$. The probability functions for the signals $S_i = 0$ and $S_i = \mu_g$ are also common knowledge, while the realized effort level, x_i, is private information of each entrepreneur.

After the contest, the venture capitalist must compete with an outside financier over the chance to finance the winning project. Unlike the venture capitalist, who observes S_A and S_B, the outside financier does not know the quality signals. He can, however, draw conclusions about the winning project's expected payoff based on the fact that it has won the business plan contest. Given the two possible realizations of the quality signal, there are three possibilities: Both projects have a low quality signal, $S_A = S_B = 0$, both projects have a high quality signal, $S_A = S_B = \mu_g$, or one has a high signal while the other has a low signal, $S_A = \mu_g \wedge S_B = 0$ or $S_A = 0 \wedge S_B = \mu_g$. Only in the latter case, there is a meaningful competition. The project with the higher quality signal will win the contest, and the outside investor can use this information to condition his financing offer upon.

This information structure implies that the realization of the signal S_i influences both the probability of winning the business plan contest, which is equivalent to the probability of getting financed, and the expected amount of finance, since winning the contest sends a quality-signal to the outside financier. All that the latter knows about the winning project is the probability distribution of its net present value, α and $1 - \alpha$, the probability of a signal distortion, and the fact that it has won. With this information, he can calculate a project's expected value contingent upon winning.

When assuming that the project with the higher quality signal will be declared winner of the contest, the question arises whether this is in the venture capitalist's best interest from a strategic point of view. She might instead declare the entrepreneur with the lower signal the winner, and make him such an unattractive financing

offer that he will choose the outside financier with certainty. Afterwards, the VC could enter into negotiations with the entrepreneur with the better signal, who was declared the loser. She could thus avoid competition from the outside financier and finance the better project at a lower cost.

There are two reasons why the VC will not strategically distort the contest in this way. Firstly, there are reputational considerations. Assuming that she stays in business, the VC will continue to finance entrepreneurs and to hold business plan contests after this one. If it were ever to be known that the best entrepreneur did not win a contest, and that there were negotiations on the side, it would not be in the interest of entrepreneurs to participate in future contests. The VC therefore has to maintain a track record of compliance with the contest rules. Secondly, if one were to assume that the VC had an interest in distorting the competition, this would be anticipated by the outside financier. He would modify his bidding strategy accordingly and also try to strike a deal with the official tournament loser. Following a similar line of argument as in chapter 4, this leads to the conclusion that in equilibrium, the VC will declare the entrepreneur with the higher quality signal the winner.

Figure 5.1 summarizes the four stages of the model.

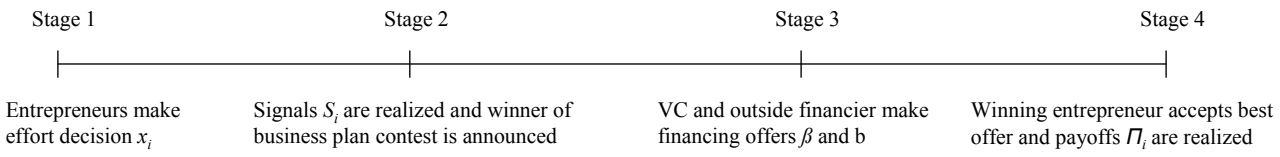

Figure 5.1: Business plan contest: stages

5.2 Analysis

In order to determine the entrepreneurs' expected payoffs, and to analyze the effects of the different financing options on their incentives to generate information, the multiple stage game is solved by backward induction. In a first step, the bidding behavior of the VC and the outside financier at the post-contest financing stage will be analyzed. In a second step, the entrepreneurs' effort decisions will be examined during the business plan contest in which they have to signal the quality of their projects. In a last step, the effects of changes in exogenous parameters on the effort decisions will be discussed, as well as their effects on bids and expected payoffs.

5.2.1 Equilibrium strategies in the financing game

In the financing game that follows the announcement of the contest's winner, two bidders (the VC and the outside financier) compete for the same object (financing the winning entrepreneur) of common but not commonly known value. Also, there is asymmetric information between the two bidders. As the contest sponsor, the VC has superior information about the true value of the entrepreneur's project, knowing the realization of his quality signal. The outside financier, on the other hand, only knows the expected return of a high-quality project, the *a priori* probabilities for high- and low-quality projects, the probability of signal distortion, and the fact that the entrepreneur has won. The bidding process is modelled as a first-price sealed bid common value auction with asymmetric information, similar to the one analyzed in Engelbrecht-Wiggans and others (1983). The formal arguments presented in this section follow their line of arguments, while also making use of the contributions of

Hendricks and Porter (1988) and Rajan (1992) to this type of model.

The equilibrium bidding strategies of the inside and the outside investor are determined by the maximization of their respective expected payoffs, given the strategy of the other investor. The VC (the inside investor) maximizes

$$\Pi_I^e = \Pr\{\text{ VC wins financing game }\} \cdot [\, E[\mu|\, S_w] - \beta(S = S_w)\,] \,. \tag{5.2.1}$$

Equation (5.2.1) combines the winning probability with the expected net payoff from becoming residual claimant of the winning project. The latter is the difference between the expected net present value of the project, given the winner's observed quality signal S_w, less the bid, which also depends on the realization of the signal. The fact that the entrepreneur has won the business plan contest is only of indirect interest to the VC, as it influences the beliefs of the outside investor on how valuable the project is. Since the exact realization of S_w is a more precise information, it constitutes a sufficient statistic for the fact that an entrepreneur has won.

Since the signal can only take on two different values (0 or μ_g), it is possible to specify the VC's expected payoffs for these two cases:

For $S_w = 0$:

$$\Pi_I^e(S = 0) = \Pr\{\text{ VC wins financing game } |\, S_w = 0\,\} \cdot [\, E[\mu|\, S_w = 0] - \beta(S_w = 0)\,] \tag{5.2.2.a}$$

For $S_w = \mu_g$:

$$\Pi_I^e(S = \mu_g) = \Pr\{\text{ VC wins financing game } |\, S_w = \mu_g\,\} \cdot [\, E[\mu|\, S_w = \mu_g] - \beta(S_w = \mu_g)\,] \tag{5.2.2.b}$$

According to the signal realization, the VC can thus distinguish between two different conditional expected payoffs, which in turn depend on his conditional probability of winning the financing game, as well as on the conditional expected net payoff from becoming residual claimant of the project.

Simultaneously, the outside investor maximizes

$$\Pi_O^e = \Pr\{\text{ outsider wins }\} \cdot [\, E[\mu|\,\text{higher S; outsider wins}\,] - b\,]\,. \tag{5.2.3}$$

Like equation (5.2.1), equation (5.2.3) combines the winning probability with the expected net payoff from becoming residual claimant of the project. However, in this case, the expected net payoff is conditioned on two pieces of information: In contrast to the inside investor, the outsider can only observe the fact that the entrepreneur in question has won the business plan contest (which in equation (5.2.3) is indicated by the expression "higher S"). Since the entrepreneur with the higher signal always wins the contest, the fact of winning influences the beliefs of the outsider concerning the value of the project. The second piece of information that the outsider can draw on is the fact that in order to win, his bid must be higher than that of the inside investor. Since the bid of the latter depends on the observed quality signal, this also conveys information about the project value. The resulting equilibrium is characterized in

Proposition 3: *(i) No equilibrium exists in pure strategies. (ii) There is an equilibrium in mixed strategies where: (a) The VC bids according to a pure strategy and conditions her bid on the observed Signal S_w. She makes positive expected profits. (b) The outside investor bids according to a mixed strategy independent of the state. He makes zero expected profits.*

Proof: See appendix.

As in chapter 4, it is intuitively clear that the outside investor cannot bid according to a pure strategy. Not knowing the true (common) value of the winning project, he is, however, aware that the VC has superior information. Therefore, if he were to bid according to a pure strategy, and bid an amount less than the expected value of a high-quality project, he would have a low probability of winning a high-quality project - since the better-informed VC would outbid him whenever she observed a good quality signal. He would win a high-quality project with positive probability only when the project generates a low quality signal, in which case the VC bids according to the expected value of the project, conditional on observing a low quality signal. In order to ever win a high-quality project, the outside investor would therefore always have to bid higher than this amount, thereby generating sure losses in the case the project is of low quality, which would more than offset his expected payoff from getting lucky and winning a high-quality project. Similarly, were he to bid a fixed amount equal to or higher than the expected value of a high-quality project, he would always realize zero or negative payoffs.[10]

The venture capitalist, on the other hand, can bid according to a pure strategy, conditioning her bid on the observed quality signal of the winning project. Knowing the randomizing strategy of the outside investor, she can shade her bid downwards when she observes a good quality signal, thereby generating a positive information rent, and when she observes a low quality signal, she bids according to the expected

[10]For a more detailed discussion of the outside investor's bidding strategy, see the appendix.

value of the project, conditional on observing this signal.

The equilibrium bidding strategies of the VC and the outside investor are described in

Corollary 3: *(a) The equilibrium bidding strategy of the VC (inside investor) is*

$$\beta(S_w = 0) = \mu_g \cdot \frac{\xi}{\hat{t}} = E[\mu | S_w = 0] \tag{5.2.4}$$

if she observes $S_w = 0$*, and*

$$\beta(S_w = \mu_g) = \mu_g \cdot (1 + \xi - \hat{t}) = E[\mu | S_w = \mu_g] \cdot (1 + \xi - \hat{t}) \tag{5.2.5}$$

if she observes $S_w = \mu_g$.[11]

(b) The equilibrium bidding strategy of the outside investor is a distribution G *on* $\Re_+$*, where* $G(\beta)$ *is the probability that he tenders a bid not exceeding* β *and* u *is the realization of a random variable* U *which is independent of* (μ, S) *and has an atomless distribution on* $[0, 1]$*:*

$$G(\beta(S_w = 0)) = u \cdot \hat{t} \tag{5.2.6}$$

and

$$G(\beta(S_w = \mu_g)) = u\,. \tag{5.2.7}$$

Proof: See appendix.

[11] With $\xi = \Pr\{S_w = 0 \cap \mu_w = \mu_g\} = [\alpha - 0,5(2 + x_A + x_B)(1 + \alpha)\alpha\gamma + (1 + x_A)(1 + x_B)\alpha^2\gamma^2]$ and $\hat{t} = \Pr\{S_w = 0\} = ((1 + x_A)\alpha\gamma - 1)\,((1 + x_B)\alpha\gamma - 1)$. From this definition, it follows that $\hat{t} \geq \xi$ and therefore $0 \leq \frac{\xi}{\hat{t}} \leq 1$ and $0 \leq (1 + \xi - \hat{t}) \leq 1$.

The winning probability of the venture capitalist, conditional on the observation of signal S_w, follows directly from Corollary 3 (b). If the informed investor observes $S_w = 0$, her bid is higher than the uninformed investor's with probability $G(\beta(S_w = 0)) = u \cdot \hat{t}$. If she observes $S_w = \mu_g$, her bid is higher with probability $G(\beta(S_w = \mu_g)) = u$. In both cases, the winning probability of the uninformed investor is the respective counterprobability.

5.2.2 Equilibrium expected payoffs from the financing game

Having determined the equilibrium bidding strategies of the VC and the outside investor, as well as the respective winning probabilities, it is now possible to determine the equilibrium expected payoffs of both investors at this stage of the game, as well as the expected payoff of an entrepreneur.

As established in chapter 3, a standard result in common value auctions with asymmetric information is the fact that the outside investor makes zero expected profits. Since he has inferior knowledge of the project's expected value, he pursues a mixed bidding strategy which generates positive expected profits in the case he gets to finance a project with a good signal ($S_w = \mu_g$) and negative expected profits in the case he finances a project with a bad signal ($S_w = 0$). Weighted with the probabilities of winning, this results in an overall expected payoff of zero for the outsider.

Also in line with the literature, and as stated in Proposition 3, the inside investor (the VC) makes positive expected profits. These are specified in

Corollary 4: *If the insider observes a winning signal $S_w = 0$, she will not shade her bid*

in equilibrium, but she will bid up to the expected gross profits from financing such a project. Therefore, her expected net payoff is $\Pi_I^e(S_w = 0) = 0$. *In the case of* $S_w = \mu_g$, *the VC will shade her bid downwards such that she can profit from her information advantage. Her expected payoff conditional on observing* $S_w = \mu_g$ *is positive and equal to* $\Pi_I^e(S_w = \mu_g) = E[u] \cdot E[\mu | \, S_w = \mu_g](\hat{t} - \xi) = \frac{1}{2}\,\mu_g\,(\hat{t} - \xi)$.

Proof: Corollary 4 immediately follows from Corollary 3.

The third relevant expected payoff in the financiers' bidding game is that of an entrepreneur with a high-quality project, who has to make an effort decision which influences his chances of winning the business plan contest in the first stage and obtaining financing in the second stage. In the standard common value auction setting, this corresponds to the expected payoff of the seller, which is not usually analyzed in the literature, since the main focus is on the bidding behavior and expected payoffs of the bidders. Engelbrecht-Wiggans and others (1983) use an indirect approach in order to provide a complete formal analysis of the equilibrium: Given the expected payoffs of the two bidders as well as the total expected revenue from the project, it is possible to deduce the expected payoff of the seller by simply subtracting the bidders' expected payoffs from the expected project value. However, their method cannot be applied here, since it requires the seller and the informed bidder to have the same information set, while in the present setup, the seller (entrepreneur) has superior information.

Since in this setting, the financing game forms only the second phase of the model setup, the expected payoff of the entrepreneur becomes an essential part of the solution: It determines the optimal effort levels in the first phase of the game, which

is the business plan contest. In this context, the relevant variable is the expected payoff of an entrepreneur with an expected project value μ_g and who has to decide which effort level x_i to choose (as the model does not allow for effort decisions when an entrepreneur has a low-quality project).

Given that the deduction method of Engelbrecht-Wiggans and others (1983) is not applicable here, a different way for determining the entrepreneurs' expected payoff needs to be found. The most straightforward solution would be to add up the expected bids of the VC and of the uninformed outside investor, which, of course, must be weighted with their respective winning probabilities. The problem in the setup of Engelbrecht-Wiggans and others (1983) is that the bid of the outside investor cannot be explicitly determined. It is this ambiguity which forces those authors to choose the above mentioned indirect approach. However, in contrast to their analysis, the present model only counts with one outside investor, such that the joint bid distribution reduces to the bid distribution of the single outside investor. With this distribution given as a piecewise defined function between 0 and μ_g, the expected bid of the outside investor - conditional on his winning the financiers' bidding game - can be calculated.[12]

In the case of a good signal, $S_w = \mu_g$, the VC bids according to $\beta(S_w = \mu_g)$, such that the outside investor can only win the financing game if his bid is higher. His expected bid conditional on winning is therefore

$$E[b \,|\, b > \beta(S_w = \mu_g)] = \frac{1}{2}\left(\mu_g + \beta(S_w = \mu_g)\right) . \tag{5.2.8}$$

[12]For the complete derivation of the outside investor's bidding strategy, see the appendix.

In the case of a bad signal, $S_w = 0$, the VC bids according to $\beta(S_w = 0)$, and the outside investor's expected bid becomes

$$E[b \,|\, b > \beta(S_w = 0)] = \frac{(1 - \hat{t})\,[\beta(S_w = \mu_g) + \beta(S_w = 0)] + [\mu_g + \beta(S_w = \mu_g)]}{4\,(1 - \frac{1}{2}\,\hat{t})}\,. \tag{5.2.9}$$

The expected profit of entrepreneur A, conditional on $\mu_A = \mu_g$ can now be determined as follows (due to symmetry, the analogous result holds for entrepreneur B):

$$\begin{aligned}
&\Pi_A^e(\mu_A = \mu_g) = \\
&\quad \Pr\{S_A = \mu_g \cap \text{A wins BP contest} \,|\, \mu_A = \mu_g\} \cdot \Big[\Pr\{\text{VC wins financing game}\} \cdot \beta(S_g = \mu_g) \\
&\qquad + \Pr\{\text{outsider wins}\} \cdot E[b \,|\, b > \beta(S_w = \mu_g)]\Big] \\
&\quad + \Pr\{S_A = 0 \cap \text{A wins BP contest} \,|\, \mu_A = \mu_g\} \cdot \Big[\Pr\{\text{VC wins financing game}\} \cdot \beta(S_g = 0) \\
&\qquad + \Pr\{\text{outsider wins}\} \cdot E[b \,|\, b > \beta(S_w = 0)]\Big] \\
&\quad - c(x_A)\,,
\end{aligned} \tag{5.2.10}$$

where $c(x_i)$ is the cost of an entrepreneur's effort, with $c'(x_i) > 0$ and $c''(x_i) \geq 0$, and which is assumed to be symmetric for both entrepreneurs.

Plugging in for the relevant probabilities and simplifying (while taking into account that $E[\,u\,] = \frac{1}{2}\,$) yields:[13]

$$\begin{aligned}
&\Pi_A^e(\mu_A = \mu_g) = \\
&\quad \frac{1}{4}\,\mu_g \cdot \Bigg[(\gamma + \gamma x_A)\left(1 - \alpha(\gamma + \gamma x_B)\tfrac{1}{2}\right)\left(4 + 3\xi - 3\hat{t}\right) \\
&\quad + (1 - \gamma - \gamma x_A)(\alpha(1 - \gamma - \gamma x_B) + (1 - \alpha)\,)\left[\frac{3}{2}\,(1 + \xi - \hat{t}\,) + \frac{1}{2}\,(\hat{t}^{\,2} - \hat{t}\,\xi + \frac{\xi}{\hat{t}})\right]\Bigg] \\
&\qquad - c(x_A)\,. \qquad (5.2.11)
\end{aligned}$$

This completes the solution of the financing game.

5.2.3 Equilibrium in the business plan contest

Having determined the strategies and the payoffs of the financing game, it is now possible to solve for the optimal strategies in the business plan contest. In this phase of the game, the two competing entrepreneurs have to decide how much effort to invest in the precision of their quality signals, given the quality of their respective projects. On the one hand, the optimal effort of an entrepreneur depends on the effect that it has on his probability of winning the business plan contest, and on the other hand, on the effect that it has on the expected payoff in case of winning, i.e. on the expected bids of the VC and the outside investor.

To determine the equilibrium effort decision of an entrepreneur with a high-quality project, the standard procedure is to compute both entrepreneurs' respective reaction

[13] Probabilities $\Pr\{S_A = \mu_g \cap \text{A wins BP contest} \mid \mu_A = \mu_g\}$ and $\Pr\{S_A = 0 \cap \text{A wins BP contest} \mid \mu_A = \mu_g\}$ can be derived from the contest probability tree (figure B.2 in the appendix), while the winning probabilities in the financing game result from the bid distributions of the outside financier, as given in equations (5.2.6) and (5.2.7).

functions in order to identify the Bayesian Nash equilibrium. Entrepreneur A's implicit reaction function is given by the first order condition for a maximum expected payoff, which was derived with the help of the program *Mathematica* (Wolfram) by taking the first derivative of entrepreneur A's expected payoff as specified in equation (5.2.11) with respect to his signaling effort x_A:

$$\begin{aligned}
&\frac{\partial \Pi_A^e(\mu_A = \mu_g)}{\partial x_A} = \\
&(\alpha - 1) \cdot \frac{\gamma \mu_g}{16} \cdot \Bigg(\frac{1 + 7\alpha}{(\alpha - 1)} - 2\alpha\gamma[10 + 3\alpha + 2x_B(2 + \alpha) + x_A(6 + \alpha)] \\
&\quad + \alpha^2\gamma^2[21 + 20x_A + 3x_A^2 + 22x_B + 14x_A x_B + 4x_B^2 + \alpha(1 + x_B)(9 + 4x_A + 5x_B)] \\
&\quad - (1 + x_B)\alpha^3\gamma^3[17 + 6x_A^2 + 3\alpha + x_B(12 + x_B + \alpha(4 + x_B)) + 2x_A(11 + \alpha + x_B(5 + \alpha))] \\
&\quad + \alpha^4\gamma^4(1 + x_A)(1 + x_B)^2(5 + 3x_A + 2x_B) \\
&\quad + \frac{(x_A - x_B)(\alpha - 1)}{(1 + x_A)((1 + x_A)\alpha\gamma - 1)^2} - \frac{(1 + x_B)(\alpha - 1)}{(1 + x_A)((1 + x_A)\alpha\gamma - 1)} \Bigg) - c'(x_A) \stackrel{!}{=} 0 \qquad (5.2.12)
\end{aligned}$$

Due to the symmetry assumption, entrepreneur B's reaction function is identical (switching only the indices). Entrepreneur A's reaction function $r_A = x_A(x_B)$ is depicted for $\alpha = \frac{1}{2}$, $\gamma = \frac{1}{2}$, $\mu_g = 1$ and $c(x_i) = \frac{1}{2}\, x_i^2$ in figure 5.2.

The reaction function is the curve which is formed by all (x_B, x_A)-combinations for which entrepreneur A's first order condition is satisfied, i.e. for which $\frac{\partial \Pi_A^e(\mu_A = \mu_g)}{\partial x_A} = 0$. The shaded area above the curve indicates the zone in which this derivative is less than zero, while it is greater than zero in the area below the curve. From the fact that the value of the first derivative of A's expected profit switches from positive to negative as x_A surpasses the critical value $x_A(x_B)$, it follows that the slope of the first derivative must be negative at this point, which means that the second derivative is

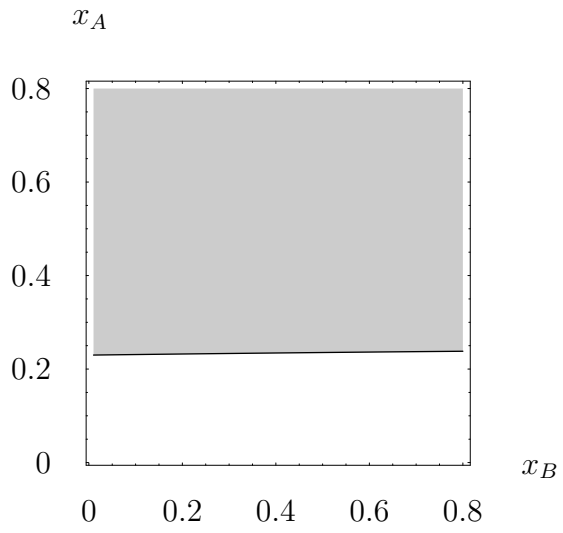

Figure 5.2: Entrepreneur A's reaction function $r_A = x_A(x_B)$

negative and, therefore, second order conditions are satisfied.[14]

The symmetric Bayesian Nash equilibrium lies at the intersection of the two reaction functions in the (x_A, x_B)-space. It is visualized in figure 5.3 for the same parameter values that were used in figure 5.2. As before, only entrepreneur A's reaction function is plotted - the symmetric Bayesian Nash equilibrium lies at the intersection with the dashed 45°-line.

For $\gamma = \frac{1}{2}$, the optimal effort levels $x_A^* = x_B^* = x^*$ lie in the interval $[0, \frac{1-\gamma}{\gamma}]$ and are therefore valid solutions to the effort decision problem. These unique symmetric equilibrium values depend on the specifications of the exogenous parameters α, γ, and μ_g. As part of the comparative statics analysis in the next section, the effects of

[14]For a formal discussion of the second order condition, see the appendix.

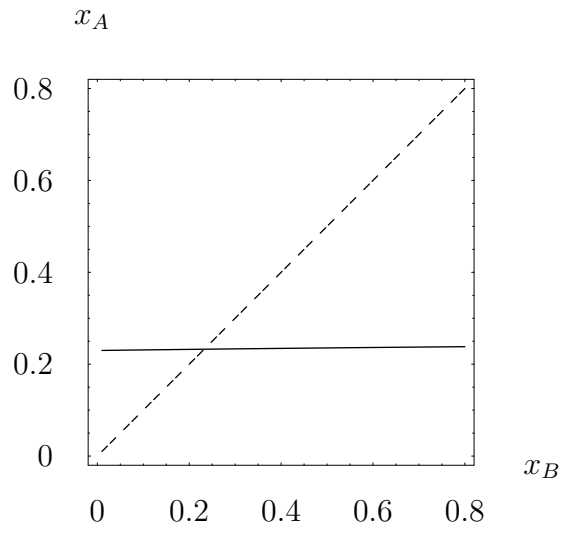

Figure 5.3: Equilibrium effort levels $x_A^* = x_B^* = x^*$

changes in these parameters on equilibrium effort levels will be analyzed.

5.3 Comparative statics

This section analyzes the effects of changes in exogenous parameters on all agents' optimal decisions and payoffs. This allows to characterize specific institutional settings which are more or less favorable for information production and conducive to a profitable participation in business plan contests. Since parameter changes affect expected returns, they also affect effort decisions in the contest, agents' beliefs, and bids in the financing game. The first step of the comparative statics analysis is to assess the effects on entrepreneurs' effort decisions of changes in α (the *a priori* probability that a project is of high quality), γ (the probability with which a high-quality project generates a good signal if no effort is exerted), and μ_g (the expected return from a

high-quality project). Once these effects are established, the impact of parameter changes on equilibrium bids in the financing game and on agents' expected payoffs can also be assessed.

5.3.1 Effects of parameter changes on equilibrium effort levels

Figure 5.4 visualizes the effects of changes in the *a priori* probability α for an entrepreneur to have a high-quality project (i.e. a project with an expected return of μ_g). The values on the ordinate give the resulting equilibrium effort levels x^* for an entrepreneur with a high-quality project. Different values of μ_g are used as location parameters. The dotted graph results from $\mu_g = 0.5$, the solid graph from $\mu_g = 1$, and the dashed one from $\mu_g = 1.5$, while the dot-dashed graph results from $\mu_g = 2$. The value of γ is fixed at $\gamma = \frac{1}{2}$. Note that the maximum level of x^* is still lower than one, which ensures that the condition $0 \leq x \leq \frac{1-\gamma}{\gamma}$ is satisfied.

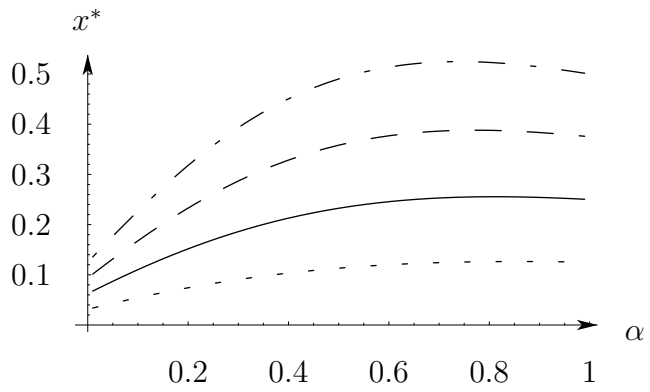

Figure 5.4: Equilibrium effort levels for different α and μ_g

The effect of variations in the location parameter μ_g on equilibrium effort levels is

straightforward: A higher expected return on a high-quality project makes winning the business plan contest more attractive for an entrepreneur, since it leads to higher expected bids from both the VC and the outside investor. Also, winning the business plan contest with a good signal $S_i = \mu_g$ is even more attractive as this raises the bid of the VC. That is, a higher μ_g widens the gap between the expected payoff given a low signal $S_i = 0$ and the expected payoff given a high signal $S_i = \mu_g$. Therefore, it is worthwhile to invest more costly effort into raising the probability for a good signal, since this raises the probability of winning the contest, and it increases the expected payoff in case of winning.

A variation of α, the *a priori* probability that a projet is of high quality, influences equilibrium effort levels in a number of ways. Depending on which effects dominate, a higher α can lead to a rise or a fall in signaling efforts and, hence, in information generation. In figure 5.4, the fall in x^* for higher values of α is most prominent in the dot-dashed graph, i.e. for $\mu = 2$. For even higher levels of μ and lower levels of γ (so as to allow for higher values of x^*), this effect becomes much more pronounced. The intuitive explanation of this phenomenon is as follows: On the one hand, a rise of α tells an entrepreneur with a high-quality project that the probability of the competing entrepreneur also having a high-quality project is rising. It therefore becomes more worthwhile for him to exert effort in order to raise his chance of generating a high signal and therefore of winning the contest. On the other hand, the investors also know α. For them, a higher α - given a constant level of signaling effectiveness γ - means a higher probability of a project being of high quality, even if it has a bad signal, $S_i = 0$. They are therefore, all else being equal, willing to bid higher

for the winning entrepreneur if they observe a higher α, independently of the quality signal.[15] This, in turn, reduces the incentive to exert costly signaling effort. For low levels of α, the first effect dominates: An initially low probability of the other entrepreneur having a high-quality project, combined with a reasonable level of γ, i.e. a reasonable chance to generate a good signal, means that an entrepreneur with a high-quality project has a high chance of winning the business plan contest, even without exerting much effort. A rise in α then means that the probability of the other entrepreneur also having a good project rises, which in turn causes an entrepreneur with a high-quality project to exert more effort. For high levels of α, the second effect dominates: Since both investors assume that even a project with a bad signal is likely to be of high quality, it is less important to generate a good signal, and therefore an entrepreneur with a good project is less eager to exert costly effort. In this situation, a further rise in α diminishes the incentives to exert effort, and x^* falls.

To determine the effects that changes in γ - the probability with which a high-quality project generates a good signal if no effort is exerted - have on x^*, consider figure 5.5. The same values of μ_g are used as location parameters, and α is set to $\frac{1}{2}$. Values above $\gamma = 0.65$ are not considered, since for higher values of the location parameter μ, the resulting x^* would violate the condition $0 \leq x \leq \frac{1-\gamma}{\gamma}$.

As before, a higher μ_g leads to higher equilibrium effort levels x^*. The intuition is the same as above and needs not be repeated.

[15] For a detailed discussion of the effect of changes in parameters on equilibrium bids, see subsection 5.3.2.

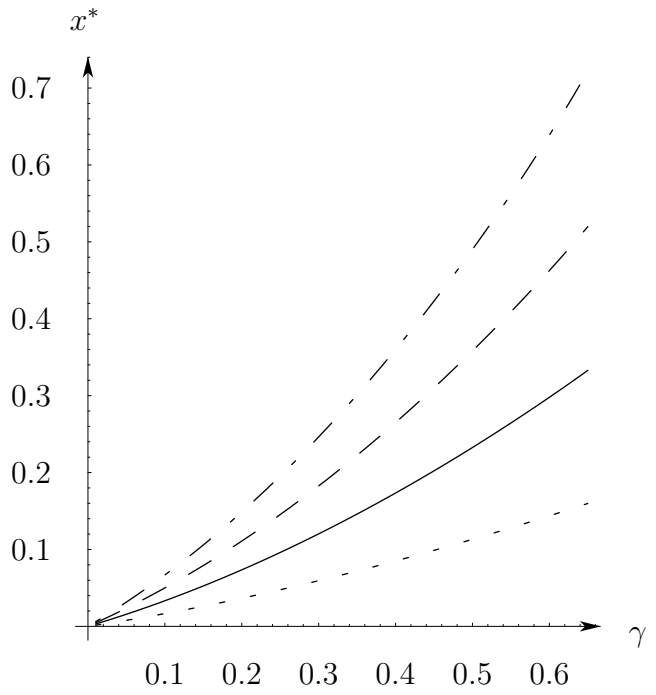

Figure 5.5: Equilibrium effort levels for different γ and μ_g

Like the variation in α above, a variation in γ affects equilibrium effort levels in a number of ways. On the one hand, a high γ implies a high probability of a good signal, even without exerting costly effort. This in turn reduces a manager's effort incentives. On the other hand, a high γ makes costly effort more worthwhile, since it raises its effectiveness (remember that the probability of a high-quality project leading to a good signal is defined as $\gamma + \gamma x_i$). In addition, an increase in the likelihood that a high-quality project generates a good quality signal affects the bidding behavior of both the VC and the outside investor. As discussed in more detail in subsection 5.3.2 below, a higher γ reduces the information asymmetry between the outside investor and the VC if the latter observes a good quality signal (and, therefore, knows for sure that the winning project is of high quality). This is owing to the fact that a higher

signaling effectiveness makes it more likely that the business plan contest is won by a high-quality entrepreneur, which in turn induces the outside investor to raise his valuation for the winning project, and to adjust his expected bid upwards. As a consequence, this approximation of beliefs and the concurrent increase in the outside investor's expected bid forces the VC to also raise her bid, thus forfeiting part of her information rent. This increase in the expected bids from both investors that goes along with a decrease in information asymmetry, results in a higher expected payoff for the winning entrepreneur, thus increasing effort incentives in the business plan contest, as entrepreneurs strive to enhance their quality signals in order to improve their chances of winning. Taken together, the described positive incentive effects clearly dominate the first, negative one, and a rise in γ therefore leads to a rise in the equilibrium effort level x^*.

Figure 5.6 depicts the relationship between μ_g and x^* explicitly for fixed $\alpha = \frac{1}{2}$ and $\gamma = \frac{1}{2}$. In line with figures 5.4 and 5.5, the relation is unambiguous: A higher expected return of the high-quality project makes it worthwhile to spend more effort on signal precision in equilibrium (i.e. on information generation).

Summarizing the results of the above comparative statics analysis, one can derive the following hypotheses on information generation and, hence, selection efficiency in the business plan contest: (a) The higher the probability of a high-quality project is, the more effort is spent on enhancing signal precision. This leads to the hypothesis that information generation is higher in more competitive and highly reputed business plan contests - which are the ones where self-selection is most likely to lead to a higher

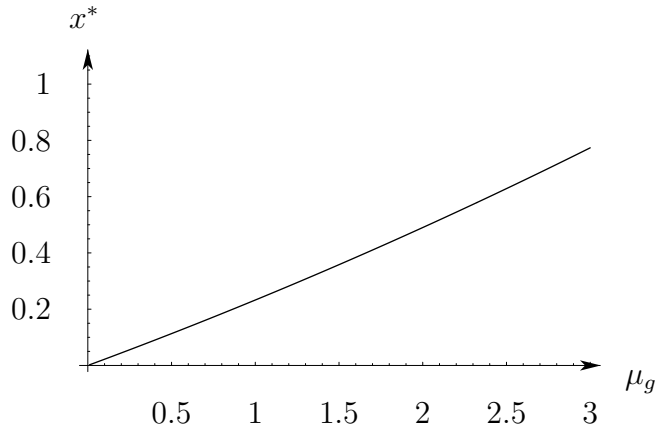

Figure 5.6: Equilibrium effort levels for different μ_g

a priori probability for high-quality projects in the competition. Only in the case of an already high level of this probability does a further increase cause the opposite effect, as the value of additional project-specific quality information becomes less important against the backdrop of the general expectation of a high-quality contestant pool. (b) The higher the probability of generating a good signal is, and the more effective the effort to enhance signal precision is, the more effort is exerted. This leads to the hypothesis that contests between projects in well-understood industries (or industries for which the VC has a special expertise) are superior in terms of information generation. (c) The higher the expected returns of a high-quality project are, the higher is the effort spent on enhancing signal precision. The corresponding hypothesis would be that a focus on projects from higher value-added industries induces more information generation in a business plan contest.

5.3.2 Effects of parameter changes on equilibrium bids in the financing game

After having determined the effects that parameter changes have on equilibrium effort levels, it is now possible to analyze the effects of such changes on equilibrium bids and payoffs in the financing game. In the next step, the analysis will turn to the effects on the bids by both the VC and the outside financier, and after that, the comparative static analysis will be concluded by assessing the effects on all agents' expected payoffs.

As shown above, the VC's optimal bid β depends on the quality signal that she observes. If she observes a low quality signal ($S_w = 0$), she will bid exactly the amount of the conditional expected value of the project, $E[\mu|\, S_w = 0] = \mu_g \cdot \frac{\xi}{\hat{t}}$, with ξ and $\hat{t}$ as defined in equation (5.2.4). An increase in μ_g clearly induces her to increase her bid. Meanwhile, the joint probability ξ that the winning project is of high quality and has a low quality signal depends on the parameters α and γ, as does the probability $\hat{t}$ that the winning project has a low quality signal.

When γ (the probability that a high-quality project generates a high-quality signal if no effort is exerted) rises, both ξ and $\hat{t}$ fall for all relevant parameter ranges. Since ξ falls faster than $\hat{t}$, the VC's bid falls with rising γ if she observes a low quality signal.[16] Intuitively, this is clear: As γ rises, it becomes ever less likely that a high-quality project generates a low quality signal. Therefore, the conditional probability for the project to be of high quality when it has a low quality signal, $\frac{\xi}{\hat{t}}$, falls. This

[16] A formal discussion of the derivative of $\beta(S_w = 0)$ with respect to α and γ can be found in the appendix.

causes the expected value $E[\mu|\, S_w = 0]$, and therefore the bid $\beta(S_w = 0)$, to fall.

When α rises, the probability $\hat{t}$ that the winning project has a low quality signal falls for all relevant parameter ranges, as both projects are more likely to be of high quality. The effect of a rise in α on the joint probability ξ is somewhat more ambiguous. On the one hand, it raises the probability that the winning project is of high quality (again simply because all projects are more likely to be of high quality). On the other hand, as stated above, it also increases equilibrium effort levels, making it more likely that a high-quality project generates a high signal. This, in turn, makes it less likely that a high-quality project should generate a low quality signal, and still win the business plan contest. However, it should be noted that this effort-increasing effect only holds for low and medium values of α, while higher values of α can actually induce a reduction in equilibrium effort levels. While the impact on ξ thus remains ambiguous, the clear-cut decline in $\hat{t}$ ensures that the combined effect on $\frac{\xi}{\hat{t}}$ is positive, causing an increase in the expected value $E[\mu|\, S_w = 0]$, and therefore in the bid $\beta(S_w = 0)$.

If the venture capitalist observes a high quality signal ($S_w = \mu_g$), she will shade her bid downward and bid less than the conditional expected value of the project: $\beta(S_w = \mu_g) = E[\mu|\, S_w = \mu_g] \cdot (1 + \xi - \hat{t})$, where $E[\mu|\, S_w = \mu_g] = \mu_g$, as stated in equation (5.2.5). Again, a rise in μ_g induces the VC to raise her bid, and changes in α and γ affect $\hat{t}$ and ξ in the same way as discussed above. For a broad set of parameter ranges, the VC's bid $\beta(S_w = \mu_g)$ therefore rises both if α rises and if γ rises.[17]

[17] A formal discussion of the derivatives of $\beta(S_w = \mu_g)$ with respect to α and γ can be found in the appendix.

Intuitively, the effect of a rise in α is clear, as it increases the *a priori* probability for high project quality and hence the expected value of the winning project. This, in turn, induces the outside investor to value the winning project higher, and increase his expected bid accordingly. While still randomizing his bid between zero and μ_g, he does so by placing more probability mass on the higher bids. As a result, this approximation of valuations between the outside investor and the VC (in case she observes a good quality signal) reduces the information rent of the latter, as she is forced to also raise her bid (i.e. to shade less).

For a rise in γ, the intuition is somewhat more complex. For the outside financier, a rise in γ means an increase in the probability that the winning entrepreneur has a high-quality project. This is so because a rise in γ makes it more likely that a high-quality project generates a good signal, and if at least one of the two entrepreneurs has a good signal, so has the winner of the business plan contest. This in turn means that the expected project quality of the winning entrepreneur rises, as a good signal guarantees that the project quality is high. Therefore, the outside investor will again put more probability mass on the higher bids, thereby raising his expected bid. As before, this in turn reduces the VC's information rent by allowing for less bid shading, and thus causing her equilibrium bid $\beta(S_w = \mu_g)$ to rise.

A closer look a the effects of parameter changes on the outside financier's equilibrium bids confirms this intuitive line of argument:

As discussed in the appendix, the outside financier always randomizes his bid between zero and μ_g. The only way in which parameter changes affect his bidding behavior is by inducing him to reallocate probability mass between these two extremes.[18] As argued in the appendix, the bidding distribution of the outside financier is piecewise uniform on three different intervals. The boundaries of these intervals are determined by the equilibrium bidding strategy of the venture capitalist: Between zero and $\beta(S_w = 0)$, the outside financier bids according to density function $g_1(b)$; between $\beta(S_w = 0)$ and $\beta(S_w = \mu_g)$, he bids according to density function $g_2(b)$; and between $\beta(S_w = \mu_g)$ and μ_g, he bids according to $g_3(b)$.

As α rises, it becomes more likely that both projects are of high quality, which in turn makes it more likely that the winning project is of high quality. Therefore, the outside financier values the winning project more highly and raises his expected bid by placing more probability mass in the higher intervals of his distribution function. In technical terms, this implies a decrease in the the slope of the distribution function in the first interval, $g_1(b)$, and an increase in the third interval. Also, the upper boundary of the first interval, $\beta(S_w = 0)$, increases, as does the upper boundary of the second interval, $\beta(S_w = \mu_g)$.[19] In more intuitive terms, it is this increase in the outside financier's expected bid which induces the venture capitalist to raise her equilibrium bid for higher values of α, thereby raising her costs, but maintaining her chances of winning the financing game.

[18] Against this backdrop, an increase in μ_g would increase the outside financier's expected bid by raising the upper boundary of the interval over which he randomizes his bid.

[19] A formal discussion of the effects of changes in α and γ on the outside financier's bidding strategy can be found in the appendix.

As γ rises, so does the the probability that a high-quality project generates a good signal. For the outside financier, this implies an increase in the probability that the winning entrepreneur has a high-quality project. Therefore, an increase in γ induces the outside investor to raise his valuation of the winning project, and to increase his expected bid by placing more probability mass in the higher intervals of his distribution function. In technical terms, this implies an increase in the slope of the distribution function in the third interval, $g_3(b)$, as well as an increase in the upper boundary of the second interval, $\beta(S_w = \mu_g)$. The changes in the slope in the first and second interval are more ambiguous, however, and the upper boundary of the first interval $\beta(S_w = 0)$ is lowered. The overall result is an increase in the outside financier's expected bid, which in turn reduces the VC's room for bid-shading and forces her to raise her equilibrium bid conditional on a good signal.

This completes the analysis of the effects of changes in the exogenous variables α and γ on the bidding strategies in the financing game. It was shown that a rise in α induces the venture capitalist to make higher equilibrium bids both if she observes a good signal and if she observes a bad signal. The outside financier in turn responds to a rise in α by placing more probability mass on higher bids. A rise in γ reduces the VC's equilibrium bid if she observes a bad signal, but raises it in case she observes a good signal, and it again induces the outside financier to place more probability mass on higher bids.[20]

[20]Note, however, that it reduces $E[b \mid b > \beta(S_w = 0)]$, the outside investor's expected bid conditional on wining the financing game. This is owing to the decrease in $\beta(S_w = 0)$.

5.3.3 Effects of parameter changes on expected payoffs

This subsection explores the effects of parameter changes on all agents' expected payoffs. A first step examines the changes in the expected payoff of an entrepreneur with a high-quality project. The second step analyzes the changes in the expected payoffs of the venture capitalist. As stated in Proposition 3, the expected payoff of the outside financier is always equal to zero; it is therefore not affected by a change in parameter values.

Changes in the expected payoffs of both a high-quality entrepreneur and of the VC follow the directions which one would expect from the changes in bids: An entrepreneur with a high-quality project benefits from rises in γ (the probability with which a high-quality project generates a good signal if no effort is exerted) and in α (the *a priori* probability that a project is of high quality), while the VC's expected payoff falls in both cases. An increase in μ_g would cause the expected payoffs of both the entrepreneur and the VC to rise. While the entrepreneur would benefit from the associated increase in the VC's bids (conditional on the observed quality signal), these bids would increase by less than the increase in μ_g, thus allowing her to appropriate part of the benefits of a higher expected return on a high-quality project.

As can be seen from equation (5.2.11), the payoff of an entrepreneur with a high-quality project depends on a set of variables, in addition to his own and his competitor's effort levels: Firstly, and most obviously, it depends on the level of μ_g, i.e. on the expected value of the high-quality project. The direction of the influence is clear: All else equal, a higher μ_g results in a higher expected payoff for the entrepreneur.

This is owing to the the assumption that both financiers (VC and outsider) know the value μ_g of a good project. An increase in this value raises the expected payoff of the winning project (as it raises the expected payoffs of all high-quality projects). This, in turn, raises both investors' valuations of the winning project and hence their expected equilibrium bids. Secondly, the entrepreneur's expected payoff also depends on the value of α, which is the *a priori* probability of a high-quality project, and on γ, which influences the probability of generating a high quality signal if the project quality is high.

In equilibrium, the signaling effort of a high quality entrepreneur is given as $x_A^* = x_B^* = x^*$. Plugging this into equation (5.2.11), and using *Mathematica* (Wolfram) to derive the partial derivative with respect to γ yields:[21]

$$\begin{aligned}\frac{\partial \Pi_A^e(\mu_A = \mu_g)}{\partial \gamma} = \frac{1}{8}\,\mu\,(\alpha-1)\Bigg[-1+\alpha\Bigg(-2+(1+x^*)\gamma\Big[-14 \\ +\alpha\Big(-6+(1+x^*)\gamma\big[18+\alpha\big(12+(1+x^*)\gamma(-4(4+\alpha) \\ +5(1+x^*)\alpha\gamma)\big)\big]\Big)\Big)\Bigg]\Bigg]\left(1+x^*+\gamma\underbrace{\frac{\partial x^*}{\partial \gamma}}_{>0}\right)-c'(x^*)\underbrace{\frac{\partial x^*}{\partial \gamma}}_{>0}\quad . \end{aligned} \tag{5.3.1}$$

To determine the sign of this expression around equilibrium values, plug in the reference values $\alpha = \frac{1}{2}$, $\gamma = \frac{1}{2}$, $\mu_g = 1$, and $c'(x^*) = x^*$. The result is positive, as should be expected from the effects of a rise in γ on bids and on the VC's expected payoff. Further numerical calculations show that the positive sign of the derivative persists throughout.[22] That is, a rise in signaling effectiveness γ leads to a rise in the

[21] As before, the discussion focuses on the expected payoff of entrepreneur A, who is assumed to have a high-quality project.

[22] As in chapter 4, it is safe to assume that the marginal cost of effort, $c'(x)$ is not prohibitively high. For the numerical calculations, it is assumed that $c'(x^*) = x^*$.

expected payoff of an entrepreneur with a high-quality project.

For the effects of a change in the *a priori* probability that a project is of high quality, α, on the expected payoff of an entrepreneur with a high-quality project, take the partial derivative with respect to α. *Mathematica* (Wolfram) yields the following result:

$$\frac{\partial \Pi_A^e(\mu_A = \mu_g)}{\partial \alpha} = \frac{1}{8}\,\mu \Bigg\{ 3 + (1+x^*)\gamma \Bigg(1 + 7(1+x^*)\gamma + \alpha\Big[-4 + (1+x^*)\gamma\Big(-8 - 12(1+x^*)\gamma$$
$$+ \alpha\big[-9 + (1+x^*)\gamma\big(6 + 12(1+x^*)\gamma + \alpha(-1+\gamma+x^*\gamma)(-16 + (1+x^*)(-4+5\alpha)\gamma)\big)\big]\Big)\Big]\Bigg)\Bigg]\Bigg)$$
$$+ (-1+\alpha)\gamma\Bigg[-1 + \alpha\Bigg(-2 + (1+x^*)\gamma\Big[-14 + \alpha\Big(-6 + (1+x^*)\gamma\big[18$$
$$+ \alpha\big(12 + (1+x^*)\gamma(-4(4+\alpha) + 5(1+x^*)\alpha\gamma)\big)\big]\Big)\Big]\Bigg)\Bigg]\frac{\partial x^*}{\partial \alpha}\Bigg\} - c'(x^*)\frac{\partial x^*}{\partial \alpha} \quad . \tag{5.3.2}$$

Again, plug in $\alpha = \frac{1}{2}$, $\gamma = \frac{1}{2}$, $\mu_g = 1$, and $c'(x^*) = x^*$ to determine the sign of the derivative near equilibrium. The result is positive and remains so for any relevant α-γ-μ_g-combination. That is, a rise in α also leads to a rise in the expected payoff of an entrepreneur with a high-quality project. This is in line with the increases in both investors' expected bids in response to an increase in α.

In sum, higher values of μ_g, γ and α lead to a rise in the expected payoff of a high-quality entrepreneur. While the argument is straightforward for μ_g, the intuition for the other two variables is as follows: Both a rise in signaling effectiveness γ and in the *a priori* probability for a project to be of high quality, α, contribute to a reduction in information asymmetry between the venture capitalist and the outside investor.[23]

[23] As discussed above, both a rise in α and a rise in γ contribute to a reduction in information

This, in turn, forces the VC to reduce her bid-shading, thereby reducing her expected payoff. The tougher competition between the VC and the outside investor thus allows the winning entrepreneur to retain a larger share of the project's expected value.

As stated in Corollary 4, the venture capitalist makes positive expected profits. Her expected payoff conditional on observing a good quality signal is given as $\Pi_I^e(S_w = \mu_g) = \frac{1}{2}\,\mu_g\,(\hat{t} - \xi)$, varying with the exogenous parameters μ_g, α and γ.[24] As stated above, a rise in a high-quality project's future returns μ_g leads to a rise in the VC's expected payoff. The effects of changes in α and γ are analyzed in more detail below.

Plugging in for $\hat{t}$ and ξ while taking into account that in equilibrium $x_A^* = x_B^* = x^*$, and simplifying yields

$$\Pi_I^e(S_w = \mu_g) = u\mu_g[(1-\alpha)(1-(1+x^*)\alpha\gamma)]\,. \tag{5.3.3}$$

Differentiating this with respect to γ results in

$$\frac{\partial \Pi_I^e(S_w = \mu_g)}{\partial \gamma} = u\,\mu_g \left[(1+x^*)(\alpha^2 - \alpha) + \underbrace{\frac{\partial x^*}{\partial \gamma}}_{>0} (\alpha^2\gamma - \alpha\gamma) \right] < 0\,. \tag{5.3.4}$$

Thus, a rise in γ unambiguously leads to lower expected profits for the VC. This is in line with the intuitive results on the reduction of her information rent, associated with an increase in signaling efficiency.

asymmetry between the outside investor and the VC. Increases in both parameters increase the probability of a high-quality project winning the business plan contest, thereby raising the outside financier's valuation for the winning project, and bringing it closer in line with the valuation of the VC in case she observes a good quality signal.

[24] Remember that her expected payoff conditional on observing a low quality signal is zero.

Differentiating equation (5.3.3) with respect to α yields a less clear cut result:

$$\frac{\partial \Pi_I^e(S_w = \mu_g)}{\partial \alpha} = u\,\mu_g \left[(1 + x^*)\gamma\,(2\alpha - 1) + \alpha\gamma\,\frac{\partial x^*}{\partial \alpha}\,(\alpha - 1) - 1 \right]. \tag{5.3.5}$$

However, plugging in the reference values of $\alpha = \frac{1}{2}$, $\gamma = \frac{1}{2}$, and $\mu_g = 1$ yields a clearly negative result:

$$\frac{\partial \Pi_I^e(S_w = \mu_g)}{\partial \alpha} = u\,\mu_g(-1 - \frac{1}{8}\underbrace{\frac{\partial x^*}{\partial \alpha}}_{>0}) < 0$$

Further numerical calculations show that this result continues to hold for all relevant parameter ranges. Again, this is in line with the intuitive results on the reduction of information asymmetry and the associated decrease in her information rent, caused by an increase in the *a priori* probability that a project is of high quality.

In sum, higher values of γ and α, which both represent lower uncertainty levels regarding the quality of the winning project, lead to a reduction in the expected payoff of the venture capitalist. This is in line with intuition, since it is this very uncertainty, combined with the informational advantage that the VC has over the outside investor, that guarantees her positive returns. If the informational advantage is reduced owing to a reduction in overall uncertainty, less bid shading is optimal, which in turn leads to lower expected payoffs.

All results of the comparative statics analysis are summarized in figure 5.7.

	$\mu_g \uparrow$	$\alpha \uparrow$	$\gamma \uparrow$
x*	$\uparrow$	$\uparrow$; $\downarrow$ (for large α)	$\uparrow$
$\beta(S_w = 0)$	$\uparrow$	$\uparrow$	$\downarrow$
$\beta(S_w = \mu_g)$	$\uparrow$	$\uparrow$	$\uparrow$
$E[b]$	$\uparrow$	$\uparrow$	$\uparrow$
$\Pi_A^e(\mu_A = \mu_g)$	$\uparrow$	$\uparrow$	$\uparrow$
$\Pi_I^e(S_w = \mu_g)$	$\uparrow$	$\downarrow$	$\downarrow$

Figure 5.7: Business plan contest: comparative statics

5.4 Conclusion

The model that was laid out and discussed in this chapter applies methods from the tournament literature to the field of venture capital finance by analyzing the workings of a business plan contest, and it endogenizes the payoffs of competing entrepreneurs by using auction theory to asses the mechanisms of a subsequent financing game.

In the business plan contest, which is modelled as a rank-order tournament, two entrepreneurs compete for winning the contest in order to secure funding for their respective projects. The analysis focuses on the incentives for entrepreneurs to generate meaningful information on their respective projects' quality. While this information is unproductive in the sense that it does not in itself enhance a project's expected payoff, it helps to convey private information about its expected future returns (i.e. its quality) to the venture capitalist who sponsors the contest. The latter has to declare a winner, based on the received quality signals, and it is shown that it is always in her best interest to declare the project with the higher quality signal as the winner of the business plan contest.

In the analysis of the post-contest financing game, equilibrium bidding strategies

and expected payoffs are established - conditional on entrepreneurs' equilibrium effort levels in the business plan contest. The financing game is modelled as a first-price sealed-bid common value auction with asymmetric information, where the venture capitalist has superior information about the winning project's expected returns, relative to an uninformed outside financier. It is shown that no equilibrium exists in pure strategies, but that there exists a unique Bayesian Nash equilibrium where the VC bids according to a pure strategy (conditional on her inside information), making positive expected profits, and the uninformed outside investor randomizes his bid, making zero expected profits.

In a second step of the analysis, entrepreneurs' equilibrium effort decisions for information generation in the business plan contest are established. It is shown that in equilibrium, an entrepreneur with a high-quality project exerts an intermediate effort level which does not guarantee full information revelation, but balances the costs of effort with the increased probability for winning the business plan contest and cashing in on his project by securing the most attractive financing offer.

In a third step, comparative statics highlight the impact of changes in exogenous parameters on the previous results. The effects on equilibrium effort levels are at the center of this assessment, since on the one hand, they affect all agent's expected payoffs, and on the other hand, they allow a direct assessment of selection efficiency in the business plan contest. This is owing to the fact that a higher effort level leads to higher signal precision, and the chance component in choosing the winner is therefore reduced (as soon as at least one project generates a high-quality signal, a high-quality

project is with certainty picked as the winner of the business plan contest).

It is shown that equilibrium effort levels, and, hence, selection efficiency, increase with an increase in all relevant exogenous parameters, with one important exception: if the probability that a project is of high quality, α, is very high, a further rise in this parameter leads to a fall in equilibrium effort levels.[25] This allows to derive hypotheses about selection efficiency in different institutional settings: (a) Selection efficiency tends to be higher in more competitive and highly reputed business plan contests, where self-selection leads to a higher overall quality level of competing projects, since an increase in the *a priori* probability that a project is of high quality increases the entrepreneurs' incentives to exert costly signaling efforts. However, in the case of an extremely competitive environment, selection efficiency will be reduced with a further increase in the level of competitiveness; (b) it also tends to be higher in industry-specific contests, where the venture capitalist has special expertise and the probability of generating a good signal is higher, since the increased probability of creating a good quality signal makes the signaling effort more worthwhile; and (c) it tends to be higher in contests that focus on higher value-added industries, with higher expected returns from a high-quality project, since this increases the entrepreneurs' incentives to exert signaling effort in order to win the business plan contest and obtain financing for their projects.

[25]This is owing to the fact that for high levels of α, financiers assume that even a project with a low quality signal is likely to be of high quality. With a further increase in this parameter, it therefore becomes less important for an entrepreneur to spend costly effort in order to generate a high quality signal.

It is further shown that the venture capitalist's equilibrium payoff from the business plan contest increases with an increase in the expected return of a high-quality project, μ_g, and it decreases with an increase both in the probability that a project is of high quality, α, and in the probability for creating a good signal, γ. The effect of a rise in μ_g is straightforward, as the venture capitalist becomes residual claimant of the project if she wins the bidding game. The negative effect of an increase in the two probabilities on the VC's expected payoff is mainly owing to corresponding changes in the uninformed outside financier's valuation. Both a rise in α and in γ reduce information asymmetry as they lead to an increase in the probability that a high-quality project wins the business plan contest. This, in turn, causes the uninformed financier to raise his expected bid in equilibrium, forcing the venture capitalist to also bid higher, while her valuation of the project remains unchanged.[26] This causes a reduction in her information rent, and, hence, in her expected payoff. The reduction in the VC's information rent which is caused by an increase in α and γ directly benefits the winning entrepreneur, who receives more lucrative financing offers for his project from both financiers.

In line with these results, the expected payoff of an entrepreneur with a high-quality project rises with an increase in all three exogenous parameters. The effect of an increase in the expected return μ_g is again straightforward, as it leads to an increase in all potential financiers' equilibrium bids. Increases in the two probabilities α, with which a project is of high quality, and γ, with which a high-quality project

[26]Note that the comparative statics analysis focuses on the VC's expected payoff conditional on observing a high-quality signal (since the expected payoff when observing a low-quality signal is equal to zero). The expected return of a project with a high-quality signal is μ_g with certainty, independently of the levels of α and γ.

generates a good signal if no effort is exerted, both contribute to a reduction in information asymmetry between the VC and the outside investor and therefore in the venture capitalist's information rent, as argued above. This reduction in information rent is fully appropriated by the entrepreneur, as the outside financier's expected payoff is always equal to zero.

As a result of the above, a business plan contest should be more attractive for entrepreneurs with high-quality projects when the level of competitiveness is higher, and when the contest sponsor has special expertise in the industry. In both cases, outside investors' valuations for the winning entrepreneur are higher, suggesting more lucrative financing offers for an entrepreneur in case he wins. On the other hand, the low level of information asymmetry that is associated with these conditions should make a business plan contest less attractive for a venture capitalist. Against this backdrop, it makes sense that such contests are still predominantly sponsored by other (mainly public) institutions, and venture capitalists often participate only in a supportive role. This allows them to minimize their expenses, while still gaining access to insider information, making use of what little informational advantage they can gain. Meanwhile, the main - public - sponsors of the contest should be happy to see the benefits transferred to the winning entrepreneur, and hope for the expected positive external effects to materialize. The empirical verification of these hypotheses as well as those on selection efficiency is left for future research.

Chapter 6

Conclusion

This thesis explores the role of tournaments as selection devices in the context of heterogeneous competitors and incomplete information. In doing so, it adds to the literature in two ways. Firstly, it introduces a novel tournament-cum-auction framework that allows to endogenize contestants' prizes, thereby accounting for the dual impact of contestants' strategic decision making on the winning probability and on the winning prize. Secondly, it contributes to the literature on internal capital markets and corporate auctions, as well as on the venture capital literature.

Rank-order tournaments are useful incentive and selection mechanisms in multiple agent settings in the the face of imperfect or incomplete information, when cardinal ranking is costly or impossible. Possible fields of application rank from corporate environments (with internal labor and capital markets), to public procurement, research, sports, and other contexts. Chapter 2 gives an overview of the theoretical and empirical literature. It first introduces the basic framework as derived by Lazear and Rosen (1981), with two risk-neutral homogeneous contestants and one risk-neutral principal (tournament sponsor) who sets the spread between fist and second prize

such as to optimize her expected payoff from the agents' combined output. While effort levels are unobservable and output is also influenced by an unobservable luck component, the optimal spread establishes incentives for both contestants to exert first-best productive efforts.

While a rank-order tournament in this basic setup is equivalent to other efficient incentive mechanisms, the remainder of chapter 2 explores circumstances under which rank-order tournaments are indeed superior mechanisms, such as in the presence of correlated external shocks and risk aversion. It introduces additional research on the existence and properties of equilibria, allowing for contestant heterogeneity, multiple contestants and prizes, as well as for different informational settings. While most of the literature takes a specific tournament structure as given, chapter 2 also presents work on the tournament design itself, deriving the optimal number of prizes/contestants, and suggesting the introduction of additional performance standards. The empirical literature, which draws its insights mainly from the realm of competitive sports, aims to verify some of the main hypotheses derived from theory. Among other things, it shows that higher prizes induce higher effort levels, while a higher degree of heterogeneity among contestants reduces effort levels (if contestants' different skill levels are common knowledge *ex ante*). In addition, it shows that in a single-prize tournament, the optimal winning prize increases with the number of contestants.

Chapter 3 lays the groundwork for endogenizing the tournament prize by introducing a post-tournament auction. In the field of research tournaments, some authors

have combined tournaments and auctions before, by introducing pre-tournament selection auctions, or a post-tournament auction such as the one presented by Fullerton and others (2002), where contestants compete in a private value auction, offering the tournament sponsor idiosyncratic combinations of innovation quality and price. In the context of this thesis, however, the framework of an asymmetric common value auction is more appropriate for modeling the post-tournament auction, where investors are assumed to compete over one division/project of common but unknown value.

The chapter presents in detail the analytical framework of a common value auction with asymmetric information, as characterized by Engelbrecht-Wiggans and others (1983). It analyzes equilibrium bidding strategies and expected payoffs, showing that the informed bidder uses a pure strategy in equilibrium, and shades her bid downwards in order to realize positive expected payoffs (her information rent). The uninformed bidder in turn randomizes his bid, and realizes zero expected profits in equilibrium. Chapter 3 also presents two applications of the Engelbrecht-Wiggans and others (1983) framework, one empirical and one theoretical, which confirm the main results and their broad applicability in different contexts.

Chapters 4 and 5 use the building blocks established in the two previous chapters to analyze two different economic selection problems with the help of the tournament-cum-auction framework. Chapter 4 applies the framework to a corporate internal capital market, where headquarters ranks two divisions according to perceived future profitability, and decides to divest the losing division, as resource constraints do not

allow her to finance both. The divestiture takes places via a corporate auction, prior to which one of the potential outside investors acquires inside information about the division's value. Chapter 5 applies the framework to a business plan contest, where two entrepreneurs compete for the quality seal that is associated with winning the tournament, allowing them to secure funding for their respective projects - either from the venture capitalist, who acquires inside information from sponsoring the contest, or from an outside financier.

Both chapters address the same two leading research questions. The first question concerns the equilibrium levels of information generation - determining the degree of selection efficiency - in the tournament phase, and the second is related to the interaction of the post-tournament auction with expected payoffs and incentives. In this context, the tournament-cum-auction framework allows to analyze how the expected impact of signal precision on the winning prize affects equilibrium signaling efforts (and, therefore, selection efficiency) beyond the direct effects that the latter have on the winning probability. Also, changes in exogenous parameters, such as the probability of a high-quality division/project, and its expected return, as well as the effectiveness of signaling efforts, affect selection efficiency in equilibrium through the same two channels - by affecting the probability of winning the tournament, and by affecting the amount of the winning prize. Since both the change in the *a priori* probability for a high-quality division/project (i.e. the quality composition of the contestant pool) and the effectiveness of signaling efforts determine the level of information asymmetry in the post-tournament auction, the main findings can be formulated in terms of the impact of information asymmetry on equilibrium strategies

and expected payoffs.

In chapter 4, the outside investors compete over financing the division that is divested after losing the internal rank-order tournament. As a general result, the better-informed investor shades her bid downwards and realizes positive expected profits only when she observes a high quality signal. It is only in this case, therefore, that information asymmetry influences strategies and expected payoffs. An increase in the quality composition of the contestant pool then decreases information asymmetry, as the uninformed investor attributes a higher probability to the event that a divested division is of high quality (as the overall probability for a division to be of high quality rises). This brings the uninformed investor's expectations closer in line with those of the informed investor who observes a high quality signal and therefore knows for sure that the divested division is of high quality. On the other hand, an increase in the effectiveness of signaling increases information asymmetry, as it induces the outside investor to attribute a lower probability to the event that a divested division is of high quality (and divested only because it was mistaken for a low-quality division).

Against this backdrop, while a decrease in information asymmetry always leads to a lower expected payoff (information rent) for the informed investor and a higher expected payoff for corporate headquarters, it does not always lead to higher selection efficiency. Indeed, a decrease in information asymmetry raises selection efficiency only when this decrease is triggered by an increase in the quality composition of the contestant pool. If it is triggered by a decrease in the effectiveness of signaling, it will

actually lead to a reduction in selection efficiency.

In chapter 5, the venture capitalist and the outside financier compete over financing the project that has won the business plan contest. In this case, both an increase in the quality composition of the contestant pool and an increase in the effectiveness of signaling lead to a decrease in information asymmetry, since they induce the outside financier to attribute a higher probability to the event that the winning project is indeed of high quality. Against this backdrop, a decrease in information asymmetry always leads to higher selection efficiency. The only exception emerges in the case of a very high quality composition of the contestant pool. In this case, a further increase of the probability that a project is of high quality would reduce selection efficiency, as it reduces the incentives for information generation.[1] This non-linearity is in line with the result of Hvide and Kristiansen (2003), who also analyze the impact of changes in exogenous parameters on selection efficiency. However, their result occurs for intermediate quality levels of the contestant pool, and it is driven by a different logic, namely that an increase in the opponent's expected quality causes a contestant to pursue a riskier strategy, thus increasing the variance of his output and decreasing selection efficiency.

The above presented findings on the impact of information asymmetries on expected payoffs and on selection efficiency, as well as the related hypotheses on the effects of different market conditions and institutional settings on selection efficiency

[1]The reduction in incentives for exerting costly signaling effort is owing to the increase in the prize for a contest winner with a bad quality signal, as the bid of the venture capitalist will be higher if a high-quality contestant pool induces her to believe that a project is likely to be of high quality in spite of a low quality signal.

in internal capital markets and in business plan contests - as presented in the conclusions of the respective chapters - lend themselves to empirical verification. For instance, the empirical observation that business plan contests are often called within narrowly defined industries in which the contest sponsor has a special expertise, and are also often confined to higher value-added industries, could be motivated by the higher selection efficiency that is associated with these types of contest. On the other hand, it was shown that this increase in selection efficiency reduces the expected payoff of the contest sponsor, which could be an explanation for the observed strong involvement of universities and public institutions in these contests, as they are more interested in supporting the entrepreneurs and in potential spillover-effects than in their own private returns. A thorough empirical verification of all the presented results and their implications is, however, beyond the scope of this thesis, and must be left for future research.

Another path for future research would consist in applying the tournament-cum-auction framework to other economic selection problems. Ready examples can be found in the realm of public contracting, where architectural or design competitions are used to determine the most desired construction project, but the work on the realization of the project itself is only determined afterwards, through public auctions in which contractors submit competing offers. It is safe to assume that the architectural or construction firm that submitted the winning design has inside information about the intricacies and potential costs of realizing the project, turning the public auction into one of asymmetric information. A further example from corporate finance could be the selection of an underwriter bank for a firm's initial public offering. In

the tournament phase, several potential underwriters compete for the contract, and in the second phase, the firm is auctioned off (in small slices) in the stock market at a price that is determined through competitive bidding and influenced by market beliefs about the characteristics of the underwriter.

Future research could also add to the theoretical understanding of the tournament-cum-auction framework. One critical assumption in the presented setup is the fact that signal distortions can occur only in the case of a high-quality division/project, and, in line with that, they can only be downward distortions. If the signal of a low-quality division/project were upward distorted, there would be no natural incentive to increase signal precision (i.e. to reduce the signal), and an additional incentive mechanism would have to be created. Furthermore, the effect of any signaling effort in the presented setup was designed in such a way that an overshooting is ruled out. Empirical observation suggests, however, that contestants frequently engage in window-dressing activities, in order to make their divisions/projects seem of higher quality. By allowing the signaling effort to overshoot, resulting in a quality signal that exceeds the true quality of the division/project, this phenomenon could also be modelled, and its implications on expected payoffs and selection efficiency could be assessed.

On the other hand, the introduction of multiple contestants, prizes, and financiers is unlikely to change the main results. As was argued in chapter 2, the optimal number of contestants under a broad range of assumptions is two, which is in line with the present setup. Also, the addition of a second prize for the loser would not affect the

incentive structure as long as the prize spread is appropriately chosen. The addition of further uninformed outside investors in the post-tournament auction would also not change the results, as all uninformed investors will bid according to the same mixed strategy in equilibrium, and realize zero expected profits. Only the addition of further informed investors would change the equilibrium results. As soon as a second bidder has the same private information about the common value object, bid shading becomes impossible and all information rent is dissipated, causing all bidders to realize zero expected profits. In effect, this would reduce the post-tournament auction to a common value auction with complete information.

Appendix A

Appendix to chapter 4

A.1 Proof of Proposition 2

The proof is analogous to that in Rajan (1992), in following Engelbrecht-Wiggans and others (1983) and Hendricks and Porter (1988).

In the corporate auction, the informed investor chooses β to maximize

$$\Pi_I^e = \Pr\{\text{ informed investor wins }\} \cdot [\, E[\mu|\, S_l] - \beta(S_l)\,] \,. \tag{A.1.1}$$

The informed investor's private information, the signal value S_l of the divested division, enters her decision problem only through $H = E[\mu|\, S_l]$. Assume without loss of generality that the informed investor observes the real valued random variable H rather than S_l. After observing the signal, she can be characterized by her information-induced *type* h. The solution method requires a one-to-one mapping between the information-induced type of the informed investor and her equilibrium bid. As H is not continuously distributed in this problem, the types must be "smoothed out" in order to obtain it. This is done by allowing the informed investor mixed

strategies. She can randomize her bid by using a random variable U which is independent of (μ, S) and has an atomless distribution on $[0,1]$. A mixed strategy β of the informed investor is a function from $\Re \times [0,1] \to [0,1]$, and $\beta(h,u)$ is the bid when $H = h$ and $U = u$. Also, assume without loss of generality that β is nondecreasing in u for fixed values of h.

With these assumptions, it is possible to deduce the informed investor's *distributional type*.[1] Let $\{(H,U) < (h,u)\}$ denote the event $\{H < h, \text{ or } H = h \text{ and } U < u\}$, let $t(h,u)$ be the probability of that event and define $T = T(H,U)$. T is called the informed investor's *distributional type* and is uniformly distributed on $[0,1]$. Letting $H(t) = \inf\{h|\, P(H \leq h) > t\}$, gives $H = H(T)$ almost surely. Therefore, the distributional type T carries all the information that H does, but it has the advantage of being a continuous distribution. It is now possible to write the informed investor's bidding strategy β as a function from the space of types $t \in [0,1]$ to the space of bids $[0,1]$:

$$\beta(t) = E[H(T)|\, T \leq t\,;\, \text{lower S}\,], \tag{A.1.2}$$

which is continuous and non-decreasing in t. As in Hendricks and Porter (1988), the "uninformed" outside investor has access to some public information about the expected value of the divested division: The fact that the division is being divested implies that it must have lost the tournament for internal financing, and is therefore likely to have a lower quality signal ("lower S"). All probabilities and expected values are therefore made contingent on this information.[2]

[1] As pointed out before, the notion of *distributional type* was first introduced by Milgrom and Weber (1985).

[2] For the better informed inside investor, this is redundant information, and it could therefore be

While the informed investor uses a mixed bidding strategy for purely technical reasons in this general framework, the uniformed investor must bid according to a mixed strategy in order to avoid sure losses, as will be shown in Lemma 1. Together with the above, Lemma 1 concludes the proof of Proposition 2.

Lemma 1: *The strategies* (β, G) *form a Bayesian Nash equilibrium if the informed investor bids:*

$$\beta(t) = E[H(T)|\, T \leq t\,;\, \text{lower S}\,] \tag{A.1.3}$$

and the distribution of the uninformed bid is

$$G(\beta(t)) = F(h,\, \text{lower S}\,), \tag{A.1.4}$$

where $F(h,$ *lower* $S)$ *is the joint distribution function of the realization of* h *and the event of losing the internal tournament.*

Proof: In a Bayesian Nash Equilibrium, the players' equilibrium strategies maximize their respective expected payoffs, conditional on their information set and taking the strategy of the other player as given. The proof consists of the following steps: (1) Show the equilibrium bids have identical support. (2) Use this to show that the uninformed investor makes zero profits in equilibrium. (3) Set the uninformed investor's profit to zero to obtain the optimal bid for the informed investor. (4) Use the optimizing behavior of the informed investor to derive the bidding strategy for the uninformed investor.

---

omitted.

Steps (1) and (2) are identical to the argument in Engelbrecht-Wiggans and others (1983), as discussed above, so they are omitted here. For step (3), remember that as the uninformed investor bids an amount b, his expected profit is

$$\Pi_U^e = \Pr\{\text{ uninf. investor wins }\} \cdot [\, E[\mu|\,\text{lower S; uninf. investor wins}\,] - b\,] \quad \text{(A.1.5)}$$

with "lower S" describing the fact that the division has lost the internal ranking contest, and "uninf. investor wins" meaning that the uninformed investor wins the corporate auction. Setting this expression equal to zero gives

$$E[\mu|\,\text{lower S; uninf. investor wins}\,] - b = 0.$$

Since the equilibrium bids of the informed and the uninformed investor have identical support, the uninformed investor's bid b can take the value $\beta(t)$. Of course, the rule that the uninformed investor's expected profit is zero also holds in this case. This yields:

$$\beta(t) = E[\mu|\,\text{lower S; uninf. investor wins}\,],$$

which, in terms of H and T is the same as[3]

$$\beta(t) = E[H(T)|\,T \le t\,;\,\text{lower S}\,]. \quad \text{(A.1.6)}$$

Using the formula for conditional expected values, this is equivalent to

$$\beta(t) = \frac{1}{F(t;\,\text{lower S}\,)} \cdot \int_0^t H(s) \cdot f(s;\,\text{lower S}\,)\,ds. \quad \text{(A.1.7)}$$

[3]The fact that the uninformed investor wins the bidding game is mirrored in the assumption that $T \le t$: If the uninformed investor bids according to $\beta(t)$, he can only win if the signal that the informed investor observes is equal to or less than t, since this induces her to bid less than or equal to $\beta(t)$.

Integration by parts results in

$$\beta(t) = H(t) - \frac{1}{F(t;\text{ lower S})} \cdot \int_0^t F(s;\text{ lower S})\, dH(s). \tag{A.1.8}$$

The bidding strategy of the informed investor maximizes her expected payoff, given the strategy of the uninformed investor. Thus, the equilibrium strategy of the uninformed investor must induce the informed investor to bid according to $\beta(t)$ in equilibrium (step (4)).

After observing t, the informed investor maximizes

$$G(\beta(t)) \cdot [H(t) - \beta(t)] \tag{A.1.9}$$

with respect to $\beta(t)$.[4] The first order condition is then given by

$$dG(\beta)\,[H(t) - \beta(t)] - G(\beta)\, d\beta \stackrel{!}{=} 0\,,$$

with

$$dG(\beta) = \underbrace{\frac{\partial G(\beta)}{\partial \beta}\frac{\partial \beta}{\partial t}}_{G'(\beta)}\, dt\,.$$

Transforming the FOC into

$$\frac{G'(\beta)}{G(\beta)} = \frac{\frac{d\beta}{dt}}{H(t) - \beta(t)}$$

and plugging in for $\frac{d\beta}{dt}$ and $\beta(t)$ yields

$$\frac{G'(\beta)}{G(\beta)} = \frac{F(t;\text{ lower S}) \cdot H(t) \cdot f(t;\text{ lower S}) - f(t;\text{ lower S}) \int_0^t H(s) f(s;\text{ lower S})\, ds}{[F(t;\text{ lower S})]^2 \cdot \frac{1}{F(t;\text{ lower S})} \int_0^t F(s;\text{ lower S})\, dH(s)}\,.$$

[4] Where $G(\beta(t))$ is the probability that the uninformed investor's bid does not exceed $\beta(t)$.

Simplifying results in

$$\frac{G'(\beta)}{G(\beta)} = \frac{f(t;\text{ lower S})}{F(t;\text{ lower S})}$$

and therefore

$$G(\beta(t)) = F(t;\text{ lower S}). \tag{A.1.10}$$

This completes the proof of Lemma 1.

Proof of Corollary 1

Given the general equilibrium strategies in Lemma 1, the specific bidding strategies for the model in the text can be derived as follows: Find the equivalents of $f(t;\text{ lower S})$, $F(t;\text{ lower S})$, and t, calculate their values for the different signals $S_l = 0$ and $S_l = \mu_g$ which the informed investor can possibly observe, and use the results to determine the equilibrium bidding strategies. Knowing the distribution of μ and of the signal values, all probabilities and conditional expected values can be calculated with the help of a "divestiture matrix" (figure A.1) and the tournament probability tree (figure A.2). While the matrix gives an overview of all possible value and signal combinations, the probability tree allows to trace the probabilities for all these possible outcomes and the resulting divestiture decisions. Also, assume without loss of generality that U is uniform on $[0,1]$ with $L(u) = u$ and $l(u) = 1$.

When the informed investor's *distributional type* is $T = t$, this means she really observes $(H, U) = (h, u)$. Therefore,

$$f(t;\text{ lower S}) = k(h; u;\text{ lower S}) = \Pr\{H = h \cap \text{ lower S}\} \cdot \underbrace{l(u)}_{=1}. \tag{A.1.11}$$

Integrating over h and u gives

$$F(t;\text{ lower S}) = K(h;u;\text{ lower S}) = \int_u \int_h \Pr\{H = h \cap \text{ lower S}\} \cdot l(u)\, dh\, du$$
$$= u \cdot \int_h \Pr\{H = h \cap \text{ lower S}\}\, dh\,. \qquad \text{(A.1.12)}$$

			Division B		
			$\mu_B = 0$	$\mu_B = \mu_g$	
			$S_B = 0$	$S_B = 0$	$S_B = \mu_g$
Division A	$\mu_A = 0$	$S_A = 0$	50 : 50 lottery (1)	50 : 50 lottery (2)	A is divested (5)
	$\mu_A = \mu_g$	$S_A = 0$	50 : 50 lottery (3)	50 : 50 lottery (4)	A is divested (6)
		$S_A = \mu_g$	B is divested (7)	B is divested (8)	50 : 50 lottery (9)

Figure A.1: Divestiture matrix

The value of $t = \Pr\{H < h, \text{ or } H = h \,\text{and}\, U < u\}$ can be calculated as follows: For $S_l = 0$, one gets $t(S_l = 0) = \Pr\{E[\mu|S_l] = E[\mu|S_l = 0] \,\text{and}\, U < u\}$, which is equivalent to

$$t(S_l = 0) = [1 - \Pr\{S_l = \mu_g\}] \cdot u \;=\; [1 - \alpha^2\gamma^2\,(1 + x_A)(1 + x_B)] \cdot u\,; \qquad \text{(A.1.13)}$$

and for $S_l = \mu_g$, this becomes $t = \Pr\{E[\mu|S_l] = E[\mu|S_l = 0]\} + \Pr\{E[\mu|S_l] = E[\mu|S_l = \mu_g] \,\text{and}\, U < u\}$, or

$$t(S_l = \mu_g) = 1 - (1 - u) \cdot\, \alpha^2\gamma^2\,(1 + x_A)(1 + x_B)\,. \qquad \text{(A.1.14)}$$

Note that for $S_l = 0$ and $u = 1$, t takes on the same value $\hat{t}$ as for $S_l = \mu_g$ and $u = 0$:

$$\hat{t} = t(S_l = 0; u = 1) = t(S_l = \mu_g; u = 0) = 1 - \alpha^2\gamma^2\,(1 + x_A)(1 + x_B)\,. \qquad \text{(A.1.15)}$$

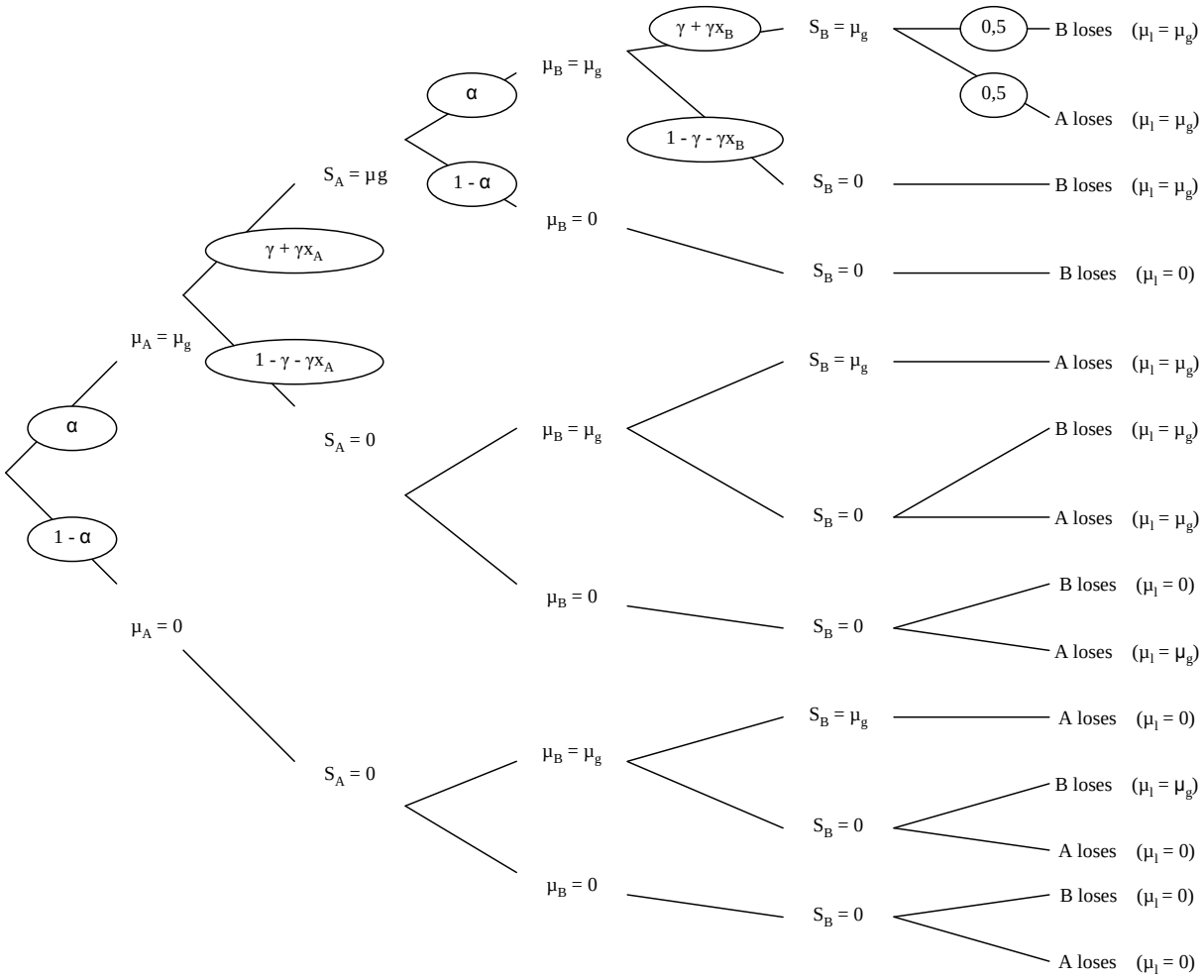

Figure A.2: Tournament probability tree

For $S_l = 0$, or, equivalently, $t \leq \hat{t}$,

$$\begin{aligned} f(t(S_l = 0);\ \text{lower S}) &= \Pr\{E[\mu \mid S_l] = E[\mu \mid S_l = 0] \cap \text{lower S}\} \\ &= 1 - \alpha^2\gamma^2(1 + x_A)(1 + x_B) \ = \ \hat{t}, \end{aligned} \tag{A.1.16}$$

and

$$F(t(S_l = 0);\ \text{lower S}) \ = \ u \cdot [1 - \alpha^2\gamma^2\,(1 + x_A)(1 + x_B)] \ = \ u \cdot \hat{t}, \tag{A.1.17}$$

while

$$H(t(S_l = 0)) \ = \ E\,[\mu \,|\, S_l = 0\,] = \mu_g \cdot \frac{\Pr\{\mu = \mu_g \cap S_l = 0\}}{\Pr\{S_l = 0\}} \ = \ \mu_g \cdot \frac{\xi}{\hat{t}}\ , \tag{A.1.18}$$

with $\xi = \alpha - 0,5(2 + x_A + x_B)(1 - \alpha)\alpha\gamma - (1 + x_A)(1 + x_B)\alpha^2\gamma^2\,.$

Plugging (A.1.13), (A.1.16), (A.1.17), and (A.1.18) into (A.1.7) and (A.1.10) yields the equilibrium bidding strategies for $S_l = 0$:

$$\beta(t(S_l = 0)) \ = \ \mu_g \cdot \frac{\xi}{\hat{t}} \ = \ H(t(S_l = 0)) \tag{A.1.19}$$

and

$$G(\beta(t(S_l = 0))) = u \cdot \hat{t}\,. \tag{A.1.20}$$

For $S_l = \mu_g$, or, equivalently, $t \geq \hat{t}$,

$$\begin{aligned} f(t(S_l = \mu_g);\ \text{lower S}) \ &= \ \Pr\{E\,[\mu \,|\, S_l\,] = E\,[\mu \,|\, S_l = \mu_g\,] \cap \text{ lower S}\,\} \\ &= \ \alpha^2\gamma^2\,(1 + x_A)(1 + x_B) \\ &= \ 1 - \hat{t}\,, \end{aligned} \tag{A.1.21}$$

and

$$\begin{aligned} F(t(S_l = \mu_g);\, \text{lower S}) &= u \cdot [\Pr\{E[\mu \,|\, S_l] = E[\mu \,|\, S_l = 0] \cap \text{loser}\} \\ &\quad + \Pr\{E[\mu \,|\, S_l] = E[\mu \,|\, S_l = \mu_g] \cap \text{loser}\}] \\ &= u \cdot [1 - \alpha^2\gamma^2 (1 + x_A)(1 + x_B) \\ &\quad + \alpha^2\gamma^2 (1 + x_A)(1 + x_B)] \\ &= u\,, \end{aligned} \tag{A.1.22}$$

while

$$H(t(S_l = \mu_g)) \;=\; E[\mu \,|\, S_l = \mu_g] \;=\; \mu_g \cdot \frac{\Pr\{\mu = \mu_g \cap S_l = \mu_g\}}{\Pr\{S_l = \mu_g\}} \;=\; \mu_g\,. \tag{A.1.23}$$

Plugging (A.1.14), (A.1.21), (A.1.22), and (A.1.23) into (A.1.7) and (A.1.10) yields the equilibrium bidding strategies for $S_l = \mu_g$:

$$\begin{aligned} \beta(t(S_w = \mu_g)) &= \frac{1}{u} \cdot \int_0^{1+(u-1)(1-\hat{t})} H(s) \cdot f(s;\, \text{lower S})\, ds \\ &= \frac{1}{u} \cdot \int_0^{u\cdot\hat{t}} H(s) \cdot f(s;\, \text{lower S})\, ds + \frac{1}{u} \int_{u\cdot\hat{t}}^{\hat{t}} H(s) \cdot f(s;\, \text{lower S})\, ds \\ &\quad + \frac{1}{u} \int_{\hat{t}}^{1+(u-1)(1-\hat{t})} H(s) \cdot f(s;\, \text{lower S})\, ds \\ &= \frac{1}{u} \cdot \mu_g \frac{\xi}{\hat{t}}\, u\,\hat{t} + 0 + \frac{1}{u} \cdot \mu_g\,(u - u\hat{t}) \\ &= \mu_g \cdot (1 + \xi - \hat{t}) = H(t(S_l = \mu_g)) \cdot (1 + \xi - \hat{t}) \end{aligned} \tag{A.1.24}$$

and

$$G(\beta(t(S_l = \mu_g))) = u\,. \tag{A.1.25}$$

This completes the proof of Corollary 1.

A.2 Determination of $Pr\{\text{win}\}$ and $Pr\{\text{MBO}\}$

The probability that division manager A wins the internal tournament, given that he has high expected future returns ($\mu_A = \mu_g$), is the sum of the probabilities that he wins with a high quality signal and with a low quality signal:

$$Pr\{\text{win}\} = Pr\{S_A = \mu_g \cap A \text{ wins} \,|\, \mu_A = \mu_g\} + Pr\{S_A = 0 \cap A \text{ wins} \,|\, \mu_A = \mu_g\} \quad \text{(A.2.1)}$$

Plugging in the conditional probabilities (compare Fig. A.2) yields

$$\begin{aligned} Pr\{\text{win}\} &= (\gamma + \gamma x_A)\left[\alpha(\gamma + \gamma x_B)\tfrac{1}{2} + \alpha(1 - \gamma - \gamma x_B) + (1 - \alpha)\right] \\ &+ (1 - \gamma - \gamma x_A)\left[\alpha(1 - \gamma - \gamma x_B)\tfrac{1}{2} + (1 - \alpha)\tfrac{1}{2}\right]. \end{aligned} \quad \text{(A.2.2)}$$

The probability that the informed investor wins the corporate auction - i.e. that a management buy-out (MBO) takes place - is the weighted sum of the probabilities that she wins after observing a high quality signal and after observing a low quality signal:[5]

$$\begin{aligned} Pr\{\text{MBO}\} &= Pr\{S_A = \mu_g \,|\, \mu_A = \mu_g\} \cdot Pr\{\text{MBO} \,|\, S_l = \mu_g\} \\ &+ Pr\{S_A = 0 \,|\, \mu_A = \mu_g\} \cdot Pr\{\text{MBO} \,|\, S_l = 0\} \end{aligned} \quad \text{(A.2.3)}$$

The probability that the informed investor wins the corporate auction is equivalent to the probability that the uninformed investor's bid does not exceed that of the informed investor. Conditional on the observed signal and together with Corollary 1 (b), this gives:

$$Pr\{\text{MBO} \,|\, S_l = \mu_g\} = G(\beta(t(S_l = \mu_g))) = u \quad \text{(A.2.4.a)}$$

[5]Since Pr{MBO} describes the probability of a management buy-out from the perspective of division manager A with $\mu_A = \mu_g$, the probabilities for each signal value are conditional on $\mu_A = \mu_g$.

and

$$Pr\{\text{MBO} \mid S_l = 0\} = G(\beta(t(S_l = 0))) = u \cdot \hat{t}\,. \tag{A.2.4.b}$$

Plugging (A.2.4.a) and (A.2.4.b) into (A.2.3), and using the conditional probabilities for the two possible realizations of S_A, results in

$$Pr\{\text{MBO}\} = (\gamma + \gamma x_A) \cdot u + (1 - \gamma - \gamma x_A) \cdot u \cdot \hat{t}\,. \tag{A.2.5}$$

The division manager does not know the realization of the random variable u; he therefore assigns to it the expected value $E[u] = \frac{1}{2}$, resulting in:

$$Pr\{\text{MBO}\} = \frac{1}{2}(\gamma + \gamma x_A) + \frac{\hat{t}}{2}(1 - \gamma - \gamma x_A)\,. \tag{A.2.6}$$

A.3 The expected bid of the uninformed investor, conditional on winning the corporate auction

From Corollary 1 (b), it is known that $G(\beta(S_l = 0)) = u \cdot \hat{t}$ and $G(\beta(S_l = \mu_g)) = u$, where $G(\beta)$ is the probability that the outside investor tenders a bid not exceeding β. Also, it is clear that the support of G must be equal to the range of β, which is $[0, \mu_g]$.[6] For an intuitive explanation of why the uninformed investor must randomize his bid between these two extremes, consider the following reasoning: If he were to bid no higher than, say, $\beta(S_l = \mu_g)$, then he would never win the bidding game in the case of a good signal ($S_l = \mu_g$), while making expected losses whenever bidding too much for a division with a bad signal. Assigning a positive probability to bids higher than $\beta(S_l = \mu_g)$ is thus the only way to assure zero expected payoffs, given the bidding strategy of the informed investor. Bidding more than μ_g, on the other hand,

[6] This results from step 1 in the proof of Proposition 2.

would not make sense as it would generate sure losses. The outside investor must also bid with positive probability between $\beta(S_l = 0)$ and $\beta(S_l = \mu_g)$, since, if this were not the case, the informed investor could lower her bid in the case of a good signal without reducing her chances to win and therefore increase her expected payoff. The same argument holds for the range between 0 and $\beta(S_l = 0)$: Any bid b lower than $\beta(S_l = 0)$ causes the uninformed investor to lose the auction, given that the informed investor does not bid below $\beta(S_l = 0)$ in equilibrium. Thus, the uninformed investor might as well bid zero with a positive probability. However, this strategy would allow the informed investor to lower her bid to a marginal $\varepsilon > 0$ without reducing her winning probability, thereby generating a positive expected return for the case of a low quality signal $S_l = 0$. Therefore, in order to support the informed investor's equilibrium bidding strategy, the uninformed investor has to bid with positive probability in the range $[0, \beta(S_l = 0)]$.

Now, given the support for the uninformed investor's randomizing strategy, his bidding strategy is determined by the distribution function $G(\beta)$. The interval $[0, \mu_g]$ is divided into three subintervals by the available information on $G(\beta)$: $G(0) = 0$, $G(\beta(S_l = 0)) = u \cdot \hat{t}$, $G(\beta(S_l = \mu_g)) = u$, and $G(\mu_g) = 1$. Assuming without loss of generality that this distribution is piecewise uniform, this leads to the following piecewise defined density function:

$$g(b) = \begin{cases} g_1(b) = \frac{u \cdot \hat{t}}{\beta(S_l=0)} & \text{for} \quad 0 \leq b \leq \beta(S_l = 0) \\ g_2(b) = \frac{u \cdot (1-\hat{t})}{\beta(S_l=\mu_g) - \beta(S_l=0)} & \text{for} \quad \beta(S_l = 0) < b \leq \beta(S_l = \mu_g) \\ g_3(b) = \frac{1-u}{\mu_g - \beta(S_l=\mu_g)} & \text{for} \quad \beta(S_l = \mu_g) < b \leq \mu_g \end{cases} \tag{A.3.1}$$

With this density function, the information needed to determine $E[b \,|\, b > \beta(S_l =$

0)] and $E[b \,|\, b > \beta(S_l = \mu_g)]$ is complete. It is now straightforward to calculate the expected bid of the outside investor, conditional on $b > \beta(S_l = \mu_g)$, respective on $b > \beta(S_l = 0)$:

$$\begin{aligned} & E[\, b \,|\, b > \beta(S_l = \mu_g)] = \\ &= \frac{1}{\Pr\{b > \beta(S_l = \mu_g)\}} \cdot \int_{\beta(S_l=\mu_g)}^{\mu_g} b \cdot g_3(b)\, db \\ &= \frac{1}{1 - G(\beta(S_l = \mu_g))} \cdot \frac{1-u}{\mu_g - \beta(S_l = \mu_g)} \cdot \int_{\beta(S_l=\mu_g)}^{\mu_g} b\, db \\ &= \frac{1}{2}\left(\mu_g + \beta(S_l = \mu_g)\right) \end{aligned} \tag{A.3.2}$$

and

$$\begin{aligned} & E[\, b \,|\, b > \beta(S_l = 0)] = \\ &= \frac{1}{\Pr\{b > \beta(S_l = 0)\}} \cdot \int_{\beta(S_l=0)}^{\mu_g} b \cdot g(b)\, db \\ &= \frac{1}{1 - G(\beta(S_l = 0))} \cdot \left[\int_{\beta(S_l=0)}^{\beta(S_l=\mu_g)} b \cdot g_2(b)\, db + \int_{\beta(S_l=\mu_g)}^{\mu_g} b \cdot g_3(b)\, db \right] \\ &= \frac{u(1-\hat{t}\,)\frac{1}{2}\left(\beta(S_l = \mu_g) + \beta(S_l = 0)\right) + (1-u)\frac{1}{2}\left(\mu_g + \beta(S_l = \mu_g)\right)}{1 - u\,\hat{t}}, \end{aligned}$$

which, owing to $E[\, u\,] = \frac{1}{2}$, becomes

$$E[\, b \,|\, b > \beta(S_l = 0)] = \frac{(1-\hat{t}\,)\left(\beta(S_l = \mu_g) + \beta(S_l = 0)\right) + \left(\mu_g + \beta(S_l = \mu_g)\right)}{4\,(1 - \frac{1}{2}\,\hat{t}\,)} \tag{A.3.3}$$

A.4 Second order condition for the optimal effort decision

In order for the second order condition to hold, the second derivative of the managers' expected utility functions, $\frac{\partial^2 \Pi_i^e}{\partial x_i^2}$, must be negative. As before, the analysis focuses only on the second derivative for manager A, as symmetry implies that the second order

condition holds for manager B if it holds for manager A. Taking the second derivative of Π_A^e with respect to x_A yields:

$$\frac{\partial^2 \Pi_A^e(\mu_A = \mu_g)}{\partial x_A^2} = v\frac{1}{2}(1+x_B)\alpha^2\gamma^3(2+(-3-3x_A+\alpha+\alpha x_B)\gamma) - c''(x_A)\,. \quad \text{(A.4.1)}$$

Equation (A.4.1) is always negative if $(2+(-3-3x_A+\alpha+\alpha x_B)\gamma) < 0$. This is ensured for high values of γ and low values of α. However, the second order condition may hold even if this strict condition is violated, since the fact that $c''(x_A) \geq 0$ eases the constraint. As demonstrated in the text, the second order condition for the optimum effort level holds for the chosen parameter values, as it does for a wide range of plausible values.

A.5 Effects of parameter changes on $Pr\{\text{MBO}\}$

After plugging in for $\hat{t}$, setting $x_A^* = x_B^* = x^*$ and simplifying, the probability of a management buy-out, as given in equation (A.2.6), becomes:

$$Pr\{\text{MBO}\} = \frac{1}{2}(1+(1+x^*)^2\alpha^2\gamma^2(-1+\gamma+x^*\gamma)) \quad \text{(A.5.1)}$$

The derivative of $Pr\{\text{MBO}\}$ with respect to α is then given as:

$$\frac{\partial Pr\{\text{MBO}\}}{\partial \alpha} = \frac{1}{2}(1+x^*)\alpha\gamma^2(2(1+x^*)(-1+\gamma+x^*\gamma))+\alpha(-2+3(1+x^*)\gamma)\frac{\partial x^*}{\partial \alpha}\ . \quad \text{(A.5.2)}$$

This expression is negative for small and medium values of γ, but turns positive as $\gamma \to \frac{1}{1+x^*}$.

The derivative with respect to γ is given as:

$$\frac{\partial Pr\{\text{MBO}\}}{\partial \gamma} = \frac{1}{2}(1+x^*)\alpha^2\gamma\underbrace{(-2+3(1+x^*)\gamma)}_{sign}(1+x^*+\gamma\frac{\partial x^*}{\partial \gamma})\ . \quad \text{(A.5.3)}$$

The sign of this expression depends on the sign of the term *sign*. For small values of γ, this term is negative, turning the whole derivative negative. For large γ, the derivative of $Pr\{\text{MBO}\}$ with respect to γ turns positive.

A.6 Effects of parameter changes on bidding strategies

When analyzing the effects of a change in α and γ on the informed investor's bid, one must also take into account the effect of the observed quality signal on the optimal bidding strategy. If she observes a low quality signal, the informed investor bids according to $\beta(S_l = 0) = \mu_g \cdot \frac{\xi}{\hat{t}} = E[\mu | S_l = 0]$. It is straightforward to take the derivative of this expression both with respect to α and with respect to γ:

$$\frac{\partial \beta(S_l = 0)}{\partial \alpha} = \mu_g \frac{\hat{t}\frac{\partial \xi}{\partial \alpha} - \xi \frac{\partial \hat{t}}{\partial \alpha}}{\hat{t}^2} \tag{A.6.1}$$

$$\frac{\partial \beta(S_l = 0)}{\partial \gamma} = \mu_g \frac{\hat{t}\frac{\partial \xi}{\partial \gamma} - \xi \frac{\partial \hat{t}}{\partial \gamma}}{\hat{t}^2} \tag{A.6.2}$$

The signs of these expressions depend on the reactions of ξ and $\hat{t}$ to changes in α and γ. These are given below, evaluated at the equilibrium effort level, $x_A^* = x_B^* = x^*$:

$$\frac{\partial \hat{t}}{\partial \alpha} = -2\,\alpha^2\gamma(1+x^*)^2 - 2\,\alpha^2\gamma^2 \underbrace{\frac{\partial x^*}{\partial \alpha}}_{>0} < 0 \tag{A.6.3}$$

$$\frac{\partial \hat{t}}{\partial \gamma} = -2\,\alpha\gamma^2(1+x^*)^2 - 2\,\alpha^2\gamma^2 \underbrace{\frac{\partial x^*}{\partial \gamma}}_{>0} < 0 \tag{A.6.4}$$

$$\frac{\partial \xi}{\partial \alpha} = 1 - \underbrace{(1+x^*)\gamma}_{1 \geq \cdot > 0} + \underbrace{2(1+x^*)(\alpha\gamma - \alpha\gamma^2)}_{>0} - \underbrace{\frac{\partial x^*}{\partial \alpha}[\alpha\gamma - \alpha^2\gamma + \alpha^2\gamma^2]}_{>0} > 0 \tag{A.6.5}$$

$$\frac{\partial \xi}{\partial \gamma} = \underbrace{-(1+x^*)\alpha(1-\alpha) - \frac{\partial x^*}{\partial \gamma}\alpha\gamma(1-\alpha)}_{<0} \underbrace{-2(1+x^*)\alpha^2 - \frac{\partial x^*}{\partial \gamma}}_{>0}\alpha^2\gamma^2 < 0 \quad \text{(A.6.6)}$$

Plugging equation (A.6.3) and (A.6.5) into equation (A.6.1) shows that $\frac{\partial \beta(S_l=0)}{\partial \alpha} > 0$.

Since $|\frac{\partial \xi}{\partial \gamma}| > |\frac{\partial \hat{t}}{\partial \gamma}|$ and $\hat{t} > \xi$, plugging equations (A.6.4) and (A.6.6) into equation (A.6.2) confirms that $\frac{\partial \beta(S_l=0)}{\partial \gamma} < 0$.

If the informed investor observes a high quality signal, she bids according to $\beta(S_l = \mu_g) = \mu_g \cdot (1 + \xi - \hat{t})$. Accordingly, the derivatives with respect to α and with respect to γ are:

$$\frac{\partial \beta(S_l = \mu_g)}{\partial \alpha} = \mu_g \Big(\underbrace{\frac{\partial \xi}{\partial \alpha}}_{>0} - \underbrace{\frac{\partial \hat{t}}{\partial \alpha}}_{<0} \Big) > 0 \qquad \text{(A.6.7)}$$

$$\frac{\partial \beta(S_l = \mu_g)}{\partial \gamma} = \mu_g \Big(\underbrace{\frac{\partial \xi}{\partial \gamma}}_{<0} - \underbrace{\frac{\partial \hat{t}}{\partial \gamma}}_{<0} \Big) < 0 \quad \text{, given that } \left|\frac{\partial \xi}{\partial \gamma}\right| > \left|\frac{\partial \hat{t}}{\partial \gamma}\right| \text{ .} \qquad \text{(A.6.8)}$$

The uninformed investor bids according to a piecewise defined density function, given in equation (A.3.1). A change in the parameters α and γ affects the boundaries of the three intervals as well as the density function within each interval.

A rise in γ leads to a decrease in the upper boundaries of the first and second interval, since it causes $\beta(S_l = 0)$ and $\beta(S_l = \mu_g)$ to fall. It also leads to a decline of

$g_3(b)$, the density function in the third interval:

$$\frac{\partial g_3}{\partial \gamma} = \frac{(1-u)\big(\overbrace{\frac{\partial \beta(S_l=\mu_g)}{\partial \gamma}}^{<0}\big)}{\big(\mu_g - \beta(S_l=\mu_g)\big)^2} < 0 \ . \tag{A.6.9}$$

The overall effect of an increase in γ on the outside investor's equilibrium bid distribution is therefore a downward shift of probability mass, causing his expected bid to fall.

A rise in α leads to an increase in the upper boundaries of the first and second interval, since it causes $\beta(S_l = 0)$ and $\beta(S_l = \mu_g)$ to rise. It also leads to a rise of $g_3(b)$, the density function in the third interval, while it causes a decrease of $g_1(b)$, the density function in the first interval:

$$\frac{\partial g_3}{\partial \alpha} = \frac{-(1-u)\big(-\overbrace{\frac{\partial \beta(S_l=\mu_g)}{\partial \alpha}}^{>0}\big)}{\big(\mu_g - \beta(S_l=\mu_g)\big)^2} > 0 \ , \tag{A.6.10}$$

$$\frac{\partial g_1}{\partial \alpha} = \frac{\beta(S_l=0)\, u\, \overbrace{\frac{\partial \hat{t}}{\partial \alpha}}^{<0} - u\, \hat{t}\, \overbrace{\frac{\partial \beta(S_l=0)}{\partial \alpha}}^{>0}}{\big(\beta(S_l=0)\big)^2} < 0 \ . \tag{A.6.11}$$

Hence, the overall effect of an increase in α on the outside investor's equilibrium bid distribution is an upward shift of probability mass, which causes his expected bid to rise.

Appendix B

Appendix to chapter 5

B.1 Proof of Proposition 3

The proof is analogous to that in Rajan (1992), in following Engelbrecht-Wiggans and others (1983) and Hendricks and Porter (1988).

In the financing game, the VC (the inside investor) chooses β to maximize

$$\Pi_I^e = \Pr\{\text{ VC wins financing game }\} \cdot [\, E[\mu|\, S_w] - \beta(S = S_w)\,]\,. \tag{B.1.1}$$

The VC's private information, the signal value S_w of the winning entrepreneur, enters her decision problem only through $H = E[\mu|\, S_w]$. Assume without loss of generality that she observes the real valued random variable H rather than S_w. After observing the signal, she can be characterized by her information-induced *type* h. The solution method requires a one-to-one mapping between the information-induced type of the VC and her equilibrium bid. As H is not continuously distributed in this problem, the types must be "smoothed out" in order to obtain it. This is done by allowing the VC mixed strategies. She can randomize her bid by using a random variable U which is independent of (μ, S) and has an atomless distribution on $[0, 1]$. A mixed strategy

β of the VC is a function from $\Re \times [0,1] \to [0,1]$ and $\beta(h,u)$ is the bid when $H = h$ and $U = u$. Also, assume without loss of generality that β is nondecreasing in u for fixed values of h.

With these assumptions, it is possible to deduce the VC's *distributional type*.[1] Let $\{(H,U) < (h,u)\}$ denote the event $\{H < h, \text{ or } H = h \text{ and } U < u\}$, let $t(h,u)$ be the probability of that event and define $T = T(H,U)$. T is called the VC's *distributional type* and is uniformly distributed on $[0,1]$. Letting $H(t) = \inf\{h|\, P(H \leq h) > t\}$, gives $H = H(T)$ almost surely. Therefore, the distributional type T carries all the information that H does, but it has the advantage of being a continuous distribution. It is now possible to write the VC's bidding strategy β as a function from the space of types $t \in [0,1]$ to the space of bids $[0,1]$:

$$\beta(t) = E[H(T)|\, T \leq t\,;\, \text{higher S}\,], \tag{B.1.2}$$

which is continuous and non-decreasing in t. As in Hendricks and Porter (1988), the "uninformed" outside financier has access to some public information about the expected value of the winning project: The fact that the project has won the business plan contest implies that it is likely to have a higher quality signal ("higher S"). All probabilities and expected values are therefore made contingent on this information.[2]

While the VC uses a mixed bidding strategy for purely technical reasons in this general framework, the outside investor must bid according to a mixed strategy in order to avoid sure losses, as will be shown in Lemma 2. Together with the above,

[1] As pointed out before, the notion of *distributional type* was first introduced by Milgrom and Weber (1985).

[2] For the better informed VC, this is redundant information, and it could therefore be omitted.

Lemma 2 concludes the proof of Proposition 3.

Lemma 2: *The strategies* (β, G) *form a Bayesian Nash equilibrium if the VC bids:*

$$\beta(t) = E[H(T)|\, T \leq t\,;\, \text{higher S}\,] \tag{B.1.3}$$

and the distribution of the uninformed bid is

$$G(\beta(t)) = F(h,\, \text{higher S}\,), \tag{B.1.4}$$

where $F(h,$ *higher* $S)$ *is the joint distribution function of the realization of* h *and the event of wining the business plan contest.*

Proof: In a Bayesian Nash Equilibrium, the players' equilibrium strategies maximize their respective expected payoffs, conditional on their information set and taking the strategy of the other player as given. The proof consists of the following steps: (1) Show the equilibrium bids have identical support. (2) Use this to show that the uninformed outside financier makes zero profits in equilibrium. (3) Set the outsider's profit to zero to obtain the optimal bid for the venture capitalist. (4) Use the optimizing behavior of the VC to derive the bidding strategy for the outsider.

Steps (1) and (2) are identical to the argument in Engelbrecht-Wiggans and others (1983), as discussed above, and are omitted here. For step (3), remember that since the outside financier bids an amount b, his expected profit is

$$\Pi_O^e = \Pr\{\,\text{outsider wins}\,\} \cdot [\, E[\mu|\,\text{higher S; outsider wins}\,] - b\,] \tag{B.1.5}$$

with "higher S" describing the fact that the entrepreneur has won the business plan contest, and "outsider wins" meaning that the uninformed outside investor wins the

financing game. Setting this expression equal to zero gives

$$E[\mu|\,\text{higher S; outsider wins}\,] - b = 0.$$

Since the equilibrium bids of the VC and the outside investor have identical support, the outsider's bid b can take the value $\beta(t)$. Of course, the rule that the outside investor's expected profit is zero also holds in this case. This results in:

$$\beta(t) = E[\mu|\,\text{higher S; outsider wins}\,],$$

which, in terms of H and T is the same as[3]

$$\beta(t) = E[H(T)|\,T \leq t\,;\,\text{higher S}\,]. \tag{B.1.6}$$

Using the formula for conditional expected values, this is equivalent to

$$\beta(t) = \frac{1}{F(t;\,\text{higher S}\,)} \cdot \int_0^t H(s) \cdot f(s;\,\text{higher S}\,)\,ds. \tag{B.1.7}$$

Integration by parts results in

$$\beta(t) = H(t) - \frac{1}{F(t;\,\text{higher S}\,)} \cdot \int_0^t F(s;\,\text{higher S}\,)\,dH(s). \tag{B.1.8}$$

The bidding strategy of the VC maximizes her expected payoff, given the strategy of the outside investor. Thus, the equilibrium strategy of the outside investor must induce the VC to bid according to $\beta(t)$ in equilibrium (step (4)).

After observing t, the VC maximizes

$$G(\beta(t)) \cdot [H(t) - \beta(t)] \tag{B.1.9}$$

[3]The fact that the outside investor wins the financing game is mirrored in the assumption that $T \leq t$: If the outsider bids according to $\beta(t)$, he can only win if the signal that the VC observes is equal to or less than t, since this induces her to bid less than or equal to $\beta(t)$.

with respect to $\beta(t)$.[4] The first order condition is then given by

$$dG(\beta)\,[H(t)-\beta(t)]-G(\beta)\,d\beta \stackrel{!}{=} 0\,,$$

with

$$dG(\beta)=\underbrace{\frac{\partial G(\beta)}{\partial \beta}\frac{\partial \beta}{\partial t}}_{G'(\beta)}\,dt\,.$$

Transforming the FOC into

$$\frac{G'(\beta)}{G(\beta)}=\frac{\frac{d\beta}{dt}}{H(t)-\beta(t)}$$

and plugging in for $\frac{d\beta}{dt}$ and $\beta(t)$ yields

$$\frac{G'(\beta)}{G(\beta)}=\frac{F(t;\text{ higher S})\cdot H(t)\cdot f(t;\text{ higher S})-f(t;\text{ higher S})\int_0^t H(s)f(s;\text{ higher S})\,ds}{[f(t;\text{ higher S})]^2\,\cdot\,\frac{1}{F(t;\text{higher S})}\int_0^t F(s;\text{ higher S})\,dH(s)}\,.$$

Simplifying results in

$$\frac{G'(\beta)}{G(\beta)}=\frac{f(t;\text{ higher S})}{F(t;\text{ higher S})}$$

and therefore

$$G(\beta(t))=F(t;\text{ higher S})\,. \tag{B.1.10}$$

This completes the proof of Lemma 2.

Proof of Corollary 3

Given the general equilibrium strategies in Lemma 2, the specific bidding strategies for the model in the text can be derived as follows: Find the equivalents of $f(t;\text{ higher S})$, $F(t;\text{ higher S})$, and t, calculate their values for the different signals

[4] Where $G(\beta(t))$ is the probability that the outside investor's bid does not exceed $\beta(t)$.

$S_w = 0$ and $S_w = \mu_g$ which the VC can possibly observe, and use the results to determine the equilibrium bidding strategies. Knowing the distribution of μ and and of the signal values, all probabilities and conditional expected values can be calculated with the help of a "winning matrix" (figure B.1) and the contest probability tree (figure B.2). While the matrix gives an overview of all possible value and signal combinations, the probability tree allows to trace the probabilities for all these possible outcomes and the resulting contest winners. Also, assume without loss of generality that U is uniform on $[0, 1]$ with $L(u) = u$ and $l(u) = 1$.

When the VC's *distributional type* is $T = t$, this means she really observes $(H, U) = (h, u)$. Therefore,

$$f(t;\text{ higher S}) = k(h;u;\text{ higher S}) = \Pr\{H = h \cap \text{ higher S}\} \cdot \underbrace{l(u)}_{=1}. \qquad \text{(B.1.11)}$$

Integrating over h and u gives

$$\begin{aligned} F(t;\text{ higher S}) = K(h;u;\text{ higher S}) &= \int_u \int_h \Pr\{H = h \cap \text{ higher S}\} \cdot l(u)\, dh\, du \\ &= u \cdot \int_h \Pr\{H = h \cap \text{ higher S}\}\, dh\,. \end{aligned} \qquad \text{(B.1.12)}$$

			Entrepreneur B		
			$\mu_B = 0$	$\mu_B = \mu_g$	
			$S_B = 0$	$S_B = 0$	$S_B = \mu_g$
Entrepreneur A	$\mu_A = 0$	$S_A = 0$	50 : 50 lottery (1)	50 : 50 lottery (2)	B wins (5)
	$\mu_A = \mu_g$	$S_A = 0$	50 : 50 lottery (3)	50 : 50 lottery (4)	B wins (6)
		$S_A = \mu_g$	A wins (7)	A wins (8)	50 : 50 lottery (9)

Figure B.1: Winning matrix

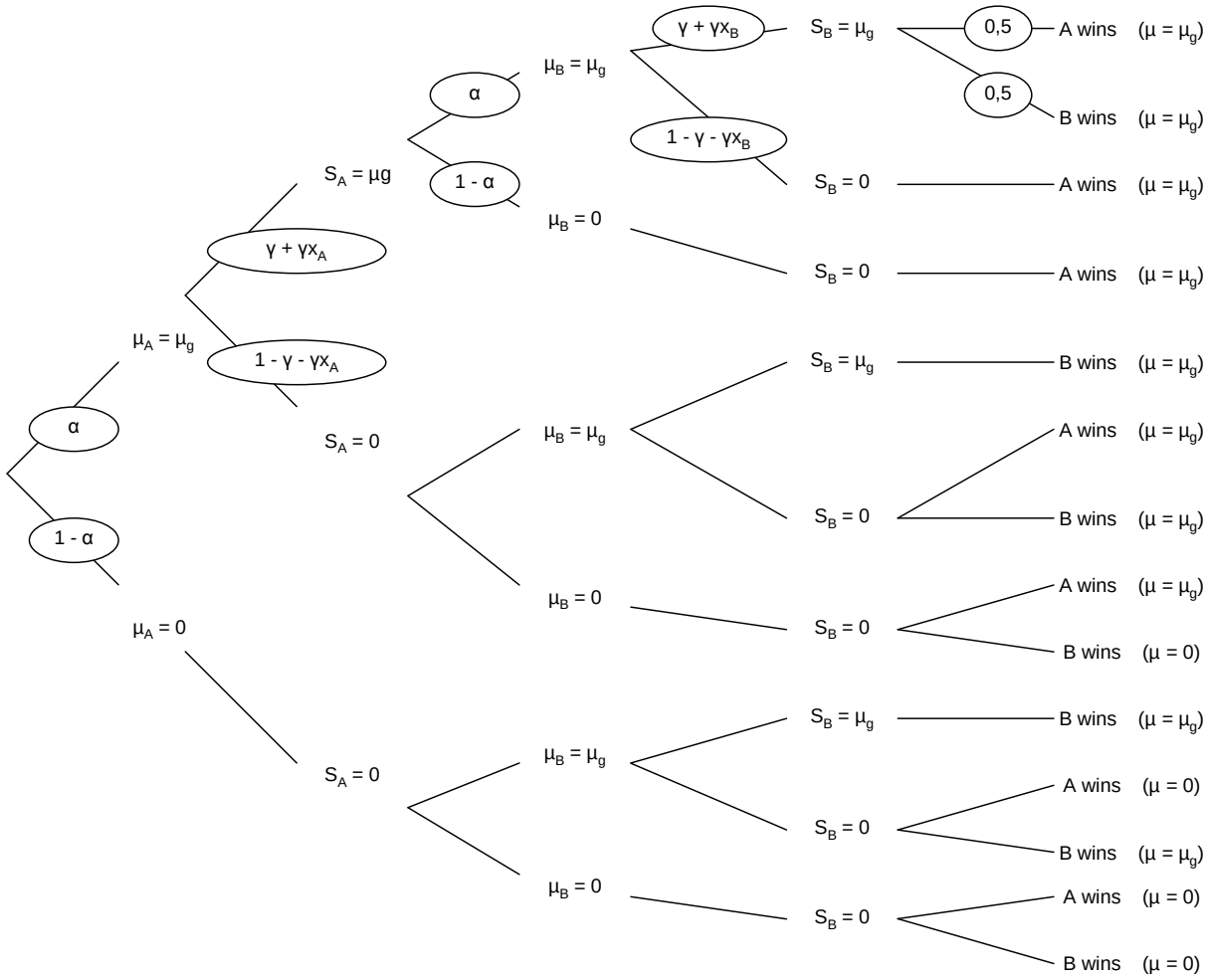

Figure B.2: Contest probability tree

The value of $t = \Pr\{H < h, \text{ or } H = h \,\text{and}\, U < u\}$ can be calculated as follows: For $S_w = 0$, one gets $t(S_w = 0) = \Pr\{E[\mu|S_w] = E[\mu|S_w = 0] \,\text{and}\, U < u\}$, wich is equivalent to

$$t(S_w = 0) = \left((1 + x_A)\alpha\gamma - 1\right)\left((1 + x_B)\alpha\gamma - 1\right) \cdot u\,; \tag{B.1.13}$$

and for $S_w = \mu_g$, this becomes $t = \Pr\{E[\mu|S_w] = E[\mu|S_w = 0]\} + \Pr\{E[\mu|S_w] =$

$E[\mu|S_w = \mu_g]$ and $U < u\}$, or

$$t(S_w = \mu_g) = 1 + (u-1) \cdot \left[1 - \left((1+x_A)\alpha\gamma - 1\right)\left((1+x_B)\alpha\gamma - 1\right)\right]. \quad \text{(B.1.14)}$$

Note that for $S_w = 0$ and $u = 1$, t takes on the same value $\hat{t}$ as for $S_w = \mu_g$ and $u = 0$:

$$\hat{t} = t(S_w = 0; u = 1) = t(S_w = \mu_g; u = 0) = \left((1+x_A)\alpha\gamma - 1\right)\left((1+x_B)\alpha\gamma - 1\right). \quad \text{(B.1.15)}$$

For $S_w = 0$, or, equivalently, $t \leq \hat{t}$,

$$\begin{aligned} f(t(S_w = 0); \text{ higher S}) &= \Pr\{E[\mu \mid S_w] = E[\mu \mid S_w = 0] \cap \text{ higher S}\} \\ &= ((1+x_A)\alpha\gamma - 1)((1+x_B)\alpha\gamma - 1) = \hat{t}, \quad \text{(B.1.16)} \end{aligned}$$

and

$$F(t(S_w = 0); \text{ higher S}) = u \cdot ((1+x_A)\alpha\gamma - 1)((1+x_B)\alpha\gamma - 1) = u \cdot \hat{t}, \quad \text{(B.1.17)}$$

while

$$H(t(S_w = 0)) = E[\mu \mid S_w = 0] = \mu_g \cdot \frac{\Pr\{\mu = \mu_g \cap S_w = 0\}}{\Pr\{S_w = 0\}} = \mu_g \cdot \frac{\xi}{\hat{t}}, \quad \text{(B.1.18)}$$

with $\xi = \alpha - 0,5(2 + x_A + x_B)(1+\alpha)\alpha\gamma + (1+x_A)(1+x_B)\alpha^2\gamma^2$.

Plugging (B.1.13), (B.1.16), (B.1.17), and (B.1.18) into (B.1.7) and (B.1.10) yields the equilibrium bidding strategies for $S_w = 0$:

$$\beta(t(S_w = 0)) = \mu_g \cdot \frac{\xi}{\hat{t}} = H(t(S_w = 0)) \quad \text{(B.1.19)}$$

and

$$G(\beta(t(S_w = 0))) = u \cdot \hat{t}. \quad \text{(B.1.20)}$$

For $S_w = \mu_g$, or, equivalently, $t \geq \hat{t}$,

$$\begin{aligned} f(t(S_w = \mu_g);\, \text{higher S}) &= \Pr\{E[\mu \mid S_w] = E[\mu \mid S_w = \mu_g] \cap \text{higher S}\} \\ &= [1 - ((1+x_A)\alpha\gamma - 1)\,((1+x_B)\alpha\gamma - 1)] \\ &= 1 - \hat{t}\,, \end{aligned} \tag{B.1.21}$$

and

$$\begin{aligned} F(t(S_w = \mu_g);\, \text{higher S}) &= u \cdot [\Pr\{E[\mu \mid S_w] = E[\mu \mid S_w = 0] \cap \text{higher S}\} \\ &\quad + \Pr\{E[\mu \mid S_w] = E[\mu \mid S_w = \mu_g] \cap \text{higher S}\}] \\ &= u \cdot [((1+x_A)\alpha\gamma - 1)((1+x_B)\alpha\gamma - 1) \\ &\quad + [1 - ((1+x_A)\alpha\gamma - 1)\,((1+x_B)\alpha\gamma - 1)]] \\ &= u\,, \end{aligned} \tag{B.1.22}$$

while

$$H(t(S_w = \mu_g)) \;=\; E[\mu \mid S_w = \mu_g] \;=\; \mu_g \cdot \frac{\Pr\{\mu = \mu_g \cap S_w = \mu_g\}}{\Pr\{S_w = \mu_g\}} \;=\; \mu_g\,. \tag{B.1.23}$$

Plugging (B.1.14), (B.1.21), (B.1.22), and (B.1.23) into (B.1.7) and (B.1.10) yields the equilibrium bidding strategies for $S_w = \mu_g$:

$$\begin{aligned} \beta(t(S_w = \mu_g)) &= \frac{1}{u} \cdot \int_0^{1+(u-1)(1-\hat{t}} H(s) \cdot f(s;\, \text{higher S})\, ds \\ &= \frac{1}{u} \cdot \int_0^{u\cdot\hat{t}} H(s) \cdot f(s;\, \text{higher S})\, ds + \frac{1}{u} \int_{u\cdot\hat{t}}^{\hat{t}} H(s) \cdot f(s;\, \text{higher S})\, ds \\ &\quad + \frac{1}{u} \int_{\hat{t}}^{1+(u-1)(1-\hat{t}} H(s) \cdot f(s;\, \text{higher S})\, ds \\ &= \frac{1}{u} \cdot \mu_g \frac{\xi}{\hat{t}}\, u\,\hat{t} + 0 + \frac{1}{u} \cdot \mu_g\,(u - u\hat{t}) \\ &= \mu_g \cdot (1 + \xi - \hat{t}) = H(t(S_w = \mu_g)) \cdot (1 + \xi - \hat{t}) \end{aligned} \tag{B.1.24}$$

and

$$G(\beta(t(S_w = \mu_g))) = u\,. \tag{B.1.25}$$

This completes the proof of Corollary 3.

B.2 The expected bid of the outside investor, conditional on winning the financing game

From Corollary 3 (b), it is known that $G(\beta(S = 0)) = u \cdot \hat{t}$ and $G(\beta(S = \mu_g)) = u\,$, where $G(\beta)$ is the probability that the outside investor tenders a bid not exceeding β. Also, it is clear that the support of G must be equal to the range of β, which is $[0, \mu_g]$.[5] For an intuitive explanation of why the outside investor must randomize his bid between these two extremes, consider the following reasoning: If he were to bid no higher than, say, $\beta(S_w = \mu_g)$, then he would never win the financing game in the case of a good signal ($S_w = \mu_g$), while making expected losses whenever bidding too much for a project with a bad signal. Assigning a positive probability to bids higher than $\beta(S_w = \mu_g)$ is thus the only way to ensure zero expected payoffs, given the bidding strategy of the VC. Bidding more than μ_g, on the other hand, would not make sense as it would generate sure losses. The outside investor must also bid with positive probability between $\beta(S_w = 0)$ and $\beta(S_w = \mu_g)$, since, if this were not the case, the VC could lower her bid in the case of a good signal without reducing her chances to win and therefore increase her expected payoff. The same argument holds for the range between 0 and $\beta(S_w = 0)$: Any bid b lower than $\beta(S_w = 0)$ causes the outside investor to lose the financing game, given that the VC does not bid below $\beta(S_w = 0)$ in equilibrium. Thus, the outside investor might as well bid zero with

[5] This results from step 1 in the proof of Proposition 3.

a positive probability. However, this strategy would allow the VC to lower her bid to a marginal $\varepsilon > 0$ without reducing her winning probability, thereby generating a positive expected return for the case of a low quality signal $S_w = 0$. Therefore, in order to support the VC's equilibrium bidding strategy, the outside investor has to bid with positive probability in the range $[0, \beta(S_w = 0)]$.

Now, given the support for the outside investor's randomizing strategy, his bidding strategy is determined by the distribution function $G(\beta)$. The interval $[0, \mu_g]$ is divided into three subintervals by the available information on $G(\beta)$: $G(0) = 0$, $G(\beta(S_w = 0)) = u \cdot \hat{t}$, $G(\beta(S_w = \mu_g)) = u$, and $G(\mu_g)) = 1$. Assuming without loss of generality that this distribution is piecewise uniform, this leads to the following piecewise defined density function:

$$g(b) = \begin{cases} g_1(b) = \frac{u \cdot \hat{t}}{\beta(S_w=0)} & \text{for} \quad 0 \leq b \leq \beta(S_w = 0) \\ g_2(b) = \frac{u \cdot (1-\hat{t})}{\beta(S_w=\mu_g) - \beta(S_w=0)} & \text{for} \quad \beta(S_w = 0) < b \leq \beta(S_w = \mu_g) \\ g_3(b) = \frac{1-u}{\mu_g - \beta(S_w=\mu_g)} & \text{for} \quad \beta(S_w = \mu_g) < b \leq \mu_g \end{cases} \tag{B.2.1}$$

With this density function, the information needed to determine $E[b \,|\, b > \beta(S_w = 0)]$ and $E[b \,|\, b > \beta(S_w = \mu_g)]$ is complete. It is now straightforward to calculate the expected bid of the outside investor, conditional on $b > \beta(S_w = \mu_g)$, and on $b > \beta(S_w = 0)$, respectively:

$$\begin{aligned} & E[\, b \,|\, b > \beta(S_w = \mu_g)] = \\ &= \frac{1}{\Pr\{b > \beta(S_w = \mu_g)\}} \cdot \int_{\beta(S_w=\mu_g)}^{\mu_g} b \cdot g_3(b)\, db \\ &= \frac{1}{1 - G(\beta(S_w = \mu_g))} \cdot \frac{1-u}{\mu_g - \beta(S_w = \mu_g)} \cdot \int_{\beta(S_w=\mu_g)}^{\mu_g} b\, db \\ &= \frac{1}{2}\left(\mu_g + \beta(Sw = \mu_g)\right) \end{aligned} \tag{B.2.2}$$

and

$$
\begin{aligned}
&E[\,b\,|\,b > \beta(S_w = 0)] = \\
&= \frac{1}{\Pr\{b > \beta(S_w = 0)\}} \cdot \int_{\beta(S_w=0)}^{\mu_g} b \cdot g(b)\, db \\
&= \frac{1}{1 - G(\beta(S_w = 0))} \cdot \left[\int_{\beta(S_w=0)}^{\beta(S_w=\mu_g)} b \cdot g_2(b)\, db + \int_{\beta(S_w=\mu_g)}^{\mu_g} b \cdot g_3(b)\, db \right] \\
&= \frac{u(1-\hat{t}\,)\frac{1}{2}\,(\beta(S_w = \mu_g) + \beta(S_w = 0)) + (1-u)\frac{1}{2}\,(\mu_g + \beta(S_w = \mu_g))}{1 - u\,\hat{t}}\,,
\end{aligned}
$$

which, owing to $E[\,u\,] = \frac{1}{2}$, becomes

$$
E[\,b\,|\,b > \beta(S_w = 0)] = \frac{(1-\hat{t}\,)\,(\beta(S_w = \mu_g) + \beta(S_w = 0)) + (\mu_g + \beta(S_w = \mu_g))}{4\,(1 - \frac{1}{2}\,\hat{t}\,)}\,. \tag{B.2.3}
$$

B.3 Second order condition for the optimal effort decision

In order for the second order condition to hold, the second derivative of the entrepreneurs' expected utility functions, $\frac{\partial^2 \Pi_i^e}{\partial x_i^2}$, must be negative. As before, the analysis focuses only on the second derivative for manager A, as symmetry implies that the second order condition holds for manager B if it holds for manager A. Taking the second derivative of Π_A^e with respect to x_A yields:

$$
\begin{aligned}
&\frac{\partial^2 \Pi_A^e(\mu_A = \mu_g)}{\partial x_A^2} = \\
&\frac{1}{8}\,(-1+\alpha)\alpha\gamma^2\mu_g \cdot \Bigg(-6 - \alpha + \alpha\gamma(10 + 3x_A + 7x_B + 2\alpha(1 + x_B)) \\
&- \alpha^2\gamma^2(1+x_B)(11 + 6x_A + \alpha + x_B(5+\alpha)) + \alpha^3\gamma^3(1+x_B)^2(4 + 3x_A + x_B) \\
&+ \frac{(-1+\alpha)(1+x_B)}{(1+x_A)(-1+\alpha\gamma(1+x_A))^2} - \frac{(-1+\alpha)(x_A - x_B)}{(1+x_A)(-1+\alpha\gamma(1+x_A))^3} \Bigg) - c''(x_A).
\end{aligned} \tag{B.3.1}
$$

Taking into account that in equilibrium, $x_A^* = x_B^* = x^*$, this expression simplifies to

$$\left.\frac{\partial^2 \Pi_A^e(\mu_A = \mu_g)}{\partial x_A^2}\right|_{x_A^*=x_B^*=x^*} =$$

$$\frac{1}{8}(-1+\alpha)\alpha\gamma^2\mu_g\Bigg(2\alpha\gamma(5+\alpha)(1+x^*)+4\alpha^3\gamma^3(1+x^*)^3$$

$$-6-\alpha-\alpha^2\gamma^2(11+\alpha)(1+x^*)^2-\frac{1-\alpha}{(-1+\alpha\gamma(1+x^*))^2}\Bigg)-c''(x^*)\,. \qquad \text{(B.3.2)}$$

Equation (B.3.2) is always negative if

$$2\alpha\gamma(5+\alpha)(1+x^*)+4\alpha^3\gamma^3(1+x^*)^3 > 6+\alpha+\alpha^2\gamma^2(11+\alpha)(1+x^*)^2+\frac{1-\alpha}{(-1+\alpha\gamma(1+x^*))^2}\,.$$

However, the second order condition may hold even if this strict condition is violated, since the fact that $c''(x_A) \geq 0$ eases the constraint. As demonstrated in the text, the second order condition for the optimum effort level holds for the chosen parameter values, as it does for a wide range of plausible values.

B.4 Effects of parameter changes on bidding strategies

When analyzing the effects of a change in α and γ on the venture capitalist's bid, one must also take into account the effect of the observed quality signal on the optimal bidding strategy. If she observes a low quality signal, the VC bids according to $\beta(S_w = 0) = \mu_g \cdot \frac{\xi}{\hat{t}} = E[\mu|\, S_w = 0]$. It is straightforward to take the derivative of this expression both with respect to α and with respect to γ:

$$\frac{\partial\beta(S_w=0)}{\partial\alpha} = \mu_g \frac{\hat{t}\frac{\partial\xi}{\partial\alpha} - \xi\frac{\partial\hat{t}}{\partial\alpha}}{\hat{t}^2} \qquad \text{(B.4.1)}$$

$$\frac{\partial \beta(S_w = 0)}{\partial \gamma} = \mu_g \frac{\hat{t}\frac{\partial \xi}{\partial \gamma} - \xi \frac{\partial \hat{t}}{\partial \gamma}}{\hat{t}^2} \tag{B.4.2}$$

The signs of these expressions depend on the reactions of ξ and $\hat{t}$ to changes in α and γ. These are given below, evaluated at the equilibrium effort level, $x_A = x_B = x^*$:

$$\frac{\partial \hat{t}}{\partial \alpha} = 2\gamma(-1 + (1 + x^*)\alpha\gamma)(1 + x^* + \alpha \cdot \underbrace{\frac{\partial x^*}{\partial \alpha}}_{>0 \text{ for low and medium levels of } \alpha}) \tag{B.4.3}$$

This term is negative for all parameter combinations where $\alpha\gamma < \frac{1}{1+x^*}$.

$$\frac{\partial \hat{t}}{\partial \gamma} = 2 \cdot [(1 + x^*)\alpha\gamma - 1] \cdot \alpha \cdot [\gamma \cdot \underbrace{\frac{\partial x^*}{\partial \gamma}}_{>0} + (1 + x^*)] \tag{B.4.4}$$

This term is also negative for all parameter combinations where $\alpha\gamma < \frac{1}{1+x^*}$.

$$\frac{\partial \xi}{\partial \alpha} = (-1+\gamma+x^*\gamma)[-1+2(1+x^*)\alpha\gamma]+\alpha\gamma[-1+\alpha(-1+2(1+x^*)\gamma)]\cdot \overbrace{\frac{\partial x^*}{\partial \alpha}}^{>0 \text{ for low and medium levels of } \alpha} \tag{B.4.5}$$

This term is negative for a wide range of parameter combinations.

$$\frac{\partial \xi}{\partial \gamma} = \alpha[-1 + \alpha(-1 + 2(1 + x^*)\gamma)](1 + x^* + \gamma \underbrace{\frac{\partial x^*}{\partial \gamma}}_{>0}) \tag{B.4.6}$$

Plugging equation (B.4.3) and (B.4.5) into equation (B.4.1) yields

$$\frac{\partial \beta(S_w = 0)}{\partial \alpha} = \mu_g \frac{1 - \gamma[1 + x^* + \alpha(1 - \alpha)\frac{\partial x^*}{\partial \alpha}]}{[-1 + (1 + x^*)\alpha\gamma]^2} \ , \tag{B.4.7}$$

which is positive for all relevant parameter ranges.

Plugging equation (B.4.4) and (B.4.6) into equation (B.4.2) yields

$$\frac{\partial\beta(S_w=0)}{\partial\gamma} = \mu_g \frac{\alpha(-1+\alpha)(1+x^*+\gamma\frac{\partial x^*}{\partial\gamma})}{[-1+(1+x^*)\alpha\gamma]^2} < 0\,, \tag{B.4.8}$$

which is always negative.

If the venture capitalist observes a high quality signal, she bids according to $\beta(S_w = \mu_g) = \ \mu_g \cdot (1+\xi-\hat{t})$. Accordingly, the derivatives with respect to α and with respect to γ are:

$$\frac{\partial\beta(S_w=\mu_g)}{\partial\alpha} = \mu_g \left(\frac{\partial\xi}{\partial\alpha} - \frac{\partial\hat{t}}{\partial\alpha}\right), \tag{B.4.9}$$

$$\frac{\partial\beta(S_w=\mu_g)}{\partial\gamma} = \mu_g \left(\frac{\partial\xi}{\partial\gamma} - \frac{\partial\hat{t}}{\partial\gamma}\right). \tag{B.4.10}$$

Plugging in equations (B.4.3), (B.4.5), (B.4.4) and (B.4.6) confirms that both expressions are positive for all relevant parameter ranges.

The uninformed outside financier bids according to a piecewise defined density function, given in equation (B.2.1). A change in the parameters α and γ affects the boundaries of the three intervals as well as the density function within each interval.

A rise in γ leads to a decrease in the upper boundary of the first interval, as it causes $\beta(S_w = 0)$ to fall. At the same time, it leads to an increase in the upper boundary of the second interval, as it causes $\beta(S_w = \mu_g)$ to rise. It also leads to an

increase of $g_3(b)$, the density function in the third interval:

$$\frac{\partial g_3}{\partial \gamma} = \frac{(1-u)\big(\overbrace{\frac{\partial \beta(S_w=\mu_g)}{\partial \gamma}}^{>0}\big)}{\big(\mu_g - \beta(S_w=\mu_g)\big)^2} > 0 \ . \tag{B.4.11}$$

With ambiguous effects on $g_1(b)$ and $g_2(b)$, the overall effect of an increase in γ on the outside financier's equilibrium bid distribution is an upward shift of probability mass, which causes his expected bid to rise.

A rise in α leads to an increase in the upper boundaries of the first and second interval, since it causes $\beta(S_w=0)$ and $\beta(S_w=\mu_g)$ to rise. It also leads to a rise of $g_3(b)$, the density function in the third interval, while it causes a decrease of $g_1(b)$, the density function in the first interval:

$$\frac{\partial g_3}{\partial \alpha} = \frac{-(1-u)\big(-\overbrace{\frac{\partial \beta(S_w=\mu_g)}{\partial \alpha}}^{>0}\big)}{\big(\mu_g - \beta(S_w=\mu_g)\big)^2} > 0 \ , \tag{B.4.12}$$

$$\frac{\partial g_1}{\partial \alpha} = \frac{\beta(S_w=0)\, u\, \overbrace{\frac{\partial \hat{t}}{\partial \alpha}}^{<0} - u\, \hat{t}\, \overbrace{\frac{\partial \beta(S_w=0)}{\partial \alpha}}^{>0}}{\big(\beta(S_w=0)\big)^2} < 0 \ . \tag{B.4.13}$$

Hence, the overall effect of an increase in α on the outside financier's equilibrium bid distribution is an upward shift of probability mass, which causes his expected bid to rise.

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