Tax Progression and the Wage Curve

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Abstract

We show that a progressive income tax in an efficiency wage model with training cost and quitting will shift the wage curve, an increase in progression reducing the pretax wage.

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This note derives a model of the wage curve, closely following Phelps (1994) and Campbell and Orszag (1998). Phelps discusses a variety of theoretical bases for a wage curve, the two major competing types being bargaining models and efficiency wage models, though Phelps prefers the term 'incentive wage' models for the latter. The model discussed here is a parametric version of one of Phelps's incentive wage models, developed by Campbell and Orszag. The innovation here is to introduce income taxation into the model, with the effect of modifying the wage curve by introducing a measure of tax progressivity into the equation of the wage curve. This result provides a link to the bargaining literature, where the same effect of progression has already been demonstrated.

1 The model

The firm maximises the present value of profits at time t_0,

$$V_{t_0} = \int_{t_0}^{\infty} e^{-\rho(t-t_0)}[f(E_t) - w_t E_t - T(h_t)E_t]dt$$

subject to the dynamic constraint

$$\frac{\dot{E}}{E} = h - q(w^*, w_A^*, R)$$

where E is employment, h the hiring rate, $f(E)$ the production function, w the firm's wage rate, w^* the firm's net of tax wage rate, w_A^* the economy-wide net of tax average wage rate, T training costs (time existing workers need to train new workers), and q is the quit rate. The net of tax wage rate is

$$\begin{aligned} w^* &= w(1-a), \text{ where } a = r(w)/w \text{ is the average tax rate} \\ \text{and } m &= \frac{\partial r(w)}{\partial w} \text{ is the marginal tax rate.} \end{aligned}$$

We assume that hours are fixed, so E corresponds to the number of employees as well as the total input of labour, and the tax is therefore levied on labour income rather than just on the wage rate. The quit rate depends on the firm net of tax wage, the economy net of tax average wage, the unemployment benefit level, and economy-wide average employment R. L is the (exogenous) labour force, so $(L-R)/L = u$, the unemployment rate. Firms behave in a Nash manner, treating economy-wide averages as given, but in equilibrium wages and employment are equated across firms.

To solve the firm's problem, set up the current value Hamiltonian:

$$H_t = f(E_t) - w_t E_t - T(h_t)E_t + \lambda_t[h_t - q(w_t^*, w_{A,t}^*, R_t)]E_t.$$

The first-order conditions are

$$\begin{aligned} \frac{\partial T(h_t)}{\partial h_t} &= \lambda_t && (1) \\ -\lambda_t \frac{\partial q}{\partial w_t} &= 1 \\ \text{i.e.} -\lambda_t \frac{\partial q}{\partial w_t^*} \frac{\partial w_t^*}{\partial w_t} &= 1 \\ -\lambda_t \frac{\partial q}{\partial w_t^*}(1-m) &= 1 && (2) \end{aligned}$$

$$\dot{\lambda}_t = \rho\lambda_t - \frac{\partial H_t}{\partial E_t}$$

$$\text{i.e. } \dot{\lambda}_t = \rho\lambda_t - [\frac{\partial f(E_t)}{\partial E_t} - w_t - T(h_t)] \tag{3}$$

Equation (1) equates marginal training costs with the shadow value of an additional worker. Equation (2) sets the wage to balance the effect of the wage on replacement costs with the effect of a change in the wage on the total wage bill. Imposing the usual transversality condition, integrating equation (3) expresses the shadow value of an additional worker in terms of the present discounted value of future cash flows from hiring an additional worker:

$$\lambda_t = \int_t^\infty e^{-\rho(n-t)}[\frac{\partial f(E_n)}{\partial E_n} - w_n - T(h_n)]dn. \tag{4}$$

The next stage is to derive the wage curve, assuming that $q(.)$ and $T(.)$ are, respectively, constant elasticity and quadratic functions. Assume the potential quitter compares his actual net-of-tax wage w^* with the expected alternative net-of-tax wage $w_A^*(1-u) + b.w_A^*.u$, where u is interpreted as the probability of remaining unemployed, and $b.w_A^*$ is the benefit level or unemployment income. Then, assuming a constant elasticity function,

$$q(w^*, w_A^*, u) = B\left[\frac{w^*}{w_A^*(1-u(1-b))}\right]^{-\eta} \tag{5}$$

$$\text{i.e. } q(w^*, w_A^*, R) = B\left[\frac{w^*}{w_A^*(1-(1-R/L)(1-b))}\right]^{-\eta}. \tag{6}$$

And let

$$T(h) = \frac{A}{2}h^2. \tag{7}$$

The first-order conditions (1) and (2) now become

$$Ah_t = \lambda_t, \tag{8}$$

$$\text{and } -\lambda_t \frac{\partial q_t}{\partial w_t^*}(1-m) = 1$$

$$\text{i.e. } \lambda_t B\eta(w_t^*)^{-\eta-1}(w_{A,t}^*)^\eta(1-u_t(1-b))^\eta(1-m) = 1. \tag{9}$$

In equilibrium, $w = w_A$, and $w^* = w_A^* = w(1-a)$, so equation (9) becomes

$$\lambda B\eta(w(1-a))^{-1}(1-u(1-b))^\eta(1-m) = 1$$

$$\text{i.e. } w = \frac{1-m}{1-a}\lambda B\eta(1-u(1-b))^\eta \tag{10}$$

Substituting equations (5) and (8) into equation (10), and imposing the steady-state condition that $h = q$, yields

$$w = \left(\frac{1-m}{1-a}\right) AB^2\eta(1-u(1-b))^{2\eta}. \tag{11}$$

This is the wage curve, the steady-state relationship between the rate of unemployment and the wage rate, where the wage rate has fully adjusted to the unemployment rate. Taking logarithms,

$$\log(w) = \log\left(\frac{1-m}{1-a}\right) + \log(AB^2\eta) + 2\eta\log(1-u(1-b)). \tag{12}$$

If the unemployment rate is relatively low, $\log(1-u(1-b)) \simeq -u(1-b)$, so the log wage curve simplifies to

$$\log(w) = \log\left(\frac{1-m}{1-a}\right) + \log(AB^2\eta) - 2\eta(1-b)u. \tag{13}$$

2 Completing the model

The wage curve plays the role of labour supply curve. To complete the model we solve equation (3) for $\dot{\lambda}_t = 0$, the steady-state condition. We set $h = q = B[1-u(1-b)]^\eta$ for $w^* = w_A^*$ (see equation(5)), obtaining

$$\rho AB(1-u(1-b))^\eta - \frac{\partial f(E)}{\partial E} + w + \frac{AB^2}{2}(1-u(1-b))^{2\eta} = 0.$$

To solve in terms of w and u, express E as $L(1-u)$, and choose a parametric form for the production function: let

$$f(E) = CE^\alpha, \text{ for } 0 < \alpha < 1.$$

Hence the demand curve is

$$w = \alpha C(L(1-u))^{\alpha-1} - \rho AB(1-u(1-b))^\eta - \frac{AB^2}{2}(1-u(1-b))^{2\eta} \tag{14}$$

The wage equals the marginal product less the marginal cost of training at the equilibrium hiring rate. To illustrate, let $A = B = C = 1$, $\alpha = 0.75$, $\eta = 0.5$, $b = 0.5$, $m = 0.5$ and 0.3, $a = 0.25$, $\rho = 0.05$ and $L = 1$.

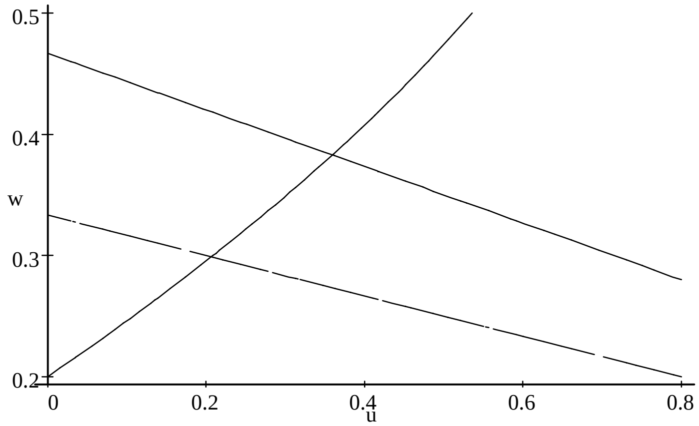

Figure 1: The wage-curves (downward sloping) for high and low progression, and the labour demand curve (upward sloping)

Changes in progression can easily be seen to have the following consequences: an increase in marginal rate of tax, for given average rate, will shift the wage curve down, reducing the wage rate and the unemployment rate; changes in the average rate will have the opposite effects. In the diagram, the upper wage curve corresponds to $m = 0.3$, the lower to $m = 0.5$.

3 Comments

The term $\frac{1-m}{1-a}$ is the well-known index of residual progression, the elasticity of after-tax income to pretax income: see Lambert (1993). It is of interest to note that the same functional form for the wage curve, complete with the residual progression index, is derived by Lockwood and Manning (1993) using a bargaining model of wage determination, as opposed to our efficiency-wage model. It therefore appears that the index of progression should be included as a matter of course in wage curve studies, since this form of equation is consistent with both of the leading theories of equilibrium unemployment. It should be noted that an increase in progressivity implies a *decrease* in this index: thus a revenue-neutral income tax reform which raised the marginal rate would have the effect of reducing the pretax wage in this model, for any given level of unemployment. Lockwood and Manning summarise recent theoretical bargaining literature thus: "....a very robust result is that increases in the marginal rate of income tax lower the pre-tax real wage, and hence unemployment, whereas an increase in the average tax usually has the opposite effect." These comments can be extended to include efficiency wage literature.

The intuition of the result is that a revenue-neutral rise in progression increases the marginal cost to an employer of inducing an employee by means of a high wage not to quit and thus generate training costs for a replacement. The employer lowers the wage and accepts a higher quit rate. The higher quits are associated with lower unemployment in equilibrium (see eq (5)). Thus the wage curve shifts to the south-west.

The introduction of the residual progression index ($RPI = \frac{1-m}{1-a}$) closely parallels the effect demonstrated by Campbell and Orszag of introducing wage and training subsidies: they show that the term

$$\frac{1-\omega}{1-s}$$

appears in the wage curve, in exactly the same way as RPI appears in equation (11) above, where ω is the rate of training subsidy and s the rate of wage subsidy (or negative payroll tax on employers). Thus the introduction of these measures is equivalent (if $s < \omega$) to the introduction of a progressive income tax levied on wage-earners; alternatively, if $s > \omega$, a progessive income tax can perfectly offset the effects of wage and training subsidies. Notice that the subsidy to training affects the marginal cost of employment by affecting only replacements for quitters, while the wage subsidy affects the average cost of employment, thus mirroring the marginal and average impacts of progressive taxation.

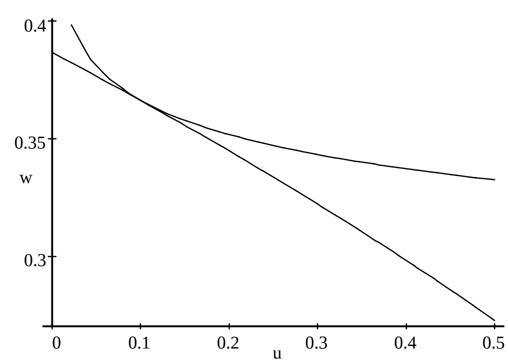

Figure 2: Alternative wage curves

The other feature of the wage curve in this paper, and those of Lockwood and Manning, and Campbell and Orszag, is that unemployment enters the logarithmic version of the equation through the term $log(1 - u(1 - b))$, whereas the conventional form is that popularised by Blanchflower and Oswald (1994) in which the term is $\log(u)$, with an appropriate change of sign. The econometric investigations so far reported (e.g. by Blanchflower and Oswald) have not focussed particularly on these alternative functional forms. This would also seem to merit further investigation. The curves are in fact rather different, as can be seen from Figure 2 in which the $log(1-u(1-b))$ version is clearly more linear, plotting both curves around the point $u = 0.1$. Finally, the effect of a higher benefit rate, b, is to reduce the slope of (i.e. flatten) the wage curve: this is another testable property.

4 Acknowledgments

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