

COMPATIBILITY AND PRODUCT DESIGN  
IN SOFTWARE MARKETS

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## ABSTRACT

*This paper analyzes the interplay between compatibility and product design decisions in a symmetric software duopoly with network effects. We show that suppliers do not always offer differentiated product designs and compete within the market. Rather, whenever both the significance of the network effects and the costs of compatibility are high, they offer homogeneous and incompatible variants and compete for the market, although this leads to Bertrand competition with zero profits. Moreover, we show that given our symmetric setting, antitrust authorities should never intervene against incompatibility, whereas compatibility arrangements should always be under their scrutiny.*

# 1. INTRODUCTION

In this paper we analyze the suppliers' decisions on (in)compatibility and horizontal product designs in a symmetric microcomputer software duopoly when network effects are present. Clear examples for microcomputer software markets with significant network effects are the markets for word processors, for spreadsheet programs, and for database management systems. In these cases, the existence of direct network effects due to file portability is obvious. Moreover, there are considerable indirect network effects due to the positive correlation between the total number of users of a software program (or of compatible programs) and the variety of complementary software for this (these) program(s). For example, the more popular a database management program is, the more complementary software for statistical analysis is offered for this program.<sup>1</sup> This paper examines the consequences of the existence of these network effects for the case that due to intellectual property rights, compatibility can only be established unanimously. As for the decision on product design, we have, for instance, in mind the decision of a software supplier on whether to design a variant which aims at a specific user group such as scientists or businessmen, or to design a variant which tries to satisfy the needs of all users. In software markets, typically, this design decision is irrevocably made before a credible commitment to compatibility is feasible. Hence, we assume that the suppliers commit themselves to their product designs in the first stage of the game and decide on (in)compatibility in the second stage; subsequently, they compete in prices, and in the fourth stage, consumers choose their variants.

We prove that whenever the costs of compatibility are low both compared with the significance of the network effects and with the heterogeneity of preferences, suppliers develop differentiated product designs and opt for compatibility, whereas when the costs of compatibility are high compared with the significance of the network effects and the latter is low compared with the heterogeneity of preferences, they develop differentiated product designs and opt for incompatibility. In both these cases, a duopolistic equilibrium occurs. However, our main new result with regard to market equilibria is that whenever both the costs of compatibility and the significance of the network effects are

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<sup>1</sup>Recent empirical studies on network effects in microcomputer software markets are Gandal (1994), Gandal (1995), Brynjolfsson and Kemerer (1996), and Gröhn (1999, pp. 115ff).

high compared to the heterogeneity of preferences, suppliers develop identical variants, opt for incompatibility, and compete for the market. This happens although such a competition for the market leads only to normal profits and irrespective of the fact that here, a competition within the market with differentiated variants could lead to strictly positive profits. As for antitrust policy, we deduce policy recommendations against the background of the fact that antitrust authorities typically cannot intervene in the decisions on horizontal product designs but only in the decisions on (in)compatibility. It turns out that given our symmetric setting, they should never intervene when suppliers opt for incompatibility but should tolerate compatibility arrangements only when the costs of compatibility are very low compared to the significance of the network effects. Here, our main new result is that competition for the market with incompatible homogeneous variants is welfare superior to compatibility irrespective of how low the costs of compatibility are. This is due to the fact that when there is competition for the market, suppliers locate their variants at the center of the consumer distribution, whereas under compatibility, they differentiate them excessively.

As for the literature on horizontal product differentiation without network effects, our model builds on Anderson, Goeree and Ramer (1997); their results hold when compatibility is given. As for the literature on compatibility decisions, our model builds on Farrell and Saloner (1992), on Chou and Shy (1996), and on Woeckener (1999a). In these articles, however, product designs are given exogenously. A first step in the analysis of compatibility decisions when product designs are endogenous was done in Baake (1995). He showed that in duopolistic equilibria, product designs do not depend on the compatibility decision. There, however, the possibility of competition for the market was not analyzed. Finally, in an accompanying paper (Woeckener [1999b]), we have taken account of this possibility in an asymmetric setting (one of the suppliers is favored by network-size expectations and, thus, earns strictly positive profits from competition for the market) and for a special version of the model presented here (among other things, without costs of compatibility).

The paper is organized as follows: In the next section, the main assumptions are presented, and subsequently, we discuss consumers' choices and price competition. The fourth section analyzes the decisions on compatibility and product designs, Section 5 presents our welfare analysis, and finally, we derive policy implications in Section 6.

## 2. THE MODEL

In our model there are two suppliers each of whom offers a variant of a software program which is produced with constant marginal costs. We assume that both the marginal costs and the fixed costs of production are the same for both variants and normalize them to zero. With regard to the decisions on product design and (in)compatibility, we make the following assumptions:

- The suppliers can locate their variants anywhere on the Hotelling line. If different locations are chosen, we call the variant located to the left  $A$ , and its address (design)  $d_A$ , and the variant located to the right  $B$ , and its address  $d_B$ . Hence,  $d_B - d_A$  is the extent of horizontal product differentiation.
- The decision on (in)compatibility is a binary choice where  $\delta = 0$  stands for a move to compatibility and  $\delta = 1$  stands for maintaining incompatibility. Establishing compatibility requires the consent of both suppliers and leads to fixed costs of compatibility of  $Q > 0$  for each supplier. We see  $Q$  as the costs of developing a built-in converter such as an import/export interface.

A consumer's maximal willingness to pay for variant  $i$  ( $i = A, B$ ) consists of the general willingness to pay for this variant  $g_i$ , and the network effect rent from this variant  $h_i$ . With regard to  $g_i$  and  $h_i$ , we make the following assumptions:

- As for  $g_i$ , we denote a consumer's address on the Hotelling line, i.e. the location of his ideal variant, as  $x$  and the general willingness to pay for the ideal variant as  $b$ . Whereas  $b$  is the same for all consumers,  $x$  is not. Moreover, a consumer's willingness to pay for an existing variant overproportionally decreases in the distance between his address (ideal variant) and the respective existing variant. Thus, individual preferences over product designs are convex. As we see no compelling reason for a certain sign of the third derivative  $\partial^3 g_i / \partial x^3$ , we use the common quadratic approach for the alienation terms. Then, the general willingnesses to pay is  $g_i(x) = b - t(x - d_i)^2$ , where  $t$  is the measure for the convexity of preferences. As for the heterogeneity of preferences among consumers,  $x$  is uniformly distributed along the interval  $] - a, a[$  with a density of  $0.5/a$ , where  $0 < |a| < \infty$  holds. Hence, the distribution function reads

$$F(x) = 0.5 + \frac{x}{2a} \quad \text{with} \quad -a \leq x \leq a, \quad (1)$$

and  $a$  is the measure for the heterogeneity of preferences. Important for market equilibria is the term  $a^2t$ . Here, a higher  $t$  is qualitatively equivalent to a higher  $a$ . Therefore, we denote this term simply as the ‘heterogeneity of preferences’, which comprises both inter-consumer heterogeneity and intra-consumer convexity. Note that with Equation (1), the total mass of consumers is normalized to one. Furthermore,  $b$  is assumed to be sufficiently high to guarantee that each consumer has at least for one variant a maximal willingness to pay which is higher than the equilibrium price. With each consumer buying only one piece of only one variant, this means that total demand is also normalized to one and that the demand for variant  $i$  equals its market share  $m_i$ .

- As for the network effect rents  $h_i$ , consumers are assumed to be homogeneous with respect to the valuation of network effects. The network size of variant  $i$  is the total number of users who buy compatible programs and is denoted as  $0 \leq z_i \leq 1$ . In the case of compatible variants,  $z_A = z_B = m_A + m_B = 1$  holds, whereas in the case of incompatible variants,  $z_i = m_i$  holds. Typically, marginal network effects are decreasing in network size but do not diminish completely. Therefore, we use the quadratic approach  $h_i(z_i) = n(z_i - cz_i^2)$  with  $0 < c < 0.5$ , so that  $\partial h_i / \partial z_i = n(1 - 2cz_i) > 0$  and  $\partial^2 h_i / \partial z_i^2 = -2nc < 0$  hold. Here,  $n$  denotes the general significance of the network effects and  $c$  is the measure for the concavity of the network effect rent. With compatible variants ( $\delta = 0$ ), our covered-market assumption implies  $h_A = h_B = n(1 - c)$ , whereas with incompatible variants ( $\delta = 1$ ),  $h_i = n(m_i - cm_i^2)$  holds. Important for market equilibria is the term  $n(1 - c)$ , which comprises both the general significance of the network effects and the degree of concavity of the network effect rent. In the following, we denote this term simply as the ‘significance of the network effects’.

Summing up our assumptions concerning the maximal willingnesses to pay, and denoting prices as  $p_i$ , we obtain

$$s_{xi}(p_i, d_i, \delta) = b - t(x - d_i)^2 - p_i + \begin{cases} n(1 - c) & \text{if } \delta = 0 \\ n(m_i - cm_i^2) & \text{if } \delta = 1 \end{cases} \quad (2)$$

as the surplus of a consumer with address  $x$  from variant  $i$ .

### 3. DEMAND FUNCTIONS AND PRICE COMPETITION

In this section, we derive the Nash equilibria of the fourth and the third stage of the game. We lay emphasis on the fact that maintaining incompatibility in the second stage can turn the market into a natural monopoly. Moreover, we restrict ourselves to analyzing long-run equilibria where network-size expectations are fulfilled and where coordination problems among consumers are solved.

#### 3.1 DEMAND FUNCTIONS

The derivation of demand equilibria for given prices, given (in)compatibility, and given locations is straightforward; equating  $s_{xA}$  with  $s_{xB}$  leads to the address of those consumers who are indifferent between the two variants  $\hat{x}$ , and by substituting this address into Equation (1), we obtain market shares  $m_1$  and  $m_2 = 1 - m_1$ . With given compatibility, obviously, the network effect rent  $n(1 - c)$  has no effect on  $\hat{x}$  and, thus, on market shares and Nash equilibria. Hence, market equilibria are the same as in the standard Hotelling model. For given incompatibility,  $\hat{x}$  depends on market shares (network sizes). Here, we are only interested in demand equilibria where network-size expectations are fulfilled. Then,  $m_i = 0.5 \pm 0.5\hat{x}/a$  holds, and we obtain<sup>2</sup>

$$m_i(p_A, p_B; d_A, d_B; \delta) = 0.5 + \frac{p_j - p_i \pm t(d_B - d_A)(d_A + d_B)}{4at(d_B - d_A) - \delta 2n(1 - c)}. \quad (3)$$

In the case of given compatibility, a duopoly equilibrium with strictly positive market shares for both suppliers is guaranteed, because then there is no reason why a supplier should accept a price and/or design disadvantage which results in zero demand for his variant. In the case of given incompatibility, this only holds if, for a given extent of consumer heterogeneity, the product differentiation dominates the significance of the network effects, i.e. if  $d_B - d_A > n(1 - c)/(2at)$  holds. Otherwise, relatively strong network effects turn the market into a natural monopoly, because then demand equilibria according to Equation (3) are unstable. This becomes clear from a look at the surplus equations; for  $n(1 - c) > 2at(d_B - d_A)$ , an exogenous shock, however small, sets off self-re-enforcing bandwagon effects which do not come to a halt until one of

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<sup>2</sup>In case of a  $\pm$  or  $\mp$ , the upper sign holds for variant  $A$  and the lower sign holds for variant  $B$ . For a model with adaptive expectations, see Woekener (1999c).

the variants has covered the whole market.<sup>3</sup>

### 3.2 PRICE COMPETITION

By comparing duopolistic demand equilibria for given compatibility with duopolistic demand equilibria for given incompatibility, it becomes clear that the price elasticity of demand is higher (in absolute terms) in the latter case. Hence, maintaining incompatibility leads to tougher price competition and to lower equilibrium prices. This is due to the fact that under incompatibility, a reduction in  $p_i$  results in a rise in  $m_i$  which *uno actu* means a rise in network size (in the variety of complementary software programs), whereas under compatibility, all consumers are in a joint network of size one irrespective of prices. Given a duopoly, maximizing  $p_i m_i$  with respect to prices leads via the FOC to<sup>4</sup>

$$p_i(d_A, d_B; \delta) = t(d_B - d_A) \left( 2a \pm \frac{d_A + d_B}{3} \right) - \delta n(1 - c), \quad (4)$$

and by substitution into Equation (3), we obtain

$$m_i(d_A, d_B; \delta) = 0.5 \pm \frac{t(d_B - d_A)(d_A + d_B)}{12at(d_B - d_A) - \delta 6n(1 - c)}. \quad (5)$$

Note that the latter is equivalent to  $p_i/[4at(d_B - d_A) - \delta 2n(1 - c)]$ , so that equilibrium profits can be formulated as  $p_i^2/[4at(d_B - d_A) - \delta 2n(1 - c)]$ . According to Equations (4) and (5), a supplier whose variant lies closer to the center of the consumer distribution than his competitor's variant has both the higher price and the higher market share. Hence, we can presume that with regard to the overall game, only equilibria which are symmetric in locations will be of relevance. When the suppliers have maintained incompatibility and the network effects dominate the product differentiation, the only stable Nash equilibria with fulfilled expectations are  $m_1 = 1$  and  $m_1 = 0$ . Then, there is competition for the market, and the outcome of this competition depends on consumers'

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<sup>3</sup>Note that this instability is indicated by a positive slope of Equation (3). Moreover, note that interior equilibria in the case of given incompatibility and  $n(1 - c) > 2at(d_B - d_A)$  are also ruled out by the fact that for these parameter constellations, the second-order conditions of profit maximization are not fulfilled (see the next subsection).

<sup>4</sup>The SOC read  $-1/[2at(d_B - d_A) - \delta n(1 - c)] < 0$ , and for given incompatibility, they are only fulfilled when the product differentiation dominates the network effects.



expectations. Here, it seems natural to assume that the equilibrium is focal which leads to a higher cumulated consumer surplus. As for the product design decisions in the first stage, this assumption implies that suppliers who opt for such a competition for the market locate their variants at the center of the consumer distribution. Choosing any other location would mean enabling the competitor to realize a product advantage and monopolize the market.<sup>5</sup> Therefore, we can anticipate that in the case of competition for the market, only the locations  $d_i = 0$  are of relevance, i.e. that the suppliers offer homogeneous variants. Then, consumers' expectations solely depend on prices, so that the suppliers are forced to set prices equal to marginal costs in order to maintain their chances of becoming the monopolist. Hence, to sum up, we can state

LEMMA 1:

1. *If the suppliers have established compatibility in the second stage, subsequently a competition within in the market takes place, and equilibrium profits (gross of the fixed costs of compatibility) amount to*

$$\Pi_i(d_A, d_B; \delta = 0) = \frac{t(d_B - d_A)}{4a} \left( 2a \pm \frac{d_A + d_B}{3} \right)^2. \quad (6)$$

2. *If the suppliers have maintained incompatibility in the second stage, subsequently a competition within the market only takes place in the case of a dominating product differentiation ( $d_B - d_A > n(1 - c)/(2at)$ ). Then, equilibrium profits amount to*

$$\Pi_i(d_A, d_B; \delta = 1) = \frac{\left[ t(d_B - d_A) \left( 2a \pm \frac{d_A + d_B}{3} \right) - n(1 - c) \right]^2}{4at(d_B - d_A) - 2n(1 - c)}. \quad (7)$$

*Otherwise, maintaining incompatibility in the second stage results in competition for the market with normal profits.*

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<sup>5</sup>Due to lower cumulated alienation effects, a standardization on the variant which is closer to the center is always pareto-superior and leads, given equal prices, to a higher cumulated consumer surplus.

## 4. COMPATIBILITY DECISIONS AND COMPETITION OVER PRODUCT DESIGNS

The most straightforward way to obtain the Nash equilibria of the first two stages of the game is to derive in a first step the profit maximizing product designs for given market structure (compatible duopoly, incompatible duopoly or natural monopoly), and to examine in a second step which of these market structures arises when product designs are fixed first. This procedure makes use of the fact that the decision on (in)compatibility is a binary choice and avoids the rather tedious discussion of the asymmetric equilibria of the second stage which are irrelevant for the overall game.

### 4.1 PROFIT MAXIMIZING PRODUCT DESIGNS FOR GIVEN MARKET STRUCTURE

As already mentioned, with given compatibility, the results of our model are the same as in the standard Hotelling model. From Anderson, Goeree and Ramer (1997, p. 116), we know that in the latter, given a symmetric unimodal logconcave distribution, locations and prices in Nash equilibria read  $0.75/f(0)$  and  $1.5t/[f(0)]^2$ , respectively, where  $f(0)$  is the density at the median.<sup>6</sup> For the uniform distribution, this means locations of  $\mp 1.5a$  and prices of  $6a^2t$ . Hence, a symmetric compatible duopoly emerges with individual profits of  $3a^2t - Q$ . At the equilibrium locations, two partial effects equalize each other: the price effect (moving closer to the center means tougher price competition) and the be-where-the-consumers-are effect (moving closer to the center means a lower average distance to consumers). Note that the profit maximizing locations lie outside the support of the consumer distribution.

With regard to a duopolistic competition between incompatible variants, Baake (1995, p. 9f) proved that compared with the standard Hotelling model, the existence of network effects again has no effect on locations. Hence, the strengthening of the price effect caused by the working of bandwagon effects is exactly compensated by the strengthening of the be-where-the-consumers-are effect caused by the fact that market shares are now network sizes. However, in an incompatible duopoly, prices are lower than under compatibility; they amount to  $6a^2t - n(1 - c)$  (see Equation [4]). Whether

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<sup>6</sup>The condition for uniqueness is  $-\partial^2 f / \partial x^2(0) / [f(0)]^3 < 8$ , and in the case of a uniform distribution, this is obviously fulfilled.

this also holds for individual profits  $3a^2t - 0.5n(1 - c)$  depends on the fixed costs of compatibility.

Finally, considering a natural monopoly, we have already seen that the only location of relevance is the center of the consumer distribution; i.e., the suppliers offer identical product designs and make (only) normal profits.

## 4.2 NASH EQUILIBRIA

From the previous subsection, we know that when the suppliers intend to establish compatibility in the second stage, they locate their variants at  $\mp 1.5a$  in the first stage, whereas when they intend to maintain incompatibility, they locate them at  $\mp 1.5a$  or at the center. In this subsection, we examine in a first step under which circumstances offering differentiated product designs is a Nash equilibrium. As we will see, whenever it is a Nash equilibrium, suppliers make supranormal profits irrespective of the compatibility decision. Hence, whenever offering identical product designs (and earning normal profits) is also a Nash equilibrium, we can take it for granted that offering differentiated variants is the focal equilibrium. Therefore, there is no need to examine under which circumstances offering identical designs is a Nash equilibrium (which would require a numerical analysis). Rather, it suffices to show that it is a Nash equilibrium whenever offering differentiated designs is not.

### *Offering differentiated product designs as a Nash Equilibrium*

If the suppliers choose locations of  $\mp 1.5a$  in the first stage, they either opt for compatibility and make profits of  $3a^2t - Q$  or opt for incompatibility and make profits of  $3a^2t - 0.5n(1 - c)$  in the second stage. Obviously, the former happens whenever the costs of compatibility are low compared with the significance of the network effects ( $Q < 0.5n(1 - c)$ ), whereas otherwise, the latter takes place. These choices are Nash equilibria if no supplier has an incentive to change his location in order to monopolize the market by subsequently maintaining incompatibility and setting the limit price. Let us assume that the supplier of variant  $A$  is the one who considers deviating. If he deviated from  $d_A = -1.5a$ , he would locate his variant at  $a$  and subsequently maintain incompatibility. Then,  $x = a$ -consumers choose his variant even for  $p_B = 0$  (given

$d_B = 1.5a$ ) if he sets his price equal to  $0.25a^2t$ . Hence, this is the limit price, which equals his profits in the case of deviating. Comparing these profits with  $3a^2t - Q$  and  $3a^2t - 0.5n(1 - c)$  makes clear that offering differentiated product designs ( $\mp 1.5a$ ) is a Nash equilibrium for  $Q < 2.75a^2t$  as well as for  $n(1 - c) < 5.5a^2t$ . As profits in the case of deviating are strictly positive, profits in duopolistic Nash equilibria are also always strictly positive.

*Offering identical product designs as a unique Nash equilibrium*

Locating the variants at the center and subsequently maintaining incompatibility and competing for the market is a Nash equilibrium whenever no supplier can deviate from the center and enforce a compatible or incompatible duopoly with strictly positive profits. Let us assume that the supplier of variant  $B$  is the one who considers deviating. Then, by using Equation (6) with  $d_A = 0$ , we can show that his profit maximizing location in the case of deviating and subsequently maintaining incompatibility is  $d_B = a + n(1 - c)/(3at) + \sqrt{[a + n(1 - c)/(3at)]^2 - n(1 - c)/t}$ . Substituting this location back into Equation (6) shows that strictly positive profits require  $n(1 - c) < 3a^2t$  to hold. Hence, offering identical product designs is a Nash equilibrium for  $n(1 - c) > 3a^2t$ . Comparing this result with the above conditions for a duopolistic Nash equilibrium makes clear that it is a Nash equilibrium whenever offering differentiated variants is not.<sup>7</sup>

Hence, summing up our results concerning Nash equilibria, we can state the following proposition:

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<sup>7</sup>Given a move to compatibility in the second stage, the profit maximizing location in the case of deviating is  $d_B = 2a$ . Then, the supplier of variant  $B$  would make profits of  $0.8a^2t$ . This, however, is only of relevance when his competitor agrees to the move to compatibility. Hence, in order to deduce all the parameter constellations for which offering identical variants is a Nash equilibrium, we would have to examine whether the deviating supplier prefers compatibility or incompatibility and whether his competitor agrees to an eventual move to compatibility. However, as already mentioned, these calculations are unnecessary because in the case of coexisting Nash equilibria, it is obvious that the duopolistic solutions are the relevant equilibria.

PROPOSITION 1:

1. *If the costs of compatibility are low both compared to the significance of the network effects and compared to the heterogeneity of preferences ( $Q < 0.5n(1 - c)$  and  $Q < 2.75a^2t$ ), the suppliers opt for differentiated product designs and subsequently establish compatibility, i.e.*

$$d_i^* = \mp 1.5a \quad \text{with} \quad \delta^* = 0 \quad (8)$$

*holds. This results in a symmetric duopolistic equilibrium ( $m_i^* = 0.5$ ) where profits amount to*

$$\Pi_i^* = 3a^2t - Q . \quad (9)$$

2. *If the costs of compatibility are high compared to the significance of the network effects and the latter is low compared to the heterogeneity of preferences ( $Q > 0.5n(1 - c)$  and  $n(1 - c) < 5.5a^2t$ ), the suppliers opt for differentiated product designs and subsequently maintain incompatibility, i.e.*

$$d_i^* = \mp 1.5a \quad \text{with} \quad \delta^* = 1 \quad (10)$$

*holds. This results in a symmetric duopolistic equilibrium ( $m_i^* = 0.5$ ) where profits amount to*

$$\Pi_i^* = 3a^2t - 0.5n(1 - c) . \quad (11)$$

3. *Finally, if both the costs of compatibility and the significance of the network effects are high compared to the heterogeneity of preferences ( $Q > 2.75a^2t$  and  $n(1 - c) > 5.5a^2t$ ), the suppliers opt for identical product designs and subsequently maintain incompatibility; then,*

$$d_i^* = 0 \quad \text{with} \quad \delta^* = 1 \quad (12)$$

*holds. This results in competition for the market ( $m_A = 1$  or  $m_A = 0$ ) with normal profits:*

$$\Pi_i^* = 0 . \quad (13)$$

The reason behind the first two parts of this proposition is obvious. If there were no costs of compatibility, the suppliers would always prefer a compatible duopoly to an incompatible duopoly because a move to compatibility softens price competition considerably. However, with fixed costs of compatibility, they have to weigh the extra

profits from a move to compatibility and its costs. The third part of Proposition 1 is our main new result with regard to market equilibria: A competition for the market with identical product designs can happen although it leads only to normal profits. It must be stressed that this result holds for parameter constellations where offering differentiated designs and competing within the market would lead to supranormal profits.<sup>8</sup> The reason behind this result is that in duopolistic Nash equilibria, the suppliers differentiate their designs so excessively that they lie outside the support of the consumer distribution. Otherwise, i.e. if the profit maximizing locations were inside this support, a profitable deviation and monopolization would not be possible. Then, competition for the market can only take place in cases where duopolistic equilibria lead to negative profits (which is a trivial result). However, as is well known, in duopolistic Nash equilibria, the variants' locations only lie outside the support of the consumer distribution if, given a fixed support, the consumer mass is not too concentrated around the center. In order to illustrate the degree of robustness of our result, we will demonstrate this by means of the family of triangular densities  $f(x) = (1 + 0.25\alpha) - \alpha|x|$  with support  $-0.5 \leq x \leq 0.5$  and with  $0 < \alpha \leq 4$ .<sup>9</sup> Here, the condition for equilibrium locations outside the support of the distribution reads  $0.75/f(0) > 0.5$ . With  $f(0) = 1 + 0.25\alpha$ , this means  $\alpha < 2$ . For  $\alpha = 2$ , we obtain  $d_i^* = 0.5$ ,  $f(0) = 1.5$ , and  $f(\pm 0.5) = 0.5$ . For a lower  $\alpha$ ,  $f(0)$  is lower,  $f(\pm 0.5)$  are higher, and the locations  $d_i^*$  lie outside the support of the distribution. For a higher  $\alpha$ , the reverse holds.

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<sup>8</sup>This is the case for  $2.75a^2t < Q < 3a^2t$  with  $n(1 - c) > 6a^2t$  (a compatible duopoly would result in supranormal profits), for  $5.5a^2t < n(1 - c) < 6a^2t$  with  $Q > 3a^2t$  (an incompatible duopoly would result in supranormal profits), and when both  $2.75a^2t < Q < 3a^2t$  and  $5.5a^2t < n(1 - c) < 6a^2t$  hold (any duopoly would result in supranormal profits).

<sup>9</sup>For  $\alpha = 0$ , we obtain a uniform distribution with  $a = 0.5$  as the lower borderline case;  $\alpha = 4$  results in the upper borderline case with  $f(\pm 0.5) = 0$  and  $f(0) = 2$ . In order to preclude problems which are caused by the fact that these densities are not differentiable at the median, we assume that they are sufficiently smoothed around  $x = 0$  (by use of a higher-order polynomial) so that existence and uniqueness of the symmetric equilibria are guaranteed.

## 5. WELFARE ANALYSIS

In this section, we discuss welfare realized in Nash equilibria and first-best welfare optima as well as second-best welfare optima. Whereas first-best optima are derived under the assumption that the social planner decides both on (in)compatibility and on product designs, second-best optima are derived under the assumption that he only decides on (in)compatibility and leaves the decision on product designs to the market. As antitrust authorities cannot and do not intervene in the decisions on horizontal product design, the second-best optima are the appropriate benchmark for deducing practicable policy recommendations.

### 5.1 REALIZED WELFARE

The calculation of realized total welfare  $W$  and cumulated consumer surplus  $S = W - \Pi$  is straightforward. For product designs  $d_i^* = \mp 1.5a$  and  $d_i^* = 0$ , we obtain by integration cumulated alienation effects of  $-1.08\bar{3}a^2t$  and  $-0.\bar{3}a^2t$ , respectively. Moreover, whereas in a natural monopoly and under compatibility, cumulated network effects amount to  $n(1 - c)$ , they only amount to  $n(0.5 - 0.25c)$  in an incompatible duopoly. Taking into account the fixed costs of compatibility of  $2Q$  and equilibrium profits according to Proposition 1, we obtain

PROPOSITION 2:

1. *If the suppliers offer differentiated ( $d_i = \mp 1.5a$ ) and compatible variants (i.e. for  $Q < 0.5n(1 - c)$  and  $Q < 2.75a^2t$ ), realized total welfare and consumer surplus amount to*

$$W^* = b + n(1 - c) - 1.08\bar{3}a^2t - 2Q \quad \text{and} \quad (14)$$

$$S^* = b + n(1 - c) - 7.08\bar{3}a^2t, \quad \text{respectively.} \quad (15)$$

2. *If the suppliers offer differentiated ( $d_i = \mp 1.5a$ ) and incompatible variants (i.e. for  $Q > 0.5n(1 - c)$  and  $n(1 - c) < 5.5a^2t$ ), realized total welfare and consumer surplus amount to*

$$W^* = b + n(0.5 - 0.25c) - 1.08\bar{3}a^2t \quad \text{and} \quad (16)$$

$$S^* = b + n(1.5 - 1.25c) - 7.08\bar{3}a^2t, \quad \text{respectively.} \quad (17)$$

3. Finally, if the suppliers offer homogeneous ( $d_i = 0$ ) but incompatible variants (i.e. for  $Q > 2.75a^2t$  and  $n(1-c) > 5.5a^2t$ ), realized total welfare and consumer surplus amount to

$$W^* = S^* = b + n(1-c) - 0.\bar{3}a^2t . \quad (18)$$

Note that the following corollary holds:

*COROLLARY 1: A move to compatibility always hurts consumers as a whole, whereas maintaining incompatibility is always in their interest.*

Given a relatively low significance of the network effects ( $n(1-c) < 5.5a^2t$ ), the suppliers have to decide whether to compete in an incompatible or in a compatible duopoly. Here, with regard to consumer surplus, the network-size advantage of compatibility is overcompensated by its price disadvantage. For a relatively high significance of the network effects ( $n(1-c) > 5.5a^2t$ ), the suppliers have to decide whether to compete in a compatible duopoly or for the market. Then, from the perspective of consumers, compatibility has the disadvantage of leading to higher prices as well as to higher cumulated alienation effects. Hence, in particular competition for the market with incompatible homogeneous variants is in the interests of consumers.

## 5.2 FIRST-BEST WELFARE

If there were no costs of compatibility, a social planner who decides on (in)compatibility as well as on product designs would obviously realize a compatible duopoly with locations  $d_i = \mp 0.5a$ . This leads to maximal cumulated network effects of  $n(1-c)$  and to minimal cumulated alienation effects of  $-0.08\bar{3}a^2t$ . However, with  $Q > 0$ , he has to weigh the network-size disadvantage of an incompatible duopoly and the costs of a move to compatibility. Moreover, the alternative of realizing a monopoly with a variant lying at the center of the consumer distribution then becomes relevant. Compared to an incompatible duopoly (with  $d_i = \mp 0.5a$ ), such a monopoly has the advantage of higher cumulated network effects and the disadvantage of higher cumulated alienation effects; compared to a compatible duopoly, the same disadvantage holds, but now it has the advantage of avoiding the costs of compatibility. By comparing these three alternatives, it is straightforward to prove the following proposition:



PROPOSITION 3:

1. *If the costs of compatibility are very low both compared to the significance of the network effects and compared to the heterogeneity of preferences ( $Q < 0.25n(1 - 1.5c)$  and  $Q < 0.125a^2t$ ), the first-best welfare optimum is a symmetric duopoly with differentiated ( $d_i = \mp 0.5a$ ) and compatible variants.*
2. *If the costs of compatibility are not very low compared to the significance of the network effects but the latter is very low compared to the heterogeneity of preferences ( $Q > 0.25n(1 - 1.5c)$  and  $n(1 - c) < 0.5a^2t$ ), the first-best welfare optimum is a symmetric duopoly with differentiated ( $d_i = \mp 0.5a$ ) and incompatible variants.*
3. *Finally, if neither the costs of compatibility nor the significance of the network effects is very low compared to the heterogeneity of preferences ( $Q > 0.125a^2t$  and  $n(1 - c) > 0.5a^2t$ ), the first-best welfare optimum is a monopoly whose variant is located at the center of the consumer distribution.*

Comparing these results with market equilibria according to Proposition 1 makes clear that against the background of the first-best optima, the market almost always fails. A noteworthy exception is a competition for the market; whenever it happens, it is first-best welfare optimal.

### 5.3 SECOND-BEST WELFARE

In reality, antitrust authorities can and do only intervene in the decisions on (in)compatibility but not in the decisions on horizontal product designs. Hence, for deducing policy recommendations, the appropriate welfare-theoretical benchmark are the welfare levels which can be attained when the decisions on horizontal product designs are left to the market. In order to derive these second-best optima, we only have to compare realized welfare in a compatible duopoly with realized welfare in an incompatible duopoly for  $n(1 - c) < 5.5a^2t$  and with realized welfare in a competition for the market for  $n(1 - c) > 5.5a^2t$ . From Proposition 2, we obtain:

PROPOSITION 4:

1. *If the costs of compatibility are very low compared to the significance of the network effects and the latter is low compared to the heterogeneity of preferences ( $Q < 0.25n(1 - 1.5c)$  and  $n(1 - c) < 5.5a^2t$ ), the second-best welfare optimum is a symmetric*

*duopoly with differentiated ( $d_i = \mp 1.5a$ ) and compatible variants.*

2. *If the costs of compatibility are not very low compared to the significance of the network effects but the latter is low compared to the heterogeneity of preferences ( $Q > 0.25n(1 - 1.5c)$  and  $n(1 - c) < 5.5a^2t$ ), the second-best welfare optimum is a symmetric duopoly with differentiated ( $d_i = \mp 1.5a$ ) and incompatible variants.*

3. *Finally, if the significance of the network effects is high compared to the heterogeneity of preferences ( $n(1 - c) > 5.5a^2t$ ), the second-best welfare optimum is competition for the market with homogeneous but incompatible variants (located at the center of the consumer distribution) irrespective of the costs of compatibility.*

The reason behind the first two parts of this proposition is simply the trade-off between the network-size advantage of compatibility and its costs. Our main new result with regard to welfare is the third part. It must be stressed that it holds for any value of  $Q$  because it is due to the fact that cumulated alienation effects are lower when there is competition for the market. Thus, the reason behind it is that the fierce competition for the market forces the suppliers to offer product designs which match the preferences of consumers as well as possible, whereas a move to compatibility softens competition over product designs drastically. As for the robustness of this result, note that in the case of the triangular distributions discussed at the end of the previous section, it holds for any  $\alpha$ , i.e. irrespective of how steep the density function is.

## 6. POLICY IMPLICATIONS

Comparing second-best welfare optima according to Proposition 4 with Nash equilibria according to Proposition 1 makes clear that there are five different parameter regimes and that in two of them, antitrust authorities should intervene in the compatibility decisions.

- If the significance of the network effects is low compared to the heterogeneity of preferences ( $n(1 - c) < 5.5a^2t$ )

- and the costs of compatibility are high compared to the significance of the network effects ( $Q > 0.5n(1 - c)$ ), the suppliers' decisions result in a duopoly with differentiated and incompatible variants, and this is second-best welfare optimal.

- and the costs of compatibility are very low compared to the significance of the network effects ( $Q < 0.25n(1 - 1.5c)$ ), the suppliers' decisions result in a duopoly with differentiated and compatible variants, and this is second-best welfare optimal.

- and the costs of compatibility are neither high nor very low compared to the significance of the network effects ( $0.25n(1 - 1.5c) < Q < 0.5n(1 - c)$ ), the suppliers' decisions again result in a duopoly with differentiated and compatible variants, but now an incompatible duopoly is second-best welfare optimal. Hence, in this case, antitrust authorities should prohibit compatibility arrangements. Here, market equilibria and welfare optima fall apart, because suppliers weigh the costs of compatibility and the higher prices under compatibility, whereas antitrust authorities should weigh the costs of compatibility and its network-size advantage.

- If the significance of the network effects is high compared to the heterogeneity of preferences ( $n(1 - c) > 5.5a^2t$ )

- and the costs of compatibility are high compared to the heterogeneity of preferences, too ( $Q > 2.75a^2t$ ), the suppliers' decisions result in a competition for the market, and this is second-best welfare optimal.

- and the costs of compatibility are low compared to the heterogeneity of preferences ( $Q < 2.75a^2t$ ), the suppliers' decisions result in a duopoly with differentiated and compatible variants, whereas the second-best welfare optimum is a competition for the market. Hence, here again, antitrust authorities should prohibit compatibility arrangements. In this case, the falling apart of market equilibria and welfare optima is due

to the fact that suppliers weigh the costs of compatibility and the higher prices under compatibility, whereas what counts for antitrust authorities is that in a competition for the market, costs of compatibility are avoided and consumers get a variant which matches preferences better than the variants in a duopoly equilibrium do.

Hence, to sum up, we can conclude from Propositions 1 and 4:

*COROLLARY 2: Antitrust authorities should never intervene when the market opts for incompatibility and should permit compatibility arrangements only when the costs of compatibility are very low compared to the significance of the network effects ( $Q < 0.25n(1 - 1.5c)$ ).*

In particular, given the symmetric setting of our model, enforcing compatibility where suppliers would otherwise opt for a competition for the market is a policy failure. Here, compatibility arrangements must be seen as cartels which aim at softening both competition in prices and over product designs.

Of course, in an asymmetric setting, policy implications can (but must not) be different; see, for example, Katz and Shapiro (1998, pp. 29ff). Moreover, policy implications can be different in a longer-run perspective, i.e. in a model with endogenous R&D expenditures, innovation, and entry; see, for example, Farrell and Katz (1998).

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