The Interdependence of Financial Markets - Econometric Modeling and Estimation

Inaugural-Dissertation
zur Erlangung des Doktorgrades
der Wirtschaftswissenschaftlichen Fakultät
der Eberhard-Karls-Universität zu Tübingen

vorgelegt von

Dirk Baur
aus Stuttgart

2004

Dekan:	Prof. Dr. rer. pol. Renate Hecker
Erstberichterstatter:	Prof. Dr. rer. pol. Gerd Ronning
Zweitberichterstatter:	Prof. Dr. rer. pol. Joachim Grammig

Tag der mündlichen Prüfung: 26. August 2003

Contents

List of Figures

List of Tables

Chapter 1

Introduction

We live in an era of interdependence.

Keohane and Nye, 2002

The above statement expresses a widespread feeling that the world we live in is now more interconnected than it was before. However, such statements do usually not deliver precise definitions of the words entailed and examples of interdependent phenomena in the medical, social, political and economic aspects of our existence, not to mention the economic structures, are infinite (see Drouet and Kotz, 2001).

We exclusively study economic interdependence and focus on the interdependence of financial markets.

It is noteworthy that the finance literature has neither provided a generally accepted definition or description for "interdependence" nor for "dependence". However, a thorough analysis of "interdependence" requires an accurate definition of the term before examining the sources and the constituting factors of this phenomenon in a static and also a dynamic sense. After the discussion of such issues, it is surely interesting to evaluate the results of such an interconnectedness. Any increased knowledge could lead to a better understanding of the functioning of the international financial system and could answer the question whether the financial markets are part of an appropriate financial architecture or not. An analysis of interdependence can potentially also assign a new role to the International Monetary Fund (IMF) and to the role that international investors play in this system.

Interdependence can be defined as mutual dependence of equally distributed or unevenly distributed entities. This definition contains two types of interdependence: The first is interdependence characterized by equality and symmetry and the latter type is adequately described by inequality and asymmetry. An extreme form of such an unequal interdependence is a pure asymmetric dependence of one entity on another, in other words: A pure asymmetric dependence is a non-mutual dependence.

An example of the latter is as follows: a small country exports lemons (among other things) to the United States. In theory, the US depends on the supply of these lemons, while the small country depends on the US market. But what does that mean in reality? If no lemons are exported from the small country, US consumers hardly notice the price change in the grocery store, while the farmers of the small country suffer severe losses. Thus US "dependence" on the small country is only nominal and the interdependence is highly asymmetric.

Apart from the question whether interdependence is mutual to any extent or not, it is also important to assess the outcomes of interdependence. Does interdependence always lead to better states of the world (synergy) as if there was no interdependence or can interdependence also result in poorer outcomes (negative synergy)? This question can also be discussed for the above example. Classical economic theory suggests that trade is beneficial for all participating parties leading to synergetic effects of interdependence. Whether this is also true for financial markets is an important issue and will be part of the analyses in this work.

The aim of this study is to assess the characteristics of varying interdependence by modeling and estimating symmetric and asymmetric linkages of different financial markets.

We analyze whether the linkages of financial markets increased in the past years, whether these linkages are persistent and whether they exhibit any asymmetric behavior depending on the shocks of the markets. We investigate factors that cause changes of the linkages and analyze how shocks are transmitted from one market to the other. We also examine periods of financial turmoil, especially the Asian crisis in 1997.

More econometric issues are also discussed, such as the relation of volatilities and correlations and the finding of spurious regressions and spillovers.

This study is organized as follows:

The second chapter describes different models to estimate time-varying volatilities in a univariate framework and builds the fundament for an extension to multivariate specifications. In the second part of the chapter, time-varying volatilities and correlations are analyzed within a multivariate GARCH framework.

The third chapter focusses on asymmetric correlations and spillovers obtained by univariate regression models with time-varying volatilities, a time-varying parameter model and a Quantile Regression model.

In terms of dependence or interdependence, the second chapter can be viewed as an analysis of the dependence of *volatilities* through time, an examination of symmetric interdependencies of the *returns* of financial markets through time and the third chapter introduces models of asymmetric (possibly non-mutual) conditional interdependence. Both returns and variances are analyzed.

We contribute to the literature in several parts.

In the first part, we introduce a new bivariate correlation estimator that is more flexible than existing multivariate GARCH models and thus not prone to potential misspecifications. Furthermore, merits and shortcomings of existing multivariate GARCH models are discussed and evaluated in a simulation study. Differences of daily and monthly returns regarding the persistence and the asymmetry of correlations are additionally analyzed.

The second part extends the literature in four ways: First, a classification of the different forms of spillovers is made, second, the potential occurrence of spurious correlations or spillovers is analyzed, third, adequate estimation frameworks to investigate varying spillovers are proposed and fourth, correlations in mean and volatility are analyzed in order to obtain insights into the existence and the causes of contagion among financial markets.

The last part briefly summarizes the main results and points to areas for future research.

Chapter 2

Symmetric Interdependence: Correlations

In this chapter we analyze the interdependence of financial markets within a symmetric model and thus assume an equally distributed dependence between two markets.

We first present an econometric framework that estimates time-varying volatilities and then use this framework to model time-varying correlations. This preliminary part that is exclusively focussing on volatilities is fundamental for the understanding of the concept of time-varying symmetric interdependence.

2.1 Univariate GARCH Models

The (univariate) autoregressive conditional heteroscedastic (ARCH) model was introduced by Engle (1982) and generalized by Bollerslev (1986).

We discuss the main univariate models of the ARCH family that build the basis for multivariate ARCH models. The selection of these models is based on the (i) frequency the models are used in the literature, (ii) the existence of multivariate counterparts and (iii) the *potential* existence of multivariate models.

We assume a simple mean equation without any exogenous regressors since the focus is on the modeling of the variance equation. The mean equation is thus given by

$$r_t = \mu + \epsilon_t \tag{2.1}$$

where μ is a constant and the innovation ϵ_t is factorized as

$$\epsilon_t = z_t h_t^{1/2} \tag{2.2}$$

where z_t is assumed to be an iid sequence with mean zero and variance one.

Engle (1982) postulated the conditional variance h_t to be a function of past squared innovations of ϵ_t:

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \tag{2.3}$$

Since the variance must be positive by definition, the conditional variance h_t is only surely well defined if the parameters satisfy the following conditions: $\omega > 0$ and $\alpha_i \geq 0$ for all i.

It is not readily clear that this model is an autoregressive process as suggested by the name. Defining $v_t = \epsilon_t^2 - h_t$, and substituting $h_t = \epsilon_t^2 - v_t$ in equation (2.3) we get

$$\epsilon_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + v_t \tag{2.4}$$

This model is obviously an autoregressive model. It is covariance stationary if and only if the sum of the positive autoregressive parameters is less than one. A model is said to be covariance stationary if it is mean reverting, i.e. the conditional variance tends to return to its unconditional mean. The unconditional (not time-varying) variance is $\sigma^2 = \omega/(1 - \sum\limits_{i=1}^{q} \alpha_i)$ and can be derived by setting $h_t = \epsilon_t^2 = \epsilon_{t-i}^2$ for all i.

Applied to empirical data, ARCH(q) models make it necessary to use long lag lengths to describe the variance process adequately (e.g. see Engle, 1982). This problem can be circumvented by using the Generalized ARCH model (GARCH) proposed by Bollerslev

(1986). The GARCH(p,q) model can be written as

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{2.5}$$

Again, for the conditional variance to be well defined, all parameters must be non-negative. A widely applied model is the GARCH(1,1):

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} \tag{2.6}$$

Using the same transformation as for the ARCH model ($\nu_t = \epsilon_t^2 - h_t$), we get:

$$\epsilon_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \epsilon_{t-1}^2 - \beta v_{t-1} + v_t \tag{2.7}$$

This is an ARMA(1,1) model for ϵ_t^2. The GARCH(1,1) process is covariance stationary if and only if the sum of the autoregressive parameters ($\alpha + \beta$) is less than one. The unconditional variance is computed by setting $h_t = h_{t-i} = \epsilon_{t-i}^2$ which yields

$$\sigma^2 = \omega/(1 - \alpha - \beta) \tag{2.8}$$

Rewriting equation (2.6) and adding the term $(\alpha h_{t-1} - \alpha h_{t-1})$ yields:

$$h_t = \omega + (\alpha + \beta) h_{t-1} + \alpha(\epsilon_{t-1}^2 - h_{t-1}) \tag{2.9}$$

The term $(\epsilon_{t-1}^2 - h_{t-1})$ has mean zero and can be thought of as a volatility shock at time t. The coefficient α measures the extent to which the variance shock at time t feeds through into the volatility of the next period. The sum of the parameters α and β measures the rate at which this effect dies out over time, i.e. it measures the persistence of shocks. It can also be shown that the GARCH(1,1) process is a nonlinear autoregressive process (of order one) with a stochastic autoregressive coefficient. To show this, we write equation (2.6) as

$$h_t = \omega + (\beta + \alpha \frac{\epsilon_{t-1}^2}{h_{t-1}}) h_{t-1}$$

Using $z_t = \dfrac{\epsilon_t}{\sqrt{h_t}}$ we get

$$h_t = \omega + (\beta + \alpha z_{t-1}^2) h_{t-1} \tag{2.10}$$

Here, the term $(\beta+\alpha z_{t-1}^2)$ is the stochastic autoregressive coefficient making the process nonlinear.

To illustrate the dynamic properties of the GARCH models, we simulate four different GARCH(1,1) processes with the following parameter values for α and β ($\omega = 0.01$): (i) $\alpha = 0.05$, $\beta = 0.9$, (ii) $\alpha = 0.05$, $\beta = 0.94$, (iii) $\alpha = 0.05$, $\beta = 0.5$ and (iv) $\alpha = 0.25$, $\beta = 0.7$. The upper plot of figure 2.1 presents the time-varying volatilities for a GARCH(1,1) process with the parameter values given by (i) and (ii) and the lower part of the figure presents the time-varying volatilities for the parameter values given by (iii) and (iv). We assume the same random innovation ϵ_t ($\sim N(0,1)$) for all processes. It is evident that process (ii) exhibits the highest persistence of shocks and a pronounced pattern of volatility clustering. This pattern is also visible for processes (i) and (iv). The clustering is not identifiable for the process given by (iii).

Recursively substituting equation (2.8) into equation (2.9) leads to the conditional expectation of volatility (j periods ahead):

$$E_t(h_{t+j}) = (\alpha+\beta)^j(h_t - \frac{\omega}{1-\alpha-\beta}) + \frac{\omega}{1-\alpha-\beta} \tag{2.11}$$

Hence, the volatility at $t+j$ reverts to its unconditional mean at rate $(\alpha+\beta)$. For $\alpha+\beta = 1$ shocks are persistent and the GARCH process is said to be integrated in volatility (of order one) and thus called integrated GARCH (IGARCH). However, the presence of an unit root in the volatility process must not be confused with an unit root in the underlying returns for example. IGARCH processes do not violate the stationarity properties (see Nelson, 1991).

The ARCH and GARCH models assume that the conditional variance h_t is a function of lagged *squared* residuals. However, Taylor (1986) and Schwert (1989) use the absolute residuals to model the time-varying variance. These modified GARCH models are less frequently used and thus not further discussed.

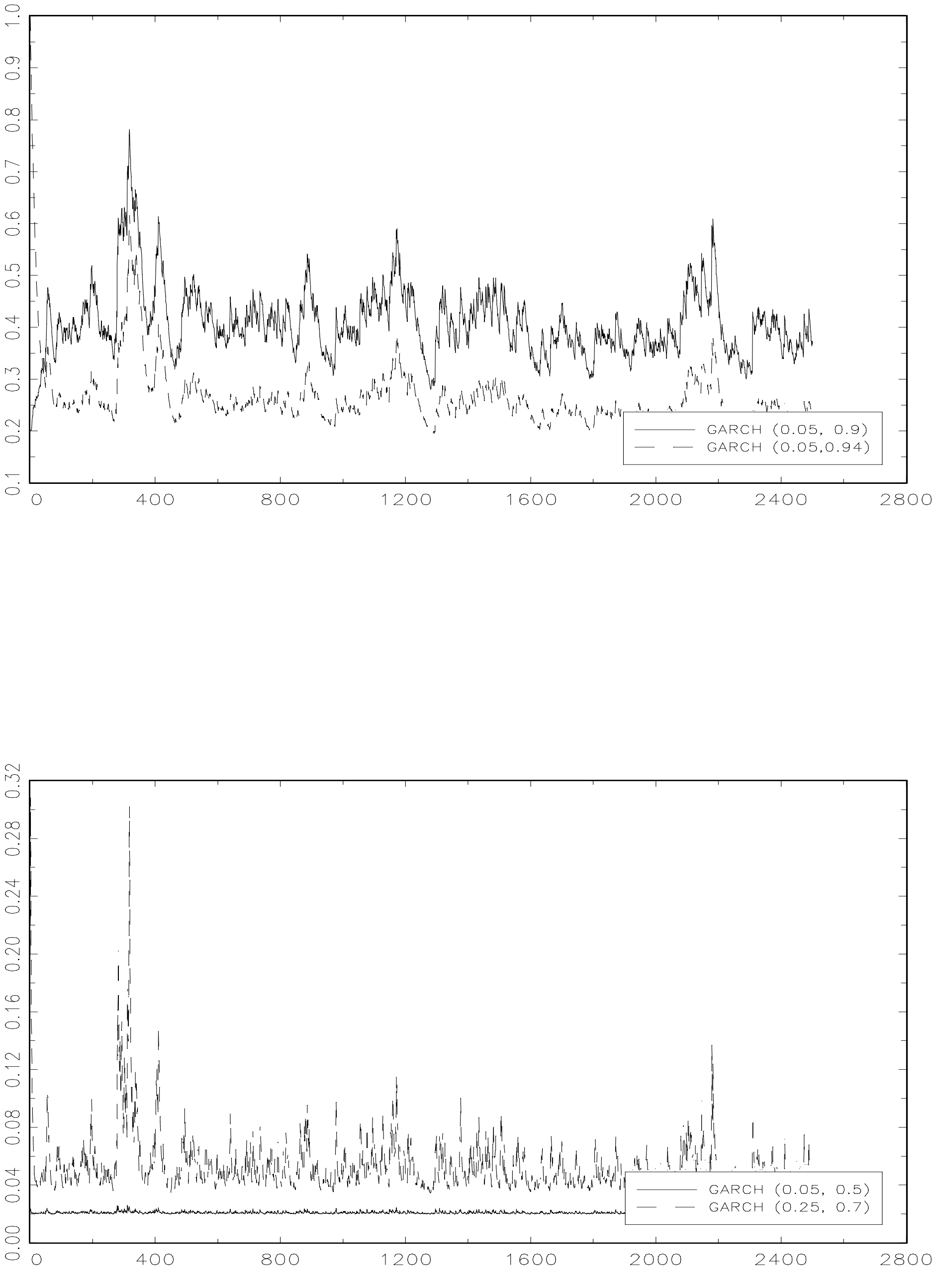

Figure 2.1: *Simulated GARCH(1,1) processes*

The GARCH models have been introduced to model time-varying volatilities and the stylized fact of volatility clustering. Another empirical regularity is the asymmetric effect of positive and negative shocks on volatility. This is discussed in the next section.

2.1.1 Asymmetric GARCH Models

Black (1976) and Christie (1982) found evidence that stock returns are negatively correlated with return volatility. This asymmetry (often related to as financial leverage or volatility feedback) means that volatility tends to rise in response to a negative shock and to fall in response to a positive shock.[1]

The GARCH model does not account for this finding and assumes symmetric impacts of positive and negative shocks on future volatility. The Asymmetric ARCH (AARCH) model of Engle (1990), the Quadratic GARCH (QGARCH) model of Sentana (1991), the Exponential GARCH (EGARCH) model of Nelson (1991) and the Asymmetric GARCH (AGARCH) model of Glosten, Jagannathan and Runkle (1993) account for these asymmetries. All these asymmetric models and the non-asymmetric GARCH model can be nested in one model (see Hentschel, 1995). However, the presentation of such a nested model would not be consistent with the focus of this section which is only an introduction to multivariate GARCH models.

A forerunner of the Asymmetric GARCH model of Glosten et al. (1993) was proposed by Engle (1990). However, only the model of Glosten et al. is now widely applied to financial data:

$$h_t = \omega + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \epsilon_{t-1}^2 D_t + \beta h_{t-1}$$

[1] Financial leverage means that a highly leveraged firm faces more uncertainty when stock prices fall since the leverage and thus volatility increases. Volatility feedback means that higher volatility results in higher risk which requires higher expected returns that force stock prices to fall (see Campbell and Hentschel, 1992).

where D_t is a dummy variable that equals one if $\epsilon_{t-1} < 0$ and zero otherwise. Hence, negative values of ϵ_{t-1} are additionally captured by the parameter α_2 (besides the parameter α_1) that measures the impact of these shocks on volatility at t. If α_2 is statistically significant different from zero there is an asymmetric effect of positive and negative shocks on future volatility. In most empirical applications of this model, α_2 is positive which implies that negative shocks increase volatility more than positive shocks. The model of Glosten et al. (1993) can also be called a Threshold GARCH model (TGARCH). For example, the dummy variable D could be used to estimate the impact of shocks larger than a multiple of the standard deviations of ϵ_t on the conditional volatility.

The Quadratic GARCH model was introduced by Sentana (1991) and the conditional variance h_t is

$$h_t = \omega + \alpha(\epsilon_{t-1} + b)^2 + \beta h_{t-1}$$

This specification produces a symmetric curve around b. If b is negative this means that negative shocks increase the conditional volatility h_t more than negative shocks. A more popular model is based on the idea of using an exponential function instead of the linear representation of the simple ARCH model and its asymmetric extensions. This was first mentioned by Engle (1982) but only proposed by Nelson (1991).

The conditional variance h_t of this exponential GARCH (EGARCH) model is given by:

$$h_t = \exp\left(\omega + \sum_{k=1}^{q} g_k(z_{t-k}) + \sum_{i=1}^{p} \beta_i \ln h_{t-i}\right) \tag{2.12}$$

where ω is the constant, g_k a function of the standardized residuals z_{t-k} ($= \frac{\epsilon_{t-k}}{\sigma_{t-k}}$) and β the parameter that describes the volatility clustering of the process.

Taking the logarithm, we get a linear model

$$\ln(h_t) = \omega + \sum_{k=1}^{q} g_k(z_{t-k}) + \sum_{i=1}^{p} \beta_i \ln h_{t-i} \tag{2.13}$$

This linear formulation is very common but only theoretical and can be misleading since the process is estimated as an exponential function given by equation (2.12).

A less common formulation is

$$h_t = \exp(\omega)\exp\left(\sum_{k=1}^{q} g_k(z_{t-k})\right)\prod_{i=1}^{p} h_{t-i}^{\beta_i} \tag{2.14}$$

This formulation shows that the parameter β must not be confused with the counterpart in linear asymmetric GARCH models since β is not a linear coefficient of h_{t-1} but an exponent. As already mentioned, equation (2.13) is therefore a rather confusing formulation (but very common).

Unlike other GARCH models the EGARCH process does not require any restrictions to ensure non-negativity of the conditional variance. Equations (2.12) and (2.14) show that h_t is a nonlinear function of h_{t-i}. This is especially important for the analysis of the persistence of shocks.

The asymmetric relation between returns and volatility changes is captured by the function $g_k(z_t)$.

$$g_k(z_{t-k}) = \theta_k z_{t-k} + \gamma_k[|z_{t-k}| - E(|z_{t-k}|)] \tag{2.15}$$

There is evidence of an asymmetric impact of shocks on conditional volatility if $\theta_k < 0$ for $q = 1$. This means that positive shocks can also reduce volatility whereas negative shocks always augment it. This effect is in contrast to all linear asymmetric GARCH models where shocks always increase volatility. This is due to the parameter restrictions that are necessary in the linear models to guarantee positive conditional variances but need not imposed in the EGARCH model. Thus, the EGARCH model can be viewed as more flexible. The magnitude effect of the process is described by the term $\gamma_k[|z_{t-k}| - E(|z_{t-k}|)]$. It could be argued that the exponential function does part of the work itself because large values augment volatility proportionally more than small values.

Equation (2.12) shows that if $q = 1$ the parameter estimates of θ_1 and γ_1 lead to **one** function $g(z, \theta_1, \gamma_1)$. For $\theta_1 < 0$ a plot of this function with $z_t = z$ would show the asymmetric impact of shocks on $g(z)$ and thus on volatility. However, if $q > 1$, q different $g_k(z)$ functions result and the interpretation of the asymmetry is not straightforward since the $g_k(z)$ functions do not depend on just one variable z_t but on different variables z_{t-k} for $k = 1, 2, 3, ...$. This means that different z_{t-k} and θ_k have different impacts on h_t. Thus, it is not sufficient to analyze only the impact of each shock on conditional volatility but also the aggregate impact of the shocks.
The same statement is true for all other asymmetric GARCH models with higher lag orders. We stressed this problem for the EGARCH model since only this model contains a function $g(z)$ explicitly modeling the asymmetric effect.

The analysis of asymmetric effects of positive and negative shocks and the persistence of shocks in general have commonly been analyzed separately. El Babsiri and Zakoian (2001) closed this gap and introduced the concept of contemporaneous asymmetry which allows different volatility processes for positive and negative return movements. In other words, if positive and negative shocks have a different impact on volatilities they might also have another persistence and conditional distribution.

We do not discuss this issue in more detail and focus on the asymmetric effect and the news-impact curve in the next section.

2.1.2 The News-Impact Curve

The news-impact curve introduced by Pagan and Schwert (1990) and Engle and Ng (1993) shows how positive and negative shocks influence conditional volatility. Engle and Ng (1993) analyzed processes of the order one for the autoregressive and the moving average term.[2] The news-impact curve measures how new information is incorporated into volatility estimates. Thus, holding constant the information dated $t-2$ and earlier it plots the implied relation between shocks at $t-1$ (ϵ_{t-1}) and conditional volatility at t (h_t).

The news-impact curve for the GARCH(1,1) process is given by

$$h_t = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma^2 \tag{2.16}$$

where σ^2 is the unconditional variance. The equation shows that positive and negative shocks have the same influence on conditional volatility h_t.

The QGARCH(1,1) model exhibits an asymmetric impact of shocks on future volatility if $b \neq 0$:

$$h_t = \omega + \alpha(\epsilon_{t-1} + b)^2 + \beta\sigma^2 \tag{2.17}$$

The news-impact curve of the AGARCH(1,1) process is given by the two equations for positive and negative shocks:

$h_t = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma^2$ for $\epsilon_{t-1} > 0$

$h_t = \omega + (\alpha + b)\epsilon_{t-1}^2 + \beta\sigma^2$ for $\epsilon_{t-1} < 0$

The EGARCH(1,1) model is also given by two different equations :

$h_t = A \cdot \exp\left(\frac{(\theta+\gamma)}{\sigma} \cdot \epsilon_{t-1}\right)$ for $\epsilon_{t-1} > 0$ and

$h_t = A \cdot \exp\left(\frac{(\theta-\gamma)}{\sigma} \cdot \epsilon_{t-1}\right)$ for $\epsilon_{t-1} < 0$

with $A = \sigma^{2\beta} \cdot \exp\left(\alpha - \gamma \cdot E(|\epsilon_{t-1}|)\right)$. Note, that the news-impact curve is a function of ϵ_{t-1} and not of z_{t-1} since the standardized residuals z_t can only be constructed by the use of the unconditional variance.

We plot the news-impact curves for the four discussed models with typical parameter values to obtain pronounced but also comparable functions. The news-impact curves are shown in figure 2.2. It is evident that the GARCH model reacts equally to positive and

[2] Note, that the order of the investigated asymmetric GARCH models is determined ad hoc and not by any selection criterion.

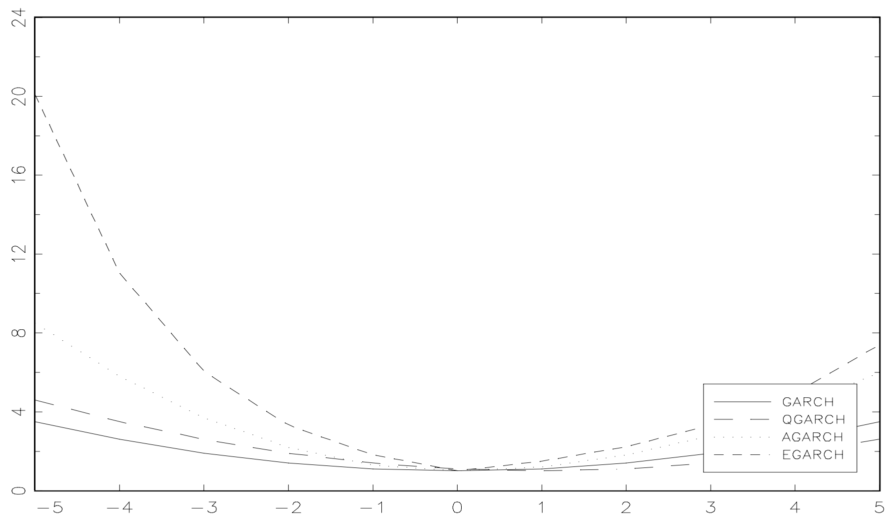

Figure 2.2: *News-Impact Curves*

negative shocks which is not true for the other asymmetric GARCH models. In the plotted example, the QGARCH model has higher variance for negative shocks but lower variance for positive shocks than the (non-asymmetric) GARCH model. The AGARCH model has higher conditional variance than the GARCH and the QGARCH model for all shocks but is always below the EGARCH where increased volatility is very pronounced for large shocks which is due to the exponential function.

Note that processes with a higher lag order are not adequately described by news impact curves since only the isolated effect of a shock at time $t-1$ on conditional volatility at time t is shown. The aggregate effect caused by the higher lag order is not revealed. In addition, this graphical concept does not show the dynamic properties of the models, i.e. the persistence of shocks is not uncovered. The dynamic behavior could be visualized with an impulse-response function. Such a function could show both the different impacts of positive and negative shocks and its persistence on volatility. This graphical approach has the advantage that it is easily implementable and also feasible for the EGARCH process for which the persistence can not be formulated analytically. We do not plot such impulse-response functions since we also do not extent this concept for to the Multivariate GARCH Models.

The previously discussed asymmetry of positive and negative shocks on conditional volatility assumes that the volatility processes for up and down moves are the same. El Babsiri and Zakoian (2001) introduce the concept of contemporaneous asymmetry which allows different volatility processes for positive and negative return movements, i.e. positive and negative shocks can not only have a different impact on conditional volatilities but can also exhibit different conditional distributions.

2.1.3 Estimation

There are three main possibilities to estimate the above described GARCH models. The most common is by Maximum Likelihood (ML) or Quasi ML (QML). Other approaches use a two-stage OLS procedure or the Generalized Methods of Moments (GMM). The latter two methods have the advantage that they do not need any distributional assumption. However, the two-stage OLS does only work well for ARCH models and both estimation procedures are rarely used. Most contributions in the literature only mention the alternatives but use Maximum Likelihood.

The estimation of the discussed ARCH and GARCH models with the Maximum Likelihood method is based on a sample of T observations of the returns vector $\boldsymbol{r}$ and can be done through numerical maximization of a likelihood function assuming a particular distribution of the returns vector (for example, Engle and Bollerslev (1986) assume a t-distribution and Nelson (1991) uses a Generalized Error Distribution (GED)). For normally distributed returns, we get the following log-likelihood function:

$$\log L(\boldsymbol{\theta}; r_1, \ldots, r_T) = -T/2\,\log(2\pi) - 1/2\sum \log(h_t) - 1/2\sum \frac{\epsilon_t^2}{h_t} \tag{2.18}$$

where $\boldsymbol{\theta}$ is a parameter vector consisting of all parameters to be estimated in the mean and the variance equation.

The two-stage OLS estimation consists of the following stages: First, the residual $\epsilon_t = r_t - x_t'\gamma$ is estimated where r_t is the returns vector as above, x_t an exogenous variable and γ the parameter to be estimated. In the second step the conditional volatility in an ARCH model is estimated: $\hat{\epsilon}_t^2 = \omega + \alpha\hat{\epsilon}_{t-1}^2$. Estimation of a GARCH model would require an additional step to estimate β.

GMM estimation of ARCH-type models was used by Glosten et al. (1993) and Rich et al.(1991) among others. However, apart from the two mentioned examples, the application of GMM is rare compared to the use of the QML method. A more detailed discussion of GMM can be found in Pagan (1996). Pagan (1996) summarized that *great care has to be exercised when applying GMM estimators* to ARCH type models (page 49). Recently, Skoglund (2001) has further explored efficiency gains by the use of GMM estimation and shown that GMM is advantageous compared to ML when excess-kurtosis, high peakedness and skewness are characteristics of the data.

2.1.4 Conclusions

We have discussed the main univariate GARCH models in order to lay the ground to extend these models to multivariate versions. In contrast to this chapter, we will focus on the covariance and correlation of the time-series and not on its volatility. However, since the covariances and correlations are similarly modeled and parameterized as volatilities, it is essential to discuss these models in such extensive form as chosen here.

We have not discussed all existing GARCH models since the aim is the description of multivariate GARCH models, its existing specifications, shortcomings, innovations and potential extensions of these models. For example, the discussion of the news-impact curve is important to understand the multivariate counterpart - the news-impact surface.

The same is true for the discussion of the asymmetric effect of positive and negative shocks, the persistence of shocks and the possible estimation procedures presented in the previous section.

2.2 Multivariate GARCH Models

The knowledge of the time-varying behavior of correlations and covariances between asset returns is an essential part in asset pricing, portfolio selection and risk management. Whereas unconditional correlations can easily be estimated, this is not true for time-varying correlations. One approach to estimate conditional covariances and correlations is within a Multivariate GARCH model. Other approaches as a moving average specification for the covariances and the variances provide time-varying correlations but do not parameterize the conditional correlations directly. We attribute the fact that correlations are considerably less frequently analyzed than variances mainly to the difficulties in the estimation process. Consequently, studies comparing the existing multivariate GARCH models are rare in relation to the existing studies that compare univariate time-varying volatility models (see Pagan and Schwert (1990) and Engle and Ng (1993) among others). For multivariate GARCH models we are only aware of the work of Kroner and Ng (1998), Engle (2000) and Engle and Sheppard (2001). While Kroner and Ng (1998) compare the main existing models within an empirical analysis, Engle (2000) and Engle and Sheppard (2001) use Monte-Carlo simulations to analyze different models with a focus on the Dynamic Conditional Correlation (DCC) estimator.

The first multivariate GARCH model is proposed by Bollerslev, Engle and Wooldridge (1988). This model uses the VECH operator and is thus referred to as VECH-model. It does not guarantee a positive-definite covariance matrix and the number of parameters is relatively large. Baba, Engle, Kroner and Kraft (1991) proposed a multivariate GARCH model, called BEKK (named after the authors), that guarantees the positive definiteness of the covariance matrix.[3] Interestingly, it seems that even restricted versions of the BEKK

[3]Restricting the BEKK model to be diagonal reduces the number of parameters that must be estimated. The Factor GARCH model (Engle et al., 1990) reduces the number of parameters and can be transformed to a BEKK model.

model have too many parameters since commonly only bivariate models are estimated (see Bekaert and Wu, 2000, Engle, 2000, Karolyi et al., 1996, Kroner and Ng, 1998, Longin and Solnik, 1995 and Ng, 2000). In addition, we are not aware of any multivariate GARCH model that is estimated with a higher lag order than GARCH(1,1).

The Constant Correlation Model (CCM) of Bollerslev (1990) does also circumvent the problem of possible non-positive definiteness of the covariance matrix but is very restrictive since it does not allow correlations to be time-varying.
Asymmetric extensions of the existing model are introduced by Kroner and Ng (1998) who proposed the general asymmetric dynamic covariance (ADC) model that nests the VECH, the Factor GARCH, the BEKK model and the Constant Correlation Model.[4]

Recently, Tse and Tsui (2000) proposed a new multivariate GARCH model that parameterizes the conditional correlation directly by using the empirical correlation and Engle (2000) proposed a time-varying correlation model, called Dynamic Conditional Correlations (DCC) that also parameterizes the conditional correlation directly but uses a two-stage estimation strategy. The Bivariate Dynamic Correlations (BDC) estimator proposed in this chapter can be assumed to be in the same class as the models by Tse and Tsui (2000) and Engle (2000) but is different in various respects which we discuss later on.

The remainder of this chapter is as follows: Section 2.2.1 discusses existing multivariate GARCH models and focusses on the full and restricted BEKK model and its asymmetric extensions. We also discuss the Constant Correlation Model of Bollerslev (1990) and use this model as a benchmark for volatility estimates. Section 2.2.2 introduces a new Bivariate Dynamic Correlation (BDC) Model that parameterizes the conditional correlation directly and guarantees positive definite covariance matrices with fewer parameters than the full BEKK model and more flexibility than the restricted BEKK model. The estima-

[4]Note that the nested ADC model requires further restrictions to guarantee a positive-definite covariance matrix.

tion of the described multivariate GARCH Models is explained in section 2.2.3 and Section 2.2.4 shows results of Monte-Carlo simulations for all discussed models. Section 2.2.5 estimates the BDC model and the diagonal BEKK model for daily and monthly data and focusses on the persistence and the asymmetry of time-varying correlations. Section 2.2.6 concludes.

2.2.1 Existing Multivariate GARCH Models

Extending the univariate GARCH model to a n-dimensional multivariate model requires to estimate n different mean and corresponding variance equations and $\frac{n^2-n}{2}$ covariance equations. We use a simple specification for the mean equation since our interest is the time-varying covariance matrix. Thus, returns are modeled as follows:

$$\boldsymbol{r}_t = \boldsymbol{\mu} + \boldsymbol{\epsilon}_t \qquad\qquad \boldsymbol{\epsilon}_t \,|\, \boldsymbol{\Omega}_{t-1} \sim \mathrm{N}(\boldsymbol{0}, \boldsymbol{H}_t) \tag{2.19}$$

where $\boldsymbol{r}_t$ is a vector of appropriately defined returns and $\boldsymbol{\mu}$ is a $(N\times 1)$ vector of parameters that estimates the mean of the return series. The residual vector is $\boldsymbol{\epsilon}_t$ with the corresponding conditional covariance matrix $\boldsymbol{H}_t$ given the available information set $\boldsymbol{\Omega}_{t-1}$.

We focus on the BEKK model since it is the only time-varying covariance model that guarantees a positive-definite covariance matrix. We also discuss the Constant Correlation Model (CCM) and a Zero Correlation Model (ZCM) which are used as benchmark models.

2.2.1.1 The VECH Model

The equivalent to an univariate GARCH(1,1) model is given as follows:[5]

$$\mathrm{vech}(\boldsymbol{H}_t) = \boldsymbol{\Omega} + \boldsymbol{A}\mathrm{vech}(\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1}) + \boldsymbol{B}\mathrm{vech}(\boldsymbol{H}_{t-1}) \tag{2.20}$$

where $\boldsymbol{H}_t$ is the time-varying $(N \times N)$ covariance matrix, $\boldsymbol{\Omega}$ denotes an $(N(N+1)/2 \times 1)$ vector and $\boldsymbol{A}$ and $\boldsymbol{B}$ are $(N(N+1)/2 \times N(N+1)/2)$ matrices. The VECH operator stacks

[5] We subsequently assume a simple mean equation as given by equation (2.19) and do exclusively focus on the covariance matrix.

the lower portion of an $(N \times N)$ symmetric matrix as an $(N(N+1)/2 \times 1)$ vector which can be done since the covariance matrix is symmetric by definition. In the bivariate VECH model the matrices are all (3×3) matrices thus leading to 27 parameters to be estimated.

$$\begin{pmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{pmatrix} = \mathbf{\Omega} + \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \epsilon^2_{1,t-1} \\ \epsilon_{t-1}\epsilon_{2,t-1} \\ \epsilon^2_{2,t-1} \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{pmatrix} \tag{2.21}$$

The diagonal VECH model reduces the number of parameters by using diagonal matrices $\boldsymbol{A}$ and $\boldsymbol{B}$. However, even for this special case a positive definite covariance matrix is not guaranteed.[6]

Hence, we do not present this model in its asymmetric extension and dispense with a discussion.

2.2.1.2 The BEKK Model

The BEKK model was introduced by Baba, Engle, Kraft and Kroner (1991) and can be seen as an improvement to the VECH model (introduced by Bollerslev, Engle and Wooldridge, 1988). First, the number of parameters is reduced and second, the positive-definiteness of the covariance matrix is guaranteed.

We initially present the full (unrestricted) BEKK model and its asymmetric extension and then restrict this model to the diagonal BEKK.[7]

The covariance matrix of the unrestricted BEKK model is

[6] A positive definite covariance matrix would imply that the determinant of

$$\boldsymbol{H}_t = \begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{12,t} & h_{22,t} \end{pmatrix}$$

is positive. That means that $h_{11,t}h_{22,t} > h^2_{12,t}$ which is not guaranteed since the parameters a_{ij} and b_{ij} are freely estimated for all $i, j = 1, 2$.

[7] The multivariate Factor GARCH model will not be presented here since it can be derived from a full BEKK model (see Kroner and Ng, 1998).

$$\boldsymbol{H}_t = \boldsymbol{A}'\boldsymbol{A} + \boldsymbol{B}'\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1}\boldsymbol{B} + \boldsymbol{C}'\boldsymbol{H}_{t-1}\boldsymbol{C} \tag{2.22}$$

$\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ are matrices of parameters with appropriate dimensions. It is obvious from the equation above that the covariance matrix is guaranteed to be positive definite as long as $\boldsymbol{A}'\boldsymbol{A}$ is positive definite. Furthermore, the parameters are squared or cross-products of themselves leading to variance and covariance equations without an univariate GARCH counterpart (see also equation (2.25)). Note that this is not true for the VECH model which is a simple extension of univariate GARCH models to a multivariate form.

The asymmetric extension of this model introduced by Kroner and Ng (1998) bases on the univariate asymmetric GARCH model proposed by Glosten et al. (1993). Here, the covariance matrix is given as follows:

$$\boldsymbol{H}_t = \boldsymbol{A}'\boldsymbol{A} + \boldsymbol{B}'\boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}'_{t-1}\boldsymbol{B} + \boldsymbol{C}'\boldsymbol{H}_{t-1}\boldsymbol{C} + \boldsymbol{D}'\boldsymbol{\eta}_{t-1}\boldsymbol{\eta}'_{t-1}\boldsymbol{D} \tag{2.23}$$

where $\eta_{i,t} = \min\{\epsilon_{i,t}, 0\}$ and $\boldsymbol{\eta}_t = (\eta_{1,t}, \eta_{2,t}, ...)'$. Thus, this extension can capture asymmetric effects of shocks by additionally including negative shocks and still guarantees the positive-definiteness of the covariance matrix.

To clarify the difficulties in interpreting the parameters of the covariance matrix we consider the general BEKK model in bivariate form. $h_{11,t}$ and $h_{22,t}$ denote the conditional variances of the underlying return series and $h_{12,t}$ is their covariance:

$$\begin{pmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{pmatrix} = A'A + \begin{pmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{pmatrix} \begin{pmatrix} \epsilon^2_{1,t-1} & \epsilon_{t-1}\epsilon_{2,t-1} \\ \epsilon_{1,t-1}\epsilon_{2,t-1} & \epsilon^2_{2,t-1} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} + \tag{2.24}$$

$$+ \begin{pmatrix} c_{11} & c_{21} \\ c_{12} & c_{22} \end{pmatrix} \begin{pmatrix} h_{11,t-1} & h_{12_t-1} \\ h_{12,t-1} & h_{22,t-1} \end{pmatrix} \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

Without using matrices (see equation above), we get the following form:

$$
\begin{aligned}
h_{11,t} &= a_{11}^2 + b_{11}^2\epsilon_{1,t-1}^2 + 2b_{11}b_{21}\epsilon_{1,t-1}\epsilon_{2,t-1} + b_{21}^2\epsilon_{2,t-1}^2 + c_{11}^2 h_{11,t-1} + \\
&\quad + 2c_{11}c_{21}h_{12,t-1} + c_{21}^2 h_{22,t-1} \\
h_{12,t} &= a_{12}a_{11} + b_{11}b_{12}\epsilon_{1,t-1}^2 + (b_{12}b_{21} + b_{11}b_{22})\epsilon_{1,t-1}\epsilon_{2,t-1} + b_{21}b_{22}\epsilon_{2,t-1}^2 + \\
&\quad + c_{11}c_{12}h_{11,t-1} + (c_{12}c_{21} + c_{11}c_{22})h_{12,t-1} + c_{21}c_{22}h_{22,t-1} = h_{21,t} \\
h_{22,t} &= a_{12}^2 + a_{22}^2 + b_{12}^2\epsilon_{2,t-1}^2 + 2b_{12}b_{22}\epsilon_{1,t-1}\epsilon_{2,t-1} + b_{22}^2\epsilon_{2,t-1}^2 + c_{12}^2 h_{11,t-1} + \\
&\quad + 2c_{12}c_{22}h_{12,t-1} + c_{22}^2 h_{22,t-1}
\end{aligned}
\tag{2.25}
$$

The latter formulation clarifies that even for the bivariate model the interpretation of the parameters may be misleading since there is no equation that does exclusively possess its own parameters, i.e. parameters that exclusively govern an equation. Hence, it is possible that a parameter is biased by the fact that it influences two equations simultaneously or by the sole number of regressors (see also Tse, 2000), e.g. the regressors $\epsilon_{2,t-1}^2$ and the regressor $h_{22,t-1}$ in the first variance equation ($h_{11,t}$) could both be viewed as a volatility spillover from the second return. In addition, the statistical significance of the parameters is also unclear due to the combinations of different parameters serving as new coefficients for particular regressors.

These critics do not all apply to the diagonal BEKK model where both parameter matrices are diagonal. Thus, the off-diagonal elements are all equal to zero (apart from the constant term $A'A$). The number of parameters to be estimated is significantly lower while maintaining the main advantage of this specification, the positive definiteness of the conditional covariance matrix. Instead of equation (2.25) we have

$$
\begin{aligned}
h_{11,t} &= a_{11}^2 + b_{11}^2\epsilon_{1,t-1}^2 + c_{11}^2 h_{11,t-1} \\
h_{22,t} &= a_{11}^2 + a_{22}^2 + b_{22}^2\epsilon_{2,t-1}^2 + c_{22}^2 h_{22,t-1} \\
h_{12,t} &= h_{21,t} = a_{11}a_{22} + b_{11}b_{22}\epsilon_{1,t-1}\epsilon_{2,t-1} + c_{11}c_{22}h_{12,t-1} \\
h_{21,t} &= h_{12,t}
\end{aligned}
\tag{2.26}
$$

This model exhibits essentially the same problems as the Full BEKK model since there is no parameter in any equation that exclusively governs a particular covariance equation. Hence, it is not clear whether the parameters for h_{12} are just the result of the parameter estimates for h_{11} and h_{22} or if the covariance equation alters the parameter estimates of the variance equations. In addition, the model is not very flexible and can consequently be misspecified. For example, assuming that the persistence of shocks to volatility is relatively high for both return series, say $b_{ii} + c_{ii} = 0.05 + 0.90 = 0.95$ for $i = 1, 2$, then the persistence of the covariance must be almost equally high, $b_{ii}b_{jj} + c_{ii}c_{jj} = 0.05 \cdot 0.05 + 0.9 \cdot 0.9 = 0.0025 + 0.81 = 0.8125$ for $i = 1$ and $j = 2$. Supposed that covariances are less persistent or equally persistent as volatilities it is clear that either volatilities or the covariance is misspecified.

2.2.1.3 Constant Correlation Model and Zero Correlation Model

The Constant Correlation Model (CCM) of Bollerslev (1990) models time-varying covariances more parsimoniously than the models discussed above.

The bivariate model is given by

$$
\begin{aligned}
h_{11,t} &= a_{11} + b_{11}\epsilon_{1,t-1}^2 + c_{11}h_{11,t-1} \\
h_{22,t} &= a_{22} + b_{22}\epsilon_{2,t-1}^2 + c_{22}h_{22,t-1} \\
h_{12,t} &= \rho\sqrt{h_{11,t}h_{22,t}} \\
h_{21,t} &= h_{12,t}
\end{aligned} \tag{2.27}
$$

where ρ is a parameter that can be estimated almost freely (ρ must be in the range $[-1,1]$) and is equal to the empirical correlation coefficient (see Bollerslev, 1990). In contrast to the BEKK model there is a parameter in the CCM (ρ) that exclusively governs the covariance equation. Note that the CCM exhibits time-varying covariances but only constant correlations.[8]

Setting ρ to zero implies a model that we call Zero Correlation Model (ZCM).

We will both use the CCM and the ZCM to analyze in which respect covariances affect variance estimates.

2.2.1.4 Asymmetric Extensions

While it is straightforward in the diagonal BEKK Model to analyze whether the covariance exhibits the same degree of persistence as the variances, the relevant parameter estimates measuring the persistence of shocks are potentially influenced by each other leading to biased parameter estimates. This is also true for the full BEKK Model and possibly more severe due to the larger number of parameters.

The same problem arises for the asymmetric extensions of the models. To illustrate this, we analyze the asymmetric extensions proposed by Kroner and Ng (1998) and focus on the diagonal BEKK model.

[8] To guarantee positive variances we use the variance equations of the diagonal BEKK model for the variance equations of the CCM as suggested by Kroner and Ng (1998).

The following asymmetric covariance equations for the bivariate case within the full BEKK model are

$$\begin{aligned} h_{11,t} &= ... + d_{11}^2\eta_{1,t-1}^2 + 2d_{11}d_{21}\eta_{1,t-1}\eta_{2,t-1} + d_{21}^2\eta_{2,t-1}^2 \\ h_{22,t} &= ... + d_{12}^2\eta_{1,t-1}^2 + 2d_{12}d_{22}\eta_{1,t-1}\eta_{2,t-1} + d_{22}^2\eta_{2,t-1}^2 \\ h_{12,t} &= ... + d_{11}d_{12}\eta_{1,t-1}^2 + (d_{12}d_{21} + d_{11}d_{22})\eta_{1,t-1}\eta_{2,t-1} + d_{21}d_{22}\eta_{2,t-1}^2 \end{aligned} \tag{2.28}$$

where $\eta_{i,t} = \min\{\epsilon_{i,t}, 0\}$ and $\boldsymbol{\eta}_t = (\eta_{1,t}, \eta_{2,t}, ...)'$. Equation (2.28) shows that the number of parameters and its combinations make it difficult to interpret any (clear) asymmetry of the impact of shocks on the conditional (co-)variance.

For the diagonal BEKK model (see equation (2.26)) the asymmetric extension is

$$\begin{aligned} h_{11,t} &= ... + d_{11}^2\eta_{1,t-1}^2 \\ h_{22,t} &= ... + d_{22}^2\eta_{2,t-1}^2 \\ h_{12,t} &= ... + d_{11}d_{22}\eta_{1,t-1}\eta_{2,t-1} \end{aligned} \tag{2.29}$$

where $\eta_{i,t} = \min\{\epsilon_{i,t}, 0\}$ and $\boldsymbol{\eta}_t = (\eta_{1,t}, \eta_{2,t}, ...)'$.

Here, the covariance reacts to negative shocks $\eta_{i,t}$ as determined by the asymmetry implied by the variance equations or vice versa. For example, assuming that variance h_{11} does not react asymmetrically to positive and negative shocks ($d_{11} = 0$) and variance h_{22} does ($d_{22} = 0.2$), the asymmetric effect for the covariance would be zero ($d_{11}d_{22} = 0$). Consequently, if there is an asymmetric effect of the covariance, either the variance equation or the covariance equation will be misspecified. Another example is the case where the asymmetry of the covariance is equal to 0.2. Then, the parameters d_{11} or d_{22} would have to be very large to capture this covariance asymmetry (e.g. $d_{11} = d_{22} = \sqrt{0.2}$).[9]

The asymmetric extension of the CCM (see equation 2.27) introduced by Kroner and Ng (1998) has the variance equations of the diagonal BEKK model and the covariance

[9] Ang and Chen (2002) report misspecifications of the asymmetric effect in an asymmetric GARCH-M model without being specific in pointing to this problem.

equation as given in the original model. Again, we could use this model to analyze how variance estimates change when correlations are modeled time-varying. This question is further examined in the simulation study in section 2.2.4.

The next section introduces a new bivariate model that reduces the number of parameters compared to the full BEKK model and extends the flexibility compared to the restricted BEKK model.

2.2.2 Bivariate Dynamic Correlations (BDC)

We propose a new bivariate model that is more flexible than the discussed models and parameterizes the conditional correlations directly.[10] We write the covariance matrix $\boldsymbol{H}_t$ in the following form:

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{R}_t \boldsymbol{D}_t \tag{2.30}$$

where $\boldsymbol{D}_t$ is a diagonal matrix with the roots of the variances on the main diagonal and $\boldsymbol{R}_t$ is a correlation matrix. In a bivariate model the correlation matrix $\boldsymbol{R}_t$ is

$$\boldsymbol{R}_t = \begin{pmatrix} 1 & \rho_t \\ \rho_t & 1 \end{pmatrix} \tag{2.31}$$

with ρ_t denoting the correlation between two series. $\boldsymbol{H}_t$ is positive definite if $\boldsymbol{R}_t$ is positive definite. This is guaranteed as long as $|\rho_t| < 1$. Thus we restrict $|\rho_t|$ to be smaller than one by using the following transformation:

$$\rho_t^\star = \frac{\rho_t}{\sqrt{1+\rho_t^2}} \tag{2.32}$$

where $\rho_t^\star$ is the correlation restricted to lie in the interval $(-1;1)$. This restriction allows to use own parameters for the correlation (covariance) equation and to include additional

[10] In most Multivariate GARCH models conditional correlations are derived from the ratio of the covariance and the product of the roots of the conditional variances (see equations (2.24) and (2.26)).

regressors without risking semi-definite or indefinite covariance matrices.

Hence, the Bivariate Dynamic Correlations model (BDC) is specified by the following equations:[11]

$$\begin{aligned} h_{11,t} &= a_{11}^2 + b_{11}^2 \epsilon_{1,t-1}^2 + c_{11}^2 h_{11,t-1} \\ h_{22,t} &= a_{22}^2 + b_{22}^2 \epsilon_{2,t-1}^2 + c_{22}^2 h_{22,t-1} \\ \rho_t &= a_{12} + b_{12} \epsilon_{1,t-1} \epsilon_{2,t-1} + c_{12} \rho_{t-1} \\ \rho_t^\star &= \frac{\rho_t}{\sqrt{1 + (\rho_t)^2}} \\ h_{12,t} &= \rho_t^\star \cdot \sqrt{h_{11,t} h_{22,t}} \end{aligned} \tag{2.33}$$

The BDC Model is a dynamic correlation model since $\rho_t^\star$ and thus $h_{12,t}$ are time-varying. The covariance does possess its own parameters and the covariance matrix is always guaranteed to be positive-definite. The model allows to assess the degree of persistence and to compare this persistence with the volatility persistence.

Apart from using the cross product $\epsilon_{1,t-1}\epsilon_{2,t-1}$ to model the correlation equation (see equation (2.33)) we additionally use the cross product of the standardized residuals $z_{1,t-1}$ and $z_{2,t-1}$ to analyze the different behavior of the correlation process.[12] Tse (2000) points out that there is no a priori reason to expect the standardized residuals to be a better specification. Contrary to this statement we expect the results to be different due to the fact that z_t is corrected for volatility movements. In addition, the use of z_t is a more natural specification for the conditional correlations (see Engle, 2000 and Tse, 2000). We refer to the model using the raw residuals $\boldsymbol{\epsilon}_t$ as BDC$_\epsilon$ and to the model using the standardized residuals z_t as BDC$_z$. The correlation equation for the BDC$_z$ model is given by:

$$\rho_t = a_{12} + b_{12} z_{1,t-1} z_{2,t-1} + c_{12} \rho_{t-1} \tag{2.34}$$

[11]The conditional variances are specified as in a BEKK model. However, the BDC model can be estimated with any other specification of the conditional variances, e.g. a EGARCH model.

[12]The standardized residuals are given by $z_t = \frac{\epsilon_t}{\sigma_t}$.

The next subsection introduces the asymmetric extension of the BDC model.

2.2.2.1 Asymmetric BDC Model

An extension of the presented BDC models can also capture asymmetric effects of the time-varying correlation. Thus, h_{11} and h_{22} and ρ_t are specified as follows:

$$\begin{aligned}
h_{11,t} &= a_{11}^2 + b_{11}^2\epsilon_{1,t-1}^2 + c_{11}^2 h_{11,t-1} + d_{11}^2\eta_{1,t-1}^2 \\
h_{22,t} &= a_{22}^2 + b_{22}^2\epsilon_{2,t-1}^2 + c_{22}^2 h_{22,t-1} + d_{22}^2\eta_{2,t-1}^2 \\
\rho_t &= a_{12} + b_{12}\epsilon_{1,t-1}\epsilon_{2,t-1} + c_{12}\rho_{t-1} + d_{12}\eta_{1,t-1}\eta_{2,t-1}
\end{aligned} \tag{2.35}$$

Again, $\eta_{i,t} = \min\{\epsilon_{i,t}, 0\}$ with $\boldsymbol{\eta}_t$ containing only the negative shocks of the returns at t. One important feature of the BDC model is that it does not require similar variance and correlation equations as this is necessary for all other multivariate GARCH models. For example, the BDC model is well defined (does not risk an indefinite covariance matrix) even if the variance equations are specified without any asymmetric regressors and the asymmetry is only modeled in the correlation equation. This feature can also be used to include additional regressors (e.g. thresholds or spillover effects) in the correlation equation without risking an indefinite covariance matrix, e.g. the conditional correlation equation could also be specified as follows independently of the variance equations:

$$\rho_t = a_{12} + b_{12}\epsilon_{1,t-1}\epsilon_{2,t-1} + c_{12}\rho_{t-1} + d_{12}\eta_{1,t-1}\eta_{2,t-1} + \boldsymbol{d}_{12}^{\star}\boldsymbol{X} \tag{2.36}$$

where $\boldsymbol{d}_{12}^{\star}$ is a vector capturing the effect of the matrix $\boldsymbol{X}$ of exogenous variables.

Note that the BDC model in its non-asymmetric and asymmetric version is different to the Dynamic Conditional Correlation (DCC) Estimator of Engle (2000) in various respects.

First, we estimate all variance and covariance equations simultaneously. Second, the BDC model can differentiate between the use of the raw residuals $\boldsymbol{\epsilon}$ and the standardized residuals $\boldsymbol{z}$ and third, the BDC model is flexibly extendable, e.g. asymmetric extensions as presented above or a threshold as suggested by Longin and Solnik (1995) can be included.

In order to clarify the differences between the BDC model and the DCC model, the key elements of the DCC model are presented in the Appendix.

2.2.3 Estimation

The estimation of the models based on a sample of T observations of the returns vector $\boldsymbol{r}_t$ is done through numerical maximization of a likelihood function assuming normally distributed returns:

$$\log L(\boldsymbol{\theta}; \boldsymbol{r}_1, \ldots, \boldsymbol{r}_T) = -T/2\ \log(2\pi) - 1/2\ \log(|\boldsymbol{H}_t|) - 1/2\boldsymbol{\epsilon}_t'\boldsymbol{H}_t^{-1}\boldsymbol{\epsilon}_t\ . \tag{2.37}$$

We use the BHHH algorithm of Berndt, Hall, Hall and Hausman (1974).

By using $\boldsymbol{H}_t = \boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t$ we can write the above likelihood function also as

$$\log L(.) = -1/2 \sum_t \left(n \log(2\pi) + 2\log|\boldsymbol{D}_t| + \log|\boldsymbol{R}_t| + \boldsymbol{z}_t'\boldsymbol{R}_t^{-1}\boldsymbol{z}_t \right) \tag{2.38}$$

This separation shows that a two-step estimation procedure is feasible and that variances and correlations can be estimated separately. The two-stage approach has mainly the advantage that the dimensionality of the maximization problem is reduced which accelerates the maximization process (see also Appendix). We will both use a one-step and a two-step estimation procedure to estimate the BDC model.

The standard errors and associated t-values reported in this article are calculated using the quasi-maximum likelihood methods of Bollerslev and Wooldridge (1992), i.e. the standard errors are robust to the density function underlying the residuals.

2.2.4 Simulations

In this section, we compare the covariance estimates of the diagonal BEKK, the BDC$_\epsilon$ model, the BDC$_z$ model, the CCM and Zero Correlation Model (ZCM). We use the CCM and the ZCM to compare the variance estimates and to analyze the impact of the covariance specification on the variance estimates (Tse (2000) suggested such an analysis.[13] The simulations and tests are partially similar to the ones undertaken by Engle (2000).[14] We simulate different bivariate GARCH models 200 times with 1000 observations. The data-generating process consists of $T = 1000$ Gaussian random numbers $\boldsymbol{\epsilon}_i$ for $i = 1, 2$ with mean zero and variance one transformed to a bivariate GARCH model with a time-varying covariance matrix $\boldsymbol{H}_t$ with a given (time-varying) correlation (see below) and the following variance equations:

$$\begin{aligned} h_{11,t} &= 0.01 + 0.04\epsilon_{1,t-1}^2 + 0.95h_{11,t-1} \\ h_{22,t} &= 0.01 + 0.20\epsilon_{2,t-1}^2 + 0.50h_{22,t-1} \end{aligned} \tag{2.39}$$

The variance given by h_{11} is highly persistent and the variance h_{22} is less persistent. Given these variances we use different correlation processes in the simulations:
(i) constant correlations: $\rho_t = 0.5$ and (ii) highly persistent time-varying correlations ($\rho_t = \alpha + \beta \sin(t/(50 * f))$ with a fast sine function given by $\alpha = 0$, $\beta = 0.5$, $f = 1$ and a slow sine given by $\alpha = 0$, $\beta = 0.9$, $f = 5$.

For the asymmetric extensions of the models we use the following variance equations:

$$\begin{aligned} h_{11,t} &= 0.01 + 0.04\epsilon_{1,t-1}^2 + 0.85h_{11,t-1} + 0.1\eta_{1,t-1}^2 \\ h_{22,t} &= 0.01 + 0.10\epsilon_{2,t-1}^2 + 0.50h_{22,t-1} + 0.2\eta_{2,t-1}^2 \end{aligned} \tag{2.40}$$

[13] In other words, are the estimates of the parameters in the conditional-variance estimates robust with respect to the constant correlation assumption? (page 109)

[14] Engle compares the DCC model with the scalar BEKK, the diagonal BEKK, a moving average process, an exponential smoother and a principle components GARCH.

The correlation processes are the same as for the non-asymmetric models.

We compare the estimates of $h_{11,t}$, $h_{22,t}$ and ρ_t with the true variance and covariance series by (i) the mean absolute deviation (MAD) and (ii) the means of the correlations of the true covariance series ($h_{ij,t}$ for $i, j = 1, 2$) with the estimated covariance series. The means of the correlations are also computed since they provide a measure of the fit of the estimated model compared to the simulated one.

2.2.4.1 Simulation Results

Table 2.1 presents the results for the simulated models (diagonal BEKK, BDC$_\epsilon$, BDC$_z$, CCM and ZCM) in a restricted (no asymmetric extension) specification. The table contains the results for the mean absolute deviation (MAD) and the mean of the correlation of the estimated process (variances and correlations) with the true simulated series. The values denoted with a star indicate the minimum (MAD) or maximum (mean of correlation) value among the estimated models and among the different correlation processes (constant correlations, fast sine function and sine function).

Constant correlations are best estimated by the CCM and time-varying correlations are best estimated by the BDC$_z$ model and the diagonal BEKK model. The diagonal BEKK model performs best for the fast sine function. However, the difference to the BDC$_z$ model is small. The good performance of the diagonal BEKK model can be explained with the similar characteristics of the correlation and the variance processes. In this case the diagonal BEKK model can be assumed to be least biased.[15] Moreover, it is important to emphasize that the BDC model performs clearly better for constant correlations compared to the diagonal BEKK model. This can be attributed to the greater flexibility of the BDC model.

[15] See discussion of the diagonal BEKK model above. In this case, the high persistence of the correlation process mainly contributes to this result.

Table 2.1: Simulation Results - Multivariate GARCH Models

MAD

	D.BEKK	BDC$_\epsilon$	BDC$_z$	CCM	ZCM
correlations					
constant	0.2011	0.0303	0.0295	0.0169*	0.4995
fast sine	0.1619*	0.2172	0.1859	0.5662	0.5670
sine	0.1734	0.1244	0.1202*	0.2199	0.2935
variance 1					
constant	0.1045	0.0617	0.0562*	0.0585	0.0636
fast sine	0.1668	0.0744	0.0997	0.0660*	0.0667
sine	0.0631	0.0601	0.0586*	0.0600	0.0587
variance 2					
constant	0.0041	0.0024	0.0023	0.0022*	0.0024
fast sine	0.0134	0.0041	0.0045	0.0026	0.0023*
sine	0.0038	0.0026	0.0029	0.0024	0.0023*

* denotes the best model (minimum value in row)

Mean of Correlations

	D.BEKK	BDC$_\epsilon$	BDC$_z$	CCM	ZCM
correlations					
constant	0.0761	0.4667	0.4751	1.0000*	
fast sine	0.9456*	0.9056	0.9267	0.0008	
sine	0.6188	0.7790	0.7795*	0.0252	
variance 1					
constant	0.9832	0.9881	0.9887*	0.9868	0.9795
fast sine	0.9620	0.9840*	0.9661	0.9796	0.9801
sine	0.9937*	0.9851	0.9876	0.9856	0.9801
variance 2					
constant	0.9543	0.9892	0.9917	0.9925	0.9926*
fast sine	0.8156	0.9809	0.9878	0.9887	0.9920*
sine	0.9467	0.9891	0.9904	0.9924*	0.9890

$T = 1000$

200 iterations

* denotes the best model (maximum value in row)

Table 2.2: Simulations Results - BDC$_z$ Model (two-step procedure)

	Asymmetric BDC$_z$ Model	
	Mean of Correlation	
	$T = 1000$	$T = 2000$
constant + asymmetry	0.8397	0.9237
fast sine + asymmetry	0.9044	0.9054
slow sine + asymmetry	0.7470	0.9289

200 iterations, variances are assumed to be perfectly estimated
Data-generating process for asymmetric correlation is: $\rho_t = ... + 0.1\eta_{1,t-1}\eta_{2,t-1}$
(ρ_t is restricted as in the BDC$_z$ model to guarantee positive-definite covariance matrices.)

The results for the variances show that the CCM, the ZCM and the BDC$_z$ model perform best. The higher correlation of the estimates of the variances with the true variances for the Zero Correlation Model are an indication that the correlation process is not relevant for the variance estimates.[16] Estimating time-varying correlations (instead of assuming a zero or constant correlation) does even seem to influence variance estimates negatively.

For the asymmetric models, results do not change considerably. Consequently, we do not report these results. However, we further analyze the behavior of the asymmetric BDC$_z$ model when a two-step procedure is used (see Engle, 2000). This model is estimated for the previous correlation processes with an asymmetric effect. Additionally, the simulations are performed for $T = 1000$ and $T = 2000$. Results in table 2.2 show that the two-step estimation leads to similar results as the one-step estimation strategy according to the mean of the correlation coefficient (of the true correlation process and its estimate). The constant correlation process is an exception since the values are considerably higher compared to table 2.1. This can be explained with the fact that the addition of an asymmetric term transforms the constant correlation process into a time-varying correlation process.

Furthermore, the performance increased for all correlation processes with the greater sample size of $T = 2000$.

[16] Tse (2000) proposed an analysis whether or to which extent correlation estimates improve or change variance estimates.

We conclude from our simulation results that correlation estimates are closest to the true values in the BDC model for time-varying correlations and constant correlations among the time-varying correlation models. However, constant correlations are best estimated by a CCM. In addition, the BDC$_z$ model performs better than the BDC$_\epsilon$ model which we attribute to the variance correction that potentially leads to less noise in the correlation process.

2.2.5 Empirical Results

We estimate the asymmetric versions of the BDC$_z$ and the diagonal BEKK model and use daily (close-to-close) continuously compounded returns of the following MSCI stock indices: Japan, UK, Germany and the US. The indices span a time-period of approximately 5 years from April 30th, 1997 until December 30th, 2001 with $T = 1176$ observations for each stock index. Non-trading days in a market are included to synchronize the data.[17] This implies that conditional correlations decrease at non-trading days.

We are aware of the fact that estimates for close-to-close daily data can be biased if trading hours differ (e.g. Japan and the US). However, this is especially a problem when modeling informational spillovers (see chapter 3).[18]

Due to these potential problems we also use monthly data spanning from December 1969 until April 2002 with $T = 389$ observations for each index. We additionally use this type of data to analyze any differences in the characteristics of time-varying correlations between daily and monthly data.

Tables 2.3 and 2.4 contain the descriptive statistics for the daily and monthly returns and tables 2.5 and 2.6 report the unconditional empirical correlation coefficient for these

[17] A non-trading day means that no information is processed in the market. Consequently, returns are zero.

[18] Existing methods to synchronize the data are only recently developed (Engle et al. (1998), Forbes and Rigobon (2002), Martens and Poon (2001).

Table 2.3: Descriptive Statistics (daily data)

	Mean	Variance	Skewness	Kurtosis
Japan	-0.026	2.729	2.137	48.669
UK	0.002	1.351	-0.184	7.327
Germany	0.000	2.441	-0.860	27.959
US	0.023	1.603	-0.517	14.943

Number of observations: 1175
$r_t = 100 \cdot \log(p_t) - \log(p_{t-1})$

Table 2.4: Descriptive Statistics (monthly data)

	Mean	Variance	Skewness	Kurtosis
Japan	0.772	42.308	0.439	6131.782
UK	0.573	42.634	129.330	15664.007
Germany	0.606	35.513	-85.396	5167.311
US	0.591	20.136	-50.565	2235.682

Number of observations: 389
$r_t = 100 \cdot \log(p_t) - \log(p_{t-1})$

return series, respectively.

Interestingly, whereas the unconditional correlations are higher for monthly data than for daily data, the correlation of Germany and the UK is an exception. In this case monthly returns exhibit a lower correlation than daily data. This counterintuitive result could be a result of the use of close-to-close returns because the trading hours of both markets are not synchronous.

Table 2.5: Unconditional Correlation (daily data)

	Japan	UK	Germany	US
Japan	1.000	0.189	0.182	0.069
UK		1.000	0.655	0.366
Germany			1.000	0.377
US				1.000

Number of observations: 1175

Table 2.6: Unconditional Correlation (monthly data)

	Japan	UK	Germany	US
Japan	1.000	0.376	0.373	0.307
UK		1.000	0.454	0.525
Germany			1.000	0.427
US				1.000

Number of observations: 389

Figures 2.3 and 2.4 additionally show the evolution of the prices of these series.

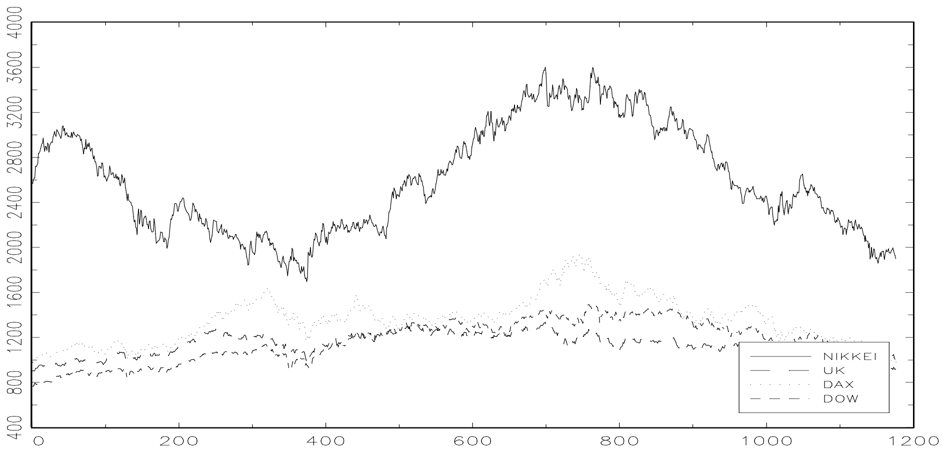

Figure 2.3: *Daily Returns (Japan, UK, Germany, US)*

We first focus on the volatility estimates of the BDC_z model and the diagonal BEKK model and then discuss the results for the correlation estimates.

Table 2.7 reports the estimates for the volatilities of the index pairs under investigation. All volatilities are highly persistent (measured by the sum $b_{ii} + c_{ii}$) and react asymmetrically to positive and negative shocks (indicated by d_{ii}). Results for the diagonal BEKK model (table 2.11) confirm this statement. However, estimates differ considerably from the results of the BDC_z model since volatilities and covariances are not freely estimated

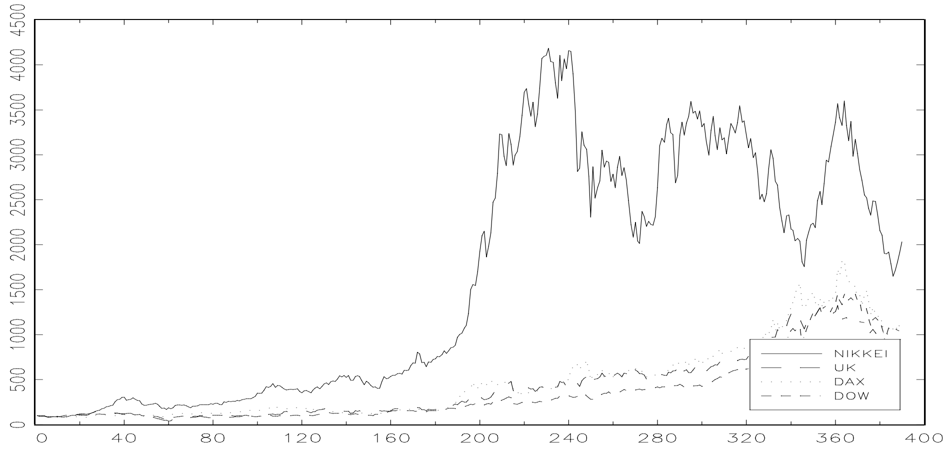

Figure 2.4: *Monthly Returns (Japan, UK, Germany, US)*

as explained above. We will further discuss this issue when interpreting the correlation estimates. An interesting result is also that volatilities are entirely driven by negative shocks for the UK and US stock indices (the parameter estimates of b_{ii} are zero).[19]

Tables 2.8 and 2.9 present the results for the correlation estimates for the BDC$_z$ model in its asymmetric specification for daily and monthly data, respectively. Comparing the values $b_{12} + c_{12}$ among the index pairs for daily data shows that the persistence of shocks varies considerably among the time-varying correlations estimated. The correlations between the UK and Germany and the UK and the US have the highest persistence while the persistence of Germany and the US is very low and the persistence between Japan and Germany is even negative. For monthly data, the persistence generally decreases for all country pairs except the correlation of Germany and the UK.[20] Any asymmetric reaction of the correlation process to positive and negative shocks is discussed in the next section.

Table 2.11 presents results for the asymmetric diagonal BEKK model.

[19]Simulations (that are not reported here) show that such a finding is not due to any misspecification or identification problem.

[20]This can be explained with the different pattern of unconditional correlations for these markets (see tables 2.5 and 2.6).

Table 2.7: Daily Data: BDC$_z$ MODEL, Asymmetric Volatility

Parameters	JAP/ UK	JAP/ GER	JAP/ US	UK/ GER	UK/ US	GER/ US
a_{11}	0.482 ***	0.482 ***	0.482 ***	0.339 ***	0.339 ***	0.429 ***
a_{22}	0.339 ***	0.429 ***	0.385 ***	0.429 ***	0.385 ***	0.385 ***
b_{11}	0.246 ***	0.246 ***	0.246 ***	0.000	0.000	0.060
b_{22}	0.000	0.060	0.000	0.060	0.000	0.000
c_{11}	0.905 ***	0.905 ***	0.905 ***	0.917 ***	0.917 ***	0.914 ***
c_{22}	0.917 ***	0.914 ***	0.888 ***	0.914 ***	0.888 ***	0.888 ***
d_{11}	0.283 ***	0.283 ***	0.283 ***	0.379 ***	0.379 ***	0.402 ***
d_{22}	0.379 ***	0.402 ***	0.494 ***	0.402 ***	0.494 ***	0.494 ***

***, **, * denotes significance at the 1, 5 and 10 percent level, respectively

Variance equation:
$h_{ii,t} = a_{ii}^2 + b_{ii}^2 z_{i,t-1}^2 + c_{ii}^2 h_{ii,t-1} + d_{ii}^2 \eta_{i,t-1}^2$

Table 2.8: Daily Data: BDC$_z$ MODEL, Asymmetric Correlations

Parameters	JAP/ UK	JAP/ GER	JAP/ US	UK/ GER	UK/ US	GER/ US
a_{12}	0.095***	0.265***	0.050**	0.049**	0.128	0.350***
b_{12}	0.037	-0.036	-0.065***	0.020	0.043	0.037
c_{12}	0.538***	-0.355	0.310	0.903***	0.740***	0.158
d_{12}	-0.013	0.112*	0.024	0.042	-0.115	-0.117***
mean (ρ_t)	0.210	0.203	0.072	0.608	0.393	0.366

***, **, * denotes significance at the 1, 5 and 10 percent level, respectively

Correlation equation:
$\rho_t = a_{12} + b_{12} z_{1,t-1} z_{2,t-1} + c_{12} \rho_{t-1} + d_{12} \eta_{1,t-1} \eta_{2,t-1}$

Table 2.9: Monthly Data: BDC$_z$ MODEL, Asymmetric Correlations

Parameters	JAP/ UK	JAP/ GER	JAP/ US	UK/ GER	UK/ US	GER/ US
a_{12}	0.231***	0.354***	0.367***	0.636***	0.180	0.456***
b_{12}	0.010	-0.021	0.021	0.220***	-0.056	0.134**
c_{12}	0.415**	0.050	-0.091	-0.231	0.712***	-0.045
d_{12}	0.116*	0.126*	0.004	-0.080	0.104**	-0.043
mean (ρ_t)	0.408	0.368	0.323	0.481	0.540	0.420

***, **, * denotes significance at the 1, 5 and 10 percent level, respectively

Correlation equation:
$\rho_t = a_{12} + b_{12} z_{1,t-1} z_{2,t-1} + c_{12} \rho_{t-1} + d_{12} \eta_{1,t-1} \eta_{2,t-1}$

Table 2.10: Daily Data: Parameter Comparison

Parameters	JAP/ UK	JAP/ GER	JAP/ US	UK/ GER	UK/ US	GER/ US
d_{12}	-0.013	0.112*	0.024	0.042	-0.115	-0.117***
$d^{\star}_{12}$	-0.044	0.030	0.035	-0.060*	-0.194***	-0.170***

***, **, * denotes significance at the 1, 5 and 10 percent level, respectively

Correlation equation:

$\rho_t = a_{12} + b_{12} z_{1,t-1} z_{2,t-1} + c_{12} \rho_{t-1} + d_{12} \eta_{1,t-1} \eta_{2,t-1}$

$d^{\star}_{12}$ denotes the estimate for a model based on non-asymmetric variance estimates ($d_{11} = d_{22} = 0$)

Table 2.11: Daily Data: Asymmetric Diagonal BEKK MODEL

Parameters	JAP/ UK	JAP/ GER	JAP/ US	UK/ GER	UK/ US	GER/ US
a_{11}	0.408 ***	0.457 ***	0.427 ***	0.321 ***	0.401***	0.381 ***
a_{22}	0.092 ***	0.106 ***	0.041	0.228 ***	0.304***	0.160 ***
b_{11}	0.245 ***	0.266 ***	0.297 ***	0.000	-0.287***	0.172 ***
b_{22}	-0.046	-0.050	-0.067 ***	-0.097	-0.133*	-0.063 *
c_{11}	0.923 ***	0.912 ***	0.918 ***	0.930 ***	0.888***	0.924 ***
c_{22}	0.883 ***	0.902 ***	0.869 ***	0.929 ***	0.843***	0.877 ***
d_{11}	0.248 ***	0.217 *	0.105	0.332 ***	-0.098	0.330 ***
d_{22}	0.418 ***	0.425 ***	0.511 ***	0.334 ***	0.468***	0.501 ***
mean (ρ_t)	0.213	0.206	0.075	0.606	0.383	0.354

***, **, * denotes significance at the 1, 5 and 10 percent level, respectively

Covariance equations:

$h_{ii,t} = a^2_{ii} + b^2_{ii} \epsilon^2_{i,t-1} + c^2_{ii} h_{ii,t-1} + d^2_{ii} \eta_{i,t-1}$

$h_{ij,t} = h_{ji,t} = a_{ii} a_{jj} + b_{ii} b_{jj} \epsilon_{i,t-1} \epsilon_{j,t-1} + c_{ii} c_{jj} h_{12,t-1} + d_{ii} d_{jj} \eta_{i,t-1} \eta_{j,t-1}$

Comparing the parameter estimates for the variance equations (a_{ii}, b_{ii}, c_{ii}, d_{ii}) among the different index pairs shows that parameters vary substantially for the same return series. This is evidence that the variance estimates are influenced by the second return series and by the estimated covariance. Thus, parameter estimates are biased. However, it seems that the bias is mainly in the variance parameters.

Since the BEKK model does not estimate the correlation process directly but by the ratio of the covariance and the squared root of the product of the variances, we can only analyze the persistence and asymmetry of the variances and the covariance which hinders us from directly comparing estimates between the BDC_z model and the diagonal BEKK model.

Figure 2.5 shows the correlation estimates for the BDC_z model.

Note that the finding of constant and non-persistent conditional correlations observed in two cases for daily returns and in five cases for monthly returns estimated with the BDC Model is unique since it could not be revealed by the single use of any standard multivariate GARCH model.

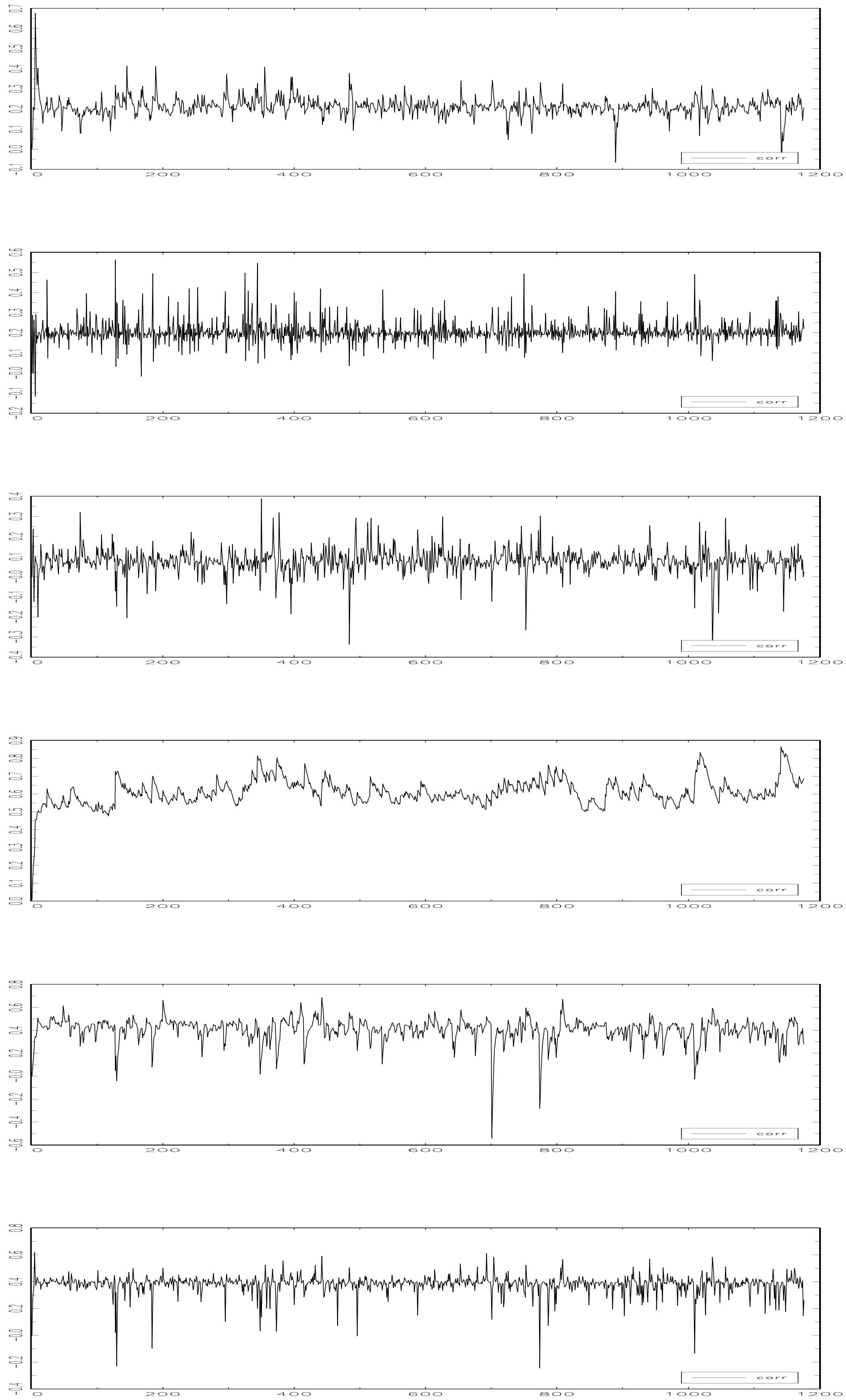

Figure 2.5: *Asymmetric Time-varying Correlations (BDC$_z$), (JAP, UK), (JAP, GER), (JAP, US), (UK, GER), (UK, US), (GER, US)*

2.2.5.1 The Asymmetry of Correlations

Asymmetric effects of volatilities to positive and negative shocks are well documented in the literature and explained with the leverage effect (Black, 1976 and Christie, 1982) and the volatility feedback effect (Campbell and Hentschel, 1992). However, little is known about the temporal behavior of stock return correlations (see Andersen et al., 2000 and Andersen et al., 2001) and even less of the potential asymmetric effects of positive and negative shocks.

If correlations are viewed as a measure of comovement, they should react in the same way (increase, stay constant or decrease) for equal values of jointly positive and jointly negative shocks. This implies that there is no asymmetric effect. Furthermore, there is no generally accepted theory based on an economic model that would predict such effects (e.g. see Karolyi and Stulz, 2001 and Connolly and Wang, 2001).

In contrast, the estimation results of the asymmetric BDC$_z$ model show that there is an asymmetric effect of correlations and that this asymmetry is not similar to the one observed for volatilities. This result differs from the findings in the literature where similar asymmetric effects of the conditional covariance are reported (see Kroner and Ng, 1998 among others). The difference can be explained with the fact that the BDC model analyzes the correlation directly whereas commonly the covariance is examined.

To interpret the asymmetric effect we focus on the parameter estimates for b_{12} and d_{12} in table 2.8 (daily data). Results reveal that correlations increase with jointly positive shocks for all country pairs except (Japan, Germany) and (Japan, US) where correlations decrease with jointly positive shocks. Whether correlations react differently to negative shocks can be assessed by the estimate for d_{12}. Correlations increase more with negative shocks than with positive shocks for (Japan, Germany), (Japan, US) and (UK, Germany) and decrease for the remaining country pairs.

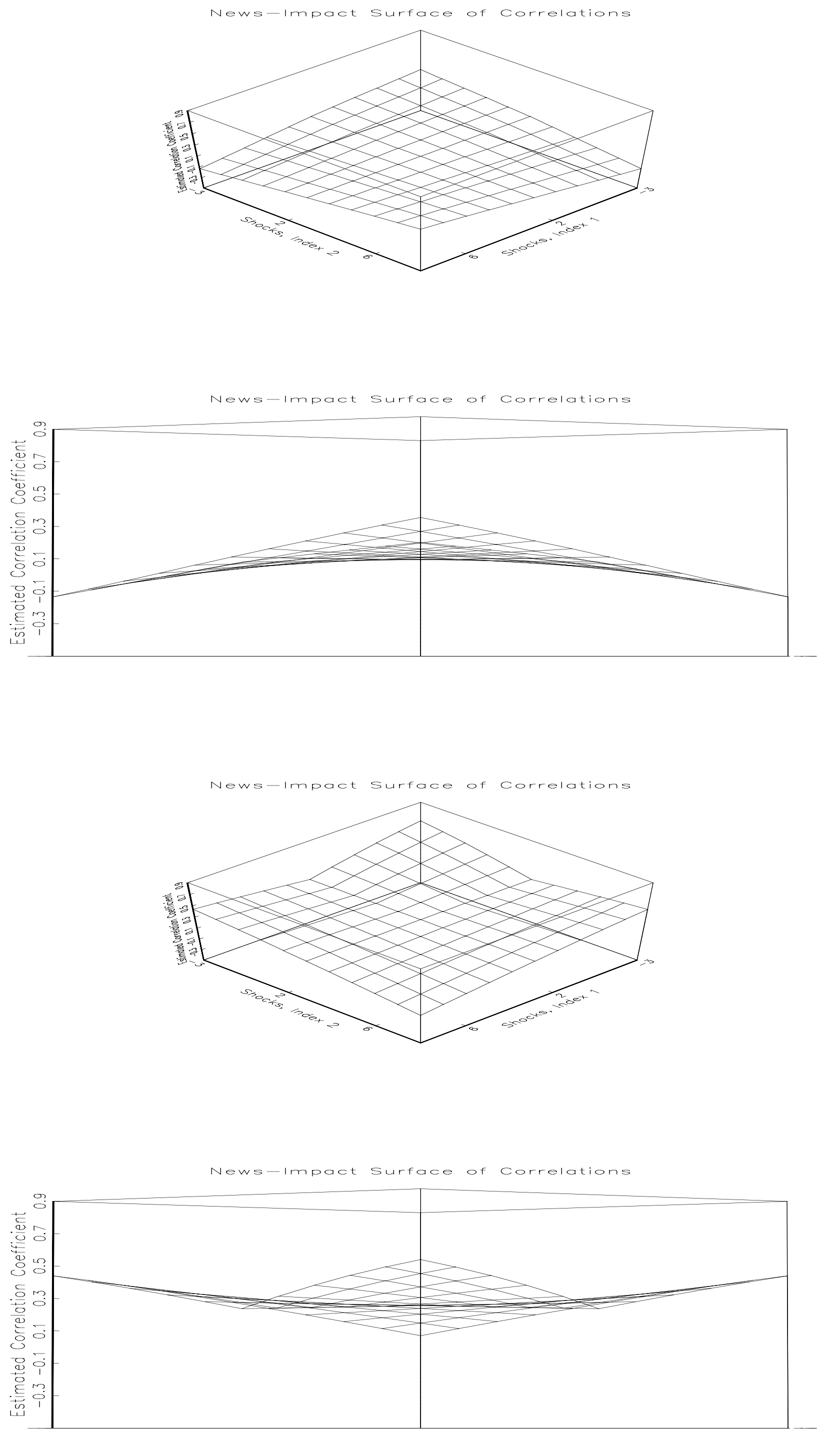

Figure 2.6: *News-Impact Surfaces and frontal views, BDC_z model (JAP/ UK) (top) and (JAP/ GER) (bottom)*

Table 2.10 shows an important finding. It tabulates the estimated parameter values of d_{12} for two different models. The first is the BDC$_z$ model as introduced above and the second is the BDC$_z$ model with the variances restricted to exhibit no asymmetric effects, i.e. $d_{11} = d_{22} = 0$. The results show that the parameter $d^{\star}_{12}$ is considerably different from the results found in the non-restricted model which is a hint that the specification of the variances can affect correlation estimates. This is not counter to the conclusions drawn from the simulation study since we have not analyzed how different volatility specifications affect correlation estimates but only how different correlation specifications can affect volatility estimates.

Furthermore, it is not surprising that the estimates of the parameters b_{12} and d_{12} for monthly data as reported in table 2.9 differ considerably from the daily data results. However, there is also asymmetry in correlations and even negative effects ($d_{12} < 0$) for two country pairs.

To clarify these findings we plot news-impact surfaces for all estimated daily correlations given by figures 2.6 to 2.8. These functions show how correlations react to different combinations of shocks of two time-series. We set the range of positive and negative shocks to $[-2.5, +2.5]$ and additionally plot a frontal view of the news-impact surfaces to ease the analysis and interpretation of the behavior of the correlations.

Due to the construction of the correlation process, all news-impact surfaces exhibit a symmetric behavior for the residuals of opposite signs, i.e. negative shocks of one stock index do not have a different impact on the correlation than negative shocks of the other stock index. The same is true for positive shocks. Such a symmetric picture is not existent if both shocks have the same sign since we account for such differences in the correlation equation (equation (2.35)).

The asymmetry of correlations is closely related to the empirical finding that correla-

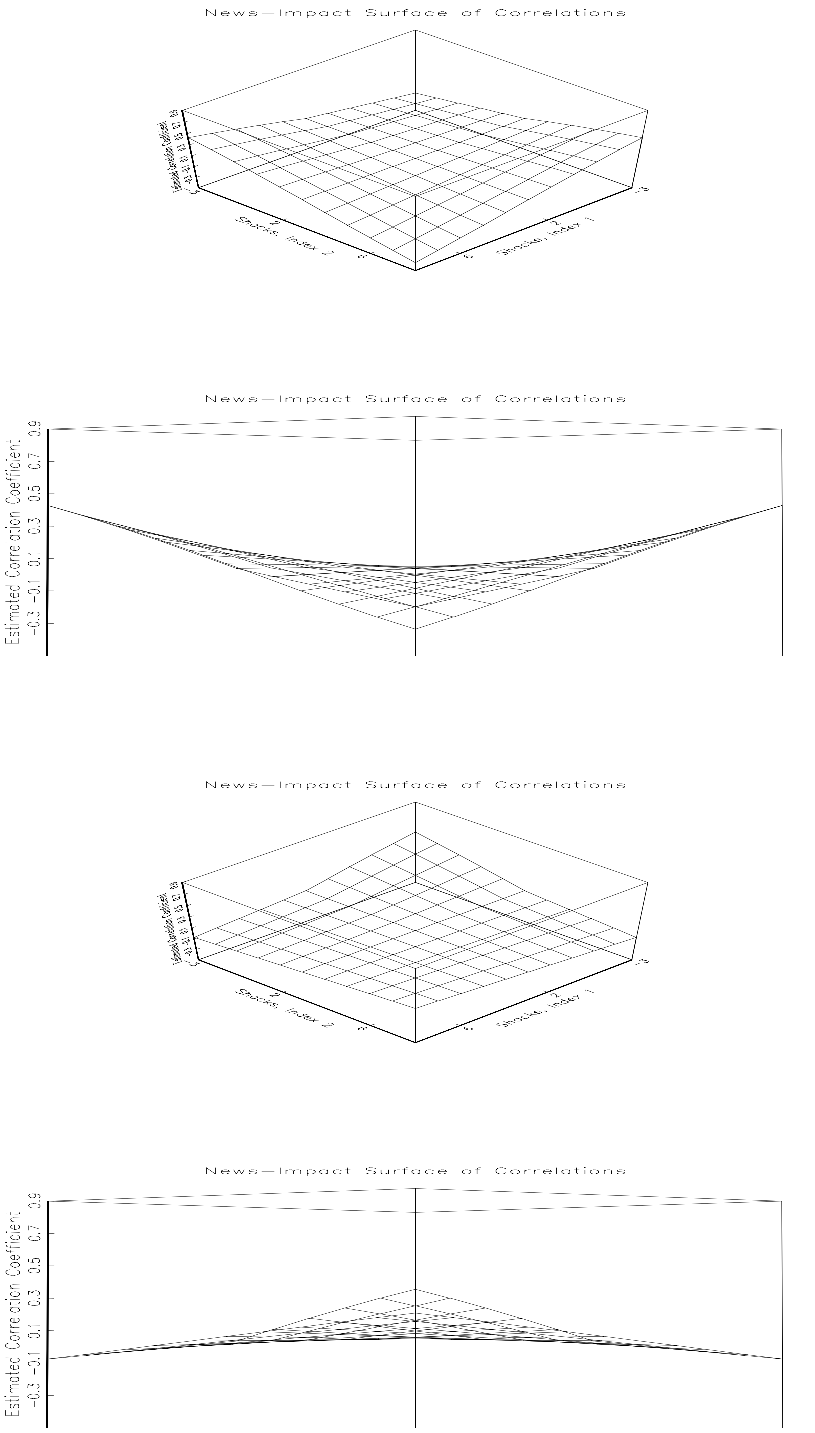

Figure 2.7: *News-Impact Surfaces and frontal views, BDC_z model (JAP/ US) (top) and (UK/ GER) (bottom)*

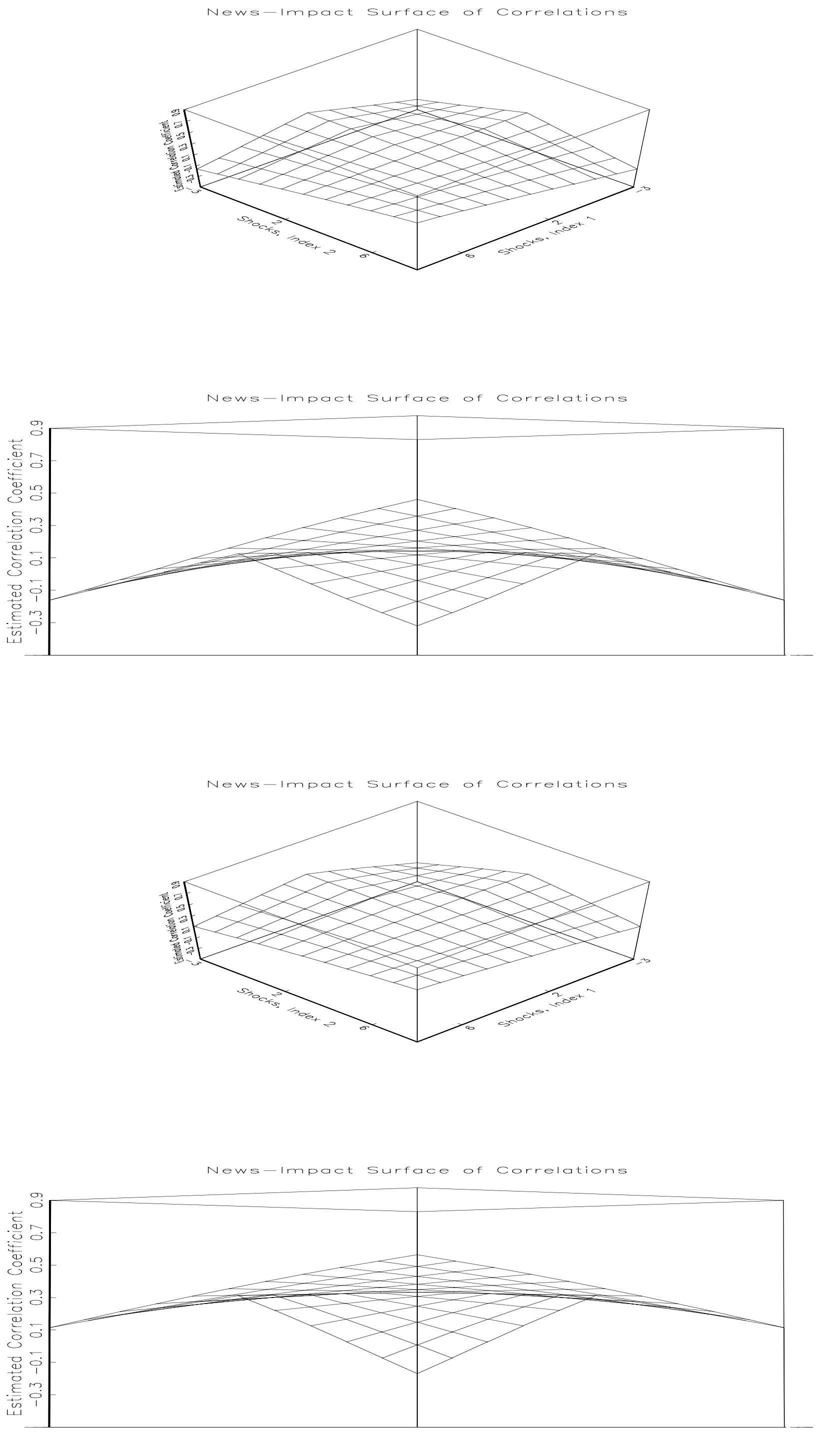

Figure 2.8: *News-Impact Surfaces and frontal views, BDC_z model (UK/ US) (top) and (GER, US) (bottom)*

tions increase with volatility. More precisely, it was often stated that correlations increase in bear markets thus calling into question the desirability of international portfolio diversification (see De Santis and Gerard, 1997, Longin and Solnik, 1995, Longin and Solnik, 2001, Ng, 2000, Ramchand and Susmel, 1998 and Susmel and Engle, 1994).

Interpreting simultaneously high positive and negative values of ϵ_1 and ϵ_2 as a high-volatility state, we can answer this question by interpreting the parameter estimate for d_{12}. The results for the correlation of the DAX with the NIKKEI and the FTSE ($d_{12} > 0$) replicate the findings in the literature, i.e. international portfolio diversification is not effective whenever it is needed most because the correlation increases with simultaneously positive and negative shocks. The result for the correlation of the DAX and the DOW is counter to the findings in the literature ($d_{12} < 0$) and further encourages international portfolio diversification between Germany and the US.

2.2.6 Conclusions

We have shown that existing multivariate GARCH models do not adequately model constant or time-varying correlations. The same is true for the asymmetric extensions of these models. The Bivariate Dynamic Correlations (BDC) Model introduced here performs clearly better in this regard.

We estimated the BDC model for four international stock market indices and found that correlations exhibit a different temporal behavior compared to volatilities, i.e. correlations are more often constant and less persistent than volatilities and the asymmetry of shocks on volatility is more pronounced and more similar among volatilities itself than the asymmetric effects of jointly positive or negative shocks on correlations.

Future research could be done by studying the impact of the distributional assumptions on the persistence and asymmetry of correlations. In addition, although mentioned, further research is necessary to answer the question how correlation estimates change with volatilities or vice versa.

2.2.7 Appendix

2.2.7.1 Dynamic Conditional Correlations

The Dynamic Conditional Correlations Model (DCC) is introduced by Engle (2000). The time-varying covariance matrix $\boldsymbol{H}_t$ can be written as in equation (2.30) ($\boldsymbol{H}_t = \boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t$). Engle presents different possibilities to estimate the correlation matrix $\boldsymbol{R}_t$. However, the main contribution is the two-step estimation strategy that provides consistent but inefficient estimates of the parameters of the model. The log-likelihood function of the DCC estimator can be expressed as

$$\log L(.) = -1/2 \sum_t \left(n \log(2\pi) + \log |\boldsymbol{H}_t| + \boldsymbol{r}_t' \boldsymbol{H}_t^{-1} \boldsymbol{r}_t \right) \tag{2.41}$$

assuming that $\boldsymbol{r}_t$ is conditionally multivariate normal distributed with zero means and covariance matrix $\boldsymbol{H}_t$ ($\boldsymbol{r}_t \sim N(\boldsymbol{0}, \boldsymbol{H}_t)$) of dimension $(n \times n)$.

Using $\boldsymbol{H}_t = \boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t$ results in

$$\log L(.) = -1/2 \sum_t \left(n \log(2\pi) + \log |\boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t| + \boldsymbol{r}_t' (\boldsymbol{D}_t\boldsymbol{R}_t\boldsymbol{D}_t)^{-1} \boldsymbol{r}_t \right) \tag{2.42}$$

Substituting $\boldsymbol{r}_t'\boldsymbol{D}_t^{-1}$ by the standardized residuals $\boldsymbol{z}_t$ yields

$$\log L(.) = -1/2 \sum_t \left(n \log(2\pi) + 2 \log |\boldsymbol{D}_t| + \log |\boldsymbol{R}_t| + \boldsymbol{z}_t' \boldsymbol{R}_t^{-1} \boldsymbol{z}_t \right) \tag{2.43}$$

Engle suggests to estimate the variance equations in a first step by setting $\boldsymbol{R}_t$ equal to the identity matrix. This yields consistent but inefficient estimates. Hence, the log-likelihood for the variance equations is given by

$$\log L_u(\boldsymbol{\theta}|\boldsymbol{R}_t = \boldsymbol{I}) = -1/2 \sum_t \left(n \log(2\pi) + 2 \log |\boldsymbol{D}_t| + 0 + \boldsymbol{z}_t' \boldsymbol{z}_t \right) \tag{2.44}$$

where $\boldsymbol{\theta}$ is a vector consisting of the variance parameters. If $\boldsymbol{\theta}$ is known, the variance estimates and thus the standardized residuals $\boldsymbol{z}_t$ can be computed.

Then the remaining part of the log-likelihood is

$$\log L_c(\boldsymbol{\phi}|\boldsymbol{\theta}, \boldsymbol{r}_t) = -1/2 \sum_t \left(\log |\boldsymbol{R}_t| + \boldsymbol{z}_t' \boldsymbol{R}_t^{-1} \boldsymbol{z}_t \right) \tag{2.45}$$

where $\boldsymbol{\phi}$ is a vector that contains all parameters of the correlation matrix $\boldsymbol{R}_t$. Note, that the matrix $\boldsymbol{R}_t$ must be positive definite since $\log|\boldsymbol{R}_t|$ and $\boldsymbol{R}^{-1}$ need to be computed.

The likelihood sacrificed by the two step estimation strategy can be written as

$$\log L(\boldsymbol{\theta}, \boldsymbol{\phi}) = L_c + L_u + 1/2 \sum_t \boldsymbol{z}_t' \boldsymbol{z}_t \tag{2.46}$$

The last term on the right hand side is almost exactly $NT/2$. This term has to be added due to the last term of the log-likelihood in equation (2.44).

As outlined in this section, the main problem in the estimation of dynamic correlations is the possible indefiniteness of the correlation matrix. The technique to guarantee the correlation matrix to be positive definite is described as follows (see Engle and Sheppard, 2001):

The dynamic covariance in a DCC(1,1) model is:

$$\boldsymbol{Q}_t = (1 - \alpha_1 - \beta_1)\overline{\boldsymbol{Q}} + \alpha_1 \boldsymbol{z}_{t-1} \boldsymbol{z}_{t-1}' + \beta_1 \boldsymbol{Q}_{t-1} \tag{2.47}$$

where $\overline{\boldsymbol{Q}}$ is the unconditional covariance of the standardized residuals. The correlation matrix is given by

$$\boldsymbol{R}_t = \boldsymbol{Q}_t^{\star^{-1}} \boldsymbol{Q}_t \boldsymbol{Q}_t^{\star^{-1}} \tag{2.48}$$

where $\boldsymbol{Q}_t^{\star^{-1}}$ is a diagonal matrix composed of the square root of the diagonal elements of $\boldsymbol{Q}_t$. A proof that equation (2.48) guarantees a positive-definite correlation matrix can be found in Engle and Sheppard (2001).

Chapter 3

Asymmetric Interdependence: Spillovers

In this chapter we depart from the Multivariate GARCH framework and its focus on the estimation of time-varying correlations and concentrate on an estimation framework that models the influence of shocks of a given market on another market in an asymmetric way. Such an asymmetric impact of shocks is often called spillover since shocks transmit (spill over) from one market to another.

Spillovers are mainly modeled to analyze information flows and its transmission between markets (e.g. Hamao et al., 1990) or around the world (see Engle et al., 1990). The investigation of spillovers is also applied to tests of the efficient market hypothesis, i.e. whether markets process information efficiently or not.

The fact that a spillover is an element of an asymmetric relation between markets is a characteristic that constitutes the fundamental difference to the correlation of markets that measures symmetric phenomena. Although this asymmetric relation can also be mutual to some extent, it is usually not the focus of a spillover analysis and often restricted by the time structure of the data. For example, the dependent and independent variables of a regression model in an analysis of the effects of the morning trading (independent variable) on the afternoon trading (dependent variable) are predetermined by the timing of the data: only the effect of the morning trading on the afternoon trading can be economically interpreted. Thus, the timing of the data determines the regression model.

Since a spillover is the result of a transmission of an innovation (or shock) from one market to another it must not contain overlapping information sets (e.g. overlapping trading hours). Otherwise a spillover could not be distinguished from any measure of the correlation of markets. The difference is therefore mainly due to the structure of the variables under investigation and not due to the measure employed.

However, the correlation coefficient is more prone to measure symmetric relations between two variables (markets) x and y because it can be viewed as a weighted measure of the variances ($\rho = \frac{cov(x,y)}{\sqrt{Var(x)Var(y)}}$). On the other hand, the regression coefficient β of a regression model $y = \beta x + \epsilon$ is more adequate to measure asymmetric relations because it is not a symmetrically weighted measure of the variances of the variables involved ($\beta = \frac{cov(x,y)}{Var(x)}$).

We advocate the use of a regression model with a dependent variable y and an independent variable x for the analysis of spillovers since it can be seen as a natural framework to model asymmetric relations. If x is not independent or exogenous[1], the direction of the spillover is not clear and the results of a spillover analysis are biased.

A typical example describes the effects of the use of overlapping and non-overlapping returns within a correlation or a spillover analysis: assume two positively correlated financial markets with different trading hours. One market opens at 9am and closes at 8pm and the other market opens at 3pm and closes at 10pm. This example could fit to a market in Europe and one in North or South America. Both markets exhibit a 5 hour trading overlap (3pm until 8pm). Analyzing the close-to-close returns[2] 8pm(t-1)-to-8pm(t)

[1] see Engle et al., 1983 for a discussion of the term *exogeneity*

[2] A close-to-close return is the return between the current closing price at t and the previous markets' closing

Table 3.1: Timing of Markets

close-to-close return (10pm-10pm)
8pm (t-1) - **10pm (t-1)** 8pm (t) - **10pm (t)**
close-to-close return (8pm-8pm)
8pm (t-1) - 10pm (t-1) **8pm (t)** - 10pm (t)
overlap of close-to-close returns is 22 hours: 10pm(t-1)-to-8pm(t)

and 10pm(t-1)-to-10pm(t) would lead to a 22 hours overlap and bias the correlation coefficient because 2 hours are missing each day (In these 2 hours trading occurs in one market while the other market is closed.). Table 3.1 shows the timing of the close-to-close returns of both markets.

On the contrary, an investigation of the 10pm(t-1)-to-8pm(t) return of the first market and the 10pm(t-1)-to-8pm(t) return of the second market would synchronize both returns and neither understate nor overstate the true correlation of these markets (see Martens and Poon, 2001).[3]

There is another possibility to show that non-synchronous trading hours yield incorrect correlation coefficients: The parameters of two different regressions are denoted as β_1 and β_2 where ($y_t = \beta_1 x_t + \epsilon_t$) and ($x_t = \beta_2 y_t + \epsilon_t$) are the associated regression equations, respectively. The parameter β_1 describes the effect of one market denoted with x_t on the other market denoted with y_t and the parameter β_2 describes the reverse effect, i.e. the influence of market y_t on market x_t. Using the square root of the product of the absolute values of β_1 and β_2 multiplied by the sign of the covariance of x_t and y_t yields the following

price at $t-1$.

[3]A literature overview concerning non-synchronous trading can be found in Campbell et al. (1997).

correlation coefficient:

$$\rho_t = \text{sign}(cov(x_t, y_t))\sqrt{|\beta_1| \cdot |\beta_2|} \tag{3.1}$$

This separation of the correlation coefficient shows that one component is the result of an incorrect specification. For example, the regression of the close-to-close return 8pm(t-1)-to-8pm(t) on the 10pm(t-1)-to-10pm(t) return is not adequate since the 10pm price at t cannot be exogenous for the 8pm price at t. On the contrary, a spillover analysis of the open-to-close returns[4] (9am(t)-8pm(t) and 3pm(t)-10pm(t)) would lead to a biased estimate of the spillover since trading hours overlap by 5 hours. The observed spillover is biased because it also contains the contemporaneous correlation of these markets. Thus, it is fundamental to isolate the returns, e.g. the open-to-3pm return and its effect on the 3pm-to-close return would be a correct spillover analysis.

Both strategies to synchronize or isolate (de-synchronize) the returns are very intuitive. However, while the spillover literature has mainly accounted for these problems (e.g. see Hamao et al., 1990), the analysis of the correlation of financial markets has only recently proposed to synchronize the data to avoid spurious results (see Burns, Engle and Mezrich, 1998 and Martens and Poon, 2001).

3.1 Mean and Volatility Spillovers

A spillover can be (i) constant or time-varying, (ii) affecting the mean or the volatility (or both) of another market and (iii) being contemporaneous or lagged. A general model that nests all these cases can be formulated as follows:

$$y_t = \mu + \beta_t x_{t-k} + \epsilon_t \,, \qquad \text{where} \quad \epsilon_t|\mathcal{F}_{t-1} \sim \text{N}(0, h_t)$$

$$\text{and} \quad h_t = a + b\epsilon_{t-1}^2 + ch_{t-1} + d_t\, x_{t-k^\star}^2 \tag{3.2}$$

[4]A open-to-close return is the return between the current closing price at t and the opening price at t.

where x_{t-k} and y_t denote the returns under investigation. The parameters μ are constants and ϵ_t is assumed to follow a GARCH(1,1)-process as indicated above. The information set available up to time $t-1$ is denoted by $\mathcal{F}_{t-1}$. The effect of return shocks from one market (x_{t-k}) to the mean and variance equations of the other market is described by $\beta_t x_{t-k}$ and $d_t x^2_{t-k^\star}$, respectively. Mean spillovers are contemporaneous for $k = 0$ and lagged for any $k > 0$. Moreover, volatility spillovers are contemporaneous for $k^\star = 0$ and lagged for any $k^\star > 0$. Both effects can differ, i.e. it is possible to model contemporaneous mean spillovers and lagged volatility spillovers or vice versa. The model given by equation (3.2) is a time-varying mean and volatility spillover model. Imposing the restrictions $\beta_t = \beta$ or $d_t = d$ would lead to a constant mean spillover model or constant volatility spillover model, respectively. Restricting d_t or β_t to be zero would imply a pure mean spillover model or a pure volatility spillover model, respectively.

It is important to note that the names *constant spillover model* and *time-varying spillover model* can be misleading since spillovers are shocks that typically vary. Thus, the *constant (time-varying) spillover model* means that the influence of varying spillovers on another market is constant (time-varying) independent of the spillover per se. We are not aware of any contribution in the literature that has adverted to this semantic problem. Apart from this fact, we follow the literature and do not alter the names since this would result in more complicated expressions.

If it is not possible to isolate a shock that spills over from one market to another it is more adequate to speak of *contemporaneous correlation* or more specifically *contemporaneous correlation in mean*. Such a correlation is not equal to the correlation coefficient since it is the result of a regression model.[5] *Correlation* is a more appropriate name in this case because there is not a clear asymmetry or exogeneity which is characteristic for spillovers.

[5] Note that *contemporaneous spillover* and *contemporaneous correlation* are sometimes used as an equivalent (see e.g. Edwards, 1998).

Table 3.2: Classification of Spillovers

Mean Spillovers

	non-overlapping returns	**overlapping returns**
$y_t = \beta x_t$	(constant) mean spillover	(constant) contemporaneous correlation in mean
$y_t = \beta_t x_t$	(time-varying) mean spillover	(time-varying) contemporaneous correlation in mean

Volatility Spillovers

$h_t = ... + dx_t^2$	(constant) volatility spillover	(constant) contemporaneous correlation in volatility
$h_t = ... + d_t x_t^2$	(time-varying) volatility spillover	(time-varying) contemporaneous correlation in volatility

lagged spillovers or correlations are not tabulated

Analogous to this *correlation in mean*, if a volatility spillover is a result of two overlapping returns we propose to name the latter *contemporaneous correlation in volatility*.

Table 3.2 summarizes the different types of spillovers and focusses on mean and volatility spillovers. It distinguishes between a spillover and the correlation. The table does not contain lagged spillovers or lagged correlations.

It is important to mention that we are not aware of recent contributions to the literature that have explicitly classified these different effects. In addition, there is no precise and explicit definition of a spillover. For example, Lin et al., 1994 only give a definition for lagged spillovers in a footnote (see footnote 2).

Table 3.2 does not only give an overview of existing types of spillovers in the literature but it is also an overview of this chapter: Constant mean and volatility spillovers are discussed in section 3.2, varying and conditional mean spillovers are modelled in section 3.3, and section 3.4 is a special case of time-varying mean and volatility spillovers. Possible parameterizations for time-varying mean spillovers (β_t) are discussed in section 3.3.1. We also discuss conditional spillovers depending on the magnitude of the dependent variable in section 3.3.2.

3.2 Constant Mean and Volatility Spillovers

This section describes a model to test if there are mean or volatility spillovers among two financial markets. We analyze synchronous and non-synchronous trading hour overlaps of markets and discuss the associated findings of spurious correlations and spurious spillovers. We also describe the problems that can arise if opening quotes are not reliable due to the content of fractions of the previous closing prices.

The main part of this section focusses on the estimation of mean and volatility spillovers between the German stock index DAX and the US stock index DOW Jones Industrial Average. We find strong contemporaneous correlations between these markets and a trade-off between effects in the mean and in the volatility. This is further investigated by a simulation study.

3.2.1 Spurious Correlations and Spillovers

We have argued that the difference between a spillover and the correlation is rooted in the overlapping or non-overlapping information sets. If innovations can be isolated (information sets do not overlap), a spillover analysis can be carried out, otherwise a correlation analysis is conducted. If the assumptions corresponding to a correct spillover or correlation analysis are violated, spurious results can occur.[6]

Spurious correlations or spillovers can occur if (i) the opening quotes contain large fractions of the previous closing quotes due to the structure and the computation of indices[7] (see Stoll and Whaley, 1990 and Baur and Jung, 2002 for more details) or if (ii) the returns under investigation contain overlapping trading hours.

[6] Spurious regressions are first analyzed by Yule (1926) and Granger and Newbold (1974). These studies warned that spurious relations may be found between the levels of trending time-series that are actually independent. We do not concentrate on trending time-series but discuss spurious relations associated with overlapping information sets.

[7] This finding is also referred to *stale prices*.

In order to avoid these problems it is necessary to construct returns that are different to the close-to-close returns. For example, separating a close-to-close return in open-to-close (day return) and close-to-open (night return) return components (see Hamao et al., 1990) can be a first step to eliminate overlapping trading hours. Furthermore, the availability of tick-by-tick data can fully eliminate trading hour overlaps and allows to study returns of higher frequency. It can also diminish or eliminate the problem of unreliable opening prices (stale prices) by using modified opening quotes (e.g. opening quote plus 5 minutes). This flexibility in constructing returns is essential to conduct an unbiased spillover analysis.

The next section presents an econometric model that is closely related to the general model presented in equation (3.2) followed by the empirical results for the German DAX and a the US DOW Jones Industrial Average stock index, explicitly accounting for the problems discussed above.

3.2.2 The Econometric Model

The empirical analysis is based on a variant of the popular aggregate-shock (AS) model as described in detail in Lin et al. (1994).

The AS-model comprises of the following set of equations:

$$r_{1,t} = \mu_1 + \epsilon_{1,t}\ , \qquad \text{where} \quad \epsilon_{1,t}|\mathcal{F}_{t-1} \sim \mathrm{N}(0, h_{1,t}) \tag{3.3}$$

$$\text{and} \qquad h_{1,t} = a_{11} + b_{11}\epsilon^2_{1,t-1} + c_{11}h_{1,t-1}$$

$$r_{2,t} = \mu_2 + \beta_1\epsilon_{1,t} + \beta_2\ r^{\star}_{2,t-1} + \epsilon_{2,t}\ , \qquad \text{where} \quad \epsilon_{2,t}|\mathcal{F}_{t-1} \sim \mathrm{N}(0, h_{2,t}) \tag{3.4}$$

$$\text{and} \quad h_{2,t} = a_{22} + b_{22}\epsilon^2_{2,t-1} + c_{22}h_{2,t-1} + d_{22}\,\epsilon^2_{1,t}$$

$r_{1,t}$ and $r_{2,t}$ denote the foreign and home returns respectively, μ_1 and μ_2 are constants while $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are assumed to be serially uncorrelated and mutually independent shocks that follow a GARCH(1,1)-process as indicated above. The term $r^{\star}_{2,t-1}$ captures any autocorrelation of the dependent variable.[8] The information set containing home and foreign returns available up to time $t-1$ is denoted by $\mathcal{F}_{t-1}$. The effect of return shocks in the foreign market to the mean and variance equations of the home market is captured in $\beta_1\epsilon_{1,t}$ and $d_{22}\,\epsilon^2_{1,t}$, respectively. We assume that $\mu_1 = 0$ since we want to analyze the raw effect of shocks of $r_{1,t}$ onto $r_{2,t}$.

Lin et al. (1994) labeled the model aggregate shock model in contrast to a signal-extraction model because the foreign return is not decomposed into a global and a local factor. As a result of this specification, the model implicitly assumes that all the information revealed in the foreign market has a global impact on the returns of the home market. Despite this limitation, the AS-model is much more popular among applied financial researchers as compared to the signal-extraction model. This is probably due to the fact that the latter model requires a more sophisticated estimation procedure[9] while the former model can easily be estimated using standard (quasi-) maximum likelihood procedures implemented in many econometric packages.

Due to the very simple specification of the mean equation we can follow Hamao et al. (1990) and Susmel and Engle (1994) and use a univariate estimation procedure whereby the foreign return is substituted directly into the mean equation of the home return in the first line of equation (3.4). Quite analogous, the squared foreign return serves as a proxy for the foreign volatility surprise and can be included in the variance equation of (3.4) which can be estimated using standard GARCH methods.

It is quite obvious from the discussion above that by setting β or d_{22} in the equation (3.4) equal to zero, it is possible to specify and estimate separate mean and volatility spillover models. This is a useful analysis due to the fact that many studies either estimate mean

[8] It is not necessary to include such a regressor in the variance equation since the structure of a GARCH(1,1) model accounts for any autocorrelation of the variance.

[9] Lin et. al (1994) e.g. employ a Kalman-filtering procedure.

and volatility effects separately or simultaneously but do not evaluate nor explicitly discuss differing results (see e.g. Chakrabarti et al. 2002, Edwards, 1998, Hamao et al., 1990, Lin et al., 1994 and Susmel and Engle, 1994). In examining the implications of the different specifications below we will be able to demonstrate potential pitfalls in the uncritical use of such reduced model specifications.

The first alternative specification under investigation is the pure mean spillover model. It comprises of the following two equations for the home returns:

$$r_{2,t} = \mu_2 + \beta_1 \epsilon_{1,t} + \beta_2 \, r^{\star}_{2,t-1} + \epsilon_{2,t} \ , \qquad \text{where} \quad \epsilon_{2,t}|\mathcal{F}_{t-1} \sim \mathrm{N}(0, h_{2,t}) \tag{3.5}$$
$$\text{and} \quad h_{2,t} = a_{22} + b_{22}\epsilon^2_{2,t-1} + c_{22}h_{2,t-1} \ .$$

The second one is the pure volatility spillover model given by

$$r_{2,t} = \mu_2 + \beta_2 \, r^{\star}_{2,t-1} + \epsilon_{2,t} \ , \qquad \text{where} \quad \epsilon_{2,t}|\mathcal{F}_{t-1} \sim \mathrm{N}(0, h_{2,t}) \tag{3.6}$$
$$\text{and} \quad h_{2,t} = a_{22} + b_{22}\epsilon^2_{2,t-1} + c_{22}h_{2,t-1} + d_{22}\,\epsilon^2_{1,t}$$

We estimate the full spillover model and its restricted versions in the following section and further explore characteristics of these models in a simulation study afterwards.

3.2.3 The Empirical Results

In this section we present and discuss the estimation results for four different hypotheses. We will differentiate between the contemporaneous correlation of returns based on overlapping time spans and a pure spillover effect which is obtained from returns based upon non-overlapping time periods. Only the second approach allows a causality test of whether the preceding foreign returns contain any additional information relevant for the home returns.

The data used in this study consist of time series of intra-day equity index returns for the period January 2, 1998, through December 29, 2000, providing us with a sample size

Table 3.3: Timing of the opening and the closing of the Frankfurt and the New York stock market

Date	Day t-1				Day t	
CET	9:00	15:30	20:00	22:00	9:00	15:30
EST	4:00	9:30	14:00	16:00	4:00	9:30
Germany	DAX *open* (t-1)		DAX *closed* (t)		DAX *open* (t)	
New York	DOW *closed* (t-1)	DOW *open* (t-1)		DOW *closed* (t)		

of 773 daily observations. We choose the DAX as the most prominent representative of the German stock market and the Dow Jones 30 Industrial Average (DJIA) as its US counterpart[10]. More details regarding the data can be found in Baur and Jung (2002). The opening and closing times of the Frankfurt and the New York market are presented in Table 3.3. To keep the table as simple as possible, the Frankfurt opening hours are given as 9.00 am (CET) to 8.00 pm (CET).

The first hypothesis is concerned with the contemporaneous dependence of the overnight return of the DAX and the previous day DJIA performance.

The second hypothesis looks at the information transmission between the intra-day DAX returns and the overnight returns of the DJIA from the same day.

The subject of the third hypothesis is the development of the DAX in the first trading hours in Frankfurt with possible spillovers from the DJIA intra-day returns from the previous trading day.

[10]The DAX data were obtained from the KKMDB (Karlsruher Kapitalmarkt Datenbank) and the DJIA data from Tick Data, Inc. (`http://www.tickdata.com`).

The fourth hypothesis analyzes the effects of the first trading hours in Frankfurt on the opening of the New York stock market on the same day.

Quasi-Maximum likelihood estimates were computed using the BHHH algorithm. As will be shown below in all cases the skewness and kurtosis of the standardized regression residuals indicate the use of robust standard errors as provided in Bollerslev and Wooldridge (1992). We report the skewness and the kurtosis of the standardized regression residuals along with a test for autocorrelation of the standardized residuals and the squared standardized residuals.

3.2.3.1 Contemporaneous correlation between daytime returns of the DOW and the DAX overnight returns

We now investigate whether the daytime returns of the previous trading day in New York are correlated with the overnight returns in Frankfurt. As pointed out by Hamao et al. (1990) as well as Lin et al. (1994) any significant finding would not violate the efficient market hypothesis but rather reflects a reaction of the German stock market to (global) information contained in the DOW returns. The DOW return is calculated from closing time Frankfurt (this varied in our sample starting from 5:00 pm (CET) over 5:30 pm (CET) to now 8:00 pm (CET)) until the closing in New York (i.e. 4:00 pm (EST)). Information accumulated during this time span has not been incorporated into the DAX at $t-1$ but will eventually emerge in the DAX opening at t. Information contained in the DOW returns from its opening up to the closing in Frankfurt are already being processed in previous days DAX returns. The overnight return of the DAX is calculated on the basis of the previous day closing price and the first DAX quote of the day plus 5 minutes[11]. As a result of this setup we expect to see some reaction of the DAX opening to (global) information contained in those DOW returns that resulted after the trading in Frankfurt.

[11]This time period is necessary to avoid stale prices in the index return.

Table 3.4: DAX overnight returns and the preceding daytime returns of DOW

	a_{22}	b_{22}	d_{22}	c_{22}	μ_2	β_1	β_2
Full Model	0.009023	0.1065	0.01855	0.8524	0.09213	0.6871	-0.1032
	(1.338)	(3.376)	(1.121)	(19.50)	(3.610)	(19.76)	(-1.330)
Mean Spillover Model	0.009377	0.1117		0.8749	0.09177	0.6872	-0.09882
	(1.656)	(3.923)		(36.83)	(3.588)	(18.32)	(-1.322)
Volatility Spillover Model	0.1806	0.07637	0.6788	0.1011	0.1293		-0.1025
	(3.921)	(1.368)	(4.929)	(1.159)	(5.695)		(-1.530)
t-values in parenthesis							
Mean/ SD of z	0.0452	1.0022					
Skewness	0.8850						
Kurtosis	13.1147						
Autocorrelation of $\hat{z}$	0.6454						
Autocorrelation of $\hat{z}^2 - 1$	0.1163						

Full Model:
$DAXNR_t = \mu_2 + \beta_1 DOWDR^{\star}_{t-1} + \beta_2 DAXDR^{\star}_{t-1} + \epsilon_{2t}$
$h_{2,t} = a_{22} + b_{22}\epsilon^2_{2t-1} + d_{22}DOWDR^2_{t-1} + c_{22}h_{2,t-1}$

Regression and testing results are provided in Table 3.4 and show that a significant correlation in mean can be identified in the data. The estimated value for β_1 implies that both markets share a large amount of information. The validity of this result is checked by running a pure mean spillover as well as a pure volatility spillover regression.

The results of the mean spillover regression are close to those of the full specification whereas those of the pure variance spillover specification differ markedly. There seems to be clear evidence of a rather pronounced correlation in volatility effect from New York to the DAX overnight returns. The omission of a mean spillover term can obviously have quite a severe impact on the regression results. Notice further that the estimated GARCH-parameter decreases dramatically. The variance spillover term now clearly captures a considerable amount of the GARCH effects.

The standardized residuals of the full model indicate the presence of positive skewness and a considerable amount of kurtosis not captured by the model yet. However, a F-test that detects remaining autocorrelation of the estimated standardized residuals and its square with 5 lags does not indicate any misspecification.

To assess the stability of the parameter estimates obtained, separate regressions for data for the years 1998 and 1999 as well as for the years 1999 and 2000 only were run with no qualitatively different results. Result are thus not reported.

3.2.3.2 Contemporaneous correlation between daytime returns of the DAX and the DOW overnight returns

The purpose of the analysis undertaken in this subsection is to contribute to the ongoing debate about the sensitivity of the New York opening to global information accrued overnight (see e.g. Lin et al. (1994) and the results discussed therein). It is important to notice that the New York markets open while trading in Germany is still under way. Taking this into account we chose the DAX open-to-3:30 pm (CET) segment as the independent variable for our analysis. The dependent variable is the overnight return of the DOW where 9:45 am (EST) is chosen as a proxy for the opening quote[12]. If the DAX returns contain any global information relevant for the New York market, the opening quote should exhibit a significant effect.

Table 3.5: DOW overnight returns and the preceding daytime returns of DAX

	a_{22}	b_{22}	d_{22}	c_{22}	μ_2	β_1	β_2
Full Model	0.05293	0.1291	0.03894	0.3904	0.04712	0.2476	0.02951
	(2.257)	(2.659)	(2.887)	(2.193)	(3.086)	(14.45)	(1.844)
Mean Spillover Model	0.01579	0.1003		0.8354	0.04911	0.2357	0.04071
	(2.639)	(3.728)		(21.01)	(3.086)	(10.64)	(2.383)
Volatility Spillover Model	0.07732	0.1061	0.1051	0.1769	0.05061		0.002418
	(4.274)	(2.125)	(5.272)	(2.091)	(3.028)		(0.1896)
t-values in parenthesis							
Mean/ SD of z	-0.0052	1.001					
Skewness	-0.0893						
Kurtosis	3.720						
Autocorrelation of $\hat{z}$	1.045						
Autocorrelation of $\hat{z}^2 - 1$	1.097						

Full Model:

$DOWNR_t = \mu_2 + \beta_1 DAXDR^{\star}_{t-1} + \beta_2 DOWDR^{\star}_{t-1} + \epsilon_{2t}$

$h_{2,t} = a_{22} + b_{22}\epsilon^2_{2t-1} + d_{22}DAXDR^2_{t-1} + c_{22}h_{2,t-1}$

[12] This time span avoids stale prices.

The results of our three specifications are tabulated in Table 3.5. The full model containing mean as well as volatility spillover terms clearly indicates the presence of such effects. The estimated reaction coefficient in the mean equation is much smaller as compared to the reaction of the German market to the previous day returns from New York (see Table 3.4). This result is qualitatively similar to the one obtained by Dornau (1998) and supports the finding of Lin et al. (1994) that the New York stock market is not immune to foreign information.

What is quite striking though is the unusual low estimate for the GARCH-coefficient. This is further analyzed in a simulation study below. As can be seen from the results, the mean spillover specification produces similar results as compared to the full model with the exception of a much higher estimate for the GARCH-coefficient now. Leaving out the mean spillover in exchange of a pure volatility spillover coefficient leads to the effect that already occurred above. The value of the estimated coefficient increases quite dramatically as compared to the full model and the estimate for the GARCH-coefficient is down even further.

The values of the skewness and kurtosis coefficients obtained for the full model point toward a much less severe deviation from the conditional normality as compared e.g. to the first hypotheses tested above. Furthermore, there is no sign of remaining autocorrelation in the standardized residuals and its square.

3.2.3.3 Spillovers from New York to the morning trading in Frankfurt

The analysis of spillovers from New York to the morning trading in Frankfurt is in the spirit of Susmel and Engle (1994). It tests part of the claim put forward in financial media that the previous day performance of the New York markets can explain the behavior of the DAX in the following trading day. If the German market is processing the global information contained in the previous days DOW returns efficiently, there should be no significant spillover effects in both mean and variance from the previous trading day in New York to the DAX morning returns (e.g. open-to-noon). Note that this specification tests a true spillover since returns do not overlap as in the previous models.

We chose the open-to-noon time span because it is quite likely to be net of any additional global information generated in the US due to the fact that it corresponds to the very early morning hours in New York. Extending the time span e.g. up to the opening of the New York markets would have tampered possible effects due to news announcements that usually take place before 9:30 am (EST) or 3:30 pm (CET).

Table 3.6: DAX morning returns: spillovers from the preceding day in New York

	a_{22}	b_{22}	d_{22}	c_{22}	μ_2	β_1	β_2
Full Model	0.01066	0.04097	0.01310	0.9330	-0.02854	0.06369	-0.1753
	(0.8646)	(2.105)	(1.139)	(23.47)	(-0.9221)	(1.231)	(-3.196)
Mean Spillover Model	0.009964	0.05103		0.9364	-0.03080	0.06285	-0.1786
	(0.9773)	(2.447)		(31.45)	(-1.091)	(1.204)	(-3.235)
Volatility Spillover Model	0.01079	0.04133	0.01383	0.9319	-0.03831		-0.1349
	(0.8707)	(1.976)	(1.079)	(22.14)	(-1.411)		(-3.324)
t-values in parenthesis							
Mean/ SD of z	-0.0128	0.9947					
Skewness	-0.1010						
Kurtosis	4.977						
Autocorrelation of $\hat{z}$	0.2220						
Autocorrelation of $\hat{z}^2 - 1$	0.5031						

Full Model:
$DAXDR^{\star}_t = \mu_2 + \beta_1 DOWDR^{\star}_{t-1} + \beta_2 DAXNR_{t-1} + \epsilon_{2t}$
$h_{2,t} = a_{22} + b_{22}\epsilon^2_{2t-1} + d_{22}DOWDR^2_{t-1} + c_{22}h_{2,t-1}$

Regression results for the full model, the mean spillover model only and the variance spillover model only are shown in Table 3.6. No significant mean or volatility spillover effects can be found and none of the problems that occurred in the last subsections are present under this setting. Thus, the German market is processing the information contained in the previous DOW return efficiently.

The reported test statistics at the bottom of Table 3.6 do not point toward misspecifications in the regression models.

3.2.3.4 Spillovers from Frankfurt to the morning trading in New York

The analysis of spillovers from the morning trading in Frankfurt on the opening of the New York stock market is the counterpart to the third hypothesis above. It tests whether the US market is processing information accrued in the German stock market efficiently. This specification also tests a true spillover since returns do not overlap.

We chose the 10am-to-12am time span for the DOW and the open-to-3:30pm (CET) for the DAX on the same day.

Regression results for the full model, the mean spillover model and the variance spillover model are shown in Table 3.7. No significant mean or volatility spillover effects can be found in the full model and its restricted versions. It can thus be conlcuded that the US market processes information contained in the German stock market efficiently.

Table 3.7: DOW morning returns: spillovers from the morning in Frankfurt

	a_{22}	b_{22}	d_{22}	c_{22}	μ_2	β_1	β_2
Full Model	0.0023	0.05066	0.004425	0.8972	0.0000	0.01831	0.1101
	(0.6292)	(3.173)	(0.8507)	(11.03)	(0.1282)	(0.9610)	(2.701)
Mean Spillover Model	0.0000	0.0554		0.9162	0.000	0.01641	0.1111
	(-0.5667)	(2.223)		(12.39)	(-0.1845)	(0.6058)	(2.669)
Volatility Spillover Model	0.0000	0.05158	0.0042	0.8975	0.0000		0.1299
	(0.6233)	(3.042)	(0.8405)	(10.21)	(0.1398)		(3.627)
t-values in parenthesis							
Mean/ SD of z	-0.0083	1.001					
Skewness	-0.0480						
Kurtosis	3.326						
Autocorrelation of $\hat{z}$	1.111						
Autocorrelation of $\hat{z}^2 - 1$	0.9731						

Full Model:
$DOWDR_t^{\star} = \mu_2 + \beta_1 DAXDR_t^{\star} + \beta_2 DOWNR_{t-1} + \epsilon_{2t}$
$h_{2,t} = a_{22} + b_{22}\epsilon_{2t-1}^2 + d_{22} DAXDR_t^2 + c_{22} h_{2,t-1}$

There is no considerable difference between the full model and the pure volatility spillover model as this is true for the first two hypotheses tested above. In addition, the persistence of the volatility process measured by the parameter c_{22} is stable among the three different models.

Again, the reported test statistics at the bottom of Table 3.7 do not point toward any misspecification in the regression models.

3.2.4 Simulations

In this section, we further explore the results found in the empirical section indicating that there is an interaction or a trade-off between a mean spillover and a volatility spillover. For example, tables 3.4 and 3.5 show that the volatility spillover is higher in a pure volatility spillover model compared to a full model. It is also evident that there are considerable changes of the GARCH parameters among the estimated models.

In order to gain more insights in the empirical findings obtained above, we assume three different data-generating processes (DGP): (i) a pure mean spillover model, (ii) a pure volatility spillover model and (iii) a simultaneous mean and volatility spillover model also referred to as full model. We simulate these models with 1000 observations and 1000 repetitions and estimate each DGP with three different models. We focus on a sample of 1000 observations to make a comparison with the empirical results (773 observations) feasible. However, we additionally run the simulations with 2500 observations in order to minimize any small sample bias.

The DGP of a simultaneous mean and volatility spillover model is given by the following equation:

$$\begin{aligned} r_{2,t} &= \beta\epsilon_{1,t} + \epsilon_{2,t} \\ h_{2,t} &= a_{22} + b_{22}\epsilon^2_{2,t-1} + c_{22}h_{2,t-1} + d_{22}\,\epsilon^2_{1,t} \end{aligned} \tag{3.7}$$

The DGP of a pure mean spillover model is given by restricting d_{22} to be zero and the DGP of a pure volatility spillover model is obtained by restricting β to be zero. The model

Table 3.8: Simulation Results - Pure Mean Spillover

DGP	$\hat{\beta}$	$\hat{d_{22}}$	$\hat{a_{22}}$	$\hat{b_{22}}$	$\hat{c_{22}}$
$\beta = 0.0$, $d_{22} = 0.0$	0.0009	0.0050	0.0081	0.0531	0.8589
	0.0009		0.0085	0.0537	0.8599
		0.0050	0.0080	0.0528	0.8609
$\beta = 0.2$, $d_{22} = 0.0$	0.1999 ***	0.0047	0.0073	0.0531	0.8685
	0.1997 ***		0.0087	0.0538	0.8579
		0.0114	0.0099	0.0515	0.8404
$\beta = 0.4$, $d_{22} = 0.0$	0.3989 ***	0.0047	0.0073	0.0539	0.8673
	0.3987 ***		0.0093	0.0550	0.8504
		0.0811	0.0318	0.0528	0.5969
$\beta = 0.6$, $d_{22} = 0.0$	0.6002 ***	0.0050	0.0083	0.0541	0.8564
	0.6000 ***		0.0108	0.0558	0.8331
		0.3315 **	0.0777	0.0416	0.1378
$\beta = 0.8$, $d_{22} = 0.0$	0.7985 ***	0.0048	0.0076	0.0538	0.8650
	0.7985 ***		0.0101	0.0550	0.8439
		0.6282 ***	0.0907	0.0296	0.0371

1000 runs
first line: full model
second line: pure mean spillover model
third line: pure volatility spillover model
*, **, *** indicate rejection of null hypothesis ($\hat{\beta} = 0$ or $\hat{d_{22}} = 0$) in more than 90 percent, 95 percent and 99 percent of all simulation runs, respectively.
GARCH parameters: $b_{22} = 0.05$, $c_{22} = 0.9$, $a_{22} = 0.005$

is simulated with different values of the parameter β (0.0, 0.2, 0.4, 0.6, 0.8) and d_{22} (0.0, 0.1, 0.2), respectively.

The innovations $\epsilon_{1,t}$ and $\epsilon_{2,t}$ are iid random variables with mean zero and variances that follow a GARCH(1,1) process with the parameter values $b_{22} = 0.05$, $c_{22} = 0.9$ and $a_{22} = (1 - b_{22} - c_{22}) \cdot \sigma$ where σ is the unconditional variance assumed to be 0.1.

In accordance with the empirical section we use the Quasi-Maximum-Likelihood procedure and the BHHH algorithm to estimate the above model.

Each DGP is estimated with a full model and its two restricted counterparts leading to nine different alternatives.

Simulation results for DGP (i) (pure mean spillover) are tabulated in table 3.8.

It can be stated that the pure mean spillover model leads to unbiased estimates ($\hat{\beta} = \beta$).

A pure volatility spillover model is estimated with a bias of the parameter d_{22} increasing with the magnitude of the mean spillover and is estimated unbiased if there is no mean spillover. Such an upward bias reflects effects on volatilities that can be explained with the fact that a mean spillover is a shock that can also affect the volatility of the underlying return series. Assume that there is a mean spillover but no volatility spillover:

$$\begin{aligned} r_{2,t} &= \beta\epsilon_{1,t} + \epsilon_{2,t} \\ h_{2,t} &= a_{22} + b_{22}\epsilon_{2,t-1}^2 + c_{22}h_{2,t-1} \end{aligned} \tag{3.8}$$

If the true DGP is assumed to contain only a volatility spillover, the appropriate estimation framework would be as follows:

$$\begin{aligned} r_{2,t}^\star &= \epsilon_{2,t}^\star \\ h_{2,t}^\star &= a_{22} + b_{22}\epsilon_{2,t-1}^{\star 2} + c_{22}h_{2,t-1}^\star + d_{22}\epsilon_{1,t}^2 \end{aligned} \tag{3.9}$$

The stars ($\star$) indicate that the return $r_{2,t}^\star$ and thus $\epsilon_{2,t}^\star$ are different from the true DGP since the mean spillover is not included.

Since the volatility of $\epsilon_{2,t}^\star$ will usually be larger than the volatility of $\epsilon_{2,t}$, the parameter d_{22} *can* pick up this shock leading to a volatility spillover. Of course, the shock *may* also be captured by the parameter b_{22} only. This possibility is also evident by analyzing the simulation results. These shocks cannot always be clearly distinguished resulting in an identification problem.

In addition (and only for DGP (i)), there is a strong effect on the GARCH parameters: the persistence is decreasing extremely, e.g. $\hat{c_{22}}$ is smaller than 0.05 for a mean spillover with $\beta = 0.8$. It is further evident that there is a general downward bias of $\hat{c_{22}}$. This is true for all DGP but is most severe for DGP (i). We attribute this to a small sample bias since the bias of $\hat{c_{22}}$ diminishes for sample sizes of 2500 observations.[13] Finally, a full (unrestricted) mean and volatility spillover estimates the mean and volatility spillover unbiased.

[13]Results are not reported.

Table 3.9: Simulation Results - Pure Volatility Spillover

DGP	$\hat{\beta}$	$\hat{d_{22}}$	$\hat{a_{22}}$	$\hat{b_{22}}$	$\hat{c_{22}}$
$\beta = 0.0$, $d_{22} = 0.1$	-0.0018	0.1087 ***	0.0072	0.0473	0.8921
	-0.0018		0.0167	0.0614	0.8829
		0.1092 ***	0.0072	0.0474	0.8917
$\beta = 0.0$, $d_{22} = 0.2$	0.0007	0.2117 **	0.0081	0.0476	0.8934
	0.0007		0.0252	0.0656	0.8835
		0.2127 ***	0.0082	0.0476	0.8931

1000 runs
first line: full model
second line: pure mean spillover model
third line: pure volatility spillover model
*, **, *** indicate rejection of null hypothesis ($\hat{\beta} = 0$ or $\hat{d_{22}} = 0$) in more than 90 percent, 95 percent and 99 percent of all simulation runs, respectively.
GARCH parameters: $b_{22} = 0.05$, $c_{22} = 0.9$, $a_{22} = 0.005$

The simulation results for the DGP (ii) (pure volatility spillover) are shown in table 3.9 and indicate that all types of models yield unbiased estimates for β and d_{22}.

Simulation results for the third DGP (iii) (full mean and volatility spillover) are given by table 3.10. The results are consistent with the previous findings and can also be derived by these. This means that a full mean and volatility spillover model leads to unbiased estimates for β and d_{22}. A pure mean spillover model leads to unbiased estimates for β and a pure volatility spillover model results in an overestimated parameter for d_{22} depending on the magnitude of β due to the simultaneous influence of a shock to the mean and to the volatility of the underlying return. For example, a simultaneous mean and volatility spillover with the parameter values $\beta = 0.8$ and $d_{22} = 0.2$ leads to estimates $\hat{d_{22}} = 0.4000$ in a pure volatility spillover model (last line, table 3.10). This is an overestimation of $\hat{d_{22}} - d_{22} = 0.2$. Generally, slightly overestimated estimates of $\hat{d_{22}}$ can be explained with underestimated conditional volatilities (GARCH parameters). This effect fades away with larger sample sizes.

Note that this trade-off between mean and volatility spillovers is not rooted in the univariate specification but only in the fact that a mean spillover can (and often does) increase

Table 3.10: Simulation Results - Mean and Volatility Spillover

DGP	$\hat{\beta}$	$\hat{d_{22}}$	$\hat{a_{22}}$	$\hat{b_{22}}$	$\hat{c_{22}}$
$\beta = 0.2$, $d_{22} = 0.1$	0.2015 ***	0.1087 *	0.0075	0.0474	0.8904
	0.2015 ***		0.0174	0.0622	0.8788
		0.1174 **	0.0079	0.0469	0.8873
$\beta = 0.4$, $d_{22} = 0.1$	0.3992 **	0.1072 ***	0.0074	0.0476	0.8919
	0.3989 ***		0.0175	0.0621	0.8792
		0.1424 **	0.0103	0.0463	0.8753
$\beta = 0.6$, $d_{22} = 0.1$	0.5983 ***	0.1095 **	0.0072	0.0472	0.8919
	0.5979 ***		0.0167	0.0618	0.8817
		0.2192 *	0.0188	0.0468	0.8302
$\beta = 0.8$, $d_{22} = 0.1$	0.7992 ***	0.1063 **	0.0073	0.0478	0.8918
	0.7977 ***		0.0171	0.0617	0.8808
		0.4330 *	0.0555	0.0515	0.6720
$\beta = 0.2$, $d_{22} = 0.2$	0.2003	0.2132 **	0.0088	0.0467	0.8922
	0.2005		0.0276	0.0663	0.8777
		0.2229 **	0.0095	0.0465	0.8895
$\beta = 0.4$, $d_{22} = 0.2$	0.4021 ***	0.2103 **	0.0081	0.0474	0.8936
	0.4024 ***		0.0248	0.0650	0.8848
		0.2452 **	0.0095	0.0465	0.8867
$\beta = 0.6$, $d_{22} = 0.2$	0.5974 ***	0.2153 **	0.0083	0.0482	0.8914
	0.5976 ***		0.0265	0.0672	0.8790
		0.3026 **	0.0134	0.0466	0.8708
$\beta = 0.8$, $d_{22} = 0.2$	0.7984 ***	0.2134 **	0.0086	0.0478	0.8917
	0.7978 ***		0.0259	0.0665	0.8812
		0.4000 **	0.0223	0.0477	0.8404

1000 runs
first line: full model
second line: pure mean spillover model
third line: pure volatility spillover model
*, **, *** indicate rejection of null hypothesis ($\hat{\beta} = 0$ or $\hat{d_{22}} = 0$) in more than 90 percent, 95 percent and 99 percent of all simulation runs, respectively.
GARCH parameters: $b_{22} = 0.05$, $c_{22} = 0.9$, $a_{22} = 0.005$

volatility.[14]

3.2.5 Conclusions

We have stated that spurious correlations and spurious spillovers can occur if there are stale prices around the opening of markets or if returns are overlapping to some extent. We have also shown that the finding of a spillover depends on the applied model potentially resulting in a trade-off between mean and volatility spillovers. This implies that the source of an estimated volatility spillover can also be a mean spillover. It further shows that studies that exclusively model volatility spillovers have to be interpreted with caution.

Economically, this means that information spilling over from a foreign market that affects the mean of a given market can also influence the volatility of this market but not vice versa, i.e. uncertainty indicated by increased volatility does not have any influence on the mean return of other markets, only on its volatility.

[14]Simulation results with a multivariate model confirm this conclusions. Results are not reported.

3.3 Varying Mean Spillovers

In the previous section, we have modeled and estimated constant influences of spillovers on other markets. The results are based on the possibly strong assumption that the impact of spillovers does not vary. In this section we relax the assumption of constant mean spillovers and introduce two different regression models in order to examine any variation of the impact of shocks (spillovers) on the mean of a given market.

The first approach is a time-varying parameter model that estimates the impact of spillovers on a given market for every time t and the second estimation framework is a Quantile Regression model that provides estimates of the impact of spillovers conditional on the value of the dependent variable.

Whereas the employed estimation procedures are standard approaches in particular strands of the literature, they are new in the context of spillovers.

In the next paragraphs we will describe the econometric framework necessary to estimate the different impact of spillovers and present empirical results for the hypotheses analyzed in section 3.2. Since we do not use a new data set, we can focus on the differences between constant and varying spillovers.

3.3.1 Time-varying Mean Spillovers

Time-varying spillovers are first estimated for volatility spillovers by the use of dummy variables indicating different regimes of volatility (see Susmel and Engle, 1994). Bekaert and Harvey (1995, 1997) and Bekaert, Harvey and Ng (2002) estimate time-varying risk parameters within an international Capital Asset Pricing Model (CAPM). Ng (2000) applies this method to model time-varying impacts of mean and volatility spillovers.[15] Time-

[15] Ng (2000) analyzes contemporaneous (and synchronous) weekly data. Thus, it is not a true spillover analysis and different to the frequency of the data used here.

varying conditional betas of a CAPM are also estimated by Ball and Kothari (1989), Braun et. al.(1995) and Cho and Engle (2001).

There are important differences regarding the parameterizations of the time-varying parameters: the last three cited articles use a bivariate EGARCH model that only includes lagged values of the variables under investigation to estimate the time-varying parameter. On the other hand, Bekaert et al. (1995, 1997, 2002) and Ng (2000) include exogenous variables to parameterize the time-varying parameters. For example, they use the sum of imports and exports of a market (country) as a proxy for world market integration. Another variable is the spread between a world market dividend yield and a particular market under investigation. This approach introduced by Bekaert and Harvey (1995) is different to the Braun et al. (1995) approach since the time-variation is driven by economic variables that are not part of the estimated system and are optimally derived from economic theory.

It is important to note that all cited contributions do not belong to the original spillover literature as introduced in this chapter. They focus on the time-varying behavior of a market index on a given portfolio or asset or do not analyze spillovers but contemporaneous correlations (see Ng, 2000). We describe the approaches by Braun et al. (1995) and Bekaert et al. (2002) in their original versions and within their original context in order to clarify the difference to the modified model in the context of time-varying spillovers that is proposed below.

The bivariate EGARCH model introduced by Braun et al. (1995) provides estimates for conditional betas of a CAPM and can be written as follows:

$$r_{pt} = \mu_{pt} + \beta_{pt} r_{mt} + \sigma_{pt} z_{pt} \tag{3.10}$$

where r_{pt} and r_{mt} denote the time t excess returns of a portfolio and a market index respectively. μ_{pt} is the conditional mean of the portfolio and β_{pt} the conditional beta of portfolio

p with respect to the market index. The product $\sigma_{pt} z_{pt}$ is the idiosyncratic shock of r_{pt} that consists of the conditional variance σ^2_{pt} and a normally distributed random variable z_{pt} with mean zero and variance one. The conditional variance follows a EGARCH process (see Nelson, 1991) and the conditional beta (β_{pt}) is parameterized as follows:

$$\beta_{pt} = \alpha_\beta + \lambda_{pm} z_{pt-1} z_{mt-1} + \lambda_m z_{mt-1} + \lambda_p z_{pt-1} + \delta_\beta(\beta_{pt-1} - \alpha_\beta) \tag{3.11}$$

where α_β is a constant, λ_{pm} captures the covariance of z_{pt-1} and z_{mt-1}, λ_m and λ_p estimate any leverage effect and δ_β indicates the persistence of shocks to the conditional beta (β_{pt}).

A simplified model of Bekaert et al. (2002) can be written as follows:

$$r_{it} = \beta^{US}_{it-1} \mu_{US,t-1} + e_{it} \tag{3.12}$$

$$\beta^{US}_{it-1} = \boldsymbol{p}' \boldsymbol{X}^{US}_{it-1} + \boldsymbol{q}' \boldsymbol{X}^{w}_{it-1} \tag{3.13}$$

where r_{it} is the excess return on the equity index of country i, $\mu_{US,t-1}$ is the conditional expected excess return on the US market based on information available at time $t-1$, e_{it} is the idiosyncratic shock of any market i and β^{US}_{it-1} is the time-varying risk parameter. This risk parameter is specified as given by equation (3.13). $\boldsymbol{X}^{US}_{it-1}$ consists of information variables that capture the covariance risk of market i with the US and $\boldsymbol{X}^{w}_{it-1}$ consists of local instruments that should capture the covariance risk of market i with the world portfolio. The parameter vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ capture the influence of the variables included. This model is very different to the (pure) time-series approach initiated by Ball et al. (1989). By using exogenous variables to parameterize β^{US}_{it}, the model does not only aim to explain the excess return of a country i by exogenous variables but also tries to explain the value of the impact of any exogenous variable. However, we focus on the model based on Braun et al. (1995) since the Bekaert et al. (1995, 1997, 2002) approach requires a more extensive discussion regarding the selection of the variables parameterizing β_t.[16]

[16] Potential variables that could be used to explain the varying impact of spillovers or correlations are the

A model based on Braun et al. (1995) is given as follows:

$$r_{2,t} = \beta_{1t} r_{1,t} + \beta_2 r^{\star}_{2,t-1} + \sqrt{h_{2,t}} z_{2,t} \tag{3.14}$$

with the conditional variance following an EGARCH process

$$h_{2,t} = \exp\left(c + \theta z_{2,t-1} + \gamma(|z_{2,t-1}| - E(|z_{2,t}|)) + \delta \log(h_{2,t-1})\right) \tag{3.15}$$

and the time-varying impact of a spillover

$$\beta_t = \alpha_\beta + \lambda_1 \epsilon_{1,t-1} \epsilon_{2,t-1} + \lambda_2 \epsilon_{1,t-1} + \lambda_3 \epsilon_{2,t-1} + \delta_\beta \beta_{t-1} \tag{3.16}$$

We use the unstandardized residuals (ϵ_{it}) to avoid additional estimations of the conditional volatility of these series. Estimation is done by the Quasi-Maximum-Likelihood method. The specification tests of these models are described and conducted in the empirical section. Interestingly, the tests for these models exclusively focus on the estimated standardized residuals but do not test the specification of β_t directly, e.g. no measure of the fit of the time-varying beta is used in the literature.

Empirical results of the modified model of Braun et al.(1995) are presented in the next section.

3.3.1.1 Empirical Results

Results for the four hypotheses put forward are shown in tables 3.11-3.14. The tables contain the parameter estimates and the associated t-values. In addition, the mean, the minimum and the maximum value of the varying parameter β_t is reported. Statistics that indicate the correct specification of the model are also tabulated.

The first hypothesis investigates the contemporaneous correlation between daytime returns of the DOW and the DAX overnight returns. Estimation results are shown in table

volume of stocks traded, GDP growth rate differentials, interest rate differentials and the sum of total exports and imports among others.

3.11 and indicate that there is a considerable variation across time: the minimum and maximum of β_t is -0.0363 and 1.264, respectively.

The varying parameter β_t is mainly characterized by a constant $\alpha_\beta = 0.5736$ and an asymmetric effect of lagged shocks of $DOWDR_{t-1}$ represented by the negative parameter λ_2. This means that negative shocks of the $DOWDR_{t-1}$ increase the impact on the dependent variable. In other words, the influence of the US market on the German market tends to increase in bear markets.

Table 3.11: Time-varying Correlation ($DAXNR_t$, $DOWDR_{t-1}$)

	c	θ	γ	δ	α	α_β	λ_1	λ_2	λ_3	δ_β	β_2
parameter	-0.1464	0.05817	0.1608	0.9778	0.1025	0.5736	0.01088	-0.1245	0.05199	0.1573	-0.01322
t-value	(-3.733)	(1.405)	(4.469)	(74.92)	(4.897)	(2.089)	(0.3943)	(-1.030)	(0.3765)	(0.3849)	(-0.1114)
Mean of β_t	0.6695										
Min/ max of β_t	-0.03634	1.264									
Mean/ SD of z	0.01477	1.009									
Skewness	0.5063										
Kurtosis	8.572										
Autocorrelations of z	0.5011										
Autocorrelations of squared z	0.3636										

$DAXNR_t = \alpha + \beta_t DOWDR^\star_{t-1} + \beta_2 DAXDR^\star_{t-1} + \epsilon_{2t}$
$h_{2,t} = \exp\left(c + \theta z_{2,t-1} + \gamma(|z_{2,t-1}| - E(|z_{2,t}|)) + \delta \log(h_{2,t-1})\right)$
$\beta_t = \alpha_\beta + \lambda_1 r_{1,t-1} r_{2,t-1} + \lambda_2 r_{1,t-1} + \lambda_3 r_{2,t-1} + \delta_\beta \beta_{t-1}$

The second hypothesis examines the contemporaneous correlation between the daytime return of the DAX and the DOW overnight return. Results are presented in table 3.12 and show a significant negative effect of the covariance (λ_1) and the persistence (δ_β) of β_t. The estimated β_t varies only between 0.0285 and 0.3277.

Table 3.12: Time-varying Correlation ($DOWNR_t$, $DAXDR_t$)

	c	θ	γ	δ	α	α_β	λ_1	λ_2	λ_3	δ_β	β_2
parameter	-0.2321	-0.03699	0.1792	0.9370	0.04507	0.3327	-0.02369	-0.02260	0.001548	-0.4019	-0.02901
t-value	(-3.280)	(-1.011)	(3.570)	(32.25)	(2.729)	(6.828)	(-2.204)	(-0.8261)	(0.1443)	(-2.425)	(-1.796)
Mean of β_t	0.2302										
Min/ max of β_t	0.02849	0.3277									
Mean/ SD of z	0.001437	1.003									
Skewness	-0.1104										
Kurtosis	3.942										
Autocorrelations of z	0.8521										
Autocorrelations of squared z	0.3859										

$DOWNR_t = \alpha + \beta_t DAXDR^\star_t + \beta_2 DOWDR_{t-1} + \epsilon_{2t}$
$h_{2,t} = \exp\left(c + \theta z_{2,t-1} + \gamma(|z_{2,t-1}| - E(|z_{2,t}|)) + \delta \log(h_{2,t-1})\right)$
$\beta_t = \alpha_\beta + \lambda_1 r_{1,t-1} r_{2,t-1} + \lambda_2 r_{1,t-1} + \lambda_3 r_{2,t-1} + \delta_\beta \beta_{t-1}$

The third hypothesis analyses if the information from the previous days closing quote in New York spills over to the next days morning trading in Frankfurt. Results are given by table 3.13 and show that there is a significant positive effect of the covariance of both markets on the conditional influence of the DAXDR. Furthermore, β_t varies over a wide range between -1.200 and 1.174 which can be attributed to the rather large values of the parameters λ_1 and δ_β.

Table 3.13: Time-varying Spillover ($DAXDR_t$, $DOWDR_{t-1}$)

	c	θ	γ	δ	α	α_β	λ_1	λ_2	λ_3	δ_β	β_2
parameter	-0.08587	0.001073	0.1079	0.9899	-0.04246	-0.02705	0.1367	0.03141	0.01699	0.3922	-0.04070
t-value	(-3.406)	(0.03219)	(3.546)	(71.09)	(-1.311)	(-1.010)	(2.163)	(0.5288)	(0.2572)	(1.508)	(-0.8057)
Mean of β_t	-0.05235										
Min/ max of β_t	-1.200	1.174									
Mean/ SD of z	-0.0119	0.9945									
Skewness	-0.1539										
Kurtosis	4.518										
Autocorrelations of z	0.5581										
Autocorrelations of squared z	0.3924										

$DAXDR^\star_t = \alpha + \beta_t DOWDR^\star_{t-1} + \beta_2 DAXNR_{t-1} + \epsilon_{2t}$
$h_{2,t} = \exp\left(c + \theta z_{2,t-1} + \gamma(|z_{2,t-1}| - E(|z_{2,t}|)) + \delta \log(h_{2,t-1})\right)$
$\beta_t = \alpha_\beta + \lambda_1 r_{1,t-1} r_{2,t-1} + \lambda_2 r_{1,t-1} + \lambda_3 r_{2,t-1} + \delta_\beta \beta_{t-1}$

The fourth hypothesis analyses if the information from the morning trading in Frankfurt spills over to the morning trading in New York on the same day. Results are given by table 3.14 and reveal a negative and significant parameter λ_1 that governs the covariance of the two returns analyzed and a positive and significant parameter λ_3 that implies that positive shocks of the DAXDR increase the influence on the DOWDR and negative shocks decrease this influence. The impact of the spillover varies between -0.8632 and 1.261.

Specification tests primarily focus on the distributional characteristics of $\hat{z}_t$ (see Nelson, 1991). We compute the mean and the variance of the estimated standardized residual $\hat{z}_t = \dfrac{\hat{\epsilon}_t}{\hat{\sigma}_t}$ that should be zero and one, respectively. In addition, we carry out a F-test for autocorrelation of $\hat{z}_t$ and $\hat{z}_t{}^2 - 1$ with 5 lags.

For all four hypotheses the indicators do not point to any systematic violation of the assumptions regarding $\hat{z}_t$. Although the skewness and the excess kurtosis are clearly dif-

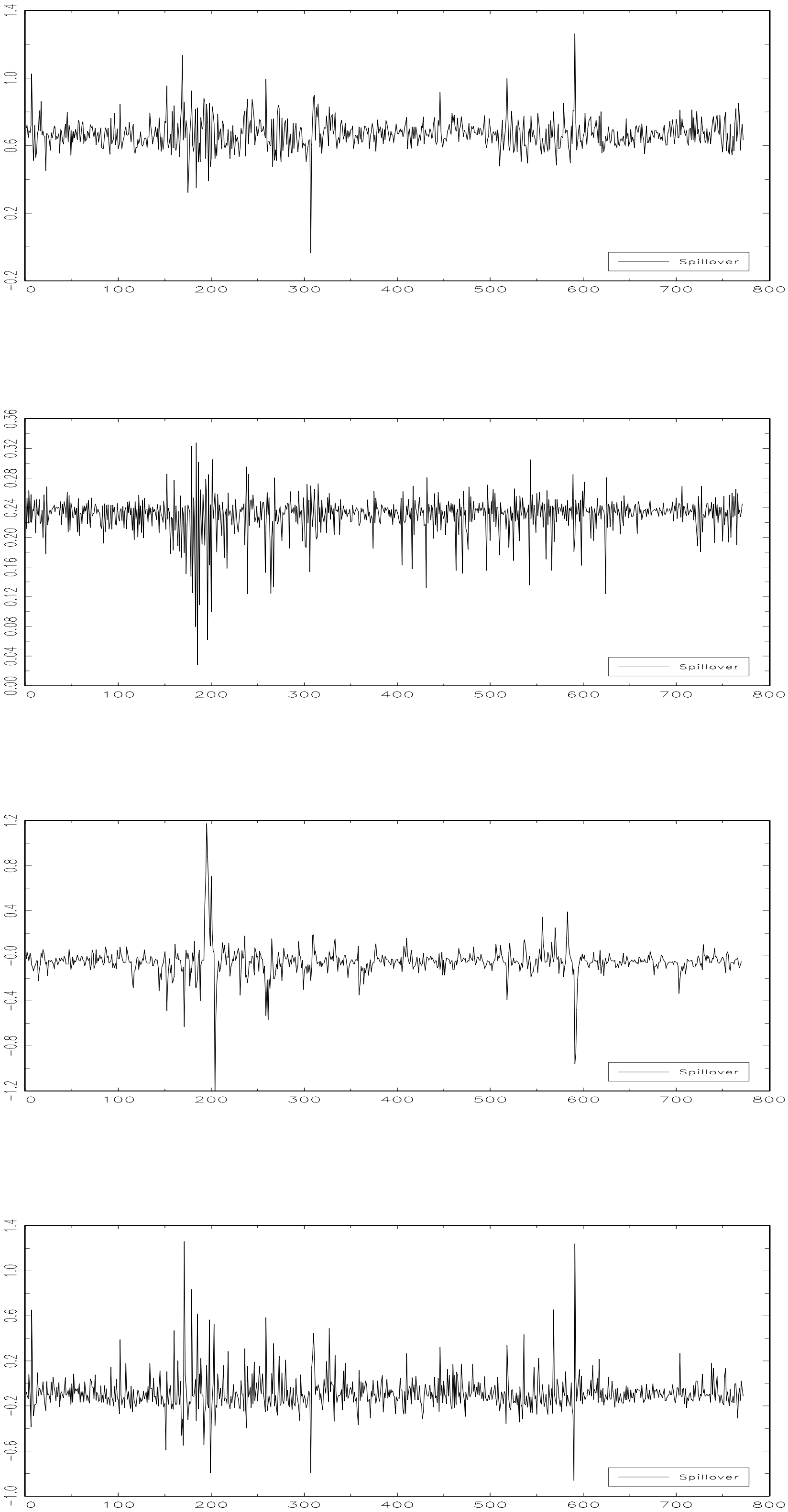

Figure 3.1: *Time-varying Correlations (Spillovers): top: ($DAXNR_t$, $DOWDR_{t-1}$), intermediate: ($DOWNR_t$, $DAXDR_t$), ($DAXDR_t$, $DOWDR_{t-1}$), bottom: ($DOWDR_t^\star$, $DAXDR_t^\star$)*

Table 3.14: Time-varying Spillover ($DOWDR_t^\star$, $DAXDR_t^\star$)

	c	θ	γ	δ	α	α_β	λ_1	λ_2	λ_3	δ_β	β_2
parameter	-0.1361	-0.02230	0.1702	0.9869	0.06674	-0.06371	-0.1126	-0.09467	0.1263	0.1806	-0.06733
t-value	(-3.279)	(-0.5045)	(3.729)	(67.69)	(2.388)	(-0.3850)	(-2.099)	(-0.5746)	(1.738)	(0.1163)	(-1.450)
Mean of β_t	-0.0853										
Min/ max of β_t	-0.8632	1.261									
Mean/ SD of z	0.02941	1.006									
Skewness	-0.05657										
Kurtosis	6.807										
Autocorrelations of z	1.146										
Autocorrelations of squared z	0.1009										

$DOWDR_t^\star = \alpha + \beta_t DAXDR_t^\star + \beta_2 DOWNR_{t-1} + \epsilon_{2t}$
$h_{2,t} = \exp\left(c + \theta z_{2,t-1} + \gamma(|z_{2,t-1}| - E(|z_{2,t}|)) + \delta \log(h_{2,t-1})\right)$
$\beta_t = \alpha_\beta + \lambda_1 r_{1,t-1} r_{2,t-1} + \lambda_2 r_{1,t-1} + \lambda_3 r_{2,t-1} + \delta_\beta \beta_{t-1}$

ferent from zero, the model can be assumed to be well-specified because the non-normality is accounted for in the estimation process by the use of the Quasi-Maximum Likelihood (QML) procedure.

The plots of β_t are given by figure 3.1 and show that the analyzed time-varying effects exhibit wide ranges of values for the first, third and fourth hypotheses and a relatively smaller range for the second hypothesis.

The plots also show that there is increased variation between $t = 150$ and $t = 200$ which could be attributed to effects of the Brazilian and Russian crises in 1998. This finding additionally justifies the estimation of a time-varying parameter model: even if the constancy of β_t for the whole sample cannot be rejected, the model can also provide anomalies for shorter periods which would not be revealed by a constant parameter model.

3.3.2 Conditional Mean Spillovers

The Quantile Regression (QR) model introduced by Koenker and Bassett (1978) (see Appendix for details) provides estimates for different conditional quantiles of the dependent variable. Applying this method to several quantiles supplies information about the varying impact of an exogenous variable x on an endogenous variable y. A simple quantile regression equation to model a mean spillover is given as follows:

$$y_t = c(q) + \beta_1(q)x_t + \beta_2 y^{\star}_{t-1} + \epsilon_t \tag{3.17}$$

where q is the desired quantile of the dependent variable y_t, x_t is the exogenous variable and ϵ_t the error term. The parameters $c(q)$, $\beta_1(q)$ and $\beta_2(q)$ are estimated with the QR model and potentially yield different values for each quantile.[17]

Moreover, the QR model can provide asymmetric results without any additional regressor which would be necessary in the standard model (see equation (3.2)) and the time-varying parameter model (see equation (3.14)). For example, if an exogenous variable x_t has a larger positive impact on the dependent variable y_t at the 10 percent quantile of y_t than at the 90 percent quantile of y_t ($\beta(10) > \beta(90)$), then the spillover is higher if y_t is small (e.g. large negative values) and lower if y_t is large (e.g. large positive values). This example would imply that simultaneous movements of returns are more pronounced in bear markets than in bull markets.[18] There is also an obvious link to the finding that correlations of stock market returns are higher for negative returns than for positive returns (see Ang and Chen, 2002 and Longin and Solnik, 2001 among others).

[17]While equation (3.17) is appropriate to model conditional mean spillovers, the QR model could also be used to estimate conditional volatility spillovers similar to an approach by Chakrabarti et al.(2002) who model volatility spillovers not within a GARCH framework but focus on innovations in variances directly.

[18]In other words, bears and bulls move differently across countries (see Lin et al., 1994).

3.3.2.1 Empirical Results

The first hypothesis investigates the contemporaneous correlation between daytime returns of the DOW and the DAX overnight returns. Estimation results are shown in table 3.15 and do not reveal a clear difference of the estimated parameters among the quantiles. Estimates for $\beta(q)$ vary slightly around 0.7 and are all significant at the 1 percent level. The measure of fit[19] denoted with Pseudo-R^2 has the highest value at the 1 percent quantile (0.3055) and the lowest value (0.2392) at the 60 percent quantile. The values for the remaining quantiles vary around 0.25.

Table 3.15: Conditional Correlation ($DAXNR_t$, $DOWDR_{t-1}$)

	q1	q10	q20	q30	q40	q50	q60	q70	q80	q90	q99	**LS**
$R^2(q)$	0.3055	0.2521	0.2660	0.2541	0.2440	0.2406	0.2392	0.2411	0.2556	0.2806	0.2677	0.4326
$\beta_1(q)$	0.694	0.709	0.712	0.719	0.705	0.708	0.723	0.690	0.696	0.712	0.769	0.705
	(4.05)***	(11.65)***	(13.53)***	(17.28)***	(18.08)***	(15.84)***	(14.35)***	(13.25)***	(18.16)***	(13.21)***	(4.96)***	(24.02)***
$\beta_2(q)$	-0.017	0.017	-0.004	0.010	0.016	-0.011	-0.034	-0.057	-0.103	-0.115	-0.371	-0.075
	(0.09)	(0.64)	(0.12)	(0.22)	(0.55)	(0.31)	(0.80)	(1.45)	(2.82)***	(2.64)***	(1.53)	(2.88)***
$c(q)$	-1.676	-0.541	-0.303	-0.144	-0.018	0.108	0.255	0.400	0.582	0.799	1.905	0.116
	(8.94)***	(12.17)***	(11.33)***	(5.25)***	(1.09)	(3.82)***	(11.44)***	(14.75)***	(20.91)***	(27.30)***	(4.51)***	(5.00)***

$DAXNR_t(q) = c(q) + \beta_1(q)DOWDR^{\star}_{t-1} + \beta_2(q)DAXDR_{t-1}$
t-values in parenthesis (calculated by bootstrapping with 2000 repetitions)

The second hypothesis examines the contemporaneous correlation between the daytime return of the DAX and the DOW overnight return. Results are presented in table 3.16 and show that there is a larger influence at the lower quantiles (especially at the 1 percent quantile) than in the higher quantiles. All quantiles larger than 20 percent exhibit a relatively constant relationship around 0.20. This finding indicates that the DAXDR has a higher impact on the DOWNR for larger negative values of the DOWNR. This can be viewed as an asymmetric effect of the DAXDR on the DOWNR as described above. Note that all estimates of $\beta_1(q)$ are highly significant at the 1 percent level. Furthermore, the R^2 is considerably higher at the 1 percent quantile (0.2924) than in all other quantiles

[19] $R^2 = 1 - \frac{\text{Sum of Weighted Deviations About Estimated Quantile}}{\text{Sum of Weighted Deviations About Raw Quantile}}$
The raw quantile is given by quantile regression with no regressor. This R^2 is analogous to the R^2 of the least square regression.

declining almost steadily to exhibit the lowest value at the 70 percent quantile (0.1095).

Table 3.16: Conditional Correlation ($DOWNR_t$, $DAXDR_t$)

	q1	q10	q20	q30	q40	q50	q60	q70	q80	q90	q99	**LS**
$R^2(q)$	0.2924	0.1982	0.1534	0.1338	0.1179	0.1104	0.1120	0.1095	0.1114	0.1120	0.1191	0.2587
$\beta_1(q)$	0.268	0.263	0.228	0.200	0.202	0.198	0.188	0.197	0.178	0.172	0.171	0.217
	(5.95)***	(10.62)***	(10.59)***	(7.96)***	(8.64)***	(8.27)***	(6.76)***	(10.90)***	(6.32)***	(5.50)***	(3.96)***	(16.32)***
$\beta_2(q)$	0.158	0.091	0.032	0.032	0.020	0.026	0.004	0.026	0.029	0.014	0.043	0.045
	(1.74)*	(2.55)**	(1.12)	(1.64)	(1.01)	(1.42)	(0.18)	(1.32)	(0.93)	(0.35)	(0.89)	(2.93)***
$c(q)$	-1.178	-0.530	-0.325	-0.174	-0.067	0.064	0.152	0.285	0.413	0.632	1.253	0.046
	(10.28)***	(15.82)***	(11.65)***	(8.74)***	(3.19)***	(3.47)***	(8.11)***	(14.18)***	(17.40)***	(18.17)***	(12.95)***	(2.60)***

$DOWNR_t(q) = c(q) + \beta_1(q)DAXDR_t^\star + \beta_2(q)DOWDR_{t-1}$
t-values in parenthesis (calculated by bootstrapping with 2000 repetitions)

Table 3.17: Conditional Spillover ($DAXDR_t$, $DOWDR_{t-1}$)

	q1	q10	q20	q30	q40	q50	q60	q70	q80	q90	q99	**LS**
$R^2(q)$	0.0016	0.0043	0.0054	0.0068	0.0047	0.0023	0.0010	0.0017	0.0024	0.0081	0.0143	0.0016
$\beta_1(q)$	0.016	-0.048	-0.067	-0.005	0.019	-0.002	-0.019	-0.041	-0.059	-0.108	0.051	-0.030
	(0.11)	(0.45)	(0.72)	(0.09)	(0.44)	(0.06)	(0.55)	(1.18)	(1.09)	(1.58)	(0.21)	(0.76)
$\beta_2(q)$	0.094	-0.122	-0.101	-0.115	-0.076	-0.051	-0.029	-0.042	0.010	-0.009	0.143	-0.032
	(0.51)	(1.10)	(1.33)	(2.38)**	(1.74)*	(1.45)	(0.75)	(0.76)	(0.15)	(0.12)	(0.67)	(0.87)
$c(q)$	-2.407	-1.092	-0.672	-0.366	-0.196	-0.029	0.110	0.312	0.555	0.929	2.292	-0.066
	(13.47)***	(17.44)***	(14.09)***	(11.13)***	(7.43)***	(1.04)	(3.36)***	(8.41)***	(11.49)***	(20.11)***	(8.69)***	(2.09)**

$DAXDR_t^\star(q) = c(q) + \beta_1(q)DOWDR_{t-1}^\star + \beta_2(q)DAXNR_{t-1}$
t-values in parenthesis (calculated by bootstrapping with 2000 repetitions)

The third hypothesis analyzes the question if the information from the previous days closing quote in New York spills over to the next days morning trading in Frankfurt. Results are given by table 3.17 and indicate that $\beta(q)$ is close to zero and insignificant for all quantiles. The insignificant estimates are also reflected in values of the Pseudo-R^2 close to zero.

Table 3.18: Conditional Spillover ($DOWDR_t^\star$, $DAXDR_t^\star$)

	q1	q10	q20	q30	q40	q50	q60	q70	q80	q90	q99	**LS**
$R^2(q)$	0.0627	0.0195	0.0165	0.0137	0.0151	0.0135	0.0109	0.0102	0.0055	0.0008	0.0165	0.0220
$\beta_1(q)$	-0.054	0.039	0.008	0.018	0.034	0.007	0.008	0.004	0.014	-0.011	0.053	0.015
	(1.15)	(0.94)	(0.29)	(0.76)	(1.18)	(0.35)	(0.23)	(0.10)	(0.41)	(0.25)	(0.66)	(0.75)
$\beta_2(q)$	0.336	0.192	0.185	0.144	0.138	0.143	0.142	0.101	0.072	0.039	0.149	0.128
	(2.25)**	(2.24)**	(2.76)***	(2.90)***	(2.09)**	(3.05)***	(2.34)**	(1.96)*	(1.06)	(0.58)	(0.60)	(3.07)***
$c(q)$	-0.014	-0.007	-0.004	-0.003	-0.001	-0.000	0.001	0.003	0.004	0.007	0.014	-0.000
	(12.43)***	(19.40)***	(14.80)***	(8.43)***	(8.35)***	(0.08)	(4.90)***	(8.27)***	(12.21)***	(16.37)***	(11.42)***	(0.13)

$DOWDR_t^\star(q) = c(q) + \beta_1(q)DAXDR_t^\star + \beta_2(q)DOWNR_{t-1}$
t-values in parenthesis (calculated by bootstrapping with 2000 repetitions)

The fourth hypothesis shows that the influence of the morning trading in Frankfurt on the opening of New York stock market is insignificant for all quantiles.

The last column of tables 3.15-3.18 show the least square results. These values are

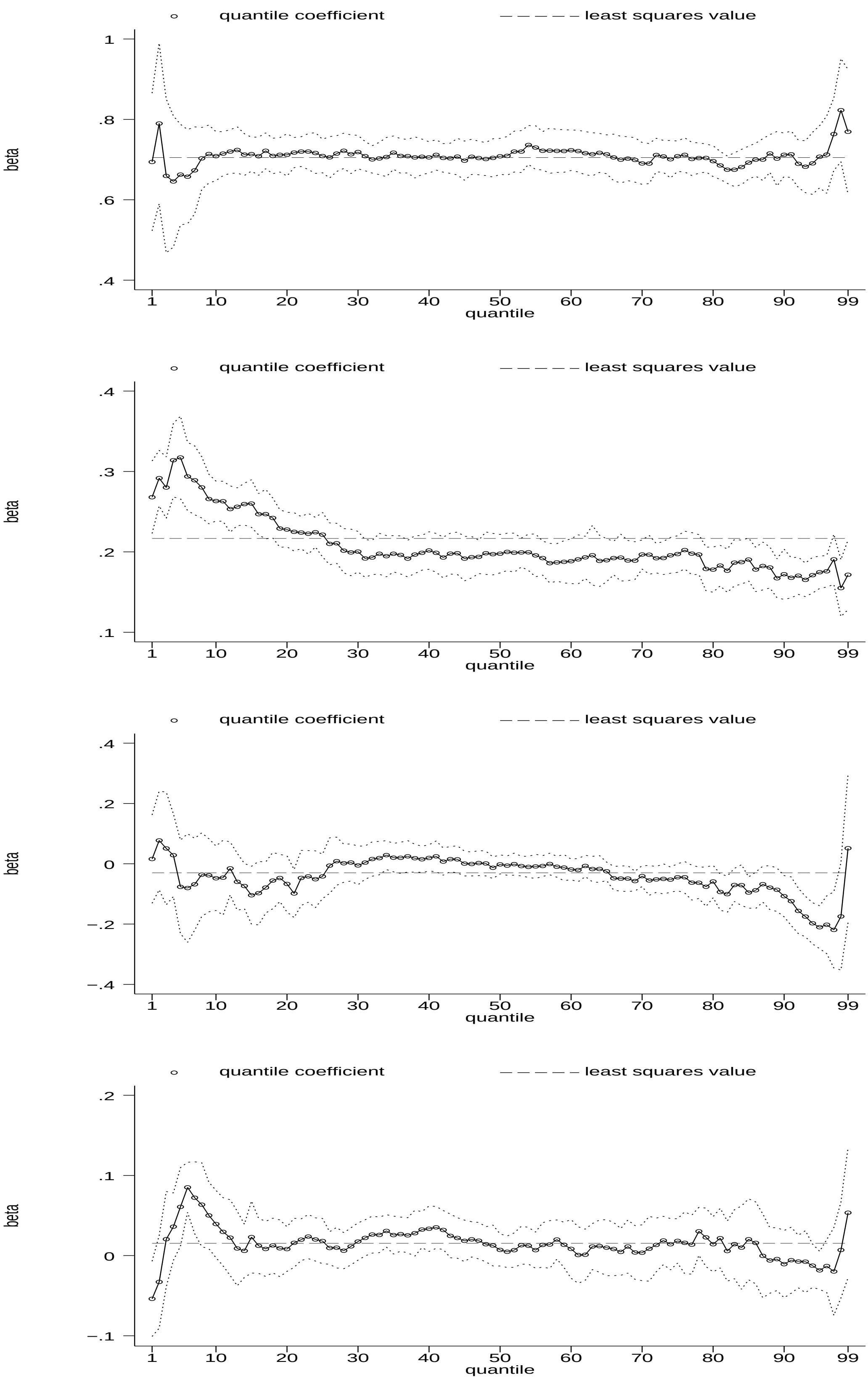

Figure 3.2: *Conditional Correlations (Spillovers): top: ($DAXNR_t$, $DOWDR_{t-1}$), intermediate: ($DOWNR_t$, $DAXDR_t$), ($DAXDR_t$, $DOWDR_{t-1}$), bottom: ($DOWDR_t^\star$, $DAXDR_t^\star$)*

slightly different compared to the results presented in the previous section since no GARCH effects are modeled.

Considerably different estimates compared to the original AS-model are only obtained for the second hypothesis. Note, that it is only tested whether these estimates are different from zero. A test whether the slope estimates are identical at every quantile is not carried out (see Koenker and Bassett, 1982) since the aim of this quantile regression analysis is to obtain additional information compared to the least square regression. Thus, we focus on the question whether the quantile estimates are different to the least square estimates.

The plots of the conditional quantile estimates are given in figure 3.2 and do not only contain the conditional parameter β (solid line) but also a 95 percent pointwise confidence band. There is also a dashed line representing the ordinary least squares estimate of the mean effect. The graphs for the QR model suggest that there is negligible variation among all quantiles for the first, third and fourth hypotheses. However, the extreme negative and positive quantiles show values that are different from the least square estimates for the second hypothesis (negative quantiles), the third hypothesis (positive quantiles) and the fourth hypothesis (negative quantiles). Given the narrow confidence bands for these values, they are clearly different from the average values of the other quantiles and the least square estimates. This finding is not surprising since changing market conditions can imply changing parameter estimates. For example, extreme market situations such as crashes or bubbles can be expected to exhibit different parameter values than market conditions that are characterized as more normal. The Quantile Regression Model accounts for such different market situations whereas the classical regression model provides only estimates of the mean effect.

3.3.3 Conclusions

This section relaxes the restriction of constant correlations and constant impacts of spillovers and extends the existing literature in two ways: it provides methods to estimate impacts of spillovers on other markets depending on the value of the dependent variable and on the time of the spillover. Results show that the assumption of an identical impact of spillovers through time and across quantiles is a strong restriction. This implies that an analysis as carried out above is more appropriate and can provide information that remains hidden in a standard spillover analysis.

3.3.4 Appendix

3.3.4.1 Quantile Regression

Quantile Regression (QR) as introduced in Koenker and Bassett (1978) may be viewed as a natural extension of classical least square estimation of conditional mean models to models for conditional quantile functions. These conditional quantile functions are estimated by minimizing an asymmetrically weighted sum of absolute errors as follows:

$$\text{argmin} \sum \rho_\theta(y_i - \epsilon) \tag{3.18}$$

$$\text{with the check function} \qquad \rho_\theta(z) \; = \begin{cases} \theta z & : \; z \geq 0 \\ (\theta - 1)z & : \; z < 0 \end{cases} \tag{3.19}$$

To obtain an estimate of the conditional quantile function, we simply replace the scalar of equation (3.18) by the parametric function $\epsilon(x_i, \beta_\theta)$ and solve

$$\min \sum \rho_\theta(y_i - \epsilon(x_i, \beta_\theta)) \tag{3.20}$$

where $z = (y_i - \epsilon)$. Instead of using the check function $\rho_\theta(z)$ we write the minimization problem more explicitly as

$$\min \left\{ \sum_{i:y_i \geq x'\beta_\theta} \theta |y_i - x_i'\beta_\theta| + \sum_{i:y_i < x'\beta_\theta} (1-\theta)|y_i - x_i'\beta_\theta| \right\} \tag{3.21}$$

A general closed solution to the minimization problem does not exist. However, it can be shown that this optimization problem can be solved with a linear programming algorithm. For example, a method by Koenker and Park (1996) is competitive to least-squares estimation even for very large data sets. Standard errors can either be computed by using asymptotic results or by performing bootstraps.

3.4 Mean and Volatility Contagion

This section does not further analyze the German DAX and the US DOW Jones stock index but examines a broader range of stock market indices. First, correlations in mean and in volatility are investigated and second, it is examined whether there is a change in the correlation during certain periods of time, particularly in periods of financial stress or financial turmoil. It is thus an analysis of time-varying correlations with different regimes of market characteristics.

The investigation of changes in correlations is crucial for portfolio and risk management since diversification can only be effective if correlations and variances do not significantly and rather abruptly change in certain time periods. Examining changing interdependencies is also important when discussing the appropriate financial architecture and the reasons for financial crises. For example, the justification for aid packages or credits to crises countries by the International Monetary Fund (IMF) is the fear that the crisis could spread to other countries, i.e. that the crisis is contagious for other countries. Consequently, it is not only important to assess the existence of contagion but also the strength of contagion. Apart from this problem, there is even widespread disagreement what the term *contagion* entails (see Forbes and Rigobon, 2002). A list of different definitions is provided, for example, by the World Bank[20] and Pericoli et al. (2001). We adapt the definition of Baig and Goldfajn (1999) and Forbes and Rigobon (2002) who define contagion as a significant increase in cross-market linkage after a shock to one country or a group of countries. Forbes and Rigobon (2002) stress that this notion of contagion excludes a constant high degree of comovement in a crisis period. In this case, markets are just interdependent.

The definition of Forbes and Rigobon (2002) uses the term *cross-market linkage* as a synonym for *correlation* and *comovement*. However, these terms describe forms of market

[20] http://www1.worldbank.org/contagion/definitions.html

association that are not explicitly defined. Furthermore, they sometimes seem to be used inflationary which is also true for *transmission*, *interdependence* and *spillover*. We use precise definitions of these terms in order to describe and estimate empirical phenomena more accurately.

Focussing on the narrow definition of Forbes and Rigobon (2002) and their test for contagion, we show that results can be misleading when (i) correlations are time-varying and not constant, (ii) heteroscedasticity is a source of contagion and (iii) the crisis period is too short, i.e. the test does not have enough power to detect contagion.

In addition, we argue that the correlation coefficient is an inadequate measure to analyze an asymmetric phenomenon such as contagion and therefore advocate to focus on the transmission mechanism of shocks directly.

The remainder of this part is organized as follows: Section 3.4.1 briefly discusses potential models of contagion, section 3.4.2 explains the test developed by Forbes and Rigobon (2002) and shows that this test severely hinges on certain assumptions. Section 3.4.3 introduces a test that concentrates on the transmission mechanism of shocks directly and not on the correlation coefficient. In section 3.4.4, we also introduce a test for volatility contagion and report empirical results for the Asian crisis in section 3.4.5. Section 3.4.6 concludes.

3.4.1 Modeling Contagion

We present two models that explain the phenomenon of contagion (i.e. increased correlation in crises times) and discuss their strengths and shortcomings. A single factor model for two markets can be written as follows (see Corsetti et al., 2001)

$$\begin{aligned} r_{1t} &= a_1 + b_1 f_t + u_{1t} \\ r_{2t} &= a_2 + b_2 f_t + u_{2t} \end{aligned} \tag{3.22}$$

where r_{it} is the return of market i, f_t is a common factor that potentially affects both markets, u_{it} is the error term of return i and a_i, b_i (for $i = 1, 2$) are the parameters to be estimated. In such a framework, correlations are time-varying due to the global factor f_t if $b_i \neq 0$ (for $i = 1, 2$)[21]. There is no transmission of shocks from one market to the other. Thus, we call this 'comovement' which is only explained by the common factor. Contagion when defined as increased correlation occurs when (i) the common factor increases, (ii) the loading b_i increases or (iii) the ratio of the variances of u_{it} and f_t increases (ceteris paribus).

Another potential model of contagion is characterized by a country-specific shock that becomes regional or global in a crisis period (see Corsetti et al., 2001). In such a setting there is a transmission of a shock from one market to the other:

$$\begin{aligned} r_{1t} &= a_1 + b_1 f_t + u_{1t} \\ r_{2t} &= a_2 + b_2 f_t + b_3 u_{1t} D_{Crisis} + u_{2t} \end{aligned} \tag{3.23}$$

Here the factor model as described by equation (3.22) is extended with the term $b_3 u_{1t} D_{Crisis}$, i.e. the error term u_{1t} transmits to the other market (r_{2t}) in the crisis period.[22] The parameter b_3 captures this transmission and the dummy D_{Crisis} ensures the shock to be influential only in the crisis period (the dummy is equal to one in the crisis period and zero otherwise).

Note that this model does not assume a constant transmission of shocks from one market to the other. There is 'only' comovement in tranquil (normal) times whereas in a crisis period there is a transmission of an idiosyncratic shock from market 1 to market 2. Con-

[21] The time-varying correlation coefficient ρ_t in a factor model is given as follows:

$$\rho_t = \frac{E(b_1 f_t + u_{1t})E(b_2 f_t + u_{2t})}{\sqrt{E(b_1^2 f_t^2 + u_{1t}^2)E(b_2^2 f_t^2 + u_{2t}^2)}} = ... = \frac{1}{\sqrt{(1 + \frac{Var(u_{1t})}{b_1^2 Var(f_t)})(1 + \frac{Var(u_{2t})}{b_2^2 Var(f_t)})}}$$

This expression shows that time-varying variances of f_t or the idiosyncratic shocks u_{1t} and u_{2t} imply variation of ρ through time.

[22] Note that $E(u_{1t})$ is not necessarily zero in the crisis period in contrast to $E(u_{1t})$ for the whole period.

tagion happens in this model if b_3 is different from zero.[23]

In the following section, we describe and discuss the test for contagion introduced by Forbes and Rigobon (2002) that is based on changes of the correlation coefficient in a crisis period relative to a tranquil period. Then, we propose a test for contagion that is based on equation (3.23) and relate this test to the Forbes and Rigobon (2002) approach.[24]

3.4.2 Excess comovement

Forbes and Rigobon (2002) base their test for contagion on a comparison of the cross-market linkage during a relatively stable period (measured as a historic average) with the linkage during a crisis period.[25] They use the correlation coefficient as a measure for the cross-market linkage. This means that linkage and correlation are viewed as an equivalent phenomenon. Since Forbes and Rigobon do not assume any model (i.e. data-generating process), the model could be given by equations (3.22), (3.23) or any other model generating contagion.

Testing for contagion as proposed by Forbes and Rigobon is based on an unconditional correlation coefficient, i.e. a constant correlation.

The null hypothesis of no contagion is $H_0 : \rho_0 \geq \rho_1$ against the alternative of contagion ($H_1 : \rho_0 < \rho_1$) where ρ_0 and ρ_1 stand for the unconditional correlation coefficients of the full period and the crisis period, respectively. This approach tests whether there is increased comovement in the crisis period compared to the full period. This increased comovement is also referred to as excess comovement. If the comovement does not increase in a crisis

[23] A common notion of contagion implies that the parameter b_3 is positive.

[24] Other testing approaches, e.g. the analysis of news spillovers (e.g. Lin et al., 1994 and Edwards, 1998), increased probabilities of a crisis in crises times (e.g. Kaminsky and Reinhart, 1999), co-integration (e.g. Cashin et al. 1995) and the coincidence of extreme returns (Bae et al., 2002) will not be discussed in further detail.

[25] This concept was first proposed by King and Wadhwani (1990) and also used by Baig and Goldfajn (1999), for example. We refer to the study of Forbes and Rigobon (2002) since it is the most recent and also most general analysis of contagion in financial markets.

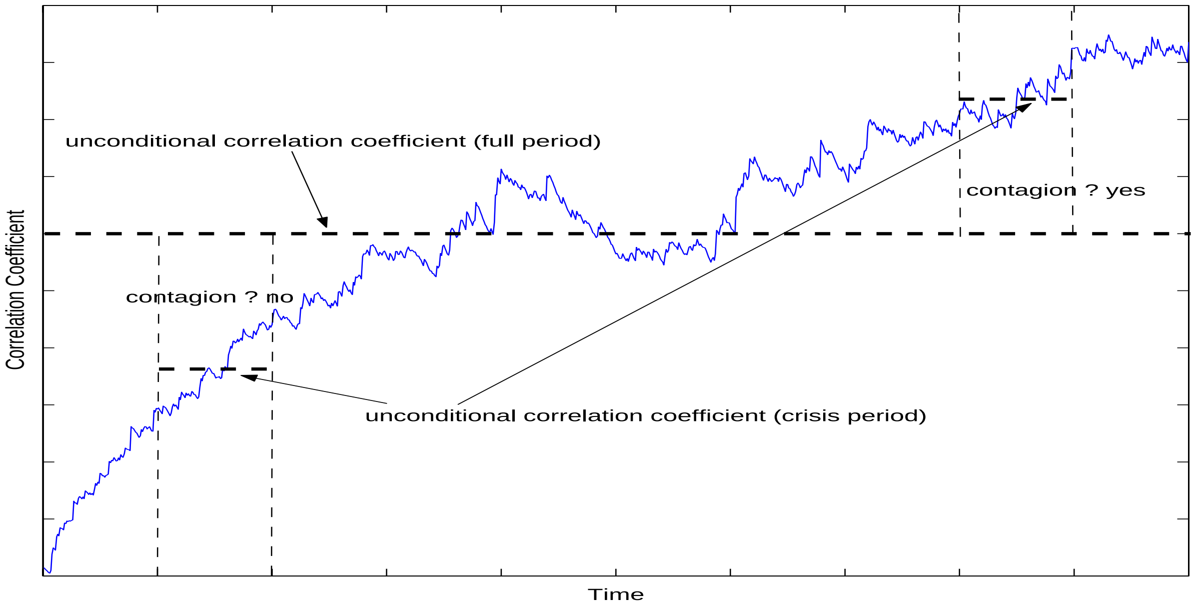

Figure 3.3: *Simulated Correlation process (1)*

period, markets are only interdependent (see Forbes and Rigobon, 2002).

We now explain that this constant (unconditional) correlation assumption can lead to false conclusions when correlations are not constant but time-varying in nature. We give three different examples of how a time-varying correlation can severely bias test results for contagion.

First, assume the correlation increases steadily in the time-period under investigation. Averaging such a trend can lead to the non-rejection of the null hypothesis of no contagion when the crisis period is in the beginning of the sample even if there is contagion. Figure 3.3 clarifies this point. Contrary, if there is no contagion but the crisis period is at the end of the sample the test would falsely detect contagion.

Second, assume the correlation is constant in nature but there is a structural break. Similar to the trend example the question whether contagion is found depends crucially on the time location of the crisis period.

Third, if the correlation between two markets varies due to different business-cycles or different periods of capital in- and outflows, correlations can exhibit a cyclical behavior

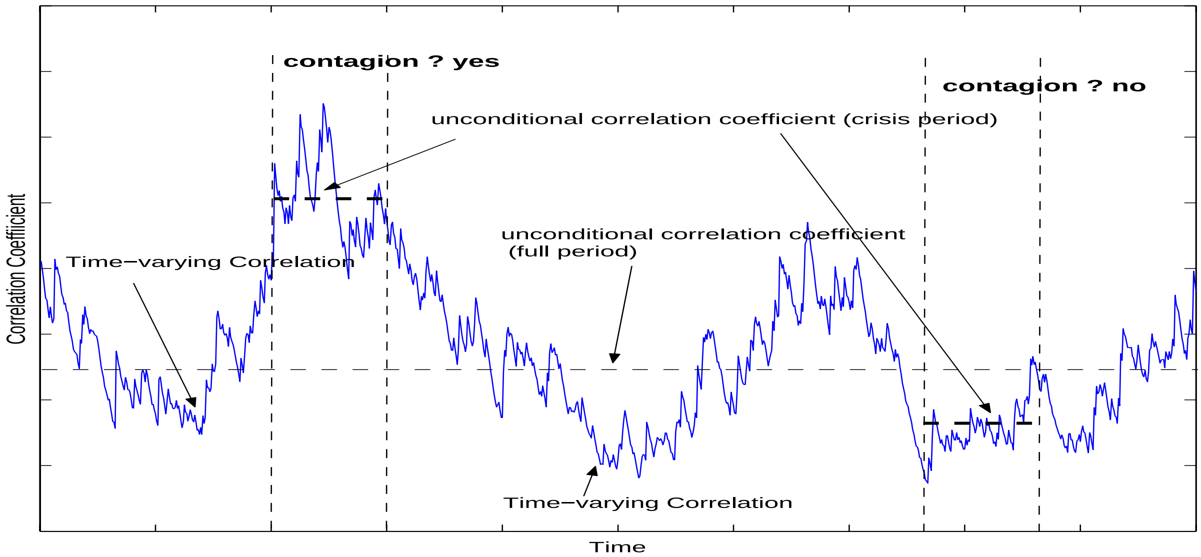

Figure 3.4: *Simulated Correlation process (2)*

with one peak or multiple extremes as depicted in figure 3.4. Again, contagion is falsely detected depending on the assumed time of a crisis.

Given these examples, we conclude that it is crucial to assess the dynamic structure of the correlation in such a test for contagion and that any test assuming a constant correlation can lead to false conclusions. The question now is whether time-varying correlations are an empirical regularity that would justify our argument or whether changing correlations are rather a rare phenomenon. Apart from studies that explicitly test for the constancy of correlations (see e.g. Karolyi and Stulz, 1996, Longin and Solnik, 1995, 2001, and Tse, 2000) and find that correlations are often not constant, we additionally focus on the fact that correlations are time-varying when the ratio of the variances of two markets is changing over time or the regression coefficient β obtained from a regression of y_t on x_t is not constant. The correlation coefficient ρ can be written as

$$\rho = \beta \frac{\sigma_x}{\sigma_y} \tag{3.24}$$

where β is the coefficient of a regression of y on x and σ_x and σ_y are the standard deviations of x and y, respectively.[26] This formulation clarifies that the correlation coefficient varies over time if β or the ratio of the standard deviations are not constant.

Thus, we focus on the estimation of β and show empirically that the assumption of a constant β as made by Forbes and Rigobon (2002) is too restrictive.

Alternatively[27], we generate two random variables with a constant correlation ($\rho = 0.5$) and time-varying volatilities with randomly chosen intervals of potential contagion. In simulations with 1000 iterations, we find that correlations vary with a standard deviation of 0.50 around its mean ($\rho = 0.5$) although the random variables exhibit a constant correlation.

In addition, there are other potential shortcomings of the concept proposed by Forbes and Rigobon (2002). First, the fact that the correlation coefficient is biased in high volatility regimes and the correction for this bias (see Appendix and Forbes and Rigobon, 2002 and Boyer et al., 1999) can be misleading if volatility is an important factor of contagion (see Baig and Goldfajn, 2000). Second, short crisis periods can lead to a test statistic with low power considerably affecting test results (see Dhungey and Zhumabekova, 2001).[28] The third and more general shortcoming is the use of the correlation coefficient which is a symmetric measure, i.e. both series (markets) are affected equally by each other. We argue that it is more adequate to model contagion in an asymmetric way since a virus is transmitted from one subject to another not in a symmetric way. Bae et al. (2002) also argue that the correlation coefficient is a linear measure and therefore not suitable given that contagion is probably a non-linear phenomenon. In the next section, we advocate the use of a different concept for contagion that eliminates these shortcomings.

[26] This formulation can be derived from the following formulas for ρ and β: $\rho = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$ where σ_{xy} is the covariance of x and y and the regression coefficient is $\beta = \frac{\sigma_{xy}}{\sigma_x^2}$

[27] See Appendix for details.

[28] Own simulations confirm this finding (see Appendix for further details).

3.4.3 Mean Contagion

We base our test on a modified model by Corsetti et. al (2001) where a shock in one market becomes a regional or global shock in that it affects at least one other market. Since the original model does not capture any change in the transmission mechanism beyond the transmission which is expected in normal times, we modify the model and allow for a change in the transmission mechanism during a crisis period relative to a tranquil period. The model can be written in the following form:

$$
\begin{aligned}
r_{1t} &= u_{1t} \\
r_{2t} &= \mu_2 + b_1 r_{1t} + b_2 r_{1t} D_{Crisis} + u_{2t}
\end{aligned}
\tag{3.25}
$$

where the parameter μ_2 is the mean, the parameter b_1 captures the normal effect (transmission) of shocks from one market (r_1) to the other market (r_2) and the parameter b_2 indicates whether there is an additional effect (beyond what is normally expected) in a particular crisis period (D_{Crisis} is a dummy variable that is equal to one in the crisis period and zero otherwise). The common factor f_t in equation (3.23) is substituted by the return r_{1t}. Thus, the parameter b_1 measures the comovement of the two markets under investigation caused by common global (regional) shocks and country-specific shocks. Consequently, the parameter b_2 measures the change of this comovement.[29]

The parameter b_1 does not necessarily measure an asymmetric relationship since r_{1t} cannot be assumed to be exogenous. On the contrary, we assume that shocks originating in the crisis country (r_{1t}) within the crisis period are exogenous. This is not a strong assumption if the crisis period is confined to a short period of time.

It is obvious that the above model does not distinguish between global and country-specific shocks. The return shocks contain global (regional) and country-specific shocks.

[29]The term $b_2 r_{1t} D_{Crisis}$ does not only account for the change in the comovement but does also avoid a bias of b_1 if the crisis period contains outliers or constitutes a structural break.

The model given by equations (3.25) can be estimated within a multivariate framework by the Maximum-Likelihood (ML) method or by Ordinary Least Squares (OLS). Time-varying variances (e.g. a GARCH process) can also be included by using the ML method. The null hypothesis of a test for contagion is that there is no increased transmission of shocks from one market to the other in the crisis period: $H_0 : b_2 \leq 0$ against the alternative hypothesis $H_1 : b_2 > 0$. A positive parameter b_2 can be viewed as excess transmission of shocks in the crisis period. In addition, H_0 tests the assumption whether the regression coefficient β is constant. Note that this assumption is made by Forbes and Rigobon (2002) and crucial for their results.

The test based on a regression model is superior to the concept based on the correlation coefficient proposed by Forbes and Rigobon (2002) in various respects: (i) the question whether the correlation coefficient varies and the associated problems need not be considered since we focus on the transmission mechanism (the regression coefficient β) directly, (ii) the test is more conservative than the Forbes and Rigobon test (see simulations in the Appendix) and (iii) this approach can explain contagion by shocks that transmit from one market to the other, i.e. it does model contagion in an asymmetric way.

The next section extends the regression based concept to volatilities since an increased variance of shocks can be contagious per se.

3.4.4 Volatility Contagion

Volatility contagion can be derived from the analysis of the body temperature of a human being during an illness accompanied by fever. In this case, not only the body temperature increases but also its volatility. A typical evolution of the temperature is depicted in figure 3.5. It shows that (i) volatility increases considerably and that (ii) the period of increased volatility can clearly be separated from the normal period (non-illness period). For finan-

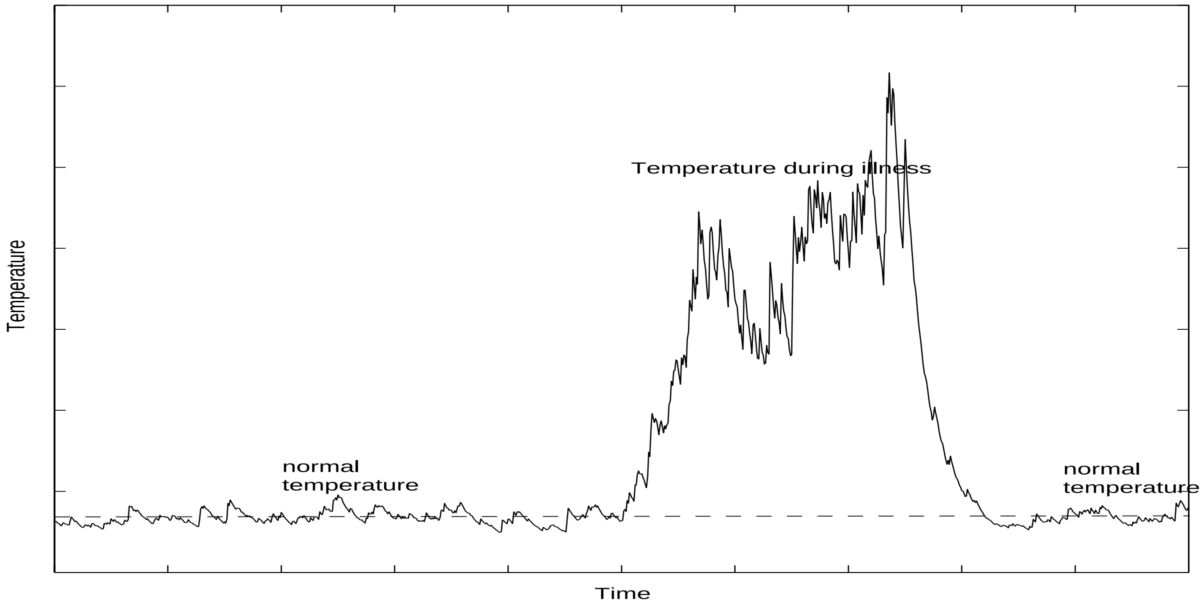

Figure 3.5: *Volatility Contagion*

cial time series one can think of an equivalent phenomenon: In times of increased uncertainty or around crises periods, volatility increases and can be distinguished from normal (less volatile) times. This analogy also matches the stylized fact of volatility clustering.

The notion of volatility contagion is closely related to the spillover literature (e.g. Chakrabarti and Roll, 2002, Edwards, 1998, Lin et al., 1990, Hamao et al., 1990 and King and Wadhwani, 1990 among others). Edwards (1998) tests for volatility contagion in the following framework:

$$\begin{aligned} y_t &= u_t = z_t\sqrt{h_t} \\ h_t &= a + bu_{t-1}^2 + ch_{t-1} + dX_{t-1} \end{aligned} \tag{3.26}$$

where y_t is the time-series under investigation, z_t is a normally distributed random variable with mean zero and variance one and h_t is the conditional volatility of y_t. This typical GARCH(1,1) specification is extended by an exogenous regressor X_{t-1} that can be any variable that affects the volatility. If this exogenous variable has any significant effect on the conditional volatility, there is evidence of *volatility contagion* or a *volatility spillover* (see Edwards, 1998, page 9 and Chakrabarti et al., 2002, page 7). Obviously, this literature does not distinguish between (volatility) contagion and (volatility) spillover.[30] However, we argue that there is a difference between a spillover and contagion: A volatility spillover (correlation in volatility) is a shock that transmits from one market to the other at every time t. In contrast, volatility contagion is an effect which changes the commonly observed volatility spillover (correlation in volatility) during a particular period of time. This notion of contagion is essentially consistent with our definition of mean contagion.

Thus, the model capable of detecting volatility contagion as defined here is given as follows:

$$\begin{aligned} r_{2t} &= u_{2t} = z_{2t}\sqrt{h_{2t}} \\ h_{2t} &= a_0 + b_0 u_{2t-1}^2 + c_0 h_{2t-1} + d_1 r_{1t-1}^2 + d_2 r_{1t-1}^2 D_{Crisis,t-1} \end{aligned} \tag{3.27}$$

where r_{2t} is the return under investigation. The conditional variance h_{2t} is a GARCH(1,1) model with two additional regressors. The first regressor captures the volatility spillover commonly observed (r_{1t-1}^2) and the second regressor reveals any departure from the normal volatility spillover in the crisis period ($D_{Crisis,t-1}$ is equal to one in the crisis period and zero otherwise).

Analogously to the effects of mean contagion, it is also possible that volatility does not increase in the crisis period but decreases. However, allowing the parameter d_2 to be negative would risk a negative volatility in the estimation process of the GARCH model. To avoid this problem, we make use of the exponential GARCH (EGARCH) model (see Nelson, 1991) in which it is not necessary to restrict the parameters to be non-negative. The EGARCH model is given as follows:

$$\begin{aligned} r_{2t} &= u_{2t} \\ h_{2t} &= \exp(c + \theta z_{2t-1} + \gamma\left(|z_{2t-1}| - E(|z_{2t-1}|)\right) + \delta log(h_{2t-1}) + d_1 r_{1t-1}^2 + \\ &\quad + d_2 r_{1t-1}^2 D_{Crisis,t-1}) \end{aligned} \tag{3.28}$$

[30] Furthermore, this literature does not distinguish between *volatility spillover* and *correlation in volatility* as proposed in previous sections.

The parameters to be estimated are a_0, b_0, c_0, d_1 and d_2. The null hypothesis of no volatility contagion is H_0: $d_2 \leq 0$ against the alternative hypothesis $H_1 : d_2 > 0$.

Since a shock coming from another market can have contagious effects on the mean of the return (first moment) and also on the variance of this return (second moment), we model both effects simultaneously.

The full model is given as follows:

$$\begin{aligned} r_{2t} &= \mu_2 + b_1 r_{1t} + b_2 r_{1t} D_{Crisis} + u_{2t} \\ h_{2t} &= \exp(c + \theta z_{2t-1} + \gamma\left(|z_{2t-1}| - E(|z_{2t-1}|)\right) + \delta log(h_{2t-1}) + d_1 r_{1t-1}^2 + \\ &\quad + d_2 r_{1t-1}^2 D_{Crisis,t-1}) \end{aligned} \tag{3.29}$$

This model is different from equation (3.28) only in the mean equation where the transmission of shocks and its crisis behavior is also modeled.

Empirical results are presented in the following section.

3.4.5 Empirical Results

We use daily (close-to-close) continuously compounded stock index returns of eleven Asian stock markets[31]: China, Hong Kong, India, Indonesia, Japan, South Korea, Malaysia, Philippines, Singapore, Taiwan and Thailand. The indices span a time-period of 4 and a half years from April 30th, 1997 until October 30th, 2001. The number of observations is $T = 1176$. All indices are denominated in US dollar. Table 3.19 presents the descriptive statistics for the stock market returns. Tables 3.20 and 3.21 present the unconditional correlations for the whole sample and the crisis periods in Hong Kong and in Thailand, respectively.

Empirical results for mean and volatility contagion based on equations (3.25) during the Hong Kong crisis are presented in table 3.22. The crisis period is October 17th until

[31] The data is provided by Morgan Stanley Capital International Inc. (MSCI) and can be retrieved under www.mscidata.com

Table 3.19: Descriptive Statistics (Asian Markets)

Country	median	mean	std dev	min	max	skewness	kurtosis	autocorr
China	-.0012658	-.0014264	.0266808	-.1444192	.1274350	.1332843	5.758194	0.1563
Hong Kong	.0000065	-.0003807	.0219682	-.1377221	.1600540	.1766766	9.637965	0.0324
India	.0000000	-.0002875	.0188786	-.0732472	.0782044	-.1556417	4.724459	0.0933
Indonesia	-.0011408	-.0018711	.0448212	-.4306072	.2381204	-.7614149	16.392310	0.1307
Japan	-.0008902	-.0002562	.0165281	-.0715819	.1227236	.4743118	6.538290	0.0203
Korea	-.0001647	-.0001964	.0359397	-.2166640	.2688081	.3417927	9.596132	0.1056
Malaysia	-.0008460	-.0008494	.0308228	-.3695134	.2568147	-.5732581	32.532380	0.0868
Philippines	-.0008455	-.0012681	.0215515	-.1035550	.2118705	1.2705870	16.081110	0.2181
Singapore	-.0005161	-.0006736	.0206082	-.1002934	.1551547	.5024695	8.756342	0.1483
Taiwan	-.0009461	-.0007620	.0203578	-.1112802	.0738537	-.0055746	5.252786	0.0391
Thailand	-.0012523	-.0013319	.0294436	-.1488675	.1644259	.6388479	7.172669	0.1638

Table 3.20: Correlations (Asian Markets)

	CHN	HON	INA	IND	JAP	KOR	MAL	PHI	SIN	TAI	THA
China	1.00										
Hong Kong	0.60	1.00									
India	0.18	0.21	1.00								
Indonesia	0.26	0.35	0.10	1.00							
Japan	0.28	0.36	0.15	0.21	1.00						
Korea	0.25	0.29	0.19	0.17	0.26	1.00					
Malaysia	0.27	0.31	0.11	0.33	0.23	0.20	1.00				
Philippines	0.31	0.37	0.14	0.38	0.22	0.21	0.26	1.00			
Singapore	0.44	0.61	0.17	0.46	0.38	0.26	0.39	0.44	1.00		
Taiwan	0.23	0.26	0.12	0.18	0.19	0.20	0.18	0.18	0.28	1.00	
Thailand	0.31	0.38	0.17	0.38	0.25	0.31	0.37	0.40	0.48	0.23	1.00

Table 3.21: Crises correlations (Asian Markets)

	CHN	HON	INA	IND	JAP	KOR	MAL	PHI	SIN	TAI	THA
Hong Kong	0.60	1.00	0.21	0.35	0.36	0.29	0.31	0.37	0.61	0.26	0.38
Hong Kong Crisis	0.81	1.00	0.10	0.63	0.47	0.18	0.58	0.67	0.79	0.11	0.01
Thailand	0.31	0.38	0.17	0.38	0.25	0.31	0.37	0.40	0.48	0.23	1.00
Thailand Crisis	-0.03	0.02	0.19	0.24	0.03	0.25	0.28	0.24	0.14	0.16	1.00

November 17th, 1997 (see Forbes and Rigobon, 2002).

Table 3.22: Mean and Volatility Contagion (Hong Kong Crisis)

	c	γ	θ	δ	μ_2	b_1	b_2	d_1	d_2
China	0.10174	0.30943	-0.076752	0.71659	-0.13619	0.77365	-0.19461	0.015031	-0.010849
	(0.77477)	(4.1096)	(-2.3721)	(6.6458)	(-2.2994)	(21.917)	(-3.2474)	(2.9304)	(-2.3926)
India	-0.081655	0.27026	-0.099753	0.88191	0.046420	0.20106	-0.14924	0.00077691	-0.00085451
	(-1.8637)	(6.5139)	(-3.3056)	(27.375)	(0.93839)	(7.0404)	(-3.4153)	(0.32479)	(-0.32971)
Indonesia	-0.073562	0.24892	-0.068447	0.95136	-0.12802	0.38009	0.18738 **C**	0.0043056	-0.0034404
	(-1.6984)	(2.1948)	(-2.3890)	(27.533)	(-1.5155)	(5.4939)	(1.1114)	(1.1031)	(-0.99744)
Japan	-0.063601	0.10038	-0.053620	0.97087	-0.052726	0.26759	-0.082709	0.0018866	-8.7079e-005
	(-2.5647)	(3.3229)	(-2.5258)	(70.950)	(-1.2206)	(9.6129)	(-1.3846)	(1.7673)	(-0.062903)
Korea	-0.051907	0.089199	-0.027907	0.98967	-0.024912	0.56473	-0.54548	0.00094654	0.0022297 **C**
	(-2.1093)	(2.7987)	(-2.0001)	(237.94)	(-0.33216)	(9.6875)	(-2.3929)	(1.0538)	(1.9735)
Malaysia	-0.10456	0.15453	-0.054406	0.98223	-0.033209	0.26680	0.14400 **C**	0.0055116	-0.0040969
	(-4.6793)	(5.9750)	(-2.7546)	(105.19)	(-0.66569)	(6.3751)	(1.3576)	(2.5952)	(-2.1005)
Philippines	-0.073043	0.14487	-0.11401	0.95842	-0.12160	0.27739	0.042211 **C**	0.0048123	-0.0043817
	(-1.2663)	(1.5711)	(-3.8996)	(55.504)	(-2.1761)	(5.8951)	(0.42018)	(2.9207)	(-2.8735)
Singapore	-0.033307	0.050437	-0.064581	0.96213	-0.078554	0.51781	-0.082064	0.0057966	-0.0036485
	(-0.64514)	(0.63290)	(-2.4592)	(35.359)	(-1.9273)	(16.141)	(-1.1234)	(2.0231)	(-1.5833)
Taiwan	-0.026239	0.10935	-0.087414	0.95723	-0.098633	0.26104	-0.13553	-0.00048356	-0.00014907
	(-0.96222)	(2.3624)	(-3.4363)	(39.349)	(-1.8258)	(8.7262)	(-0.95736)	(-0.41834)	(-0.12056)
Thailand	-0.063445	0.16277	-0.020449	0.94978	-0.13973	0.50910	-0.33007	0.0068014	-0.0035349
	(-1.7061)	(2.2294)	(-1.0058)	(40.124)	(-2.1297)	(10.898)	(-2.0286)	(3.4421)	(-1.9133)

$r_{1t}(\text{Hong Kong}) = u_{1t}$
$r_{2t}(\text{Country})_i = \mu_2 + b_1 r_{1t} + b_2 r_{1t} D_{Crisis} + u_{2t}$
$h_{2t}(\text{Country})_i = \exp\left(c + \theta z_{2t-1} + \gamma(|z_{2t-1}| - E(|z_{2t-1}|)) + \delta log(h_{2t-1}) + d_1 r^2_{1,t-1} + d_2 r^2_{1,t-1} D_{Crisis,t-1}\right)$

Crisis origin is Hong Kong and the crisis period is October 17th, 1997
until November 17th, 1997
t-values in parenthesis
C denotes contagion in mean or volatility

Table 3.23 presents results based on the same model only alternatively assuming that the crisis origin is the Thailand stock market. We assume the crisis period to begin on July 2nd and to last until September 2nd, 1997 (approximately one month before the Hong Kong crisis occurs).

Figure 3.6 is a plot of the prices of the first 180 trading days for all Asian markets.[32] The common downward movement is especially evident for the period after $t = 60$ (Thailand crisis) and for the period around $t = 130$ (Hong Kong crisis).

Estimation results of mean contagion for the Hong Kong crisis (table 3.22) indicate that the transmission mechanism of shocks (parameter b_2) is not constant for most countries. Results show that there is a decreasing transmission mechanism of shocks for China, India, Japan, Korea, Singapore, Taiwan and Thailand. An increase in the transmission mechanism of shocks from Hong Kong to the other markets can be found for Indone-

[32]The initial prices are set to 100 in order to show the common evolution of the prices.

Table 3.23: Mean and Volatility Contagion (Thailand Crisis)

	c	γ	θ	δ	μ_2	b_1	b_2	d_1	d_2
China	0.020874	0.24077	-0.10694	0.87642	-0.16861	0.27582	-0.30553	0.0011605	0.0077967 **C**
	(0.19941)	(1.7717)	(-3.6345)	(7.5288)	(-2.4150)	(5.720)	(-3.4251)	(0.70761)	(1.7313)
Hong Kong	-0.065238	0.21464	-0.10776	0.90760	-0.067618	0.27182	-0.41820	0.0018403	-0.0014799
	(-1.5169)	(6.0388)	(-3.2571)	(20.550)	(-1.3447)	(10.784)	(-7.4516)	(1.1310)	(-0.33263)
India	-0.028248	0.24706	-0.11007	0.87505	0.0027945	0.076066	-0.012966	-0.0018628	-0.0041274
	(-0.50308)	(5.7332)	(-3.4726)	(22.430)	(0.056166)	(4.2038)	(-0.21706)	(-1.4909)	(-1.1994)
Indonesia	-0.060750	0.34592	-0.10096	0.91557	-0.17247	0.38437	-0.29925	0.0025752	-0.0062681
	(-0.92418)	(4.1993)	(-3.0227)	(27.816)	(-1.8982)	(7.6645)	(-3.1314)	(1.5828)	(-1.1688)
Japan	-0.092533	0.15883	-0.057212	0.96071	-0.051183	0.13332	-0.072198	0.00057673	-0.0023068
	(-3.5875)	(5.1582)	(-2.3963)	(48.247)	(-1.1556)	(6.7664)	(-7.3278)	(0.84305)	(-1.0867)
Korea	-0.014660	0.14526	-0.078655	0.95004	-0.061823	0.38369	-0.39227	0.0023418	-0.012180
	(-0.38122)	(3.3534)	(-3.4070)	(46.588)	(-4021.8)	(8.5932)	(-6.1362)	(2.3521)	(-2.7322)
Malaysia	-0.11512	0.17297	-0.061342	0.99031	0.052216	0.21801	-0.25209	0.00098919	-0.00014449
	(-4.9391)	(6.7283)	(-2.8060)	(96.991)	(0.82521)	(7.1143)	(-4.0029)	(0.96012)	(-0.063537)
Philippines	-0.13035	0.29811	-0.15266	0.89146	-0.10644	0.27278	-0.24507	0.0039714	0.0044827 **C**
	(-2.9880)	(3.2415)	(-3.3618)	(12.624)	(-2.2840)	(6.2695)	(-2.7000)	(1.2977)	(0.68572)
Singapore	-0.10882	0.22241	-0.084153	0.91480	-0.058604	0.27830	-0.33945	0.0028419	0.00022997 **C**
	(-2.9058)	(2.9752)	(-3.5965)	(16.630)	(-1.2915)	(11.441)	(-7.3151)	(1.2672)	(0.052197)
Taiwan	-0.0084177	0.099611	-0.10030	0.95178	-0.068903	0.16379	-0.056934	-0.00095753	0.0017585 **C**
	(-0.28852)	(2.2818)	(-4.4087)	(50.880)	(-1.2419)	(7.5196)	(-1.1774)	(-2.3108)	(1.3035)

$r_{1t}(\text{Thailand}) = u_{1t}$
$r_{2t}(\text{Country})_i = \mu_2 + b_1 r_{1t} + b_2 r_{1t} D_{Crisis} + u_{2t}$
$h_{2t}(\text{Country})_i = \exp\left(c + \theta z_{2t-1} + \gamma(|z_{2t-1}| - E(|z_{2t-1}|)) + \delta log(h_{2t-1}) + d_1 r^2_{1,t-1} + d_2 r^2_{1,t-1} D_{Crisis,t-1}\right)$

Crisis origin is Thailand and the crisis period is July 2nd, 1997
until September 2nd, 1997
t-values in parenthesis
C denotes contagion in mean or volatility

sia, Malaysia and the Philippines. The increase is close to 50 percent for Indonesia and Malaysia and approximately 20 percent for the Philippines. The total transmission ($\hat{b_1}+\hat{b_2}$) of shocks is close to zero for India and Korea which could be interpreted as immunity to shocks originating in Hong Kong.

Volatility contagion (parameter d_2) can only be found for Korea. Interestingly, the volatility transmission decreased in all other countries in the crisis period. However, a decreased volatility transmission does not necessarily imply lower volatility.

The results for a crisis assumed to originate in Thailand can be summarized as follows: the transmission mechanism (table 3.23) is not constant in the crisis period for all countries except India and Taiwan. Again, there are countries that can be viewed to be isolated from shocks coming from Thailand in the crisis period which can be assessed by the total transmission ($b_1 + b_2$) of shocks. Here, China and Korea exhibit values of $b_1 + b_2$ close to zero. For Hong Kong, Malaysia and Singapore the total transmission is clearly negative in the crisis period.

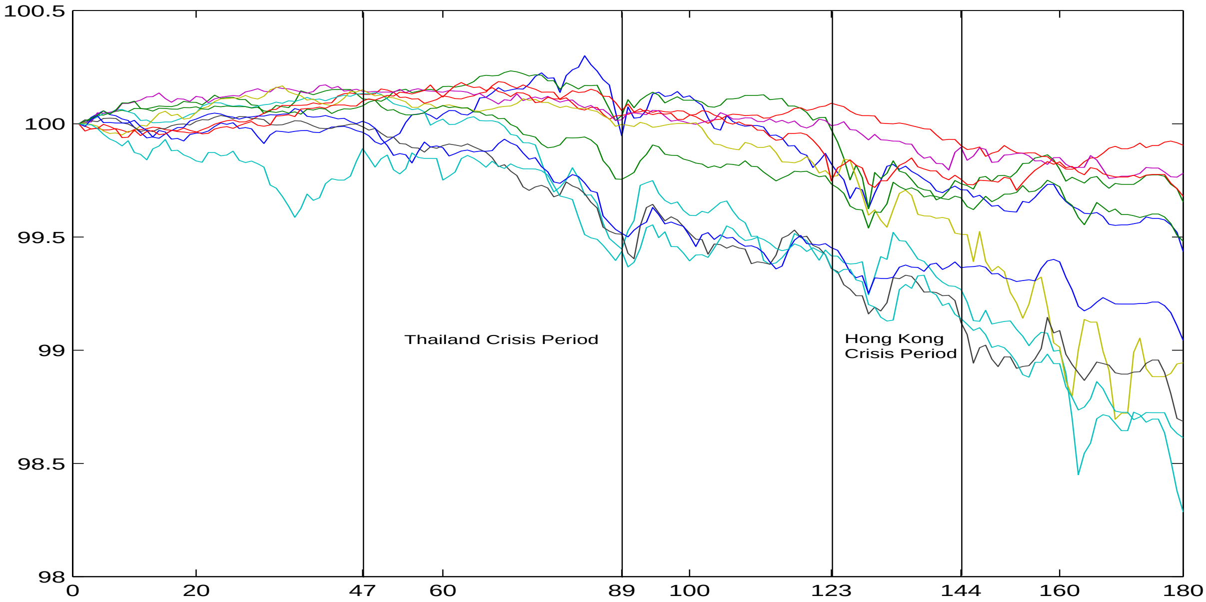

Figure 3.6: *Asian markets, first 180 trading days*

Volatility contagion can be found for China, the Philippines, Singapore and Taiwan.

Different results for mean and volatility contagion are not surprising since an increased effect of shocks to the mean of a return does not necessarily need to increase the impact on the volatility and increased shocks to the volatility do not need to increase the influence on the underlying returns.

Summarizing the results reveals that there is mean and volatility contagion during the Asian crisis. In addition, the assumption of a constant transmission mechanism must be rejected for almost all countries.

The finding of mean and volatility contagion is counter to the results obtained by Forbes and Rigobon (2002) who find contagion for the Hong Kong crisis in Indonesia, Korea and the Philippines only for an unadjusted correlation coefficient. Using the adjusted correlation coefficient proposed by Forbes and Rigobon, no contagion is found in any Asian market analyzed.

The remaining question is now whether the degree of interdependence influences the degree of contagion. Since this issue is closely related to the medical sciences, the following example aims to clarify the relation between interdependence and contagion.

A disease affecting a human being can only be contagious by direct or indirect contact with the disease. Thus, the disease can only be transmitted from one person to another if their existence is interdependent to some extent, e.g. they live or work together. In analogy to this, it would be natural to assume that contagion among financial markets depends on the degree of interdependence (e.g. measured by the correlation coefficient or the transmission mechanism). In particular, the higher the interdependence, the higher the contagious effect and the lower the interdependence, the lower the contagious impact. However, the empirical results do not support such a hypothesis. High exposures to shocks in normal times do not have any systematic influence on the occurrence or the degree of contagion in crises times.

3.4.6 Conclusions

We have argued that existing tests for contagion based on the correlation coefficient can lead to false conclusions if the correlation is changing over time, the crisis period is too short or volatility is a factor of contagion per se.

Empirically, we find that the transmission mechanism of shocks to the mean is not constant and mainly decreases in the crisis period relative to a tranquil period. This is also true for the transmission of volatility which is not constant and often decreases in the crisis period.

Whereas mean contagion is more frequent in the Hong Kong crisis, volatility contagion is more common in the Thailand crisis period. These findings suggest that markets first transmitted volatility within the months before the Hong Kong crisis and then started to transmit shocks also to the mean making the Hong Kong crisis more severe.

Future research could focus on the investigation of the decreasing transmission mechanism since even a lower percentage of raw shocks coming from another market in a crisis period can cause contagion if the magnitude of these shocks increases considerably.

3.4.7 Appendix

3.4.7.1 Simulations: Correlations are time-varying

We transform two independent, normally distributed random variables z_1 and z_2 to two variables that exhibit a constant correlation ($\rho = 0.5$) and changing volatilities generated with a GARCH(1,1) process ($h_{i,t} = 0.01 + 0.05\epsilon^2_{i,t-1} + 0.90h_{i,t-1}$ for $i = 1, 2$):

$$\begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} = (\boldsymbol{D}_t \boldsymbol{R} \boldsymbol{D}_t)^{-1/2} \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} \tag{3.30}$$

where $\boldsymbol{D}_t$ is a diagonal (2×2) matrix with the root of the conditional variances on the diagonal and $\boldsymbol{R}$ is given as follows:

$$\boldsymbol{R} = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix} \tag{3.31}$$

Due to the changing volatilities the series ϵ_{1t} and ϵ_{2t} do not exhibit a constant correlation if short subperiods (i.e. crisis periods) are analyzed. Simulations with randomly selected intervals of 2.5 percent of the full period (T=1000) show that correlations vary with a standard deviation of 0.50 around its mean ($\rho = 0.5$).

3.4.7.2 Simulations: Contagion Tests

According to the data length that is commonly used (e.g. see Forbes and Rigobon, 2002) we simulate the return series with $T = 500$ and $T = 1000$. The beginning of the crisis period is randomly chosen within the full period every run of the simulation. The length of the crisis period is assumed to be 2.5 (as in Forbes and Rigobon, 2002 for example), 5 and 10 percent of the full period (T).

Table 3.24: Simulation Results - Contagion Tests

percent of rejection ($t = 500$)									
crisis period	$0.025T$			$0.05T$			$0.10T$		
b_2	$b_2 \leq 0$	BG*	FR**	$b_2 \leq 0$	BG	FR	$b_2 \leq 0$	BG	FR
0.0	0.099	0.314	0.296	0.095	0.380	0.366	0.106	0.411	0.394
0.1	0.125	0.447	0.430	0.134	0.559	0.542	0.181	0.664	0.651
0.2	0.186	0.570	0.555	0.263	0.730	0.718	0.389	0.841	0.833
0.3	0.274	0.682	0.671	0.435	0.847	0.841	0.636	0.941	0.940

percent of rejection ($t = 1000$)									
crisis period	$0.025T$			$0.05T$			$0.10T$		
b_2	$b_2 \leq 0$	BG	FR	$b_2 \leq 0$	BG	FR	$b_2 \leq 0$	BG	FR
0.0	0.105	0.383	0.365	0.101	0.412	0.397	0.106	0.439	0.421
0.1	0.144	0.559	0.544	0.184	0.672	0.660	0.247	0.776	0.767
0.2	0.260	0.727	0.717	0.404	0.854	0.846	0.591	0.940	0.938
0.3	0.442	0.849	0.844	0.668	0.948	0.947	0.871	0.990	0.990

$\star$ Baig and Goldfajn (1999)
$\star\star$ Forbes and Rigobon (2002)

10.000 runs
$r_{1t} = u_{1t}$
$r_{2t} = 0.25u_{1t} + b_2 u_{1t} D_{Crisis} + u_{2t}$

level of significance is 5 percent
Note that estimates of b_2 are unbiased (estimates are not tabulated)

The data-generating process is as follows:

$$
\begin{aligned}
r_{1t} &= u_{1t} \\
r_{2t} &= b_1 u_{1t} + b_2 u_{1t} D_{Crisis} + u_{2t}
\end{aligned}
\tag{3.32}
$$

where $u_{it} = z_{it}\sqrt{h_{it}}$ for $i = 1, 2$ and $z_{it} \sim N(0,1)$ and $h_{it} = 0.01 + 0.05u_{it-1}^2 + 0.9h_{it-1}$ is the conditional variance of series i. Thus, the returns r_1 and r_2 exhibit time-varying variances and are correlated through the transmission of shocks from market 1 to market 2.

To assess the power of the contagion test, we compute a scenario with no contagion ($b_1 = 0.25$, $b_2 = 0$) and with contagion ($b_2 = 0.1$, 0.2, 0.3). The level of significance is 5 percent.

We also compute the test proposed by Forbes and Rigobon for this model in its unconditional (corrected for heteroscedasticity) and in its conditional (not corrected for heteroscedasticity) version. Simulation results are presented in table 3.24.

Simulations of 10.000 runs (only the mean equations are estimated) show some interesting results: (i) the test proposed by Forbes and Rigobon falsely rejects the null hypothesis of no contagion (type-1 error) in approximately 40 percent of all simulation runs. The approach based on the transmission mechanism only leads to a type-1 error in less than 10 percent, (ii) the Forbes Rigobon test (also the Baig and Goldfajn approach) have very low power for short crises periods (2.5 and 5 percent of the total number of observations) (see also Dhungey and Zhumabekova, 2001). Note that the low power is also a regularity for the concept based on the transmission mechanism. However, estimates are unbiased which means that conclusions need not to be based on the t-values as this is necessary for the Baig and Goldfajn (1999) and the Forbes and Rigobon (2002) test procedure.

3.4.7.3 Correction for heteroscedasticity

Dividing a given sample into two sets so that the variance of x_t is lower in the first group (l) and higher in the second group (h), Forbes and Rigobon (2002) define the following correlation coefficients ρ^h and ρ^l:

$$\rho^h = \frac{\sigma^h_{xy}}{\sigma^h_x \sigma^h_y} \tag{3.33}$$

$$\rho^l = \frac{\sigma^l_{xy}}{\sigma^l_x \sigma^l_y} \tag{3.34}$$

Defining $1 + \delta = \frac{\sigma^h_{xx}}{\sigma^l_{xx}}$, the following formula is obtained:

$$\rho^h = \rho^l \sqrt{\frac{1+\delta}{1+\delta(\rho^l)^2}} \tag{3.35}$$

A simple example

Assume the following volatilities in the high and low variance regime for x: $\sigma^h_{xx} = 0.2$ and $\sigma^l_{xx} = 0.1$

This yields $1 + \delta = \frac{0.2}{0.1} = 2$ and $\delta = \frac{0.2}{0.1} - 1 = 1$.

Also assume that the volatility of y is the same in both regimes ($\sigma^h_{yy} = \sigma^l_{yy} = 0.1$) and that the covariances for the low variance regime and the high variance regime are $\sigma_{xy}^l = 0.05$ and $\sigma_{xy}^l = 0.1$. Thus, we get the following correlation coefficients for the two different regimes:

$\rho^l = 0.05/(\sqrt{0.1 \cdot 0.1}) = 0.5$ and $\rho^h = 0.1/(\sqrt{0.2 \cdot 0.1}) = 0.71$

However, the corrected correlation coefficient is given by

$$\rho^{h^*} = 0.5\sqrt{\frac{2}{1 + 1 \cdot 0.5^2}} = 0.63$$

which is clearly smaller.

Chapter 4

Concluding Remarks

The subject of this study is the econometric modeling and estimation of interdependent relations among financial markets.

The first part of this work is a discussion of univariate GARCH models that describe the empirical regularities of volatility clustering and the persistence of volatility. It is also discussed whether volatility reacts asymmetrically to positive and negative shocks. Finally, a graphical approach to visualize the persistence and the asymmetry of volatilities is described and evaluated.

The second part is an extension of univariate GARCH models to multivariate GARCH models that provide time-varying volatility estimates and also time-varying covariance and correlation estimates. These correlations are measures of symmetric interdependencies of the returns of two markets. The analysis of the persistence and the asymmetry of the time-varying correlations is essentially not different to an investigation of these characteristics for volatilities in econometric terms. However, in comparison to the existence of economic explanations for the findings of persistent and asymmetric reactions of conditional volatilities, there is no generally accepted interpretation regarding the persistence or the asymmetry of time-varying correlations. It is also important to emphasize that the analysis of the asymmetry of time-varying correlations focusses on different effects between simultaneously negative and simultaneously positive shocks. It does not analyze asymmetries of the influence of shocks of different financial markets on their conditional correlations.

The third part extends the analysis of interdependence in two ways: First, interdependence is modelled in an asymmetric form and second, not only the dependence of returns is examined but also their volatilities. The last part of this work discusses theoretical models of contagion and introduces appropriate estimation frameworks to analyze whether there is increased interdependence in financial crises or not.

The discussion of the multivariate GARCH models shows that the existing models depend on rather strong restrictions to guarantee well-defined results. These strong restrictions suggest that future research is still necessary and that other measures of dependence should also be considered. The correlation coefficient cannot only be viewed as a problematic measure due to the difficulties associated with the estimation of multivariate GARCH models, it is also problematic that the correlation coefficient is a linear measure which is not necessarily adequate for all possible market reactions. Furthermore, although the knowledge of the relation between correlations and volatilities is very important for the theory of finance, the results obtained so far provide only mixed evidence.

The analysis of spillovers as carried out in this study is mainly an investigation of shock transmissions without the examination of the source of these shocks. It is implicitly assumed that the spillovers originate in one of the (two) markets under investigation. In order to obtain more results regarding the functioning of the financial system and the processing of (global) information, it is necessary to extend the number of markets under investigation and to model the interaction of all these markets simultaneously. The last section of the third chapter points in this direction but is only an initial step to reach this goal. This section has further shown that the correlation coefficient is an inappropriate measure to describe forms of co-relation or comovement for short periods of time or periods of increased volatility.

In the introduction, we raised questions regarding the adequate architecture of the financial system. The last section showed that interdependence can have negative impacts on the markets involved in periods of financial turmoil or stress. The finding that negative shocks and increased volatility are transmitted from some countries to other countries is a logical result of interdependence. Thus, the analysis of the degree of interdependence is the basis for further investigations regarding the question whether the degree of interdependence can or should be reduced in certain periods of time without causing persistent harm to the functioning of financial markets in other periods of time, e.g. periods of average volatility.

Moreover, it could be interesting to investigate and measure the degree of interdependence not only between pairs of countries but also for many countries simultaneously. It is well possible that the degree of interdependence is too high between some countries and much too low between other countries resulting in a non-optimal equilibrium. The degree of interdependence could be explained with the degree of diversified portfolios that are held by investors. High interdependence between a given number of markets can be due to the high diversification between these markets but also due to the low diversification with other countries.

Unfortunately, the econometric focus of this study did not provide the scope to discuss theoretical explanations for interdependence and its consequences in more detail. However, the study made clear that the estimation of the evolution of the interdependence of financial markets is an essential stage in order to approach theoretical solutions to this problem and to draw policy related conclusions.

Chapter 5

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