Signals and the Structure of Societies

D i s s e r t a t i o n
zur
Erlangung des akademischen Grades
Doktor der Philosophie
in der Philosophischen Fakultät

der Eberhard Karls Universität Tübingen

vorgelegt von

Roland Mühlenbernd

aus

Lemgo

2013

Gedruckt mit Genehmigung der Philosophischen Fakultät
der Eberhard Karls Universität Tübingen

Dekan: Prof. Dr. Jürgen Leonhardt

Hauptberichterstatter: Prof. Dr. Gerhard Jäger
Mitberichterstatter: Prof. Dr. Simon Huttegger

Tag der mündlichen Prüfung: 23. Juli 2013

TOBIAS-lib, Tübingen

Contents

Acknowledgements

Networks have played an important role not only in this dissertation but also in my life. It has therefore been an honor to be a node in the larger network of collaborators, all of whom have supported me in completing this dissertation. Although there is no way to make an exhaustive list, I want to thank Anton Benz, Armin Buch, Stefano Demichelis, Christian Ebert, Fritz Hamm, Elliott Wagner and all my colleagues for support and enlightening discussions, especially in the fields of game theory and pragmatics, language evolution and change, network theory and phenomena from sociolinguistics. I should also mention our very supportive and lively department (Special thanks to Christl!). My thanks go to them not only for their professionalism but also for their congeniality. In that regard, I would also like to thank my students, especially those from courses like SLANG and PENG, courses that both inspired some of the ideas seen herein.

In closing, special thanks go to three persons that were not only the most prominent supporters of this work, but also good friends. First, I want to thank my doctoral advisor Gerhard Jäger. It is a privilege to have an advisor who has not only a research record of astonishing diversity, but also an excellent nose for new research directions. I am, in particular, thankful for i) the encouragement to break new ground with my research and ii) the freedom to do so. Both of these factors resulted in an excellent work environment. Second, I want to thank Michael Franke for a productive and engaging collaboration, especially during his stay in Tübingen. He was an excellent mentor and the reason for much of my thesis coming into being. Thanks for your encouragement, Micha! And finally, I want to thank Jason Quinley for amazing support in matters both professional and personal. Our track of job-related and private journeys through Europe and the United States was extensive and full of exciting times. Thanks for all your help!

Introduction

> *"A name is a spoken sound significant by convention... I say 'by convention' because no name is a name naturally but only when it has become a symbol."*
>
> *Aristotle 1984, De Interpretatione*

> *"[w]e can hardly suppose a parliament of hitherto speechless elders meeting together and agreeing to call a cow a cow and a wolf a wolf."*
>
> *Russell 1921, The Analysis of Mind*

My Ph.D. project investigates the question of how linguistic conventions arise and stabilize. To that end I will examine the emergence of conventional signaling in the spirit of Lewis (1969) who sought an answer to the question: how can linguistic meaning emerge as a convention among language users, assuming no previous explicit agreements? I would like to elucidate this puzzle here. Consider the following three propositions:

1. A convention is exclusively determined by an explicit agreement
2. Human language is a system that consists of linguistic conventions
3. Language, even in the simplest case, is needed to make agreements

Standing by these three propositions, we would come to a paradox: language is needed for language to emerge. As a consequence, one of these propositions must be wrong. While philosophers like Quine took the view that the second proposition must be wrong and thus language cannot be conventional, Lewis paved another way for a solution by claiming that the first of these three propositions is wrong: conventions are not exclusively determined by explicit agreements; conventions, and particularly conventional linguistic meaning, can arise without such agreements. He illustrated his claim by constructing and providing a model that formalizes the way conventions can arise without explicit agreements, namely by the emergence of regularities in (linguistic) behavior.

The Character of Conventions

What characterizes a convention? A natural starting is that it is determined by an *explicit agreement*. Promises, contracts or rules accepted and followed by all parties among whom the convention holds are examples. In other words: a convention is characterized by its origin. This characterization is e.g. according to Quine (1966), who once asked: "What is convention when there can be no thought of convening?" In fact, there are many conventions that originated by explicit agreement (e.g. to drive on the right or left lane of the street), even for linguistic meaning (e.g. to christen a person or a ship, to invent a word for a new product), but apart from that, conventions in language use are usually not the offspring of agreements once made. This is supported by Russell (1921), who, while dealing with the puzzle of the evolution of linguistic meaning, once argued: "we can hardly suppose a parliament of hitherto speechless elders meeting together and agreeing to call a cow a cow and a wolf a wolf."

Thus while philosophers like Quine questioned the fact that linguistic meaning is based on conventions, Lewis questioned the way these philosophers defined conventions. Should a convention really be characterized by its origin? He denied this claim and suggested that while the origins of a convention can be multifarious, there are other conditions that uniquely characterize a convention. Lewis's definition of a convention looks roughly as follows:

Definition 0.1 (Convention)**.** *A convention C is a behavioral regularity among a population with the following properties:*

1. common knowledge*: every member conforms to C and every member expects everybody else to conform to C*
2. arbitrary*: C is not determined by nature, but by society*
3. alternative*: C has at least one excluding (and probably competing) alternative C', it could be replaced with*

To give an example: a behavioral regularity that fulfills these conditions is *to drive on the right lane of the street*: i) it is common knowledge among the population that everybody behaves according to it, ii) it is not determined by nature and iii) it has an alternative: driving on the left lane of the street. Thus according to Lewis's definition, it's a convention. A counterexample that fulfills at least the first condition is *to breathe*: i) it is common knowledge that every living being breathes. But condition 2 and 3 are violated: ii) it is determined by nature: from birth on we breathe; and iii) the only excluding alternative would be not to breathe which is not a serious alternative.

Let's see if the conditions hold for linguistic meaning: to use a specific verbal expression for describing a matter i) is used by all members of the same language area; and it is also known by all members that everybody else inside this language area uses it; ii) is not given by nature, but in general completely arbitrary; and iii) has practically an unlimited number of alternatives, namely any other verbal expression a society could conventionally use for that matter. Thus according to the three conditions linguistic meaning is a convention.

Furthermore, as you can see in the conditions in Definition 0.1, the origin of a convention does not play a role. Conventions can arise with or without an explicit agreement. Dealing with the puzzle of language evolution, Lewis was particularly interested in the question of *how* conventions can arise *without* explicit agreements. He argues that all it needs for conventions to arise is a specific process, an "exchange of manifestations of a propensity to conform to a regularity." (Lewis 1969, pages 87-88). To give a formal framework for modeling such a process Lewis introduced a game-theoretic model, called the *signaling game*.

Signaling Games & Signaling Systems

A signaling game is a game-theoretic model that Lewis introduced to explain the emergence of linguistic meaning as a convention. It basically models a communication situation between two players, a sender and a receiver, where the sender has the role of encoding an information state with an expression, here called message, then the receiver decodes the message with an interpretation state. To communicate most successfully, sender and receiver should use compatible coding/encoding patterns. In terms of a game these are also called contingency plans or strategies. Furthermore, a compatible pair of strategies depicts an unique map between information states and messages, and can be seen as a meaning allocation. By assuming that the message is a verbal expression, such a strategy pair represents linguistic meaning.

For (linguistic) meaning to arise among sender and receiver, both have to find a compatible strategy pair, also called a *signaling system*. Once they have (directly or indirectly) agreed on one, not only is successful communication guaranteed, but also such a signaling system constitutes a strict *Nash equilibrium* (c.f. Myerson 1991). This roughly means that both participants do not have any interest in changing their strategies, so long as they know that the other player would not change either. This implies indirectly that such a signaling system fulfills the first condition of a convention according to Definition 0.1, when considering both players as the population in question.[1]

In the standard settings, a signaling game has at least two information states and two messages. This leads to the realization that there is more than just one signaling system. Consequently, it guarantees that a signaling system also fulfills the third condition of Definition 0.1 since there are alternatives. In addition, a mapping from information state to message and from message to interpretation state is not defined in the game, thus each mapping is possible; there is no definite or nature-given mapping integrated, thus the second condition is also fulfilled. This means that a signaling system is a communicative behavior that i) determines (linguistic) meaning and ii) fulfills all conditions of a convention among both players according to Definition 0.1.

Let us consider a concrete example: when a person wishes to affirm or negate something, *'shaking one's head'* or *'nodding'* are messages, *yes* and *no* are two information states. One possible sender strategy s is: i) 'nodding' for *yes*, and ii) 'shaking one's head' for *no*. A receiver strategy r is: i) construing 'nodding' with *yes* and ii) construing 'shaking one's head' with *no*. Then the pair $\langle s, r\rangle$ is a compatible pair of strategies and has the following meaning attribution: 'nodding' means *yes* and 'shaking one's head' means *no*. In addition, the strategy pair $\langle s, r\rangle$ is a signaling system. And it is a Nash equilibrium: none of the participants would have an interest to change the behavior as long as they know that the communication partner behaves according to this convention; e.g. if the sender knows that the receiver construes 'nodding' with *yes*, why should she change her strategy

[1]The fact that signaling systems are strict Nash equilibria implies that they are *evolutionary stable* in terms of a population-based and evolutionary perspective, as will be shown in Chapter 2.

and e.g. shake her head for *no*? In conclusion, the strategy pair $\langle s, r \rangle$ is a convention since also the other conditions are fulfilled: it is obviously not given by nature and there are alternative conventions, e.g. to behave exactly the other way around and form a strategy pair for which 'nodding' means *no* and 'shaking one's head' means *yes*. Another term for such a convention is a *signaling convention*.

To analyze how conventional linguistic meaning arises, I examine the way in which signaling systems and accompanying signaling conventions arise. More precisely, I analyze the minimalistic version of a signaling game, called the *Lewis game*. This game has only two information states and two messages (like the 'nodding'/'shaking one's head' example), which implies that it bears only two signaling systems. Both players have to agree on one of both in order to communicate successfully and establish a linguistic convention. In the Lewis game both signaling systems are equally good. In the following paragraph I will argue that such parity is an exception rather than the rule.

Horn's Rule

A signaling game contains a set of messages. In general, it is highly idealized to assume that the players have the same preference for all messages; they may be biased for or against one or the other. A sender could be biased against a message just because it is harder to produce, to pronounce; or the message is just longer than the alternative; a receiver could be biased against a message because the interpretation is harder or takes longer. Such negative biases can be modeled as message costs. Furthermore, it can be assumed that information states are not equally frequent topics of communication. This case can be modeled by assigning so-called prior probabilities to information states to define which one is more or less frequent.

By applying signaling games with uneven probabilities for information states and different costs for messages, it is possible to analyze a linguistic phenomenon that is known as the *division of pragmatic labor* (Horn 1984), also known as *Horn's rule*. This rule says that, according to evolutionary forces optimizing linguistic usage, it is expected that i) the unmarked message is used for the frequent information state and ii) the marked message is used for the infrequent one. By considering a signaling game that has unequal prior probabilities for information states and different message costs, exactly one of both possible signaling systems of such a game depicts Horn's rule. Therefore, I will call such a game a *Horn game*. In addition to the Lewis game, I will analyze the Horn game to get insights into what circumstances support or weaken the emergence of linguistic conventions according to Horn's rule.

Psychology & Sociology

It is important to note that models of meaning evolution may make precise the exact conditions under which semantic meaningfulness can emerge. This may occur if the habit of using a particular sound or gesture to communicate a certain meaning arises and is amplified and sustained in a population. The *psychology* of language users plays a pivotal role here: among other things, the particulars of agents' perception and memory, their disposition to adapt their behavior, including the extent to which they make rational choices, will heavily influence the way linguistic behavior evolves over time.

To find insights for the puzzle of the evolution of linguistic meaning and how specific conventions may arise without explicit agreement, I analyze *repeated* signaling games. To model a process depicting conventions arising because of regularities in behavior, like coordinating on strategies that constitute a signaling system, I will factor in agents in repeated interactions that update their behavior in dependence of previous plays: they behave according to *update dynamics*. I will introduce different types of update dynamics for repeated signaling games that are the most well-regarded in recent research (for an overview see Huttegger and Zollman, 2011). In that regard I will present a study where I analyze two kinds of *learning dynamics*: *reinforcement learning* and *belief learning*.

It is not only the psychology of language users that plays a role in determining the time course and outcome of evolutionary processes. The *sociology* of language-using populations is also a key factor. For that reason many evolutionary models make explicit assumptions about the interaction patterns within a population of language users, such as who interacts with whom (c.f. Nettle 1999, for an early model). Different interaction structures may, for instance, lead to different predictions about uniformity or diversity of language (c.f. Zollman 2005; Wagner 2009; Mühlenbernd 2011).

Consequently, another focus in my research is on the interaction structure of populations of agents. For that purpose I will introduce basic concepts of network theory and apply them in my experiments to i) model more realistic interaction structures for multi-agent networks and ii) have the means to analyze the resulting structural patterns. The goal is not only to analyze, what kind of conventions evolve and how they are spread among the society, but also to determine how specific environmental features, in terms of network properties, influence the temporal and spatial evolution of language conventions.

Overview

The thesis is structured as follows: in Chapter 1 I will give the definition of a signaling game central to this thesis and its related concepts. Further, I will introduce the two specific types of signaling games that are objects of study in this work: the *Lewis game* and the *Horn game*. In Chapter 2 I will give an overview of the models and literature related to *update dynamics* in combination with signaling games; these are basically classified as *evolutionary dynamics*, *imitation dynamics* and *learning dynamics*. In Chapter 3 I will introduce basic concepts from *network theory* needed for employing and analyzing network structures in subsequent chapters. In Chapter 4 I will present experiments of multi-agent populations. These agents i) interact on a toroid lattice structure, ii) communicate via Lewis or Horn game, and iii) update their behavior by using learning dynamics. I will further analyze results like the influence of agents' internal as well as environmental circumstances on the resulting spatial structure of conventions among the society. In Chapter 5 I will present experiments of simulation runs similar to those of Chapter 4, but this time agents are placed on more realistic network structures, so-called small-world networks. Here, the analysis includes the examination of the relationship between a) the structural features of the network and its members and b) properties of the agents' learning behavior and social integration. In Chapter 6 I will give a summary of the most relevant results of this study and a final conclusion.

Chapter 1

Signaling Games

He said to his friend, "If the British march
By land or sea from the town to-night,
Hang a lantern aloft in the belfry arch
Of the North Church tower as a signal light,
One if by land, and two if by sea;
And I on the opposite shore will be,
Ready to ride and spread the alarm
Through every Middlesex village and farm,
For the country folk to be up and to arm."

Longfellow 1863, Paul Revere's Ride

"Conventions are like fires: under favourable conditions, a sufficient concentration of heat spreads and perpetuates itself. The nature of the fire does not depend on the original source of heat. Matches may be the best fire starters, but that is no reason to think of fires started otherwise as any the less fires."

Lewis 1969, Convention

I would like to begin with Lewis's first example of a signaling game, drawn from Longfellow's (1863) poem *Paul Revere's Ride* which depicts a scene in the American revolution. For the imminent battles of Lexington and Concord between Americans and British forces, it was of essential importance that the Americans were previously informed of how the British forces arrived: by land or by sea. The early information should allow the American

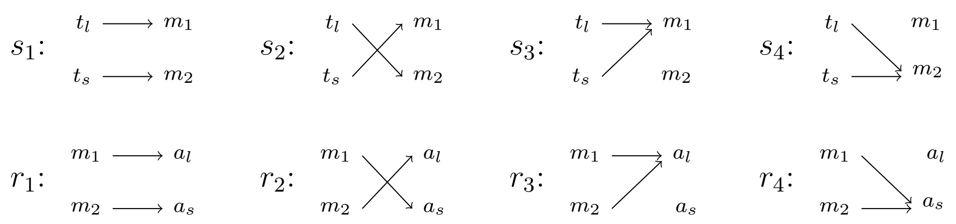


Figure 1.1: Lewis's original example: the sexton's and Revere's admissible contingency plans.

messenger Paul Revere sufficient time to warn others in preparation for the imminent attack. Revere's plan was to let the sexton of the Old North Church keep watch in the tower at night, from where he had a wide view over the area. When identifying enemy forces, the sexton had to signal Paul Revere if the Redcoats are coming by land or by sea. His signals were to hang out one or two lanterns in the belfry.

In this example there are two information states and two possible signals. Let's call the information states t_l and t_s (for *land* and *sea*), the signals m_1 and m_2 (one or two lanterns). Thus the sexton should have a kind of code; Lewis called this an *admissible contingency plan*: a one-to-one allocation between information states and signals. Accordingly, Revere should also have an admissible contingency plan, a one-to-one allocation between the signals and appropriate actions a_l and a_s (a_i is the appropriate response to information state t_i). As depicted in Figure 1.1 s_1, s_2, s_3 and s_4 are the four possible admissible contingency plans the sexton could have, and r_1, r_2, r_3 and r_4 are Revere's four possible admissible contingency plans.

How successful communication is (in a probabilistic way) depends on the combination $\langle s, r\rangle$ of admissible contingency plans, also called *pure strategies*. E.g. if the sexton behaves according to s_1, but Revere according to r_3, communication succeeds only if the redcoats are coming by sea. Thus, communication is successful in one of two possible information states; with the assumption that all states are equiprobable, the expected communicative success probability is .5. In the same way $\langle s_2, r_2\rangle$ would lead to an expected communicative success probability of 1, $\langle s_1, r_2\rangle$ to 0. Table 1.1 depicts the values of expected communicative success probabilities of all possible combinations along these 4×4 admissible pure strategies.

The strategies s_3 and s_4 as well as r_3 and r_4, also called *pooling strategies*, seem to be negligible because they don't transfer any information in a distinctive way. This is visible in Table 1.1 because each pair of contingency

	r_1	r_2	r_3	r_4
s_1	1	0	.5	.5
s_2	0	1	.5	.5
s_3	.5	.5	.5	.5
s_4	.5	.5	.5	.5

Table 1.1: Expected communicative success for all combinations of the sexton's and Revere's four contingency plans.

plans with at least one pooling strategy leads to an expected communicative success probability of .5, which means that successful communication is a matter of chance. Nevertheless, these are some of the possible strategies within the formal framework introduced in the coming section, and therefore they are not to be neglected. The combinations of particular interest are the ones that allow for perfect communication, the strategy pairs $\langle s_1, r_1 \rangle$ and $\langle s_2, r_2 \rangle$, also called *signaling systems*.

Combinations that form signaling systems are also called *coordination equilibria* (Lewis 1969) because they constitute (strict) Nash equilibria in such a coordination game. These combinations not only ensure perfect communication, but also distinctively attribute meanings to signals: they establish a meaning function between information and signal. E.g. the signaling system $\langle s_2, r_2 \rangle$ makes common knowledge that one lantern means *attack by sea* and two lanterns mean *attack by land*.

Note that this example shows us how communication succeeds *with* previous explicit agreements. Revere and the sexton have to agree on one of the two coordination equilibria beforehand, either on $\langle s_1, r_1 \rangle$ or $\langle s_2, r_2 \rangle$. And for those who know the poem, it is known that they agreed on $\langle s_1, r_1 \rangle$.

Let's consider the research question, I initially presented from Lewis, namely "How can linguistic meaning emerge as a convention among language users, by assuming no previous explicit agreements?". It would be interesting to apply the framework of a signaling game to analyze how coordination equilibria that form *signaling conventions* can evolve *without* previous explicit agreements; thus I want to find answers for the following questions: what leads individuals to coordinate and therefore behave in a conventional way without previous explicit agreement, or even more, without any prior knowledge? And how can we adapt these concepts to explain conventions in whole populations instead among two individuals, as modeled in the standard game? There are a lot of open questions to be answered, and the framework of a signaling game seems to be a promising starting point for analyzing the puzzle of the evolution of conventional linguistic meaning in human societies.

1.1 Definition of a Signaling Game

Let's formalize the model of a signaling game that is analyzed in this thesis. A simple version, called the *vanilla signaling game*, is defined in the following way:

Definition 1.1 (Vanilla Signaling Game). $VSG = \langle (S,R), T, M, A, P, U \rangle$ *is a* vanilla signaling game *with a 'first move' player S, called the sender, a 'second move' player R, called the receiver, a set of information states T, a set of messages M, a set of interpretation states A, a prior probability function over information states* $P \in \Delta(T)$ *and an utility function* $U : T \times A \to \mathbb{R}$

In the Lewisean spirit a signaling game is grounded on a coordination game that shapes the preferences in the utility table. To define a signaling game in this spirit and, additionally, to extend the game for allowing *message costs* that diminish utility values, I impose the following conditions:

- the number of states $|T|$ and interpretation states $|A|$ is equal and each information state $t_i \in T$ has its companion interpretation state $a_j \in A$ marked by the index, thus: $|T| = |A|$ and $\forall t_i \in T\ \exists a_j \in A : i = j$
- the messages have cost values, defined by a cost function $C : M \to \mathbb{R}$
- the utility function is based on a coordination game with value 1 for coordination and 0 for miscoordination; it is decreased by the costs of the used message; thus it is defined as $U : T \times M \times A \to \mathbb{R}$

With these extensions and conditions the signaling game considered as standard in the realm of this thesis is defined as follows:

Definition 1.2 (Signaling Game). $SG = \langle (S,R), T, M, A, P, C, U \rangle$ *is a* signaling game *with a 'first move' player S, called the sender, a 'second move' player R, called the receiver, a set of information states T, a set of messages M, a set of interpretation states A, a prior probability function over information states* $P \in \Delta(T)$*, a cost function* $C : M \to \mathbb{R}$ *with*

$\forall m \in M : 0 \leq C(m) < 1$[1] *and a utility function $U : T \times M \times A \to \mathbb{R}$. Furthermore $|T| = |A|$ and each element in T has a counterpart in A, denoted by the same index: $\forall t_i \in T\ \exists a_j \in A : i = j$. The utility function is based on a coordination game and affected by the cost value, given as:*

$$U(t_i, m, a_j) = \begin{cases} 1 - C(m) & \text{if } i = j \\ 0 - C(m) & \text{else} \end{cases}$$

A round of a signaling game is played in the following way: nature N picks an information state $t \in T$ with probability $P(t)$ which the sender S wants to communicate to the receiver R. For that purpose the sender chooses a message $m \in M$ and sends it to the receiver. Now the receiver R construes the message m with an interpretation state $a \in A$. If t and a correspond to each other, coordination and therefore communication is successful, as expressed by the standard utility function given in Definition 1.2. In this sense, both participants have aligned preferences and an interest for successful communication for gaining a maximal utility value.

Such a signaling game is a *dynamic game* because both players act in sequence. I want to illustrate this fact by Lewis's example: the sexton has to signal if redcoats are coming by land or by sea by hanging one or two lanterns in the belfry. *After* the sexton has sent a signal, Revere can perceive the signal and construe it appropriately. Generally, such a game with two information states and two messages depicts Lewis's example and is therefore called the *Lewis game*. This game is the most simple case of a signaling game, and it forms the object of study in this work. It is defined in the following way:

Definition 1.3 (Lewis Game)**.** *A Lewis game is a signaling game $SG = \langle (S,R), T, M, A, P, C, U \rangle$ with the following settings:*

- $T = \{t_l, t_s\}$, $M = \{m_1, m_2\}$, $A = \{a_l, a_s\}$
- $P(t_l) = P(t_s) = .5$
- $C(m_1) = C(m_2) = 0$

[1] van Rooij (2008) mentioned that costs should be *nominal*, thus never exceed the benefit of successful communication and that according to Blume et al. (1993) we're then still in the realm of *cheap talk* games.

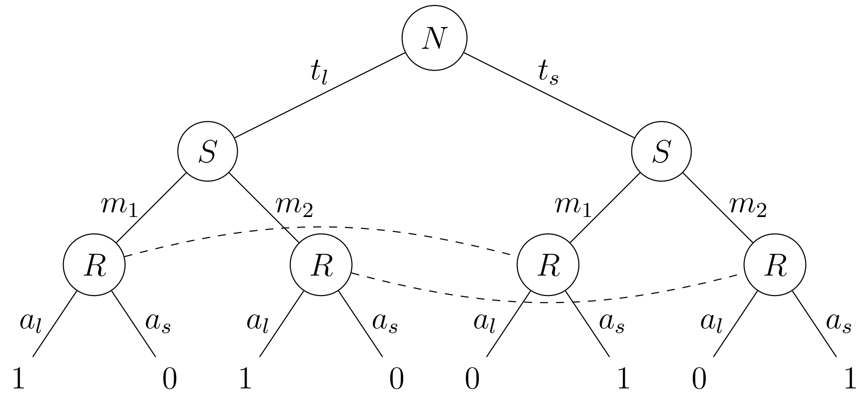


Figure 1.2: The extensive form of the Lewis game.

A Lewis game has two information states, messages and interpretation states. Both information states are equiprobable, and both messages are costless. U is the predefined utility function giving one point if communication was successful and 0 if not. The possible paths for such a game can be displayed as a tree, also known as the *extensive form game*, as depicted in Figure 1.2. It highlights the dynamic sequential nature of the game.

Because of the facts i) that both players act asynchronously and therefore in a dynamic way and ii) that the receiver doesn't know the information state nature has picked, a signaling game SG belongs to the class of *dynamic games of incomplete information* (also *dynamic Bayesian games*, see e.g. Ely and Sandholm 2005). In Figure 1.2 the receiver's incomplete information is displayed by the dashed lines which denote that he cannot differentiate the connected situations from each other because he only knows the message he receives, but not the information state the sender has. Each leaf of the tree indicates the resulting utility value both players obtain for the corresponding path, indicating if communication was successful or not by 1 point or 0 points, respectively.

1.2 Strategies, Equilibria & Signaling Systems

Lewis's contingency plan is formally a total function $s : T \to M$ for the sender and a total function $r : M \to A$ for the receiver. These functions are pure strategies. In the following, the function s is called *pure sender strategy* and the function r is called *pure receiver strategy*. For Lewis a contingency plan is admissible if and only if it is a one-to-one allocation, formally a bijective function. Such an admissible contingency plan is also called an *equilibrium strategy* and defined as follows:

Definition 1.4 (Equilibrium Strategy). *A pure sender strategy s or pure receiver strategy r, respectively, is called an equilibrium strategy if and only if it is a bijective function.*

Figure 1.1 (page 10) depicts all different pure sender and receiver strategies for the Lewis game, where only s_1, s_2, r_1 and r_2 are equilibrium strategies. To explain why those strategies are called equilibrium strategies, I first have to introduce the *expected utility* $EU(s, r)$, the value you would expect on average by playing a pure sender strategy s against a pure receiver strategy r. It is defined as follows:

$$EU(s,r) = \frac{\sum_{t \in T} U(t, r(s(t)))}{|T|} \tag{1.1}$$

An equilibrium strategy is defined by the concept of a *Nash equilibrium*, a strategy combination for which no player can score better by solely deviating from it. Formally, a Nash equilibrium is defined in the following way:

Definition 1.5 (Nash Equilibrium). *Given a static 2-player game $G = \{P_1, P_2, \mathbf{S_1}, \mathbf{S_2}, U\}$ with player P_1 having a set of strategies $\mathbf{S_1}$ and player P_2 having a set of strategies $\mathbf{S_2}$ and a utility function $U : \mathbf{S_1} \times \mathbf{S_2} \to \mathbb{R}$, a pair of strategies $\langle s_1, s_2 \rangle$ with $s_1 \in \mathbf{S_1}$, $s_2 \in \mathbf{S_2}$ forms a* Nash equilibrium *if and only if the following two conditions hold:*

- $U(s_1, s_2) \geq U(s_1, s') \ \forall s' \in \mathbf{S_2}$
- $U(s_1, s_2) \geq U(s', s_2) \ \forall s' \in \mathbf{S_1}$

For the analysis of signaling conventions, also the *strict Nash equilibrium* plays an important role. Here a player scores worse by deviating from such an equilibrium. A strict Nash equilibrium is defined as such:

Definition 1.6 (Strict Nash Equilibrium). *Given a static 2-player game $G = \{P_1, P_2, \mathbf{S_1}, \mathbf{S_2}, U\}$ with player P_1 having a set of strategies $\mathbf{S_1}$ and player P_2 having a set of strategies $\mathbf{S_2}$ and a utility function $U : \mathbf{S_1} \times \mathbf{S_2} \to \mathbb{R}$. Then a pair of strategies $\langle s_1, s_2 \rangle$ with $s_1 \in \mathbf{S_1}$, $s_2 \in \mathbf{S_2}$ forms a* strict Nash equilibrium *if and only if the following two conditions hold:*

- $U(s_1, s_2) > U(s_1, s') \ \forall s' \in \mathbf{S_2} \setminus \{s_2\}$
- $U(s_1, s_2) > U(s', s_2) \ \forall s' \in \mathbf{S_1} \setminus \{s_1\}$

Table 1.1 (page 11) depicts the *expected utility table* for all possible strategy combinations of the Lewis game. Let's think of this table as a utility function of a static 2-players game, where the sender is player P_1 with a strategy set $\mathbf{S_1} = \{s_1, s_2, s_3, s_4\}$ and the receiver is player P_2 with a strategy set $\mathbf{S_2} = \{r_1, r_2, r_3, r_4\}$. Then only the strategy combinations $\langle s_1, r_1 \rangle$ and $\langle s_2, r_2 \rangle$ constitute strict Nash equilibria according to Definition 1.6. Such an equilibrium of a coordination game is called a *coordination equilibrium* which can only be formed by equilibrium strategies, as Lewis (1969) was able to show. In addition, a coordination equilibrium in a signaling game forms, what Lewis called a *signaling system*, which can be defined in the following way:

Definition 1.7 (Signaling System)**.** *For a signaling game SG, the strategy combination $\langle s, r \rangle$ is a* signaling system *if and only if it is a strict* Nash equilibrium *of the expected utility table of SG.*

For a signaling system $\langle s, r \rangle$ of a signaling game SG there is a corresponding interpretation state $a_j \in A$ for each information state $t_i \in T$ that guarantees successful communication: $a_j = r(s(t_i))$ if and only if $i = j$. In addition, each signaling system has the same allocation of information state to interpretation state. Signaling systems only differ in the way messages are used. Taken as a whole, a signaling system has the following properties:

1. Acting according to a signaling system guarantees successful communication.

2. A signaling system uniquely attributes messages to corresponding pairs of information state and interpretation state.

3. A signaling system forms a strict Nash equilibrium on the expected utility table of the signaling game.

These properties reveal a combination of interesting features, namely that according to 1.) a signaling system is the most efficient way to communicate, according to 2.) a signaling system ascribes a meaning to each message and according to 3.) a signaling system incorporates a high degree of stability since players have no incentive to change their behavior by considering expected utilities. Thus, a signaling system reveals efficiency and stability by ascribing meanings to messages.

1.3 The Horn Game

In communication situations, where you have the choice between alternative expressions, your decision of which one to choose may hang on all sort of things, but, all else being equal, it may depend on the expression's functional bias. For instance, an expression is biased if it is easier to acquire, produce or receive in comparison to its alternative. This is to say that you would prefer the biased expression. For a signaling game, where the messages constitute expressions, this fact can be incorporated by integrating *message costs* in a way that the more biased a messages is, the smaller is the value of its costs. In this sense, van Rooij (2008) mentioned that "...costly messages can be used to turn games in which the preferences are not aligned to ones where they are." (page 268). By expecting that a language user is biased according to one or the other expression in almost any case, the Lewis game is an exception or a highly idealized version since the message costs of both messages are equal.[2] Thus, factoring in unequal message costs seems to be a plausible generalization.

Furthermore, what makes the Lewis game also quite specific is the fact that both information states have the same prior probability. If the prior probability describes the sender's preference for or against a specific information state, then the sender is completely unbiased about the information state she wants to communicate. It can be expected that in general a sender is at least a little bit biased for one or the other information state she wants to communicate. Consequently, I claim that a non-equal prior probability for all information states is rather the norm than the exception.

A second interpretation is that the prior probability describes the *occurrence probability* of an information state in comparison to the others. Then the Lewis game would predict that both information states' occurrences have the same probability. E.g. Zeevat and Jäger (2002) suggest that the prior probability of the contents can be derived from corpus-based statistical relations between forms and meanings. As you can imagine, even in this case, a flat prior probability would be a glaring exception.

Note that a game with two information states, two messages, two interpretation states, unequal prior probabilities and uneven message costs constitutes a framework that has been recently applied to analyze a linguistic phenomenon known as *the division of pragmatic labor* (Horn 1984), also known as *Horn's rule*. This rule says that the unmarked message is used

[2] In fact, the message costs are 0, but even if the costs are the same value c $(0 < c < 1)$ for both messages, the game would be strategically equivalent.

for the prototypical information state, while the marked message is used for the rare information state. Instances of this phenomenon are results of evolutionary forces of language change and become apparent in various linguistic constructions. Jäger (2004) gave the following examples to present instances of Horn's rule in different linguistic domains:

(1) a. John went to church/jail (prototypical)
  b. John went to the church/jail (literal)

(2) a. I need a new driller/cooker.
  b. I need a new drill/cook.

Example (1a.) describes a prototypical situation, namely that John is a prisoner in jail or going to the masses organized by the church, respectively. On the other hand (1b.) describes a more specific situation, namely that John literally went to the jail or church building. In either case, the longer and therefore marked version is (1b.) because of the additional 'the'. It describes a specific situation, whereas the shorter version (1a.) describes the prototypical and more frequent situation.

Example (2) depicts an interesting fact. The suffix -er has two opposed roles: e.g. while the drill is a tool, the suffix changes its meaning to a person who uses this tool. On the other hand the cook is a person and the suffix changes it to a tool. Thus, the additional suffix did not arise because of a specific function like "if X is a person, X + -er is a tool" or the other way around. The rule is more like: "X and X + -er are related things and X is more frequent or prototypical than X + -er." This shows that the simpler form is used for the more prototypical meaning. And Jäger (2004) was able to show a quantitative hint supporting this fact. His results for Google searches of the expression of (1a.) "went to church" got 88,000 hits, whereas "went to the church" got 13,500 hits. He observed similar results for "cook" (712,000 hits) and "cooker" (25,000 hits).

As I initially indicated, a signaling game as defined in Definition 1.2 (page 12) with strongly unequal prior probabilities and uneven message costs drives the foundation for the analysis of Horn's rule. If t_f denotes the frequent information state and t_r the rare one, then the property of uneven frequencies can be modeled by uneven prior probabilities in the following way: $P(t_f) > P(t_r)$. Further, the distinction between an unmarked message m_u and a marked message m_m can be made by different message costs: $C(m_u) < C(m_m)$. Because of the relation between Horn's rule and a signaling game with these properties, I'll call such a game *Horn game*,

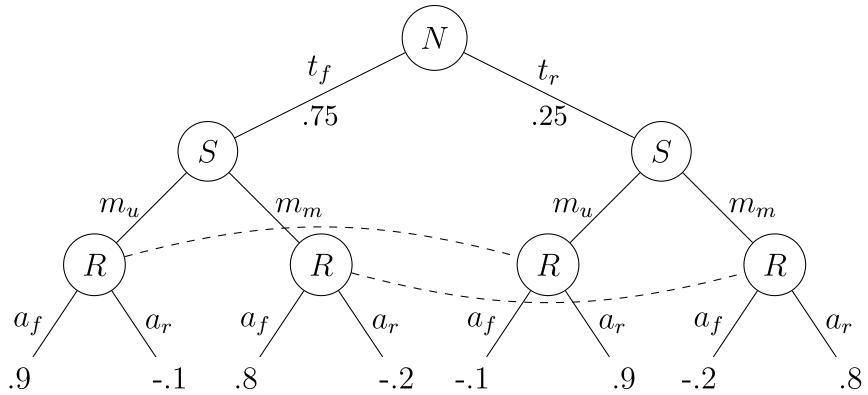


Figure 1.3: The extensive form of a Horn game with $P(t_f) = .75$, $P(t_r) = .25$, $C(m_u) = .1$ and $C(m_m) = .2$.

defined as follows:

Definition 1.8 (Horn Game). *A Horn game is a signaling game $SG = \langle (S, R), T, M, A, P, C, U \rangle$ with the following settings:*

- $T = \{t_f, t_r\}$, $M = \{m_u, m_m\}$, $A = \{a_f, a_r\}$
- $P(t_f) > P(t_r)$
- $C(m_u) < C(m_m)$

An instance HG of a Horn game according to Definition 1.8 can be given as follows: $HG = \langle (S, R), \{t_f, t_r\}, \{m_u, m_m\}, \{a_f, a_r\}, P, C, U \rangle$, whereby S is a sender and R is a receiver, $P(t_f) = .75$, $P(t_r) = .25$, $C(m_u) = .1$, $C(m_m) = .2$ and U is the predefined utility function that returns 1 if communication was successful and 0 if not, minus the message costs, respectively. The extensive form game of such a Horn game is given with Figure 1.3.

Let's take a look at the different strategy profiles and the resulting expected utility table this game offers. Since a Horn game has the same number of information states, messages and interpretation states as the Lewis game, it has also four sender strategies and four receiver strategies with the same mapping, as depicted in Figure 1.4: s_h and r_h depict the sender's and receiver's behavior according Horn's rule, also called *Horn strategies*. s_a and r_a are the strategies that depict the exact opposite behavior, also called *anti-Horn strategies*. In addition, s_s and r_s depict behavior according to avoidance of complexity. The sender does not use the more complex

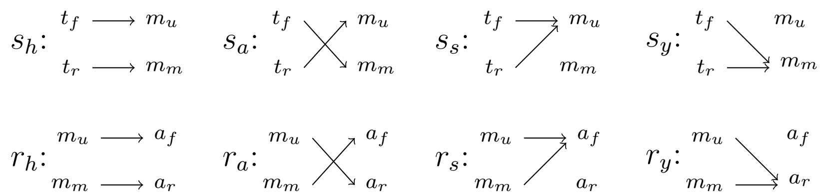


Figure 1.4: All pure strategies of the Horn game.

	r_h	r_a	r_s	r_y
s_h	.875	-.125	.625	.125
s_a	-.175	.825	.575	.075
s_s	.65	.15	.65	.15
s_y	.05	.55	.55	.05

Table 1.2: Expected utilities for all combinations of pure sender and receiver strategies for a Horn game with $P(t_f) = .75$, $C(m_u) = .1$ and $C(m_m) = .2$.

marked message, and the receiver does not take into account the interpretation state that matches the rare or less general information state. These strategies are called *Smolensky strategies*.[3] To round things out, s_y and r_y are called *anti-Smolensky strategies* since they depict the opposite behavior of the Smolensky strategies.

The resulting table of expected utilities is depicted in Table 1.2. The *EU* table of the Horn game has two strict Nash equilibria $\langle s_h, r_h \rangle$ and $\langle s_a, r_a \rangle$ which correspond to both signaling systems, just like the *EU* table of the Lewis game (Table 1.1, page 11). But while the *EU* table of the Lewis game has four non-strict Nash equilibria, the *EU* table of the Horn game has only one non-strict Nash equilibrium, namely $\langle s_s, r_s \rangle$. This is a first indicator that Horn's rule is a rule with exceptions since at least in this simple model even the opposite of Horn's rule, the anti-Horn strategy, forms a strict Nash equilibrium and therefore a situation that can be stable among players.[4] And as I will show in subsequent analysis, even the Smolensky strategy can be a notable alternative for a signaling convention that stabilizes inside a population.

[3]The Smolensky strategy is named in reference to Tesar and Smolensky (1998), who applied this strategy as a starting strategy for players in their simulations with regards to the assumption that previous generations possibly weren't aware of the more complex marked message.

[4]Indeed a convention obeying Horn's rule cover the majority of examples you can find in the literature. But you can also find examples depicting a conventions obeying the opposite of Horn's rule. Such examples are given e.g. by Schaden (2008).

It is important to note that the solution concept like the Nash equilibrium is not satisfactory since it admittedly expresses a degree of stability for the case when both players are involved in a signaling system. It fails to reveal how players come to it in the first place. In order to analyze evolutionary paths leading to signaling conventions I will reconsider signaling games as *repeated games* and combined with *update dynamics*: players play a game repeatedly and update their behavior by taking into account the performances or results of previous plays.

1.4 Repeated Games

As Franke (2009) puts it: "It is not the game but a *solution concept* that describes actual reasoning and/or decision making." (page 13). We already saw the Nash equilibrium as a solution concept, where its most common interpretation is a *steady state* in players' behavior by repeatedly playing the game. What crucially lacks at this point is a description of the mechanism that leads players to a situation of choices that forms a Nash equilibrium. Update dynamics can fill this gap: the usage of information of previous plays to optimize behavior. It can be shown for a range of games that highly unsophisticated update dynamics can lead to and remain in an optimal and stable situation of choices; i.e. a Nash equilibrium. The most common classes of such update dynamics in combination with repeated signaling games are *evolutionary dynamics*, *imitation dynamics* and *learning dynamics* which will be introduced and discussed in Chapter 2.

This mechanism gives a solution concept at hand that can explain the adjustment of players' strategies to form a signaling convention, not by previous explicit agreements, but by small steps of (unsophisticated) optimization. This might be in light of experiences of previous interaction among the community or, for the simplest case, with the direct participant. As I showed in the previous sections, there are more Nash equilibria than only the ones that form signaling systems. By finding update dynamics that leads the players to only *strict* Nash equilibria, we would have solution concepts explaining the emergence of signaling systems. Further, such dynamics seem to be explanatory solution concepts in the spirit of Lewis's idea: a process of conventionalization without explicit agreements.

To give a first idea about how update dynamics work, let's think about a really simple update rule for a repeatedly played 2-player signaling game, called *myopic best response* that is defined as follows: make a random choice as first move and then play the *best response*[5] against what the other player has played in the last round. Now let's say that players play contingency plans as strategies and try to maximize their expected utility, thus their game table is the *EU* table (e.g. Table 1.1 on page 11 for the Lewis game and Table 1.2 on page 20 for a Horn game). With the myopic best response rule, players can escape non-strict Nash equilibria. E.g. let's say that the players play the Lewis game and start with strategy $\langle s_3, r_4 \rangle$, i.e. a non-strict Nash equilibrium of the Lewis game. In the next round both players can switch to any other strategy because all receiver strategies are a best response to s_3 and all sender strategies are a best response to r_4. Thus, it is highly probable that at one point they would switch to equilibrium strategies. At this point, they are trapped in a strict Nash equilibrium and will never change their strategies. A similar argumentation is given for the Horn game. As a consequence, according to myopic best response, players wouldn't finally stick in the non-strict Nash equilibrium $\langle s_s, r_s \rangle$, but either in $\langle s_h, r_h \rangle$ or $\langle s_a, r_a \rangle$.

Note that the players might be trapped in a cycle of miscommunication. For the Lewis game it would be $\langle s_1, r_2 \rangle \rightarrow \langle s_2, r_1 \rangle \rightarrow \langle s_1, r_2 \rangle \rightarrow \ldots$, for the Horn game $\langle s_h, r_a \rangle \rightarrow \langle s_a, r_h \rangle \rightarrow \langle s_h, r_a \rangle \rightarrow \ldots$. It means that both players played the exact opposite equilibrium strategies last round and want to adapt to the other player by switching to the other equilibrium strategy. Since both players are doing this simultaneously, they will never reach a signaling system.

Now let's modify the update rule to the rule *probabilistic myopic best response*, defined in the following way: make a random choice as first move; then play the best response against what your participant has played in the last round with probability p; and stay with your old strategy with probability $1 - p$. This modification gives both participants the chance to escape from such a cycle of miscommunication and finally end up in a signaling system with dead certainty.

This example shows how a simple mechanisms of adapting behavior can lead to signaling conventions. There is need neither for a previous explicit agreement, nor for any kind of highly sophisticated rational deliberation. All that is needed is the will of both participants to communicate successfully and the possibility to learn from previous interactions by incorporating former results. And as I was able to show with the last example, integrating

[5]To play the best response means to play the move that maximizes expected utility for a given move of the other player.

probabilistic choices can avoid a situation, where players are trapped in an unwanted situation: a cycle of miscommunication.

1.5 Conclusion

In this chapter I defined the signaling game. I introduced i) the Lewis game, i.e. the simplest variant of a signaling game and ii) the Horn game, i.e. an extended variant and a formal model to analyze Horn's division of pragmatic labor. In either case, the challenge is to explain the emergence of linguistic meaning as a convention without previous explicit agreements. The solution concept of a Nash equilibrium explains the stability of signaling systems since no rational participant has an interest to deviate. This provides a good explanation for the stability of how conventions remain stable once emerged. But the crucial question is still how such conventions emerge in the first place.

The initial example of the Lewis game showed that it works *with* explicit agreement: Revere and the sexton previously agreed on a code in form of a clear information-signal mapping. But how does such a stable situation emerge *without* explicit agreement? With a simple update rule, *myopic best response* for repeatedly played games, I was able to show how players can find such stable situations; and by integrating probabilistic choices, they will finally end up in one signaling system, and that without explicit agreements.

Nevertheless, the *probabilistic myopic best response* rule i) involves an assumption of rationality and ii) were applied for a two-player game. One might ask: is an assumption of rationality necessary to explain the emergence of signaling systems, or are lesser assumptions sufficient? And how does behavior stabilize among more than only two players, namely among a whole population? Furthermore, how can the emergence of the Horn strategy in favor of anti-Horn be explained since both are signaling systems and strict Nash equilibria?

Basic answers to these questions can be found in Chapter 2, where I'll show that *update dynamics* in *repeated games* have the key role of explaining the conventionalization process of linguistic meaning without previous explicit agreements. In general, I'll show how conventions emerge through the dynamics of optimization in cultural evolution.[6] In Chapter 3 I'll introduce

[6] According to the Horn game, this is in the spirit of van Rooij (2004), who suggests that Horn's division of pragmatic labor involves not only language use but language organization, thus one should look at signaling games from an evolutionary point of view.

some basic notions from network theory that provide tools for modeling (realistic) structures of populations. The combination of both i) update dynamics to model processes of cultural evolution and ii) network theory to shape realistic social network structures constitutes the innovation in my research and will be analyzed in subsequent chapters.

Chapter 2

Update Dynamics

> *"Two savages, who had never been taught to speak, but had been bred up remote from the societies of men, would naturally begin to form that language by which they would endeavor to make their mutual wants intelligible to each other, by uttering certain sounds, whenever they meant to denote certain objects."*
>
> *Smith 1761, Considerations Concerning the First Formation of Languages*

> *"...the replicator dynamics is a natural place to begin investigations of dynamical models of cultural evolution, but I do not believe that it is the whole story."*
>
> *Skyrms 1996, Evolution of the Social Contract*

In Chapter 1 I discussed the properties of a signaling game. This included the possible solution concepts for determining signaling conventions and their stability. I also mentioned that by considering evolutionary paths or processes for the emergence of such conventions, repeated games have to be considered. But this alone is not enough. Such a process can only emerge if agents' *experiences* of already played rounds of a game guide their decision for following rounds; i.e. the performance or the outcome of one or more previously played rounds of a game influences the way agents *update* their behavior. Recently three branches of such *update dynamics* have gained attention considering this issues: *evolutionary dynamics*, *imitation* *dynamics* and *learning dynamics*. To proceed chronologically, I will start this chapter with the first of these three since *evolutionary dynamics* were to my knowledge the earliest account of applying update dynamics on repeated games. This concept originated in the field of *evolutionary game theory*.

While classical game theory found its first applications in the field of economics, evolutionary game theory originally applied to biological contexts. As Alexander (2009) noted, evolutionary game theory has recently inspired social scientists for three reasons:

1. evolution can be considered as *cultural evolution*, e.g. to explain the emergence of conventions, cooperation and norms

2. the *rationality assumptions* of evolutionary game theory are in general more suitable for social systems than those of classical game theory

3. evolutionary game theory is an *explicit dynamic theory* and therefore provides an important element, missing from traditional game theory

Furthermore, while classical game theory deals with one-shot games between two agents, evolutionary game theory has considered populations of agents and repeated games from the beginning. In this chapter I'll introduce three popular dynamics types by i) giving the formal definition, ii) presenting the latest noteworthy literature and iii) showing results of my own research for these dynamics in combination with signaling games. In Section 2.1 I'll introduce the most popular dynamics in the framework of evolutionary game theory that is called *replicator dynamics* (Taylor and Jonker 1978), with which we take a *macro-level* perspective: update steps are defined in dependence of proportions of the population without taking a particular agent's decision process into account. In Section 2.2 I'll switch to the *micro-level* perspective of populations of individual 'behavior updating' agents, whereas I analyze *imitation dynamics*, whereof one variant is shown to be closely related to the replicator dynamics. In Section 2.3 I'll analyze agents updating their behavior by *learning dynamics* which basically differ from imitation dynamics by the fact that agents can collect information from multiple previous plays to guide their decisions.

2.1 Evolutionary Dynamics

As seen in Chapter 1, Lewis's definition of a convention requires assumptions about the players' common knowledge to let them make their decisions according to a signaling system, a fact also supported by e.g. Vanderschraaf (1998). But this requirement bears a question: where should this common knowledge come from? If the emergence of language should be explained without previous agreements, why should previous common knowledge be assumed? Considering the agents' knowledge about interlocutors, it would be preferable to start with as few prerequisites as possible. Evolutionary game theory gives a clue to this puzzle, as Huttegger (2007) pointed out: "The adaption of the evolutionary viewpoint implies that we do not follow Lewis (1969) and Vanderschraaf (1998) in invoking any common knowledge assumption to define conventions. ... We assume neither that the individuals in the population reach a convention by explicit agreement, nor that they have a preexisting language or common knowledge of the game. Indeed, they may not have much knowledge at all." (page 9).

With this point of view, it is not required that members of a population have preexisting common knowledge to agree on a behavioral pattern that constitutes a convention. A convention can be established in a society through the mechanism of selection and mutual adaption. van Rooij (2004) emphasizes the point that a convention can have its own internal dynamics. In addition, he highlights the relationship to biological evolution: "A linguistic convention can be seen as a behavioral phenomenon that develops through the forces of evolution. Indeed, a linguistic convention can be thought of as a typical example of what Dawkins (1976) calls *memes*: cultural traits that are subject to natural selection. In contrast with genes, memes are not replicated - transmitted and sustained - through genetic inheritance, but through imitation, memory, and education. ... But linguistic conventions thought of as memes share a crucial feature with genes: if they do not serve the needs of the population, evolutionary forces will act to improve their functioning." (page 516).

In the following I want to present a number of different accounts that deal with signaling games in evolutionary frameworks on a population level. Furthermore, I want to show how conventions can arise without assuming much, if any, knowledge of individuals in the population. For that purpose it is necessary to consider a modified version of a standard signaling game SG (as defined in Chapter 1) which I call a *static signaling game SSG*. With respect to the majority of literature about signaling games and evolutionary dynamics, there is particularly one distinction that separates a SSG into two classes: the distinction between *symmetric* and *asymmetric* static signaling games.

2.1.1 Symmetric and Asymmetric Static Signaling Games

To adapt signaling games to established evolutionary accounts, let us consider a *static* variant: sender or receiver can play all pure strategies and the virtual game is the table of expected utilities among these strategies. Games like that are i) *asymmetric* because sender and receiver have a different set of strategies and ii) *static* because agents play their strategies simultaneously. Hence, an asymmetric static signaling game SSG^a is defined in the following way:

Definition 2.1 (Asymmetric Static Signaling Game). *Given a signaling game $SG = \langle (S, R), T, M, A, P, C, U' \rangle$ with a sender S, a receiver R, a set of information states T, a set of messages M, a set of interpretation states A, a probability function over states $P \in \Delta(T)$, a cost function $C : M \to \mathbb{R}$ and a utility function $U' : T \times M \times A \to \mathbb{R}$, the corresponding asymmetric static signaling game $SSG^a = \langle (S, R), \mathbf{S}, \mathbf{R}, U \rangle$ is defined as follows:*

- *S is a sender, R is a receiver*
- *$\mathbf{S} = \{s | s \in T \to M\}$ is the set of pure sender strategies*
- *$\mathbf{R} = \{r | r \in M \to A\}$ is the set of pure receiver strategies*
- *$U : \mathbf{S} \times \mathbf{R} \to \mathbb{R}$ is the utility function over sender and receiver strategies, defined as $U(s, r) = \sum_t P(t) \times (U'(t, r(s(t))) - C(s(t)))$*

Accordingly, for the Lewis game the payoff table of the corresponding symmetric static signaling game is depicted in Table 4.5a, whereas the payoff table representing the symmetric static signaling game for a Horn game with $P(t_f) = .75$, $C(m_u) = .1$ and $C(m_m) = .2$ is depicted in Table 4.5b. Notice that these are asymmetric games by definition because for a symmetric game both players have the same set of strategies to choose from, and the utility doesn't depend on the position of the players. There are two possibilities for integrating a static game in an evolutionary setup, as Skyrms (1996) pointed out: "In an evolutionary setting, we can either model a situation where senders and receivers belong to different populations or model the case where individuals of the same population at different times assume the

	r_1	r_2	r_3	r_4
s_1	1	0	.5	.5
s_2	0	1	.5	.5
s_3	.5	.5	.5	.5
s_4	.5	.5	.5	.5

(a) *EU*-table Lewis Game

	r_h	r_a	r_s	r_y
s_h	.875	-.125	.625	.125
s_a	-.175	.825	.575	.075
s_s	.65	.15	.65	.15
s_y	.05	.55	.55	.05

(b) *EU*-table Horn Game

Table 2.1: The payoff tables of asymmetric static signaling games for the Lewis game (Table 4.5a) and the Horn game (Table 4.5b) with $P(t_f) = .75$, $C(m_u) = .1$ and $C(m_m) = .2$.

role of sender and receiver." (page 87). It is easy to see that a SSG^a in an evolutionary setting can only be applied on two disjoint populations, one with only senders, the other with only receivers. Accordingly, agents can only interact with members of the other population.

The second possibility that Skyrms mentioned, allows us to adapt signaling games to one homogeneous population. Here agents can switch roles: every agent can be sender and receiver, each role with a probability of .5 (as a general setup). A game with such a setting is also called the *role-conditioned version* (Huttegger 2007). I will call such a game a symmetric static signaling game SSG^s. An agent's strategy consists of a sender strategy s_i and a receiver strategy r_j. I will call such a strategy pair (s_i, r_j) a language L_{ij} (or the abbreviation L_i if $i = j$), defined as follows:

Definition 2.2 (Language)**.** *Given a static signaling game SSG with a set of sender strategies* $\mathbf{S}$ *and receiver strategies* $\mathbf{R}$ *for players that take up sender and receiver role, each player's strategy pair* (s_i, r_j) *with* $s_i \in \mathbf{S}$ *and* $r_j \in \mathbf{R}$ *is called her* language L_{ij}*, or* L_i *iff* $i = j$*.*

A symmetric static signaling game is defined in the following way:

Definition 2.3 (Symmetric Static Signaling Game)**.** *Given an asymmetric static signaling game* $SSG^a = \langle (S, R), \mathbf{S}, \mathbf{R}, U' \rangle$ *with a sender* S*, a receiver* R*, a set of pure sender strategies* $\mathbf{S}$*, a set of pure receiver strategies* $\mathbf{R}$ *and a utility function* $U : \mathbf{S} \times \mathbf{R} \to \mathbb{R}$*. The corresponding symmetric static signaling game* $SSG^s = \langle (S, R), \mathbf{L}, U \rangle$ *is defined as follows:*

- S *is a sender,* R *is a receiver*
- $\mathbf{L} = \{L_{ij} | L_{ij} = \langle s_i, r_j \rangle \forall s_i \in \mathbf{S}, r_i \in \mathbf{R}\}$ *is the set of languages*

	r_1	r_2	r_3	r_4
s_1	L_1	L_{12}	L_{13}	L_{14}
s_2	L_{21}	L_2	L_{23}	L_{24}
s_3	L_{31}	L_{32}	L_3	L_{34}
s_4	L_{41}	L_{42}	L_{43}	L_4

(a) Languages of the Lewis game

	r_h	r_a	r_s	r_y
s_h	L_h	L_{ha}	L_{hs}	L_{hy}
s_a	L_{ah}	L_a	L_{as}	L_{ay}
s_s	L_{sh}	L_{sa}	L_s	L_{sy}
s_y	L_{yh}	L_{ya}	L_{ys}	L_y

(b) Languages of a Horn game

Table 2.2: 4×4 strategy pairs constitute 16 languages for the Lewis game (Table 2.2a) and the Horn game (Table 2.2b) as well.

- $U : \mathbf{L} \times \mathbf{L} \rightarrow \mathbb{R}$ *is the utility function over languages, defined as* $U(L_{ij}, L_{kl}) = \frac{1}{2}(U'(s_i, r_l) + U'(s_k, r_j))$

Instead of a SSG^a that distinguishes between a population of senders with a set of strategies **S** and receivers with another set of strategies **R**, a SSG^s can be integrated in a 'one population' model, where each agent has the same set of languages **L**. Note that for a 'SSG^s one population model', each agent has 16 different languages for the Lewis game and the Horn game as well. These languages are depicted in Table 2.2a for the Lewis game and Table 2.2b for the Horn game.

The research of this chapter considers evolutionary dynamics for static signaling games in populations. With regard to this account I would like to answer two pressing questions (according to Huttegger 2007, page 6):

1. How is a conventional language maintained in a population?
2. And how might a conventional language established in the first place?

2.1.2 Evolutionary Stability

The first question can be answered with the concept of the *evolutionarily stable strategy* (ESS) (c.f. Maynard Smith and Price 1973; Maynard Smith 1982) which is defined as follows:

Definition 2.4 (Evolutionarily stable strategy)**.** *For a symmetric signaling game SSG^s a strategy s_i is said to be a* evolutionarily stable strategy *if and only if the following two conditions hold:*

1. $U(s_i, s_i) \geq U(s_i, s_j)$ *for all* $s_j \neq s_i$
2. *if* $U(s_i, s_i) = U(s_i, s_j)$ *for some* $s_j \neq s_i$*, then* $U(s_i, s_j) > U(s_j, s_j)$

These conditions guarantee that a population of agents playing strategy s_i will be resistant against the invasion of a small proportion of mutants with another strategy $s_j \neq s_i$. For such an invasion, selection will move the population back to a state with only s_i players. The number of mutants against which an ESS is resistant is also called the *invasion barrier*.

In addition, the first condition requires an ESS to be a Nash equilibrium. The second condition says that if a strategy s_j can survive with the evolutionary stable strategy s_i, then s_i must be able to successfully invade s_j. It follows immediately that any strategy that forms a strict Nash equilibrium is evolutionarily stable. All in all, the concept of an ESS is strongly connected to the concept of a Nash equilibrium in the following manifestation:

- if a strategy s_i constitutes a strict Nash equilibrium, then s_i is an ESS
- if a strategy s_i is an ESS, then s_i is a Nash Equilibrium

This leads to the following inclusion relation:

$$\textit{Strict Nash Equilibria} \subset \textit{ESS} \subset \textit{Nash Equilibria}$$

Further, Wärneryd (1993) was able to show that for a symmetric signaling game, a strategy is an evolutionarily stable state if and only if it is a signaling system. This explains the fact that once signaling systems emerge in a population, they are stable (in fact evolutionarily stable).

Note that for the asymmetric static signaling game with two populations, the second condition of the ESS definition doesn't matter: here a strategy s_i of the first population and strategy r_j of the second population constitute an ESS if $U(s_i, r_j) > U(s_k, r_j)$ and $U(s_i, r_j) > U(s_i, r_l)$ for all $s_k \neq s_i$ and $r_l \neq r_j$. In this case, both populations have an invasion barrier. In addition, for an assymetric static signaling game an ESS and a strict Nash equilibrium coincide: Selten (1980) was able to show that a strategy pair (s_i, r_j) constitutes an ESS if and only if (s_i, r_j) is a strict Nash equilibrium.

The concept of an ESS provides a good answer to the first question of stability for a signaling system once it has emerged, but it doesn't answer the question of how signaling systems emerge in the first place. Here the *replicator dynamics* should give us a preliminary answer.

2.1.3 Replicator Dynamics

The replicator dynamics in its general specification is a dynamics that models replication in populations, like biological reproduction. It is defined for an infinite population of agents playing a game by choosing among strategies, where the *fitness* is defined by agents' utility values and *selection* is generated by the choice behavior of the population. In detail: given is an infinite population of agents playing a game with a set of strategies $s \in \mathbf{S}$ randomly against each other, and furthermore:

- $U(s_i, s_j)$ is the utility of playing strategy s_i against s_j
- $p(s_i)$ is the proportion of agents in the population playing strategy s_i
- $U(s_i) = \sum_{s_j \in S} p(s_j)U(s_i, s_j)$ is the expected utility for playing s_i
- $U = \sum_{s_i \in S} p(s_i)U(s_i)$ is the average utility of the whole population

According to the replicator dynamics, the proportion $p'(s_i)$ of the population playing a strategy s in the next generation depends on i) its proportion $p(s_i)$ of the current generation and ii) its success in form of overall utility $U(s_i)$ in comparison to the population's average utility U. By considering that time intervals between generations are arbitrarily small and that the population size goes towards infinity, the development of the relative frequency of the different strategies within the population converges towards a deterministic dynamics. This dynamics is called *replicator dynamics* and can be described by the following differential equation:

Definition 2.5 (Replicator Dynamics)**.** *For a given set of strategies $s_i \in \mathbf{S}$ and the predefined utility of a strategy $U(s_i)$, proportion of agents playing that strategy $p(s_i)$ and the population's average utility U, the replicator dynamics is defined by the following differential equation:*

$$\frac{dp(s_i)}{dt} = p(s_i)[U(s_i) - U]$$

This equation was first introduces by Taylor and Jonker (1978) and later studies by e.g. Zeeman (1980), Bomze (1986) and Schuster and Sigmund (1983). Note that there are only two cases for a strategy s_i not to change it's proportion $p(s_i)$ over time ($\frac{dp(s_i)}{dt} = 0$): first, the strategy is as good as the populations average ($U(s_i) = U$) and second, the strategy is extinct ($p(s_i) = 0$). Furthermore, a strategy proportion $p(s_i)$ increases ($\frac{dp(s_i)}{dt} > 0$) if and only if it is better than the average ($U(s_i) > U$), and deceases ($\frac{dp(s_i)}{dt} < 0$) if and only if it is worse than the average ($U(s_i) < U$).

As initially mentioned, the replicator dynamics was originally used to capture biological phenomena. There are some later studies that reasonably apply the replicator dynamics in a cultural context (c.f. Björnstedt and Weibull 1996; Harms 2004). For example, Björnstedt and Weibull (1996) showed that replicator dynamics describe a learning process governed by imitation. Thus the replicator dynamics seems to be a good point of departure for analyzing signaling games in a context of cultural evolution.

2.1.4 The Lewis Game in Evolution

Skyrms (1996) made replicator dynamics simulations for populations of agents playing a symmetric static variant of the Lewis game. By starting from all possible combinations of sender and receiver strategies, he showed that signaling systems always evolve. Furthermore, Skyrms (2000) gave a mathematical analysis of the replicator dynamics applied on simple cases of signaling games, inter alia the Lewis game, for which he was able to show that signaling systems emerge without fail. It is important to note that which of both signaling systems is finally selected depends on the initial proportions. To highlight this fact, let's take a look at the whole utility table of the symmetric static variant of the Lewis game, as depicted in Table 2.3. In the following I'll denote the symmetric static variant of the Lewis game with LG^s.

As visible in Table 2.3 LG^s has two strict Nash equilibria and therefore at least two evolutionarily stable strategies: languages L_1 and L_2. There are four Nash equilibria which are not evolutionarily stable: L_3, L_{34}, L_{43} and L_4. In addition, there are two languages which are bad against themselves but perfect against each other: L_{12} and L_{21}. Finally, the remaining languages fail to be Nash equilibria and are attracted by either L_1 or L_2. Because LG^s has a 16×16 utility table, it's difficult to display dependencies of different languages for the whole game. To highlight relationships between specific languages, I'll break down the game to sub-games that are defined for a subset of the given set of the languages. A sub-game for a given static game G is defined in the following way:

Definition 2.6 (Sub-game)**.** *Given a static game* $G = \langle (P_1, P_2), \mathbf{S_1}, \mathbf{S_2}, U\rangle$ *with players* P_1 *and* P_2*, a set of strategies* $\mathbf{S_1}$ *for the first and* $\mathbf{S_2}$ *for the second player and a utility function* $U : \mathbf{S_1} \times \mathbf{S_2} \to \mathbb{R}$*, a corresponding sub-game of game* G *restricted to strategy sets* $\mathbf{S'_1} \subseteq \mathbf{S_1}$ *and* $\mathbf{S'_2} \subseteq \mathbf{S_2}$ *is defined as* $sub(G, \mathbf{S'_1}, \mathbf{S'_2}) = \langle (P_1, P_2), \mathbf{S'_1}, \mathbf{S'_2}, U'\rangle$ *with:*

	L_1	L_{12}	L_{13}	L_{14}	L_{21}	L_2	L_{23}	L_{24}	L_{31}	L_{32}	L_3	L_{34}	L_{41}	L_{42}	L_{43}	L_4
L_1	**1**	.5	.75	.75	.5	0	.25	.25	.75	.25	.5	.5	.75	.25	.5	.5
L_{12}	.5	0	.25	.25	1	.5	.75	.75	.75	.25	.5	.5	.75	.25	.5	.5
L_{13}	.75	.25	.5	.5	.75	.25	.5	.5	.75	.25	.5	.5	.75	.25	.5	.5
L_{14}	.75	.25	.5	.5	.75	.25	.5	.5	.75	.25	.5	.5	.75	.25	.5	.5
L_{21}	.5	1	.75	.75	0	.5	.25	.25	.25	.75	.5	.5	.25	.75	.5	.5
L_2	0	.5	.25	.25	.5	**1**	.75	.75	.25	.75	.5	.5	.25	.75	.	.5
L_{23}	.25	.75	.5	.5	.25	.75	.5	.5	.25	.75	.5	.5	.25	.75	.5	.5
L_{24}	.25	.75	.5	.5	.25	.75	.5	.5	.25	.75	.5	.5	.25	.75	.5	.5
L_{31}	.75	.75	.75	.75	.25	.25	.25	.25	.5	.5	.5	.5	.5	.5	.5	.5
L_{32}	.25	.25	.25	.25	.75	.75	.75	.75	.5	.5	.5	.5	.5	.5	.5	.5
L_3	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
L_{34}	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
L_{41}	.75	.75	.75	.75	.25	.25	.25	.25	.5	.5	.5	.5	.5	.5	.5	.5
L_{42}	.25	.25	.25	.25	.75	.75	.75	.75	.5	.5	.5	.5	.5	.5	.5	.5
L_{43}	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
L_4	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5

Table 2.3: Utility table of the symmetric static Lewis game LG^s.

	L_1	L_2	L_3
L_1	1	0	.5
L_2	0	1	.5
L_3	.5	.5	.5

(a) $sub(LG^s, \{L_1, L_2, L_3\})$

	L_1	L_{12}	L_{21}
L_1	1	.5	.5
L_{12}	.5	0	1
L_{21}	.5	1	0

(b) $sub(LG^s, \{L_1, L_{12}, L_{21}\})$

	L_1	L_3	L_{41}
L_1	1	.5	.75
L_3	.5	.5	.5
L_{41}	.75	.5	.5

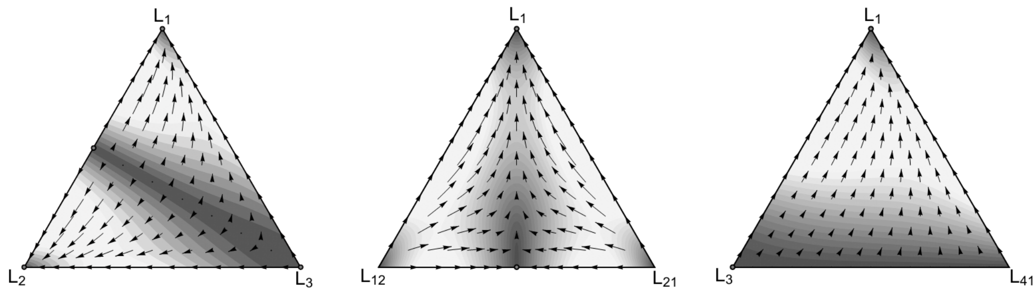
(c) $sub(LG^s, \{L_1, L_3, L_{41}\})$

Table 2.4: Utility tables for different sub-games of LG^s

- *players* P_1 *and* P_2
- *strategy set* $\mathbf{S_1'} \subseteq \mathbf{S_1}$ *for* P_1 *and* $\mathbf{S_2'} \subseteq \mathbf{S_2}$ *for* P_2
- $U' : \mathbf{S_1'} \times \mathbf{S_2'} \rightarrow \mathbb{R}$ *as the utility function over strategies, defined as* $U'(s_i, s_j) = U(s_i, s_j)$ *for all* $s_i \in \mathbf{S_1'}, s_j \in \mathbf{S_2'}$

Note that for a symmetric game and a sub-game $sub(G, \mathbf{S_1}, \mathbf{S_2})$ with $\mathbf{S_1} = \mathbf{S_2}$, the sub-game is abbreviated as $sub(G, \mathbf{S_1})$.

To illustrate different dependencies between these languages I will pick out three *sub-games* with three languages each, as depicted in Table 2.4. The global dynamics between both evolutionarily stable languages L_1 and L_2 and a third non-evolutionarily stable language L_3 (equal to L_{34}, L_{34}



(a) $sub(LG^s, \{L_1, L_2, L_3\})$ (b) $sub(LG^s, \{L_1, L_{12}, L_{21}\})$ (c) $sub(LG^s, \{L_1, L_3, L_{41}\})$

Figure 2.1: Global dynamics pictures of different sub-games of LG^s

or L_4) can be analyzed in sub-game $sub(LG^s, \{L_1, L_2, L_3\})$, as depicted in Table 2.4a. The corresponding *global dynamics picture* under the replicator dynamics is depicted in Figure 2.1a.[1] All three languages are stable rest points, but a population of L_3 players can be easily invaded by L_1 or L_2 players: they score as good as L_3 players in such a population, but better against themselves. The smallest deviation from 100% of L_3 players drifts the population state away, attracted by either a population state of only L_1 or of only L_2 players. These two population states are *attraction points* and the spectrum of population states attracted by them is called their *basin of attraction*. It is observable that in sub-game $sub(LG^s, \{L_1, L_2, L_3\})$ the whole space of population states is equally divided in a basin of attraction for L_1 and one for L_2. Only the population states with $p(L_1) = p(L_2) > 0$ drives the population state to a state with 50% of L_1 and 50% of L_2 players which is a mixed Nash equilibrium.

The global dynamics of sub-game $sub(LG^s, \{L_1, L_{12}, L_{21}\})$ (Table 2.4b) are displayed in Figure 2.1b: here the whole space is a basin of attraction for L_1, except the bottom line of only L_{12} and L_{21} players. On this line the replicator dynamics drives the population state to a stable rest point of 50% of L_{12} and 50% of L_{21} players. The average utility is .5 for such a population state. Only an L_1 invader would score the same but better against himself. Thus the smallest deviation from a population state of only

[1] Such a global dynamics picture displays the global dynamics between three strategies. Each vertex of the triangle corresponds to 100% of the population playing the corresponding strategy, all coordinates in between are mixed population states. The arrows represent the directions of change of the population state, the shading illustrates the speed of change: the lighter the color, the higher the speed. The gray dots represent stable rest points.

L_{12} and L_{21} players to L_1 invaders drives the population to a state of only L_1 players.

The global dynamics of sub-game $sub(LG^s, \{L_1, L_3, L_{41}\})$ (depicted in Table 2.4c) are displayed in Figure 2.1c: here the whole space is a basin of attraction for L_1, except the population state of only L_3 players. A population state of only L_{41} players is unstable because L_1 players score better against L_{41} players than L_{41} players against themselves.

All in all, it can be shown that with a few exceptions of some rest points, the whole population state space of LG^s is equally divided in two basins of attraction, one for L_1 and one for L_2. Consequently, dependent on the initial population state, the replicator dynamics drives the population to a state of only players of one of those two languages that constitute signaling systems.

2.1.5 The Horn Game in Evolution

At the beginning of this section I gave reasonable arguments for the way to analyze signaling games under an evolutionary point of view. This view can be augmented in particular for the Horn game, as van Rooij (2004) pointed out: "According to a tradition going back to Zipf (1949), economy considerations apply in the first place to languages. Speakers obey Horn's rule because they use a conventional language that, perhaps due to evolutionary forces, is designed to minimize the average effort of speakers and hearers." (page 494).

Beside Skyrms (1996), a vast number of further scholars analyzed standard signaling games like the Lewis game in combination with replicator dynamics, but comparatively a lot less has been done for the Horn game. Note that two adjustments distinguish a Horn game from the Lewis game: uneven state probabilities and message costs that differ among the messages. On the one hand, signaling games with an uneven probability distribution of both states are analyzed e.g. by Nowak et al. (2002), Huttegger (2007) or Hofbauer and Huttegger (2007), but without reconsidering message costs. One the other hand, games with costly signaling were analyzed by economists (c.f. Spence 1973) and biologists (c.f. Grafen 1990), but uneven state probabilities were not considered. Research for signaling games with the combination of both is rare. In the following, I'll give an overview about a couple of studies dealing with Horn games.

There is a study of Jäger (2008) that deals with a mathematical analysis of Horn games by incorporating both an uneven probability distribution for

	r_h	r_a	r_s	r_y
s_h	.875 ; 1	−.125 ; 0	.625 ; .75	.125 ; .25
s_a	−.175 ; 0	.825 ; 1	.575 ; .75	.075 ; .25
s_s	.65 ; .75	.15 ; .25	.65 ; .75	.15 ; .25
s_y	.05 ; .25	.55 ; .75	.55 ; .75	.05 ; .25

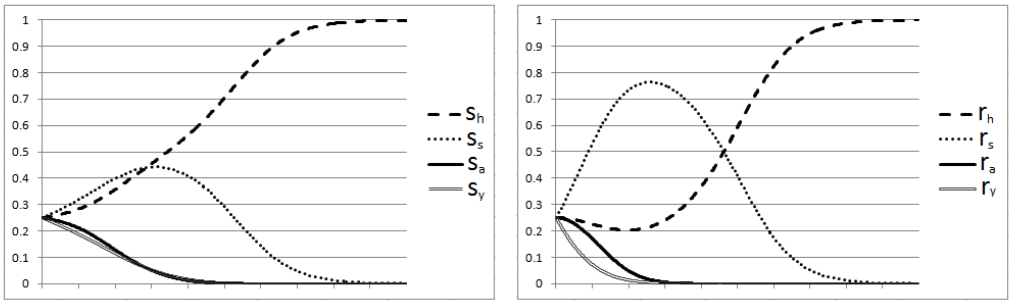
Table 2.5: The asymmetric static Horn game HG^a with $P(t_f) = .75$, $C(m_u) = .1$ and $C(m_m) = .2$, whereby costs are only considered for senders.

the states and different message costs. This study is not restricted to games with only two states, two messages and two actions. The results show that the signaling systems are evolutionarily stable for signaling games with an equal number of states, messages and actions. Jäger also found an infinity of *neutrally stable states*[2] that are not included in the set of evolutionarily stable states, but attract a set of states with a positive measure.

Furthermore, Benz et al. (2005) used simulations to analyze the asymmetric variant of a Horn game ($P(t_f) = .75$, $C(m_u) = .1$, $C(m_m) = .2$) under the replicator dynamics for a 'two population' model, similar to the one depicted in Table 4.5b, but message costs are only considered for the agents being senders, not for being receivers. The corresponding SSG^a is depicted in Table 2.5. The resulting times series is depicted in Figure 2.2 for a starting population with all strategies having 25% proportion of the society, Figure 2.2a for the sender population, Figure 2.2b for the receiver population. As you can see, with this initial setting the resulting population will use the Horn strategy s_h for all senders and r_h for all receivers as well. Apparently, the Smolensky strategy is initially really successful particularly among the receiver population, but it is finally driven to extinction. In addition, it can be shown that for the same game but with considering message costs also for receivers (Table 4.5b, page 136), the time series looks roughly the same: neglecting or incorporating message costs for receivers basically doesn't seem to make a difference.

Nevertheless, it can be shown that the anti-Horn strategies s_a and r_a and Smolensky strategies s_s and r_s are also attractors for specific starting populations. Thus Horn strategy, anti-Horn strategy and Smolensky strategy all have a basin of attraction, where the one of the Horn strategy is

[2] Neutrally stable states differ from evolutionarily stable states by relaxing the second condition to a non-strict inequality (see Definition 2.4): if $U(s_i, s_i) = U(s_i, s_j)$ for some $s_j \neq s_i$, then $U(s_i, s_j) \geq U(s_j, s_j)$. Note that the Smolensky language is a neutrally stable state for a variant of the static Horn game.



(a) sender population (b) receiver population

Figure 2.2: The time series of the Horn game for sender and receiver population.

always larger than the other two.[3] For instance, for the predefined Horn game ($P(t_f) = .75$, $C(m_u) = .1$, $C(m_m) = .2$) the basin of attraction for the Horn strategy spans almost 50% of the space of population states. Figure 2.3 depicts the basins of attraction segmentation of the population state space as a 3-simplex with uniform population states at the four corners. The position coordinates of each sphere corresponds to the initial population state[4], the color represents the final uniform population state.

These results show what van Rooij (2004) pointed out, namely that the basin of attraction and therefore the initial population state plays a significant role for the way strategies evolve and stabilize in the population. While the simulation example of Benz et al. (2005) started with a population state of the same proportion for each strategy, de Jaegher (2008) argues for an initial population of agents playing only the Smolensky strategy which he labeled with a more general term as *pooling equilibrium*: "...the evolution of an equilibrium that selects Horn's rule follows straightforwardly from the fact that a signaling equilibrium must at some point have evolved from a pooling equilibrium." (page 276).

Like Benz et al. (2005), de Jaegher (2008) also examined the asymmetric static Horn game with a sender and a receiver population which I'll denote with HG^a. In his first experiment he considered the sub-game $sub(HG^a, \{s_h, s_s\}, \{r_h, r_s\})$ (Table 2.6a) to analyze the evolutionary drift from the Smolensky to the Horn strategy. The resulting phase diagram for the replicator dynamics is depicted in Figure 2.4a. As you can see, the pop-

[3]To what extend these basins of attraction differ depends of the parameter settings of the Horn game: state probability P and the difference between the costs $C(m_u)$ and $C(m_m)$.

[4]Initial states of sender population and receiver population are identical.

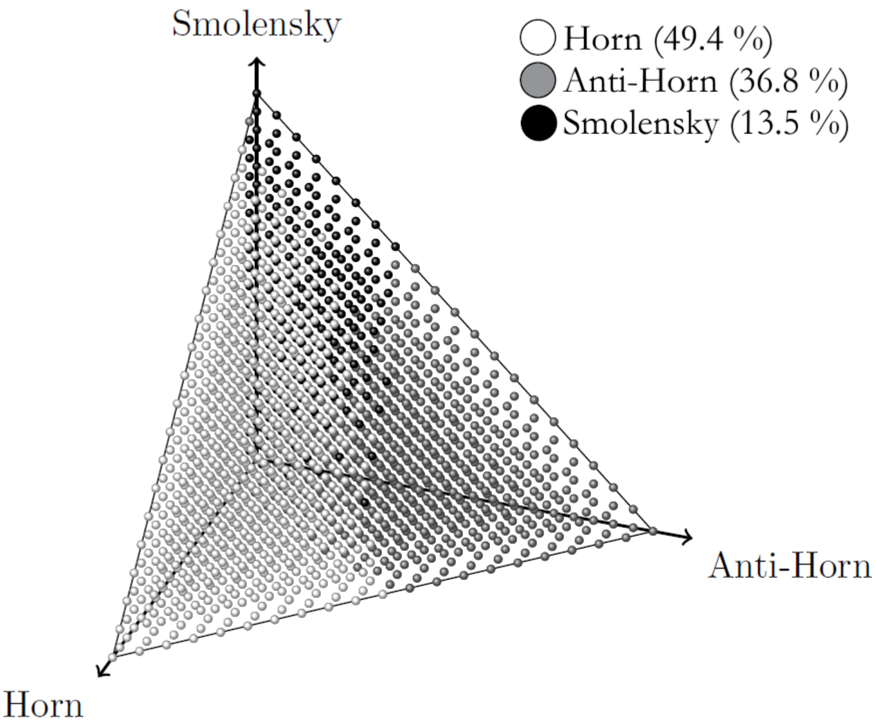


Figure 2.3: Basins of attraction segmentation of a discrete population state space as 3-simplex. The four corners are the uniform population states for Horn (bottom left), anti-Horn (bottom-right), Smolansky (top) and anti-Smolensky (bottom backside).

	r_h	r_s
s_h	.875 ; 1	.625 ; .75
s_s	.65 ; .75	.65 ; .75

(a) $sub(HG^a, \{s_h, s_s\}, \{r_h, r_s\})$

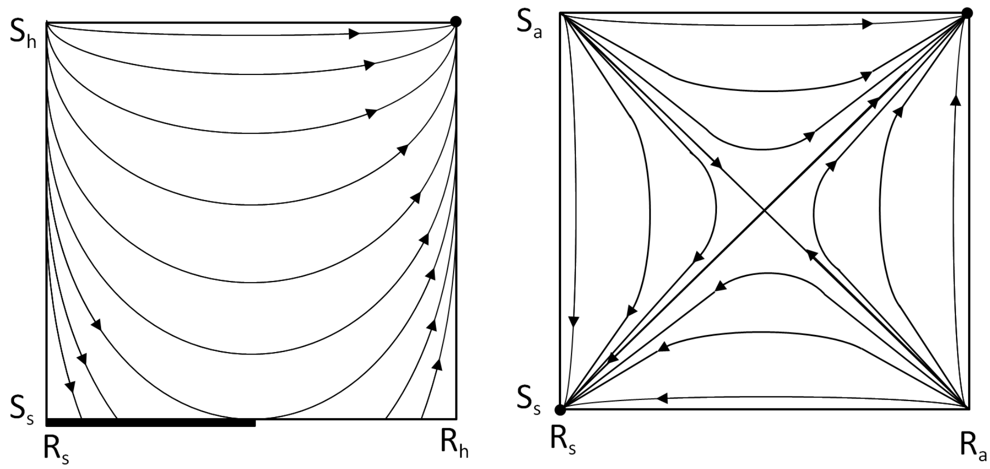
	r_a	r_s
s_a	.825 ; 1	.575 ; .75
s_s	.15 ; .25	.65 ; .75

(b) $sub(HG^a, \{s_a, s_s\}, \{r_a, r_s\})$

Table 2.6: Utility tables for different sub-games of LG^s

ulation state remains close to the pooling equilibrium (s_s, r_s) (bold line at the bottom) as long as less than half of the receivers interpret the costly signal with the infrequent interpretation state. In any other case the dynamics drives the population to the signaling equilibrium (s_h, r_h).

In addition, de Jaegher analyzed the sub-game $sub(HG^a, \{s_a, s_s\}, \{r_a, r_s\})$ (Table 2.6b). The resulting phase diagram for the replicator dynamics is depicted in Figure 2.4b. This time there is no evolutionary path from the pooling equilibrium (s_s, r_s) to the signaling equilibrium (s_a, r_a). But de Jaegher (2008) was able to show that there is an evolutionary path from



(a) $sub(HG^a, \{s_h, s_s\}, \{r_h, r_s\})$ (b) $sub(HG^a, \{s_a, s_s\}, \{r_a, r_s\})$

Figure 2.4: Global dynamics pictures of the two sub-games

the other pooling equilibrium (s_y, r_y) to the signaling equilibrium (s_a, r_a).

Furthermore, Jäger (2004) simulated the Horn game with replicator dynamics and integrated a specific degree of *noisy mutation*: a randomly chosen proportion of the population is randomly replaced by invaders that play a randomly chosen strategy. In such a model no strategy is evolutionarily stable because every mutation barrier can be overcome, even with a very small probability. Although the invasion barrier is high for populations of Horn players to be invaded by anti-Horn players (and the other way around), an invasion is probable. Jäger (2004) simulated such a run for the Horn game as specified in Table 2.5. He showed that the system switched from time to time between populations either with predominantly Horn or anti-Horn players. The result also revealed that the system spent 67% of its time in a population with predominantly Horn players and 26% of its time in a population with predominantly anti-Horn players. This indicates that the Horn strategy being more probable is the only *stochastically stable strategy* which is a refinement of evolutionary stability: a strategy is stochastically stable if its probability converges to a value $Pr > 0$ as the mutation rate approaches 0.[5] For the symmetric variant of the Horn game van Rooij (2004) had the same observation and pointed out that "in case the mutation rate is small, the system spends most of the time at the 'good'

[5]For more details and e.g. a formal definition of stochastically stable strategies, see e.g. Vega-Redondo (1996) or Young (1998).

equilibrium, with probability 1 in the long run." (page 523).

Lentz and Blutner (2009) examined a symmetric variant of the Horn game for a 'SSG^s one population' model under evolutionary dynamics slightly different from the replicator dynamics: here a population of agents is given, wherein each agent plays against every other agent and acquires a total score as the sum of all played games. The total outcome of each agent in relation to all other agents' outcomes defines the number of her offspring. The higher the score, the more offspring. But only couples of agents which are randomly chosen at each simulation step can have offspring. Furthermore, if one agent of this couple uses language L_{ij} and the other agent uses language L_{kl}, then the offspring in the next round plays either L_{il} or L_{kj}. In other words, the offspring uses the sender strategy of one parent and the receiver strategy of the other. After each step 85% of the population is replaced with offspring with a 1% chance of mutation.

In comparison to the replicator dynamics, this dynamics is in some ways *innovative*: new or already extinct languages can evolve just by the fact that the offspring's language is a combination of parts of the parents' languages. Admittedly, these new languages are also limited to possible combinations of already given sender and receiver strategies. But combined with the chance of mutation this dynamics can lead to strong shifts in an initially homogeneous population, a property that is really useful for the kind of experiments Lentz and Blutner (2009) performed: e.g. by starting with only agents playing the Smolensky language L_s, in 98% of all simulation runs the resulting stable society consisted of agents playing the Horn language L_h. This result is congruent with the one of de Jaegher (2008), who showed for the asymmetric variant of the Horn game that there is an evolutionary path from Smolensky strategy to Horn strategy (Figure 2.4a).

van Rooij (2004) also analyzed the symmetric variant of the Horn game (HG^s) and remarked that the only two evolutionarily stable states are the Horn and the anti-Horn language. This is apparent from taking a look on the utility table of HG^s (with $P(t_f) = .7$ and costs $C(m_u) = .1$ and $C(m_m) = .2$) that is depicted in Table 2.7. It shows that the only languages that are strict Nash equilibria against themselves and therefore ESS's are the languages L_h and L_a. In addition, L_s is a non-strict Nash equilibrium, but not evolutionarily stable: first, L_s players score as good against L_{sh} players as against themselves, thus the first condition of Definition 2.4 isn't satisfied. Second, L_{sh} players score as good against themselves as against L_s players, thus the second condition of Definition 2.4 isn't satisfied. Furthermore, L_{sh} itself is not even a Nash equilibrium, and those players can easily be

	L_h	L_{ha}	L_{hs}	L_{hy}	L_{ah}	L_a	L_{as}	L_{ay}	L_{sh}	L_{sa}	L_s	L_{sy}	L_{yh}	L_{ya}	L_{ys}	L_y
L_h	**.87**	.37	.72	.52	.35	-.15	.2	0	.735	.235	.585	.385	.485	-.015	.335	.135
L_{ha}	.37	-.13	.22	.02	.85	.35	.7	.5	.535	.035	.385	.185	.685	.185	.535	.335
L_{hs}	.72	.22	.57	.37	.7	.2	.55	.35	.735	.235	.585	.385	.685	.185	.535	.335
L_{hy}	.52	.02	.37	.17	.5	0	.35	.15	.535	.035	.385	.185	.485	-.015	.335	.135
L_{ah}	.35	.85	.7	.5	-.17	.33	.18	-.02	.215	.715	.565	.365	-.035	.465	.315	.115
L_a	-.15	.35	.2	0	.33	**.83**	.68	.48	.015	.515	.365	.165	.165	.665	.515	.315
L_{as}	.2	.7	.55	.35	.18	.68	.53	.33	.215	.715	.565	.365	.165	.665	.515	.315
L_{ay}	0	.5	.35	.15	-.02	.48	.33	.13	.015	.515	.365	.165	-.035	.465	.315	.115
L_{sh}	.735	.535	.735	.535	.215	.015	.215	.015	.6	.4	.6	.4	.35	.15	.35	.15
L_{sa}	.235	.035	.235	.035	.715	.515	.715	.515	.4	.2	.4	.2	.55	.35	.55	.35
L_s	.585	.385	.585	.385	.565	.365	.565	.365	.6	.4	**.6**	.4	.55	.35	.55	.35
L_{sy}	.385	.185	.385	.185	.365	.165	.365	.165	.4	.2	.4	.2	.35	.15	.35	.15
L_{yh}	.485	.685	.685	.485	-.035	.165	.165	-.035	.35	.55	.55	.35	.1	.3	.3	.1
L_{ya}	-.015	.185	.185	-.015	.465	.665	.665	.465	.15	.35	.35	.15	.3	.5	.5	.3
L_{ys}	.335	.535	.535	.335	.315	.515	.515	.315	.35	.55	.55	.35	.3	.5	.5	.3
L_y	.135	.335	.335	.135	.115	.315	.315	.115	.15	.35	.35	.15	.1	.3	.3	.1

Table 2.7: Utility table for all languages of a Horn game with $P(t_f) = .7$ and costs $C(m_u) = .1$ and $C(m_m) = .2$.

invaded by L_h players. Note that this possible shift $L_s \to L_{sh} \to L_h$ realizes indirectly the so-called *intuitive criterion* (Cho and Kreps 1987): with the first switch $L_s \to L_{sh}$ the receiver strategy changes from the Smolensky language to the Horn language which is equivalent to construing a previously unused message with the interpretation matching the prototypical state; and with the second switch $L_{sh} \to L_h$ the sender strategy switches from the Smolensky language to the Horn language.

Furthermore, van Rooij (2004) mentioned that two factors can explain a predominance of L_h to L_a: *mutation* and *correlation*. First, he mentioned that if mutation is involved, L_h is the only stochastically stable equilibrium, a fact that I already observed and discussed for the asymmetric variant HG^a. The second factor is correlation: instead of random pairing it is more probable that agents interact with interlocutors using the same strategy than with others. Skyrms (1994) was able to show that if correlation is (nearly) perfect, the strictly efficient strategy (the one with the highest payoff) is the unique equilibrium of the replicator dynamics. And the only strictly efficient strategy of HG^s is L_h. And van Rooij emphasizes that "for linguistic communication, positive correlation is the rule rather than the exception: we prefer and tend to communicate with people that use the same linguistic conventions as we do..." (page 522).

All in all, and independent of symmetric or asymmetric games, there are at least three possible reasons that strongly support and therefore explain the emergence of Horn's rule in favor of anti-Horn under the replicator dynamics, even though the latter one is a strict Nash equilibrium and evolutionarily stable in all ways of modeling this game:

- By assuming the *Smolensky strategy as the initial population* state (e.g. because of prior absence of the complex form) a small mutation rate brings the population to the Horn strategy.
- By assuming a strong mutation rate, the system switches between population states of predominantly Horn strategy or anti-Horn strategy players. The system stays most of the time in the former populations state. Consequently, the Horn strategy is the only *stochastically stable equilibrium.*
- By assuming correlation instead of random pairing, it can be shown that if *correlation is (nearly) perfect*, the unique equilibrium is the Horn strategy.

Some properties of HG^s remain to be analyzed. To do this, I'll focus on a sub-game of HG^s by considering only languages forming *plausible* strategy pairs. Plausibility is a property that incorporates the relationship of sender and receiver strategy.

2.1.6 The Plausible Symmetric Horn Game

By considering all the studies so far, there is one thing conspicuous: sender and receiver strategy are completely unrelated by definition. Not noteworthy are the asymmetric games since there are different populations of sender and receiver. But for symmetric games we have agents that are sender and receiver at the same time. And since these games allow for all possible strategy pairs, there is no restriction to combinations accounting for a particular relation or dependency between sender and receiver strategy. But is that really realistic? I don't think so. Let me make the case for a restriction to specific *plausible strategy pairs* that I define in the following way:

Definition 2.7 (Plausible Strategy Pair)**.** *Given is a static game* $G = \langle (P_1, P_2), \mathbf{S_1}, \mathbf{S_2}, U : \mathbf{S_1} \times \mathbf{S_2} \to \mathbb{R} \rangle$ *with player* P_1 *with a set of strategies* $\mathbf{S_1}$ *and player* P_2 *with set of strategies* $\mathbf{S_2}$*. A strategy pair* (s_i, s_j) *with* $s_i \in \mathbf{S_1}$*,* $s_j \in \mathbf{S_2}$ *is* plausible *if and only if the following two conditions hold:*

	r_1	r_2	r_3	r_4
s_1	1	0	.5	.5
s_2	0	1	.5	.5
s_3	.5	.5	.5	.5
s_4	.5	.5	.5	.5

(a) *EU*-table Lewis Game

	r_1	r_2	r_3	r_4
s_1	.875	-.125	.625	.125
s_2	-.175	.825	.575	.075
s_3	.65	.15	.65	.15
s_4	.05	.55	.55	.05

(b) *EU*-table Horn Game

Table 2.8: All strategy pairs for the Lewis game (Table 2.8a) and the Horn game (Table 2.8b) with $P(t_f) = .75$, $C(m_u) = .1$ and $C(m_m) = .2$. The plausible strategy pairs are shaded in gray.

1. $U(s_i, s_j) \geq U(s_i, s_k)$ *for all* $s_k \in \mathbf{S_2}$

2. $U(s_i, s_j) \geq U(s_l, s_j)$ *for all* $s_l \in \mathbf{S_1}$

In other words: a strategy pair (s_i, s_j) is plausible if and only if s_i is a best response to s_j and the other way around. With this definition there are 6 plausible strategy pairs for the Lewis game and 3 plausible strategy pairs for the Horn game, as highlighted in the payoff tables 2.8a and 2.8b. Not surprisingly, plausible strategy pairs are exactly the Nash equilibria of the payoff table of the asymmetric signaling games.

With this definition it is reasonable to restrict symmetric signaling games to plausible strategy pairs. I'll call such a signaling game a plausible symmetric static signaling game SSG^s_p, defined in the following way:

Definition 2.8 (Plausible Symmetric Static Signaling Game)**.** *Given an asymmetric static signaling game* $SSG^a = \langle (S, R), \mathbf{S}, \mathbf{R}, U' \rangle$ *with a sender* S*, a receiver* R*, a set of pure sender strategies* $\mathbf{S}$*, a set of pure receiver strategies* $\mathbf{R}$ *and a utility function* $U : \mathbf{S} \times \mathbf{R} \rightarrow \mathbb{R}$*, the corresponding plausible symmetric static signaling game* $SSG^s_p = \langle (S, R), \mathbf{L}, U \rangle$ *is defined as follows:*

- S *is a sender,* R *is a receiver*

- $\mathbf{L} = \{L_{ij} = \langle s_i, r_j \rangle | s_i \in \mathbf{S}, r_i \in \mathbf{R}, \forall s' \in \mathbf{S} : U(s_i, r_j) > U(s', r_j), \forall r' \in \mathbf{R} : U(s_i, r_j) > U(s_i, r')\}$ *is the set of languages that form plausible strategy pairs*

- $U : \mathbf{L} \times \mathbf{L} \rightarrow \mathbb{R}$ *is the utility function over languages, defined as* $U(L_{ij}, L_{kl}) = \frac{1}{2}(U'(s_i, r_l) + U'(s_k, r_j))$

	L_1	L_2	L_3	L_{34}	L_{43}	L_4
L_1	1	0	.5	.5	.5	.5
L_2	0	1	.5	.5	.5	.5
L_3	.5	.5	.5	.5	.5	.5
L_{34}	.5	.5	.5	.5	.5	.5
L_{43}	.5	.5	.5	.5	.5	.5
L_4	.5	.5	.5	.5	.5	.5

(a) Plausible Lewis Game

	L_h	L_a	L_s
L_h	.87	−.15	.585
L_a	−.15	.83	.365
L_s	.585	.365	.6

(b) Plausible Horn Game

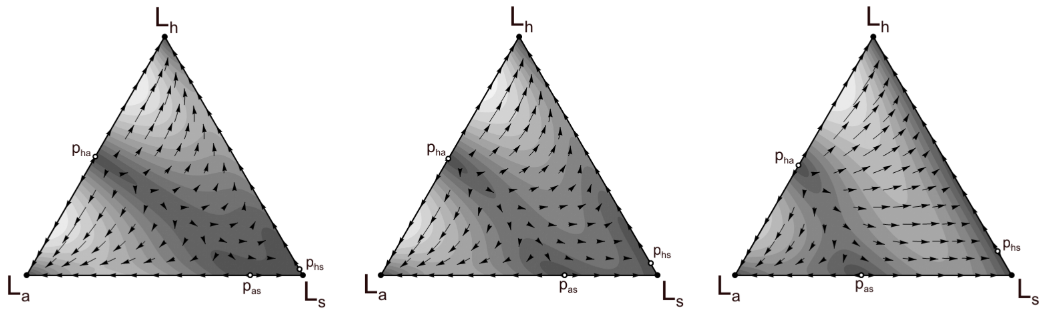
Table 2.9: The plausible Lewis game LG^s_p and Horn game HG^s_p for $P(t_f) = .75$, $C(m_u) = .1$ and $C(m_m) = .2$.

It is easy to show that each SSG^s_p is a sub-game of the respective SSG^s, just restricted to the plausible strategy pairs. The corresponding plausible symmetric static signaling games for the Lewis game LG^s_p and the Horn game HG^s_p are depicted in Table 2.9.

Admittedly, the plausible Lewis game is not really an exciting case to analyze. After all, the structure is really similar to the asymmetric game LG^a or the symmetric game LG^s. It has two strict Nash equilibria and therefore evolutionarily stable states L_1 and L_2 and four languages that are non-strict Nash equilibria and not evolutionarily stable. But the plausible Horn game reveals an interesting property: the Smolensky language L_s is a strict Nash equilibrium and therefore evolutionarily stable.

Figure 2.5 depicts the global dynamics of plausible Horn games with different parameters which I'll call the *weak* Horn game ($P(t_f) = .6$, $C(m_u) = .05$, $C(m_m) = .1$), the *normal* Horn game ($P(t_f) = .7$, $C(m_u) = .1$, $C(m_m) = .2$) and the *strong* Horn game ($P(t_f) = .9$, $C(m_u) = .1$, $C(m_m) = .3$). While the dynamics doesn't seem to differ strongly among all three games, an important detail is the position of the three rest points, marked as white dots. At such a point the smallest deviation brings the population state in one or the other direction, dependent of the direction of the deviation. There is a rest point p_{ha} between population states of Horn and anti-Horn players, a rest point p_{hs} between population states of Horn and Smolensky players and a rest point p_{as} between population states of anti-Horn and Smolensky players.

The fact that p_{hs} is close to the corner of the population state of only Smolensky players in all three games reveals that, even the Smolensky strategy is evolutionarily stable, it has a low invasion barrier against the Horn strategy. While this invasion barrier is the lowest for the weak Horn game



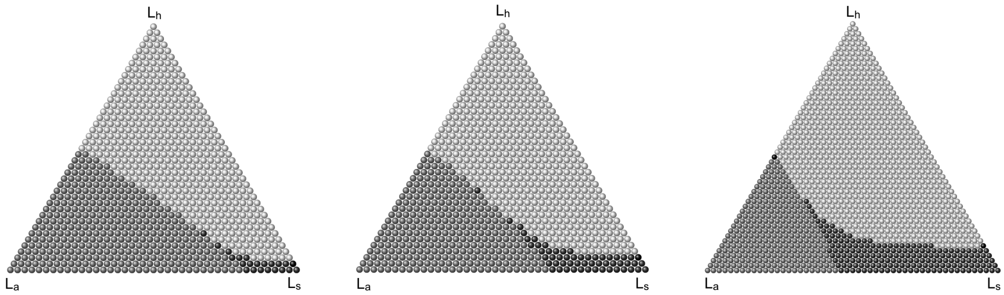
(a) Weak Horn game (b) Normal Horn game (c) Strong Horn game

Figure 2.5: Global dynamics pictures of the three sub-games

among this three games, it is higher for the strong Horn game, but nevertheless pretty low. In contrast, the rest point p_{as} is changing its position strongly in dependence of the different games. While the Smolensky language has a relatively low invasion barrier against the anti-Horn language for the weak Horn game, it has a pretty high one for the strong Horn game: in fact here L_s has a higher invasion barrier against L_a than the other way around.

Another interesting fact is that these rest points mark roughly the basins of attraction. This is easy to see by taking a look at Figure 2.6 which depicts the basins of attraction, where the position of a dot represents the initial population state and its shading the final population state of entirely one language (light: L_h, medium: L_a, dark: L_s). The differences between the three Horn games are remarkable: from weak to normal to strong Horn game the size of the basin of attraction of the Horn language is slightly increasing (55% → 59% → 64%), while the one of the anti-Horn language is strongly decreasing (43% → 35% → 22%). The basin of attraction of the Smolensky language is very strongly increasing; it triples from the weak to the normal Horn game and more than doubles from the normal to the strong Horn game (2% → 6% → 14%). Furthermore, it is nice to see how the basin of attraction of the Smolensky language more or less spans the three rest points of the global dynamics pictures in Figure 2.5.

These figures show even more clearly that, while populations states of Horn or anti-Horn players have a relatively high invasion barrier and *system shaking* invasions are necessary to shift the population state to another stable state, the population state of Smolensky language players needs a relatively small invasion at least of Horn players to change the whole situation



(a) Weak Horn game's basin of attraction sizes are L_h: 55%, L_a: 43%, L_s: 2%

(b) Normal Horn game's basin of attraction sizes are L_h: 59%, L_a: 35%, L_s: 6%

(c) Strong Horn game's basin of attraction sizes are L_h: 64%, L_a: 22%, L_s: 14%

Figure 2.6: The basins of attraction of the three Horn games.

of the population. Nevertheless, the Smolensky strategy is still evolutionarily stable for the plausible variant of the Horn game. Thus the concept of an ESS doesn't seem to be a satisfying explanation for a predominant emergence of Horn's rule in human language since not only anti-Horn, but the Smolensky language is evolutionarily stable for a particular setup. Maybe something more is necessary than only the population-based replicator dynamics to find an explanation for the previously mentioned predominance of the Horn language.

In exploring the adequacy of update dynamics to explain the emergence of signaling systems in repeated signaling games, Huttegger and Zollman (2011) argued that the following three questions are of particular interest (page 169):

1. How little cognitive ability is needed to learn a signaling system?

2. Is the replicator dynamics an appropriate approximation for models of individual learning?

3. Do all models that have limited memory converge to signaling systems? What about all those that remember the entire history?

Regarding the first question, Huttegger and Zollman (2011) pointed out that by applying replicator dynamics as an usually simpler model than other learning rules, a process like natural selection can result in the emergence of signaling systems. Thus there seems to be no need for modeling more cognitive abilities to explain the emergence of signaling systems. But, by reconsidering the second question, would we get different, maybe more explicable results by applying more sophisticated and/or detailed individual-based dynamics? If not, the replicator dynamics, despite lacking explanatory potential, seems to be fit enough. To find an answer to this question, I'll take a look at more individual-based types of update dynamics: *imitation dynamics* and *learning dynamics*. The third question highlights the importance of the agents' memory. Does this play a role for the way signaling systems emerge? I will analyze this by comparing learning dynamics with *limited* and *unlimited* memory.

2.2 Imitation Dynamics

The replicator dynamics describes how strategy distributions in a population of agents develop over time if relative fitness of a strategy directly influences relative future proportions of the population using that strategy. This is a *macro-level perspective* because the updates are defined by proportions of the population without factoring in the way a particular agent behaves. But it is possible to link the replicator dynamics to a *micro-level perspective*: it can be shown that the replicator dynamics describes the most likely path of strategy distributions in a virtually infinite and *homogeneous* population[6], when every agent updates her behavior by a specific imitation dynamics, called *conditional imitation*. To put it the other way around: if agents' behavior is guided by conditional imitation, the aggregate population behavior can be approximated by the replicator dynamics (c.f. Helbing 1996; Schlag 1998). In the following, I'll introduce conditional imitation which is a generalized version of the quite popular imitation dynamics *'imitate the best'*.

2.2.1 Imitate the Best

Following a micro-level perspective, the imitation dynamics defines the particular behavior of an agent by an update rule, also called *behavioral rule*. The behavioral rule 'imitate the best' is a generalization of the rule 'imitate if better' (c.f. Ellison and Fudenberg 1995; Malawski 1990): here an agent interacts with another agent and switches to the opponent's strategy if this one reflects a higher payoff. 'Imitate the best' is more general in the sense

[6] A population is *homogenous* if every agent repeatedly interacts with everybody else with the same frequency.

that here an agent interacts and compares her payoff with a set of other agents and switches to that agent's strategy with the maximal payoff among all agents in the set if scoring better than herself. It is easy to show that using 'imitate the best' and restricting all sets of interlocutors to singletons reproduces 'imitate if better'.

'Imitate the best' is defined in the following way: agents play a game against some other agents in a population. Let's label the utility value of agent z playing against agent z' as $U_{z,z'}$, the used strategy of agent z as s_z. First, it is possible to restrict the access of an agent's potential interaction partners. The set of accessible interlocutors of agent z is called her *neighborhood* NH_z, whereas any $z' \in NH_z$ is called a *neighbor* of z.[7] Further, let's say that in each round of a repeated game each agent z is playing against all her neighbors and her income is the average utility value over all interactions among her neighborhood. Thus, for an agent z I define

- her income as $I_z = \frac{\sum_{z' \in NH_z} U_{z,z'}}{|NH_z|}$
- her set of incomes for neighborhood NH_z as $I(NH_z) = \{I_{z'} | z' \in NH_z\}$
- her set of neighbors with maximal income as $NH_z^* = \{z' \in NH_z | I_{z'} \in \max(I(NH_z))\}$

The dynamics 'imitate the best' works as follows: each agent z switches to the strategy of a neighbor z^* with maximal income if the income of z^* is higher than the one of z. Otherwise agent z keeps her strategy. To put it formally: let $I^* \in \max(I(NH_z))$ be the maximal income among NH_z and let $z^* \in NH_z^*$ be a randomly chosen agent of NH_z with maximal income. Further, let s_z be the old strategy of agent z and s_z^u the strategy after the update, then the 'imitate the best' behavioral rule is given as follows:

$$s_z^u = \begin{cases} s_z & \text{if } I_z \geq I^* \\ s_{z^*} & \text{else} \end{cases} \tag{2.1}$$

Note that 'imitate the best' incorporates (i) an *ignorance of previous interactions* and (ii) a *bounded rationality assumption.* Property (i) describes the fact that the memory access of the behavioral rule is limited to the current interaction, while previous interactions are ignored. In comparison: the learning dynamics which I'll introduce later, incorporate behavioral rules

[7] Notice that a population of agents without restriction is also defined, namely if $\forall z \in P : NH_z = P \setminus \{z\}$, in other words if each agent's neighborhood consist of all other agents in the population.

that are defined with access to the past n interactions or even the whole history of previous interactions. Property (ii) describes the fact that the behavioral rule is *myopic* in the sense that it inhibits the agents to anticipate how the decision may effect future interactions. This property is in fact also given for all further dynamics that I'll introduce in this thesis.

Furthermore, Schlag (1998) calls a behavioral rule *improving* if it (weakly) increases expected payoffs in any decision situation, under the assumption of ignorance of previous interactions. Schlag mentioned that this property is important by comparing agent-based imitation models with population-based models like the replicator dynamics. I'll introduce a further imitation rule related to 'imitate the best' which (i) is improving and (ii) is, in terms of systemic behavior, strongly connected to the replicator dynamics: it is called *conditional imitation*.

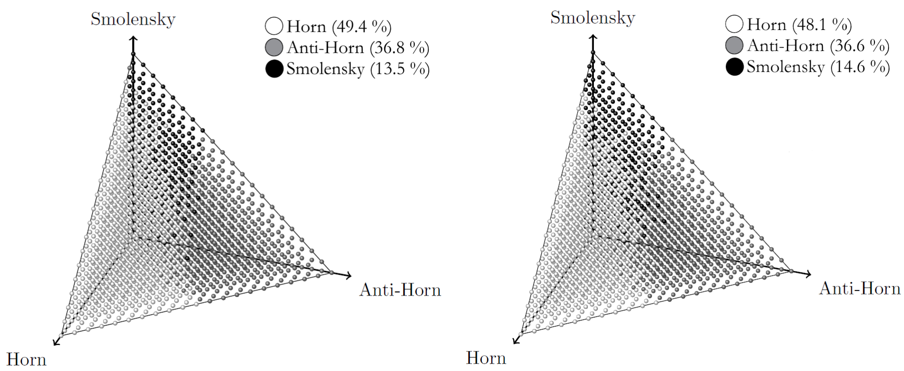
2.2.2 Conditional Imitation

The behavioral rule of 'imitate the best' lets an agent *always* switch to a strategy yielding a higher payoff; conditional imitation lets an agent switch *only with a specific probability*. I call $P(s \to s') \in (\Delta(S))^S$ the probability that an agent is switching from s to s'. Let $I^* \in \max(I(NH_z))$ be the maximal income among NH_z and let $z^* \in NH_z{}^*$ be a randomly chosen agent of NH_z with maximal income; let s_z be the old strategy of agent z and s_{z^*} the old strategy of agent z^*. In addition, let α be an arbitrary scaling factor, P_{min} the minimal and P_{max} the maximal payoff value of the game table, then the behavioral rule for conditional imitation is given as follows:

$$P(s_z \to s_{z^*}) = \begin{cases} 0 & \text{if } I_z \geq I^* \\ \alpha \times \frac{I^* - I_z}{P_{max} - P_{min}} & \text{else} \end{cases} \tag{2.2}$$

The behavioral rule exhibits that the probability $P(s \to s')$ of switching from strategy s_z yielding income I_z to a strategy s_{z^*} yielding a higher income I^* depends on the difference between I_z and I^*: the higher the positive difference between the incomes of agent z and agent z^* ($(I^* - I_z) \geq 0$), the higher the probability that agent z switches to strategy s_{z^*}.

I mentioned in the beginning of this section that the following claim could be confirmed by e.g. Helbing (1996) or Schlag (1998): it can be shown that the replicator dynamics describes the most likely path of strategy distributions in a virtually infinite and homogeneous population, when every agent updates her behavior by conditional imitation. But neither Helbing



(a) Replicator Dynamics (b) Conditional Imitation

Figure 2.7: Comparing the basins of attraction for the replicator dynamics (Figure 2.7a) and the conditional imitation rule by numerical simulation for a population of 50 randomly interacting agents (Figure 2.7b).

nor Schlag considered i) signaling games and ii) various population structures. In the following section I'll analyze the similarity of conditional imitation and replicator dynamics for populations of agents playing a signaling game; additionally I'll take different heterogeneous population structures into account to analyze how spatial features may change the aggregate population behavior.

2.2.3 Comparing Replicator Dynamics and Imitation

I showed in Section 2.1 that the Horn game has three relevant attractor states for the replicator dynamics (e.g. Figure 2.3 on page 39, Figure 2.6 on page 47). In addition, these attractor states have different sizes of basins of attraction. Given the connection between replicator dynamics and the conditional imitation update rule explained above, the similar experiments for conditional imitation are expected to show a strong conformity of the basins of attraction for both dynamics. For that purpose I started simulation runs of 50 agents interacting randomly by playing the asymmetric static Horn game HG^a ($P(t_f) = .75$, $C(m_u) = .1$ and $C(m_m) = .2$) and updating via conditional imitation for different initial population states. Here each agent used the same sender and receiver strategy at the beginning. Figure 2.7b shows the resulting basins of attraction of the simulations whose extents are similar to the ones of the replicator dynamics, depicted in Figure 2.7a.

First, the differences of the sizes of the basins of attraction for both dynamics are minute: while the replicator dynamics' basin of attraction of

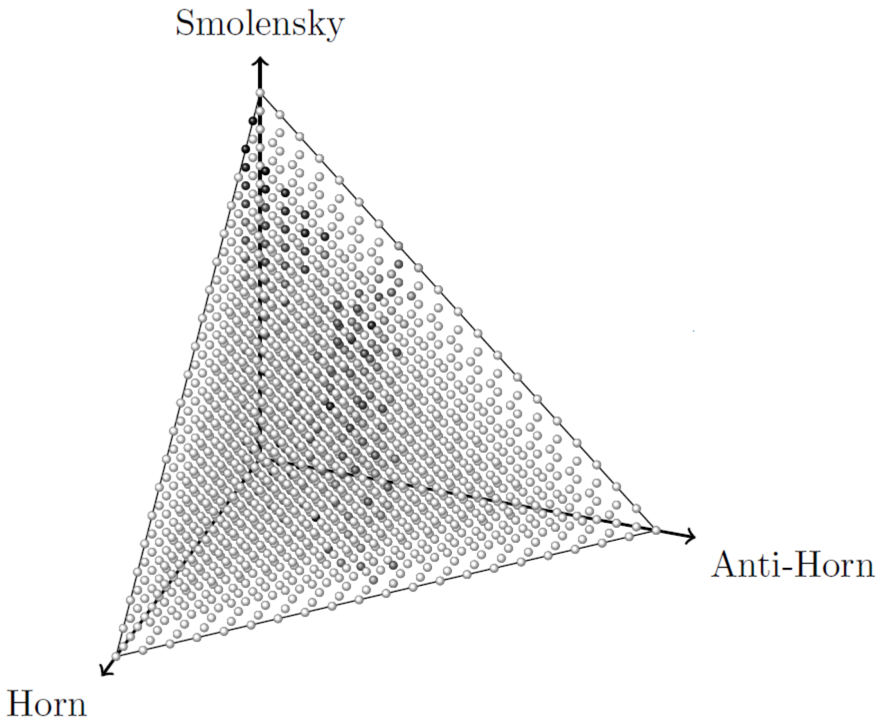


Figure 2.8: Similarity of the spreadings of basins of attraction for the replicator dynamics and the conditional imitation rule by numerical simulation for a population of 50 randomly interacting agents. The darker the spheres, the higher the deviation.

Horn occupies a share of 49, 4%, the basin of attraction of Horn for the conditional imitation update rule for 50 agents amounts to 48, 1%. Further, the basins of attraction for anti-Horn (36, 8% to 36, 6%) and Smolensky (13, 5% to 14, 6%) reveal a similar size comparing both dynamics. Nevertheless, there are small differences; in this regard it would be interesting to see where and how strongly the basins of attraction differ.

Figure 2.8 shows the similarity of both dynamics' spreading of their basins of attraction, computed in the following way: for each initial population I made 10 simulation runs with 50 randomly interacting agents playing the Horn game and updating via conditional imitation. Now, the more final population states of these ten runs corresponded to the final population of the replicator dynamics, the higher the similarity of both dynamics for that initial population state; and the lighter the appropriate spheres shading in Figure 2.8. E.g. if all 10 simulations' final population states corresponded to the result of the replicator dynamics, the sphere is white, if none of them, the sphere is black. For a number between 0 and 10 the sphere has an appropriate gray shading.

Figure 2.8 depicts the resulting pattern that displays where the basins of attraction conform or differ. Not surprisingly the deviations are at the borders of the three basins of attraction, whereas most of the space of the simplex coincides among both dynamics: this experiment reveals a similarity of 93.8% for both replicator dynamics' and conditional imitation's spreading of basins of attraction.

2.2.4 Integrating Network Structure

While the population-based replicator dynamics abstracts from a specific interaction structure, the individual-based imitation dynamics allows for different arrangements of interaction. As a first illustration that interaction structure matters, I would like to examine what happens to the sizes of these basins of attraction when we look at network games with different kinds of social interaction structures. While Figure 2.7b (page 51) shows the resulting pattern for a completely connected and therefore homogeneous network, I conducted the same experiment for other types of heterogeneous network structures to get a good glimpse at the consequences of the assumption or omission of homogeneity for evolutionary dynamics.

For this purpose, I made the following experiments: 100 agents are placed on a network structure, so that each agent is connected to a subset of all other agents in the population. Agents can only interact with those other agents with whom they are connected. A simulation run ends when all agents play the same strategy. This happened in each simulation run after a couple of simulation steps. I applied different network structures which I will introduce and define in Chapter 3: two types of so-called *small-world networks*, one that is called a *scale-free network* and one that is called *β-graph*; and furthermore a *grid network*.[8] For each network type I conducted 20 simulation runs.

Figure 2.9 depicts the average sizes of basins of attraction for a complete network, the scale-free network, the β-graph and the grid network, each involving a population of 100 agents playing the asymmetric static Horn game and updating via conditional imitation among connected agents. The

[8]See Chapter 3, Definition 3.22 (page 87) for the small-world network, Definition 3.23 (page 87) for the scale-free network and Figure 3.1b (page 85) for an example of a grid network. Furthermore, in my simulations I used a β-graph with $k = 4$, $\beta = .1$ (see Section 3.2.3), for the scale-free network I used the Holme-Kin algorithm with $m = 3$ and $P_t = .8$ (see Section 3.2.3), the grid network is a 10×10 toroid lattice (see Section 3.2.1).

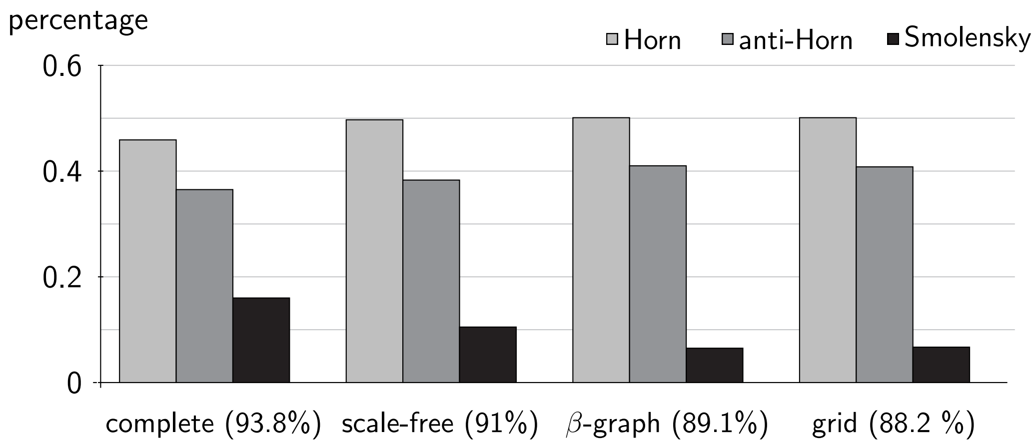


Figure 2.9: Comparing distributions of basins of attraction for the conditional imitation dynamics and its conformity to the replicator dynamics (percentage in brackets), for different network topologies: complete, scale-free, small-world and grid (10×10) with 100 agents each.

results differ not only in the distributions of the basins of attraction, but also in the correspondence to the basin of attraction of the replicator dynamics, noted in brackets behind the particular network names. In addition, the results let one assume that a network structure that supports a fast spread of conventions like the scale-free network promotes the emergence of the Smolensky strategy and displays a higher RD-correspondence.

Note that the Smolensky strategy has a considerable significance for the Horn game in evolutionary dynamics, as I showed in Section 2.1 for the replicator dynamics: here the Smolensky strategy has a considerable basin of attraction for the asymmetric and symmetric static Horn game; it is further evolutionarily stable by considering only plausible strategy pairs. In addition, I showed in this section that the conditional imitation dynamics reveals an almost identical result for the Horn game's basins of attraction. And now, by considering more heterogeneous network structures for the conditional imitation dynamics, the basin of attraction of the Smolensky strategy is shrinking with the *locality* (see Chapter 4) of the network structure. I will show in further experiments of Chapter 4 and 5 that the degree of locality of network structures has a strong influence on the way different strategies evolve and stabilize.

In general, these results are evidence enough for the simple, but important conceptual point I would like to make here: paying attention to social factors of interaction has non-trivial effects on the results of evolutionary processes; this is particularly true for the emergence or non-emergence of unexpected resulting population states for the Horn game. Note that for this experiment the final outcome of each simulation run was a population that uses one strategy society-wide. In the following experiment, I will show cases with more varied results.

2.2.5 Imitation on a Grid: Some Basic Results

To get a better impression of how a resulting society structure looks like for simulating 'imitate the best' (as introduced in Section 2.2.1) on a grid network, I reproduced a study by Zollman (2005). He analyzed the behavior of agents playing the Lewis game on a toroid lattice by updating with 'imitate the best'. Considering that i) signaling games are a model for simulating the emergence of conventions and ii) conventions arise in populations usually consisting of more than two persons, in accordance with my argumentation, Zollman argued for a more realistic framework than the existing *replicator dynamics* accounts, where each agent communicates with any other at random. As a consequence, I performed experiments similar to those of Zollman.

First, I started the following experiment: 1600 agents are placed on a 40×40 toroid lattice and playing the symmetric static Lewis game, thus they can choose among 16 languages, as depicted in Table 2.3; and they update by the 'imitate the best' dynamics. The resulting pattern is depicted in Figure 2.10: like in Zollman's experiments regions of two regional meanings emerge and stabilize, each of agents with solely using L_1 or L_2, thus exactly both strategy pairs that constitute a signaling system.

In the next experiment, I analyzed the behavior of a society of agents playing the asymmetric static Lewis game. Here an agent is sometimes a) in the sender role and chooses among the four pure sender strategies s_1, s_2, s_3 and s_4 or b) in the receiver role and chooses among the four different pure receiver strategies r_1, r_2, r_3 and r_4. Again, each agent updates by the 'imitate the best' dynamics, this time for each role. Each agent tries to find the optimal sender and the optimal receiver strategy as well. I started the following experiment: 900 agents are placed on a 30×30 toroid lattice and play the asymmetric static Lewis game. The resulting patterns are depicted in Figure 2.11, where each cell is marked by the agent's sender strategy with the color of the cell's top left half and by the agent's receiver strategy with the color of the cell's bottom right half.

Figure 2.11a depicts an exemplary pattern of a start situation where ev-

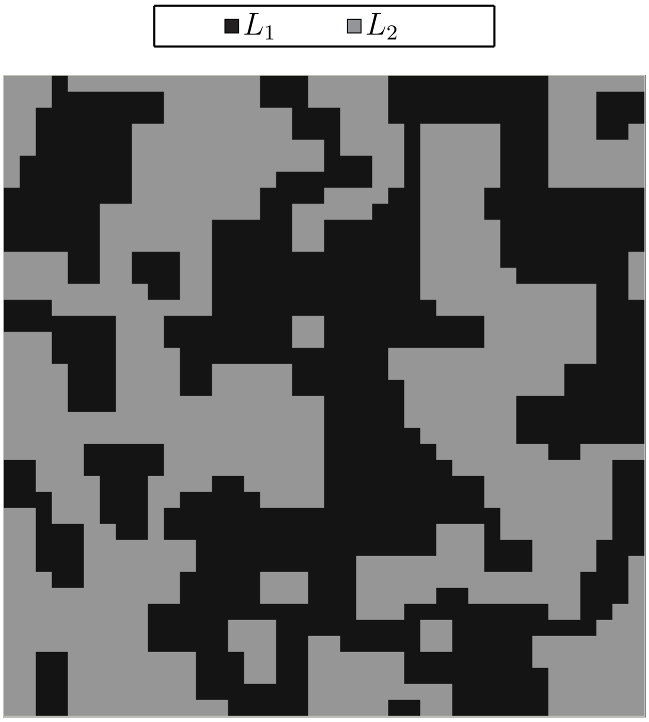
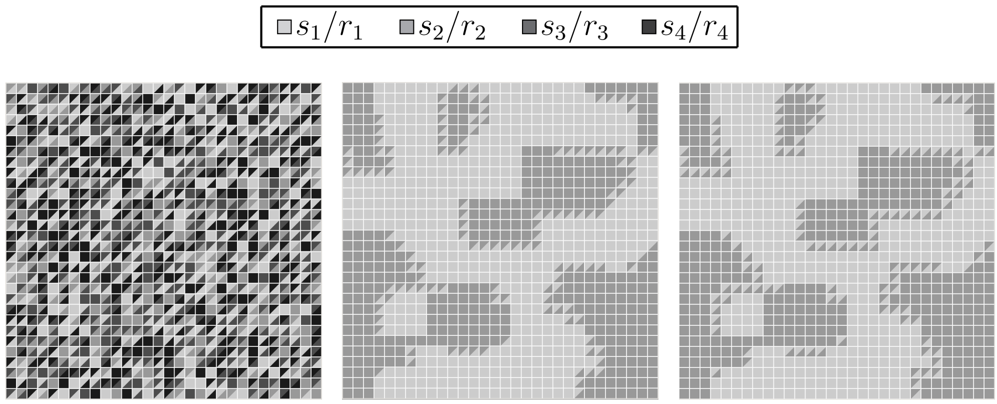


Figure 2.10: Exemplary resulting pattern for agents playing a symmetric static Lewis game and updating via 'imitate the best', on a 40×40 toroid lattice.

ery agent initially plays a randomly assigned strategy $s_i \in \{s_1, s_2, s_3, s_4\}$ as sender and also a randomly assigned strategy $r_i \in \{r_1, r_2, r_3, r_4\}$ as receiver. Figures 2.11b and 2.11c both depict an exemplary pattern for a final situation. Regions of agents playing the strategy pair $\langle s_1, r_1 \rangle$ or $\langle s_2, r_2 \rangle$ emerged. Between these regions border agents emerged switching in each step between $\langle s_1, r_2 \rangle$ and $\langle s_2, r_1 \rangle$. Figure 2.11b depicts the final pattern for each even simulation step, Figure 2.11c depicts the final pattern for each odd simulation step which shows that border agents switch each simulation step. It is important to note that the behavior of those switching agents on the border between two regions is a result of *synchronous update* (everybody adapts his behavior at the same time); this possibly converges to more realistic results without switching agents by using a more general *asynchronous update*. Nevertheless, this result gives a hint for the particular role of agents at the border between different language regions.

To conclude at this point: in this section I switched from a macro-level to a micro-level perspective by changing from replicator dynamics to imitation dynamics. Furthermore, I showed i) the similarity of replicator and



(a) Initial pattern (b) Resulting pattern 1 (c) Resulting pattern 2

Figure 2.11:

imitation dynamics and ii) the remarkable influence of network structure. In the next section, I will stay at a micro-level perspective, but switch from static to dynamic signaling games, as known from Chapter 1, and combine them with two kinds of learning dynamics: *reinforcement learning* and *belief learning*.

2.3 Learning Dynamics

The learning dynamics are a next step in the direction of considering a more fine-grained setup. Note that by switching from replicator dynamics to imitation dynamics, I switched from population-based dynamics to agent-based dynamics: instead of considering population proportions, the behavior of each particular agent is modeled. This allows me i) to give up the idealization of an infinite population size and ii) to integrate heterogeneous population structures. Furthermore, the switch from imitation to learning dynamics allows that a (possibly unlimited) number of previous interaction steps guides an agent's decision. The number of steps an agent can keep in mind is called her memory size. In this sense, imitation has a memory size of 1. In contrast, the essence of learning dynamics is to factor in a larger number of previously acquired inputs/observations.

But there is an additional substantial difference between the imitation dynamics and learning dynamics employed in this thesis: while the introduced types of imitation dynamics are applied on static signaling games (in accordance with the replicator dynamics), the learning dynamics are applied on dynamic/sequential signaling games; i.e. the standard account

dynamics type	replicator	imitation	learning
perspective	macro-level (population-based)	micro-level (agent-based)	micro-level (agent-based)
population	homogeneous	variable	variable
population size	infinite	finite	finite
memory size	-	1	$n \in \mathbb{N}$
game type	static	static	dynamic/sequential
strategy	pure	pure	behavioral

Table 2.10: Overview of structural and environmental differences of replicator, imitation and learning dynamics.

of a signaling game as given by Lewis and as defined in Chapter 1. This is an important conceptual difference that leads to the fact that agents play so-called *behavioral strategies* (Definition 2.10 on page 59) instead of pure ones. An overview of correspondences and differences between the three types of dynamics is displayed in in Table 2.10.

In a learning dynamics account for signaling games sender and receiver act according to *response rules* that are adjusted over time by *update rules* which are defined by an *update mechanism*. All in all, a learning dynamics account can be defined as follows:

Definition 2.9 (Learning Dynamics)**.** *A learning dynamics for a signaling game is given as* $D = \langle SG, \sigma, \rho, M_U \rangle$*, where*

- $SG = \langle (S, R), T, M, A, P, C, U \rangle$ *is a dynamic signaling game*
- σ *is the response rule of sender* S
- ρ *is the response rule of receiver* R
- M_U *is the update mechanism*
- σ *and* ρ *constitute behavioral strategies*

In the following I'll i) introduce *behavioral strategies*, ii) define the notions of *language learner* and *long-term stability*, and iii) present *response rules* and *update mechanisms* for the learning dynamics *reinforcement learning* and *belief learning*.

2.3.1 Languages, Learners and Stability

Recall that a static signaling game is defined by the *EU* tables of all pure strategy combinations (see Table 2.1), for which sender and receiver choose simultaneously one of the pure strategies, or contingency plan, and get the corresponding payoff. Huttegger and Zollman (2011) argued that the fact that agents' behavior is committed to whole contingency plans isn't plausible for learning (page 169). Here, as opposed to static games, agents play a dynamic signaling game, and, instead of selecting among pure strategies, they probabilistically choose a particular move (sender's message or receiver's interpretation state) for a given *choice point*. This can be modeled by a *behavioral strategy* which is a function that maps choice points to probability distributions over moves available in that choice point. A behavioral sender and receiver strategy can be defined as follows:

Definition 2.10 (Behavioral strategy)**.** *Given is a signaling game $SG = \langle (S, R), T, M, A, P, C, U \rangle$ with a sender S, a receiver R, a set of information states T, a set of messages M, a set of interpretation states A, a probability function over information states $P \in \Delta(T)$, a cost function $C : M \to \mathbb{R}$ and a utility function $U : T \times M \times A \to \mathbb{R}$. A behavioral sender strategy σ is a function that maps information states $t \in T$ to probability distributions over messages $m \in M$; a behavioral receiver strategy ρ is a function that maps messages $m \in M$ to probability distributions over interpretation states $a \in A$, thus:*

- $\sigma : T \to \Delta(M)$
- $\rho : M \to \Delta(A)$

Note that the pure strategies of a signaling game constitute a subset of the set of behavioral strategies which is infinite. To see an example, four different behavioral strategies for the Lewis game are depicted in Figure 2.12, two sender (σ_a, σ_b) and two receiver (ρ_a, ρ_b) strategies. Note also that the strategies σ_b and ρ_b constitute exactly the pure strategies s_1 and r_3 of the Lewis game (see Figure 1.1 on page 10).

Language Learner

It is possible to compute the proximity between two behavioral strategies via a measure known as the *Hellinger similarity* which, in general, measures

$$\sigma_a = \begin{bmatrix} t_l \mapsto & \begin{bmatrix} m_1 \mapsto & .9 \\ m_2 \mapsto & .1 \end{bmatrix} \\ t_s \mapsto & \begin{bmatrix} m_1 \mapsto & .5 \\ m_2 \mapsto & .5 \end{bmatrix} \end{bmatrix} \qquad \rho_a = \begin{bmatrix} m_1 \mapsto & \begin{bmatrix} a_l \mapsto & .3 \\ a_s \mapsto & .7 \end{bmatrix} \\ m_2 \mapsto & \begin{bmatrix} a_l \mapsto & 1 \\ a_s \mapsto & 0 \end{bmatrix} \end{bmatrix}$$

$$\sigma_b = \begin{bmatrix} t_1 \mapsto & \begin{bmatrix} m_1 \mapsto & 1 \\ m_2 \mapsto & 0 \end{bmatrix} \\ t_2 \mapsto & \begin{bmatrix} m_1 \mapsto & 0 \\ m_2 \mapsto & 1 \end{bmatrix} \end{bmatrix} \qquad \rho_b = \begin{bmatrix} m_1 \mapsto & \begin{bmatrix} a_1 \mapsto & 1 \\ a_2 \mapsto & 0 \end{bmatrix} \\ m_2 \mapsto & \begin{bmatrix} a_1 \mapsto & 1 \\ a_2 \mapsto & 0 \end{bmatrix} \end{bmatrix}$$

Figure 2.12: Four different behavioral strategies, two sender and two receiver strategies. σ_b and ρ_b constitute the pure strategies s_1 and r_3 of the Lewis game.

the similarity of two probability distributions. It is defined in the following way:

Definition 2.11 (Hellinger similarity)**.** *For two given probability distributions P and Q over the same set X, the Hellinger distance between them is defined as follows:*

$$sim_H(P, Q) = 1 - \sqrt{1 - \sum_{x \in X} \sqrt{P(x) \times Q(x)}}$$

The Hellinger similarity can range between 0 and 1, where 1 is given for identical strategies. By regarding that a language is a pair of pure strategies $L_{ij} = \langle s_i, r_j \rangle$, the Hellinger similarity can be used to describe how close a pair of behavioral sender and receiver strategy $\langle \sigma, \rho \rangle$ of a role-switching agent is to one of the pure strategies, by just calculating the average of the distance of both behavioral strategies to the pure strategies of a language. I call the similarity between a behavioral strategy pair and a language *language similarity* and define it as follows:

Definition 2.12 (Language Similarity)**.** *For a given pair of behavioral sender and receiver strategy $\langle \sigma, \rho \rangle$ its* language similarity *sim_L to a language $L_{ij} = \langle s_i, r_j \rangle$ is defined in the following way:*

$$sim_L(\langle \sigma, \rho \rangle, L_{ij}) = \frac{sim_H(\sigma, s_i) + sim_H(\rho, r_j)}{2}$$

The more similar an agent's pair of behavioral sender and receiver strategy is to a given language L, the more consistent is her behavior to act according to the language L. Consequently, I stipulate that if the language similarity between an agent's current behavioral strategy pair and one of the possible languages is above a given threshold[9], the agent's current behavior is close enough to be called as behaving according to the appropriate language L; i.e. I allege that the agent has learned or is using this language L. Thus a *language learner* is defined as follows:

Definition 2.13 (Language Learner)**.** *If an agent is playing a dynamic signaling game SG repeatedly by using a learning dynamics and is switching between sender and receiver role, her behavior can be characterized by a behavioral strategy pair* $\langle\sigma, \rho\rangle$. *If for a language L the language similarity* $sim_L(\langle\sigma, \rho\rangle, L)$ *is above a given threshold* h_ϵ*, then the agent is called a* language learner *of language* L.

This indirectly includes that if the the language similarity of current behavioral strategy pair isn't above the threshold h_ϵ for all possible languages $L_{ij} \in \mathbf{L}$[10], thus $\forall L_{ij} \in \mathbf{L} : sim_L(\langle\sigma, \rho\rangle, L_{ij}) < h_\epsilon$, then the agent hasn't learned a language; i.e. she is not a learner of any language.

With these prerequisites it is possible to compare a) dynamic processes and resulting states in populations of agents playing dynamic signaling games by using learning dynamics with b) our former results for populations of agents playing static signaling games by using replicator or imitation dynamics. While for replicator and imitation dynamics agents switch among languages, for learning dynamics agents can learn languages. They learn it by playing behavioral strategy pairs that approximate pure strategy pairs depicting these languages. But since measures of evolutionary stability are not applicable for behavioral strategies, an important question has to be answered for learning dynamics: when can a language learner considered as stable in her behavior?

Long-Term Stability

As I will show in my experiments in Chapter 4 and Chapter 5, agents generally become language learners after a while. It is not easy to find a formal definition for the stability of those agents' behavior once they have learned a language. Thus I simply define their stability as a consistent long-term behavior that I call *long-term stability*. The basic idea is the

[9]In general the threshold should be close to 1, but at least higher than .5 to ensure that an agent can be a learner of not more than one language at the same time.

[10]$\mathbf{L}$ is the set of all possible languages of a given signaling game SG.

following: a whole society is defined as constituting a stable pattern if at least a specific proportion p of the society has learned a language and no member of this proportion changes it - at least for the length t of a long-term simulation run. The question is how to define the values p and t? First, my simulation results revealed that if around $\frac{2}{3}$ of a society have learned a language, the pattern doesn't change that much, e.g. almost all of those agents will never change their strategy during the whole simulation run. To control the long-term consistency I checked this with a couple of long-term simulation runs, arranged in the following way: when at least 66.6% of all agents have learned a language at simulation step τ, I conducted the simulation run until simulation step $10 \times \tau$. It revealed that in all of such runs above two thirds of the population stayed with the learned language. Consequently, these experiments let me come up with the notion of *long-term stability*, defined as follows:

Definition 2.14 (Long-Term Stability)**.** *Given is a population of agents $z \in Z$. If there is a subset $Z_s \subseteq Z$ at simulation step τ with the following properties*

- $\frac{|Z_s|}{|Z|} \geq \frac{2}{3}$
- $\forall z \in Z_s$ *: z has learned a language L*
- $\forall t$ *with* $\tau \leq t < 10 \times \tau : \forall z \in Z_s$ *: agent z uses the same language L at simulation step t*

then each agent $z \in Z_s$ is long-term stable at simulation step τ.

Note that long-term stability does not guarantee that these agents will never change their behavior. But since i) I have no mutants or external influences that change circumstances in my experiments, ii) p is high enough that a strong modification of the language pattern among the society is hard to expect (where should it come from?) and iii) there was no change to observe for a multiple of the number of considered simulation steps, then it is highly improbable that a long-term stable agent will change at a later point in time. In fact, all my experiments revealed that in every experiment each long-term stable agent stayed stable for the whole simulation run.

In conclusion, notions like language learner and long-term stability are essential for the analysis and commensurability of my experiments with the two in this thesis applies learning dynamics, described in the next sections: *reinforcement learning* and *belief learning*.

2.3.2 Reinforcement Learning

Reinforcement learning can be captured by a simple model based on urns, also known as *Pólya urns* (Roth and Erev 1995; Skyrms 2010). An urn models a behavioral strategy in the sense that the probability of making a particular decision is proportional to the number of balls in the urn that correspond to that action choice. By adding or removing balls from an urn after each encounter, an agent's behavior is gradually adjusted.

For a given signaling game $SG = \langle (S,R), T, M, A, P, C, U \rangle$ the sender has an urn $\mho_t$ for each state $t \in T$ which contains balls for different messages $m \in M$. The number of balls of type m in urn $\mho_t$ is designated with $m(\mho_t)$, the overall number of balls in urn $\mho_t$ with $|\mho_t|$. If the sender is faced with a state t, she draws a ball from urn $\mho_t$ and sends message m if the ball is of type m. In compliance, the receiver has an urn $\mho_m$ for each message $m \in M$ which contain balls for different interpretation states $a \in A$. The number of balls of type a in urn $\mho_m$ is designated with $a(\mho_m)$, the overall number of balls in urn $\mho_m$ with $|\mho_m|$. If the receiver wants to construe a message m, he draws a ball from urn $\mho_m$ and uses the interpretation a if the ball is of type a.

The resulting *response rules* for reinforcement learning are given in Equation 2.3 for the sender and 2.4 for the receiver.

$$\sigma(m|t) = \frac{m(\mho_t)}{|\mho_t|} \quad (2.3) \qquad \rho(a|m) = \frac{a(\mho_m)}{|\mho_m|} \quad (2.4)$$

The learning dynamics' *update mechanism* is realized by changing the urn content dependent on the communicative success. Let's call $y(\mho_x)^\tau$ the number of balls y in urn x at time τ and $U(t,m,a)^\tau$ the utility of the realized played signaling game at time τ, then for the classical account of reinforcement learning, called *Roth-Erev* reinforcement learning (c.f. Roth and Erev 1995), the update mechanism $M_U = \langle SG, u_\tau \rangle$ for a signaling game SG is realized by the update rule u_τ, as given by Equation 2.5 and 2.6.

$$m(\Omega_t)^{\tau+1} = \begin{cases} m(\Omega_t)^\tau + 1 & \text{if } U(t,m,a)^\tau > 0 \\ m(\Omega_t)^\tau & \text{else} \end{cases} \tag{2.5}$$

$$a(\Omega_m)^{\tau+1} = \begin{cases} a(\Omega_m)^\tau + 1 & \text{if } U(t,m,a)^\tau > 0 \\ a(\Omega_m)^\tau & \text{else} \end{cases} \tag{2.6}$$

Note that, according to the signaling games considered in this thesis, $U(t,m,a) > 0$ if and only if communication via $t \to m \to a$ is successful. In other words, Roth-Erev reinforcement adds one appropriate ball to sender and receiver urn in case of successful communication which makes the choice of such a combination more probable in subsequent plays. This relation between success and increasing probability defines the reinforcement effect.

The update mechanism I applied in my experiments is an extended version of Roth-Erev reinforcement. First, I integrated *lateral inhibition*, so that, for successful communication, not only does the number of the appropriate balls in the appropriate urn increase, but also does the numbers of all other balls in the same urn decrease. Furthermore, I integrated *negative reinforcement* so that the number of balls are decreased for unsuccessful communication.

In detail: the update mechanism $M_U = \langle SG, u_\tau, \alpha, \beta, \gamma \rangle$ is given by a signaling game SG, an update rule u_τ, a *reward value* $\alpha \in \mathbb{N}$, a *punishing value* $\beta \in \mathbb{N}$ and an *inhibition value* $\gamma \in \mathbb{N}$. If communication via t, m and a is successful, the number of balls in the sender urn $\mho_t$ is increased by α balls of type m and reduced by γ balls of type $m' \neq m$, $m' \in M$. Similarly, the number of balls in the receiver urn $\mho_m$ is increased by α balls of type a and reduced by γ balls of type $a' \neq a$, $a' \in A$. In this way successful communicative behavior is more probable to reappear in subsequent rounds.

Furthermore, unsuccessful communication via t, m and a is punished by reducing the number of balls of type m in the sender urn $\mho_t$ and the number of balls of type a in the receiver urn $\mho_m$ by β. Here the inhibition value γ is used as an inverse force by increasing the numbers of balls of type $m' \neq m$ in the sender urn $\mho_m$ and the numbers of balls of type $a' \neq a$ in the receiver urn $\mho_m$ by γ balls.

The account should warrant that the number of balls of each type cannot be negative. For this purpose a *lower limit value* φ is integrated to ensure this property. Given these predefinitions, the update rule u_τ is defined by sender update (Equations 2.7), sender inhibition (Equations 2.8), receiver update (Equations 2.9) and receiver inhibition (Equations 2.10).

$$m(\Omega_t)^{\tau+1} = \begin{cases} m(\Omega_t)^\tau + \alpha & \text{if } U(t,m,a)^\tau > 0 \\ \max(m(\Omega_t)^\tau - \beta, \varphi) & \text{else} \end{cases} \tag{2.7}$$

$$\forall m' \neq m : m'(\Omega_t)^{\tau+1} = \begin{cases} \max(m'(\Omega_t)^\tau - \gamma, \varphi) & \text{if } U(t,m,a)^\tau > 0 \\ m'(\Omega_t)^\tau + \gamma & \text{else} \end{cases} \tag{2.8}$$

$$a(\Omega_m)^{\tau+1} = \begin{cases} a(\Omega_m)^\tau + \alpha & \text{if } U(t,m,a)^\tau > 0 \\ \max(a(\Omega_m)^\tau - \beta, \varphi) & \text{else} \end{cases} \tag{2.9}$$

$$\forall a' \neq a : a'(\Omega_m)^{\tau+1} = \begin{cases} \max(a'(\Omega_m)^\tau - \gamma, \varphi) & \text{if } U(t,m,a)^\tau > 0 \\ a'(\Omega_m)^\tau + \gamma & \text{else} \end{cases} \quad (2.10)$$

All in all, reinforcement learning with lateral inhibition and punishment can be defines in the following way:

Definition 2.15 (Reinforcement Learning with Inhibition and Punishment)**.** *A reinforcement learning dynamics with lateral inhibition and punishment is given as a learning dynamics account* $RLIP = \langle SG, \sigma, \rho, M_U \rangle$*, where*

- $SG = \langle (S,R), T, M, A, P, C, U \rangle$ *is the signaling game*
- σ *is the response rule of sender* S *as given with Equation 2.3*
- ρ *is the response rule of receiver* R *as given with Equation 2.4*
- $M_U = \langle SG, u_\tau, \alpha, \beta, \gamma \rangle$ *is the update mechanism for signaling game* SG*, with parameters* $\alpha, \beta, \gamma \in \mathbb{N}$ *for the update rule* u_τ*, as given by the Equations 2.7, 2.8, 2.9 and 2.10*

2.3.3 Belief Learning

There are some preeminent properties that distinguish belief learning from reinforcement learning, as Skyrms (2010) pointed out: "Reinforcement learners do not have to know their opponent's payoff; they do not have to know the structure of the game. If acts are reinforced, they do not have to know that they *are* in a game. But now we move up a level. Individuals know that they are in a game. They know the structure of the game. They know how the combination of others' actions and their own affect their payoffs. They can observe actions of all the players in repeated plays of the game. They can think about all this." (page 90).

In addition, with reinforcement learning agents act non-rationally. I.e. they are just biased more and more in one or the other direction, whereas in this account of belief learning agents act rationally in that they play a *best response* by maximizing their *expected utilities*. This combination of learning information about other players' behavior, forming a belief out of it, and using this belief to play a best response is also known as *fictitious play*, introduced by Brown (1951).

Belief learning can be formalized as follows: I call $EU_s(m|t)$ the sender's expected utility for sending message m in state t and $EU_r(a|m)$ the receiver's expected utility for construing message m with interpretation a. A rational sender that wants to communicate state t will use the message m that maximizes her expected utility $EU_s(m|t)$. Accordingly, a rational receiver who received message m will construe it with the interpretation a that maximizes his expected utility $EU_r(a|m)$. If there are more choices which maximize those EU's, then each choice is equiprobable. Formally: if $MAX(t) = arg\max_m EU_s(m|t)$ is the set of messages where each one maximizes the sender's expected utility for a given state t and $MAX(m) = arg\max_a EU_r(a|m)$ is the set of interpretation states where each one maximizes the receiver's expected utility for a given message m, then the sender's response rule σ and the receiver's response rule ρ both can be given as follows:

$$\sigma(m|t) = \begin{cases} \frac{1}{|MAX(t)|} & \text{if } m \in MAX(t) \\ 0 & \text{else} \end{cases} \tag{2.11}$$

$$\rho(a|m) = \begin{cases} \frac{1}{|MAX(m)|} & \text{if } a \in MAX(m) \\ 0 & \text{else} \end{cases} \tag{2.12}$$

The sender's expected utility $EU_s(m|t)$ returns the utility value the sender can expect for sending message m in state t. But this expected value depends on what she believes the receiver would play. Her *belief about the receiver's strategy* $B_r(a|m)$ is a function returning the sender's believed probability that the receiver construes message m with a. Given this belief, the sender's expected utility is defined in the following way:

$$EU_s(m|t) = \sum_{a \in A} B_r(a|m) \times U(t, m, a) \tag{2.13}$$

The receiver's expected utility $EU_r(a|m)$ returns the value the receiver can expect for construing a received message m with interpretation a. Thus he needs to have a *belief about the sender's strategy* $B_s(t|m)$ that returns the receiver's believed probability that the sender is in state t by sending message m. Accordingly, the receiver's expected utility is defined as follows:

$$EU_r(a|m) = \sum_{t \in T} B_s(t|m) \times U(t, m, a) \tag{2.14}$$

But where do these beliefs B_r and B_s come from? The belief learning account of my model engenders a process of acquiring these beliefs by observation. Concretely, a player's belief is a mixed strategy representing all the interlocutor's observed past plays. E.g. assume that sender and receiver had the same kind of communicative situation many times before and that function $\sharp_{S,R}(m)$ is a counter that returns the number of times the sender S has sent message m to the receiver R. Likewise $\sharp_S(a|m)$ returns the number of times the sender has observed the receiver interpreting the sender's sent message m with a. Because of these observations the sender has the following belief $B_r(a|m)$ about the receiver:

$$B_r(a|m) = \begin{cases} \frac{\sharp_S(a|m)}{\sharp_{S,R}(m)} & \text{if } \sharp_{S,R}(m) > 0 \\ 1/|A| & \text{else} \end{cases} \tag{2.15}$$

In the same way an evaluation of the receiver's observations $\sharp_R(m|t)$ about the sender's behavior leads to belief $B_s(t|m)$ about the sender:

$$B_s(t|m) = \begin{cases} \frac{\sharp_R(t|m)}{\sharp_{S,R}(m)} & \text{if } \sharp_{S,R}(m) > 0 \\ 1/|T| & \text{else} \end{cases} \tag{2.16}$$

Notice that both equations contain the condition that the denominator $\sharp_{S,R}(m)$ must be greater than zero. This is not only to avoid a division by zero. It has also a descriptive reason: if there has never been a communicative situation by using message m, then both participants cannot have beliefs through past observations. In this case the probabilities for this message are uniformly distributed, for the sender given by a uniform distribution over all possible interpretations $a \in A$ ($1/|A|$) and for the receiver accordingly over all possible states $t \in T$ ($1/|T|$).

I define $\sharp(m)^\tau$ as the number of times m was topic until time τ. Further, I define $\sharp(a|m)^\tau$ and $\sharp(t|m)^\tau$ in the same way. Then the update mechanism $M_U = \langle SG, u_\tau \rangle$ for a signaling game SG is given by update rule u_τ that simply increments the number of observed situations of the sender (Equation 2.17), the receiver (Equation 2.18), and the number of situations the message is conveyed among both (Equation 2.19), in the case that communication happened via $t \rightarrow m \rightarrow a$:

$$\sharp_S(a|m)^{\tau+1} = \sharp_S(a|m)^\tau + 1 \tag{2.17}$$

$$\sharp_R(t|m)^{\tau+1} = \sharp_R(t|m)^\tau + 1 \tag{2.18}$$

$$\sharp_{S,R}(m)^{\tau+1} = \sharp_{S,R}(m)^\tau + 1 \tag{2.19}$$

After every communication situation in which message m is used, the sender's belief $B_r(a|m)$ and the receiver's belief $B_s(t|m)$ as well will be updated. The belief of the interlocutor's strategy results from previous communications with a dialogue partner. All in all, a belief learning account that integrates i) learning by observation and ii) best response behavior can be defined as follows:

Definition 2.16 (Belief Learning)**.** *A belief learning dynamics is given as a learning dynamics account* $BL = \langle SG, \sigma, \rho, M_U \rangle$ *where*

- $SG = \langle (S, R), T, M, A, P, C, U \rangle$ *is the signaling game*
- σ *is the response rule of sender* S *as given with Equation 2.11*
- ρ *is the response rule of receiver* R *as given with Equation 2.12*
- $M_U = \langle SG, u_\tau \rangle$ *is the update mechanism for signaling game* SG *and update rule* u_τ*, realized by the Equations 2.17, 2.18 and 2.19*

To sum up, both learning accounts presented are i) made for being applied on dynamic signaling games and ii) defined by sender and receiver response rules and an update rule. Note: while the update rule of reinforcement learning incorporates and reconsiders the *success* of previous communicative situations, the update rule of belief learning does not. Here it only reconsiders exclusively the previous *behavior* of the interlocutors. Generally speaking: while reinforcement learners adapt directly to successful behavior, belief learners i) adapt to interlocutors' behavior by learning beliefs and ii) compute successful behavior by playing what maximizes their expected utility. In this sense, only belief learning demands rationality, whereas for reinforcement learning agents make decisions randomly and biased by former results.

2.3.4 Memory Size and Forgetting

A general observation of both learning dynamics is that learned behavior manifests itself very early and ingrains itself in the dynamics. Barrett and Zollman (2009) showed that forgetting experiences increases both the dynamics of the system and the probability of an optimal language evolving. They introduced different learning accounts based on reinforcement learning and extended by different types of forgetting. I extend both reinforcement and belief learning accounts with a simple forgetting rule, informally described as follows.

Both learning accounts' response rules at time τ depend indirectly on a history of updates $H = \{u_1, u_2, \ldots, u_\tau\}$, where $u_i \in H$ is an update at time i via an appropriate update rule. Now let's assume that each agent has a *memory size* $\mu \in \mathbb{N}$ that affects her update mechanism as follows: at time τ undo all updates u_i with $i < \tau - \mu$. Thus, all updates that happened more than μ time steps ago are canceled and therefore have no influence on the response rule. In other words, an agent can't remember that they ever happened; i.e. she has forgotten them.

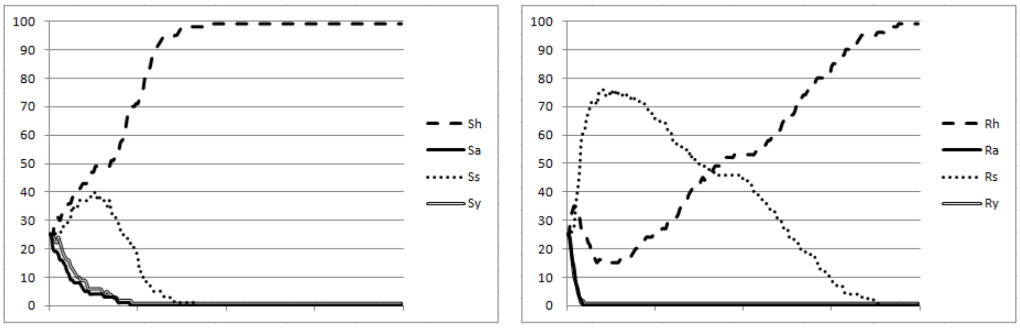
2.3.5 Learning in Populations: Some Basic Results

Recent studies have revealed significant similarities between reinforcement learning and the replicator dynamics (c.f. Beggs 2005; Hopkins and Posch 2005). E.g. Börgers and Sarin (1997) argue that the replicator dynamics can be seen as a limited case of reinforcement learning. But the application of reinforcement learning to model evolutionary processes in combination with signaling games has yet to be studied extensively. Most of the literature considers reinforcement learning for a simple two players account: a repeated game between a sender and a receiver. It has been proven for the Lewis game between a sender and a receiver that update by reinforcement learning ($\alpha = 1$, $\beta = 0$, $\gamma = 0$) that the participants' behaviors will converge (almost surely) to a signaling system (Argiento et al. 2009). In addition, it was shown for reinforcement learning that simulations of signaling games with non-equiprobable states leads to the learning of pooling equilibria which was also shown for replicator dynamics (Barrett 2006; Skyrms 2010).

But how similar are replicator dynamics and reinforcement learning if initial population states and basins of attraction are considered? And in what sense do basins of attraction differ by analyzing the Horn game? To find answers to these questions, I made some simulations with populations of agents playing the Horn game and updating via reinforcement learning. To emulate the random pairing characteristics of replicator dynamics, agents choose their partners randomly. But each agent can switch between sender and receiver, though each agent has sender urns and receiver urns.

To define initial populations states comparable to those of the experiments with the replicator dynamics, I define the value of *initial tendency* in the following way:

Definition 2.17 (Initial Tendency)**.** *If an agent's initial sender urn settings are as such that* $\forall t \exists m : \sigma(m|t) = \kappa$ *and the initial receiver urn settings are*



(a) sender strategy tendencies (b) receiver strategy tendencies

Figure 2.13: The time series of the Horn game for tendencies of sender strategies and receiver strategies.

as such that $\forall m \exists a : \rho(a|m) = \kappa$ *for a value* $0.5 < \kappa < 1$, *then the agent has the* initial tendency *of degree* κ *to a language* $L = (s, r)$ *that depicts the corresponding pure strategies with* $\kappa = 1$.[11]

For instance, if an agent that plays the Horn game has an initial urn setting that bears the behavioral strategies $\sigma(m_u|t_f) = .7$, $\sigma(m_m|t_r) = .7$, $\rho(a_f|m_u) = .7$, $\sigma(a_r|m_m) = .7$, then this agent has an initial tendency of degree .7 to language L_h: his urn setting depicts a tendency for playing the Horn strategy. With this definition it is possible to define population states of agents playing a particular language with a specific initial tendency.

In the experiment I started a simulation run with a population of 100 agents playing the Horn game, where 25% of all agents had an initial tendency for L_h, 25% for L_a, 25% for L_s and 25% for L_y, all tendencies with a degree of $\kappa = .7$. The reinforcement parameters were set to $\alpha = U(t, m, a)$, $\beta = 0$ and $\gamma = 0$, in other words the reward equals the utility value of a played round, whereas punishment and lateral inhibition are neglected. The initial number of balls were 50 per type. The resulting course of strategy learners over all agents in the population is depicted in Figure 2.13, Figure 2.13a for the sender strategy, 2.13b for the receiver strategy.[12]

The resulting patterns are quite similar to the results for the experiments in Section 2.1.5, where I analyzed the asymmetric static version of the Horn game for a 'two population' model under replicator dynamics. This reveals new insights in the similarity of both accounts. The experiments with the replicator dynamics showed that the sender population reveals an initial

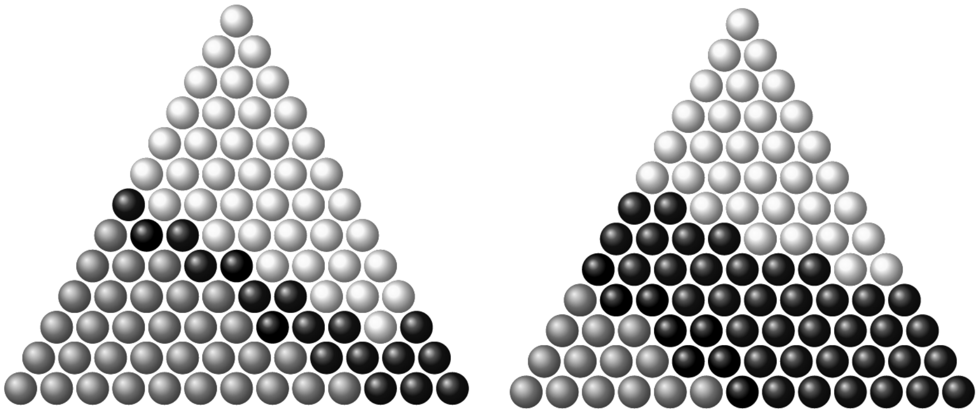
[11]Note that since a language depicts a pair of pure strategy, its κ-value is always 1.
[12]For this experiment the threshold h_ϵ for the Hellinger similarity was set to .65.

increase in the number of Horn strategy and Smolensky strategy players and a fast decrease in the number of anti-Horn and anti-Smolensky strategy players. And at one point the number of Smolensky players decreases to 0% and the number of Horn players is continuing increasing, finally to 100% (Figure 2.2a on page 38). A similar pathway is observable for the learned sender strategies of all players in the population of agents updating via reinforcement learning (Figure 2.13a). The same is true for the receiver population in the replicator dynamics experiments (Figure 2.2b on page 38) comparing with the learned receiver strategies for the reinforcement learning account (Figure 2.13b). In particular, the initially strong increase of the Smolensky strategy to almost 80% is observable in both cases.

While reinforcement learning and replicator dynamics reveal quite different properties and capabilities (see Table 2.10 on page 58), these results show the apparent similarities of both accounts. This puts forward the question of other scenarios in which similarities might exist. For that purpose I started simulation runs for different initial population states by changing the initial distribution of strategy tendencies. Like in the experiment of Section 2.1.6 for the plausible Horn game, I only considered the Horn language, the anti-Horn language and the Smolensky language and computed the final population states for different initial population states. The resulting simplexes for the weak and the normal Horn game[13] are depicted in Figure 2.14a and 2.14b, respectively.

The basins of attraction of this experiment are quite similar to those of the experiment with the replicator dynamics (Figure 2.6). However, it is apparent that the Smolensky strategy has a quite larger basin of attraction. Even though it can be argued that the resulting patterns are not that suitable for a general proposition because of the high number of parameters for the reinforcement learning account, it shows that the probability of the emergence of the Smolensky strategy still exists for populations of agents updating via reinforcement learning. And it is in fact quite larger than for the replicator dynamics account, at least for the parameters used in this experiment.

[13]The weak, normal and strong Horn game are defined by game parameters and were introduced on page 45.



(a) weak Horn game (b) normal Horn game

Figure 2.14: Basin of attractions for the reinforcement learning dynamics for initial population states of agents with different strategy tendencies.

2.4 Conclusion

In this chapter I introduced different update dynamics for repeated games and highlighted the similarities and differences in a case dependent way. I analyzed them applied on two basic variants of a signaling game that are objects of study in this thesis: the Lewis game and the Horn game. The classical account in the field of evolutionary game theory is the replicator dynamics. In Section 2.1 I tried to give an overview of the relevant literature that is as accurate as possible.

The basic results in research and analysis are the following: for the Lewis game a final population state with a homogeneous population playing exactly one of both signaling systems is expected. Which one evolves is first of all a question of the initial population state since the population state space is equally divided in two basins of attraction, one for each of the two signaling systems. Only these two states are evolutionarily stable and have a substantial invasion barrier which can be overcome by allowing for mutation, but only by switching to the other signaling system.

For the Horn game there are three strategies that are of special interest: the Horn and anti-Horn strategy which form the only two signaling systems; and the Smolensky strategy which depicts pooling with the cheapest message and the interpretation state matching the most likely information state. While the two signaling systems are evolutionarily stable, the Smolensky strategy is a non-evolutionarily stable Nash equilibrium and has a considerable basin of attraction. Furthermore, for a symmetric Horn game that is restricted to plausible strategy pairs the Smolensky strategy is evolutionarily stable, but with a relatively low invasion barrier against the Horn strategy. Nevertheless, the results show that the expected Horn strategy has not only with the anti-Horn strategy but also with the Smolensky strategy a formidable strategy that weakens the universality of Horn's rule. Like for the Lewis game, it is also the case for the Horn game that the initial population state and the possibility for mutation plays a decisive role as to whether one or the other strategy finally evolves.

In Section 2.2 I analyzed Lewis and Horn game by applying imitation dynamics on agents in a finite population. Though replicator dynamics and imitation dynamics strongly resemble each other in the way populations behave over time, the main difference is the switch from macro to micro perspective: replicator dynamics is a population-based dynamics, imitation dynamics an agent-based dynamics. By analyzing the Horn game, the conformity of basins of attraction and stable states among both dynamics is remarkable. But the agent-based perspective of imitation dynamics allows for structural variation. I was able to show that more local network topologies of agent populations reduce the basin of attraction of the Smolensky strategy. Furthermore, experiments with the Lewis game on a lattice structure revealed the emergence of multiple language regions that constitute *regional meaning*. This all gives a first impression that network structures can have a considerable impact on the course of a convention's evolution.

In Section 2.3 I introduced two types of learning dynamics: reinforcement learning and belief learning. For the former one agents make probabilistic decisions biased by previous success. For the latter one agents make decisions by playing best responses based on beliefs about their participants, where these beliefs are formed by memorizing previous behavior of them. In this sense, belief learning incorporates rationality according to its choice behavior.

Furthermore, there is an important technical difference between both types of learning dynamics and imitation dynamics: while for imitation dynamics (and also replicator dynamics) agents play a static signaling game and choose among contingency plans, for the learning dynamics agents play dynamic signaling games and use behavioral strategies: they choose between moves for given choice points. In this sense, learning dynamics is more fine grained and less abstract in comparison with imitation dynamics. Finally, experiments with reinforcement learning revealed that the resulting patterns for learning course and basin of attraction are quite similar to the results of experiments with the replicator dynamics.

All my experimental results reveal that by comparing all three dynamics types, the resulting population forces and dynamics are quite similar. Consequently, the question arises as to why I should analyze these games with more sophisticated dynamics if they do not bring any remarkable new insights in comparison with the replicator dynamics? In fact, one factor which turned out to make a difference, is worth analyzing more in detail: *the population structure*. While the results of the experiments with different topologies for the imitation dynamics revealed a considerable impact on the dynamics, it seems worthwhile to analyze different topologies for the learning dynamics as well.

In Chapter 3 I'll introduce concepts from network theory to have basic techniques to constitute and analyze more complex interaction structures. In Chapter 4 and Chapter 5 I'll analyze in depth the behavior of agents i) placed on a complex network structure, ii) interacting by signaling games and iii) acting according to learning dynamics.

Chapter 3

Social Networks

> *"Society does not consist of individuals, but expresses the sum of interrelations, the relations within which these individuals stand."*
>
> *Marx, Grundrisse: Foundations of the Critique of Political Economy*

The last chapter began with the premise that Lewis's signaling game originated in a framework of language evolution. Nevertheless, there are reasons to assume that the emergence of linguistic meaning modeled by signaling games are also appropriate to be considered under a sociolinguistic point of view. If we proceed on the assumption that actual language use is conventional and that *forces of linguistic change* realize the process of replacing old linguistic conventions with new ones, then signaling games are an adequate tool for analyzing processes of language change. Furthermore, the division between language evolution and language change is academically constructed, as the notions arose in different disciplines. By discounting such a sharp line I'll take a look at studies of language change that inspired me to take the social environment into account by analyzing the emergence of linguistic meaning via signaling games.

The idea to use social networks for the analysis of the emergence of linguistic meaning was inspired by studies that analyze language change via simulations on a quite general level. Several of these focused on the role of social structure (c.f. Nettle 1999; Ke et al. 2008; Fagyal et al. 2010). While Nettle (1999) took a *toroid lattice* grid structure into account, Ke et al. and

Fagyal et al. considered more *realistic small-world* network structures since these resemble interaction structures of real human networks. In addition, an early analysis of signaling games on social structures was made by Zollman (2005) for toroid lattices and then by Wagner (2009) on small-world networks. I'll present basic results from similar analysis in Chapter 4 for toroid lattices and in Chapter 5 for social networks.

In this chapter I'll introduce different notions from network theory. In Section 3.1 I'll introduce common *network properties*, in Section 3.2 I'll present several types of network structures that resemble more or less realistic human interaction structures. In Section 3.3 I'll give the basic idea of *network games*: applying signaling games on network structures. And finally in Section 3.4 I'll give a final conclusion.

3.1 Network Properties

A network is formally represented as an *undirected graph* $G = (N, E)$ where $N = \{1, \ldots, n\}$ is the set of nodes, and $E \subseteq N \times N$ is an irreflexive and symmetric ordering on N, also called the set of edges.[1] For such a given network G it is possible to compute specific properties of networks, subnetworks or particular nodes. In this sense, I want to distinguish between a) *structural properties* of whole (sub-)networks that consist of multiple nodes and edges and b) *node properties* that describe the position of one particular node in dependency of its environment.

3.1.1 Node Properties

Node properties describe the particular characteristics pertaining to the position of a given node inside a network on a quantitative and qualitative level. For further analysis the *centrality* and *embeddedness* of a node will play an important role. Before defining these properties I'll first introduce the following basic notions: *path* (between two nodes), *neighborhood* and *degree*.

[1] Although a couple of relation types could reasonably be modeled with asymmetric edges (directed graphs), I consider relationships as symmetric and therefore I take only undirected graphs into account.

Paths between nodes

A couple of important features of a network are based on the definition of a path between two nodes. Such a path can be defined in the following way:

Definition 3.1. *(Path) For a given graph* $G = (N, E)$ *with a set of nodes* $N = \{1, \dots, n\}$ *and a set of edges* $E = \{\{i, j\} | i, j \in N\}$*, a path* $P(i_1, i_m)$ *between node* i_1 *and node* i_m *is defined as a sequence of nodes* $P(i_1, i_m) = (i_k, i_{k+1}, \dots, i_m)$ *for each* $k \in \{1, \dots, m-1\}$*, where the following two conditions hold:*

1. $\forall k \in \{1, \dots, m-1\} : \{i_k, i_{k+1}\} \in E$
2. $\forall k, j \in \{1, \dots, m\} : i_k \neq i_j$

Thus, a path depicts a sequence of consecutively connected nodes, where each node is unique in this sequence. Furthermore, each path has a length that displays the number of nodes and can be defined in the following way:

Definition 3.2. *(Path Length) For a given path* $P(i_1, i_m) = (i_k, i_{k+1}, \dots, i_m)$ *with* $k \in \{1, \dots, m-1\}$*, the path length* $|P(i_1, i_m)|$ *is* $m - 1$*.*

Indeed, there may be multiple paths between two nodes. In particular, the *shortest path* between two nodes plays an important role for further definitions and can be defined as follows:

Definition 3.3. *(Shortest Path) For a given graph* $G = (N, E)$ *and two nodes* $i, j \in N$*, a path* $P(i, j)^*$ *between* i *and* j *is a shortest path between them if and only if for all paths* $P(i, j)$ *between* i *and* j*:* $|P(i, j)^*| \leq |P(i, j)|$*.*

Furthermore, the *shortest path length* between two nodes is given as follows:

Definition 3.4. *(Shortest Path Length) For two nodes* $i, j \in N$ *in a given graph* $G = (N, E)$*: if there is a shortest path* $P(i, j)^*$ *between* i *and* j*, then the shortest path length* $SPL(i, j) = |P(i, j)^*|$*, otherwise* $SPL(i, j) = \infty$*.*

Neighborhood and Degree

The neighborhood $NH(i)$ of a node is given by the following definition:

Definition 3.5. *(Neighborhood) For a node* $i \in N$ *in a given graph* $G = (N, E)$*, its neighborhood* $NH(i)$ *is given as*

$$NH(i) = \{j \in N | \{i, j\} \in E\}$$

Consequently, a neighbor of a given node can be defined as follows:

Definition 3.6. *(Neighbor) For a node $i \in N$ in a given graph $G = (N, E)$, a node $j \in N$ is a neighbor if and only if $j \in NH(i)$.*

An important property of a node i is its *degree* $d(i)$ that displays the number of i's neighbors and is simply defined as follows:

Definition 3.7. *(Degree) For a node $i \in N$ in a given graph $G = (N, E)$, its degree $d(i)$ is defined by its number of neighbors:*

$$d(i) = |NH(i)|$$

Centrality

There are a couple of interesting measures depicting the *centrality* of a given node regarding to different characteristics. The most local of all centrality measures is the *degree centrality.* It considers a node's degree in relation to the total number of nodes in the network and is defined as follows:

Definition 3.8. *(Degree Centrality) For a node $i \in N$ in a given graph $G = (N, E)$, its degree centrality $DC(i)$ is defined by its degree in relation to the possible maximal degree of being connected to all other $|N| - 1$ nodes, though:*

$$DC(i) = \frac{d(i)}{|N| - 1}$$

The following centrality values are more global in the sense that they interrelate a node's position to *all* other nodes in the network. E.g. *closeness centrality* describes the average shortest path length of a node to all other nodes in the network and can be defined as follows:

Definition 3.9. *(Closeness Centrality) For a node $i \in N$ in a given graph $G = (N, E)$, its closeness centrality $CC(i)$ is defined as the average shortest path length to all other nodes in the network:*

$$CC(i) = \frac{\sum_{j \in N} SPL(i, j)}{|N| - 1}$$

Finally, *betweenness centrality* of a node i describes the fraction of shortest paths among all other nodes in the graph of which i is a member. It can be defined as follows:

Definition 3.10. *(Betweenness Centrality) The function $NSP(j,k)$ returns the number of shortest paths between node j and k. Furthermore the function $MSP(j,k,i)$ returns the number of shortest paths between j and k of which node i is a member. Then for a node $i \in N$ in a given graph $G = (N,E)$, its betweenness centrality $BC(i)$ is defined as follows:*

$$BC(i) = \frac{\sum\limits_{j,k \in N} MSP(j,k,i)}{\sum\limits_{j,k \in N} NSP(j,k)}$$

Note that closeness and betweenness centrality both describe a node's position in dependence of all other nodes in the network. Consequently, both values depict a *global* centrality measure. Degree centrality takes only direct neighbors into account. It depicts a *local* centrality measure.

Embeddedness

The embeddedness of a node inside a local structure can be represented by its *individual clustering value.* It is given by the ratio of edges between a node's neighbors to all possible edges in the neighborhood (given by ${}^{d(i)\times(d(i)-1)}/_{2}$). The individual clustering value is defined as follows:

Definition 3.11. *(Individual Clustering) For a node $i \in N$ in a given graph $G = (N,E)$, its individual clustering $CL(i)$ is defined as ratio of edges between all nodes $j,k \in NH(i)$ to all possible edges in $NH(i)$:*

$$CL(i) = \frac{|\{\{j,k\} \in E | j,k \in NH(i)\}|}{{}^{d(i)\times(d(i)-1)}/_{2}}$$

In the next section I'll introduce basic structural properties of whole (sub-)networks.

3.1.2 Structural Properties

While node properties delineate a node's position in relationship to its environment, structural properties depict characteristics of whole (sub-)networks. It is important to mention that I consider only *connected* (sub-)graphs in my analysis, thus *connectedness* is a requirement for further investigations. Furthermore, the *degree distribution* of a graph resembles its homogeneity. In particular, I'm interested in measuring the *density* of such a graph, where I distinguish between *local* and *global* density measures. Furthermore, the *average path length* plays an important role as a property for a realistic human network structure.

Connectedness

In a connected graph there exists a path between each two nodes:

Definition 3.12. *(Connected Graph) A graph $G = (N, E)$ is connected if and only if the following condition holds:*

$$\forall i, j \in N : \exists P(i, j)$$

Note that according to Definition 3.4 (page 77), the shortest path length between two nodes is ∞ if there is no path between them. This is not possible for connected graphs because by definition, there exists a path between any two nodes. By considering only connected graphs, the shortest path length between two nodes is always a natural number. Furthermore, closeness centrality (Definition 3.9) and betweenness centrality (Definition 3.10) are defined by the shortest path length. It is easy to see just by implication that these values are always positive real numbers for connected graphs.

Since I want to understand regional variation, connected *sub-graphs* are of particular interest for my study. A connected sub-graph G' of graph G is defined as follows:

Definition 3.13. *(Connected Sub-Graph) A graph $G' = (N', E')$ is connected subgraph of a connected graph $G = (N, E)$ if and only if the following conditions hold:*

- $N' \subset N$
- $E' = \{\{i, j\} \in E | i, j \in N'\}$
- *G' is a connected graph*

Degree Distribution

The degree distribution is a fundamental characteristics of a graph and depicts the relative frequencies of degrees of nodes. A degree distribution for a graph G can be described as a vector P_G, where the position d of the vector stands for node degree d and the value $P_G(d)$ for the relative frequency of nodes with degree d. Thus the degree distribution P_G can be defined as follows:

Definition 3.14. *(Degree Distribution) Given a graph* $G = (N, E)$ *with maximal degree* $d^* = \max\{d(i)|i \in N\}$*. The degree distribution* P_G *of graph* G *is given as a vector of length* $d^* + 1$*, where the following condition holds:*

$$\forall d \leq d^* \in \mathbb{N}_0 : P_G(d) = \frac{|\{i \in N|d(i) = d\}|}{|N|}$$

It is easy to see that all entries of P_G are between 0 and 1 and sum up to 1, thus the following two conditions hold:

1. $\forall d = 0 \ldots d^* : 0 \leq P_G(d) \leq 1$
2. $\sum_{d=0}^{d^*} P_G(d) = 1$

There are some particularly interesting degree distributions. E.g. a regular distribution is where each node has the same degree. Therefore it can be defined as follows:

Definition 3.15. *(Regular Degree Distribution) A graph* G *has regular degree distribution* P_G *of degree* d^* *if the following properties hold:*

- $\forall d \neq d^*, d \in \mathbb{N}_0 : P_G(d) = 0$
- $P_G(d^*) = 1$

Furthermore, a *scale-free* distribution is defined as follows:

Definition 3.16. *(Scale-Free Degree Distribution) A graph* G *has scale-free degree distribution* P_G *of degree* γ *if the following property holds:*

- $P_G(d) \propto cd^{-\gamma}$

where $c > 0$ *is a scalar that normalizes the support of the distribution to sum to 1.*

Density

For structural density values I distinguish between *global* and *local* density values. I consider the classical *density* value as global, where values like *average clustering* and *transitivity* represent a local density: structures for which these values are high incorporate dense local sub-structures.

The classical *density* is the ratio of edges in a graph to the maximal number of edges a graph with $|N|$ nodes could have (what is $|N|\times(|N|-1)/2$), defined as follows:

Definition 3.17. *(Density) For a graph $G = (N, E)$, the density $dens(G)$ is defined as follows:*

$$dens(G) = \frac{|E|}{|N| \times (|N|-1) / 2}$$

The *average clustering* is the average individual clustering value of all nodes, defined as follows:

Definition 3.18. *(Average Clustering) For a graph $G = (N, E)$, the average clustering value $clust(G)$ is defined as follows:*

$$clust(G) = \frac{\sum_{i \in N} CL(i)}{|N|}$$

Finally, the *transitivity* value of a graph $G = (N, E)$ represents the ratio of triads and triangles. Triads are pairs of edges sharing the same node, given by the set $TRIADS(G)$. Triangles are three fully connected nodes, given by the set $TRI(G)$. Both are defined as follows:

- $TRIADS(G) = \{\{\{i,j\},\{i,k\}\} \subseteq E | i,j,k \in N, i \neq j \neq k\}$
- $TRI(G) = \{\{\{i,j\},\{i,k\},\{j,k\}\} \subseteq E | i,j,k \in N, i \neq j \neq k\}$

With these prerequisites, the transitivity of a graph G is defined as follows:

Definition 3.19. *(Transitivity) For a graph $G = (N, E)$, the transitivity value $trans(G)$ is defined as follows:*

$$trans(G) = \frac{|TRI(G)|}{|TRIADS(G)|}$$

Note that average clustering and transitivity depict similar characteristics. Both values are higher when more pairs of neighbor nodes are connected.[2] Nevertheless, it can be shown that for particular graph structures both values can be quite different.[3] For instance, I'll show in Section 3.2 that so-called scale-free networks can strongly differ in average clustering and transitivity value.

[2] Sometimes transitivity is called *overall clustering*, see e.g. Jackson (2008), p. 35.

[3] According to Jackson (2008) average clustering gives more weight to low-degree nodes that transitivity does.

Average Path Length

In the context of realistic human network structures, the *average path length* plays an important role. The average path length defines the shortest path length between two nodes averaged over all pairs of nodes given in the network. It can be defined as follows:

Definition 3.20. *(Average Path Length) For a graph $G = (N, E)$, the average path length $APL(G)$ is defined as follows:*

$$APL(G) = \frac{\sum_{i,j \in N} SPL(i, j)}{|N| \times (|N| - 1)}$$

3.2 Network Types

The most basic graph structure is a *completely connected network*: every node is connected with all others. A completely connected network is also inside the class of *regular networks*, introduced in the following section.

3.2.1 Regular Networks

A graph where each node has the same degree is called a *regular network*. Such a network has a *homogeneous* interaction structure. A regular network is formally defined as follows:

Definition 3.21. *(Regular Network) A graph $G = (N, E)$ is a regular network if and only if the following condition holds:*

$$\forall i, j \in N : d(i) = d(j)$$

Regular networks have received a fair amount of attention in the literature because they are comparatively easy to handle in implementations and proofs. Note that a regular network has a regular degree distribution since each node has the same degree.

One of the most popular regular network is a *k-ring network*, where all nodes are ordered and connected in a circular way to their k nearest neighbors. Figure 3.1a depicts a 4-ring network of 8 nodes. In general, a graph $G = (N, E)$ arranged as a k-ring with $n > k$ nodes has the following properties:

- $\forall i \in N : d(i) = k$

- $dens(G) = \frac{k}{n-1}$
- $clust(G) = \frac{3 \times (k-2)}{4 \times (k-1)}$
- $APL(G) = \frac{n}{2 \times k}$

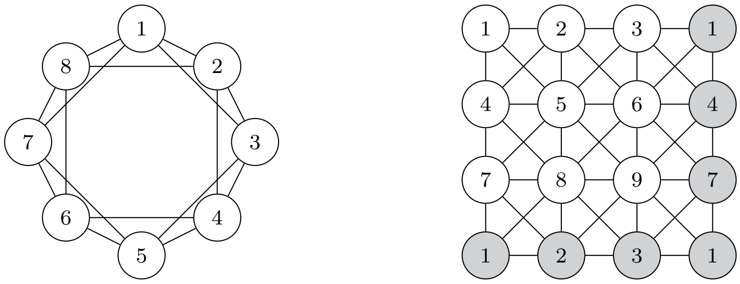
Notice that the clustering value is quite high for a ring, indeed it can be shown that $clust(G) \geq .5$ for $k \geq 4$, thus the clustering is high even for huge ring networks and furthermore completely independent of n. This is quite different for the average path length: here $APL(G)$ growth with the number of nodes n. It is therefore strongly dependent of n and quite high for huge networks, assuming a fixed $k \ll n$.

Another popular graph structure is a *toroid lattice*, where the nodes are arranged on an $n \times m$ toroid structure and every node is connected with the 8 nearest nodes on the lattice.[4] Figure 3.1b depicts a 3×3 toroid lattice: the gray nodes are doubles of some white nodes to highlight the fact that there is no border and each node has exactly 8 neighbors (e.g. the right neighbor of node 3 is node 1, the top-left neighbor of node 7 is node 6). In general, such a toroid lattice has the following properties:

- $\forall i \in N : d(i) = 8$
- $dens(G) = \frac{8}{n-1}$
- $clust(G) = \frac{3}{7}$
- $APL(G) = \frac{n}{3}$

Similar to the k-ring, the toroid lattice also has a clustering value that is independent of the lattice size and quite high, while the average path length grows with the lattice size and can be extremely high for huge toroid lattices. As we will see later, realistic human networks are assumed to have a short average path length. This is a property that neither the k-ring, nor the toroid lattice can come up with if we consider a large populations size. In the following, I will introduce a network class that has a low average path length in general, but doesn't exhibit any kind of regularity: random networks.

[4]This constitutes a structure that is also known as *Moore neighborhood*.



(a) A 4-ring network (b) A 3 × 3 toroid lattice

Figure 3.1: A 4-ring network of 8 nodes that are arranged as a ring and connected to their 4 nearest neighbors (Figure 3.1a) and a toroid lattice of 3 × 3 nodes that are connected to their 8 nearest neighbors (Figure 3.1b). Note that the lattice is a toroid because there is no border, as highlighted by doubles that are marked as gray nodes.

3.2.2 Random Networks

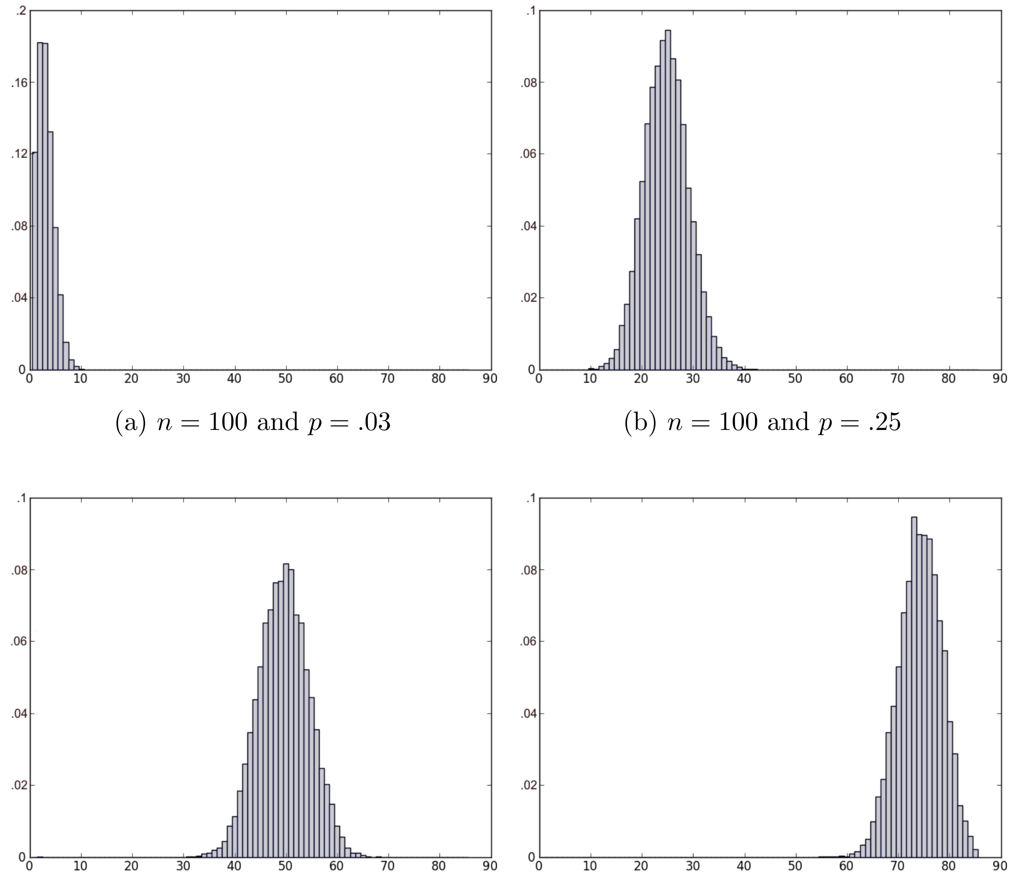
Random networks are created by a process of a completely random formation. A simple variant, also known as the Erdős-Rényi (ER) random graph model, can be described as follows: given an initial graph $G = (N, E)$ with a fixed set of n nodes $N = \{1, \ldots, n\}$ and an empty set of edges $E = \{\}$, take any pair of nodes $i, j \in N$ and add edge $\{i, j\}$ to set E with probability p $(0 \leq p \leq 1)$.[5]

Such a random network has interesting pattern. E.g. the degree distribution of a random network can be approximated by a *Poisson distribution* with the expected average degree[6] at the center of the curve. Figure 3.2 depicts averaged degree distributions for a network with 100 nodes created by the ER algorithm for different probability values .03, .25, .5 and .75.

Figure 3.2 also reveals another phenomenon: for small p-values the graph has highly probably nodes with degree 0; in other words: the graph has isolated nodes and is not connected in such a case. In general, the lower the p-value, the higher the probability that the resulting graph is non-connected. It can be shown that for a graph with n nodes, the probability value should be $p \geq \frac{log(n)}{n}$ for isolated nodes to disappear. In fact, this is exactly the threshold value for the probability that a network is connected converges to 1, as n grows.

Both of the two major assumptions of the ER-model, namely that edges

[5]Note that for $p = 1$ we would get a completely connected network.
[6]The expected average degree for a node is given by $p \times (n - 1)$.



(a) $n = 100$ and $p = .03$ (b) $n = 100$ and $p = .25$

(c) $n = 100$ and $p = .5$ (d) $n = 100$ and $p = .75$

Figure 3.2: Degree distributions of random networks for a graph of 100 nodes and different probabilities (.03, .25, .5, .75), created by the ER algorithm; each data point averaged over 100 simulation runs.

are independent and that each edge is equally likely, may be inappropriate for modeling real-life phenomena. In particular, a graph created by the ER algorithm does not have a degree distribution with heavy tails[7], as is the case in many real networks. Moreover, it has low clustering, unlike many social networks. For modeling alternatives that realize networks with such real-world characteristics, I'll introduce the most popular variants of *small-world networks*.

[7]A degree distribution with a heavy tail has a quite high number of extreme values, e.g. a high number of nodes with low degree, or with a high degree, or both.

3.2.3 Small-World Networks

Recent studies on large-scale complex networks in the real world reveal that the topology of most *real-world* networks (evolved by dynamics of self-organizing systems) is neither regular nor completely random, but somewhere between these two extremes (c.f. Watts and Strogatz 1998; Barabási 2002). Watts and Strogatz (1998) revealed that these networks are highly clustered[8] like e.g. regular lattices, but have a much shorter average path length, similar to random networks. Watts and Strogatz called networks with such structural properties *small-world* networks since a short average path length embodies the *small-world phenomenon*, also known as *the six degrees of separation*. To check if a graph G has properties of a small-world network, one has i) to measure its average path length and average clustering and ii) to compare it with those values of a connected random network G' with the same number of nodes and edges. While the average path length of G should be of similar length to that of G', the average clustering should be much higher. Thus a graph can be defined as a small-world network in the following way:

Definition 3.22. *(Small-World Network) A graph $G = (N, E)$ is called a small-world network if, in comparison with a random network $G' = (N', E')$ with $|N'| = |N|$ and $|E'| = |E|$, the following conditions hold:*

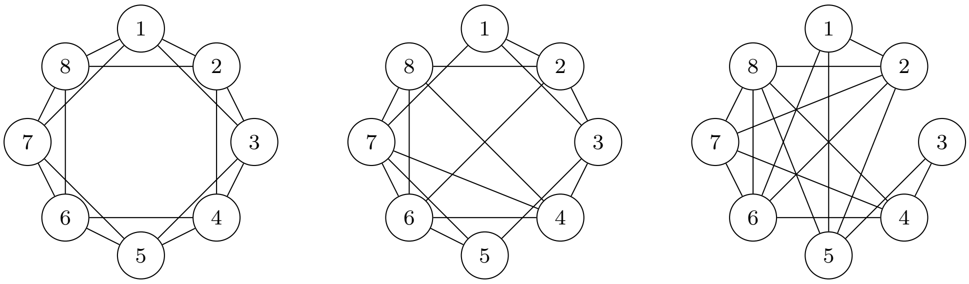
- $clust(G) \gg clust(G')$
- $APL(G) \approx APL(G')$

A further property found in a number of real-world networks is a scale-free degree distribution (see Definition 3.16). Barabási and Réka (1999) called small-world networks with this property *scale-free networks*. A scale-free network can be defined in the following way:

Definition 3.23. *(Scale-Free Network) A graph $G = (N, E)$ is called a scale-free network if its degree distribution P_G is scale-free.*

These properties often emerge as a result of human behavior. There are multiple ways to construct network structures that have small-world or even scale-free properties. In the following, I'll present some of the most popular ones from recent developments that will be object of study in subsequent chapters of this thesis.

[8]These networks have a high values of individual clustering, average clustering and/or transitivity.



(a) Initial 4-ring network (b) After rewiring 3 edges (c) After rewiring 8 edges

Figure 3.3: Example for the Watt-Strogatz algorithm: starting with a 4-ring with 8 nodes (Figure 3.3a), the edges are rewired randomly. E.g. after rewiring $\{8,1\} \rightarrow \{8,4\}$, $\{4,5\} \rightarrow \{4,7\}$ and $\{2,4\} \rightarrow \{2,6\}$, the resulting structure is less regular and has a smaller APL, accomplished by the new edges that provide shortcuts; thus it has small-world properties (Figure 3.3b). After 5 more rewired edges, the network shows probably less small-world characteristics by decreased average clustering (Figure 3.3c).

β-**Graphs**

One of the best-known algorithms for constructing small-world networks is defined by Watts and Strogatz (1998). Its resulting graph is called a *β-graph*. A β-graph is obtained by i) starting with a regular k-ring network and ii) subsequently, for each node, rewiring its $k/2$ left neighbors to a random node n with probability β, as depicted in Figure 3.3. As you can see, the resulting graph is neither regular ($\beta = 0$) nor random ($\beta \approx 1$), but somewhere between these extremes if β is centrally arranged between 0 and 1.

A graph that is somewhere between being regular and random is a small-world network; if it has the high clustering value of regular networks and the short average path length of random networks, it therefore has small-world properties. Figure 3.4 shows the clustering value (clustering coefficient) and average path length (characteristic path length) in dependence of different β-values by starting with a k-ring of 1000 nodes with $k = 12$, each data point averaged over 50 simulation runs. As you can see, such a β-graph has small-world properties particularly in the range of $.01 \leq \beta \leq .1$.

Another popular account for β-graphs is to start with a toroid lattice instead of a ring and rewire the edges with a probability β in the same way. The effect is similar. A very low β-value results in a relatively high

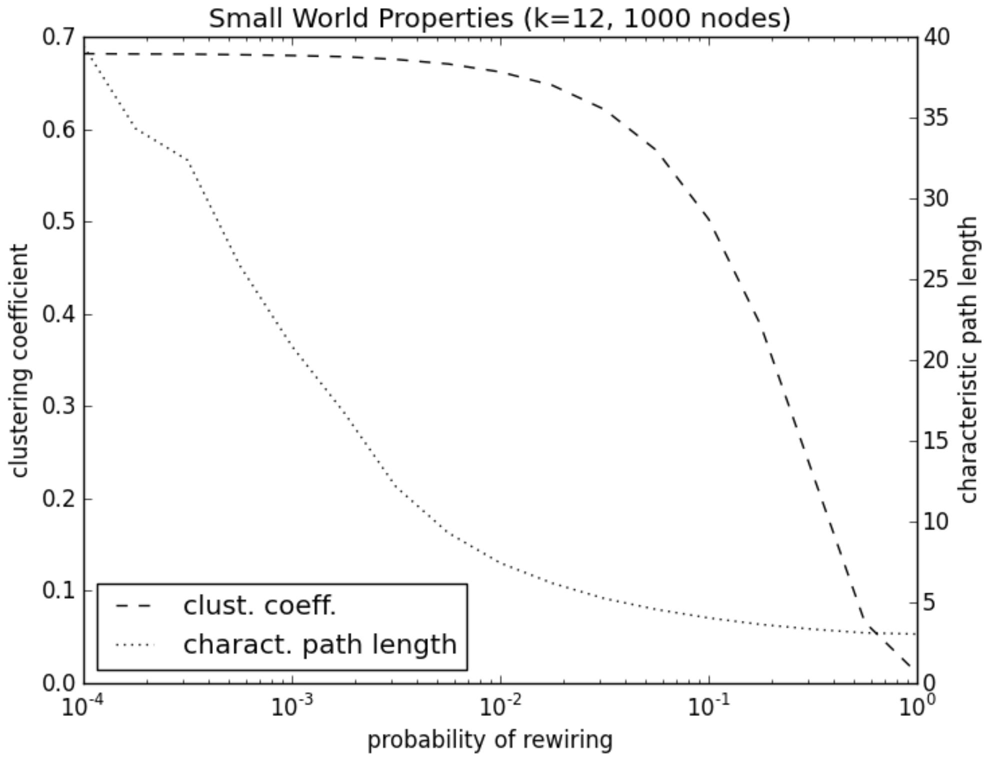


Figure 3.4: Average clustering and average path length (y-axis) of network structures created by the Watts-Strogatz algorithm for different β-values (probability of rewiring, x-axis), started with an initial 12-ring of 1000 nodes, averaged over 50 simulation runs. The typical small world characteristic is a combination of a high average clustering value and a low average path length. This is approximately given for $.01 < \beta < .1$.

average path length[9] and also a highly regular pattern, whereas a high β-value diminishes the average clustering value. Only an intermediate β-value brings along small-world properties.

Scale-Free Networks

It can be shown that β-graphs have a quite small range of different degrees of nodes. This might be acceptable for models of small societies, where interaction is fairly local and focused on immediate kinsmen. But it might be unrealistic for more open societies, where we would expect a more diverse degree distribution: most agents interact with a smaller number of agents,

[9] A toroid lattice with dimension $n \times n$ has the average path length of $n/3$, thus it is quite high for huge lattices.

but there are also agents that interact with many. The inappropriateness of β-graphs to model large, open and dynamic societies is also noted by Barabási and Réka (1999). They criticize that the β-graph model assumes i) starting with a fixed number of nodes, while most real-world networks are open and formed by continuously adding new nodes and edges to the system and ii) that the probability of two nodes being reconnected is random and uniform, while in real-world networks a new edge is more probably made with a more highly connected node.

Barabási and Réka (1999) presented an algorithm that creates a network without this inappropriateness. This *BA algorithm* can be informally described as follows:

1. Initial Setting
   - initially given is a small number of n nodes
2. Expansion
   - for every time step add a new node i_n and connect it randomly to m already given nodes ($m \leq n$)
   - the probability $Pr(i_n, i)$ of i_n being connected to a given node i depends on its degree $d(i)$, formally: $Pr(i_n, i) = {}^{d(i)}/\sum_{j \in N} d(j)$
3. Final Setting
   - after t time steps we have a network with $n + t$ nodes and $m \times t$ edges
   - the resulting network has a scale-free degree distribution

The resulting network is scale-free, but it lacks an important small-world property: a high average clustering. Holme and Kin (2002) mentioned that such a network has an average clustering value $clust(G) \approx 0$ and therefore fails to resemble a type of networks I'm interested in: *social networks* that typically have a high clustering value.

Holme and Kin (2002) presented an algorithm that creates a scale-free network with small-world properties like a high clustering. They modify the BA algorithm by changing the expansion step in the following way:

2. Expansion
   - for every time step add a new node i_n and connect it with m already given nodes in one of the two following ways:

a) connect i_n with any node i according to probability $Pr(i_n, i)$
b) connect i_n with any node $j \in NH(i)$ according to $Pr(i_n, j)$, where i is the node chosen in step a)

- each time start with 'step a)' and then follow with 'step b)' with probability P_t or 'step a)' with probability $1 - P_t$, until either i_n is connected to m nodes or i_n is connected to all $j \in NH(i)$. In the latter case follow with 'step a)'.

Notice that in 'step b)' the new node i_n is connected to a neighbor of a node it is already connected with. In other words, 'step b)' realizes a triangle formation and therefore increases the average clustering. Consequently, the clustering depends on the probability parameter P_t. And the average number of 'step b)' accomplishments is given by $m_t = (m-1)P_t$ which is the control parameter of the model.[10]

Holme and Kin (2002) were able to show that the algorithm produces a scale-free network for arbitrary m_t values. Furthermore, the relation between m_t and $clust(G)$ is almost linear for a sufficiently large number of nodes, and the average clustering increases with m_t. All in all, the algorithm creates a scale free network with small-world properties for a range of m_t values.

3.3 Network Games

In my work I will use network structures to present an explicit *interaction structure* that determines which agents can interact with one another in a population of multiple agents. In general: given is a set of n agents $X = \{x_1, x_2, \ldots, x_n\}$ and a network structure $G = \{N, E\}$ with $N = \{1, 2, \ldots, n\}$. Then every agent $x_i \in X$ corresponds to node $i \in N$. In addition, any two agents $x_i, x_j \in X$ can interact with each other if and only if $\{i, j\} \in E$.[11] One interaction step is realized by playing one round of a dynamic signaling game. Furthermore, agents interact repeatedly and their decisions are guided by *learning dynamics*, as already introduced in Chapter 2. I refer to a signaling game SG that is i) played by a population of agents X placed on a fixed interaction structure G and ii) combined with an update dynamics D, as a *network game*.

[10]Notice that for $m_t = 0$ we get the BA algorithm.

[11]This condition is weakened by applying the *social map* selection algorithm, introduced in Section 3.3.2.

Formally, a network game $NG = \langle SG, X, D, G\rangle$ is given by a signaling game SG, a set of agents X, a learning dynamics D and a graph structure G and can be defined in the following way:

Definition 3.24. *(Network game) For network game $NG = \langle SG, X, D, G\rangle$ a population of agents $X = \{x_1, \ldots, x_n\}$ is placed on a graph structure $G = \{N, E\}$ with $N = \{1, \ldots, n\}$, where each agent plays repeatedly a signaling game $SG = \langle (S, R), T, M, A, P, C, U\rangle$ and behaves according to the learning dynamics $D = \langle SG, \sigma, \rho, M_U\rangle$. Consequently, each round of play is realized in the following way:*

1. *choose randomly a pair of connected nodes $i, j \in N$ with $\{i, j\} \in E$*
2. *take agent x_i as the sender S and agent x_j as the receiver R for a round of play of signaling game SG*
3. *choose a state $t \in T$ according to prior probability $P(t)$*
4. *S sends message $m \in M$ according to behavioral strategy $\sigma(m|t)$ of learning dynamics D*
5. *R chooses interpretation state $a \in A$ according to behavioral strategy $\rho(a|m)$ of learning dynamics D*
6. *after the round is played, S and R update their learning level according to update mechanism M_U of learning dynamics D*

Note that the network structure plays a role for the first step in Definition 3.24: the choice of a communication partner. Such a choice can be realized in different ways. This can have a crucial influence on the probability of how often an agent is chosen. In the following I'll present and analyze different ways.

3.3.1 The Choice for Communication Partners

By analyzing the behavior of agents playing network games, it can be important to consider how often an agent is chosen to interact in comparison to how often other agents are chosen. The probability of a node i being chosen generally depends on the node selection algorithm and the node's degree $d(i)$. I'll illustrate this with the following different selection algorithms *random sender & neighbor*, *random receiver & neighbor*, *random edge* and *weighted edge*. They can be defined in the following way:

Definition 3.25. *(Random Sender & Neighbor) For the network game $NG = \langle SG, X, D, G = \{N, E\}\rangle$, the selection algorithm* random sender & neighbor *(RSN) selects a sender S and a receiver R in the following way:*

1. *select randomly a node $i \in N$; $x_i \in X$ is sender S*
2. *select randomly a node $j \in NH(i)$; $x_j \in X$ is receiver R*

Definition 3.26. *(Random Receiver & Neighbor) For the network game $NG = \langle SG, X, D, G = \{N, E\}\rangle$, the selection algorithm* random receiver & neighbor *(RRN) selects a sender S and a receiver R in the following way:*

1. *select randomly a node $i \in N$; $x_i \in X$ is receiver R*
2. *select randomly a node $j \in NH(i)$; $x_j \in X$ is sender S*

Definition 3.27. *(Random Edge) For the network game $NG = \langle SG, X, D, G = \{N, E\}\rangle$, the selection algorithm* random edge *(RE) selects a sender S and a receiver R in the following way:*

1. *select randomly an edge $\{i, j\} \in E$*
2. *select randomly case i) x_i is sender S and x_j is receiver R or case ii) exactly the other way around*

Definition 3.28. *(Weighted Edge) For the network game $NG = \langle SG, X, D, G = \{N, E\}\rangle$, the selection algorithm* weighted edge *(WE) selects a sender S and a receiver R in the following way:*

1. *select an edge $\{i, j\} \in E$ with probability $\frac{4}{|N| \times (d(i)+d(j))}$*
2. *select randomly case i) x_i is sender S and x_j is receiver R or case ii) exactly the other way around*

Note that the probability of an agent x_i being sender or receiver is different for each of these selection algorithms, as depicted in Table 3.1: while for algorithm RSN the sender is chosen completely randomly among all $|N|$ nodes, the receiver is more probably chosen with a higher degree centrality. This is because the higher the degree centrality of node i, the more probable it is for i to be the neighbor of another node. The same consideration exactly the other way around leads to the probabilities of the RRN algorithm. Furthermore, a node i is more probable to be an element

	RSN	RRN	RE	WE
$Pr(x_i = S)$	$1/\|N\|$	$d(i)/(2\times\|E\|)$	$d(i)/(2\times\|E\|)$	$1/\|N\|$
$Pr(x_i = R)$	$d(i)/(2\times\|E\|)$	$1/\|N\|$	$d(i)/(2\times\|E\|)$	$1/\|N\|$

Table 3.1: The probabilities of an agent being selected as a sender or receiver for different selection algorithms.

of a randomly chosen edge with a higher degree $d(i)$. Thus for algorithm RE both sender and receiver have a higher chance of being selected with a higher degree. According to algorithm WE edges with nodes with high degrees are less frequently selected then edges with nodes with low degrees. On average, each node is selected with a probability of $1/|N|$ and therefore completely randomly, as shown in Proof A.1 in Appendix A.

Thus, in a model where agents can only communicate with their direct neighbors, the probability of communication partners being selected is randomly and, dependent on the selection algorithm, biased by the degree of a node. In the following section I'll relax the restriction that agents only interact with direct neighbors. The probability of agent x_i having a communication partner x_j depends on the shortest path length between i and j.

3.3.2 The Social Map

In this section, I want to i) regard a toroid lattice as interaction structure and ii) relax the condition that possible communication partners are constrained to the direct neighborhood. The idea is as follows: the probability that agent x_i chooses a particular communication partner x_j is positively correlated to the shortest path length between i and j according to a correlation function. This transforms the toroid lattice into what I call a *social map* since spatial distance (shortest path length) depicts social distance (probability of interaction). Furthermore, for a correlation function with a strongly increasing slope, the social map structure approximates direct neighbor communication. On the other hand, for a linear correlation function the social map patterns a network where each pair of interlocutors is chosen with the same probability. In the latter case, spatial effects are eliminated and the interaction structure resembles a completely connected network.

Interlocutor Allocation and the Degree of Locality

In this section, I place agents on a *social map* (see Nettle (1999) as a comparable account): all agents $x \in X$ are placed on a lattice. The *distance* $d(x_i, x_j)$ between two agents x_i and x_j plays a crucial role and is defined as follows:

Definition 3.29. *(Distance) Given is a network game $NG = \langle SG, X, D, G = (N, E)\rangle$. For two agents $x_i, x_j \in X$, the distance $d(x_i, x_j)$ between them is defined by the shortest path length $SPL(i, j)$, $i, j \in N$:*

$$d(x_i, x_j) = SPL(i, j)$$

To realize a social map structure, an agent x_i's probability $P_\gamma(d)$ to interact with an agent x_j depends on the distance $d = d(x_i, x_j)$ between both agents. Furthermore, this dependency is influenced by a parameter γ which is called the *degree of locality*. The idea is as follows: the higher γ, the more local is the interaction structure. A maximal local interaction structure is direct neighbor communication, whereas a minimal local and therefore a maximal global interaction structure is random interaction. In other words: for $\gamma = 0$, agents interact randomly and by increasing γ, the interaction structure approximates direct neighbor communication and the social map approximates a toroid grid structure.

With this account it is possible to analyze interaction structures between the local interaction structure of a toroid lattice network and the global interaction structure of a population of randomly chosen communication partners which, in terms of network theory, equals a completely connected network.

The social map account can be realized as follows: given is a $n \times n$ toroid grid structure. I denote the set of all agents of distance d to an agent x_i with $N_d(x_i)$, formally $N_d(x_i) = \{x_j \in X | d(x_i, x_j) = d\}$. Two agent x_i, x_j are chosen as communication partners by the following *Social Map* selection algorithm:

Definition 3.30. *(Social Map) For a network game $NG = \langle SG, X, D, G = \{N, E\}\rangle$ the selection algorithm* Social Map *selects a sender S and a receiver R in the following way:*

1. *select randomly a node $i \in N$; $x_i \in X$ is sender S*
2. *select a distance d with probability $P_\gamma(d)$*

3. *select randomly an agent x_j from the set $N_d(x_i)$; x_j is receiver R*

The probability $P_\gamma(d)$ of choosing distance d in dependence of the degree of locality γ is defined as follows:

$$P_\gamma(d) = \frac{8 \times \frac{d}{d^\gamma}}{\eta(\gamma, d)} \quad \eta(\gamma, d) \text{ is normalizer function}^{12} \tag{3.1}$$

Furthermore, it is evident that for an $n \times n$ toroid lattice each agent has $8 \times d$ neighbors of distance d; to put it formally: $\forall x \in X : \forall d \leq \frac{n}{2} : |N_d(x)| = 8 \times d$. Consequently, the probability of choosing a random agent from the set $N_d(x)$ is $\frac{1}{8 \times d}$. This leads to the following interaction probability of an agent x_i interacting with an agent x_j:

Definition 3.31. *(Interaction probability on a Social Map) For a network game $NG = \langle SG, X, D, G = (N, E)\rangle$, where G is a $n \times n$ toroid lattice and agents use the selection algorithm* Social Map*, the probability $Pr(x_i, x_j)$ that an agent $x_i \in X$ interacts with agent $x_j \in X$ is given as follows:*

$$Pr(x_i, x_j)_\gamma = P_\gamma(d(x_i, x_j)) \times \frac{1}{8 \times d(x_i, x_j)}$$

It can be shown that for $\gamma = 0$, the probabilities of an agent choosing his communication partner is equiprobable among all other agents: $\forall x_i, x_j \in X : Pr(x_i, x_j)_0 = \frac{1}{|X|-1} = \frac{1}{n^2-1}$ (see Proof A.2). Thus, allocation is completely random and independent of distance d. The interaction structure equals a completely connected network. But by increasing γ, a close distance is more and more important for frequent communication.

Figure 3.5 shows the probability distributions $P_\gamma(d)$ for different γ-values over $d \in D = \{m \in \mathbb{N} | 1 \leq m \leq 10\}$. As you can see, for $\gamma = 0$ the probability of choosing a distance d increases linearly with the distance. Because also $|N_d(x)|$ increases linearly with the distance, each possible communication partner is chosen with the same probability, independently of the distance. This probability is exactly $\frac{1}{n^2-1}$, as I showed in Proof A.2.

For $\gamma = 8$ the probability of choosing a communication partner with distance 1 is $P_8(1) \approx 0.992$ for a maximal distance of 10 ($n = 21$). Thus, for $\gamma = 8$ the probability is almost 1 that agents choose a direct neighbor as a partner (see Proof A.3). This choice probability implies a selection

[12] I set $\eta(\gamma, d) = (\sum_{d=1}^{\lfloor n/2 \rfloor} 8 \times d/d^\gamma) + (\lfloor \frac{n+1}{2} \rfloor - \frac{n+1}{2}) \times (4n + 2)$ to guarantee that $P_\gamma(d)$ is a probability measure, in other words that holds: $\forall d \in D : 0 \leq P_\gamma(d) \leq 1$ and $\sum_{d \in D} P_\gamma(d) = 1$ with $D = \{1, 2, \ldots \lceil n/2 \rceil - 1\} \subseteq \mathbb{N}$.

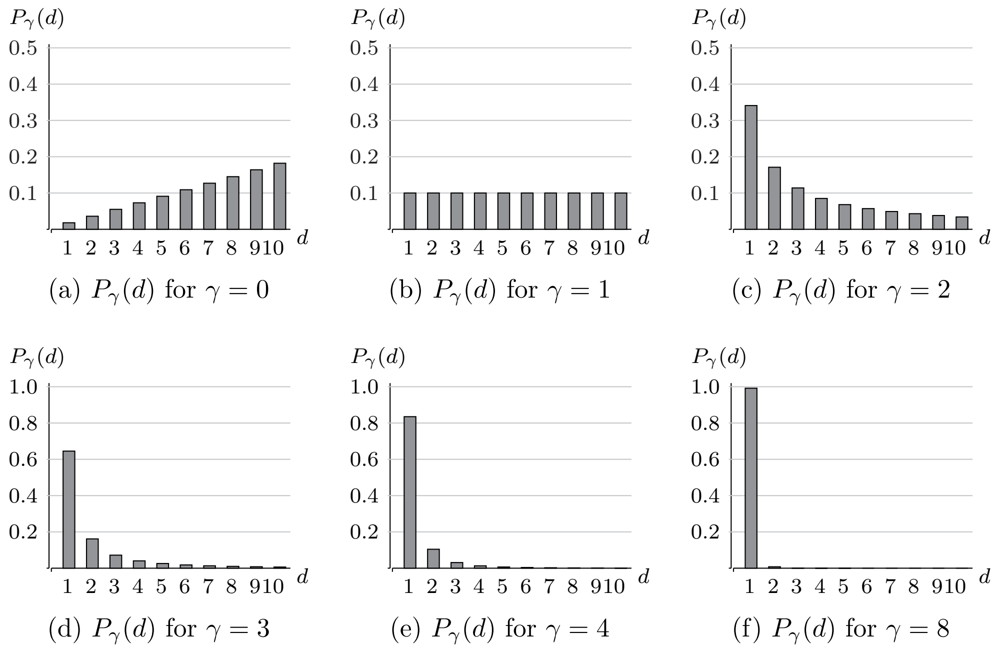


Figure 3.5: Degree distributions with a maximal degree of 10 for different γ-values. $\gamma = 0$ depicts random communication. By increasing the γ-value, agents' behavior approximates neighborhood communication. For $\gamma = 8$ the probability of choosing a direct neighbor is ca. 0.992.

strategy that is close to neighborhood communication and approximates it by increasing γ.

For my social map experiments in Chapter 4, the overall system's behavior for different γ-values is of special interest. As I already mentioned, the behavior of choosing a partner completely randomly ($\gamma = 0$) realizes a completely connected network structure, whereas the behavior of choosing one in an agent's direct neighborhood ($\gamma \to \infty$) realizes a toroid lattice structure. Both are extreme points of the degree of locality γ. One of my tasks is to find out how the degree of locality affects the emergence of signaling languages in the multi-agent system; in other words, I want to examine the behavior of societies with interaction structures between those two extreme points.

3.4 Conclusion

The task for the subsequent chapters is to simulate and analyze the behavior of agents on a network structure interacting repeatedly by playing signaling games with accessible interlocutors, where decisions are guided by learning dynamics. I termed such a configuration a *network game*, as introduced in Section 3.3. In Section 3.2 I introduced regular networks, random networks and so-called *small-world* networks that emulate realistic human network structures. The latter ones will be used as interaction structure for my simulation experiments with network games in Chapter 5. In Section 3.1 I introduced network measures describing i) *structural properties* of (sub-)networks and ii) *node properties* of particular agents. These measures will be used for the analysis of my experimental results.

All in all, the prerequisites are given. In the following Chapters 4 and 5 I'll present experiments with network games and analysis of the results. In detail, I will i) analyze the global behavior of populations under different circumstances, ii) compare such global behavior with former results (e.g. replicator and imitation dynamics), iii) analyze particular agents' behavior more in detail and in dependence of their spatial properties and iv) try to interpret the results in the light of matters from sociolinguistics.

Chapter 4

Emergence of Regional Meaning

> *"Construing agreement generously, maybe all conventions could, in principle, originate by agreements. What is clear is that they need not. And often they do not."*
>
> *Lewis 1969, Convention*

> *"However, it seems that the simpler models have not said all there is to say about cooperation and spatial structure as explanations of social cooperation."*
>
> *Zollman 2005, Talking to Neighbors: The Evolution of Regional Meaning*

The ur-mission of developing signaling games was to explain the emergence of conventions arising without explicit previous agreements. As demanded in Chapter 1 and depicted in Chapter 2, such conventions can and probably must arise over time and in populations. In Chapter 2 some basic results were obtained for different dynamics and initial population setups. In particular, for the Horn game analysis I was able to show that the only possibility for agents to learn one of the other two promising strategies, anti-Horn or Smolensky, instead of the predominant Horn strategy was an appropriately shifted initial population state. But one could argue that such a shifted population state violates the requirement for an initial unbiased situation since it might imply a sort of previous agreement.

I would like to stress this point: let's assume an initial situation for a population that behaves in a way that no previous agreement among the members was made. Such a situation should have the following properties for the different three dynamics types:

- evolutionary dynamics: the initial population state is unbiased/neutral
- imitation dynamics: the initial start strategies are randomly chosen or uniformly distributed among the society
- learning dynamics: a) the initial tendencies of a population's members to a specific strategy are uniformly distributed among the society or b) all population members are completely unbiased/unexperienced

We saw earlier that for an unbiased initial population state the replicator dynamics leads to a final population state with all members playing the Horn strategy (see e.g. Figure 2.2). In addition, I showed for the imitation dynamics that in a population of randomly acting agents starting with an initial uniform distribution of every strategy, the following result holds: in the final population all agents play the Horn strategy (see e.g. Figure 2.7). And finally, for reinforcement learning it was shown that for simulations starting with uniform initial strategy tendencies, the resulting population behaves according Horn's rule (see e.g. Figure 2.13).

I want to examine the evolution of convention by applying learning dynamics and having an initial state with completely unbiased and unexperienced agents. This means that all agents i) start with a random choice and ii) have a memory completely empty of experience.[1] This guarantees an initial state that rescind the assumption that any explicit previous agreement was made since agents do not have any knowledge about the past.

In addition, I would like to stress a second point here: I showed in earlier experiments that the interaction structure can play an important role in the way strategies evolve and stabilize. Experiments for different network topologies revealed that the basins of attraction for different strategies are shifted (see e.g. Figure 2.9). Furthermore, some basic experiments with imitation dynamics on a toroid lattice (see e.g. Zollman, 2005) showed that final population states can be significantly divergent in terms of the emergence of regional meaning (see e.g. Figure 2.10). This sketches out a possible loophole for strategies other than Horn to emerge.

[1]This means a) for reinforcement learning that all ball types of the agents' urns are uniformly distributed; and b) for belief learning that all experiences are empty or uniformly distributed.

These observations bring me to the following question: if agents a) are playing a Horn game repeatedly by using learning dynamics and starting with an initial unbiased and unexperienced status, and b) are placed on an interaction structure that allows for multiple strategies to emerge, are there circumstances that lead to an emergence of strategies different from Horn's rule? This chapter deals with exactly this question by analyzing a model that includes individuals with the following properties:

- the individuals are initially unbiased regarding any strategy
- the individuals are connected among a specific interaction structure
- the individuals interact by playing pairwise - the Lewis game or a Horn game
- the individuals adapt their behavior over time; or, more precisely: they use learning dynamics to learn strategies of communicative behavior

My model has the following features: i) individuals as artificial agents whose decision mechanism is defined by learning dynamics, ii) an interaction structure is given by a toroid lattice[2] and iii) a communication protocol achieved by playing a signaling game, the Lewis game or a variant of the Horn game.

This chapter contains the analysis of this model and how different parameter settings have an impact on the resulting society structure of language regions. In particular, it focuses on circumstances that lead to situations of agents learning strategies different from the Horn strategy. To foreshadow one general result: for a lot of circumstances the experiments lead to an emergence of regional meaning, what plays a crucial role for the question of how multiple or unexpected strategies may emerge. Thus, the more general question for getting an insight in how different language learners arise might be: what causes the emergence of regional meaning? What circumstances support the emergence of multiple language regions inside the population?

The chapter is structured in the following way: in Section 4.1 I analyze the different factors that cause the emergence or non-emergence of multiple

[2]The justification for choosing a lattice structure is as follows: i) it is a locally connected interaction structure, thus resembling a highly idealized human population structure and ii) it is regular and therefor easy to analyze and iii) its regular and strongly local structure can easily be relaxed by applying the *social map* algorithm and therefore a more realistic structure is immediately available.

language regions. In Section 4.2 I examine phenomena that emerged during my experiments in a more detailed fashion, to get a better insight in the dynamic forces inside the population. Finally, in Section 4.3 I give a conclusion about the main results of my experiments.

4.1 Causes for Regional Meaning

I want to analyze the influence of specific parameters and how they impact the learning process and resulting regional structure. I consider parameters that are values from three different *dimensions*. These dimensions are i.) the signaling game itself, ii.) the learning dynamics and iii.) the interaction structure. Basically, I analyze these three dimensions as follows:

- *signaling game* by game parameters (prior probability, message costs)
- *interaction structure* by degree of locality and lattice size
- *learning dynamics* by learning type (BL or RL) and memory size

With this analysis I want to get more detailed answers for the question of how different circumstances of the whole model, like game parameters, update dynamics, memory size or the spatial arrangement, have a specific influence on the resulting population structure. More precisely, I want to find answers for questions like the following:

- in what sense do results of these experiments differ from previous results of experiments with other dynamics and/or population structures?
- under which circumstances do multiple language regions evolve?
- if multiple language regions evolve, how do agents on the border between language regions behave?
- particularly for the Horn game: under what circumstances do anti-Horn players or Smolensky players evolve?

In most of my experiments I used a population structure of a 30×30 toroid lattice, thus a population size of 900 agents. With this setup I

game	abbr.	probabilities	message costs
Lewis game	LG	$P(t_1) = P(t_2) = .5$	$C(m_1) = C(m_2) = 0$
weak Horn game	HG_w	$P(t_f) = .6$, $P(t_r) = .4$	$C(m_u) = .05$, $C(m_m) = .1$
normal Horn game	HG	$P(t_f) = .7$, $P(t_r) = .3$	$C(m_u) = .1$, $C(m_m) = .2$

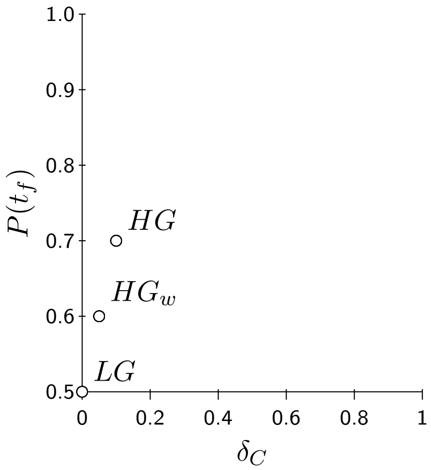
Table 4.1: The different games analyzed in the experiments.

made a reasonable compromise of a manageable runtime duration and the possibility of the emergence of global effects.[3]

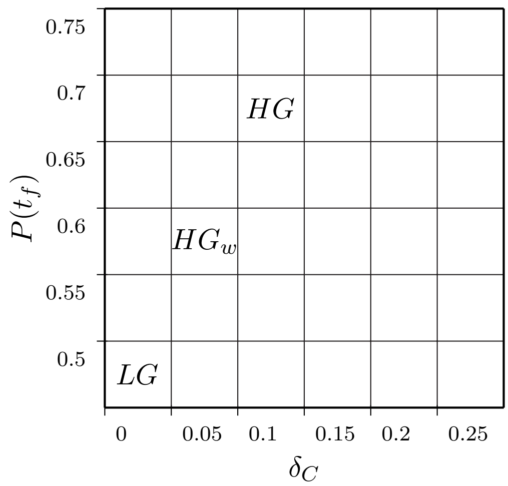
The signaling games that I analyzed in most of the experiments are the Lewis game LG as well as two Horn games with different parameters which I call (in accordance with Chapter 2) the *weak Horn game* HG_w and the *normal Horn game* HG. The appropriate game parameters are given in Table 4.1. It is important to note that all three games are inside a range of what I call the *Horn game spectrum* that defines the unlimited set of signaling games with two information states, two messages and two interpretation states, for different combinations of prior probability $P(t_f)$ and message costs difference $\delta_C = C(m_m) - C(m_u)$. Note that $P(t_r)$ is uniquely defined by $1 - P(t_f)$, and that $0 \leq C(m_u) \leq C(m_m) \leq 1$. Thus, the Horn game spectrum for different parameter combination is given as depicted in Figure 4.1a, where LG, HG_w and HG are positioned according to their parameters. In my experiments I often analyzed, next to the three predefined games LG, HG_w and HG, the outcome of experiments for further games inside this spectrum, to get a better insight in what way or extend game parameters influence the agents' behavior. These experiments are made for discrete sections that divide a subset of the spectrum, exemplary depicted in Figure 4.1b.

I was not only interested in analyzing the way game parameters influence the outcome, but also in comparing the impact of different update dynamics. In Chapter 2 I introduced i) the replicator dynamics, ii) the prominent imitation dynamics 'imitate the best' and conditional imitation and iii) the two learning dynamics reinforcement learning and belief learning. I showed some basic results for preliminary experiments for some of these

[3]If the population is too large, the runtime for simulation runs until interesting results like stable structures emerge is too high for practical research. On the other hand, if the population size is too small, the distinction between local and global phenomena is hard to make. In fact, a resulting composition of local formations are highly improbable. As corroboration, see the results of Experiment IX: the analysis of different population sizes. The smaller the population size, the less probable the emergence of multiple language regions is.



(a) Horn game spectrum



(b) Segmentation of a subset of the spectrum

Figure 4.1: The Horn game spectrum: signaling games with different combinations of prior probabilities $P(t)$ and the difference of message costs δ_C (Figure 4.1a). For my analysis I used a discrete segmentation of a subset of this spectrum, as exemplified in Figure 4.1b.

dynamics. In this chapter I will focus on learning dynamics. In Section 4.1.1 I'll focus on the comparison of reinforcement learning and belief learning, each considered with unlimited and limited memory.

In my experiments I used network games with the following settings:

- the agents use a selection algorithm *Random Sender & Neighbor* (Definition 3.25), except for Experiments V-VIII: here they use selection algorithm *Social Map* (Definition 3.30)

- for the belief learning experiments I applied the BL account (Definition 2.16) with initially empty beliefs and random choice

- for the reinforcement learning experiments I applied the $RLIP$ account (Definition 2.15) with an initial urn settings of 100 balls per urn, 50 for each type and the following update values: $\alpha = 10$, $\beta = 0$ and $\gamma = 2$

- an agent is a language learner if the Hellinger similarity sim_L of her current strategy is above threshold $h_\epsilon = 0.65$ (see Definition 2.13)

- a cohesive area of agents using the same language L is called a *language region* of language L[4]
- agents' languages and corresponding language regions are considered as stable according to *long-term stability* (Definition 2.14)

With this setup I'll analyze the three dimensions: game parameters, interaction structure and learning dynamics, by varying their parameter values and measuring the impact on the resulting language structure of the society. In Section 4.1.1 I'll analyze the impact of different dynamics settings and game parameters inside the Horn spectrum. In Section 4.1.2 I'll extend the analysis by varying the interaction structure, i.e. using the *social map* algorithm (Definition 3.30) and taking different degrees of locality γ into account. In Section 4.1.3 I'll examine the influence of different sizes of a) agents' memory and b) population members. Finally, in Section 4.1.4 I'll give an overview of the experiments and an interpretation of their results.

4.1.1 Dynamics and Game Parameters

In my experiments agents update their behavior by using different learning dynamics: reinforcement learning (RL) or belief learning plus best response dynamics (BL), as introduced in Section 2.3. In addition, for both dynamics I distinguish between agents with unlimited memory (RL^{∞}, BL^{∞}) or a limited memory of 100 (RL^{100}, BL^{100}).

The Experiments I-IV contain simulation runs, each for a particular combination of dynamics type and memory size. Each series encompasses detailed analyses of all three games plus a more general analysis for a part of the Horn game spectrum. Table 4.2 shows an overview of the 4 experiment series with 4 experiments each. Consequently, there are 16 different experiments in total.

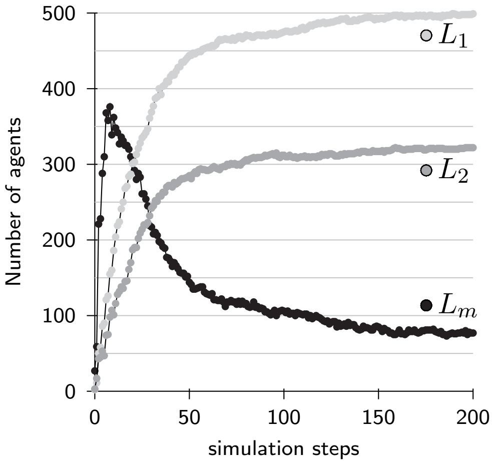
Experiment I: BL Agents with Unlimited Memory

In the first Experiment I(a) I analyzed the behavior of BL agents playing the Lewis game on a toroid lattice: I performed 100 simulation runs of the network game $NG_{I(a)} = \langle LG, X, BL^{\infty}, G_{l30}\rangle$, where $X = \{x_1, \dots x_{900}\}$ is a set of 900 agents and G_{l30} is a 30×30 toroid lattice. The fundamental result was that all simulation runs ended with a society, where the lattice is split into local signaling languages of both types L_1 and L_2, i. e. regional

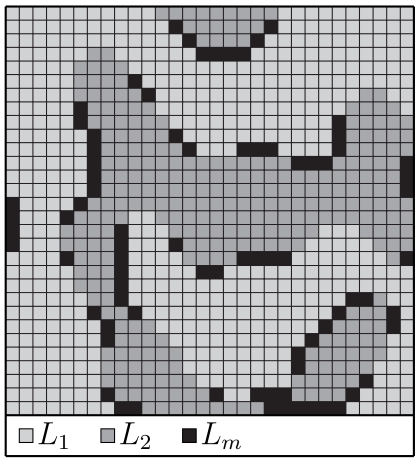
[4] A formal definition for a language region is given in Definition 5.1.

dynamics	LG	HG_w	HG	**Spectrum**
BL^∞	Exp. I(a)	Exp. I(b)	Exp. I(c)	Exp. I(d)
RL^∞	Exp. II(a)	Exp. II(b)	Exp. II(c)	Exp. II(d)
BL^{100}	Exp. III(a)	Exp. III(b)	Exp. III(c)	Exp. III(d)
RL^{100}	Exp. IV(a)	Exp. IV(b)	Exp. IV(c)	Exp. IV(d)

Table 4.2: Experiments for different games and different dynamics on a 30×30 toroid lattice.



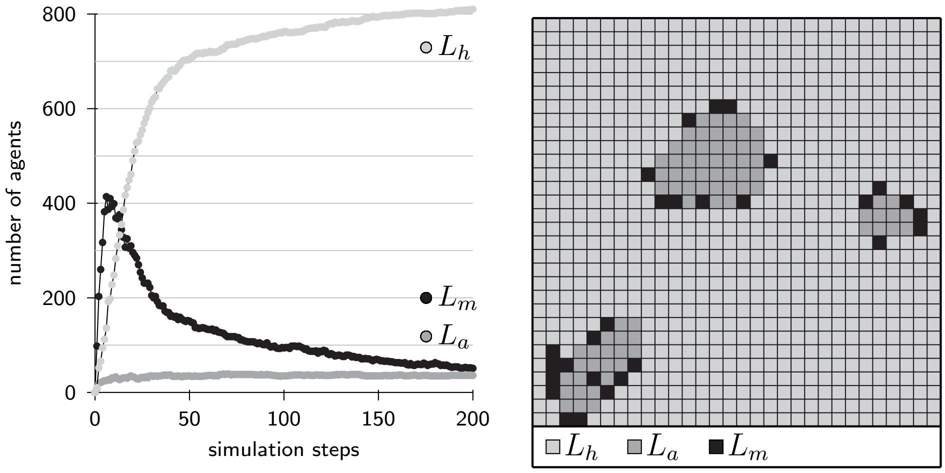
(a) Sample learning curves



(b) Sample pattern

Figure 4.2: Sample learning curves over 200 simulation steps (4.2a) and a sample of the resulting pattern after 3000 simulation steps (4.2b) for BL^∞ agents playing the Lewis game on a 30×30 toroid lattice.

meaning emerged in every simulation run. Furthermore, language regions were separated by *border agents*, who were not proficient in either of the two signaling languages. The phenomenon of such border agents is a consequence of the fact that both signaling languages are highly incompatible. A more detailed analysis of border agents is given in Section 4.2 and will show that such border agents use miscommunication languages, thus are either L_{12} or L_{21} learners (see Table 2.2a). I'll label agents that learned one of both miscommunication languages as L_m learners or miscommunication language learners. Sample learning curves of agents, who have learned L_1, L_2 or L_m are depicted in Figure 4.2a. Furthermore, Figure 4.2b shows a sample of the resulting pattern of agents that have learned L_1, L_2 or L_m, distributed on the toroid lattice.



(a) Sample learning curves

(b) Sample pattern

Figure 4.3: Sample learning curves over 200 simulation steps (4.3a) and a sample of the resulting pattern after 3000 simulation steps (4.3b) for BL^∞ agents playing HG_w on a 30×30 toroid lattice.

The Experiments I(b) and I(c) cover 100 simulation runs of the network games $NG_{I(b)} = \langle HG, X, BL^\infty, G_{l30}\rangle$ and $NG_{I(c)} = \langle HG_w, X, BL^\infty, G_{l30}\rangle$, where $X = \{x_1, \ldots x_{900}\}$ is a set of 900 agents and G_{l30} is a 30×30 toroid lattice. As a basic result and in contrast to the Experiment I(a), in these experiments it was not the case that every simulation run ended with multiple local languages: for the weak Horn game HG_w in Experiment I(b) in almost all runs (99%) both languages L_h and L_a emerged, in the remaining one run language L_h spread over the whole lattice. For the normal Horn game HG in Experiment I(c) the result was almost exactly the other way around: only 4% of all runs ended in a society with multiple language regions, the remaining runs resulted in a society of only L_h learners. Furthermore, the average number of L_a learners was 32.68 for HG_w, and only 5.75 for HG, averaged over all runs ending with multiple languages. Figure 4.3a depicts sample learning curves for a simulation run that ended up with multiple language regions. Figure 4.3b shows a sample of the resulting pattern after 3000 steps for the weak Horn game.

As one can note, the game parameters play a crucial role for the emergence of multiple language regions: while for the weak Horn game HG_w multiple language regions emerged in almost every simulation run, for the normal Horn game multiple language regions emerged in only 4% of all simulation runs. In all other cases only L_h emerged as society-wide language.

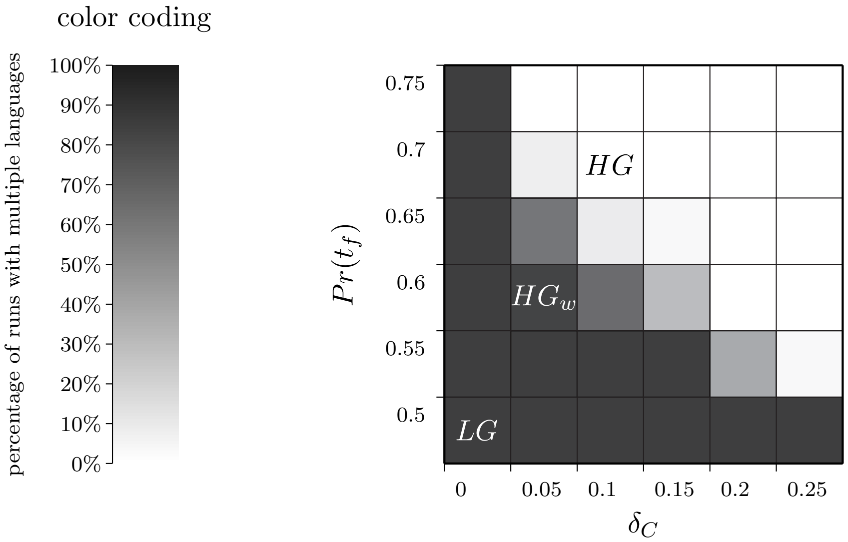


Figure 4.4: 36 different experiments of each kind of combination of prior probability and message costs of the marked message. Each experiment was conducted 50 times and the shade of gray depicts the percentage of simulation runs ending with multiple language regions: the darker the shade, the more often did multiple language regions and therefore anti-Horn language learners emerge and stabilize.

To get a more proper insight into these dependencies between game parameters and simulation results, I started Experiment I(d) for a discrete continuum of the Horn spectrum: I started 50 runs each for all possible combinations of $P(t_f) \in \{0.5, 0.55, 0.6, 0.65, 0.7, 0.75\}$ ($P(t_r) = 1 - P(t_f)$) and cost difference $\delta_C = C(m_m) - C(m_u)$ with $\delta_C \in \{0, 0.05, 0.1, 0.15, 0.2, 0.25\}$. For each combination I measured the percentage of simulation runs that ended with the emergence of multiple language region. Figure 4.4 shows the resulting graph with the continuum of message costs difference δ_C on the x-axis and the continuum of prior probability $Pr(t_f)$ on the y-axis, where the shade of gray of each combination depicts the percentage of multiple language regions as follows: the darker the shade, the higher the percentage of runs with multiple language regions.

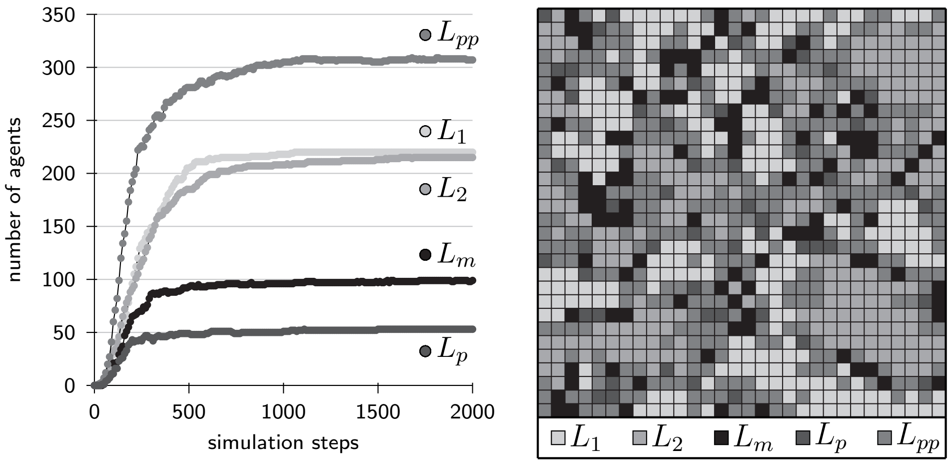
These results show different facts: first, if $Pr(t_f) = .5$ or $\delta_C = 0$, then all runs end up with multiple language regions. In either case one condition is violated for defining a Horn game. Furthermore, neither of the two possible signaling languages is superior. Thus if we have two equally good signaling systems, both form regions in each simulation run (as we'll see, this does not necessarily hold for smaller populations).

Second, if $Pr(t_f) > .5$ or $\delta_C > 0$, then we have a variant of a Horn game. Here the percentages of trials ending with the emergence of multiple language regions depend on the difference of both parameters. Roughly speaking, the higher $Pr(t_f) + \delta_C$, the *stronger* the Horn game is and the less probable multiple language regions emerge. Furthermore, the rule of thumb appears to be that if $Pr(t_f) + \delta_C \geq 0.8$, then the percentage of runs ending with the emergence of multiple language regions is (almost) zero (at least according to the experiment results). Additionally, if $Pr(t_f)+\delta_C \leq 0.7$, then the percentage of runs ending with the emergence of multiple language regions is (almost) 1. Consequently, for the given setup it is expected that i) almost all simulation runs bear multiple language regions for the weak Horn game and ii) almost all simulation runs end up without the emergence of multiple language regions for the weak Horn game.

Note that this is only valid for the given dynamics and the given lattice size and not necessarily. The results cannot be applied one-to-one to different settings. But in general it can be seen that for other lattice sizes and dynamics: the higher the value of $Pr(t_f) + \delta_C$ (and therefore the more unequal both signaling systems are), the more unlikely it is that multiple language regions emerge; and the more likely it is that only one society-wide language of the stronger signaling system emerges: the Horn language.

Let's summarize the results of Experiment I: the results of Experiment I(a) showed that multiple language regions of both signaling language learners emerged in each run. On the contrary, it was observed for the Horn games (Experiments I(b) and I(c)) that multiple language regions of L_h and L_a learners did not emerge in each run. In some runs only one society-wide language emerged which was always L_h. Consequently, the emergence of learners of the anti-Horn language L_a was only observed, when multiple language regions emerged. A comparison of the results of I(b) with I(c) shows that the percentage of runs ending with regions of L_a learners is 99% for the weak Horn game, but only 4% for the normal Horn game (according to the experimental results), everything else being equal.

An analysis of the influence of different game parameters, thus for a subset of the Horn spectrum, was conducted with Experiment I(d). It could be shown that the Lewis game and the weak Horn game are inside a range of parameters, for which the emergence of multiple language regions can be expected. Furthermore, the normal Horn game is outside this parameter range (Figure 4.4). In the next Experiment II I will examine if the same facts hold for RL^∞ dynamics.



(a) Sample learning curves

(b) Sample pattern

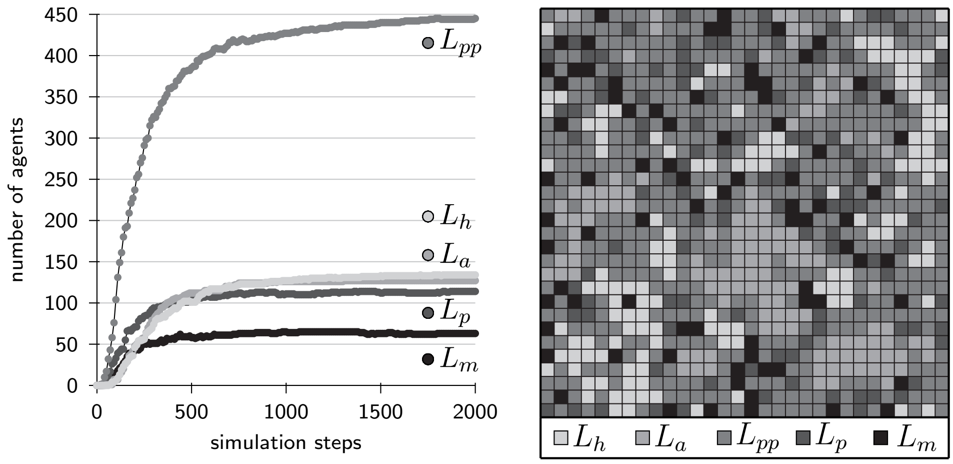
Figure 4.5: Sample learning curves over 2000 simulation steps (4.5a) and a sample of the resulting pattern after 2000 simulation steps (4.5b) for RL^{∞} agents playing the Lewis game on a 30×30 toroid lattice.

Experiment II: RL Agents with Unlimited Memory

In Experiment II(a) I conducted 100 simulation runs of the network game $NG_{II(a)} = \langle LG, X, RL^{\infty}, G_{l30} \rangle$, where $X = \{x_1, \ldots x_{900}\}$ and G_{l30} is a 30×30 toroid lattice. Here the stable society structure turned out to be more diverse. Not only L_1, L_2 and L_m learners emerged, but also learners of partial pooling and pooling languages. I labeled all partial pooling languages $L \in \{L_{13}, L_{14}, L_{23}, L_{24}, L_{31}, L_{32}, L_{41}, L_{42}\}$ with L_{pp} and all holistic pooling languages $L \in \{L_3, L_{34}, L_{43}, L_4\}$ with L_p. Figure 4.5 depicts sample learning curves and a sample of the resulting pattern of Experiment II(a).

As Figure 4.5a shows, only around 50% of the whole society become stable learners of a signaling language L_1 or L_2. In particular, a non-trivial fraction of the stable learners learned one of the partial pooling languages, in total around 33% of the society. Figure 4.5b reveals that (similar to the results in Experiment I(a)) signaling language learners stabilize in local regions, circumvented by border agents. But the border agents are mostly L_{pp} learners. Furthermore, those borders are much broader.

For Experiment II(b) I conducted 100 runs of the network game $NG_{II(b)} = \langle HG_w, X, RL^{\infty}, G_{l30} \rangle$, thus agents play the weak Horn game. Again, all partial pooling languages $L \in \{L_{hs}, L_{hy}, L_{as}, L_{ay}, L_{sh}, L_{sa}, L_{yh}, L_{ya}\}$ are labeled with L_{pp} and all pooling languages $L \in \{L_s, L_{sy}, L_{ys}, L_y\}$ are labeled with



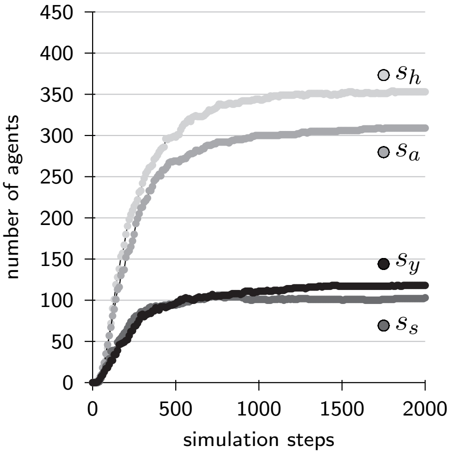
(a) Sample learning curves (b) Sample pattern

Figure 4.6: Sample learning curves over 3000 simulation steps (4.6a) and a sample of the resulting pattern after 3000 simulation steps (4.6b) for RL^{∞} agents playing the weak Horn game on a 30×30 toroid lattice.

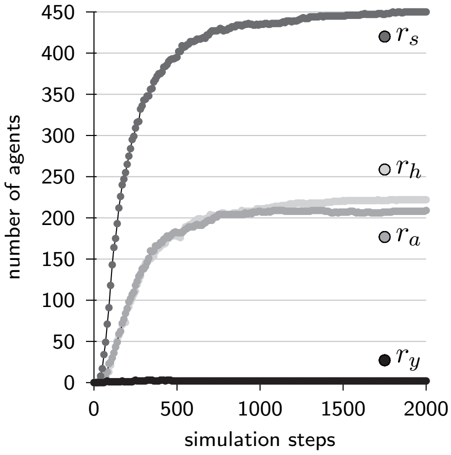
L_p. In comparison to Experiment II(b), here the number of partial pooling language learners is much larger, almost half of all agents, whereas learners of signaling languages L_h or L_h turned out to be only a small number, less than 150 for each group (Figure 4.6a). Horn and anti-Horn language learners formate in local language groups, but these groups are small and rather arranged like islands in the see of partial pooling and pooling language learners (Figure 4.6b). Note that here Smolensky language learners are not depicted explicitly, but entailed in the group of L_p learners.

Because of the high number of partial pooling language learners, it would be interesting to get a more detailed picture of the agents' behavior by isolating sender and receiver strategy. Figure 4.7a shows the course for the change of sender strategies of all agents, Figure 4.7b of receiver strategies. As you can see, while the number of sender strategies depicting behavior according to Horn and anti-Horn is quite high, the majority of agents learn the receiver strategy depicting behavior according to Smolensky. These facts explain the high number of L_{pp} learners since Horn as sender and Smolensky as receiver strategy forms a partial pooling language.

This can be highlighted by Figure 4.8a. This figure depicts the final pattern already depicted in Figure 4.6b, but now each agent's sender and receiver strategy is depicted independently: the top-left triangle of each square depicts the sender strategy, the bottom-right one the receiver strat-



(a) Senders' learning curves



(b) Receiver' learning curves

Figure 4.7: Sample learning curves over 2000 simulation steps for sender strategies (4.7a) and receiver strategies (4.7b) for RL^∞ agents playing the weak Horn game on a 30×30 toroid lattice.

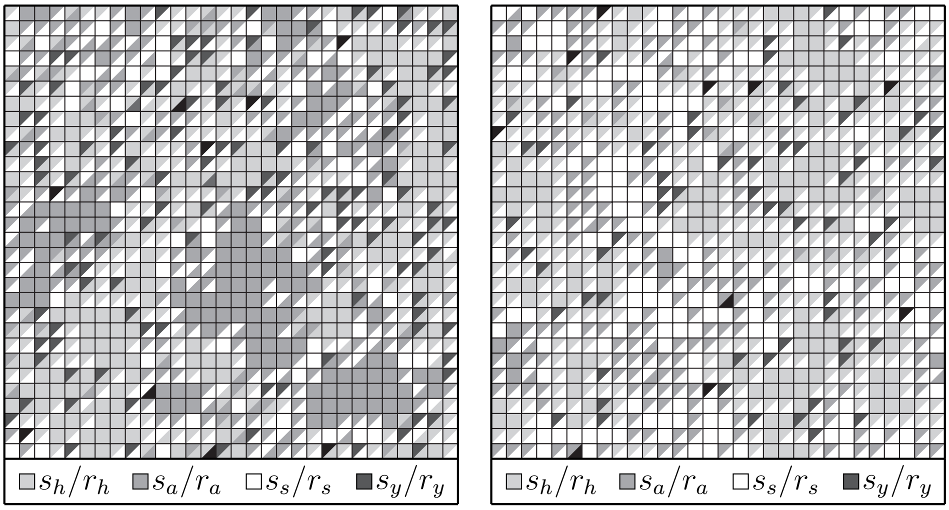
egy. Again, it is clear to see that the emergence of the high number of L_{pp} learners is a result of the fact that a lot of agents learns Smolensky as receiver strategy.

Similar results can be seen for the normal Horn game. Figure 4.8b depicts the resulting pattern, again both sender and receiver strategy as well. As you can see, the emerged number of Smolensky strategy learners, especially as receiver strategy, is high, in fact much higher than for the weak Horn game, whereas there is essentially no emergence of anti-Horn strategy learners, neither as sender, nor as receiver strategy.

The focal question seems to be: why do so many agents learn Smolensky as receiver strategy? The answer can be found in two facts: first, agents seem to have a strong initial tendency to learn Smolensky as receiver strategy as already seen in previous experiments (e.g. Figure 2.2 for replicator dynamics, Figure 2.13 for basic experiments with reinforcement learning). And second, the RL^∞ dynamics seems to be not flexible (see Table 4.4) enough for agents to change strategies learned early on in a repeated game.[5]

In Experiment II(d) I was interested in the way, game parameters influence the distribution of resulting strategies. I made 50 simulation runs each for the Horn game spectrum of $Pr(t_f) \in \{0.55, 0.6, 0.65, 0.7, 0.75\}$ and

[5]In terms of learning dynamics RL^∞ seems to be too *cold* to overcome this initial tendency: agents learning too fast and stuck in initial tendencies.



(a) Resulting pattern of HG_w

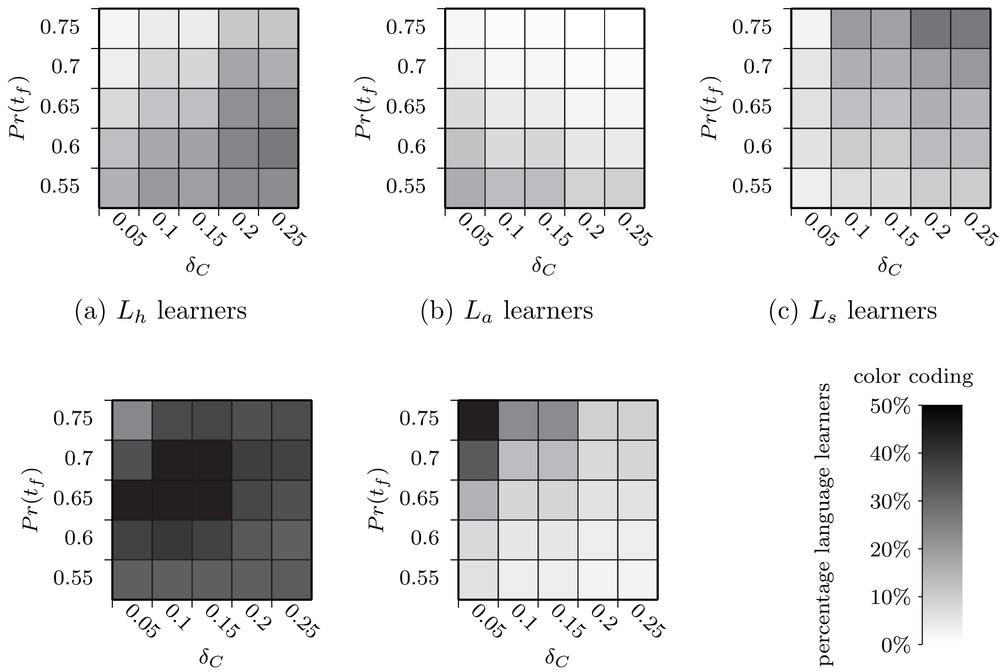
(b) Resulting pattern of HG

Figure 4.8: Sample of the resulting pattern after 3000 simulation steps of HG_w (4.8a) and HG (4.8b), each square explicitly depicts the sender strategy (top-left triangle) and the receiver strategy (bottom-right triangle).

$\delta_C \in \{0.05, 0.1, 0.15, 0.2, 0.25\}$. The average number of agents that have learned a specific language is depicted in Figure 4.9, Figure 4.9a for L_h, Figure 4.9b for L_a, Figure 4.9c for L_s and Figure 4.9d for L_{pp} learners; and finally Figure 4.9e for agents that haven't learned a language.

At least two interesting facts are evident: first, it is clear to see that by increasing the Horn parameters, the number of L_a learners decreases and the number of L_s learners increases. That is not surprising since strong Horn parameters support the Smolensky language, but prohibit anti-Horn language learners to emerge. The second observation is that for low δ_C values combined with high $P(t_f)$ values, the number of Horn language learners decreases, while the number of L_{pp} learners plus non-learners increases. For significantly high $P(t_f)$ values the number of non-learners increases dramatically (e.g. more than half of all agents were non-learners for the combination $P(t_f) = 0.75$, $\delta_C = 0.05$).

The explanation for the increasing number of L_{pp} learners is result of the fact that the higher the prior probability $P(t_f)$, the more probable it is that agents initially learn Smolensky as receiver strategy. The explanation for the suddenly increasing number of non-learners can probably be found in the following fact: because of the low probability of $P(t_r)$, agents haven't



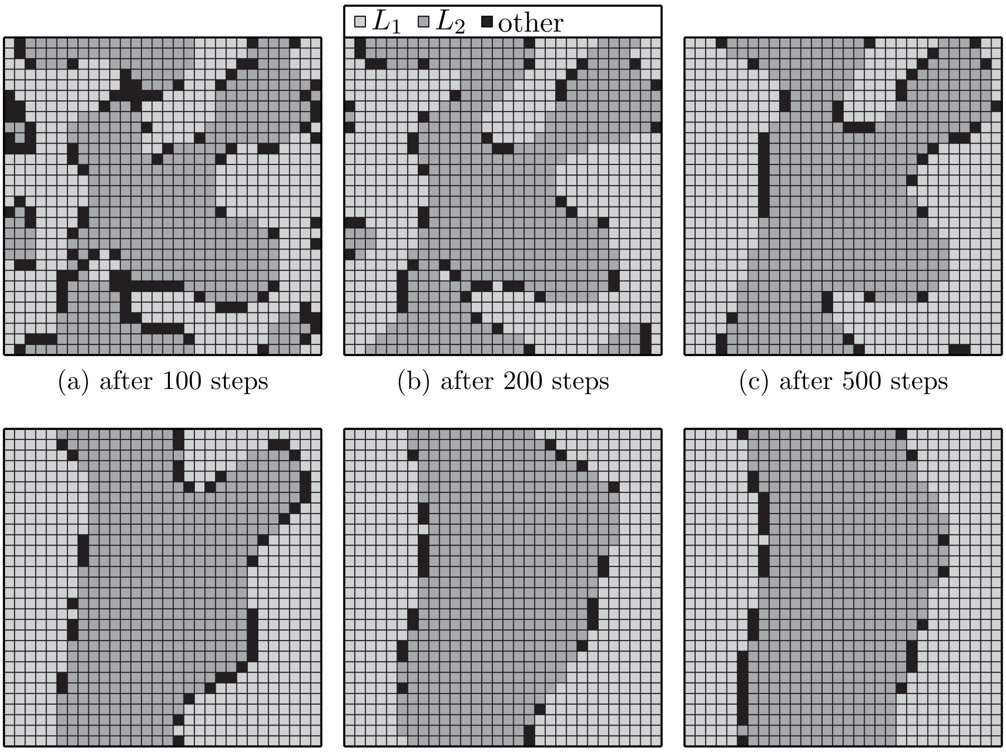
(a) L_h learners (b) L_a learners (c) L_s learners

(d) L_{pp} learners (e) no learners

Figure 4.9: The fraction of agents that have learned L_h, L_a, L_s, L_{pp} or no language at all for a subset of the Horn spectrum: all game parameter combinations for $0.55 \leq P(t_f) \leq 0.75$ and $0.05 \leq \sigma_C \leq 0.25$.

played this state often enough to have learned a language after 1000 steps and just need more time. All in all, we observe a nice negative image relationship between L_a and L_s learners on the one hand (Figures 4.9b and 4.9c), and (by counting non-learners as prospective L_{pp} learners) L_h and L_{pp} learners on the other hand (Figures 4.9a and 4.9d).

To sum up Experiment II: RL^∞ agents are less flexible than their BL^∞ cousins. While for BL^∞ agents ,language regions spread quickly and broadly, (only a small number of border agents didn't learn a signaling language), for RL^∞ agents regions of signaling languages spread much more slowly and are smaller, divided by much broader borders. Furthermore, these border agents are primarily learners of partial pooling or pooling languages. This fact is particularly visible for both Horn games, where more than half of all agents learn a L_{pp} language. That is because the majority of agents learn the Smolensky strategy as receiver and stick to it. The explanation for this phenomenon seems to be a result of two facts: i) agents that play the Horn game in a population have an initial tendency to learn Smolensky as re-



(a) after 100 steps (b) after 200 steps (c) after 500 steps

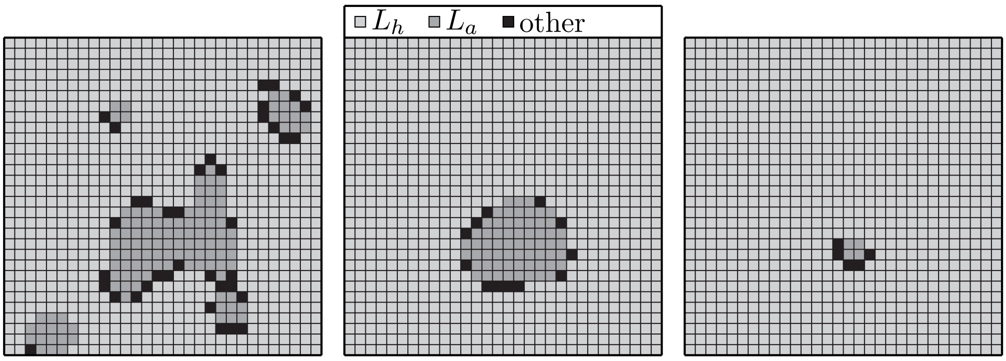
(d) after 1000 steps (e) after 2000 steps (f) after 3000 steps

Figure 4.10: The convex border melting phenomenon: language regions with convex nor concave borders can stabilize, but only with linear ones.

ceiver strategy, as also earlier studies showed; and ii) the RL^∞ dynamics is not flexible enough; the agents learn too fast and stick to strategies learned earlier. A possibility to make the agents' behavior more flexible is to give them a *limited memory*, as the next experiments will show.

Experiment III: BL Agents with Limited Memory

In Experiments III I analyzed the behavior of BL agents with limited memory. In Experiment III(a) I conducted 100 runs of the network game $NG_{III(a)} = \langle LG, X, BL^{100}, G_{l30}\rangle$ with $X = \{x_1, \ldots x_{900}\}$ and G_{l30} is a 30×30 toroid lattice. This setup brings a phenomenon into being which I call the *convex border melting phenomenon*. Figure 4.10 shows a sample course of patterns of the lattice after different numbers of simulation steps, here after 100, 200, 500, 1000, 2000 and 3000 steps. The different patterns display the convex border melting phenomenon: a border can stabilize in the end, only



(a) after 1000 steps (b) after 2000 steps (c) after 4000 steps

Figure 4.11: Whole convex regions are driven to extinction by the convex border melting phenomenon. This can lead to a final situation with only one global language, what is seen almost always for the Horn games, and sometimes for the Lewis game.

if it is linear, i.e. neither concave nor convex.[6]

It was not always the case that two regions, separated by a linear border, appear and stabilize. In fact, that happened only in 43% of all 100 simulation runs. In all the other cases, only one global language emerged at the end. This happened if one of the two signaling languages shapes solely convex regions which were driven to extinction by convex border melting. Figure 4.11 shows a sample course of patterns of the lattice after different numbers of simulation steps, here after 1000, 2000 and 4000 steps. In this simulation run the final situation was the emergence of one global language.

What does the convex border melting phenomenon mean for the Horn games? The simulation results revealed: if regions of L_a learners emerged, they were generally smaller and highly probably convex. Consequently, they were driven to extinction. Experiment III(b) and III(c) (conducting the network games $NG_{III(b)} = \langle HG_w, X, BL^{100}, G_{l30} \rangle$ and $NG_{III(c)} = \langle HG, X, BL^{100}, G_{l30} \rangle$) revealed that in all 100 simulation runs finally all agents became L_h learners and a development like depicted in Figure 4.11 was observed.

[6]As a matter of fact, a non-linear border is convex for one language region, and concave for the other one. As we'll see later in a more detailed analysis in Section 4.2, because of the superior number of the agents of the region with the concave border, these agents replace the agents of the region with the convex border. Furthermore, as we saw in the results of Experiment I, this didn't happen for BL^∞ agents. But here the limitation of the memory makes these agents more *flexible*, a term which we will define in Section 4.1.4.

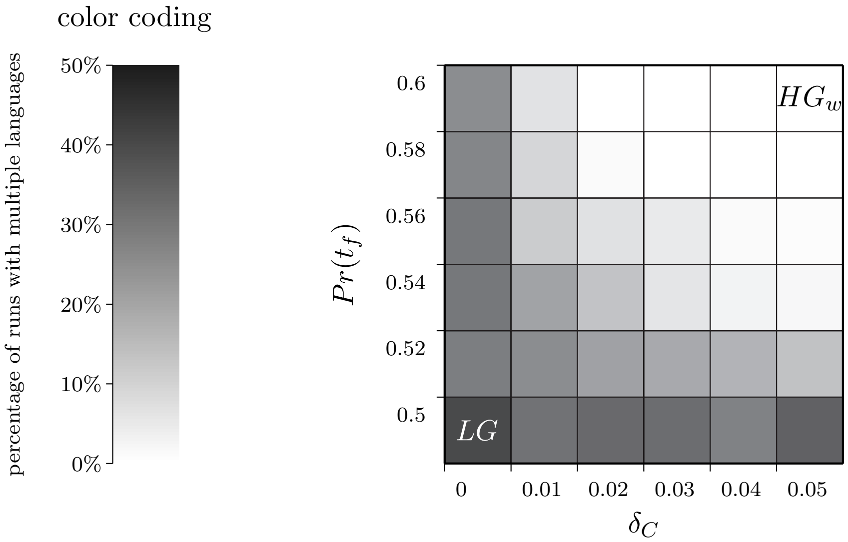
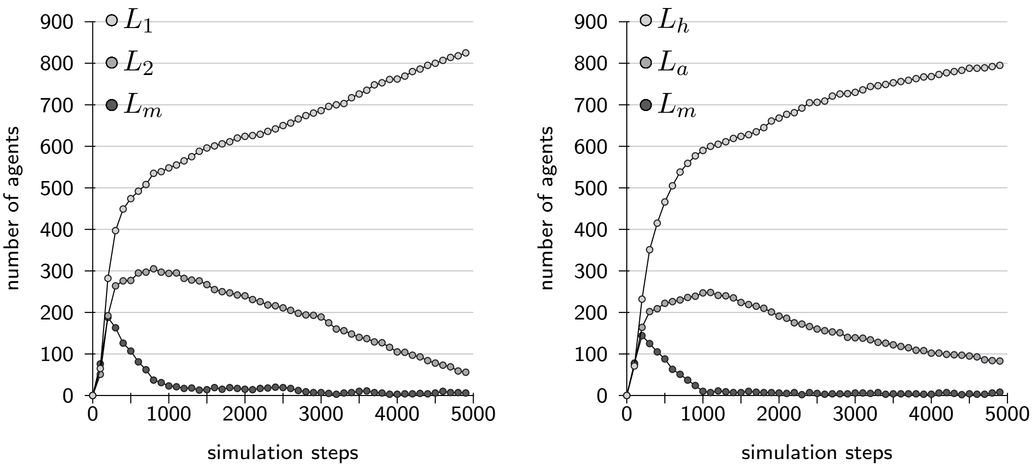


Figure 4.12: 36 different experiments of each kind of combination of prior probability and message costs of the marked message. Each experiment was conducted 50 times and the shade of gray depicts the percentage of simulation runs that ended with multiple language regions: the darker the shade, the more often did multiple language regions stabilize.

This result was supported by Experiment III(d), where I analyzed for a subset of the Horn game spectrum how probable multiple language regions emerge and stabilize. Since there was a percentage of 43% for the Lewis game and of 0% for the weak Horn game, I analyzed games between values of these two games. Thus, I made simulations for all combinations of $P(t_f) \in \{0.5, 0.52, 0.54, 0.56, 0.58, 0.6\}$ and $\delta_C \in \{0, 0.01, 0.02, 0.03, 0.04, 0.05\}$, 50 runs for each combination. The result is depicted in Figure 4.12: each square represents a particular parameter combination and the shade of gray depicts the percentage of runs with the emergence of multiple language regions, where the darker the shade, the higher the percentage.

The result was as expected: the stronger the Horn parameters, the less likely did multiple language regions stabilize. Note that in each simulation run multiple language regions emerged temporarily, but only stabilized if they were separated by a linear border (like in Figure 4.10). The percentage of runs was already low for the Lewis game (around 43%) and decreases with stronger Horn parameters.

To wrap up: BL^{100} agents have a high level of flexibility (c.f. Section 4.1.4) that causes the convex border melting phenomenon. Convex border regions are melting over time until they disappear completely. Since lan-



(a) Lewis games (b) Horn games

Figure 4.13: Sample learning curves for the Lewis (4.13a) and the Horn game (4.13b) for RL^{100} agents on a 30×30 toroid lattice, where in each case finally one society-wide language emerges.

guage regions of L_a learners are generally smaller for agents playing the weak or normal Horn game, these regions disappear and solely regions of L_h learners remain. This was seen in all 100 simulation runs for both Horn games. And even for the Lewis game one of both signaling languages spread society-wide and crowded out the other one in more than half of all runs. In the remaining runs local regions of both signaling languages emerged, separated by a linear border.

Experiment IV: RL Agents with Limited Memory

In Experiment IV I conducted simulation runs for reinforcement learning with limited memory. In detail, I simulated 100 runs each for the network games $NG_{IV(a)} = \langle LG, X, RL^{100}, G_{l30} \rangle$, $NG_{IV(b)} = \langle HG_w, X, RL^{100}, G_{l30} \rangle$ and $NG_{IV(c)} = \langle HG, X, RL^{100}, G_{l30} \rangle$ (Experiments IV(a)-IV(c)). The results of these experiments were quite similar to those of their BL^{100} cousins: convex language regions finally disappeared. Sample learning curves for the Lewis game (Experiment IV(a)) and the normal Horn game (Experiment IV(c)) are depicted in Figure 4.13.

Both sample learning curves show how the convex border melting phenomenon erases convex language regions of L_2 learners (Figure 4.13a), or L_a learners (Figure 4.13b), respectively. That happened in all simulation

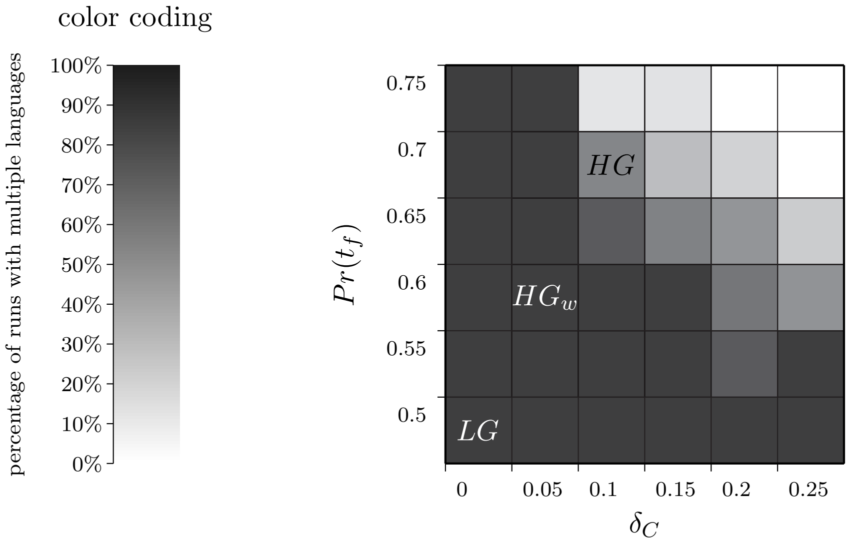


Figure 4.14: 36 different experiments of each kind of combination of prior probability and message costs of the marked message. Each experiment was conducted 50 times and the shade of gray depicts the percentage of simulation runs with the temporary emergence of multiple language regions after 1000 simulation steps: the darker the shade, the more often did multiple language regions and therefore anti-Horn players emerge.

runs for both Horn games: solely L_h learners survived finally, having assimilated temporary emerged language regions of L_a learners. In contrast to Experiment III, here it took a high number of simulation steps until the convex border melting was clearly noticeable. As evident from Figure 4.13a and refrl-melting:horn the regions of minorities start to shrink after roughly 1000 simulation steps. Up to this point, a structure of multiple language regions evolved, separated by border agents of miscommunication languages. This temporary patterns were quite similar to the final patterns of Experiment I (see e.g. Figure 4.2b and 4.3b).

To analyze such temporary patterns, I conducted Experiment IV(d). For a subset of the Horn spectrum I determined the percentage of runs, where multiple language regions emerged temporarily . Figure 4.14 shows the resulting pattern: the percentage of 100 runs for each parameter combination that multiple language regions emerged after 1000 simulation steps; where the darker the shade of gray, the higher the percentage.

Finally, I should remark that BL^{∞} agents and RL^{100} agents exhibit a similarity in emerging patterns. Obviously, the convex border melting phenomenon is only seen for the latter and therefore the final results of both

	LG	HG_w	HG
BL^{∞}	Exp. V(a)	Exp. V(b)	Exp. V(c)
RL^{∞}	Exp. VI(a)	Exp. VI(b)	Exp. VI(c)
RL^{100}	Exp. VII(a)	Exp. VII(b)	Exp. VII(c)
BL^{100}	Exp. VIII(a)	Exp. VIII(b)	Exp. VIII(c)

Table 4.3: Experiments for different games and different dynamics on a 30×30 toroid lattice.

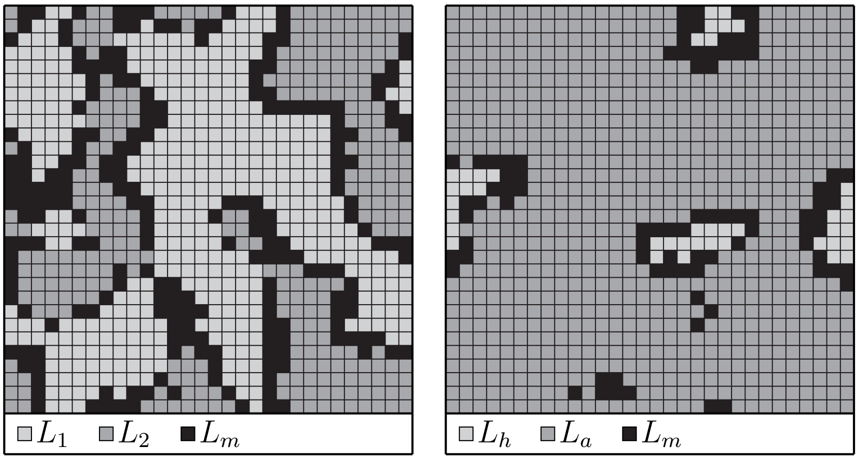
types are quite different. Nevertheless, it can be shown that the intermediate patterns of RL^{100} agents look quite similar to the final patterns of BL^{∞} agents.

4.1.2 The Degree of Locality

While in the last section I analyzed the behavior of agents communicating with their direct neighbors on a toroid lattice, in this section I want to relax the condition that possible communication partners are constrained to the direct neighborhood. The basic idea is as follows: the probability of an agent x choosing a particular communication partner x' is positively correlated to the spatial distance between x and x'. This choice behavior transforms the toroid lattice into a *social map* (see Nettle (1999) as a comparative account).

Similar to the last section, I analyzed network games for all possible combinations of dynamics and games, as depicted in Table 4.3.[7] To realize a social map structure, I used the the selection algorithm *Social Map* as given in Definition 3.30. This selection algorithm has a parameter variable γ, called the *degree of locality*, that influences the interaction structure: the higher γ, the more local is the interaction structure. A maximal local interaction structure is direct neighbor communication, whereas a minimal local and therefore a maximal global interaction structure is random interaction. In detail: for $\gamma = 0$, agents interact randomly and by increasing γ, the interaction structure approximates neighborhood communication and the social map approximates a grid structure (for a detailed description of these dependencies see Section 3.3.2). This account makes it possible to analyze interaction structures that are somewhere between the local interaction structure of a grid network and the global interaction structure of a population of randomly chosen communication partners. The latter structure is, in terms of network theory, equal to a complete network.

[7] These experiments and there results are already published in Mühlenbernd (2011).



(a) Lewis game (b) weak Horn game

Figure 4.15: Sample of the resulting pattern of learners with different strategies for $\gamma = 6$, for the Lewis and the weak Horn game.

Experiment V: BL Agents with Unlimited Memory

In Experiment V I examined the influence of the degree of locality γ on the population dynamics of BL^∞ agents. I performed 25 simulation runs each, in Experiment V(a) of the network game $NG_{V(a)} = \langle LG, X, BL^\infty, G_{l30}\rangle$, in Experiment V(b) of the network game $NG_{V(b)} = \langle HG_w, X, BL^\infty, G_{l30}\rangle$ and in Experiment V(c) of the network game $NG_{V(c)} = \langle HG, X, BL^\infty, G_{l30}\rangle$, where $X = \{x_1, \ldots x_{900}\}$ is a set of 900 agents and G_{l30} is a 30×30 toroid lattice. As initially mentioned, the agents use the Social Map selection algorithm.

The main task was to examine how the degree of locality γ influences the emergence of multiple language regions. Thus I started simulation runs with different γ-values. A sample of the resulting pattern for $\gamma = 6$ is depicted in Figure 4.15, Figure 4.15a for the Lewis game, Figure 4.15b for the weak Horn game. As evident, the runs resulting in a split societies produced signaling language regions separated by borders agents of miscommunication language learners, quite similar to the results of Experiment I, but with slightly broader borders.

Such a resulting population structure was a typical outcome for high γ values (local interaction structures), whereas for low γ values (global interaction structures) agents tend to agree on one global language. Figure 4.16 shows the percentage of 25 trials producing a society with multiple

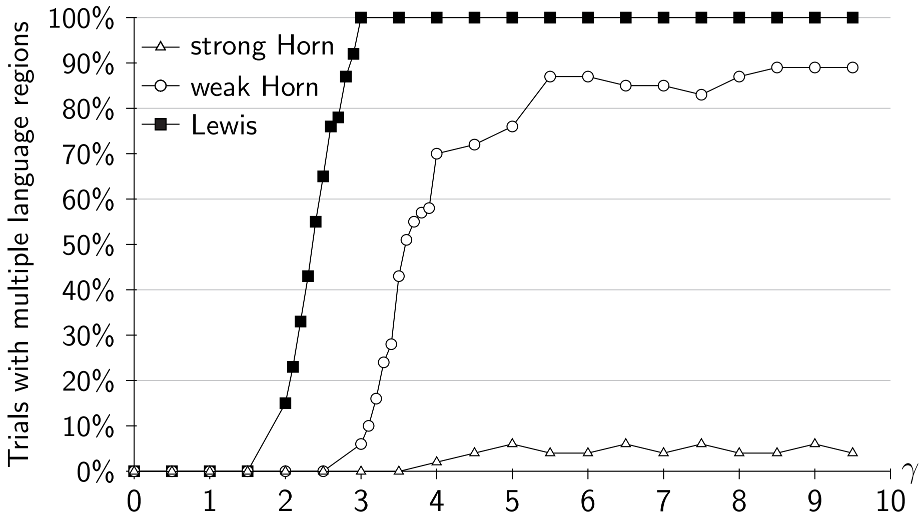


Figure 4.16: The percentage of trials resulting in a society with multiple languages regions. For the Lewis game, every trial culminated with both signaling languages if $\gamma \geq 3$. For the weak Horn game about 90% of the trials produced both Horn and anti-Horn language learners if $\gamma \geq 4$. And for the normal Horn game not more than 5% of the trials resulted with both Horn and anti-Horn language learners, even for high γ-values.

local signaling languages for different γ-values between 0 and 9.5. The results indicate that the probability of the emergence of multiple language regions strongly depends on the degree of locality γ. Remember: with $\gamma = 0$ we have random communication and expect only one global signaling language to emerge, whereas for a high γ-value we're close to neighborhood communication and expect multiple local signaling languages regions to emerge.

Figure 4.16 shows for the Lewis game: every trial resulted in a society with only one global signaling language if $\gamma < 2$. For $\gamma \geq 3$ every trial led to a society with multiple language regions of L_1 and L_2 learners. In the range $2 \leq \gamma < 3$ the percentage of trials ending with multiple language regions increases with γ. All in all, these results show: the probability that multiple languages regions emerge increases with respect to the degree of locality γ.

A similar dependency can be seen for the weak Horn game: for $\gamma \leq 3$ one society-wide signaling language emerged in every run, and each time it was L_h. For $\gamma \geq 4$ the percentage of trials that ended with a society of both, Horn and anti-Horn language learners, levels out at around 90%. In the range $3 < \gamma < 4$ the percentage of trials that rendered a society with multiple language regions increases with the γ-value. For the normal Horn game the percentage of trials that ended with multiple language regions was low in any case: for $\gamma < 4$ only L_h learners emerged, but even for $\gamma \geq 4$ the percentage of trials that finished with both Horn and anti-Horn language learners is only around 5%.

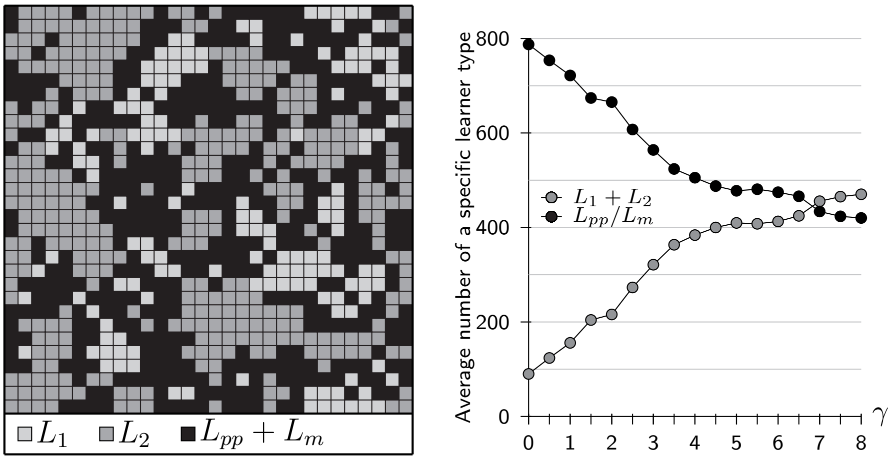
Let's summarize the results of Experiment V. For a population of BL^∞ agents playing a Lewis or Horn game, the following holds: the higher the degree of locality γ, the higher the probability that multiple language regions emerge. In any case, however, this probability is still very low for the normal Horn game.

Experiment VI: RL Agents With Unlimited Memory

In Experiment VI I examined the influence of the degree of locality γ on the behavior of RL^∞ agents: I started simulation runs of the network games $NG_{VI(a)} = \langle LG, X, RL^\infty, G_{l30}\rangle$, $NG_{VI(b)} = \langle HG_w, X, RL^\infty, G_{l30}\rangle$ and $NG_{VI(c)} = \langle HG, X, RL^\infty, G_{l30}\rangle$ (Experiment VI(a)-(c)), where $X = \{x_1, \ldots x_{900}\}$ is a set of 900 agents and G_{l30} is a 30×30 toroid lattice. The agents use the social map selection algorithm.

The results of Experiment VI(a) for RL^∞ agents playing the Lewis game contrasts to the BL^∞ agents' results in the following way: first, agents learned both signaling languages L_1 and L_2 in each trial, independently of the γ-value. Second, only a fraction of the agents learned signaling languages. Similar to Experiment II, a lot of agents learned a partial pooling languages $L_{pp} \in \{L_{13}, L_{14}, L_{23}, L_{24}, L_{31}, L_{32}, L_{41}, L_{42}\}$, as displayed in Figure 4.17a: a sample of the resulting pattern for a simulation run with $\gamma = 7.0$. Furthermore, the degree of locality γ did influence the number of agents who learned a signaling language or a partial pooling language. Figure 4.17b shows the number of agents (averaged over 25 trials) who learned a signaling or a partial pooling language. As you can see, the number of agents who learned a signaling language increases with the γ-value and levels off at around 450 signaling language learners. This is exactly 50% of all agents of the population.

In these simulation runs, signaling language learners stabilized in groups, where the higher the degree of locality γ was, the stronger those groups grew. Since RL^∞ agents became more inert as the simulation ran longer, the growing eventually stopped, and the remaining agents stabilized at a pooling language. In most of the cases the pooling language learners learned the Smolensky strategy as receiver strategy for reasons that were already



(a) Sample pattern (b) Average number of learner types

Figure 4.17: A sample of the resulting pattern of RL^{∞} agents playing the Lewis game on a 30×30 toroid lattice with $\gamma = 7.0$ (Figure 4.17a) and the average number of different learner types over 25 simulation runs each for different γ-values between 0 and 8 (Figure 4.17b).

discussed for the analysis of Experiment II(b).

Now let's take a look at the results for Experiment VI(b): RL^{∞} agents playing the weak Horn game. The resulting attribution of the average number of learner types to different γ-values is depicted in Figure 4.18a. Each data point represents the average number of particular language learner types over 25 simulation runs, each for an appropriate γ-value. As you can see, the average number of learners of partial pooling languages increases with the γ-value, from 300 to 450 agents. Further, the average number of anti-Horn language learners also increases with the γ-value, but stays below an average of 100 agents. In addition, the average number of Smolensky language learners seems to be independent of γ since it is constantly around 100 agents. An interesting fact is seen for Horn language learners: for $0 \leq \gamma < 3$, there evolved around 100 to 150 of them on average, but around 250 for higher values. It looks like a γ-value of 3 is a tipping point that doubles the average number of Horn language learners. Also note that the remaining agents were non-learners (not depicted here) which average number decreases with an increasing γ-value.

Last, the results for RL^{∞} agents playing the normal Horn game are depicted in Figure 4.18b. As you can see, for $\gamma \leq 2$ no agent learned any

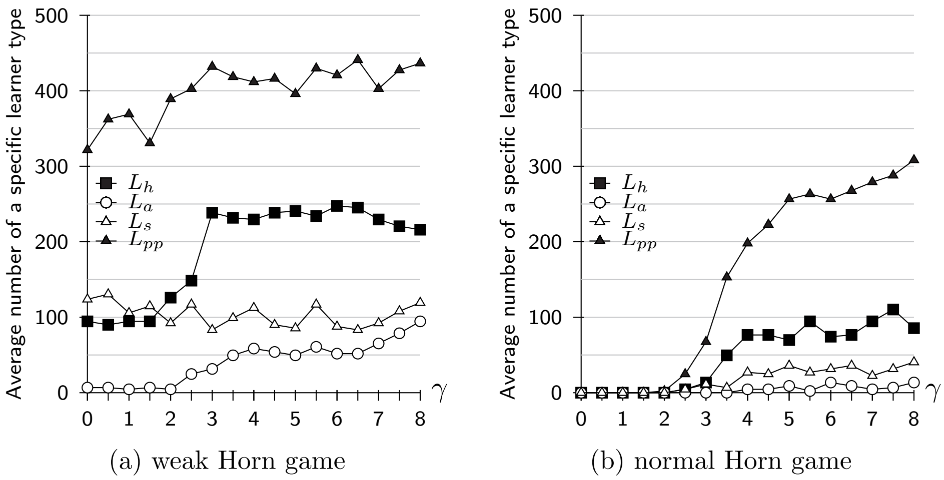


Figure 4.18: The average number of different language learner types over 25 trials each for different γ-values each, for the weak and the normal Horn game as well.

language. With increasing γ-value, the average numbers of Horn, anti-Horn, Smolensky and pooling language learners increase. For $\gamma \geq 2$ the average number of Horn language learners stabilized around 80-100, the average number of anti-Horn language learners around 10 and the average number of Smolensky language learners around 40. Nevertheless, the number of pooling language learners permanently increases with γ.By summing this up, the overall average number of agents that learned a target language is less than half of all agents even for high γ-values. Further, the emergence of anti-Horn language learners is probable, but those groups are minute in general.

All in all, it is important to recognize that because of being less flexible, even for high γ-values less or maximally half of all RL^∞ agents learned a signaling language for the Lewis and weak Horn game on average, and even maximally 100 for the normal Horn game. These results fit to those of Experiment II. Furthermore, it was seen for each game that an increasing γ-value raises the probability of the emergence of signaling languages. And with respect to the Horn game, a higher γ-value supports the probability that anti-Horn language learners emerge.

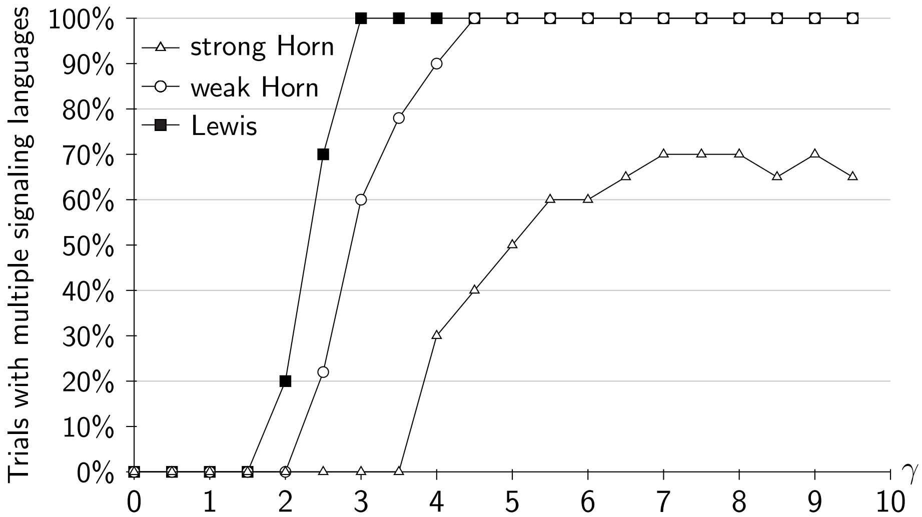


Figure 4.19: The percentage of trials resulting in a society with multiple signaling languages. For the Lewis game, every trial culminated with both signaling languages if $\gamma \geq 3$. For the weak Horn game about 80% of the trials produced both Horn and anti-Horn language learners if $\gamma \geq 4$. And for the normal Horn game less than 10% of the trials resulted with both Horn and anti-Horn language learners.

Experiment VII: RL Agents with Limited Memory

In Experiment VII I examined the influence of the degree of locality γ on the behavior of RL^{100} agents: I started simulations of the network games $NG_{VII(a)} = \langle LG, X, RL^{100}, G_{l30} \rangle$, $NG_{VII(b)} = \langle HG_w, X, RL^{100}, G_{l30} \rangle$ and $NG_{VII(c)} = \langle HG, X, RL^{100}, G_{l30} \rangle$ (Experiment VII(a)-(c)), where $X = \{x_1, \ldots x_{900}\}$ is a set of 900 agents and G_{l30} is a 30×30 toroid lattice. The agents use the social map selection algorithm.

Figure 4.19 shows the results. Each data point displays the percentage of 25 simulation runs that ended with multiple signaling languages. The different curves depict results for the Lewis game, the weak Horn game, and the normal Horn game. This result is considerably similar to the result of Experiment V for BL^{∞} agents (see Figure 4.16), apart from slightly shifted values.

As you can see, the results for the Lewis game are as follows: for $\gamma < 2$ the whole society learned solely one of both signaling languages in every simulation run. For $2 \leq \gamma < 3$ the percentage of runs with the emergence of multiple languages regions increases. For $\gamma \geq 3$ multiple languages regions emerged in every run.

For the weak Horn game, $\gamma < 2.5$ induced all agents learning solely the Horn language in every run. For $2.5 \leq \gamma \leq 4$ the percentage of simulation runs that ended with multiple language regions increases with the γ-value, and for $\gamma > 4$, each simulation run ended with multiple language regions of Horn and anti-Horn language learners. Note that this result is quite similar to Experiment V, but with the slight difference that, while in Experiment V for $\gamma \geq 4$ the percentage of those final situation levels out at 90%, here it increases to 100%.

Finally the results for the normal Horn game are as follows: for $\gamma < 4$ all agents learned solely the Horn language in every simulation run. For $4 \leq \gamma < 7$ the percentage of simulation runs with multiple language regions increases with the γ-value. For $\gamma \geq 7$ the percentage of runs with regions of Horn and anti-Horn language learners levels off at around 70%. Again, in accordance with Experiment V the tipping point of the emergence of multiple language regions is $\gamma \geq 4$, but here the percentage levels off at around 70%, while in Experiment V it was only around 5%.

These results show that by increasing the γ-value, the probability of the emergence of multiple languages regions increases. In addition, samples of the resulting patterns are quite similar to those of Experiment V. Furthermore, by comparing these results with Experiment IV, it is also a matter of fact that for a γ-value not too high (≤ 7), there was no *convex border melting phenomenon* observable, at least not in a manageable runtime. Thus, for an intermediate γ value the structuring effect of RL^{100} agents is similar to the structuring effect of BL^{∞} agents. This is an important insight that shows both dynamics have an similar degree of flexibility, as I will define in Section 4.1.4.

Experiment VIII: BL Agents with Limited Memory

In Experiment VIII I examined the influence of the degree of locality γ on the behavior of BL^{100} agents: I started simulations of the network games $NG_{VIII(a)} = \langle LG, X, BL^{100}, G_{l30}\rangle$, $NG_{VIII(b)} = \langle HG_w, X, BL^{100}, G_{l30}\rangle$ and and $NG_{VIII(c)} = \langle HG, X, BL^{100}, G_{l30}\rangle$ (Experiment VIII(a)-(c)), where $X = \{x_1, \ldots x_{900}\}$ is a set of 900 agents and G_{l30} is a 30×30 toroid lattice. The agents use the social map selection algorithm.

For each experiment I started 25 simulation runs for γ-values between 0 and 9.5. Each simulation run resulted in a society with only one signaling language independent of the γ-value. In Experiment IV(b) and IV(c) for the Horn games all agents learned the Horn language at the end of a simulation run. But by taking a closer look at the simulation runs you can see that in a lot of trials for high γ-values substantial islands of anti-Horn language learners emerged *during* a simulation run, but for these cases the convex border melting phenomenon drove them to extinction (similar to the results of Experiment III). In sum, a population of BL^{100} agents almost always ended up with one unique signaling language, independently of the γ-value.

4.1.3 Further Influences

As I showed in the last Section 4.1.2, the population's interaction structure has a strong influence on the way, the arrangement of language regions evolves and stabilizes. A high degree of locality supports the emergence of regional meaning. But not only the interaction structure, but also the size of the population may have an influence, as Wagner (2009) was able to show. Consequently, to analyze the impact of population size on the final structures, I will conduct Experiment IX that entails simulation runs for different lattice sizes.

In addition, in previous experiments I always distinguished between unlimited memory and a memory size of 100. It would be interesting to see how a smaller or larger memory size would influence the agent's learning performance. Thus, with Experiment X I will perform simulation runs for agents with different memory sizes to analyze the impact of memory size on the learning performance.

Experiment IX: Population Size

As already mentioned, Wagner's (2009) experiments showed that the number of language regions increases with the size of the population. Thus, I made similar experiments with the expectation that the lattice size is proportionally related to the probability that multiple language regions emerge.[8]

To examine this suspicion, I started Experiment IX: I conducted simulation runs for the network games $NG_{I(a)}$, $NG_{I(b)}$ and $NG_{I(c)}$ (see Experiment I), thus BL^{∞} agents playing the games LG, HG_w and HG. For each game I conducted 100 simulation runs each for different lattice sizes between 5×5

[8] Since I still analyze small lattices, the number of language regions did not play any role; in general, if multiple language regions evolved, there were only two of them: one of each signaling language.

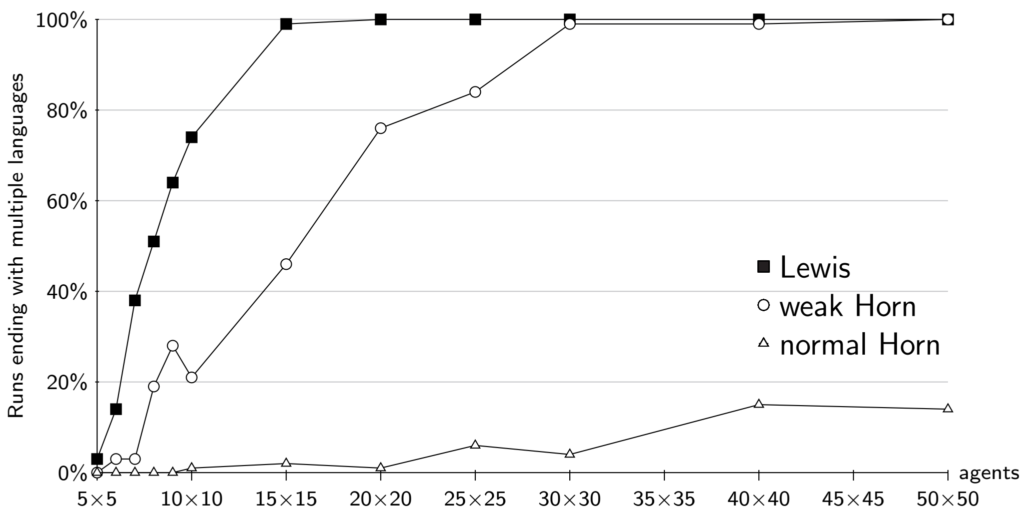


Figure 4.20: The percentage of the emergence of multiple language regions over 100 runs for different population sizes.

and 50 × 50;[9] and compared the percentages of runs that ended with multiple language regions. The result of Experiment IX is depicted in Figure 4.20.

As you can see, the following holds at least for the Lewis game and the weak Horn game: for the most part the percentage of runs that ended with multiple language regions increases with the number of agents. Furthermore, the Lewis game is content with smaller populations than the weak Horn game for the emergence of multiple language regions. The normal Horn game's percentages are small in all cases and don't increase strongly. Thus the population size effects the probability of multiple language regions for agents playing the games LG and HG_w in a proportional manner, while this effect is not strong for the normal Horn game.

In a second analysis of the data of both Horn games I calculated the average number of L_a language learners for different lattice sizes. The results are depicted in Figure 4.21: for the weak Horn game the number of emerged and stabilized anti-Horn language learners is clearly increasing with the population size in a linear manner, whereas for the normal Horn game the number of L_a learners is hardly increasing and small even for large population sizes.

Now let's summarize the results of Experiment IX: for a population of

[9] I simulated 100 runs on a $n \times n$ lattice for n = 5, 6, 7, 8, 9, 10, 15, 20, 25, 30, 40, 50 in each case.

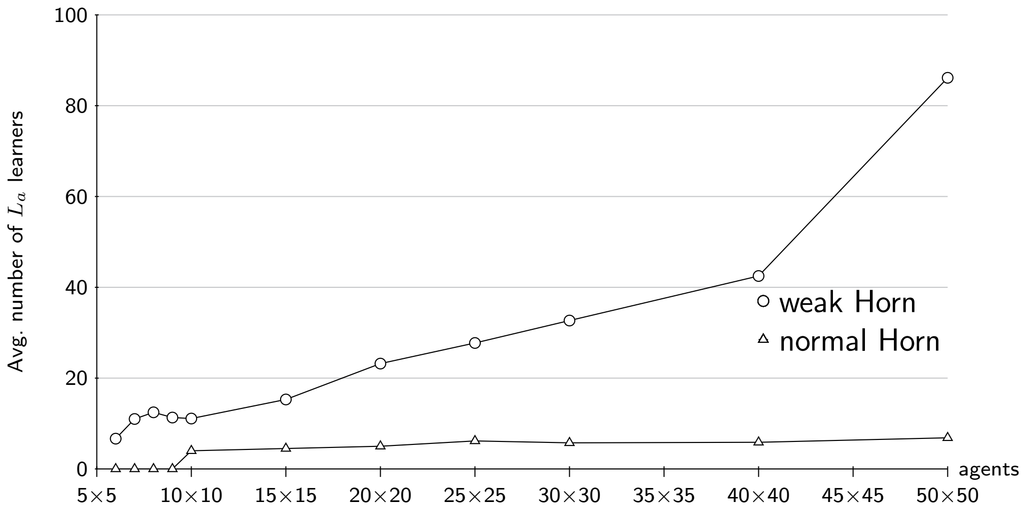


Figure 4.21: The average number of L_a learners over all runs for which L_a learners emerge and stabilize; in dependence of different population sizes.

BL^{∞} agents playing the Lewis or Horn game on toroid lattices with different sizes, the following holds: the larger the lattice/population size, the higher the approximated probability that multiple language regions emerge. These language regions consists either of L_1 or L_2 learners (Lewis game), or either of L_h or L_a learners (both Horn games), respectively. In addition, for agents playing the weak Horn game, the lattice size has a clear influence i) on the probability of the emergence of L_a learners, and ii) on the number of emerging L_a learners. This dependency is much weaker for the normal Horn game, if existent at all. I was able to obtain similar results for the other dynamics, so that the simulations revealed the following two correlations: i) population size is proportional to the probability of the emergence of multiple language regions; and ii) by considering Horn games, there is a direct proportion between population size and the number of L_a learners, at least for the weak Horn game. Furthermore, these correlations are apparently weakened by increasing Horn parameters $Pr(t)$ and δ_C, as the results for the normal Horn game inferred.

Experiment X: Memory Size

By comparing the results of RL^{∞} and RL^{100} dynamics, I found out that the first dynamics is too cold or inflexible for efficient signaling systems to spread out, whereas the second dynamics supports signaling systems to spread society-wide. Thus, it seems reasonable that the memory size has an

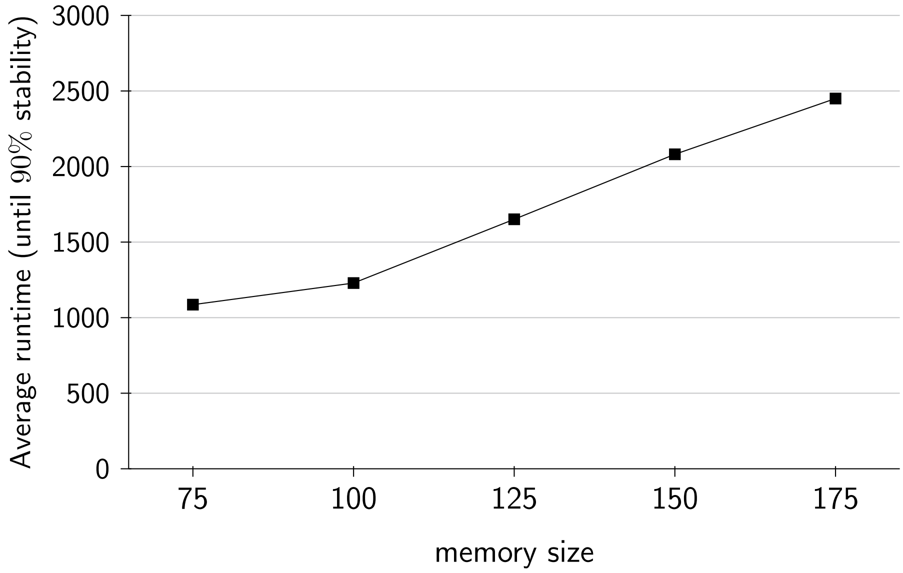


Figure 4.22: The average runtime until 90% of the population is stable - over 20 simulation runs each for every memory size between 75 and 175.

influence of the agents flexibility in the following way: the larger the memory size, the less flexible the agent's learning behavior. The agents' flexibility can be measured in the runtime, the agents need to stabilize a society of (almost) only signaling language learners. Thus I started Experiment X to determine the influence of the agents' memory size on the number of simulation steps required until a signaling language has captured 90% of the whole society. Consequently, I started simulations with RL^m agents with a memory size $m \in \{75, 100, 125, 150, 175\}$ for the Lewis game and measured the average runtime over 20 trials each. As you can see in Figure 4.22, the runtime for the whole society to learn the unique signaling language increases linearly with the memory size.

4.1.4 Interpretation of the Results

A multitude of experiments were made to extract the influences of the dimensions *game parameters*, *interaction structure* and *learning dynamics* on the final population structure, particularly on the probability of the emergence of multiple language regions. The following list summarizes the results:[10]

[10]Each point in the list assumes that all else being equal.

1. signaling game
    - game parameters: increasing game parameters inside the Horn spectrum ($Pr(t_f)$, δ_C) generally decreases the probability of the emergence of multiple language regions
2. interaction structure
    - degree of locality γ: increasing the degree of locality generally increases the probability of the emergence of multiple language regions
    - lattice size: increasing the lattice size generally increases the probability of the emergence of multiple language regions
3. learning dynamics
    - memory size: increasing the memory size generally increases the probability of the emergence of multiple language regions
    - learning type: both learning types' (BL and RL) impact can only be judged in combination with memory size (see the analysis in the following section about the *degree of flexibility*)

At this point, it is important to note that the emergence of multiple language regions for the Horn game is equivalent with the emergence of anti-Horn language learners. Consequently, the results clearly reveal i) how anti-Horn language can evolve in a structured society of learning agents: namely as local language regions next to other language regions of Horn language learners; and ii) what supports the emergence of such local language regions and therefore the emergence of anti-Horn language learners: low game parameters of the Horn spectrum, a local interaction structure, a large population and a large memory size of the agents. In addition, it is not possible to describe the influence of the learning dynamics in such a straight-forward way since both learning dynamics behave quite diverse for limited and unlimited memory. To handle this complexity, I will introduce the *degree of flexibility* to classify types of learning dynamics in combination with memory size.

Degrees of Flexibility

In my experiments I compared four different types of agents, arising out of the combinations of learning dynamics (RL and BL) and memory settings

Flexibility	Dynamics	Resulting population
Level 0	RL^{∞}	no population-wide spread of signaling language(s); emergence of (partial) pooling languages
Level 1	BL^{∞}, RL^{n}	dependent of game parameters and interaction structure the outcome is a) the emergence of several local regions of signaling languages stretched out over the whole population b) the emergence of one population-wide signaling language
Level 2	BL^{n}	the emergence of one population-wide signaling language

Table 4.4: Three different levels of flexibility

(unlimited and limited). By comparing these results, I am inclined to introduce a property that signifies the distinction of the systemic behavior of each agent type. I call this property *flexibility*. The simulation results suggest a classification of three different levels of flexibility (see Table 4.4).

Flexibility level 0 describes an extremely inert behavior like those of RL^{∞} agents. As the Experiments II and VI showed, at least roughly half of the agents did not learn any signaling language because their behavior is not flexible enough for a successful language to spread society-wide. The resulting society is a conglomeration of all possible languages, e.g. partial pooling languages emerge, caused by the fact that agents tend to learn the Smolensky strategy during an initial phase as a receiver strategy and stay with it. This is also a result of the low degree of flexibility that let them resist in initial tendencies of learned behavior.

Flexibility level 2 describes the behavior of BL^{n} agents: it is the most flexible case, so that even if language regions emerge, convex regions cannot stay stable and are driven to extinction, initiated by the convex border melting phenomenon: only one global signaling language finally emerges for the whole society in almost any case.[11]

Flexibility level 1 is the most interesting case since the agents are intermediate flexible and the resulting outcome of the society strongly depends on the other game parameters. Depending on the degree of locality γ, lat-

[11]The only exception was the emergence of linear borders between language regions. Since this is a quite artificial case, I do not consider it as a realistic outcome of a social structure.

tice size, and the game parameters, we see that either one global signaling language emerges or local groups of signaling languages regions emerge and stabilize, where only the border agents between local groups fail to learn a signaling language. This behavior is seen for BL^∞ agents as well as RL^n agents. For the latter also the convex border melting phenomenon was observed, but it is weakened by higher memory size and vanished for e.g. intermediate γ values.

These results depict the circumstances for or against the emergence of specific languages. Nevertheless, to get a deeper insight in the dynamics and interdependencies of populations in my experiments, I will analyze the phenomena that emerged in my experiments in a more detailed fashion: the emergence of border agents and the convex border melting phenomenon.

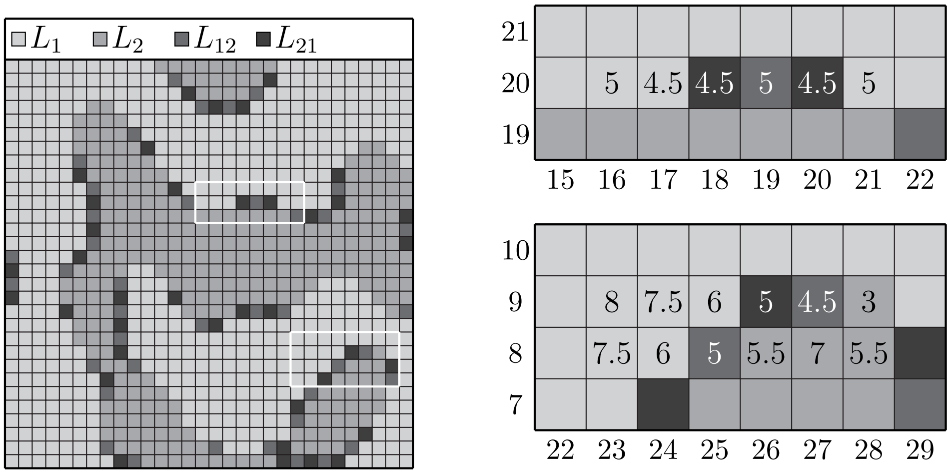
4.2 Border Agents Analysis

There are particularly interesting phenomena that emerged in the experiments of Section 4.1. These are worth to analyze in a more detailed fashion. In a couple of experiments *border agents* emerge; agents that constitute the borders between language regions of signaling languages. These border agents are learners of miscommunication languages[12] that are highly incompatible with themselves. This yields the question of how stable these border agents are. In the following I will take a more detailed look in the way border agents behave, survive or getting replaced. In addition, for populations of agents that use learning dynamics with *limited memory*, a phenomenon seems to depict an instability of border agents: the *convex border melting phenomenon*. I will take a closer look at i) why convex border melting emerges, ii) what it says about the agents' stability and iii) why it is restricted to dynamics with limited memory.

4.2.1 Border Agents Arrangement

To give an example how border agents are arranged, I extended the sample of the resulting pattern of BL^∞ agents playing the Lewis Game (as in Figure 4.2b, page 106) by distinctively displaying the language learners of both miscommunication languages L_{12} and L_{21}. This more fine-grained depiction is displayed in Figure 4.23a. As you can see, all border agents are

[12]Exceptions are RL^∞ agents playing a Horn game, for which L_p learners, L_{pp} learners and non-learners emerge at the borders, see Section 4.1.1, Experiment II(b) and II(c).



(a) Language learners on a 30 × 30 toroid lattice.

(b) Detailed clippings of Figure 4.23a with averaged EU values.

Figure 4.23: Language learners (after 3000 steps) on a 30×30 toroid lattice of BL^{∞} agents playing the Lewis Game are depicted in Figure 4.23a. Two detailed clippings of Figure 4.23a (white frames) with averaged EU values for the inner agents are depicted in Figure 4.23b.

learners of miscommunication languages, thus either L_{12} or L_{21} learners.[13] In addition, they are noticeably alternating arranged along the borders. This is not a coincidence, but the result of a particular requirements for the arrangements of border agents. This fact is more comprehensible by taking a look at two detailed clippings of Figure 4.23a, highlighted by the white frames and depicted in Figure 4.23b. In both clippings each inner cell's number represents the *neighborhood expected utility* EU_N which is the sum of all expected utilities among each agent's neighborhood, defined in the following way:

Definition 4.1 (Neighborhood Expected Utility)**.** *Given i) the expected utility* $EU(L', L'')$ *of two languages* L' *and* L'' *and ii) the information* $L(x)$ *that agent* x *has learned language* L*, then the* Neighborhood Expected Utility $EU_N(x)$ *of an agent* x *among her neighborhood* $N(x)$ *is:*

$$EU_N(x) = \sum_{y \in N(x)} EU(L(x), L(y)) \tag{4.1}$$

To get a better insight in how these values are achieved, the particular expected utilities between the strategies L_1, L_2, L_{12} and L_{21} of the Lewis game are depicted in Table 4.5a (Table 4.5b for the normal Horn game).

[13]Note: for the Horn game the miscommunication languages are L_{ha} and L_{ah}.

	L_1	L_2	L_{12}	L_{21}
L_1	1	0	.5	.5
L_2	0	1	.5	.5
L_{12}	.5	.5	0	1
L_{21}	.5	.5	1	0

(a) *EU* table Lewis game

	L_h	L_a	L_{ha}	L_{ah}
L_h	.87	−.15	.37	.35
L_a	−.15	.83	.35	.33
L_{ha}	.37	.35	−.13	.85
L_{ah}	.35	.33	.85	−.17

(b) *EU* table normal Horn game

Table 4.5: Expected utilities among different language users, Table 4.5a for the Lewis game, Table 4.5a for the normal Horn game.

Let's take a closer look at the clippings of Figure 4.23b: the top clipping shows a linear border with an $L_{21}-L_{12}-L_{21}$ alternation. To reconstruct the fact that miscommunication language learners can stabilize, let's imagine that agent $x_{(18,20)}$ would switch to a L_1 learner. Then changes of the inner agents' EU_N values would happen as follows:

- $EU_N(x_{(16,20)})$: $5 \rightarrow 5$
- $EU_N(x_{(17,20)})$: $4.5 \rightarrow 5$
- $EU_N(x_{(18,20)})$: $4.5 \rightarrow 4.5$
- $EU_N(x_{(19,20)})$: $5 \rightarrow 4.5$
- $EU_N(x_{(20,20)})$: $4.5 \rightarrow 4.5$
- $EU_N(x_{(21,20)})$: $5 \rightarrow 5$

As you can see, after such a switch the EU_N of agent $x_{(18,20)}$ wouldn't change. In addition, the left neighbor of $x_{(18,20)}$ would have a higher EU_N by 0.5, whereas the right neighbor's would have a lower one by 0.5. Consequently, the sum of EU_N values along this border would be the same in total. This would also happen if $x_{(17,20)}$ would switch to a L_{12} border agent: while she keeps her EU_N, the right neighbor would fare better by 0.5, whereas the left neighbor would loose 0.5.

This shows that once an alternating structure of border agents exists, agents around the border region don't fare better or worse (in terms of the total sum of EU_N values along the border) if that switch to a signaling language. Furthermore, if there would be no border agents at all, the agents of the clipping would fare better in total because everybody would have an EU_N value of 5 then. Thus, a single border agent couldn't survive because she would have an EU_N of 4.5, while a signaler one of 5. This reveals that a border agent with a particular miscommunication language can only stabilize if she has at least one neighbor of the other miscommunication language.

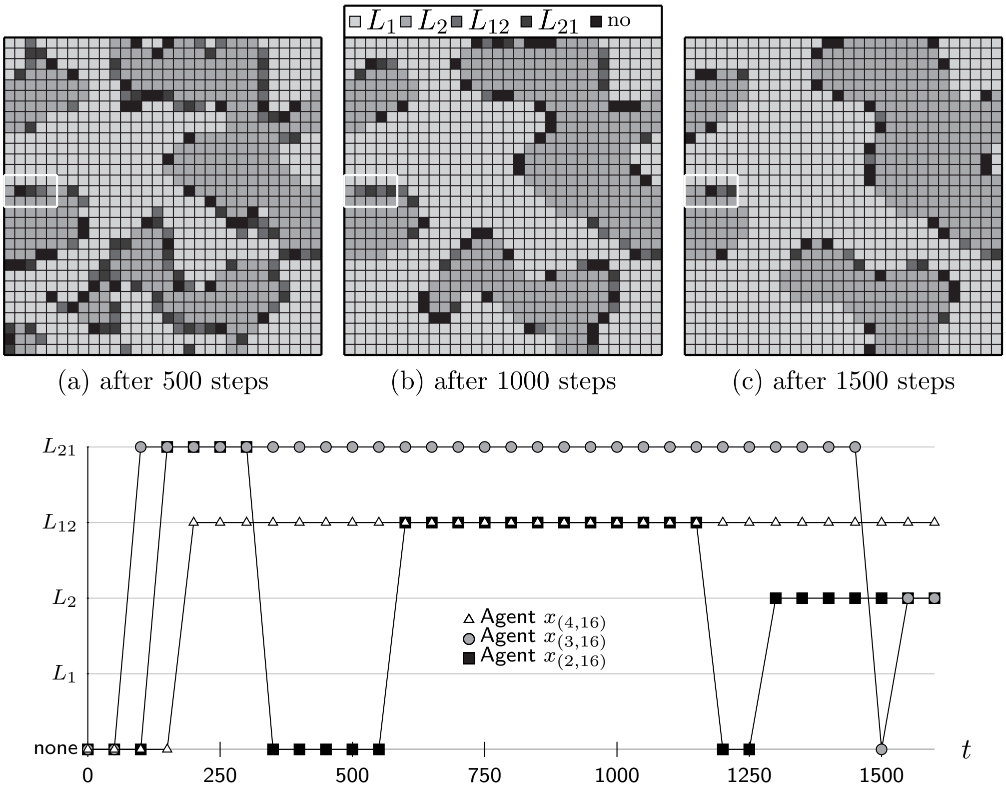
Now let's take a look at the situation of the second clipping. Imagine agent $x_{(26,9)}$ would switch to a L_1 signaler. Then her EU_N value would still be 5. Furthermore, her three top neighbors' and her left neighbor's EU_N each would increase by 0.5, whereas her three bottom neighbors' and her right neighbor's EU_N each would decrease by 0.5. Thus, the total sum of EU_N's along the border wouldn't change. If agent $x_{(26,9)}$ would switch to a L_2 signaler, she herself would loose an EU_N of 2.0. Thus, this switch can be excluded.[14] Nevertheless, $x_{(26,9)}$ as an L_1 signaler scores as good as a border agent. Thus, in many cases it is a question of chance if border agents switch to signalers and the other way around. But if they substantially emerge, then they are preferably arranged in an alternating chain of both miscommunication languages.

4.2.2 Border Agents Behavior

The following analysis shows a more detailed insight into border agents' properties by extracting the behavior of particular agents over time. Figures 4.24a - 4.24c show the sample patterns of RL^{100} agents playing the Lewis game on a 30×30 toroid lattice, after 500, 1000 and 1500 simulation steps, respectively. The agents under observation are agents $x_{(2,16)}$, $x_{(3,16)}$ and $x_{(4,16)}$ which are the three agents in the center of the white square (central left of the lattice). Figure 4.24d displays the particular learned languages for these agents over time. In detail: agent $x_{(3,16)}$ switches between L_2 and L_{21} and agent $x_{(2,16)}$ has first learned L_{21} and finally switches between L_{12} and L_2. This exemplary result reveals two facts, namely i) that agents at the border switch from time to time between a miscommunication language and a signaling language of a contiguous language region and ii) that the potential miscommunication languages are generally arranged in an alternating manner along the agents (here: the line of agents $x_{(2,16)}$ - $x_{(3,16)}$ - $x_{(4,16)}$ applies L_{12} - L_{21} - L_{12} at simulation step 1000).

Furthermore, it can be seen that between a switch from a signaling to a miscommunication language or the other way around agents are non-learners for a while. This can hold for a longer time, like for agent $x_{(2,16)}$ during simulation steps 350 - 550, or a short time, like for agent $x_{(3,16)}$ around simulation step 1500. And the fact of being a non-learner between switches from one language to another makes perfect sense for RL agents, who need this time to rearrange the urn settings and finally obtain a Hellinger dis-

[14] It can be shown for both learning dynamics that an agent never switches to a language, for which she gains much less than for the current one if her neighborhood consists of language learners.



(a) after 500 steps (b) after 1000 steps (c) after 1500 steps

(d) temporal development of 3 particular agents

Figure 4.24: Sample patterns for different numbers of simulation steps are depicted in Figure 4.24a - 4.24c. The behavior over time for the three agents $x_{(2,16)}$, $x_{(3,16)}$ and $x_{(4,16)}$ which are the ones in the middle of the white frame of all three sample patterns, is depicted in Figure 4.24d.

tance sufficiently close to a language. Therefore, while the overall behavior of BL agents is quite the same, these phases of being non-learners between switches is never observed for BL agents (c.f. Section 2.3.1).

4.2.3 Extreme Initial Conditions

To analyze where border agents emerge or remain, I started experiments with *extreme initial conditions*, so that either i) all agents are miscommunication language (L_m) learners from the start or ii) all agents are signaling language learners from the start. The goal of these settings is to analyze i) if L_m learners can really only survive on the border between two regions and ii) if L_m learner will necessarily emerge at border regions.

With Experiment XI(a) I would like to demonstrate that border agents

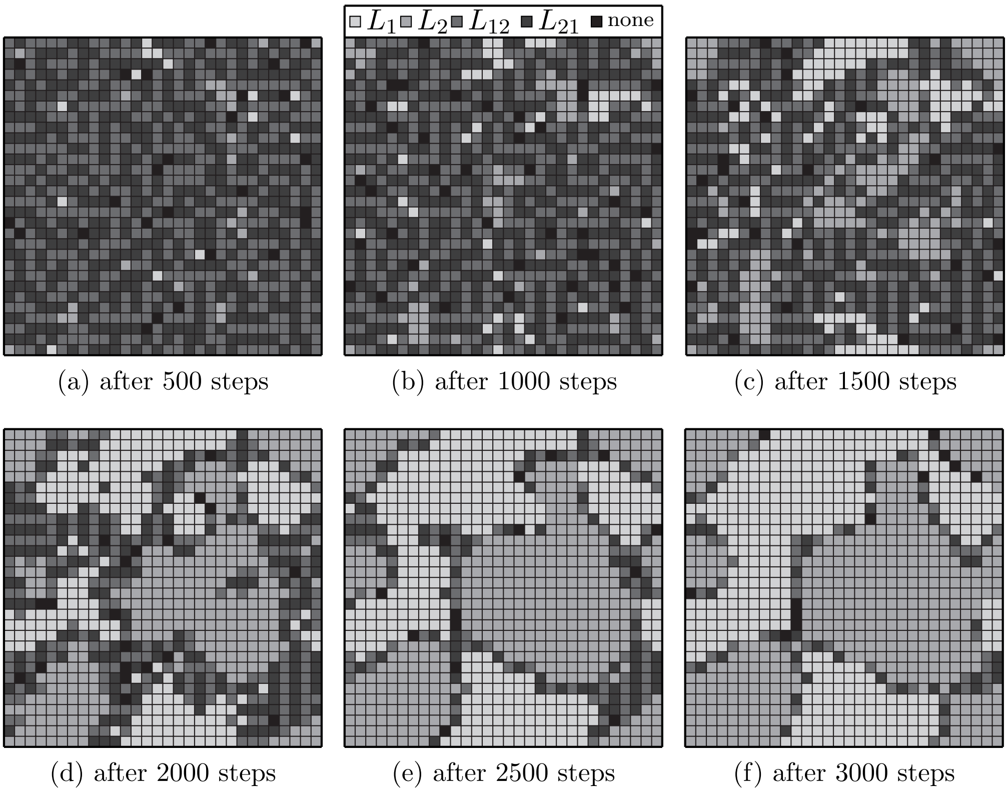
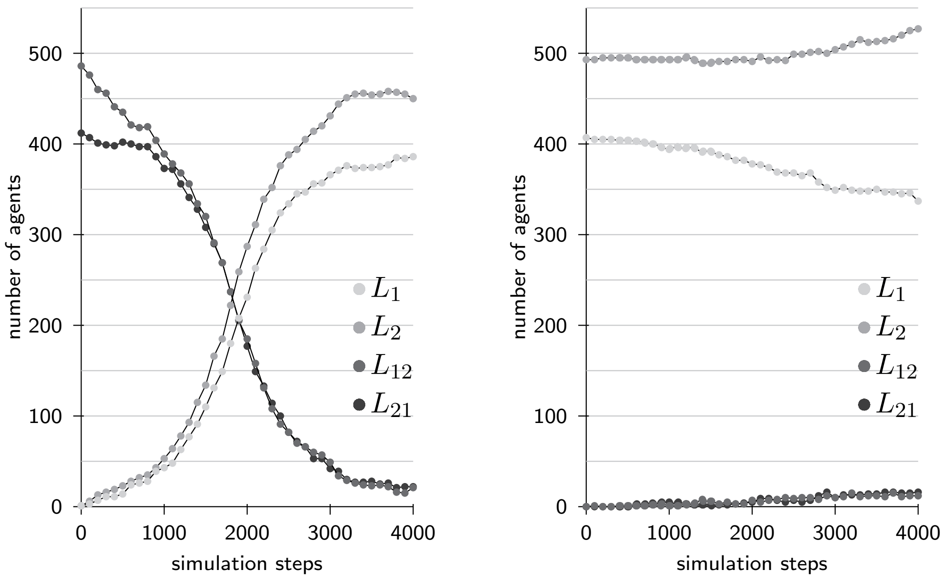


Figure 4.25: By starting with only L_{12} and L_{21} learners, signaling language regions emerge and signalers push them to the border.

can only remain on border regions between two language regions. Therefore I conducted the network game $NG_{XI(a)} = \langle LG, X, RL^{100}, G_{l30} \rangle$, where $X = \{x_1, \ldots x_{900}\}$ is a set of 900 agents and G_{l30} is a 30×30 toroid lattice. In addition, all agents initially play a miscommunication language, either L_{12} or L_{21}, by setting the agents' urns' content randomly to one of both languages. The patterns of a simulation run for different numbers of simulation steps is depicted in Figure 4.25.

As you can see, by starting with randomly distributed L_{12} and L_{21} learners, regions of both signaling languages emerge and the L_{12} and L_{21} learners continue to exist only on the borders between these regions. All 100 simulation runs with these initial settings showed similar results with exactly this dynamic behavior. Thus, it can be taken for granted, at least for this learning dynamics that miscommunication language learners only remain on the border between regions of signaling languages and nowhere else.

Figure 4.26a shows the appropriate learning curve: the number of sig-



(a) Sample learning curves for a run with only initial L_{12} or L_{21} learners

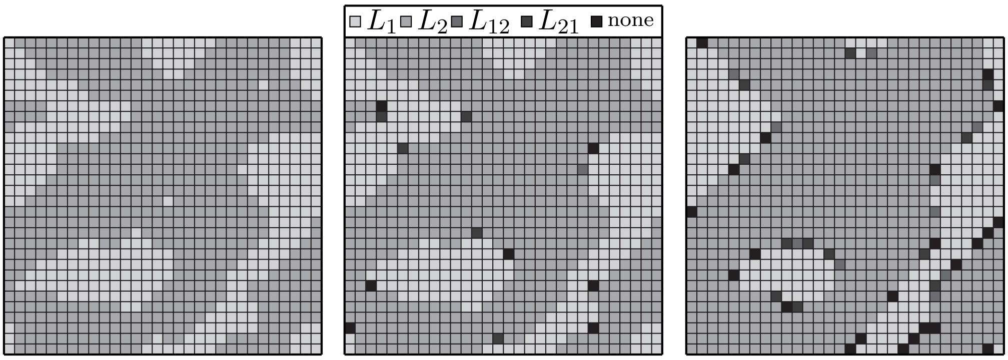
(b) Sample learning curves for a run with only initial L_1 or L_2 learners

Figure 4.26: Sample learning curves for RL^{100} agents playing the Lewis game on a 30×30 toroid lattice by having specific starting conditions.

naling language learners increases in a S-shape curve, while the number of miscommunication language learners decreases in an inverse way to a small number of final L_m learners which are the remaining border agents. Multiple runs reveals the same: L_m learners exclusively survive between signaling language regions. In addition, all language regions are generally completely separated by L_m learners.

In Experiment XI(b) I started simulations where all agents initially start with a signaling languages. Thus, I started experiments with the network game $NG_{XI(b)} = \langle LG, X, RL^{100}, G_{l30} \rangle$, where $X = \{x_1, \ldots x_{900}\}$ is a set of 900 agents and G_{l30} is a 30×30 toroid lattice. Furthermore, all agents initially have learned a signaling language, arranged as randomly emerged but coherent language regions like in Figure 4.27a. As you can see in Figure 4.27b and Figure 4.27c, after a while L_m language learners emerge between those regions, but nowhere else. Figure 4.26b shows a sample course of language learners. As you can see, the number of learners of a miscommunication language, L_{12} or L_{21} increases slightly.[15] All in all, the important

[15] Note that because of the convex border melting phenomenon, the number of L_2 learners increases, while the number of L_1 learners decreases slowly.



(a) after 0 steps (b) after 1000 steps (c) after 4500 steps

Figure 4.27: By starting with only L_1 and L_2 learners, miscommunication language learners emerge only at the border regions, but it takes a long time.

observation is the emergence of a small group of miscommunication language learners that are positioned on the border between the language regions. In all 100 simulation runs border agents emerged in any case. Thus, it can be taken for granted, that border agents necessarily emerge between regions of signaling languages.

4.2.4 The Convex Border Melting Phenomenon

As it was seen for dynamics with limited memory: convex border regions are melting. To get a better insight for this phenomenon, it is worth to take look at the EU_N values for convex borders. Figure 4.28 shows different examples for border regions with perfect alternating border structures of thickness 1 and the appropriate EU_N values for each agent by reconsidering that they are playing the Lewis game.

First, it is important to note the linear borders constitute a *strict Nash equilibrium situation* of EU_N values between both language regions. This is seen by considering the two examples for linear borders in Figure 4.28b and Figure 4.28c. In both examples all border agents gain an EU_N of 5. This is the value each L_m learner archives, who is as member of an alternating border of thickness 1. Furthermore, two learners of a signaling language regions gain exactly the same if they have the same distance to the border. In addition, since all signalers score better than the border agents, a switch of any signaler to a border agent would decrease the EU_N value of this agent: in Figure 4.28b a signaling language learner adjacent to the border

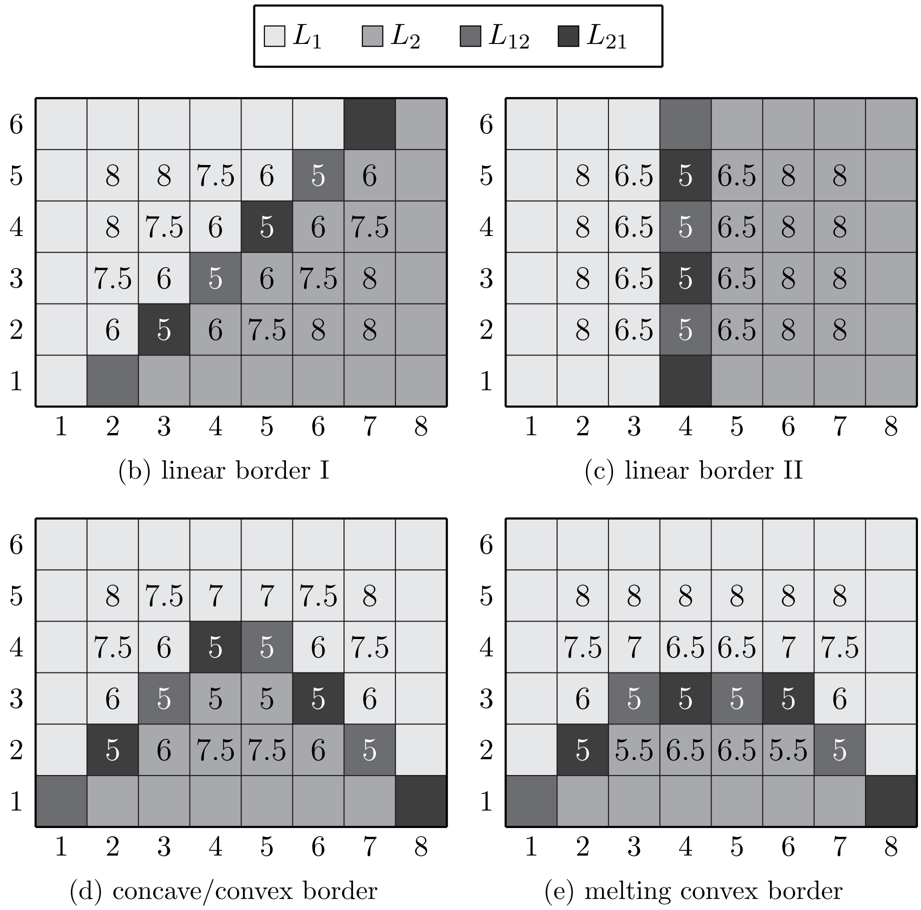


Figure 4.28: Different examples for borders between signaling language regions and the appropriate EU_N values for each agent according to the Lewis game.

would change her EU_N value from 6 to 5. The same is true for the situation in for Figure 4.28c: if an signaler that is adjacent to the border switches to a L_m language, her EU_N value would change from 6.5 to at most 5.5.

The same holds the other way around: if on of the border agents switches to a signaling language, her EU_N value would decrease from 5 to 4 (in situation of Figure 4.28b and Figure 4.28c, as well). Thus, a linear border forms a situation in which any switch to another strategy would decrease an agent's utility. In that sense, a linear border of alternating L_m learners of thickness 1 constitutes a strict Nash equilbrium in respect to all involved agents' EU_N value: each agent would score worse by switching to any other language.

As a matter of fact, this is not the case for convex borders. As you can see in Figure 4.28d, all border agents gain an EU_N value of 5. But this is also the case for two agents of the convex signaling language region, namely agent $x_{(4,3)}$ and agent $x_{(5,3)}$. This is the weak spot of the convex region. If these two agents would switch to L_m language learners, they would score the same, as depicted in Figure 4.28e. Furthermore, the agents $x_{(4,4)}$ and $x_{(5,4)}$ would score much better by subsequently switching to signaling language learners of the concave region: they increase the EU_N from 5 to 6.5. Thus, a convex border in not a strict Nash equilibrium for all involved agents.

This example gives a visual representation of the mechanisms of the convex border melting phenomenon. A question remains however: why does this happen exclusively for dynamics with limited memory? The answer can be informally given as follows: the exemplary transition from situation in Figure 4.28d to the situation in Figure 4.28e showed that to realize convex border melting, a signaling language learner at the border has to switch to a L_m learner for a situation, where she scores the same EU_N in either case. In other words, she is unbiased (in terms of maximizing expected utility) between keeping the signaling language or switching to the L_m language. Moreover, note that agents not only reconsider the actual situation, but also last encounters. Thus, agents with unlimited memory will most likely never reach the point of being unbiased since they will never forget even the first observations/encounters and therefore are biased by an initial tendency. The convex border melting phenomenon as depicted in the transition from Figure 4.28d to Figure 4.28e only emerges for agents with limited memory because as members of convex signaling language regions they can unbiased between the actual signaling language and a L_m language and, at one point, switch to it.

4.2.5 Summary

In summary, the following claims about border agents can be made with confidence:

- border agents are biased to an alternating structure of L_{12} and L_{21} learners (for a Horn game: L_{ha} and L_{ah})
- border agents can only survive with neighbors being border agents
- border agents shape a border with a maximal thickness of one unit[16]

[16]Exception is the social map structure since agents also interact with agents beyond the neighborhood. The defined EU_N value is not applicable. An appropriate value for

- border agents never stabilize in terms of long-term stability since they reveal an alternating behavior
- border agents always emerge between language regions of signaling languages
- border agents never go extinct between language regions of signaling languages
- linear borders of alternating L_m learners of thickness 1 between signaling language regions are strict Nash equilibria of all agents' EU_N (except social maps, only on grids)
- in case of dynamics with limited memory: border agents replace agents of convex signaling language regions, thus accuse the convex border melting phenomenon

4.3 Conclusion

Let's come back to this chapter's initial question: given unbiased agents, under what circumstances can we expect languages other than the Horn strategy to emerge? In the experiments of Chapter 2 I showed that under circumstances depicting unbiased starting conditions (like a neutral population state for replicator dynamics, or uniformly distributed initial strategies for imitation and learning dynamics) the only final outcome is a society of all agents acting according to Horn's rule. The other two noticeable strategies that (i) reveal stability properties and (ii) have a salient basin of attraction are anti-Horn and Smolensky strategy, but both only emerge and stabilize for biased starting points. Thus, the task was directed towards finding out if there are additional circumstances supporting the emergence of those strategies in initially unbiased settings. I did so by reconsidering more individual-based learning dynamics for agents arranged in two types of spatial structures: a toroid lattice and a social map.

It turned out as a general phenomenon that the Smolensky strategy emerges particularly at the beginning of simulation runs and mainly as receiver strategy. That happened for all dynamics types I analyzed, but disappeared in almost all of them for subsequent simulation steps. Only learning dynamics with a degree of flexibility level 0 led to the situation that

an expected utility for the social map is much more complicated to compute and goes beyond this thesis' work.

Smolensky strategy survived and stabilized, as seen for the RL^{∞} dynamics. This is the result of the fact that agents are too inflexible or, in terms of learning, their learning mechanism is too cold. Consequently, they stick in initial tendencies. Furthermore, Smolensky language learners never evolved as language regions, but as isolated learner types inside a conglomeration of multiple language learners.

The second fact is that anti-Horn language learners emerged in local regions, sharing the society with regions of Horn language learners, where those regions where zoned by border agents: agents that learned a miscommunication language, a mixture of Horn and anti-Horn language. A more detailed analysis revealed that border agents i) emerge in an alternating pattern of both miscommunication languages and ii) emerge necessarily and only on the border between language region. All in all, an emergence and stabilization of regions of anti-Horn language learners goes hand in hand with the emergence of multiple language regions.

Hence, the question of the emergence of multiple language regions in general, thus also for the Lewis game, was strongly involved in the analysis of Chapter 4: under what circumstances do multiple language regions evolve instead of one society-wide signaling language? The experiments showed that a specific dynamics class of flexibility level 1 is a necessary condition for such an outcome. This class contains belief learning with unlimited and reinforcement learning with limited memory (BL^{∞}, RL^{n}). A lower flexibility level (like for RL^{∞} dynamics) prohibits the spread of local language regions, a higher flexibility level (like for BL^{n} dynamics) leads to only one society-wide language, basically triggered by the *convex border melting phenomenon*. But learning dynamics with flexibility level 1 alone is only a necessary condition for the emergence of multiple language regions. Sufficient conditions are particular combinations of (i) *external circumstances* like a large population size and a local communication structure and (ii) *game parameters* of the signaling game: prior probabilities and message costs.

In my experiments I analyzed the impact of the *game parameters* by comparing the results of specific signaling games with different parameters, called the Lewis game LG, the normal Horn game HG and the weak Horn game HG_w. Further, the analysis includes experiments for a subset of the *Horn spectrum*, a scope of different game parameter combinations. The results are indicative of a negatively correlation between the emergence of multiple language regions and the divergence of the Horn case representing parameters, namely state probabilities and message costs. By increasing the difference of both message costs and/or prior probabilities, the probability of the emergence of multiple language regions decreases. Thus the probability of the emergence of a diverse final pattern is weakened by *strong* Horn parameters.

In my experiments both parameters of *external circumstances*, population size and locality of communication structure, were varied. I showed that a large size of the population basically supports the emergence of multiple language regions, everything else being equal. Furthermore, the locality of the communication structure was analyzed in a more detailed fashion; in the *social map experiment*: here the interaction structure of the population could be manipulated by the *degree of locality* γ and set from random interaction ($\gamma = 0$) to basically neighborhood interaction ($\gamma > 8$). This setting implicitly allows to analyze interaction structures between a complete network and a toroid lattice. And as already mentioned, multiple language regions emerged only for a sufficient dense local structure (high degree of locality), given a dynamics class of flexibility level 1. This result highlights the importance of the society's interaction structure for stable patterns of language regions.

All in all the BL^∞ and RL^n dynamics belong both to the same class of flexibility level 1 since both cause similar circumstances: the resulting structure of language region(s) mainly depend on external factors like the interaction structure of the network (see Table 4.4). This shows that both dynamics generate behavior that is sensitive to network structure. Thus BL^∞ and RL^n dynamics both seem to be excellent candidates for the experiments in subsequent Chapter 5: experiments on more realistic network structures, so-called *small-world* networks.

Chapter 5

Conventions on Social Networks

"...network studies in sociolinguistics can provide a starting point for theoretical models of social interactions underlying the spread of novel linguistic variants."

Fagyal et al. 2010, Center and peripheries: Network Roles in Language Change

"Computer models can provide an efficient tool to consider large-scale networks with different structures and discuss the long-term effect on individuals' learning and interaction on language change."

Ke et al. 2008, Language Change and Social Networks

In this chapter I will present results and analyses of experiments that are quite similar to those of Chapter 4: simulations of populations of agents that communicate via signaling games and update behavioral strategies by learning dynamics. The crucial difference, however, is the population structure. While in Chapter 4 I conducted experiments on a lattice structure and therefore a *regular network*, in this chapter I will conduct experiments on social networks structures, so-called *small-world networks*. This is switch to more heterogeneity.

The switch to social network structures has specific consequences for my analysis. First, it brings some restrictions: for a regular lattice structure I was able to make general statements for clippings of the network; take for example the *border agents analysis* in Section 4.2. Such generalized analysis is not possible for small-world networks, since they lack regular patterns. In contrast, it brings new opportunities: now each agent has an unique position in a network. Consequently, specific *node properties* mark her off from any other agent in the same network. Furthermore, also connected regions of a network, also called *sub-networks*, have specific *structural properties* that distinguish them from other sub-networks of the same network. With this comes the opportunity for substantially incorporating facets of network theory into the analysis.

As already introduced in Section 3.1, network theory offers a couple of tools for measuring and labeling both node and (sub-)network properties. Consequently, the analysis of this chapter includes new research questions. The fundamental one is as follows: in what way do specific network properties influence the way languages emerge? Or more in detail: how do the node properties of an agent influence her way of learning and using language? And in what way is the emergence of a language region influenced by it's structural properties?

While previous related work has focused on studying, which global network structures are particularly conducive to innovation and its spread (Ke et al. 2008; Fagyal et al. 2010), this work investigates more closely the local network properties associated with (regions of) agents that have successfully learned a language or not. In contrast with Zollman (2005) and Wagner (2009), but parallel to Mühlenbernd (2011), I focus not on imitation, but learning dynamics. In particular, I will focus on those learning dynamics that have an intermediate flexibility level of 1: BL^{∞} and RL^{n} dynamics.[1]

5.1 Small-World Experiments

The following experiments include simulation runs of populations of agents that repeatedly play the Lewis game or the normal Horn game, as given in Table 4.1, and update their behavior according to learning dynamics. I only consider learning dynamics of *flexibility level 1*, since I am interested in the analysis of the emergence of multiple and connected language regions. As I was able to show in Chapter 4, flexibility level 1 ensures such outcomes at least for grid structures and social maps; and, as the experiments will

[1] Note that many of these experiments are part of a joint work with *Michael Franke*. Results were already published in Mühlenbernd and Franke (2012a), Mühlenbernd and Franke (2012b), and Mühlenbernd and Franke (2012c).

game	**network structure**	BL^{∞}	RL^{100}
LG	β-graphs	Exp. I(a)	Exp. I(b)
LG	scale-free networks	Exp. II(a)	Exp. II(b)
HG	β-graphs	Exp. III(a)	Exp. III(b)
HG	scale-free networks	Exp. IV(a)	Exp. IV(b)

Table 5.1: Experiments of populations of agents playing the *Lewis* or *Horn* game for different dynamics and different network types .

reveal, this also holds for small-world network structures. Consequently, the learning dynamics that I reconsidered in the experiments are belief learning with unlimited memory and reinforcement learning with limited memory: BL^{∞} and RL^{100}. Finally, the two types of small-world networks under observation are *β-graphs* and *scale-free networks*, as introduced in Section 3.2.3. The corresponding eight experiments are depicted in Table 5.1.

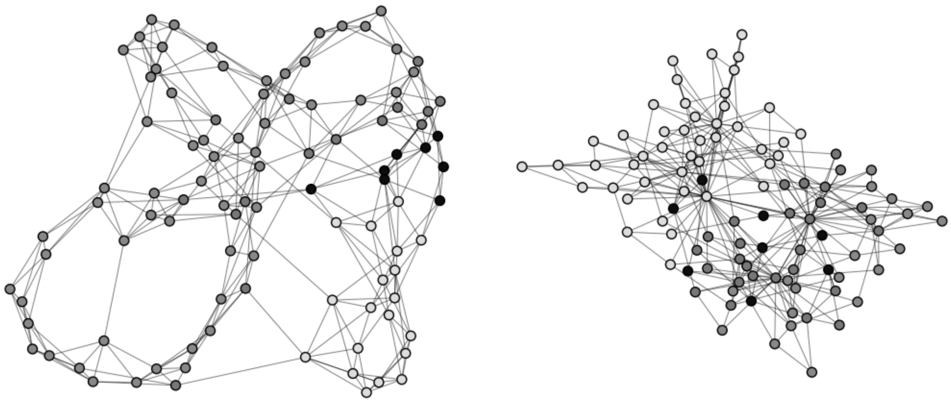
5.1.1 Language Regions & Agent Types

In this chapter I am interested in the analysis of the behavior of not only particular agents, but also groups of connected agents that learned the same language. Such a groups is called a *language region*. Furthermore, I am interested in a classification of agents referring to a) their learning behavior and b) their node properties in the networks. Consequently, I will sort them into different groups of *agent types*. In the following I will give a formal definition for both, language regions and agent types.

Language Regions

A language region of a given graph G is a *connected sub-graph* G' (see Definition 3.13), of which all nodes belong to agents that have learned the same language L. Furthermore, all agents outside the language region that are connected to at least one member of the language region have not learned language L. This feature can be formalized by the fact the the connected sub-graph $G' = (N', E')$ is *maximal* according to language L: there is no connected sub-graph $G'' = (N'', E'')$ with $N' \subset N''$, where G'' is a language region. All in all, a language region is defined as follows:

Definition 5.1. *(Language Region) A graph $G' = (N', E')$ is a language region of a network graph $G = (N, E)$ if and only if the following conditions hold:*



(a) β-graph (b) scale-free network

Figure 5.1: Sample results for a β-graph and scale-free network (100 nodes each) with a large language region for each of the languages L_1 and L_2 (white and gray nodes) and a small number of non-learners (black nodes).

- *G' is a connected sub-graph of G*
- *$\forall n \in N'$: all agents at node n have learned the same language L*
- *G' is maximal: $\neg\exists G'' = (N'', E'')$ with the following features:*
  - *G'' is a language region*
  - *$N' \subset N''$*

To anticipate a general result for experiments with the *Lewis game*: in most of the simulation runs two large language regions evolved, more or less independently of the type of learning dynamics or network structure. To get a first impression, Figure 5.1 depicts a snapshot of a simulation run of agents playing the Lewis game on a β-graph (Figure 5.1a) and on a scale-free network (Figure 5.1b), with 100 nodes each. In each case two large language regions evolved, one of language L_1, one of language L_2, plus a remaining group of agents that are non-learners at that simulation step.

Agent Types

In this section I will characterize agent types according to two classes of properties: *static properties* and *dynamic properties*. Static properties are defined in terms of network theory by the node properties of an agent. These properties are called *static*, since they are given by environmental features

connectedness	global (BC, CC)	local (CL)	individual (DC)
family man	low	high	-
globetrotter	high	low	high

Table 5.2: Properties of family man and globetrotter

and do not change during a simulation run. Thus, an agent has the same static properties before, during and after an experiment, since the network structure does not change over time. On the contrary, dynamic properties are properties that integrate the language that an agent learns. Therefore, they can evolve and change over time.

Let's start with the definition of agent types by static properties. The theoretical challenge here lies in adequately characterizing local network roles in terms of formal notions of network connectivity which can never be clean-cut, but must necessarily be of a probabilistic nature. For our present purposes, however, a rather straightforward cross-classification based on whether an agent is globally, locally or individually well connected turned out to have high explanatory value. For that purpose I reconsider the node properties *betweenness centrality* (BC), *closeness centrality* (CC), *degree centrality* (DC) and *individual clustering* (CL), as defined in Section 3.1.1.

Using suggestive terminology, I will be primarily concerned with two types of agents, *family men* and *globetrotters*. The former have tight local connections (high CL value), with less global connections (low BC and CC values); the latter show the opposite pattern (low CL value, high BC and CC values) plus a high degree of connectivity (high DC value). All in all, family men and globetrotters are characterized as depicted in Table 5.2.

The next step is to characterize agents by dynamic properties. Here we consider the entire network and its segmentation of language regions that evolve during a simulation run. The first type of dynamics properties reflects, whether an agent is positioned on the margin of a language region of language L or inside of it. In the first case such an agent is called a *marginal agent*, according to the fact that not all of his neighbors have learned the same language L. As opposed to this, an *interior agent* is positioned inside a language region, since all of her neighbors have learned the same language L. Taken together, both types are defined as follows:

Definition 5.2. *(Marginal Agent) An agent $x_i \in X$, positioned on node $i \in N$ inside a graph $G = (N, E)$, is called a* marginal agent *if the following conditions hold:*

- x_i *is member of a language region* R_L *of language* L
- $\exists j \in NH(i)$: *agent* x_j*, positioned on node* j*, is not member of language region* R_L

Definition 5.3. *(Interior Agent) An agent* $x_i \in X$*, positioned on node* $i \in N$ *inside a graph* $G = (N, E)$*, is called an* interior agent *if and only if the following conditions hold:*

- x_i *is member of a language region* R_L *of language* L
- $\forall j \in NH(i)$: *agent* x_j*, positioned on node* j*, is member of language region* R_L

The second type of dynamic properties reflects, whether an agent has learned a language at the end of a simulation run. It is important to note at this point that the simulation runs of my experiments will not run, until all agents have learned a language, but stop, when at least 90% of all agents have learned a language. Consequently, at the end of the run the remaining agents haven't finally learned a language. An agent of the former group is called a *learner*, an agent of the latter one is called a *non-learner*. Both are defined as follows:

Definition 5.4. *(Learner) An agent* $x_i \in X$ *that has learned a language* L *at the end of a simulation run is called a* learner*.*

Definition 5.5. *(Non-Learner) An agent* $x_i \in X$ *that has not learned a language* L *at the end of a simulation run is called a* non-learner*.*

These notions of language region and agent types are used for the analyses of the subsequent experiments of agents that are playing the Lewis game and the normal Horn game on β-graphs and scale-free network structure, where they update their behavior by BL^{∞} and RL^{100} dynamics.

5.1.2 Lewis Games on β-Graphs

The settings for Experiment I were as follows: I modeled a structured populations as β-graph with 300 nodes, constructed by the the algorithm of Watts and Strogatz (1998), as described in Section 3.2.3, with the parameter $k = 6$ and a varying β-value: $\beta \in \{.08, .09, .1\}$. These parameter choices ensured the small-worldliness of our networks that I had to keep small for obtaining enough data points at manageable computation costs. Furthermore, it ensures that such a β-graph and a scale-free network of subsequent experiments both have similar values of structural network properties.

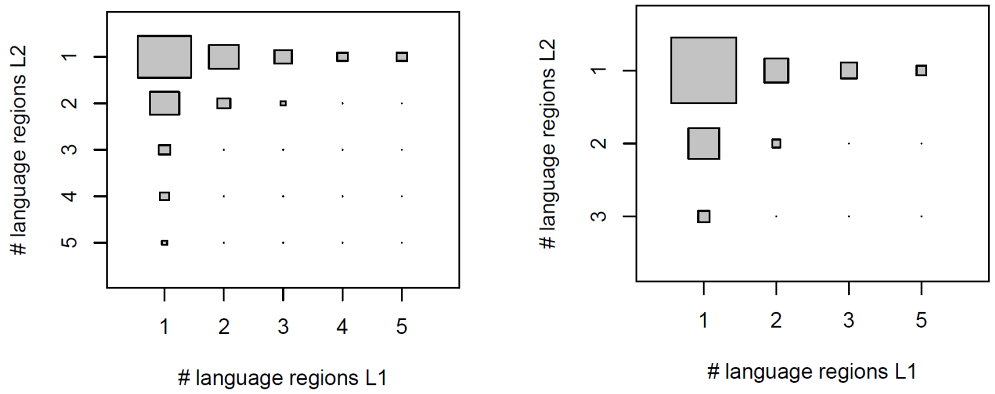
In each simulation run of the experiment, interactions happened according to the selection algorithm *Random Sender & Neighbor* (Definition 3.25, page 93) and each agent's behavior was updated separately after each round of communication in which the agent was involved. In addition, each simulation run ran until more than 90% of all agents had acquired a language, or each network connection had been used 3000 times in either direction. The latter case was to ensure a compromise between a short running time and sufficient time for learning, but also because I was interested in the results of learning after a realistic time-span, not in limit behavior.

In Experiment I(a) I simulated BL^∞ agents playing the Lewis game (LG) on a β-graph: I performed 200 simulation runs of the network game $NG_{I(a)} = \langle LG, X, BL^\infty, G_\beta \rangle$, where $X = \{x_1, \ldots x_{300}\}$ is a set of 300 agents and G_β is a β-graph of 300 nodes. The fundamental result was that all simulation runs ended with a society, where the network is split into local language regions of both types L_1 and L_2, i. d. regional meaning emerged in every simulation run. In general, it was the case that two large language regions emerged, similar to the sample pattern of Figure 5.1a (page 150), but this time with a network of 300 nodes.

To compare these results with experimental results of RL^{100} agents, I started Experiments I(b): I performed 200 simulation runs of RL^{100} agents playing the Lewis game on a β-graph, thus I applied network game $NG_{I(b)} = \langle LG, X, RL^{100}, G_\beta \rangle$. As a basic result, all simulation runs ended with a society, where the network is split into local language regions of both types, with in most cases two large language regions, similar to the exemplary resulting pattern of Figure 5.1b (page 150), but with a network of 300 nodes.

Analysis of Language Regions

As already mentioned, by comparing results of Experiment I(a) and I(b), most of the time two large language regions formed, one of language L_1, one of language L_2. But the results of Experiment I(a) (BL^∞ dynamics), due to its slightly higher flexibility, revealed a little more regional variability than results of Experiment I(b) (RL^{100} dynamics). This fact is displayed in Figure 5.2 that depicts a *Hinton diagram* of Experiment I(a) (Figure 5.2a)



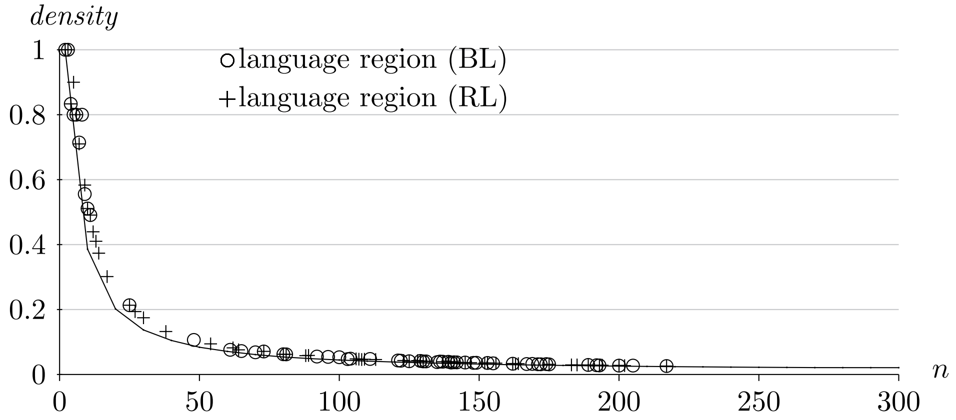
(a) BL^{∞} dynamics (b) RL^{100} dynamics

Figure 5.2: Hinton diagrams for the combination of language regions of L_1 and L_2 language learners on a β-graph for BL^{∞} and RL^{100} dynamics. For both dynamics in most of the cases two language regions emerged. This trend was stronger for the RL^{100} dynamics, while for BL^{∞} dynamics the results were more varied.

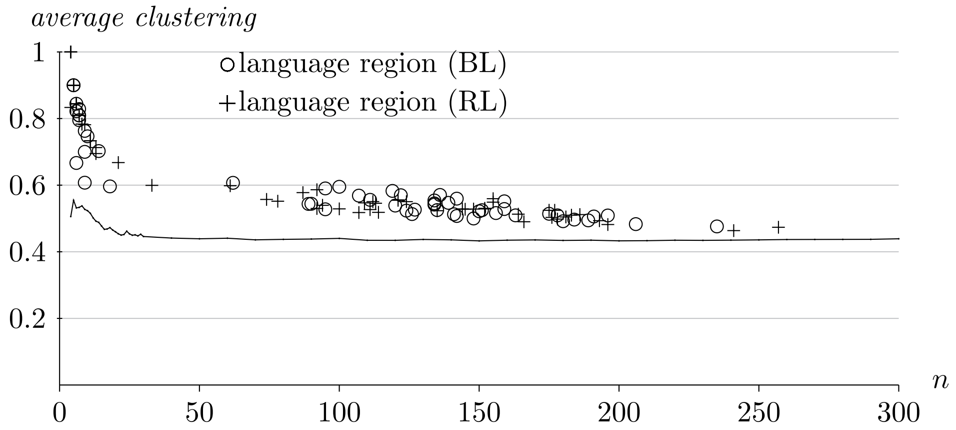
and Experiment I(b) (Figure 5.2b).[2] As observable, the share of simulation runs, where only one of each language region evolved, is higher for RL^{100} than BL^{∞} dynamics.

To analyze the properties of language regions on the β-graphs for both dynamics, I applied suitable notions from network theory which describe structural properties of (sub-)graphs: *density* (Definition 3.17, page 82), *average clustering* (Definition 3.18, page 82) and *transitivity* (Definition 3.19, page 82). To compare the different values of structural properties of language regions with a standard value, I computed the *expected value* of each property for any size n (number of nodes) of a sub-network. Figure 5.3 shows the results of my analysis in the following way: each data point depicts a language region. Its position is defined by its number of nodes n (x-axis) and its value of the appropriate structural property (y-axis). The data points labeled with 'o' are language regions that evolved in the first 20 simulation runs of Experiment I(a) (BL^{∞} dynamics); those data points labeled with '+' are language regions that evolved in the first 20 simulation runs of Experiment I(b) (RL^{100} dynamics). The solid line depicts the expected value of the structural property for different sizes n, where each data point is the average value over 100 randomly chosen connected regions

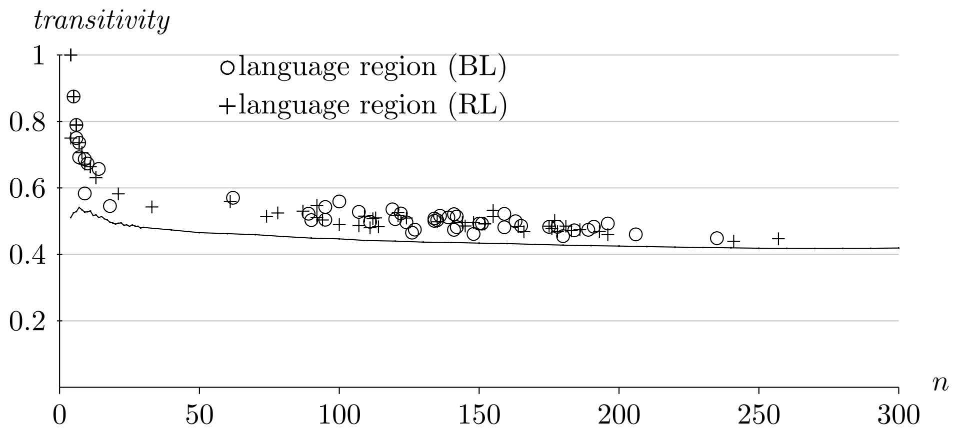
[2]In a Hinton diagram the size of each square represents the magnitude of the share of a specific combination of two values, here the numbers of L_1 and L_2 language regions.



(a) Density of resulting language regions



(b) Average clustering of resulting language regions



(c) Transitivity of resulting language regions

Figure 5.3: Density, average clustering and transitivity (y-axis) of the resulting language region in comparison with the average values from randomly chosen subgraphs (solid lines, sub-graph size n along the x-axis).

in the same network.

The results reveal that each connected language region of a given type had always a higher average clustering and transitivity value than the expected value for a connected subgraph with the same size n (Figure 5.3b and 5.3c), whereas the density value didn't exhibit such a divergence (Figure 5.3a). One may conclude from this that local density on an individual level supports the evolution of a local language, whereas global density does not. In addition, all results were by and large the same for both learning dynamics.

Agent Analysis

The next step was to investigate the relationship between an agent's a) individual learning profile as figured by her dynamic properties and b) positioning in the network as given by her static network properties. For that purpose, I started one analysis for each learning dynamics: in Analysis I(a) I partitioned the set of agents according to dynamic properties in learners and non-learners (Definition 5.4 and 5.5) of the final populations of all 200 simulation runs; in Analysis I(b) I partitioned the set of agents according to dynamic properties in interior agents and marginal agents (Definition 5.3 and 5.2) of the final populations of all 200 simulation runs. For each partition I computed the average values of the agents' network properties: individual clustering (CL), closeness centrality (CC), betweenness centrality (BC) and degree centrality (DC). The resulting values with a *t-test*[3] comparison for different partitions are depicted in Table 5.3, the top table for the results of Experiment I(a) (BL^{∞} dynamics), the bottom table for the results of Experiment I(b) (RL^{100} dynamics).

The results display no significant difference between both dynamics. To elaborate, by considering the significant differences of node properties for the appropriate partitions, the following can be observed: first, learners have a significant higher CL value and significant lower CC and BC values than non-learners, while the DC values are not significantly different. Second, marginal agents have a significant lower CL value and significant higher CC, BC and DC values than interior agents. These points are clearly displayed by representing the data as box plots (Figure 5.4).

By taking into account the characterization of agents by network properties (see Table 5.2), the following conclusions can be drawn for agents

[3] A t-test can be used to determine if two sets of data are significantly different from each other. For more details, I refer to David and Gunnink (1997); Zimmerman (1997).

	BL^{∞} dynamics					
	learners		non-learners	marginal		interior
CL	0.452	$>$	0.427	0.404	$<$	0.488
CC	0.205	$<$	0.206	0.210	$>$	0.201
BC	0.013	$<$	0.014	0.017	$>$	0.010
DC	0.020	$\approx$	0.020	0.021	$>$	0.019
	RL^{100} dynamics					
	learners		non-learners	marginal		interior
CL	0.453	$>$	0.423	0.405	$<$	0.487
CC	0.205	$<$	0.207	0.209	$>$	0.201
BC	0.013	$<$	0.015	0.017	$>$	0.010
DC	0.020	$\approx$	0.020	0.021	$>$	0.019

Table 5.3: Average local network properties of learners vs. non-learners, and of marginal vs. interior agents by different learning dynamics. Symbols $<$, $>$, $\approx$ indicate whether differences in means are considered significant by a t-Test.

playing the Lewis game on β-graphs:

1. Interior agents tend to be family men; marginal agents show the mark of globetrotters

2. In comparison to non-learners, learners tend to be family men

Intuitively speaking, this means that in order to successfully learn a language in a diffuse social network structure like a β-graph, an agent would have to be a family man, well embedded in a dense *local* structure and an interior agent of a language region. Globally well connected agents like globetrotters, on the other hand, have difficulties learning a language early on, because they might be torn between different locally firmly established languages and are often found on the margin of language regions.

In addition, a basic result of these experiments is as follows: different learning dynamics, BL^{∞} or RL^{100}, does not have remarkably different impacts on the emergence of language regions or the history of individual learning. *Where* language regions emerged was basically influenced by global structural network properties; and individual learning was strongly

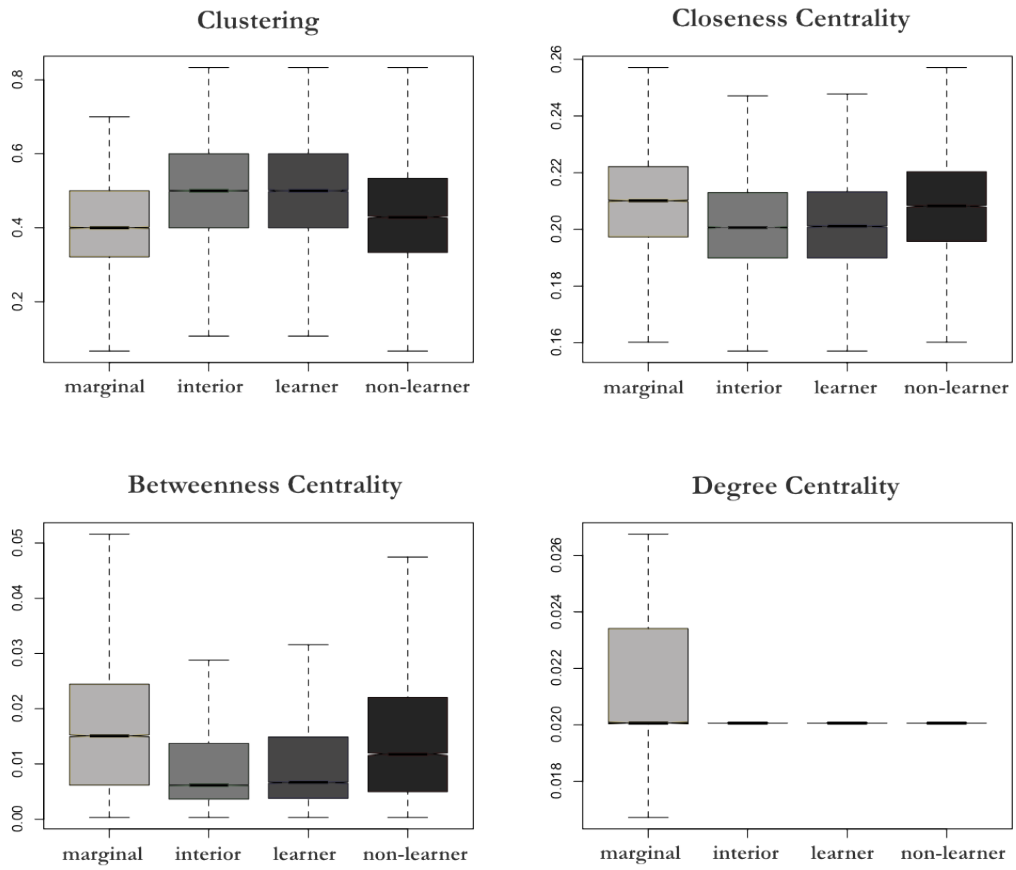


Figure 5.4: Box plot of data from Analysis I(a) and I(b)

influenced by local network properties, but both almost completely independent of the learning dynamics. This result resembles the conclusion I drew after the experiments on a toroid lattice in Section 4.1. That study also showed: BL^{∞} and RL^{100} dynamics both have a similar impact on the resulting structure. In the following I will show that both dynamics differ strongly in another way: the *temporal development* of learning.

Analysis of Temporal Development

A fundamental result of the temporal development analysis is as follows: BL^{∞} agents settle into conventions much faster than RL^{100} agents. Figure 5.5 depicts the number of language learners (averaged over all 200 simulation runs) over the number of simulation steps for each dynamics. As observable: after 10 simulation steps ca. 90% of BL^{∞} agents have learned a

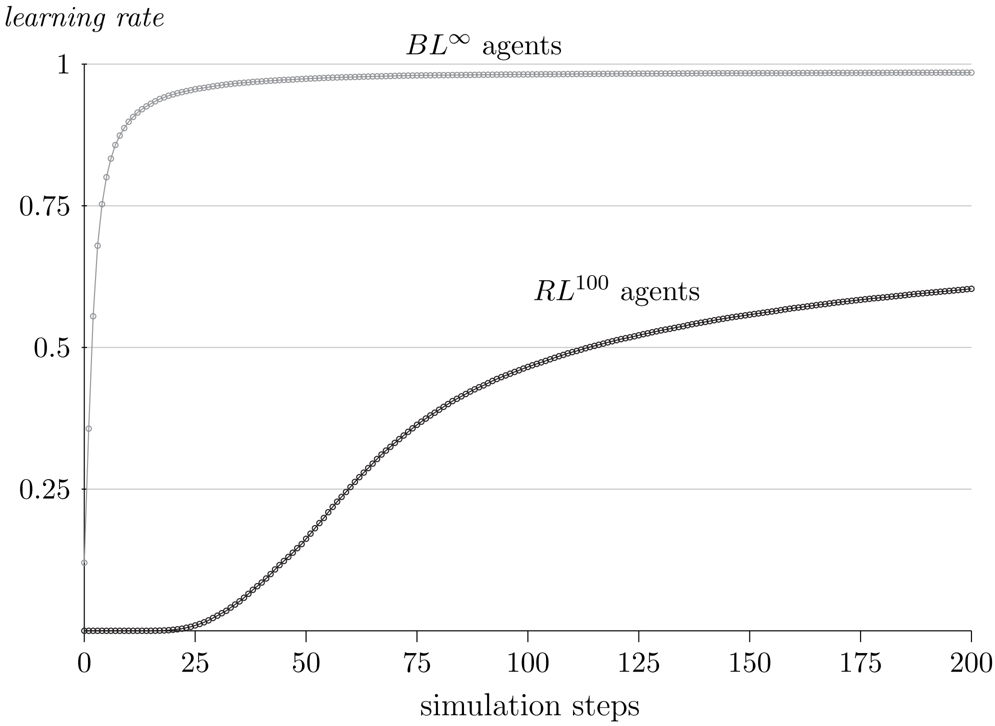


Figure 5.5: Number of BL^∞ and RL^{100} agents that have learned a language over the number of simulation steps. The developmental process of BL^∞ agents is much faster than the one of their RL^{100}-cousins. The latter one forms a so-called *S-shaped curve*.

language, whereas even after 100 simulation steps less than half of all RL^{100} agents have settled into a language.

Note that the development process of RL^{100} agents resembles the canonical *S-shaped curve* that is assumed to represent the time course of language change (c.f. Weinreich et al. 1968; Chambers 2004): the initial stage reveals a quite low incremental rate and is referred to as the *innovation phase* (c.f. Croft 2000). The rate of change increases to a maximum, when the majority of individuals are using the appropriate linguistic variant, generally close to the midpoint of S-shaped curve. This stage is called the *selection and propagation phase* (c.f. Fagyal et al. 2010). The process slows down, when the linguistic variant is used by a (nearly) majority of the speech community. This final stage of the change is called the *establishment phase* (c.f. Fagyal et al. 2010).

According to this segmentation, the developmental process of the RL^{100} dynamics forms a S-shaped curve with an innovation phase around the first

30 simulation steps, a selection and propagation phase around simulation steps 30-80, and an establishment phase that starts after 80 simulation steps. Such a development process was not detectable for BL^{∞} agents, in general because they simply learn too quickly.[4]

But the learning speed is not only influenced by the agents' learning dynamics, but also by their structural properties. Thus, in a further analysis I was interested in finding answers to the following question: what kind of node properties cause agents to learn a language early or late during a simulation run? For that purpose, I partitioned all agents into different time intervals, in which they have acquired their final language. Consequently, I define a *learning time group* as follows:

Definition 5.6. *(Learning time group) A learning time group $X_t(f, c) \subseteq X$ is the group of agents that learned their final language between simulation step f (floor) and simulation step c (ceiling).*

For my analysis I partitioned all agents that learned a language in the first 300 simulation steps in different learning time groups, where each group involves 25 simulation steps. Hence, I considered the following learning time groups: $X_t(1, 25)$, $X_t(26, 50)$, $X_t(51, 75)$, $X_t(76, 100)$, ..., $X_t(251, 275)$, $X_t(276, 300)$. To measure the node properties for learners over time, I computed the average CL, BC, CC and DC values for each learning time group, also averaged over all simulation runs. As the analysis revealed, the slower RL^{100} dynamics show a very interesting connection between the temporal development of language learning and an agent's node properties.[5] The result is depicted in Figure 5.6.

The results basically reveal that average values of global connectivity (BC and CC) of learners slightly increase over time, while the average clustering value (CL) decreases over time. In other words: early learners are family men, and the later agents learn a language, the more do they have the mark of globetrotters. This result perfectly matches with another result of Analysis I(a), namely that learners tend to be family men.

But there is another interesting fact: the degree centrality value decreases strongly for the first 75 simulation steps, and then it increases con-

[4]More precisely, because of the dynamics of the best response choice function, agents decide for one or the other language even without an established belief. Thus, they don't undergo a *learning process* that includes a long phase of being undecided and finally converging to a stable language. That makes it hard for BL^{∞} agents to detect a manifestation of the population's learning status during the learning process.

[5]Such a connection was not detectable for BL^{∞} agents because of that point already made to explain the lack for an S-shaped curve development process of learning.

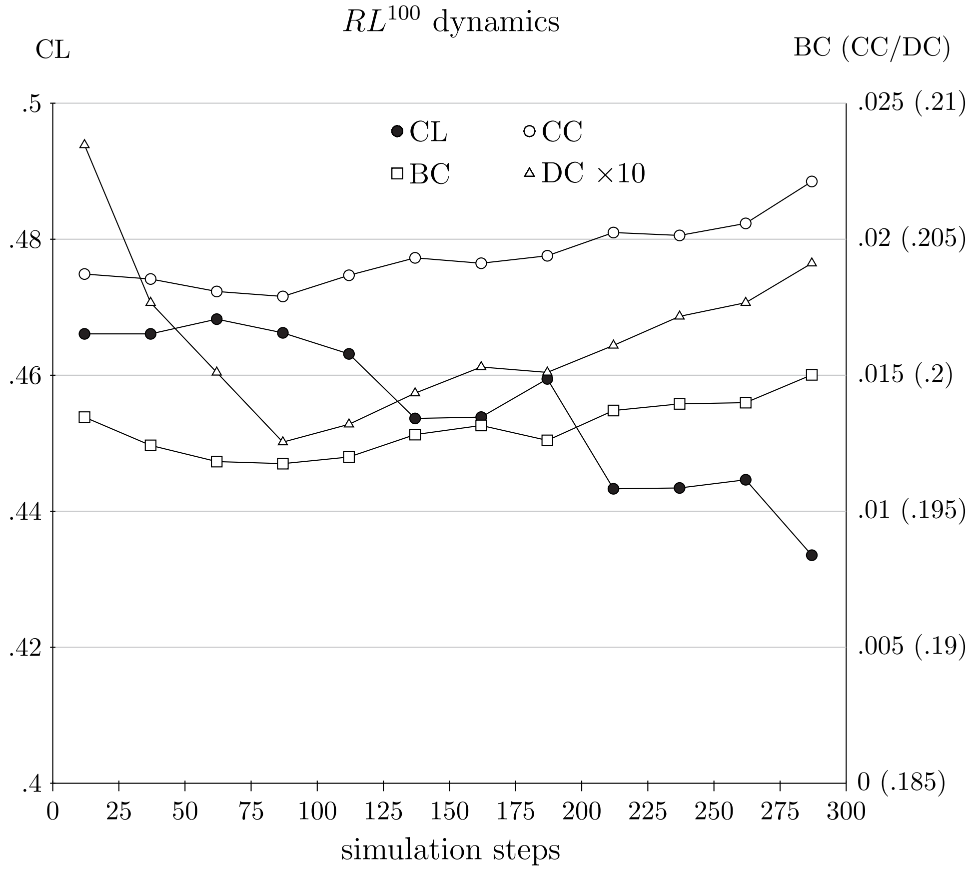


Figure 5.6: The average CL, BC, CC and DC values for RL^{100}-learners in different learning time groups (specified by an interval of simulation steps).

stantly. This additional observation tempts me to define a further classification of agent roles that depict the influence on the development process of language regions:

Definition 5.7. *(Classification of Developmental Agent Roles) Given a society of agents situated on a social network, where agents interact via a signaling games and update their strategic behavior via learning dynamics. Agents can be classified in three different roles that depict the influence on the development process of language regions:*

- Founding Fathers*: These agents are the innovators. They are the first that learn a language ($\sim$ during the first 25 simulation steps) and are possible initializers of a language region.*

- Stabilizers*: These are agents that, after a language has evolved, adopt and spread it within their local community (∼ simulation steps 25-75).*
- Late Learners*: These agents are left after core areas of language regions have already evolved ($\gtrsim$ 75 simulation steps). Late learners are most probably marginal agents of final language regions.*

Note that this classification of agents is (almost) completely in line with the three stages of language change (see Figure 5.5): founding fathers sketch the *innovation phase*, stabilizers accomplish the *selection and propagation phase* and late learners learn, when languages are essentially established, at the *establishment phase*. Furthermore, the results reveal the following relationships between the developmental agent roles and the agent types characterized by node properties (see Table 5.2) for β-graphs:

- *Founding Fathers* can be characterized as highly connected family men (high DC and IC values, low BC and CC values).
- *Stabilizers* have also the characteristics of family men. But, as opposed to founding fathers, they have a much lower DC value which makes them more likely to adopt a language from a nearby founding father.
- *Late Learners* reveal a specific pattern: the later they learn, the more do they have characteristics of globetrotters (high DC, BC and CC values, low IC value), which holds also the other way around.

It is important to note that the share of *founding fathers* constitutes less than 5% of the whole population, while the share of *stabilizers* is around 35%. Thus core areas of language regions are established after around 40% of all agents have learned a language. The slightly more than 60% of all remaining agents are labeled as *late learners*.

5.1.3 Lewis Games on Scale-Free Networks

For Experiment II we move from β-graphs to a different class of realistic network structures, so-called scale-free networks. As mentioned before, these capture the realism of preferential attachment; i.e. humans form social bonds based on preferences like friendship or economics. In this line of inquiry, I modeled structured populations as scale-free networks with 300 nodes, constructed by the algorithm of Holme and Kin (2002), as described in Section 3.2.3, with control parameter $m_t = 3$ and probability parameter $P_t = .8$. These parameters ensure that these scale-free networks and the β-graphs of Experiment I have similar values of *density* and *average clustering*, whereas scale-free networks inherently come with a lower *transitivity* value. The further experimental settings are also exactly like those of Experiment I.

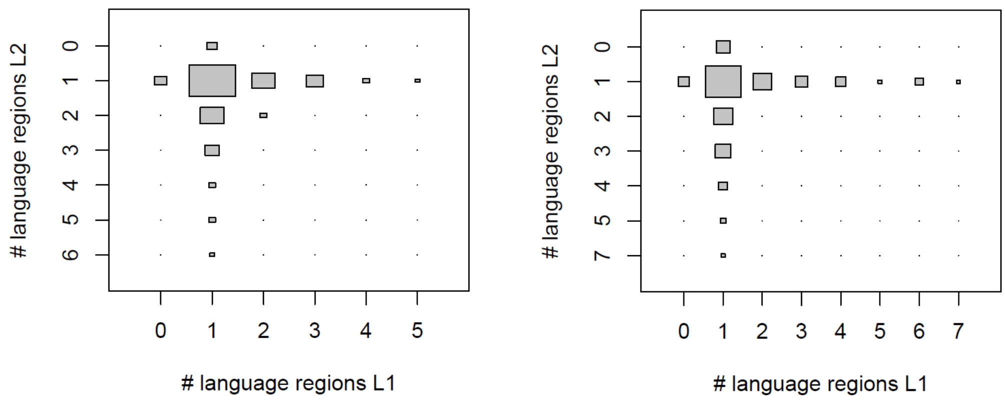
In Experiment II(a) I simulated BL^{∞} agents playing the Lewis game LG on a scale-free network: I performed 200 simulation runs of the network game $NG_{II(a)} = \langle LG, X, BL^{\infty}, G_{sf} \rangle$, where $X = \{x_1, \ldots x_{300}\}$ is a set of 300 agents and G_{sf} is a scale-free network. To compare these results with the behavior of RL^{100} agents, I started Experiment II(b): 200 simulation runs of RL^{100} agents playing the Lewis game on a scale-free network, thus I applied network game $NG_{II(b)} = \langle LG, X, RL^{100}, G_{sf} \rangle$.

Analysis of Language Regions

As a general result, in every run exactly one language region emerged for one of both languages, where the second language either a) expended over multiple language regions, b) formed also exactly one language region, or c) was driven to extinction and therefore did not form any language region. More than half of all simulation runs ended with a society, where the network was split into exactly two language regions of both types L_1 and L_2. This fact is displayed in Figure 5.7 that depicts the Hinton diagram of Experiment II(a) (Figure 5.7a) and Experiment II(b) (Figure 5.7b).

In comparison with the results of Experiment I (see e.g. Figure 5.2), the scale-free network produces much more regional variability than a β-graph. Furthermore, regional meaning did not emerge in every trial since some simulation runs ended with a society of agents that have all learned the same language.

In a further analysis, similar to Experiment I for the β-graphs, I analyzed the structural properties of language regions for scale-free networks. Thus, I measured density, average clustering and transitivity of all language regions for the first 20 simulation runs of both experiments. To compare the different values of each language region's structural properties with a standard value, I computed the *expected value* of each property for any size n (number of nodes) of a sub-network inside a scale-free network. Figure 5.8 shows the results of my analysis in the following way: each data point depicts a language region. Its position is defined by the number of nodes n (x-axis) and the value of the appropriate structural property (y-axis). The



(a) BL^{∞} dynamics (b) RL^{100} dynamics

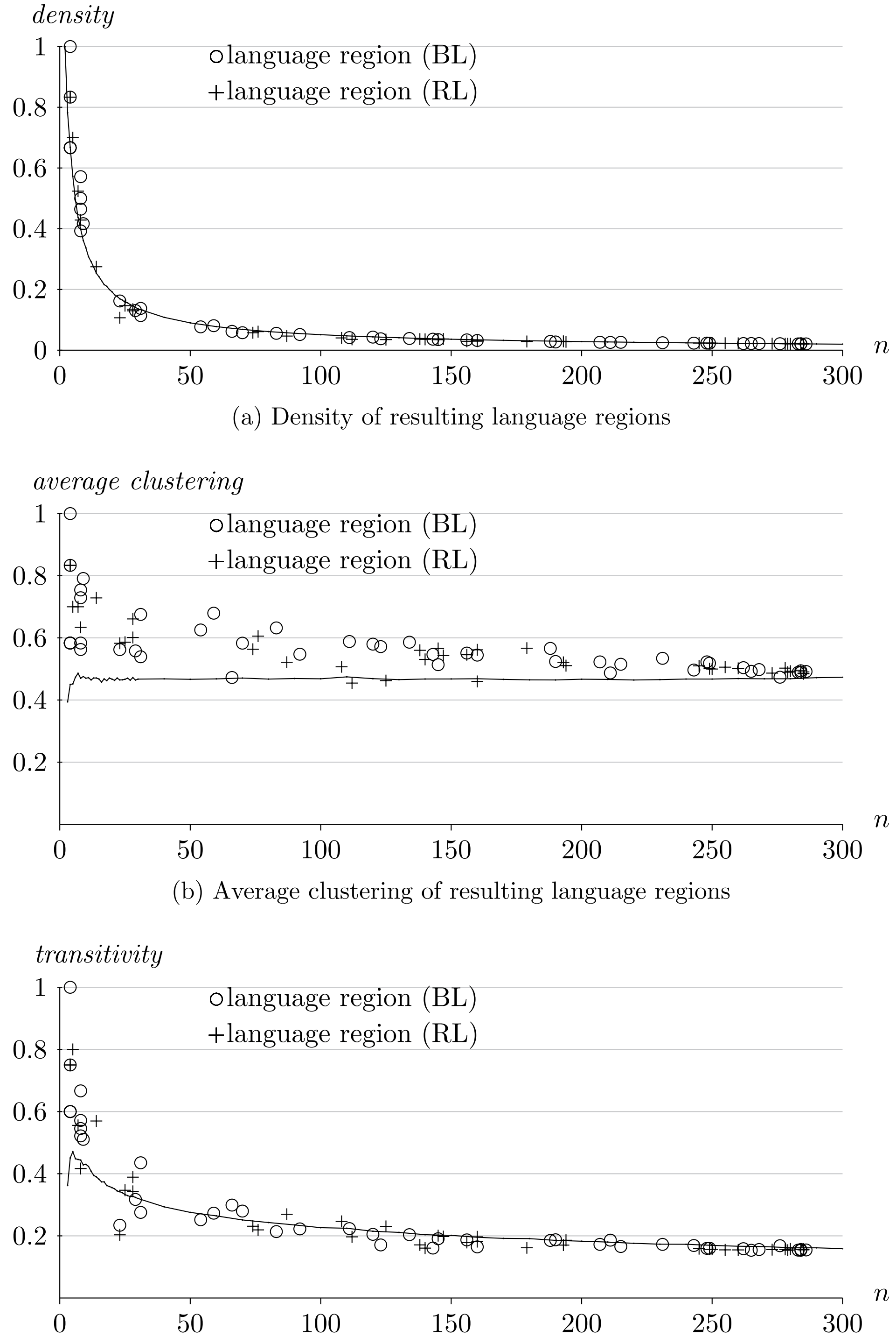
Figure 5.7: Hinton diagrams for the combination of language regions of L_1 and L_2 language learners on a small-world network for BL^{∞} and RL^{100} dynamics. For both dynamics two language regions emerged in most cases, in particular for the RL^{100} dynamics, while for BL^{∞} dynamics the results were more varied.

data points labeled with 'o' are language regions that evolved in Experiment II(a) (BL^{∞} dynamics); the data points labeled with '+' are language regions that evolved in Experiment II(b) (RL^{100} dynamics). The solid line depicts the expected value of the structural property for different sizes n, where each data point is the average value over 100 randomly chosen connected regions in the same network.

The results reveal that each connected language region of a given type had, without fail, a higher average clustering value than the expected average value for a connected subgraph with the same size n (Figure 5.8b), whereas the density and transitivity value didn't exhibit such a divergence (Figure 5.8a and and 5.8c).

Thus the results slightly contrast with the experimental results for β-graphs (see Figure 5.3, page 155): both results correspond to the two fact i) that the average clustering values were always higher than the expected average value, and ii) that the density values roughly coincide with the expected average. But the difference is that the transitivity values are always higher than expected for β-graphs, whereas for scale-free networks the transitivity values are distributed around the expected average, some above, some below.

This divergence results from the nature of both network types: in diffuse networks like β-graphs, transitivity and clustering roughly correspond



(a) Density of resulting language regions

(b) Average clustering of resulting language regions

(c) Transitivity of resulting language regions

Figure 5.8: Density, average clustering and transitivity of the resulting language region (y-axis) in comparison with average values from randomly chosen subgraphs (solid lines, sub-graph size n along the x-axis).

to each other, thus enhanced individual clustering values bring along enhanced transitivity values. This is not the case for scale-free networks: they can have enhanced individual clustering values because of the cliquishness of sub-communities, whereas the more global transitivity value stays low because unions of those communities are connected by only few hubs.

In general, this divergence hints to the fact that phenomena that can be observed in one small-world network are probably not seen in another. In reference to the given case, we can see that individual clustering and transitivity have different expressiveness: individual clustering is a *more local* value, transitivity is *more global*. And scale-free networks that are constructed by the algorithm of Holme and Kin (2002) reveal indeed an ordinary individual clustering value, but a quite low transitivity value in comparison to β-graphs.

Agent Analysis

In a next step, like in Experiment I, I investigated the relationship between an agent's a) individual learning profile as represented by her dynamic properties and b) positioning in the network depicted by her static network properties. For that purpose I started two analyses for each learning dynamics: in Analysis II(a) I partitioned all agents in learners and non-learners (Definition 5.4 and 5.5), in Analysis II(b) in interior agents and marginal agents (Definition 5.3 and 5.2) of the final populations of all runs. For each partition I computed the average values of the agents' network properties: CL, DC, CC and BC. The resulting values didn't reveal any difference between both dynamics. The values for RL^{100} dynamics are depicted as box plots in Figure 5.9.

The results resemble by and large the results of Analysis I(a) and I(b) for β-graphs (see Figure 5.4), with one small exception. While on β-graphs learners and non-learners revealed the same average degree centrality, here non-learners have a clearly higher one. Thus, on scale-free networks the non-learners clearly depict the mark of globetrotters in comparison with learners. By taking into account the characterization of agents by network properties (see Table 5.2), the following conclusions can be drawn for agents playing the Lewis game on scale-free networks:

1. Interior agents tend to be family men; marginal agents display the signs of globetrotters

2. Learners tend to be family men; non-learners tend to be globetrotters

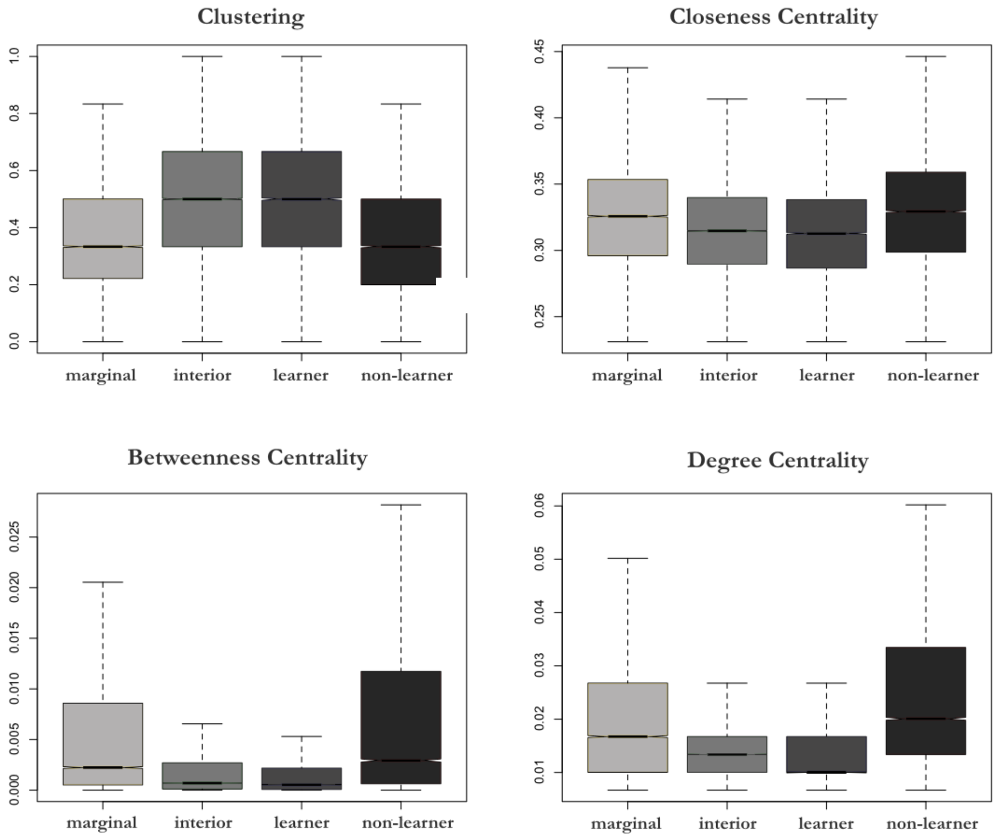


Figure 5.9: Box plot of data from Analysis II(a) and II(b)

The conclusion that can be drawn resembles the conclusion for β-graphs: to successfully learn a language (particularly at early stages), an agent has to be a family man, well embedded in a dense *local* structure inside a language region.

Analysis of Temporal Development

In accordance with Experiment I, it was shown that different learning dynamics (BL^∞ or RL^{100}) do not have clearly divergent impacts on the emergence of language regions or the history of individual learning. But they have a strong impact on the learning speed. Accordingly, on scale-free networks the learning speed differs strongly between BL^∞ agents and RL^{100} agents: BL^∞ agents learn much faster, and roughly as fast as on β-graphs. Furthermore, the learning speed of RL^{100} agents is slightly faster on β-graphs than on scale-free networks, as depicted in Figure 5.10.

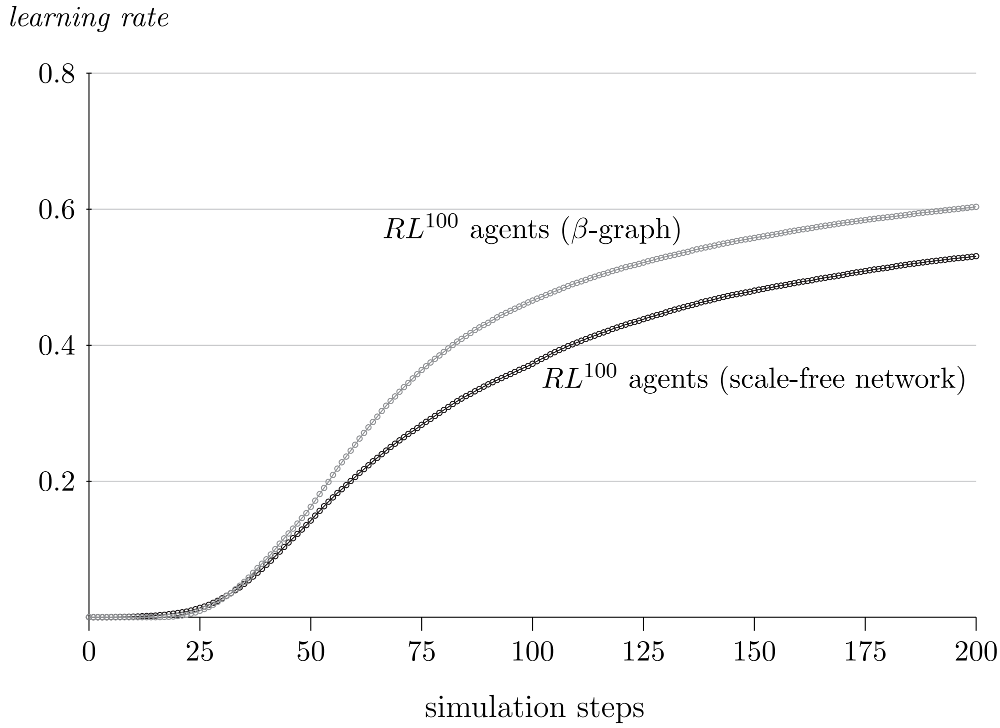


Figure 5.10: The number of RL^{100} agents that have learned a language over the number of simulation steps for both network types: the learning speed on β-graphs is a bit faster than on scale-free networks.

Much as was done for the β-graphs, I wanted to find out what kind of node properties cause agents to learn a language early or late during a simulation run on scale-free network structures. For that purpose I partitioned the agents in different learning time groups for all 25 simulation steps over the first 300 simulation steps (exactly like in Experiment I) and I measured the average values of the node properties CL, CC, BC and DC for each learning time group, averaged over all simulation runs. The slower RL^{100} dynamics showed a very interesting connection between the temporal development of language learning and agent's node properties, as depicted in Figure 5.11.

The results look quite different on scale-free networks in comparison with the results of the β-graph experiments (see Figure 5.6). Remember: on β-graphs founding fathers are highly connected family men, whereas lower connected family men stabilize the language region and late learners show more and more the mark of globetrotters. As observable in Figure 5.11 on scale-free networks, if we again characterize *founding fathers* by the

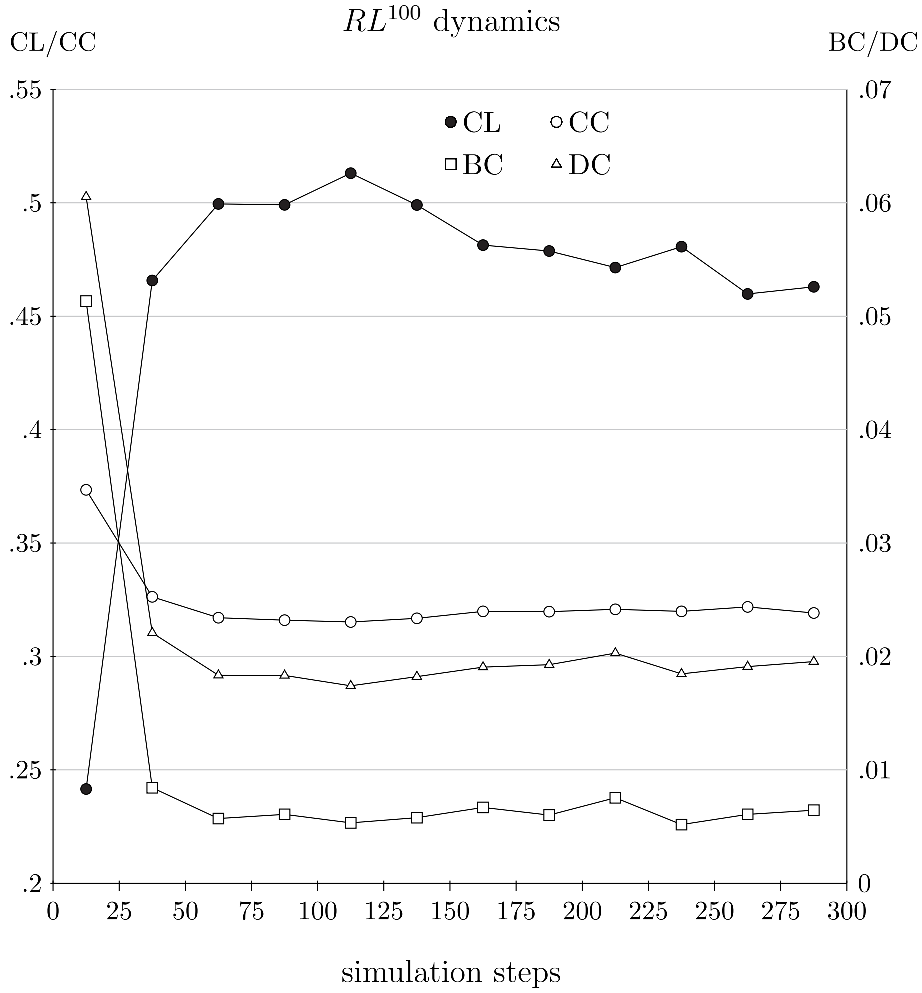


Figure 5.11: The average CL, BC, CC and DC values for RL^{100}-learners in different learning time groups (specified by an interval of simulation steps).

early learners and therefore innovators in the network ($\sim$ during the first 25 simulation steps), then these agents have completely different properties. Here founding fathers are super-globetrotters: they have extremely low CL values, but extremely high local and global centrality values (DC, BC and CC) in comparison with agents that learn a language afterwards. The *stabilizers* ($\sim$ simulation steps 25-75) and *late-learners* ($\gtrsim$ 75 simulation steps) differ basically in their CL values since the former ones reveal a higher individual clustering value.

The results reveal the following relationships between the developmental agent roles and the agent types, characterized by node properties for scale-free networks:

- *Founding Fathers* can be characterized as super-globetrotters (very high DC and BC values, high CC and low CL value).
- *Stabilizers* have the typical characteristics of low connected family men (high CL and low centrality values).
- *Late Learners* have similar characteristics like stabilizers, but the later they learn, the lower is their CL value, and the other way around.

The share of *founding fathers* constitutes less than 5% of the whole population. The share of *stabilizers* is no more than around 35%. Thus, core areas of language regions are established after around 40% of all agents have learned a language. The remaining more than 60% of all agents are labeled as *late learners*.

All in all, these results show that in more diffuse networks with relatively homogeneously distributed influence like β-graphs, the emergence of language regions is induced by local big-shots, whereas on networks following a power law distribution like scale-free networks, some super-influential global agents initiate language regions.

These circumstances explain the fact that agents on β-graphs learn faster, as depicted in Figure 5.10: the crucial difference between both S-shaped curves is the learning speed of the stabilizers during the *selection and propagation phase*. This can be explained by the fact that founding fathers on β-graphs are embedded in a dense local structure, stabilizers around them form a perfect foundation for languages to spread and stabilize very quickly. On the opposite side of the spectrum, founding fathers on scale-free networks are global players, connected to different distant[6] regions. Consequently, they are not embedded in an optimal foundation for languages to spread and stabilize fast.

Finally, the difference between stabilizers and late-learners on β-graphs is remarkable since the former are low connected family men, and the latter show the mark of globetrotters. Thus, both types are exactly inverted in terms of node properties. Furthermore, there is not a clear difference between stabilizers and late-learners on scale-free networks.[7] This can be explained by the fact that there are not many typical globetrotters in a scale-free network. And these few ones are super-influential ones that adopt the role of founding fathers.

[6] Distant in terms of shortest path length.

[7] They only noticeable observation is a slightly decreasing CL value for late-learners.

5.1.4 Horn Games on β-Graphs

Proceeding with Experiment III, I turn to the analysis of Horn games on β-graphs. Except of the game, all conditions for interaction and settings for simulation runs are given as for Experiment I in Section 5.1.2.

In Experiment III(a) I simulated BL^∞ agents playing the normal Horn game on a β-graph: I performed 200 simulation runs of the network game $NG_{III(a)} = \langle HG, X, BL^\infty, G_\beta \rangle$, where $X = \{x_1, \ldots x_{300}\}$ is a set of 300 agents and G_β is a β-graph. To compare these results with experimental results of RL^{100} agents I started Experiment III(b): I performed 200 simulation runs of RL^{100} agents playing the Horn game on a β-graph, thus I applied network game $NG_{III(b)} = \langle HG, X, RL^{100}, G_\beta \rangle$.

Analysis of Language Regions

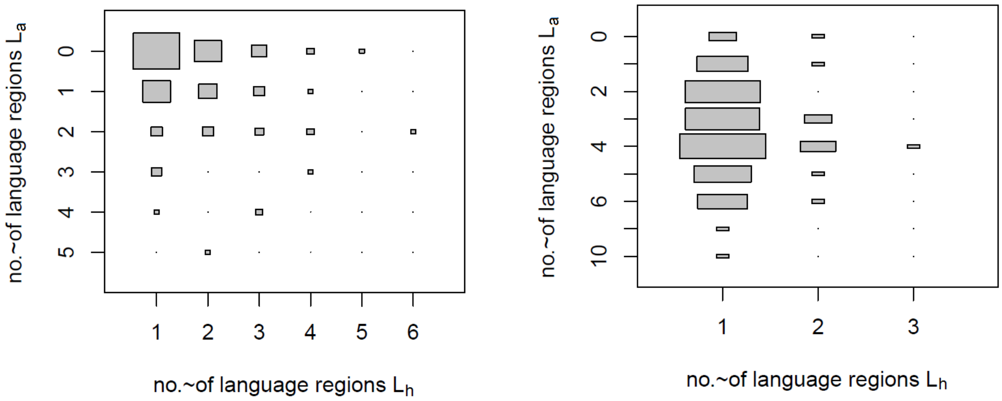
As a general observation, the resulting combinations of number of language regions for both signaling languages, here L_h and L_a, look quite different in comparison with the experiments for the Lewis games (Section 5.1.2), but also the results of RL^{100} and BL^∞ dynamics differ strongly.

The results for Experiment III(b) (RL^{100} dynamics) reveal that in almost every run exactly one language region of the Horn language L_h emerged, whereas the language L_a expended over multiple language regions, generally between 2 and 4 local regions. In the remaining simulation runs it happened that either i) no language region of L_a emerged or ii) two language regions of L_h emerged.[8] The resulting Hinton diagram of Experiment III(b) is depicted in Figure 5.12b.

The results for Experiment III(a) (BL^∞ dynamics) look quite different: here in more than half of all simulation runs one language region of the Horn language L_h emerged, whereas the language L_a failed to form a language region. In a noteworthy number of simulation runs either i) one language region for each language emerged, or ii) two L_h and one L_a language region emerged, or iii) two language regions of L_h, but none of L_a emerged. The resulting Hinton diagram of Experiment III(a) is depicted in Figure 5.12a.

In a next step, I analyzed the structural properties of language regions: I measured the density, average clustering and transitivity of all language regions for the first 20 simulation runs of both experiments; and compared the different values of structural properties of language regions with the expected average value. Figure 5.13 shows the results of my analysis, the

[8] Actually, in one simulation run 3 language regions of L_h emerged.



(a) BL^{∞} dynamic (b) RL^{100} dynamic

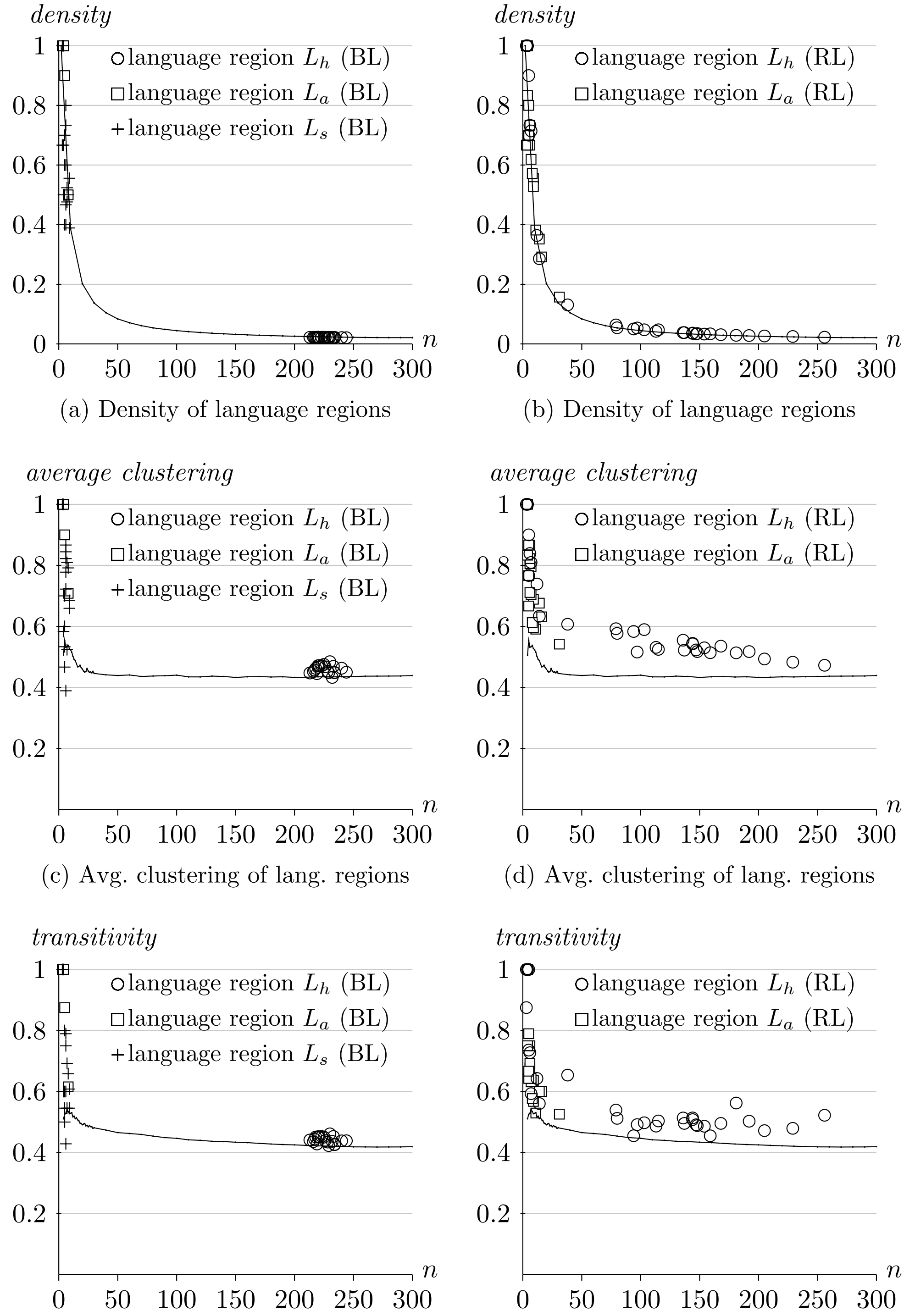
Figure 5.12: Hinton diagrams for the combination of language regions of L_h and L_a language learners on a β-graph for BL^{∞} and RL^{100} dynamics. For both dynamics in most of the cases one language region of language L_h emerged, while the number of L_a language regions varied more over all simulation runs.

left figures for the BL^{∞} dynamics (Figure 5.13a, 5.13c and 5.13e), the right figures for the RL^{100} dynamics (Figure 5.13b, 5.13d and 5.13f) Data points labeled with 'o' are L_h language regions, data points labeled with '□' are L_a language regions, and data points labeled with '+' are L_s language regions.

The results show the same pattern that were already seen for Experiment I: each connected language region of a given type had always a higher average clustering and transitivity value than the expected average value for a connected subgraph with the same size n, whereas the density value didn't exhibit such a divergence.

But the results reveal even more: the language regions of language L_h is (almost) always larger than those of L_a language regions. The results for RL^{100} agents show that most of the L_h language regions have a size of between 80 and 260 nodes, whereas all of the L_a language regions have a size of less than 40 nodes. This divergence is even stronger for the results for BL^{∞} agents: all of the L_h language regions have a size of between 220 and 250 nodes, whereas all of the L_a language regions have a size of less than 10 nodes. Furthermore, small language regions of the Smolensky language L_s emerged.

The divergence of language region sizes between L_h and L_a is a result that resembles the results for experiments on a toroid lattice in Section 4.1.1. Furthermore, the fact that BL^{∞} agents learn faster leads to two effects: first,



(a) Density of language regions

(b) Density of language regions

(c) Avg. clustering of lang. regions

(d) Avg. clustering of lang. regions

(e) Transitivity of language regions

(f) Transitivity of language regions

Figure 5.13: Density, average clustering and transitivity of the resulting language region (y-axis) in comparison with expected average values from randomly chosen subgraphs (solid lines, sub-graph size n along the x-axis).

the more efficient L_h language can spread faster and therefore leaves less room for the L_a language to spread. Second, the condition that simulation runs stop after at least 90% of all agents have learned a language is fulfilled quite early for the fast BL^∞ dynamics (basically after 10 simulation steps, see e.g. Figure 5.5 on page 159). This leads to a final population, where a small number of agents still stick with initial strategy profiles that would probably disappear over time. This explains the appearance of L_s language learners in Experiment III(a).[9]

Agent Analysis

To analyze the relationship between an agent's a) individual learning profile and b) static network properties, I started two analyses for each learning dynamics: in Analysis III(a) I partitioned all agents in learners and non-learners, in Analysis III(b) in interior agents and marginal agents of the final populations of all runs. For each partition I computed the average values of the the agents' network properties. The resulting values for RL^{100} dynamics are depicted as box plots in Figure 5.14.[10]

The results resemble by and large the results of Analysis I(a) and I(b) for agents playing the Lewis game on β-graphs (see Figure 5.4), with one small exception: in Experiment I the non-learners' degree centrality didn't show any variation, while in this Experiment III the non-learners' degree centrality values drift above the average value. By taking into account the characterization of agents by network properties (see Table 5.2), the following conclusions can be drawn for RL^{100} agents playing the Horn game on β-graphs:

1. Interior agents tend to be family men; marginal agents tend to be globetrotters

2. Learners have the characteristics of family men; non-learners those of globetrotters

[9]Note: I could show in former experiments that agents have a strong tendency to initially learn the Smolensky strategy for different dynamics and population structures. See e.g. Figure 2.2 for replicator dynamics (page 38), Figure 2.13 for basic experiments with reinforcement learning (page 70) or Figure 4.8b for reinforcement learning on a toroid lattice (page 113).

[10]It is noteworthy to mention that the results for the BL^∞ dynamics looked quite different: there was almost no deviation of any groups average value in any direction. The reason for this result is probably the fast learning speed combined with the uniformity of the final population. More precise analyses go beyond this work.

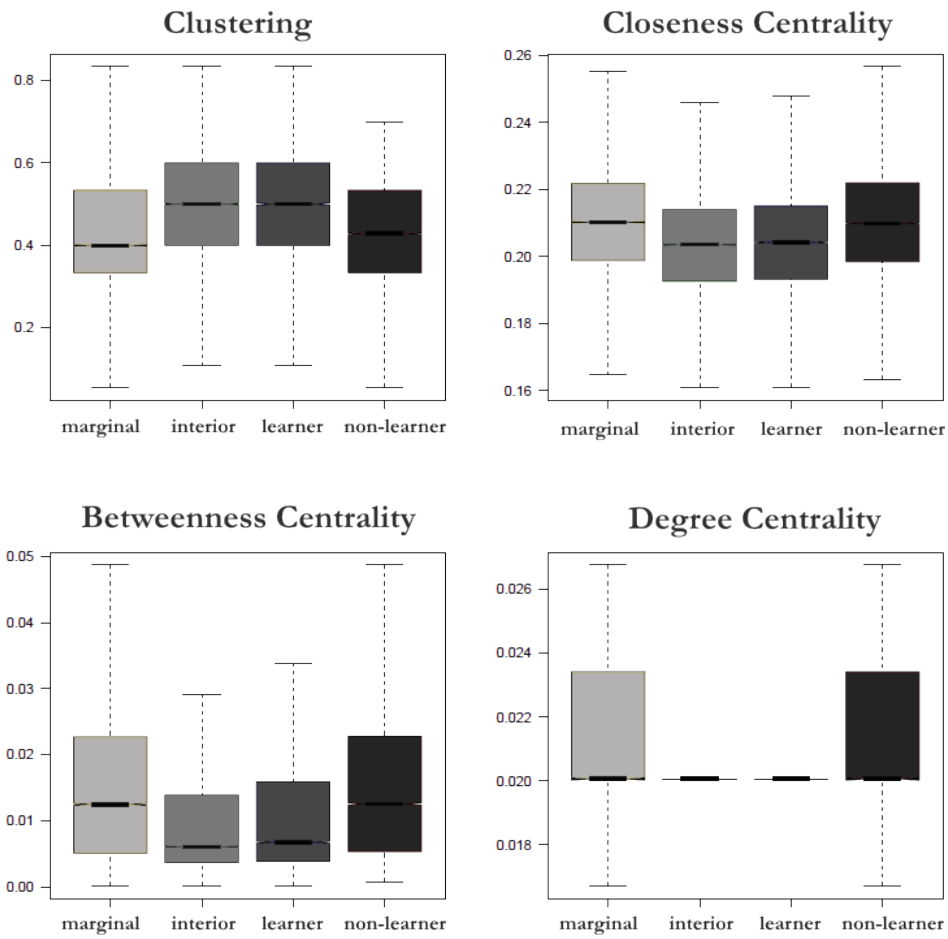


Figure 5.14: Box plot of data from Analysis III(a) and III(b)

Note that this result basically doesn't deviate from former results. Further, the analysis of temporal development didn't reveal any new insight that weren't already obtained from Experiment I with the Lewis game on β-graphs. Thus I will continue with the analysis of Experiment IV: Horn games on scale-free networks.

5.1.5 Horn Games on Scale-Free Networks

We complete our analysis of costly signaling equilibria on realistic social structures in this part. Here I conducted experiments for agents playing the Horn game on scale-free networks. Except of the game, all conditions for interaction and settings for simulation runs are given as for Experiment II in Section 5.1.3.

In Experiment IV(a) I simulated BL^{∞} agents playing the Horn game

HG on a scale-free network: I performed 200 simulation runs of the network game $NG_{IV(a)} = \langle HG, X, BL^{\infty}, G_{sf} \rangle$, where $X = \{x_1, \dots x_{300}\}$ is a set of 300 agents and G_{sf} is a scale-free network as introduced. To compare these results with experimental results of RL^{100} agents, I started Experiment IV(b): I performed 200 simulation runs of RL^{100} agents playing the Horn game on a scale-free network, thus I applied network game $NG_{IV(b)} = \langle HG, X, RL^{100}, G_{sf} \rangle$.

Analysis of Language Regions

As a general observation, the resulting combinations of language regions for both signaling languages, here L_h and L_a, look quite different in comparison with both the results for Lewis games on scale-free networks (Section 5.1.2) and the results for Horn games on β-graphs (Section 5.1.4).

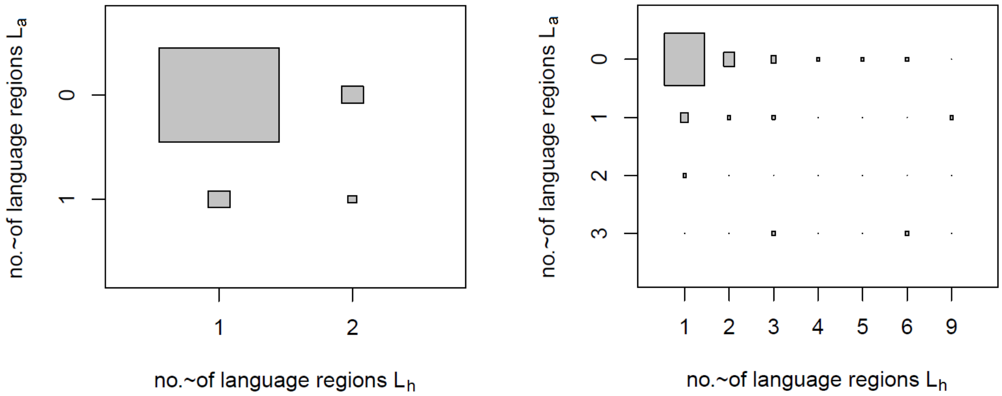
The results for Experiment IV(a) (BL^{∞} dynamics) reveal that only one language region of the Horn language L_h emerged in almost every run, whereas no such regions of language L_a emerged, as depicted in the resulting Hinton diagram (Figure 5.15a).

The results for Experiment IV(b) (RL^{100} dynamics) also reveal that in most of the simulation runs one language region of language L_h and no language region(s) of language L_a emerged, but in comparison with Experiment IV(a), the few exceptions are a little bit more varied, as observable in the resulting Hinton diagram (Figure 5.15b).

In a next step I analyzed the structural properties of language regions: I measured density, average clustering and transitivity of all language regions for the first 20 simulation runs of both experiments; and compared the different values of structural properties of language regions with the expected average value. Figure 5.16 shows the results of my analysis, the left figures for the BL^{∞} dynamics (Figure 5.16a, 5.16c and 5.16e), the right figures for the RL^{100} dynamics (Figure 5.16b, 5.16d and 5.16f), where data points labeled with 'o' are L_h language regions, and data points labeled with '□' are L_a language regions.

The resulting characteristics of the language regions reveal what the Hinton diagrams already gave us reason to expect: for both dynamics there is one huge L_h language region that spreads over the whole network. The RL^{100} agents show a little bit more variety in two ways: i) the sizes of the regions vary stronger and ii) for some runs, tiny regions of the L_a language emerged.[11]

[11]Note that these are the data of the first 20 simulation runs of each experiment. The



(a) BL^{∞} dynamics (b) RL^{100} dynamics

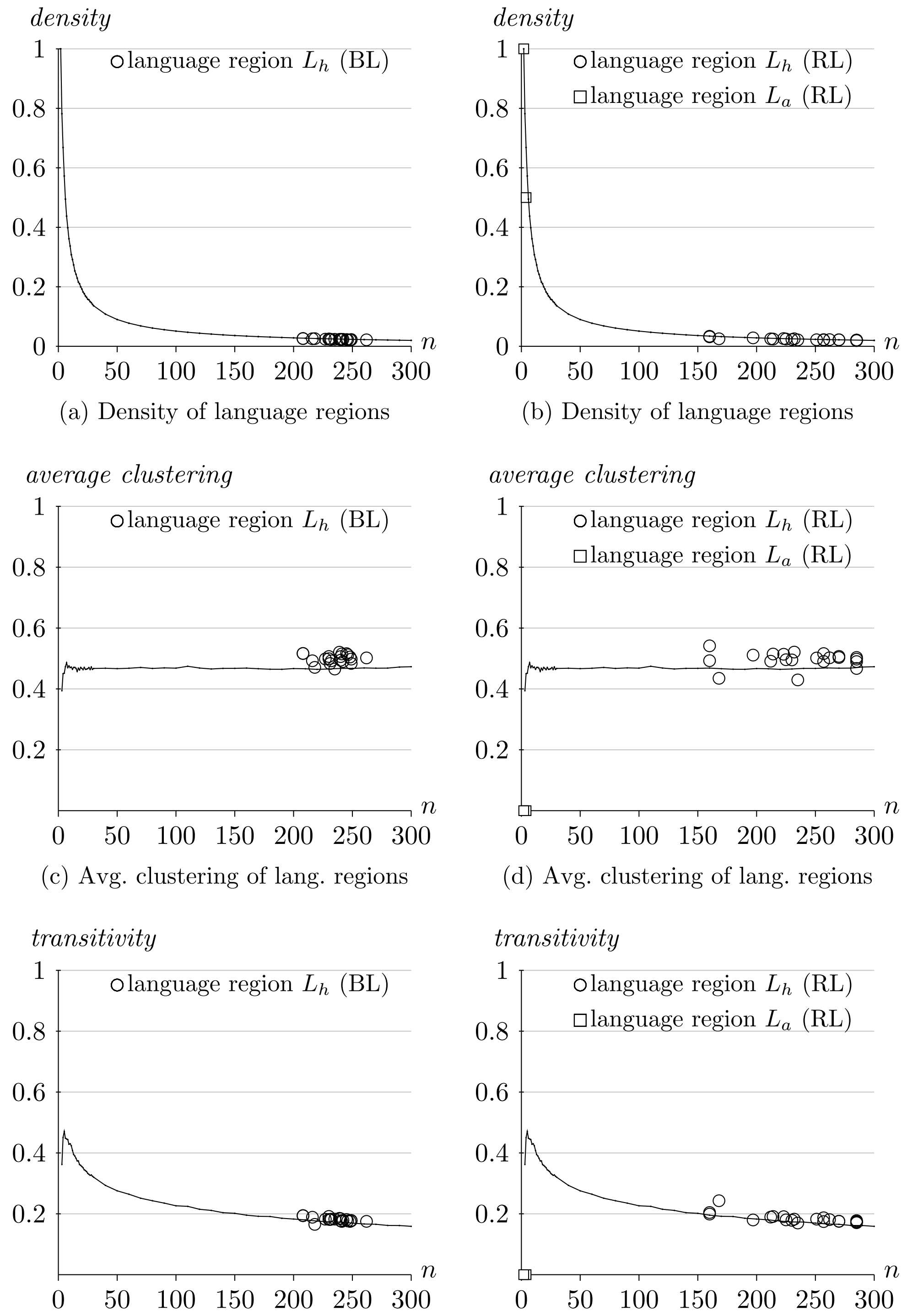
Figure 5.15: Hinton diagrams for the combination of language regions of L_h and L_a language learners on a scale-free network for BL^{∞} and RL^{100} dynamics. For both dynamics one language region of language L_h emerged in almost every case, while L_a language regions either never emerged, or failed to stabilize, thus eventually disappeared.

Both experiments reveal that the resulting transitivity and average clustering values don't distinctively differ from the expected average values. Note that this is inevitable since a sub-network's structural features approach the structural features of the whole network with increasing size.

The salient result is the extreme dominance of the L_h language on scale-free networks that outperforms the result for Horn games on β-graphs (Experiment III). The reason for this phenomenon can be deduced from the fact that founding fathers on β-graphs and scale-free networks differ strongly. On β-graphs there are more than twice as many founding fathers in local areas that can establish their language in the *innovation phase*. Consequently, the probability of founding fathers that initialize a L_a language region is higher. Furthermore, such starters of innovation operate in local terrain that provides a fast local spread and stabilization of the language. Thus, the initialized L_a language regions often establish a strong barrier against other languages. Consequently, they have a higher chance to survive in a network that is dominated by the L_h language.

On the contrary, many fewer of the founding fathers evolve in scale-free networks, thus the probability of initiating an L_a language region is small

whole data set reveals that L_a language regions emerged also for BL^{∞} dynamics, but much less frequent than the already infrequent occurrences of L_a language regions for RL^{100} dynamics.



(a) Density of language regions

(b) Density of language regions

(c) Avg. clustering of lang. regions

(d) Avg. clustering of lang. regions

(e) Transitivity of language regions

(f) Transitivity of language regions

Figure 5.16: Density, average clustering and transitivity of the resulting language region (y-axis) in comparison with expected average values from randomly chosen subgraphs (solid lines, sub-graph size n along the x-axis).

in the first place. Furthermore, those agents are locally connected and probably strongly influenced by other founding fathers since all of them are super-influential globetrotters. In addition, since local clustered terrain is rare, a strong barrier against a L_h majority is hard to establish. Thus even a spark of such an L_a language region is readily extinguished.

Agent Analysis

To analyze the relationship between an agent's a) individual learning profile and b) static network properties, I started two analyses for each learning dynamics: in Analysis IV(a) I partitioned all agents in learners and non-learners, in Analysis IV(b) in interior agents and marginal agents of the final populations of all runs. For each partition I computed the average values of the the agents' network properties. The resulting values for RL^{100} dynamics are depicted as box plots in Figure 5.17.

The results resemble by and large the results of Analysis II(a) and II(b) for agents playing the Lewis game on scale-free networks (see Figure 5.9), but with an interesting salient deviation. The non-learners revel outstanding high DC and BC values and an outstanding low clustering value CL. Note that these characteristics resemble those of super-globetrotters, the type of agents that established the founding fathers in Experiment II (see e.g. Figure 5.11). This leads to the following conclusions for agents playing the Horn game on scale-free networks:

1. Interior agents tend to be family men; marginal agents tend to be globetrotters

2. Learners have characteristics of family men; non-learners those of super-globetrotters

A reasonable explanation for how super-globetrotters are finally non-learners is as follows: Experiment II revealed that super-globetrotters constitute founding fathers. But in those experiments the agents played the Lewis game and both languages survived and stabilized. As opposed to this, in Experiment IV agents played the Horn game and while L_h takes over the whole population, L_a is driven to extinction in the end. Thus, if some of these super-globetrotters initially learned the language L_a, they became *temporary* founding fathers. But at one point language L_h has been acquired by the majority of the society; and this majority drives the minority of L_a learners to extinction. As a consequence, the temporary founding

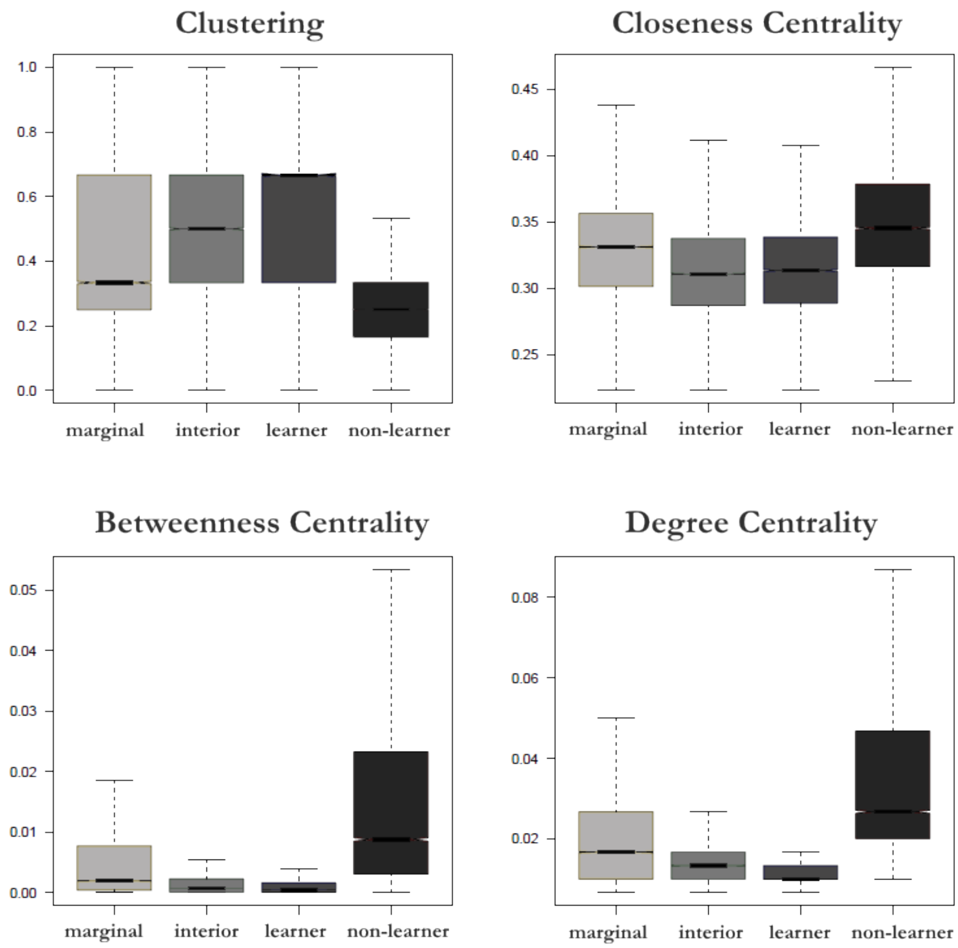


Figure 5.17: Box plot of data from Analysis IV(a) and IV(b)

fathers of L_a have to *relearn* and adopt L_h, a process that takes generally long and turns them finally into non-learners, when 90% of the society has already learned L_h.

All in all, for agents playing the Horn game on a scale-free network, the group of super-globetrotters is split into two in a temporal sense completely opposed groups: i) founding fathers, thus *early-learners*, and ii) non-learners, thus, in a technical sense, *late-learners* as well.

5.2 Conclusion

First, I want to peer at the results of Chapter 5 through the lens provided by the research question that arose in Chapter 4: under what circumstances can we expect languages other than the expected *Horn language* to emerge,

	BL^∞	RL^{100}
β-graphs	26%	63%
scale-free networks	1%	6%

Table 5.4: Percentage of simulation runs where L_a language regions stabilized for different combinations of network structure and learning dynamics.

under the precondition of initially unbiased agents? In Chapter 4 I gave an initial answer, namely that for experiments on an idealized society like a toroid lattice structure, the only language that can emerge and stabilize next to the Horn language is its counterpart, the *anti-Horn language*. Furthermore, those anti-Horn language learners emerged in local regions, sharing the society with regions of Horn language learners. Thus, the emergence of a language other than the Horn language goes hand in hand with the emergence of multiple signaling language regions. I was further able to show that the emergence of multiple language regions for Horn game experiments is, among other things, also supported by a moderately flexible learning dynamics (Level 1), weak game parameters and a local communication structure.

In this chapter I conducted experiments with flexibility level 1 alone (BL^∞ and RL^{100} dynamics) and only with the standard parameters of the Horn game (normal Horn game HG). I was chiefly interested in the ways different social network structures are conducive or detrimental to the emergence of local meaning. My results of Experiment III (Horn game on β-graphs) and Experiment IV (Horn game on scale-free networks) revealed the following causal relationships: the emergence of multiple language regions for Horn games, and therefore the emergence of L_a language regions, is stronger supported by a more locally structured small-world network (namely a β-graph), than by a globally and hierarchically organized small-world network (namely a scale-free network). The corresponding results of Experiment III and IV are depicted in Table 5.4: the percentage of simulation runs where also L_a language regions emerged and stabilized.

The results of Table 5.4 also reveal that the RL^{100} dynamics is more supportive for the emergence of multiple language regions in comparison with the BL^∞ dynamics. The more substantial difference, however, is made by the network structure. The supporting contribution of the more locally structured β-graph to the emergence of multiple language regions is analogous to the results of the social map experiments of Chapter 4: a high the degree of locality supported the probability of multiple language regions emerging. Thus, the analysis of the Horn game in this chapter did not reveal crucially new insights to explain the emergence of anti-Horn language regions that weren't already featured in the analyses of Chapter 4.

In addition, the analyses of this chapter revealed a more general question from the very beginning of this thesis: how do linguistic conventions arise and stabilize? In Experiment I and II this research question was investigated by examining, in particular, structural properties of real-world social networks. I showed that for a given small-world network the emergence of local language regions is supported by specific *structural network properties* and the developmental process is guided by specific agent roles that are correlated to particular *node properties*.

In detail, the analysis of structural properties revealed that sub-networks that constitute language regions have higher local density values[12], but a similar global density value in comparison with randomly chosen sub-networks of the same size. This result implies that a locally dense structure supports the emergence and stability of local conventions. I was able to support this result with Analysis I: the analysis of agent types on β-graphs. The results showed that the stabilization of a language region depends on the so-called *family men*: agents that are strongly embedded in a dense local structure, but without strong global connections or a central position in the network.

The comparison of Analysis III and IV revealed that those family men are important as *stabilizers* for the preservation of small local speech communities, e.g. like L_a language regions. This result is in accord with studies from sociolinguistics that deal with the influence of network structure on language variation and change in human societies. For example, Milroy and Margrain (1980) came to the conclusion that "closeness to vernacular speech norms correlates positively with the level of individual integration into local community network." (page 44).

The role of family men in the developmental process of language regions is to constitute the *selection and propagation phase*. This phase triggers a quick convergence to adopting a convention on a local scale. Studies in sociolinguistics purport that this type has a catalytic role in linguistic change. E.g. Labov (1994) postulates that language change emerges when other speakers start adopting and using innovations conventionally; or as Wolfram and Schilling-Estes (2003) pointed out: "it is not the act of inno-

[12] To be precise: language regions have higher *individual clustering* values on scale-free networks and additional higher *transitivity* values on β-graphs.

vation that changes language, but the act of influence that instantiates it." (page 733).

But before family men select, propagate and stabilize, another type of agent initiates language spread: the so-called *founding father* constitutes the *innovation phase* of the developmental process of a language region. I showed in Analysis I and II that the properties of those founding fathers depend on the structure of the network. In more locally structured small-world networks like β-graphs the founding fathers of language regions were highly connected family men, while in globally and hierarchically organized scale-free networks the founding fathers were super-globetrotters: agents with high local and global centrality values and a very low clustering value.

Both types of founding fathers are in accordance with findings of studies in sociolinguistics. E.g. Fagyal et al. (2010) mention that i) "charismatic leaders with strong ties to the local community have also been identified as innovators..." (page 4), and furthermore ii) "The near-equivalents of such central figures in other studies (Labov 2001; Mendoza-Denton 2008) led to the proposal that leaders of language change are centrally connected, highly visible individuals whose influence can extend beyond their own personal networks." (page 5).

To sum up, the results suggest the following conclusion: while the type of innovators for language emergence or change strongly depends on the structural properties of the network, the type that spreads and stabilizes local language is generally found in a dense local structure, regardless of the type of the social network.

Chapter 6

Summary

This Ph.D. project focused on the evolution and emergence of conventions and the search for an answer to the question posed by Lewis (1969): *how can linguistic meaning arise and become a convention without prior agreements?* By considering that agreements need language, we would come to a paradox: language is needed for language to emerge. Lewis paved the way for refuting this paradox by showing that, with the game-theoretic model of a signaling game, conventional linguistic meaning can arise without such agreements.

In this project I considered two specific kinds of signaling games: the *Lewis game* and the *Horn game*. These games models have, respectively, neutral and biased preferences towards selecting signaling conventions. They were chosen based on their broad coverage of phenomena in language, as some conventions are driven purely by the need to arbitrarily coordinate on meaning, and others arise with a bias built in towards some form or meaning. By taking these two games into account, I passed through the forests of literature dealing with population-based accounts of signaling games in search for the answer as to how certain signaling conventions could arise in societies of interacting agents. My journey started with concepts from *evolutionary game theory* via more individual-based *imitation dynamics* through to the core field of my research: *learning dynamics* on *heterogeneous network structures*.

In tandem with the basic question of *How do linguistic conventions arise?*, further questions arose during my research, in particular the questions of *What causes regional meaning?* and *Why is the Horn strategy predominant?* In my experimental results and analyses I tried to find satisfying

answers.

6.1 How Do Linguistic Conventions Arise?

The initial question of the introduction was as follows: how can linguistic meaning emerge as a convention among language users, even by no explicit agreements? In Chapter 1 I introduced the *signaling game* as the linchpin of my analysis of the emergence of convention. With the Lewis game I presented an initial example that Lewis used to introduce his model. Lewis's example shows that a successful signaling strategy represents a code of coordinated signaling behavior: a *signaling system*. And such a signaling system has interesting properties since it i) forms a Nash equilibrium and therefore reveals a high degree of stability, ii) renders successful communication and finally iii) ascribes a meaning to a signaling message. Thus a signaling system can model a linguistic convention that is optimally efficient and stable.

Lewis's example itself illustrates a one-shot game of a signaling game with explicit agreement about the signaling strategy. Thus, a signaling system can illustrate how a convention may stabilize, but to answer the question of how a linguistic convention might *emerge*, it is necessary to look for a model that represents a *process* rather than a single situation. This led me to models of *repeated signaling games* combined with *update dynamics*. In addition, Lewis's example depicts a situation where a successful communication strategy depends on previously made explicit agreements. Thus the challenge lies in the question of how might such a strategy emerge without such an agreement?

As a first example for repeated games with update dynamics, I introduced the update rule *myopic best response* in Chapter 1. I showed that the probabilistic version of this dynamics eventually leads to an equilibrium: a final situation of communication via a signaling system. Myopic best response represents an update rule that demands that players have a degree of rationality. Consequently, I posed the following question: is an assumption like rationality necessary to explain the emergence of signaling systems, or are lesser assumptions sufficient?

In Chapter 2 I wanted to answer this question by applying an update dynamics that considers no assumption of rationality. I started with a well-known *evolutionary dynamics* account that elides the behavior of individual players in favor of the behavior of shares of a whole population: the *replicator dynamics*. Various analyses revealed that a population playing the Lewis game and updating by the replicator dynamics results in a state where the whole population adapts the same *evolutionary stable strategy*. Such a strategy is necessarily a signaling system since it can be shown that signaling systems are the only evolutionary stable strategies of the Lewis game.

Thus it seems that I can abort my research at this point since the results for signaling games in combination with replicator dynamics reveal that the process by which signaling systems emerge and constitute linguistic conventions can be explained without cognitive ability, let alone rationality. It is here that we move to a quote by Huttegger and Zollman (2011): is the replicator dynamics an appropriate approximation for models of individual learning?

To answer this question I switched from a population-based *macro-level perspective* to an individual-based *micro-level* perspective by applying *imitation dynamics* as update mechanism for agents playing repeated signaling games. To be more concrete, I applied the imitation rule *conditional imitation*: with a specific probability that an agent adopts the strategy of a neighbor in the next round of play, but only if the latter scored better in the current round. Consequently, I started experiments with population of agents that interact with randomly chosen partners by playing the Lewis game and updating with conditional imitation. The results were akin to results of experiments with the replicator dynamics. This resembles what also other studies revealed: the replicator dynamics describes the most likely path of strategy distributions in a virtually infinite and *homogeneous* population when every agent updates her behavior by conditional imitation.

Notice that this accordance between replicator dynamics and imitation dynamics holds for homogeneous populations. Thus one further direction was to conduct experiments on *heterogeneous* population structures, in part because language change occurs in human societies, none of which are uniformly connected. For that purpose I analyzed imitation dynamics on a *toroid lattice* structure with the result that *regional meaning* emerged, some for one, some for the other signaling system.

Another direction was the switch to a more elaborated class of update dynamics: *learning dynamics*. While imitation dynamics incorporate a feature called *ignorance of previous interactions*, learning dynamics incorporate a history of previous interactions for the decision finding process and additionally allow for a more fine-grained version of a signaling game: the *dynamic* or *sequential signaling game*. Furthermore, with the goal of analyzing the importance of rationality, I applied two learning dynamics: i) *reinforcement learning* which decisions are made in a non-rational way, and ii) *belief learning* which is combined with the rational *best response* dynamics.

My experiments confirmed that there are significant similarities between reinforcement learning and the replicator dynamics. The same holds for belief learning by presupposing a *homogeneous* population structure. However, similar to the results for imitation on a lattice structure, I also found that for learning dynamics, the results differ strongly when adapting them to more *heterogeneous* population structures: the emergence of regional meaning occurs. We can draw from this that the structure of a society has a considerable impact on the evolutionary progression of the emergence of conventions. This brought with it a new question: what causes regional meaning to emerge in heterogeneous population structures.

6.2 What Causes Regional Meaning?

In Chapter 4 I dealt with the question of what *circumstances* cause regional meaning to emerge. For that purpose I conducted experiments of agents playing the Lewis game on a toroid lattice and updating via learning dynamics. To examine the impact of rationality I applied reinforcement learning and belief learning as well. To ensure that agents are initially completely neutral to any decision, the simulation runs of my experiments started with completely unbiased agents. This excluded any tendency that might have delineated a previously made explicit agreement

In general, the resulting structure was either i) a society of multiple local language regions of both signaling systems, or ii) a uniform society, where every member behaves according to one of both signaling systems. There were principally two factors that affected the probability of regional meaning emerging. The first factor concerns the *psychology* of agents: it turned out that the highest probability of the emergence of regional meaning is triggered by an intermediate *degree of flexibility* of the agents' update dynamics.[1] The second factor concerns the *sociology* of the community. Additional experiments on a *social map*, an innovation allowing for a constantly changing web of communication partners, revealed that the emergence of

[1]Note that the choice of the learning dynamics, if reinforcement or belief learning, has an indirect influence since, all else being equal, belief learning has a higher degree of flexibility. But in combination with limited/unlimited memory both dynamics can be classed with the same degree of flexibility.

regional meaning is strengthened by a high *degree of locality* supporting local communication structures.

In Chapter 5 I conducted similar experiments, but on more realistic network structures: so-called small-world networks. Again, I wanted to analyze the circumstances for the cause of regional meaning, but the focus here was the way in which specific *structural network properties* and *agent types* are involved in particular phases of the developmental process of the emergence of regional meaning. The general results underlined the results of Chapter 4: a local communication structure supports the emergence of regional meaning. Because of the heterogeneous network structures, concepts from network theory were applied, allowing for a more detailed analysis of structural patterns.

It turned out that specific agent types, in terms of individual network properties, undertake particular roles for the process of the emergence of regional meaning: on more locally structured small-world networks regional meaning is i) initialized by highly connected local leaders and ii) stabilized by sparsely connected locally embedded agents. On globally and hierarchically organized small-world networks, the stabilizers of regional meaning are also sparsely connected locally embedded agents, but this time the initializers are super-connected global players.

6.3 Why is the Horn Strategy Predominant?

While the *Lewis game* represents a symmetric variant of a signaling game with two equally probable information states, two messages and two interpretation states, I was also interested in how an *asymmetric* version of such a signaling game may change the path of a linguistic convention's emergence. Therefore I introduced a game that has distinct probability values and message costs. This game i) models some of the incentives behind the linguistic phenomenon known as *the division of pragmatic labor* (a.k.a. *Horn's rule*), and ii) is consequently called the *Horn game*. Much like before, the Horn game has two signaling systems. One system describes a system of communication that associates frequent states with cheaper signals, and is therefore called a Horn strategy in reference to Horn's parallel claim on the division of pragmatic labor. The other system depicts exactly the opposite behavior, thus it is called *anti-Horn strategy*.

Both possible signaling systems, Horn and anti-Horn strategy, have different manifestations of efficiency, but both ensure successful communication and a specific degree of stability. A third strategy also plays a notable role in the process of the emergence of convention: the so-called *Smolensky strategy*. Thus while it is assumed that Horn's rule is a determinative factor for the evolution of meaning, the appropriate Horn strategy seems to have with the anti-Horn and Smolensky strategy notable competitors. I was thus interested in the reasons that explain the predominance of Horn's rule. At the same time I wanted to find exactly which circumstances support the evolution of meaning according to strategies other than Horn's rule.

In Chapter 2 I gave an overview of a wide range of literature dealing with analyses of the Horn game in evolutionary game theory, in particular via the replicator dynamics. As a basic result, it turned out that at least three factors support emergence of conventions according to the Horn strategy: i) specific starting conditions, ii) a high mutation rate and iii) correlation of interactions.

In my own experiments I conducted the Horn game in combination with imitation dynamics on different network structures and analyzed how the share of initial population space, the so-called *basin of attraction*, was distributed among the Horn, anti-Horn and Smolensky strategy. The basic result was as follows: the more local the network structure, i) the lower the conformity to results of the replicator dynamics, ii) the lower the basin of attraction of Smolensky strategy, and iii) the higher the basin of attraction of anti-Horn strategy. Interestingly, the basin of attraction of the Horn strategy was (almost) completely independent of the network structure and always around 50%. Nevertheless, the results revealed some of the first hints of how the structure of society may change the results already obtained by applying evolutionary dynamics like replicator dynamics.

In Chapter 4 I was able to show that factors supporting the emergence of multiple areas of regional meaning for the Lewis game, namely an intermediate degree of flexibility and a high degree of locality, also support the emergence of multiple regions for the Horn game. Such a resulting structure is segmented in regions for Horn and anti-Horn strategy as well, where the Horn strategy players were generally superior in number. Moreover, a third factor influenced the emergence of regional meaning: the *strength of Horn parameters* that delineated the differences between probability and message cost values in a Horn game. The experiments revealed that the stronger the Horn parameters are, the lower the probability that regional meaning evolves is. In cases where no regional meaning emerges, the whole society behaves according to Horn's rule. All in all, the results showed that circumstances supporting the emergence of regional meaning support the emergence of conventions according to the anti-Horn strategy.

The small-world network experiments of Chapter 5 revealed similar results: a more locally dense network supports the emergence of regional meaning and therefore conventions according to the anti-Horn strategy. Interestingly, in almost every experiment of Chapter 4 and Chapter 5, behavior according to the Smolensky strategy also evolved, but this was (almost) always temporary and in initial phases of the simulation runs. Thus, with regard to heterogeneous network structures, the anti-Horn strategy features a specific degree of stability, although its emergence and stabilization process depends on several environmental factors. The Smolensky strategy, on the other hand, is adopted all along and nearly independently of environmental factors, but it lacks stability and will therefore die out. In the end, the Horn strategy turned out to be the strategy used predominantly and, in dependence of the aforementioned factors, it was either the only society-wide strategy or at least used by the majority of the population.

6.4 Conclusion

The technical outcome of my dissertation is an extended and refined game model of signaling games concerned with the analysis of language conventions. Only this model combines features that fulfill *two essential preconditions* that were missing in Lewis's original account. These must be kept for analyzing the way linguistic meaning arises and becomes a convention without prior agreements:

1. The first precondition is that we must analyze signaling games for a whole population of agents and not only among two players of the game. Such an analysis is essential to cope with the nature of conventions that generally emerge in societies of multiple individuals. In this dissertation, I introduced *network games* and applied them on artificial and social network structures to meet this precondition.

2. The second precondition is that we must model a process of the emergence of meaning without prior agreements. In this dissertation I applied *repeated* signaling games to model a process rather than an instance. In addition, I applied *learning dynamics* that permit agents to be initially unbiased in mind and behavior. Such an initial situation is essential to eliminate any possibility that previous influences like prior agreements existed.

The analytical outcome of this dissertation is a line of experimental results that revealed particular dependencies between factors that impact on the way linguistic meaning may arise. In short, these experiments showed two things:

1. First, the *psychology* of agents, in terms of update dynamics that guide their decision making process, does not have an essential impact on the final outcome. Whether evolutionary dynamics, imitation dynamics, or learning dynamics are applied, the result is a similar process of how linguistic meaning evolve. This holds exclusively for homogeneous populations.

2. Second, the *sociology* of agents plays an essential role. The way agents are structured and arranged inside a population has an impact on the way linguistic meaning evolves that cannot be ignored. In particular, I showed that locally organized interaction structures support the emergence of multiple meanings. On top of that, agents in heterogeneous population structures initiate different types of linguistic change based on their in the social network.

The second result highlights the importance of social structure in language evolution. By understanding how agents connect with each other, we can learn something about how language change. My hope for this dissertation is that I will motivate other researchers to understand the connections between games, networks, and the fields of pragmatics, sociolinguistics, and language evolution.

Chapter 7

Zusammenfassung

Diese Doktorarbeit behandelt schwerpunktmäßig den Entstehungsprozess von Konventionen und diesbezüglich die Suche nach einer Antwort auf eine Frage, die sich bereits Lewis (1969) stellte: *Wie kann sprachliche Bedeutung entstehen und zur Konvention werden, wenn davon auszugehen ist, dass es im Vorfeld keine Absprache gab, die diese Konvention festlegte?* Denn von der Annahme ausgehend, dass Absprache Sprache verlangt, würde es zu einem Paradoxon führen, Absprachen als Anfangspunkte der Sprachevolution zu verorten. Lewis ebnete den Weg, dieses Paradoxon aufzulösen, indem er mithilfe des spieltheoretischen Modells des *Signalspiels* aufzeigte, wie konventionelle sprachliche Bedeutung ohne Absprachen entstehen kann.

In dieser Dissertation entwickelte ich Lewis' Modell weiter, um detailliertere Einsichten zu bekommen, welche Faktoren den Entstehungsprozess von Konventionen in vielschichtiger Hinsicht beeinflussen können. Das technische Resultat ist ein erweitertes und verfeinertes spieltheoretisches Modell des *Signalspieles.* Mein Modell vereinigt Eigenschaften, welche zwei Bedingungen erfüllen, die im Bezug auf den Entstehungsprozess von Sprachkonventionen essentiell sind:

- Die erste Bedingung lautet: *Signalspiele sollten auf Populationen angewendet werden, da es der Natur der Sprachkonvention entspricht, in solchen zu entstehen.* Daher überführte ich die Standard-2-Spieler-Signalspiele in Netzwerkspiele, um Simulationsläufe auf künstlichen sozialen Netzwerkstrukturen durchzuführen.

- Die zweite Bedingung lautet: *Konventionen müssen in einem Prozess* *entstehen, der vorherige Absprachen vollständig ausschließt.* Um einen *Prozess* zu simulieren, verwendete ich in meinen Experimenten *wiederholte Signalspiele* in Kombination mit *Lerndynamiken.* Um eine Tendenz möglicher vorheriger Absprachen auszuschließen, startete ich die Simulationen mit Agenten, die anfänglich völlig gleichgültig hinsichtlich ihrer Präferenzen waren und ihre erste Entscheidung zufällig trafen.

Das Ergebnis meiner Experimente und Analysen ist eine Entschlüsselung von Zusammenhängen, die den Entstehungsprozess von Sprachkonventionen maßgeblich beeinflussen. Die Experimente zeigten Folgendes:

- Die *Psychologie* der Agenten in Form von Update-Dynamiken, die zur Entscheidungsfindung beitragen, spielt keine große Rolle im Entstehungsprozess von Sprachkonventionen: Experimente mit Update- Dynamiken aus verschiedenen Feldern, wie etwa Evolutionsdynamiken, Imitationsdynamiken und Lerndynamiken, zeigten keine maßgeblichen Unterschiede in den Ergebnissen hinsichtlich des Entstehungsprozesses. Allerdings galt diese Beobachtung nur für Experimente auf homogenen Populationsstrukturen.

- Die *Soziologie* der Agenten in Form ihrer Populationsstruktur dagegen spielt eine sehr wichtige Rolle im Entstehungsprozess von Sprachkonventionen: Es zeigte sich in Experimenten auf Torus-förmigen *Gitternetzwerken* und sogenannten *Small-World*-Netzwerken, dass lokale Interaktionsstrukturen die Entstehung regionaler Sprachkonventionen unterstützen. Des Weiteren nahmen Agenten unterschiedlich Rollen und Aufgaben im Entstehungsprozess von Sprachkonventionen ein, die im hohen Maße abhängig von der individuellen Position im Netzwerk waren.

Das zweite Ergebnis unterstreicht die Wichtigkeit von sozialer Struktur in Bereichen wie Sprachwandel und Sprachevolution. Um besser zu verstehen, wie Sprache entstanden ist, lohnt es sich, einen Blick darauf zu werfen, wie Gesellschaften strukturiert sind. Ich hoffe mit dieser Dissertation tiefere Erkenntnisse über die Zusammenhänge a) von Methoden aus Spieltheorie und Netzwerktheorie und b) zwischen verschiedenen Feldern, wie Pragmatik, Soziolinguistik und Sprachevolution aufzuzeigen.

Bibliography

Alexander, Jason McKenzie. Evolutionary game theory. In Zalta, Edward N., editor, *The Stanford Encyclopedia of Philosophy*. Fall 2009 edition, 2009.

Argiento, Raffaele; Pemantle, Robin; Skyrms, Brian, and Volkov, Stanislav. Learning to signal: Analysis of a micro-level reinforcement model. *Stochastic Processes and their Applications*, 119:373–390, 2009.

Aristotle. De interpretatione. In *The Complete Works of Aristotle*. Princeton University Press, 1984.

Barabási, Albert-László. *Linked: the New Science of Networks*. Perseus Publishing, 2002.

Barabási, Albert-László and Réka, Albert. Emergence of scaling in random networks. *Science*, 286:509–512, 1999.

Barrett, Jeffrey A. Numerical simulations of the lewis signaling game: Learning strategies, pooling equilibria, and the evolution of grammar. Technical report, University of California, 2006.

Barrett, Jeffrey A. and Zollman, Kevin J. S. The role of forgetting in the evolution and learning of language. *Journal of Experimental and Theoretical Artificial Intelligence*, 21(4):293–309, 2009.

Beggs, Alan. On the convergence of reinforcement learning. *Journal of Economic Theory*, 122:1–36, 2005.

Benz, Anton; Jäger, Gerhard, and van Rooij, Robert. An introduction to game theory for linguists. In *Game Theory and Pragmatics*, pages 1–82. Macmillan Publishers Limited, 2005.

Björnstedt, Johan and Weibull, Jörgen. Nash equilibrium and evolution by imitation. In *The Rational Foundations of Economic Behaviour*, pages 135–171. Macmillan Publishers Limited, 1996.

Blume, Andreas; Kim, Yong-Gwan, and Sobel, Joel. Evolutionary stability in games of communication. *Games and Economic Behavior*, 5:547–575, 1993.

Bomze, Immanuel M. Non-cooperative two-person games in biology: a classification. *International Journal of Game Theory*, 15(1):31–57, 1986.

Börgers, Tilman and Sarin, Rajiv. Learning through reinforcement and replicator dynamics. *Journal of Economic Theory*, 77(1):1–14, 1997.

Brown, George W. Iterative solution of games by fictitious play. *Activity Analysis of Production and Allocation*, pages 374–376, 1951.

Chambers, Jack K. Patterns of variation including change. In Chambers, J. K.; Trudgill, P., and Schilling-Estes, N., editors, *The Handbook of Language Variation and Change*, pages 349–372. Blackwell, 2004.

Cho, In-Koo and Kreps, David M. Signaling games and stable equilibria. *The Quarterly Journal of Economics*, 102(2):179–222, 1987.

Croft, William. *Explaining Language Change: An Evolutionary Approach.* Longman, 2000.

David, Herbert A. and Gunnink, Jason L. The paired t test under artificial pairing. *The American Statistician*, 51(1):9–12, 1997.

Dawkins, Richard. *The Selfish Gene.* Oxford University Press, 1976.

de Jaegher, Kris. The evolution of Horn's rule. *Journal of Economic Methodology*, 15(3):275–284, 2008.

Ellison, Glenn and Fudenberg, Drew. Word-of-mouth communication and social learning. *The Quarterly Journal of Economics*, 110(1):93–125, 1995.

Ely, Jeffrey C. and Sandholm, William H. Evolution in Bayesian games I: Theory. *Games and Economic Behavior*, 53(1):83–109, 2005.

Fagyal, Zsuzsanna; Swarup, Samarth; Escobar, Anna Maria; Gasser, Les, and Lakkaraju, Kiran. Center and peripheries: Network roles in language change. *Lingua*, 120:2061–2079, 2010.

Franke, Michael. *Signal to Act: Game Theory in Pragmatics.* PhD thesis, Universiteit van Amsterdam, 2009.

Grafen, Alan. Biological signals as handicaps. *Journal of Theoretical Biology*, 144:517–546, 1990.

Harms, William F. *Information and Meaning in Evolutionary Processes.* Cambridge University Press, 2004.

Helbing, Dirk. A stochastic behavioral model and a 'microscopic' foundation of evolutionary game theory. *Theory and Decision*, 40(2):149–179, 1996.

Hofbauer, Josef and Huttegger, Simon M. Selection-mutatuion dynamics of signaling games with two signals. *Proceedings of the ESSLLI 2007 Workshop on Language, Games, and Evolution*, 2007.

Holme, Petter and Kin, Beom Jun. Growing scale-free networks with tunable clustering. *Physical Review E*, 65(2):026107–1–026107–4, 2002.

Hopkins, Ed and Posch, Martin. Attainability of boundary points under reinforcement learning. *Games and Economic Behavior*, 53:110–125, 2005.

Horn, Larry. Towards a new taxonomy of pragmatic inference: Q-based and R-based implicature. In *Meaning, Form, and Use in Context: Linguistic Applications*, pages 11–42. Edited by Deborah Schiffrin, Washington: Georgetown University Press, 1984.

Huttegger, Simon M. Evolution and the explanation of meaning. *Philosophy of Science*, 74(1):1–27, 2007.

Huttegger, Simon M. and Zollman, Kevin J. S. Signaling games: dynamics of evolution and learning. In Benz, Anton; Ebert, Christian; Jäger, Gerhard, and van Rooij, Robert, editors, *Language, games, and evolution*, pages 160–176. Springer-Verlag, 2011.

Jackson, Matthew O. *Social and Economic Networks.* Princeton University Press, 2008.

Jäger, Gerhard. Evolutionary game theory for linguists. a primer. Technical report, Stanford University and University of Potsdam, 2004.

Jäger, Gerhard. Evolutionary stability conditions for signaling games with costly signals. *Journal of Theoretical Biology*, pages 131–141, 2008.

Ke, Jinyun; Gong, Tao, and Wang, William S-Y. Language change and social networks. *Communications in Computational Physics*, 3(4):935–949, 2008.

Labov, William. *Principles Of Linguistic Change: Internal Factors*. Blackwell, 1994.

Labov, William. *Principles Of Linguistic Change: Social Factors*. Blackwell, 2001.

Lentz, Tom and Blutner, Reinhard. Signalling games: Evolutionary convergence on optimality. In *Papers on Pragmasemantics*, pages 95–110. ZAS (Zentrum für Allgemeine Sprachwissenschaft), 2009.

Lewis, David. *Convention. A philosophical study*. Havard University Press, 1969.

Longfellow, Henry Wadsworth. Paul Revere's ride. In *Tales of a Wayside Inn*, pages 18–25. Ticknor and Fields, 1863.

Malawski, Marcin. *Some Learning Processes in Population Games*. Instytut Podstaw Informatyki Polskiej Akademii Nauk, 1990.

Marx, Karl. *Grundrisse: Foundations of the Critique of Political Economy*. Penguin UK, 2005.

Maynard Smith, John. *Evolution and the Theory of Games*. Cambridge University Press, 1982.

Maynard Smith, John and Price, George. The logic of animal conflict. *Nature*, (146):15–18, 1973.

Mendoza-Denton, Norma. *Home Girls*. Blackwell, 2008.

Milroy, Lesley and Margrain, Sue. Vernacular language loyalty and social network. *Language in Society*, 9:43–70, 1980.

Mühlenbernd, Roland. Learning with neighbours. *Synthese*, 183(S1):87–109, 2011.

Mühlenbernd, Roland and Franke, Michael. Signaling conventions: who learns what where and when in a social network. In *Proceedings of EvoLang IX*, pages 242–249, 2012a.

Mühlenbernd, Roland and Franke, Michael. Simulating the emergence of conventions in small-world networks. In *Proceedings of the 21st Annual Conference on Behavior Representation in Modeling and Simulation 2012*, pages 37–43, 2012b.

Mühlenbernd, Roland and Franke, Michael. Signaling conventions in small-world networks. In *Pre-Proceedings of Linguistic Evidence 12*, pages 283–288, 2012c.

Myerson, Roger B. *Game Theory: Analysis of Conflict*. Harvard University Press, 1991.

Nettle, Daniel. Using social impact theory to simulate language change. *Lingua*, 108(2–3):95–117, 1999.

Nowak, Martin A.; Komarova, Natalia L., and Niyogi, Partha. Computational and evolutionary aspects of language. *Nature*, 417:611–617, 2002.

Quine, Willard Van Orman. Truth by convention. In *The Ways of Paradox and other Essays*, pages 70–99. 1966.

Roth, Alvin E. and Erev, Ido. Learning in extensive-form games: Experimental data and simple dynamic models in the intermediate term. *Games and Economic Behaviour*, 8:164–212, 1995.

Russell, Bertrand. *The Analysis of Mind*. Unwin Brothers Ltd, 1921.

Schaden, Gerhard. Say hello to markedness. In *Proceedings of DEAL II*, pages 73–97, 2008.

Schlag, Karl H. Why imitate, and if so, how? *Journal of Economic Theory*, 78(1):130–156, 1998.

Schuster, Peter and Sigmund, Karl. Replicator dynamics. *Journal of Theoretical Biology*, 100:535–538, 1983.

Selten, Reinhard. A note on evolutionarily stable strategies in asymmetric animal conflicts. *Journal of Theoretical Biology*, 84:93–101, 1980.

Skyrms, Brian. Darwin meets the logic of decision: Correlation in evolutionary game theory. *Philosophy of Science*, 61(4):503–528, 1994.

Skyrms, Brian. *Evolution of the Social Contract*. Cambridge University Press, 1996.

Skyrms, Brian. Stability and explanatory significance of some simple evolutionary models. *Philosophy of Science*, 67(1):94–113, 2000.

Skyrms, Brian. *Signals: Evolution, Learning and Information*. Oxford University Press, 2010.

Smith, Adam. Considerations concerning the first formation of languages. *Reprinted (2000) in: The Theory of Moral Sentiments*, pages 505–538, 1761.

Spence, Michael. Job market signaling. *Quarterly Journal of Economics*, 87(3):355–374, 1973.

Taylor, Peter D. and Jonker, Leo B. Evolutionarily stable strategies and game dynamics. *Mathematical Biosciences*, 40:145–156, 1978.

Tesar, Bruce and Smolensky, Paul. Learnability in optimality theory. *Linguistic Inquiry*, 29:229–268, 1998.

van Rooij, Robert. Signaling games select Horn strategies. *Linguistics and Philosophy*, 27:493–527, 2004.

van Rooij, Robert. Games and quantity implicatures. *Journal of Economic Methodology*, 15(3):261–274, 2008.

Vanderschraaf, Peter. Knowledge, equilibrium and convention. *Erkenntnis*, 49:337–369, 1998.

Vega-Redondo, Fernando. *Evolution, Games, and Economic Behaviour*. The MIT Press, 1996.

Wagner, Elliott. Communication and structured correlation. *Erkenntnis*, 71(3):377–393, 2009.

Wärneryd, Karl. Cheap talk, coordination, and evolutionary stability. *Games and Economic Behaviore*, 5:532–546, 1993.

Watts, Duncan J. and Strogatz, Steven H. Collective dynamics of small-world networks. *Nature*, 393:440–442, 1998.

Weinreich, Uriel; Labov, William, and Herzog, Marvin. Empirical foundations for a theory of language change. In *Directions for Historical Linguistics*, pages 95–195. Austin: University of Texas Press, 1968.

Wolfram, Walt and Schilling-Estes, Natalie. Dialectology and linguistic diffusion. In *The Handbook of Historical Linguistics*, pages 509–528. Blackwell, 2003.

Young, H. Peyton. *Individual Strategy and Social Structure: An Evolutionary Theory of Institutions*. Princeton University Press, 1998.

Zeeman, Erik C. Population dynamics from game theory. In *Global Theory of Dynamical Systems: Proceedings of an International Conference Held at Northwestern University*, pages 471–497, 1980.

Zeevat, Henk and Jäger, Gerhard. A reinterpretation of syntactic alignment. In *Proceedings og the Forth Tbilisi Symposium on Language, Logic and Computation*, pages 1–15. 2002.

Zimmerman, Donald W. A note on interpretation of the paired-samples t test. *Journal of Educational and Behavioral Statistics*, 22(3):349–360, 1997.

Zipf, George K. *Human behavior and the principle of least effort*. Addison-Wesley, 1949.

Zollman, Kevin J. S. Talking to neighbors: The evolution of regional meaning. *Philosophy of Science*, 72(1):69–85, 2005.

Appendices

Appendix A

Proofs

Proof A.1. *For the decision algorithm* weighted edge *(WE) by choosing an edge $\{i, j\} \in E$ of a network $G = (N, E)$ with probability $\frac{4}{|N| \times (d(i)+d(j))}$, the probability for a node i and therefire agent z_i to be chosen as sender or receiver is $\frac{1}{|N|}$ and therefore equiprobable for each node.*

Proof. First of all note that the probability of choosing a node i by a randomly chosen edge $\{i, j\}$ depends on the degree and is therefore:

$$Pr(i) = \frac{d(i)}{2 \times |E|}$$

Furthermore by reconsidering that an edge $\{i, j\}$ has two nodes and each node is chosen equiprobable either as sender S or receiver R, we can write:

$$\begin{aligned} Pr(z_i = S) = Pr(z_i = R) = \; & \frac{1}{2} \times \left(\frac{d(i)}{2 \times |E|} + \frac{d(j)}{2 \times |E|} \right) \\ & \frac{1}{2} \times \frac{d(i) + d(j)}{2 \times |E|} \\ & \frac{d(i) + d(j)}{4 \times |E|} \\ & \frac{1}{E} \times \frac{d(i) + d(j)}{4} \end{aligned}$$

Now by reconsidering that the WE-algorithm does not choose an edge equiprobably randomly, but with probability $\frac{4}{|N| \times (d(i)+d(j))}$, we just have to substitute $\frac{1}{E}$ with this probability and thus get:

$$\begin{aligned} Pr(z_i = S) = Pr(z_i = R) &= \frac{4}{|N| \times (d(i) + d(j))} \times \frac{d(i) + d(j)}{4} \\ &= \frac{4 \times (d(i) + d(j))}{|N| \times (d(i) + d(j)) \times 4} \\ &= \frac{1}{|N|} \end{aligned} \tag{A.1}$$

□

Proof A.2. *For a toroid lattice of* $n \times n$ *agents and the predefined probability function of communication partner allocation* $Pr(x, y)_\gamma$ *the probability of agents* x *to be allocated with any agent* $y \in Y = X \setminus \{x\}$ *is equiprobable among* Y *iff* $\gamma = 0$ *and therefore:* $\forall x, y \in X : Pr(x, y)_0 = \frac{1}{|Y|} = \frac{1}{n^2-1}$.

Proof.

$$\begin{aligned} Pr(x, y)_0 &= P_0(d) \times \frac{1}{8 \times d} \\ &= \frac{8 \times \frac{d}{d^0}}{\eta(0, d)} \times \frac{1}{8 \times d} && \text{(see Definition 3.31)} \\ &= \frac{8 \times d}{\eta(0, d)} \times \frac{1}{8 \times d} \\ &= \frac{1}{\eta(0, d)} \\ &= \frac{1}{n^2 - 1} && (\forall d : \eta(0, d) = n^2 - 1, \text{ see Proof A.4}) \end{aligned} \tag{A.2}$$

□

Proof A.3. *For a toroid lattice of* 21×21 *agents and the predefined probability function of communication partner allocation* $P_\gamma(d)$ *the probability of agents an* x *to be allocated with any direct neighbor* $y \in |N_1(x)|$ *is* > 0.99, *if* $\gamma = 8$*; and therefore the interaction structure is close to neighborhood communication.*

Proof. $\forall x \in X : \forall y \in N_1(x), n = 21 :$

$$\begin{aligned}
P_8(1) &= \frac{8 \times \frac{1}{1^0}}{\eta(8,1)} \\
&= \frac{8}{\eta(8,1)} \\
&= \frac{8}{\left(\sum_{d=1}^{\lfloor 21/2 \rfloor} 8 \times \frac{d}{d^8}\right) + \left(\lfloor \frac{21+1}{2} \rfloor - \frac{21+1}{2}\right) \times (4 \times 21 + 2)} \\
&= \frac{8}{8 \times \left(\sum_{d=1}^{10} \frac{1}{d^7}\right) + (11 - 11) \times 86} \\
&= \frac{1}{\sum_{d=1}^{10} \frac{1}{d^7}} \\
&= \frac{1}{1.008349155} \\
&= 0.99172
\end{aligned} \tag{A.3}$$

□

Proof A.4. *Given is a $n \times n$ social map. If the degree of locality γ is 0, then η (the denominator and normalizer of P_γ) is $n^2 - 1$ for all distances $d \leq \lfloor n/2 \rfloor$, thus: $\forall d : \eta(0, d) = n^2 - 1$*

For this proof it is necessary to distinguish between n is even or odd. Remark: if n is even: $\lfloor n/2 \rfloor = n/2$, if n is odd: $\lfloor n/2 \rfloor = (n-1)/2$.

Proof. a) n is even

$$\begin{aligned}
\eta(0,d) = & \left(\sum_{d=1}^{\lfloor n/2 \rfloor} 8 \times d/d^0\right) + \left(\left\lfloor \frac{n+1}{2} \right\rfloor - \frac{n+1}{2}\right) \times (4n+2) \\
= & \ 8 \times \left(\sum_{d=1}^{\lfloor n/2 \rfloor} d\right) + \left(\left\lfloor \frac{n+1}{2} \right\rfloor - \frac{n+1}{2}\right) \times (4n+2) \\
= & \ 8 \times \left(\frac{\lfloor n/2 \rfloor}{2} \times (\lfloor n/2 \rfloor + 1)\right) + \left(\frac{n}{2} - \frac{n+1}{2}\right) \times (4n+2) \\
= & \ 8 \times \left(\frac{n}{4} \times \left(\frac{n}{2} + 1\right)\right) + \left(-\frac{1}{2}\right) \times (4n+2) \\
= & \ 8 \times \left(\frac{n^2}{8} + \frac{n}{4}\right) - 2n - 1 \\
= & \ n^2 + 2n - 2n - 1 \\
= & \ n^2 - 1
\end{aligned}$$

□

Proof. b) n is odd

$$\begin{aligned}
\eta(0,d) = & \left(\sum_{d=1}^{\lfloor n/2 \rfloor} 8 \times d/d^0\right) + \left(\left\lfloor \frac{n+1}{2} \right\rfloor - \frac{n+1}{2}\right) \times (4n+2) \\
= & \ 8 \times \left(\sum_{d=1}^{\lfloor n/2 \rfloor} d\right) + \left(\left\lfloor \frac{n+1}{2} \right\rfloor - \frac{n+1}{2}\right) \times (4n+2) \\
= & \ 8 \times \left(\frac{\lfloor n/2 \rfloor}{2} \times (\lfloor n/2 \rfloor + 1)\right) + \left(\frac{n+1}{2} - \frac{n+1}{2}\right) \times (4n+2) \\
= & \ 8 \times \left(\frac{n-1}{4} \times \left(\frac{n-1}{2} + 1\right)\right) + 0 \times (4n+2) \\
= & \ 8 \times \left(\frac{n^2-2n+1}{8} + \frac{n-1}{4}\right) \\
= & \ n^2 - 2n + 1 + 2n - 2 \\
= & \ n^2 - 1
\end{aligned}$$

□