Globalisation and its Effect on Inequality and Labour Markets

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David Christopher Kurfess
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Dekan:	Prof. Dr. rer. soc. Josef Schmid
1. Gutachterin:	Prof. Dr. oec. publ. Valeria Merlo
2. Gutachter:	Prof. Dr. rer. soc. oec. Wilhelm Kohler

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Chapter 1

Introduction

It is common understanding that production chains, goods and services markets as well as human movement, institutions and political relations have become more and more interlinked in the last decades (Dreher et al., 2008). This did neither go unnoticed by the public and political debate nor the academic discussion. There is no singular definition of globalisation and it can be understood as all of the different things mentioned above and more. Following the approach taken by many economists this dissertation understands the concept of globalisation mainly as an increasing integration of international markets, a reduction in trade cost and rising linkages of production chains across countries (Friedman, 1999).

For many researchers and politicians alike, particularly in the field of economics, there is little to no doubt that globalisation is beneficial on a global level (Bhagwati, 2004). This consensus is reflected in many bilateral and multilateral trade deals. The Regional Comprehensive Economic Partnership (RCEP), signed in November 2020, which covers almost 30% of GDP and around one third of the world population, is the latest trade agreement. At the same time, not only since the era of Donald Trump and its controversial trade and migration policies, it is clear that globalisation comes with winners and losers and affects inequality.[1] In particular, labour market consequences in terms of employment and wages are one of the important topics in this field and are of special interest in the public debate.[2] It is also evident that the rising linkages within and across nations imply that national policy no longer only affects the national actors, but has international consequences because cross country productions chains imply spillover effects.

[1]See Felbermayr et al. (2020) for a recent study of the effect of GATT/WTO membership. They find that trade among member countries is larger and country-specific estimates vary widely across the countries.

[2]Goldberg and Pavcnik (2004), Egger and Kreickemeier (2009), Helpman and Itskhok (2010), Davis and Harrigan (2011) and Helpman et al. (2016), are only some of the academic contributions highlighting both theoretically and empirically the heterogeneous effects of trade for the different labour market variables.

This dissertation consists of three self-contained essays and contributes to the discussion on heterogeneous effects of globalisation. Chapter 2 and 3 highlight how differences on the labour market imply heterogeneous effects when markets and countries become more integrated. Chapter 2 and 4 emphasis heterogeneous regional implications of a globalised world. Chapter 2 stresses theoretically as well as empirically that at the firm level the employment effects of trade liberalisation can depend on the labour market situation of the regional labour market a firm is situated in. As such a national trade liberalisation can have regionally different consequences, since regions differ in terms of the labour market situation. Chapter 3 emphasises the possibility of firms to influence their bargaining power in the intra firm wage bargaining process and highlights the interplay of both the export and the bargaining power improvement decision of a firm. Chapter 4 takes a somewhat different approach, which ties in with the general theme of globalisation and inequality nonetheless. Calibrating a general equilibrium, spatial quantitative model Chapter 4 studies the region and sector specific effects of changes in national corporate tax policies in a globalised world. Thereby it is emphasised how the linkages in the production structure as well as sectoral and regional differences imply spillovers and heterogeneous effects of a homogeneous national policy. A more detailed chapter description is provided in the following.

Chapter 2, titled *The Employment Effects of Trade Liberalisation and the Importance of the Labour Market Situation* is a joint work with Tobias Brändle. The focus of this study is to emphasise that regions might differ in terms of their regional labour market. This in turn implies that globalisation, more precisely trade liberalisations, can have different effects on firms and the number of workers employment dependent on the regional labour market and the respective labour market situation a firm faces. A theoretical model based on Helpman and Itskhok (2010) and Helpman et al. (2010), which features three channels, how trade liberalisations can affect the employment of firms, is used. Firstly, there is a positive market access effect of trade liberalisation which is only present for exporting firms. The second, negative competitition effect relates to the increased competition for labour due to trade liberalisation. Those two effects are also present in a standard Melitz (2003) framework. Due to labour market imperfections in form of search frictions a third channel exists. Trade liberalisations increase the labour market tightness, from a firm's perspective, and thus it is more expensive to find workers. This third effect, which is negative, we call labour market effect of trade liberalisation and its extent depends on the labour market situation which can differ across different regional labour markets. This chapter contributes to the literature by examining the interaction between trade liberalisations and the labour market situation. A better labour market situation will decrease the absolute size of the negative labour market effect of trade liberalisation. Differentiating between firms that export and those that do not, the negative employment effect for domestic firms if trade is liberalised is dampened if they face a better labour market situation. Exporting firms will hire relatively more workers due to trade liberalisations if they face a better labour market situation. Therefore, depending on the export status of firms and depending on the labour market situation they face, the framework predicts heterogeneous employment effects of trade liberalisations. To test our theoretical predictions we make use of the German linked employer-employee data (LIAB) for the years 1996 to 2010. We augment our data with industry export shares from the World Input-Output Database (WIOD) and regional information on regional labour markets. We estimate a panel fixed-effects-model and show that trade liberalisation has a positive employment effect on the exporting firms. We can also show that the regional unemployment rate has a negative effect on firm-level employment, especially for domestic firms. Furthermore, regarding the predictions of our model, we find some support that the effects of trade liberalisation depend on the regional labour market conditions. Firms in regional labour markets with a high regional unemployment rate seem to benefit more from export liberalisation, the effects are insignificant though.

Chapter 3, titled *Wage Bargaining Improvements and the Export Decision*, is single-authored. This chapter highlights the interplay of a firm's export decision and its bargaining improvement decision in the intra firm bargaining process. The chapter develops a theoretical model of international trade with labour market imperfection, ex ante heterogeneous workers and two occupation types differing in the way firms can influence their bargaining position in the wage bargaining process. The developed framework contributes to the literature by introducing the endogenous possibility for firms to influence their bargaining power with respect to workers in the wage bargaining process. This implies that firms have an additional extensive decision to make. Firms cannot only differ in their export decision but also in their decision whether to improve their bargaining power or not. In particular it is shown that firms will decide to export and/or improve their bargaining power based on their respective productivity. The most productive firms will export and improve their bargaining power, while the least productive firms do not improve their bargaining power and only sell domestically. The framework also allows for the existence of an intermediate productivity range, in which firms either export or improve their bargaining power without doing the other. Firms that export and, or improve their bargaining power choose workers with on average higher ability, sample and hire more workers, and generate higher revenues and profits. The theoretical framework predicts that the possibility and the amount of improvement in the bargaining power can rise the share of exporting firms. At the same time, trade liberalisations can increase the share of firms improving their bargaining power. As such, this suggests a new additional channel how trade liberalisations can affect within group wage inequalities. The chapter therefore adds to the understanding of several empirical studies which find that within group wage inequality rises in the context of trade liberalisations (Attanasio et al., 2004; Menezes-Filho et al., 2008; Song et al., 2018). Furthermore, the theoretical framework allows for heterogeneity in terms of the possible bargaining improvement across occupations. This implies heterogeneous effects of both globalisation and bargaining improvement across occupations. The chapter considers a situation where only the bargaining power for one of the two occupations can be influenced by the firm. In this case, the share of overall revenues belonging to the workers in the occupation facing the bargaining power improvement of the firm falls. The effect on overall wage income for this worker type depends on whether the falling wage share or the effect of the rising overall revenues dominates. At the same time, the overall wage income of workers in the occupation facing no bargaining power change rises due to the possibility of improving the bargaining power, both in absolute terms and relative to overall wage income of the other worker type.

Chapter 4 is co-authored by Peter Egger, Oliver Krebs, Valeria Merlo and Georg Wamser and titled *Regional Implications of National Tax Policy*. The chapter studies the region and sector specific effects of changes in national corporate tax policies. We are the first to do so in a new, general equilibrium, spatial quantitative model that features heterogeneous responses to national policies due to region and sector specific production structures, including varying usage of deductible capital asset types, and spillovers through a full set of input-output relations and mobility. Calibrating this model based on a unique collection of data sets for 12 sectors across 1306 European NUTS3 regions, we find that there is substantial heterogeneity in local responses to national tax policy, which is driven by the different production structures and linkages. Specifically, across the EU, the regional real consumption response to a one percentage point increase of the respective country's national tax rate ranges from -0.08% to 0.06%. Geographically, the most adverse effects are felt in regions that are the nations' manufacturing centres, such as the north of Italy, the north of Spain, German car manufacturing regions or the areas around Rotterdam and Amsterdam. Less productive regions benefit from higher redistribution of national tax income. Varying dependence on endowment with different capital asset types as well as differences in their deductibility also have a large influence on the derived heterogeneity. With respect to two prominently discussed tax policies, the adoption of a common EU corporate tax and capital asset deduction scheme as well as the introduction of a cash-flow taxation in which capital assets are fully deductible, we find that the former has a slight welfare increasing effect, whereas the latter leads to welfare losses. In both cases, however, heterogeneities across regions are very strong. Overall our results clearly point to the importance of considering regions when evaluating the effects of national tax policy. The derived strong heterogeneities are of vital importance for policy makers and understanding the underlying mechanisms for the varying responses is crucial for economists trying to project the effects of tax policies.

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Chapter 2

The Employment Effects of Trade Liberalisation and the Importance of the Labour Market Situation*

Abstract: *It has long been shown that trade liberalisation has important employment effects. These effects can differ not only between, but also within countries. Some regions inside a country might benefit more, depending on, e.g. industry composition. We show theoretically as well as empirically that the employment effects of trade liberalisation can depend on the labour market situation of the regional labour market a firm is situated in. Our results can be used to analyse the heterogeneous effect of trade liberalisation across labour markets and across firms.*

*This chapter is based on joint work with Tobias Brändle.

2.1 Introduction

Motivation. The increase in international trade, both on the final product and on the input or intermediate product level, has shaped markets across the world, especially during the last 25 years. Among the consequences caused by this development, labour market effects are among the most important ones discussed in the public and in politics. The disappearance of U.S. manufacturing jobs has, for instance, played a large role in the last presidential elections (Autor et al., 2013; Freund and Sidhu, 2017). Similarly, the fact that almost 30% of German jobs depend on exports (Aichele et al., 2013; BMWi, 2019; iwd, 2019) shapes the political discussion in Germany, and in Europe as a whole (The Economist, 2017).

For a long time the international trade literature did not consider employment effects or the observed rise in inequality, both between and within countries. The reason for rising inequality was detected in skill biased technological change (Acemoglu and Autor, 2011). This view has crucially changed in the last fifteen years. Introducing firm heterogeneity (Melitz, 2003) has allowed to capture heterogeneous export, wage and employment effects of exporting activity across firms (sectors). Developing models of intermediate input sourcing has further allowed to compare heterogeneous effects between local producers and sourcing firms. Furthermore, the blindfolded view of perfect labour markets has changed and imperfections and unemployment are considered. Different approaches modelling labour market imperfections have been established, each suggesting heterogeneous labour market effects from international trade.

Most theoretical models nowadays contain firms which differ in terms of their productivity and, as a result, in terms of their trading behaviour. Firms are, however, not only heterogeneous in terms of productivity, but they also differ in terms of their location. This implies that their regional labour market conditions in which they operate differ. This heterogeneity combined with some degree of immobility of the workforce creates a monopsony situation (Manning, 2003), where firms react differently to trade shocks depending on their location, i.e. labour market situation.

Several studies have recently analysed the employment effects of trade, see Capuano and Schmerer (2015) for an overview. While most of these studies acknowledge firm heterogeneity, equilibrium unemployment, they usually do not consider regional components. To elaborate our contribution, we think of location as being part of a regional labour market or commuting zone, where individuals are quite mobile within the boundaries, but not between them (Autor et al., 2013). Comparing the labour market situations of different locations within a country, there are usually wide differences in terms of workforce composition, sector composition, childcare services and several other factors influencing labour supply and demand. As a consequence, regional labour markets differ vastly in terms of their equilibrium unemployment rates. In Germany, for example, unemployment rates in 2017 differ between 1.4% in the county of Eichstätt (Southern Bavaria) and up to 12.6% or 13.9% in the cities of Bremerhaven (North Sea coast) or Gelsenkirchen (Ruhrgebiet), or 11.6% in the rural area of Uckermark (northeast of Berlin). While season and business cycle effects play a major role in determining unemployment rates, local differences are persistent and are not a simple variation due to short-run imbalances (Kropp and Schwengler, 2017).[1] The argument is as follows. Even though the workforce can, in principal, travel freely within a country and firms can also chose their location, moving comes at a cost. Given the reluctance of movements between regional labour markets and the differences in terms of the regional labour market situation, the location of firms (and their potential employees) is likely to influence firm decisions. In our paper, we analyse the decision of firms of how to engage in international trade, as well as the resulting effects, depending on the local labour market situation.

Therefore, we try to determine whether firms in different regional labour markets are affected differently by international trade, depending on the regional labour market situation they face. We use a theoretical model to show how the effects from trade liberalisation can differ between labour market conditions and try to validate the theoretical predictions with help of an empirical analysis using German linked employer-employee and regional data.

Theoretical Contribution. The theoretical framework has the objective to explain the implications of different labour market situations for the employment effects firms face when trade frictions change. Instead of modelling separate regions within the different countries which differ in their labour market situation we restrict our analysis to a two country framework. Some aspects of the labour market situation in the countries are exogenous and their implications can be analysed in a comparative static analysis. We thus abstract from any interaction affects between different regions present in a geographical economy framework but are able to compare trade liberalisation effects dependent on the labour market situation a firm faces, which is the core of this analysis. The framework is based on the work by Helpman and Itskhok (2010) and the following work by Helpman et al. (2010).[2] We allow for income effects of trade liberalisation in the general equilibrium. Thus we deviate from the general equilibrium implementation of Helpman and Itskhok (2010) who use quasi linear preferences and a homogenous good sector and

[1]For example, even though the number of commuters and the distances of home-to-work travel have increased, the patterns of commuting have been mostly stable over time. In Germany, most residents belong to the same regional labour market as they did in 1993.

[2]We prefer the imperfect labour market trade model with search frictions over the fair wage approach suggested by Egger and Kreickemeier (2009) or the efficiency wages approach proposed by Davis and Harrigan (2011) as it allows a rather simple way to consider differences in the labour market situations in form of different search frictions.

use, as Helpman et al. (2010) suggest in one of their general equilibrium specifications, a single sector economy. Helpman et al. (2010) allow for a heterogenous work force which implicates wage differences across firms. In order to derive a model as simple as possible we stick with the homogenous workforce assumption of Helpman and Itskhok (2010).

Our model uses the standard Diamond-Mortensen-Pissarides search approach as proposed by Helpman and Itskhok (2010) in order to introduce labour market imperfections in a (Melitz, 2003) heterogeneous trade model. A firm has to pay search costs in order to match with workers. Search costs are a function of the endogenously determined labour market tightness as well as an exogenous labour market characteristics which we refer to as labour market situation. By setting the vacancy posting costs as well as the technology parameter to one we simplify the search and matching model used by Helpman et al. (2010) to the simplest possible framework able to capture the relevant mechanism. Due to search fictions wages paid by a firm are determined via a multilateral bargaining game. As such our theoretical contribution is to simpliyfy and combine the aproaches of Helpman and Itskhok (2010) and (Helpman et al., 2010) in order to perform a detailed analysis of the trade liberalisation and labour market effect as well the interaction of the two on the employment of firms.

Theoretical Prediction. The model features three channels through which trade liberalisations can affect the employment of firms. There is a positive market access effect which is only present for exporting firms. The second negative competitive effect relates to the increased competition for labour. Those two effects are also present in a standard (Melitz, 2003) framework. The search frictions introduce a third channel. Trade liberalisations increase the labour market tightness and thus it is more expensive to find workers. This third negative effect we call labour market effect of trade liberalisation. Overall the three effects imply that domestic firms will hire fewer workers after trade liberalisation. Different to a situation where there are no labour market imperfections the model allows for parameter constellation where the positive market access effect is dominated and the employment effect for exporting firms turns negative. In such a situation the overall employment still rises due to a larger number of more productive firms in the market but each individual firm will hire fewer workers. A key contribution of our analyses is the examination of the interaction between the trade liberalisation effect and the labour market situation. A better labour market situation will decrease the absolute size of the third negative labour market effect of trade liberalisation. The negative employment effect for domestic firms if trade is liberalised is dampened if they face a better labour market situation. Exporting firms will hire relatively more workers due to trade liberalisations if they face a better labour market situation.

Empirical Approach. To test our theoretical predictions we make use of the German linked employer-employee data (LIAB) for the years 1996 to 2010. This data is of high quality and well-established in the literature, e.g. Egger et al. (2020) have evaluated the wage effects of a foreign take-over using a dynamic treatment effects estimator. Similar to Andersson et al. (2017), who look at changes of firm level demand for labour due to imports and exports of intermediates using Swedish firm level data , we have detailed survey information on firms' employment, their export behaviour, their location, as well as other firm-specific variables and official register data on the workforce employed. For the empirical validation of our theoretical predictions we use the general argumentation that firms' changes in export status (or shares) are only a reflection of their (endogenous) reaction to trade shocks (for instance Amiti and Davis (2012); Autor et al. (2013)). We therefore augment our data with industry export shares from the World Input-Output Database (WIOD) similar to Dauth et al. (2014) and regional information on regional labour markets. We estimate a panel fixed-effects-model and show that trade liberalisation has a positive employment effect on the exporting firms. We can also show that the regional unemployment rate has a negative effect on firm-level employment, especially for domestic firms. Furthermore, regarding the predictions of our model, we find some support that the effects of trade liberalisation depend on the regional labour market conditions. Firms in regional labour markets with a high regional unemployment rate seem to benefit more from export liberalisation, the effects are insignificant though.

The reminder of this paper is organized in the following manner. First, we will introduce the theoretical model and derive the theoretical predictions in Section 2.2. In a second step, we will describe the data used and the empirical identification strategy in Section 2.3. Section 2.4 presents our empirical results. We close with a short summary with concluding comments in Section 2.5. The Appendix 2.A covers crucial proofs and additional illustrations, while a full derivation of the framework can be found in the Supplementary Appendix 2.B and the Nomenclature 2.C lists all variables and their definitions.

2.2 Model Framework

In the aim of constructing a simple analytically traceable model which allows to determine the effect of the interaction between trade liberalisation and the labour market situation on firm variables we restrict our analysis to a symmetric two country framework. Even though countries are symmetric this framework allows to derive predictions on how firms react differently dependent on the labour market situation they face. In the empirical investigation the labour market situation will differ across regional labour markets. Throughout results are depicted for the home country and an asterisk indicates when that variables refer to the foreign country.

2.2.1 Preferences

Each country is populated by L ex ante identical and risk neutral workers. A continuum of horizontally differentiated varieties ϑ is produced where $q(\vartheta)$ depicts consumption of variety ϑ. Assuming constant elasticity of substitution between varieties the real consumption index Q is defined over the set of varieties M as:

$$Q = \left[\int_0^M q(\vartheta)^\beta \, d\vartheta\right]^{1/\beta} \qquad 0 < \beta < 1, \tag{2.1}$$

where β controls the elasticity of substitution between different varieties. Within this single sector economy firms have monopoly power for their unique variety. Given the preference structure and the resulting demand, the revenue of a firm in equilibrium is given by $r(\vartheta) = Aq(\vartheta)^\beta$. Each firm is supplying one of a continuum of varieties thus they take the demand shifter $A \equiv Y^{1-\beta}P^\beta$ as given when making decisions. P denotes the price index dual to Q which we use as numéraire and normalize to one. Total expenditure of individuals in home is denoted by Y. The prices of variety ϑ is denoted by $p(\vartheta)$ and $q(\vartheta) = A^{\frac{1}{1-\beta}} p(\vartheta)^{\frac{-1}{1-\beta}}$ is the quantity sold by a firm on the domestic market.[3]

2.2.2 Labour Market

The labour market is modelled in close analogy to Helpman and Itskhok (2010). Transition form one job to another is assumed to be not free of frictions. These search fictions are modelled following the Diamond-Mortensen-Pissarides approach. A firm has to pay bh units of the numéraire in order to match with a measure h of workers. A single firm takes the measure for the search costs b as given. Search costs are increasing in the endogenously determined labour market tightness and decreasing in the exogenous labour market situation. The term labour market situation refers to how the labour market tightness translates into the search costs. Labour market tightness $x = H/L < 1$ is the ratio between the number of individuals hired in a country H and the size of potential work force L. It is defined from a firm's perspective implicating that a tighter labour market implies less potential employees for a given number of jobs. As such the unemployment rate is given by $u = 1 - x$.

Following Blanchard and Galí (2010) search costs can be derived from a constant returns to scale Cobb-Douglas search technology, including the number of vacancies reported and the number of workers searching for employment, together with costs of posting vacancies. Search costs can be written as $b = \frac{x^\alpha}{\alpha_0}$.[4] The exogenous search technology parameter $\alpha > 1$ is the search cost elasticity with respect to the labour market tightness. It is an inverse measure of the importance of vacancies in the search technology. We assume that

[3] Detailed derivation of the theoretical framework can be found in the Supplementary Appendix 2.B.
[4] For a detailed derivation see the Appendix 2.A.1.

search costs are convex in the labour market tightness. Thus, the same absolute change in the labour market tightness increases the search costs more the higher the starting level of the labour market tightness. The exogenous search technology parameter α_0 is a function of the costs of posting vacancies and the technology parameter in the search function. Both parameters can be interpreted as measures for a better labour market situation where higher parameter values indicate smaller search costs. In the aim of using the simplest possible search technology able to generate interactions between trade liberalisation effects and the labour market situation we set the technology parameter $\alpha_0 = 1$.[5] This simplification does not alter the theoretical implications derived in this paper and simplifies the approach crucially. Chapter 3 considers both search technology parameters and their different effects on the firm outcomes in a more general setup with worker heterogeneity. Search costs simplify to $b = x^{\alpha}$. In terms of the terminology in this simple framework one can use labour market situation and search cost elasticity interchangeably. In a broader interpretation α can depict all those factors which change the degree to which the labour market tightness influences the search costs such as good institutions, workforce composition, sector composition, childcare services and several other factors influencing the labour market (Dengler et al., 2016).

2.2.3 Technology, Export Decision and Wage Bargaining

Production Technology. The production side and trade beween countries is modelled in analogy to Melitz (2003). A continuum of potential market entrants can pay up front entry costs f_E in order to learn about their firm specific productivity θ. We assume, as standard in this kind of literature, that productivities are independently distributed and drawn from a pereto distribution $G_{\theta}(\theta) = 1-(\theta_{min}/\theta)^z$ for $\theta \geq \theta_{min} > 0$ with $z > 1$ being the distribution shape parameter. As firms are uniquely identified by their productivity we use θ as an firm index. Domestic production involves fixed costs $f_d > 0$. Deviating from Melitz (2003) the final good is produced under diminishing returns to labour $0 < \gamma < 1$:

$$y(\theta) = \theta h^{\gamma}. \tag{2.2}$$

Export Decision. When a firm decides to export it has to pay fixed costs of exporting f_x as well as variable iceberg type trade costs τ. Firms that choose to supply both markets have to allocate their output $y(\theta)$ between the two markets. They do so by choosing the amount produced for the domestic and export market ($y_d(\theta)$ and $y_x(\theta)$) such that the marginal revenue is the same in both markets. Given the revenue equation resulting from the preference structure, revenues of demotic sales are given by $r_d(\theta) = Ay_d(\theta)^{\beta}$.

[5] If the costs of posting vacancies and the technology parameter in the search function is set to one this satisfies $\alpha_0 = 1$.

Taking into account that the quantity sold on the export market translates into the units produced for the foreign market in the following fashion $y_x(\theta) = \tau q^*(\theta)$ revenue from exporting are given by $r_x(\theta) = A^* \left(\frac{y_x}{\tau}\right)^\beta$. The equality of marginal revenues then implies the following relationship between the production for the domestic and the export market $y_x(\theta) = \tau^{-\frac{\beta}{1-\beta}} y_d(\theta)$.

Revenues. We use $I_x(\theta) = \{0, 1\}$ as a firm specific indicator variable being equal to one, if the firm exports, otherwise it is zero. Using the relationship between the production for the domestic and the export market in combination with the assumption that a firm sells all it's production $y(\theta) = y_d(\theta) + I_x(\theta) y_x(\theta)$ one can solve for total revenues of a firm as a function of total production:

$$r(\theta) \equiv r_d(\theta) + r_x(\theta) = A\theta^\beta h^{\beta\gamma} \Upsilon_x^{1-\beta}, \tag{2.3}$$

$$\text{where } \Upsilon_x(\theta) \equiv 1 + I_x(\theta)\tau^{-\beta/(1-\beta)}. \tag{2.4}$$

Firm revenues are continuous, increasing, and concave in the number of workers hired as $\beta\gamma < 1$. The variable $\Upsilon_x(\theta)$ can be interpreted as the firm's market access, which increases when trade costs are low and the firm is exporting $\frac{\partial \Upsilon_x}{\partial \tau} < 0$. We will use the following convention $\Upsilon_x \equiv \Upsilon_x(\theta)|_{I_x(\theta)=1}$ to indicate market access of an exporting firm independent of the productivity of a specific firm. For the later analysis it is also helpful to indicate that the market access is independent of the labour market situation $\frac{\partial \Upsilon_x}{\partial \alpha} = 0$ and also the effect of a change in trade costs on the market access term is not influenced by the labour market situation $\partial \left(\frac{\partial \Upsilon_x}{\partial \tau}\right) / \partial \alpha = 0$. Apart from the decision of a firm whether to export or not the market access term is a function of exogenous parameters only.[6]

Wage Bargaining. When a firm has observed its productivity, it has to choose whether or not to produce, whether or not to export, and the measure of workers to hire. The existence of search costs imply that a firm is not able to replace a worker free of cost. As such workers have a bargaining power after being hired by the firm.

The workers and the firm engage in strategic wage bargaining with equal weight in the manner as proposed by Stole and Zwiebel (1996a,b). This is a natural extension of the Nash bargaining to the case of multiple workers. A firm takes into account the feedback effect of not hiring one worker, for the wages of all other workers. When the firm and its workers bargain over the wage the different fixed costs as well as the search costs are all sunk. Thus the firm bargains bilaterally with every worker over the division of revenues. Hired worker's outside option is given by unemployment. For simplicity we set its value to zero. The bargaining game implies that in equilibrium the change in operating profits due

[6] Allowing for asymmetries implies that the market access term will also depend on the relative size of the two countries in terms of demand shifters. The Supplementary Appendix 2.B shows the functional relationship.

to a marginal change in the number of workers hired must be exactly the wage paid to the workers. The firms thus take into account how a change would effect the wage for all other workers. Solving this optimality condition $\partial\left[r(\theta,h)-w(\theta,h)h\right]/\partial h = w(\theta,h)$, the firm chooses the number of workers such that it receives the fraction $1/(1+\beta\gamma)$ of revenue,while $1 > \kappa_w \equiv \beta\gamma/(1+\beta\gamma) > 0$ can be interpreted as the share of revenues belonging to the workforce. The revenue shares are solely determined by exogenous preference and production parameters.

2.2.4 Profit Maximisation and Cutoff Productivity

Profit Maximisation. Anticipating the outcome of the bargaining game, firms maximize profits choosing whether to export or not as well as the number of workers to hire. The profit maximization problem of a firm takes the following form:

$$\max_{\substack{h(\theta)\geq 0,\\ I_x(\theta)\in\{0,1\},}} \pi(\theta) = \frac{1}{1+\beta\gamma} r(\theta) - bh(\theta) - f_d - I_x f_x. \tag{2.5}$$

Using the derived revenue equation (2.3), the profit maximization problem yields the following labour demand as a function of the revenues of a firm:

$$h(\theta) = \kappa_w \frac{r(\theta)}{b} = \kappa_w \frac{r(\theta)}{x^\alpha}. \tag{2.6}$$

As such a firm with higher revenues, facing a better labour market situation or a looser labour market ceteris paribus hires more workers. Wages are determined by the overall wage costs of a firm divided by the number of workers hired $w(\theta) = \kappa_w r(\theta)/h(\theta) = b = x^\alpha$. Firms facing a tighter labour market or a worse labour market situation ceteris paribus will pay larger wages while all firms within a country pay the same wage.[7] Using the relationship between the optimal number of workers in a firm and its revenues one can rewrite profits as follows:

$$\pi(\theta) = \kappa_f r(\theta) - f_d - I_x f_x, \tag{2.7}$$

where $\kappa_f \equiv \frac{1-\beta\gamma}{1+\beta\gamma}$ can be interpreted as the share of revenues belonging to the firm after paying wages and search costs. Solving for the amount of workers hired using the optimal revenues yields the number of workers and allows to rewrite employment and revenues in

[7]Following the approach by Helpman et al. (2010) one can introduce worker heterogeneity and a screening technology which results in wage differences across firm.

the following way:

$$h(\theta) = \left[A\kappa_w \Upsilon_x(\theta)^{1-\beta}\theta^{\beta}x^{-\alpha}\right]^{\frac{1}{1-\beta\gamma}}, \tag{2.8}$$

$$r(\theta) = \left[A\kappa_w^{\beta\gamma} \Upsilon_x(\theta)^{1-\beta}\theta^{\beta}x^{-\beta\gamma\alpha}\right]^{\frac{1}{1-\beta\gamma}}. \tag{2.9}$$

The labour market tightness x and the demand shifter A will be determined in the general equilibrium.

Firm Variables as a Function of the Cutoff Productivity. For the further analysis it is helpful to express firm variables as a function of the cutoff productivity determining the marginal firm which is indifferent between domestic production and exiting the market. Using the profit equation following from the firm optimisation (2.7) one can state that the marginal domestic firm, which earns zero profits ($\pi_d(\theta_d) = 0$) generates revenues of $r(\theta_d) = \frac{f_d}{\kappa_f}$. The export cutoff is determined by the firm which is indifferent between selling solely domestically and also supplying the export market ($\pi(\theta_x)|_{I_x=1} = \pi(\theta_x)|_{I_x=0})$). Using those two cutoff conditions allows to solve for the relationship between the two cutoffs. We assume that parameters are such that all exporting firms also sell domestically:

$$\frac{\theta_d}{\theta_x} = \left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right)^{\frac{1-\beta\gamma}{\beta}} \left(\frac{f_d}{f_x}\right)^{\frac{1-\beta\gamma}{\beta}} < 1. \tag{2.10}$$

Using the fact that the domestic cutoff firm is not exporting, allows to rewrite firm variables as a function of the domestic cutoff productivity:

$$r(\theta) = \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} \Upsilon_x(\theta)^{\frac{1-\beta}{1-\beta\gamma}}\, r(\theta_d), \tag{2.11}$$

$$h(\theta) = \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} \Upsilon_x(\theta)^{\frac{1-\beta}{1-\beta\gamma}}\, h(\theta_d) \quad \text{with} \quad h(\theta_d) = \frac{\kappa_w}{\kappa_f}\frac{f_d}{x^{\alpha}}. \tag{2.12}$$

This second equation is crucial for our analysis as it allows to determine employment responses to both changes in trade costs and in the labour market situation. Before determining the overall effect of changes in trade costs and changes in the labour market situation the cutoff productivity and the endogenous labour market tightness need to be determined.

Free Entry Condition. Free Entry implies that new firms will enter the market as long as expected profits are larger than the entry costs. As such the sum of expected domestic and expected export profits must equal the entry costs in equilibrium. It is helpful to split profits into profits that a firm would realise if it only would supply the domestic market $\pi(\theta)|_{I_X=0}$ and into profits that a firm would make selling on possibly both markets while excluding profits this firm would make if it only sold on the domestic market $\pi(\theta)|_{I_X=1} - \pi(\theta)|_{I_X=0}$. The free entry condition then takes the following form:

$$\int_{\theta_d}^{\infty} \left[\pi(\theta)|_{I_X=0}\right] \, dG_\theta(\theta) + \int_{\theta_x}^{\infty} \left[\pi(\theta)|_{I_X=1} - \pi(\theta)|_{I_X=0}\right] \, dG_\theta(\theta) = f_E. \tag{2.13}$$

Using the profit equation (2.7) together with revenues as a function of the cutoff productivity (2.11) as well as the revenues of the cutoff firm (2.B.15) and the relationship between the domestic and export cutoff (2.10) the free entry condition is given by the following expression:

$$f_d \int_{\theta_d}^{\infty} \left[\left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} - 1\right] \, dG_\theta(\theta) + f_x \int_{\theta_x}^{\infty} \left[\left(\frac{\theta}{\theta_x}\right)^{\frac{\beta}{1-\beta\gamma}} - 1\right] \, dG_\theta(\theta) = f_E. \tag{2.14}$$

Cutoff Productivity. Using the productivity distribution assumption and the derived relationship between the domestic and export cutoff (2.10) one can solve for the domestic cutoff:

$$\theta_d = \left[\frac{\frac{\beta}{1-\beta\gamma}}{z - \frac{\beta}{1-\beta\gamma}} \frac{f_d + f_x \left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right)\right)^{\frac{z(1-\beta\gamma)}{\beta}}}{f_E}\right]^{\frac{1}{z}} \theta_{min}. \tag{2.15}$$

In order to ensure that the productivity cutoff is positive the following parameter condition must hold $z > \frac{\beta}{1-\beta\gamma}$. The cutoff is a function of exogenous parameters only. An increase in the market access results in an increase in the domestic cutoff $\frac{\partial \theta_d}{\partial \Upsilon_x} > 0$, forcing the least productive firms out of the market. Using the result that the market access term rises when trade costs fall this implies that a decrease in trade costs implies a higher cutoff productivity $\frac{\partial \theta_d}{\partial \tau} < 0$, as in the standard Melitz (2003) framework. The labour market situation does not influence the domestic cutoff $\frac{\partial \theta_d}{\partial \alpha} = 0$. The effect of a market access change on the cutoff is also not affected by the labour market situation $\partial \left(\frac{\partial \theta_d}{\partial \Upsilon_x}\right) / \partial \alpha = 0$.

2.2.5 General Equilibrium

In order to derive the general equilibrium conditions, it is helpful to use the expected wage income which is given by the probability of being hired times the paid wage $\omega = wH/L = bx$ where we use that in this framework wages paid by firms are equal to the search costs $w = b$. Using the definition of the search costs $b = x^\alpha$ allows to write labour market tightness and search costs as a function of the expected wage:

$$x = \omega^{\frac{1}{1+\alpha}} \ , \qquad\qquad b = \omega^{\frac{\alpha}{1+\alpha}}. \tag{2.16}$$

In equilibrium total income equals the overall value of production $Y = PQ$. The domestic demand shifter can be written as $A = Y^{1-\beta}P^{\beta} = Q^{1-\beta}P = Q^{1-\beta}$. The last equality comes from the normalizing of the price index. Using the relationship between the consumption index and the demand shifter one can rewrite the zero profit condition of the cutoff firm (2.B.29) and solve for the consumption index Q using the relationship between the search costs and the expected wage (2.16):

$$Q = QP^{\frac{1}{1-\beta}} = A^{\frac{1}{1-\beta}} = \left(\frac{f_d}{\kappa_f}\right)^{\frac{1-\beta\gamma}{1-\beta}} \kappa_w^{-\frac{\beta\gamma}{1-\beta}} \theta_d^{-\frac{\beta}{1-\beta}} \omega^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}}. \tag{2.17}$$

It is important to highlight that the consumption index is still a function of the endogenous expected wage. In equilibrium overall expected wage income in a country has to equal the sum of all wages paid by firms in that country:

$$\omega L = M \int_{\theta_d}^{\infty} w(\theta)h(\theta)\ dG_\theta(\theta) = \kappa_w M \int_{\theta_d}^{\infty} r(\theta)\ dG_\theta(\theta) = \kappa_w Q, \tag{2.18}$$

where we use that the consumption index can be written as $Q = QP = M \int_{\theta_d}^{\infty} r(\theta)\ dG_\theta(\theta)$. Using the two general equilibrium conditions (2.17) and (2.18) the equilibrium expected wage and the consumption index can be written as a function of exogenous parameters only:

$$\omega = \kappa_o^{-\frac{1}{\Delta}} \theta_d^{\frac{\beta}{\Delta}} L^{\frac{1-\beta}{\Delta}}, \tag{2.19}$$

$$Q = \kappa_o^{-\frac{1}{\Delta}} \kappa_w^{-1} \theta_d^{\frac{\beta}{\Delta}} L^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}}, \tag{2.20}$$

where $\kappa_o \equiv \kappa_w^{1-\beta-\beta\gamma}\left(\frac{f_d}{\kappa_f}\right)^{1-\beta\gamma}$ and $\Delta \equiv -\left(1-\beta-\frac{\alpha}{1+\alpha}\beta\gamma\right) > 0$, which needs to be positive in order to ensure a stable equilibrium. Thus, the elasticity of substitution between varieties and the elasticity of search costs has to be sufficiently high. The equilibrium search costs as well as equilibrium labour market tightness and the number of active firms can be derived using the equilibrium expected wage (2.19) in combination with the general equilibrium condition ensuring that overall expected wage income equals the sum of all wages paid in a country (2.18) and the relationship between the expected wage and the labour market tightness and the search costs (2.16):

$$x = \omega^{\frac{1}{1+\alpha}} \qquad = \kappa_o^{-\frac{1}{(1+\alpha)\Delta}} \theta_d^{\frac{\beta}{(1+\alpha)\Delta}} L^{\frac{1-\beta}{(1+\alpha)\Delta}}, \tag{2.21}$$

$$b = \omega^{\frac{\alpha}{1+\alpha}} \qquad = \kappa_o^{-\frac{\alpha}{1+\alpha}\frac{1}{\Delta}} \theta_d^{\frac{\alpha}{1+\alpha}\frac{\beta}{\Delta}} L^{\frac{\alpha}{1+\alpha}\frac{1-\beta}{\Delta}}, \tag{2.22}$$

$$M = \frac{\omega L}{\beta\gamma f_E} \qquad = \frac{\kappa_o^{-\frac{1}{\Delta}}}{\beta\gamma f_E} \theta_d^{\frac{\beta}{\Delta}} L^{\frac{\alpha\beta\gamma}{\Delta}}. \tag{2.23}$$

Changes in the trade costs and thus in the market access do influence all five depicted equilibrium variables only through the cutoff productivity. As the cutoff will rise with a higher market access also the consumption index, the expected wage, labour market tightness, search costs and the number of firms will rise.

Employment Effects of Trade Liberalisation. In order to facilitate the understanding of the employment effects from changes in trade costs we restate the equation determining the employment of a firm:

$$h(\theta) = \frac{\beta\gamma f_d}{1-\beta\gamma} \theta^{\frac{\beta}{1-\beta\gamma}} \ \Upsilon_x(\theta)^{\frac{1-\beta}{1-\beta\gamma}} \theta_d^{-\frac{\beta}{1-\beta\gamma}} x^{-\alpha}. \tag{2.12}$$

Bilateral trade liberalisation can influence the employment of a firm via three separate channels. First, they will affect the market access term given a firm is exporting. Secondly, as argued before they will affect the cutoff productivity and thus the competitive situation a firm faces. Thirdly, they will influence the market tightness and thus the search costs which matter for the firm's employment. Using the equilibrium employment of a firm (2.12) in combination with the cutoff productivity (2.15) as well as the equilibrium market tightness (2.21) and the market access term (2.4) allows to determine the following comparative static result:

$$\frac{\partial h(\theta)}{\partial \tau} \Big/ h(\theta) = \left[I_X(\theta) \frac{1-\beta}{1-\beta\gamma} \frac{1}{\Upsilon_X} - \frac{\beta}{1-\beta\gamma} \frac{1}{\theta_d} \frac{\partial \theta_d}{\partial \Upsilon_X} - \alpha \frac{1}{x} \frac{\partial x}{\partial \Upsilon_X} \right] \frac{\partial \Upsilon_X}{\partial \tau}. \tag{2.24}$$

We consider the relative change in employment with respect to the employment size of that firm in order to be as close as possible to the empirical specification and to account for size differences in terms of employment due to productivity differences. The relative employment effect is only dependent on a firm's productivity to the extent that effects differ for domestic and exporting firms. For the following argumentation we consider a bilateral decline in trade costs and thus a trade liberalisation. Opposite effects would follow from protectionism and thus higher trade costs. Trade liberalisations will raise the market access in both countries $\frac{\partial \Upsilon_X}{\partial \tau} < 0$. Thus, it is sufficient to consider how the employment of a firm is influenced by an increased market access. All firms which export or start to export face a positive market access effect. Having better market access to the export market, implies a larger production and thus more workers are necessary for production. This positive effect is represented by the first part in the square brackets. A rising market access implies that only exporting and thus more productive firms have new profit opportunities. This also implies additional incentives for entry due to the higher potential return if the productivity draw is high. As a consequence the more productive firms and the additional firm entrants will demand more labour which increases the real wage and thus forces the least productive firms out of the market $\frac{\partial \theta_d}{\partial \Upsilon_X} > 0$. This second effect, reducing the employment of a firm due to increased competition on the labour market, we will refer to as the competition effect. It is important to highlight that it refers to the competition for the factor labour on the domestic market and not to competition due to increased imports. This effect is not specific to the search cost imperfections. Both exporting and domestic firms face this negative employment effect from trade liberalisation. These two effects are also present in a standard (Melitz, 2003) framework and while for the domestic firms only the negative employment effect is present for the exporting firms the positive market access effect dominates the competition effect on the labour market.[8] In addition, there is a third effect which we call the search cost effect. A change in the market access will also change the labour market tightness. In particular a larger market access will imply higher labour market tightness $\frac{\partial x}{\partial \Upsilon_X} > 0$ and as such this implies a lower employment of firms. While this enforces the negative employment effect of trade liberalisation for domestic firms it dampens the positive employment effect due to an increased market access for exporting firms. There exists a range of parameter constellations for which the negative search cost effect in combination with the negative competition effect dominates the positive market access effect and the overall employment effect of trade liberalisation is negative. We will use our empirical analysis to determine which of the two cases is present in the data. It is important to mention that even in such a case the labour market tightness increases with the market access and thus overall more workers are hired. This is the case because more firms enter the market. The parameter condition ensuring a positive employment effect for exporting firms if trade is liberalised is a function of the preference parameter β production parameter γ and the labour market situation α as well a the trade parameters (τ, f_d, f_x, z):[9]

$$\frac{\alpha(1-\beta\gamma)}{(1+\alpha)\Delta} < \frac{\left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right)\right)^{1-\frac{z(1-\beta\gamma)}{\beta}} - 1}{\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}}. \tag{2.25}$$

Low productivity dispersion, a high export to domestic fixed costs ration, high iceberg trade costs, large decreasing returns to labour and a better labour market situation c.p

[8]Proof of the positive employment effect in a situation without the employment effect due to search costs can be found in the Appendix 2.A.2.

[9]The derivation of the sign condition can be found in the Appendix 2.A.2.

increase the likelihood that the above sign condition is satisfied. Summing up one can generalize the before in the following proposition:

Proposition 1. *Domestic firms face a negative employment effect due to more competition on the labour market and higher search costs and thus employ fewer workers when trade costs fall. Exporting firms and those starting to export face the same negative employment effects but in addition they have a direct positive market access effect of a trade cost reduction. While without the negative search cost effect the employment effects are always positive there exist parameter constellations for which the overall employment effect can turn negative for exporting firms as well.*

Employment Effects of the Labour Market Situation. In our empirical approach we will compare different labour market regions with different labour market situations. When we take a comparative static perspective we are thus interested in the effect of the labour market situation on the employment of a firm. At the same time we want to abstract from any relative changes in terms of market size in the two countries. As such it is reasonable to consider an equal bilateral change in the labour market situation in both countries instead of an unilateral change which will affect the relative relationship between the two countries.

The market access and the cutoff productivity are not influenced by the bilateral change in the labour market situation as they are both only functions of exogenous parameters. A change in the labour market situation can influence the employment of a firm via two different channels both influencing the search costs of firms. The following comparative static result relative to the employment size of a firm can be derived for the labour market situation effect:

$$\frac{\partial h(\theta)}{\partial \alpha} \Big/ h(\theta) = -\ln(x) - \frac{\alpha}{x}\frac{\partial x}{\partial \alpha} = \left[\frac{1-\beta}{(1+\alpha)\Delta}\right]\ln(x) < 0. \tag{2.26}$$

The first result depicts the two effects at work. There is a direct positive labour market effect. A better labour market situation makes it c.p. less costly for firms to hire workers, thus the number of workers hired rises. The second effect is a general equilibrium effect through the labour market tightness. As described before a better labour market situation result in additional firm entry and a tighter labour market $\frac{\partial x}{\partial \alpha} > 0$. Thus, there is less unemployment, but at the same time also higher search costs for a firm. The general equilibrium effect dominates the direct effect. This at first glance surprising result that in a world with a better labour market situations each firm is hiring fewer workers is caused by the increased number of firms. In such a situation the decline of employment by individual firms is overcompensated by a rise in the number of firms and thus overall employment rises.

Proposition 2. *There is a direct positive employment effect of a bilateral improvement in the labour market situation through the decline in search costs. This positive effect is overturned by a negative general equilibrium effect through the increased labour market tightness. As such a better labour market situation implies that overall more workers are hired (more firms enter the market) but a given firm hires fewer workers.*

2.2.6 Interaction of Trade Costs and Labour Market Situation

The cutoff productivity as well as the market access and the interplay between the two are not influenced by the labour market situation. Thus, the positive market access effect and the negative pro competition effect of trade liberalisation described in equation (2.24) are not influenced by a change in the labour market situation. The employment effect of trade liberalisation is influenced solely via the search costs channel. Taking a cross derivative of the trade cost effect on relative employment with respect to the labour market tightness yields the following comparative static result:

$$\frac{\partial\left(\frac{\partial h(\theta)}{\partial \tau}/h(\theta)\right)}{\partial \alpha} = \frac{(1-\beta)}{((1+\alpha)\Delta)^2}\frac{\beta}{\theta_d}\frac{\partial \theta_d}{\partial \tau} < 0. \tag{2.27}$$

The negative search cost effect of trade liberalisation is reduced if the labour market situation is better. This implies that the negative employment effect for domestic firms is dampened if the labour market situation is better. Given parameter condition (2.25) holds, the positive employment effect from trade liberalisation is amplified if the labour markets situation is better. Otherwise the negative trade liberalisation effect for exporting firms is dampened.

Proposition 3. *If trade liberalisation results in a higher employment of exporting firms, the positive employment effect for exporting firms is reinforced while the negative trade liberalisation effects on domestic firms is dampened if the labour market situation is better. All firms thus hire relatively more workers due to the trade liberalisation in a better labour market situation.*

Disentangle the Interaction Effect. To understand the mechanisms through which the labour market situation influences the employment effect of trade liberalisation it is helpful to take a closer look at the search cost effect of trade liberalisation. Trade liberalisation increases the labour market tightness $\frac{\partial x}{\partial \Upsilon_x} > 0$ and thus c.p. search costs rise and employment of firms falls. As argued before this effect is smaller if the labour market's situation is better. One can disentangle this aggregate interaction effect into two channels, which themselves both have two components:[10]

$$\begin{aligned}\frac{\partial\left(\frac{\partial h(\theta)}{\partial\tau}/h(\theta)\right)}{\partial\alpha} &= -\frac{\partial\left(\frac{\alpha}{x}\frac{\partial x}{\partial\Upsilon_x}\right)}{\partial\alpha}\frac{\partial\Upsilon_x}{\partial\tau} \\ &= -\left[\frac{\partial\left(\frac{\alpha}{x}\right)}{\partial\alpha}\frac{x}{(1+\alpha)\Delta}+\frac{\partial\left(\frac{x}{(1+\alpha)\Delta}\right)}{\partial\alpha}\frac{\alpha}{x}\right]\frac{\beta}{\theta_d}\frac{\partial\theta_d}{\partial\tau} \\ &= -\frac{1+\Delta_2\ln(x)-\Delta_2\ln(x)-\left(1+\frac{1-\beta}{(1+\alpha)\Delta}\right)}{(1+\alpha)\Delta}\frac{\beta}{\theta_d}\frac{\partial\theta_d}{\partial\tau}<0. \qquad (2.28)\end{aligned}$$

The labour market situation can on the one hand affect how the increased labour market tightness caused by trade liberalisation translates into a change in the search costs and thus the employment of firms. We call this the transmission channel. On the other hand the labour market situation can influence how trade liberalisation affects the labour market tightness itself. This effect we refer to as the direct channel

The transmission channel is influenced by two components, the labour market situation α and indirectly by the labour market tightness x. The increase in the labour market tightness due to trade liberalisation will have a more pronounced employment effect the higher the labour market situation and the lower the labour market tightness. The more elastic the search costs react to the labour market tightness the larger the employment response due to a change in the labour market tightness caused by trade liberalisation. Thus, a better labour market situation α implies that a given change in the market tightness has a larger negative employment effect through the search cost channel. This is the first component of the transmission channel. The second component is that a better labour market situation also implies a higher labour market tightness which itself implies that the transmission channel is smaller and thus a change in the labour market tightness due to trade liberalisation has a smaller effect on the search costs and thus also employment. This channel decreases the negative search cost effect of trade liberalisation.

The direct channel depicts how the labour market tightness is differently affected by trade liberalisation due to a change in the labour market situation. The direct channel is influenced by two components, indirectly through the labour market tightness x and by the labour market situation α. A higher labour market tightness due to a better labour market situation implies that trade liberalisations will raise the labour market tightness more and thus increase the negative search cost effect of trade liberalisation. This indirect effect of the labour market situation through the higher labour market tightness and the same, but opposite effect on the transmission channel cancel out. The second component is a direct effect of the labour market situation which decreases the impact the trade liberalisation has on the labour market tightness and thus reduces the negative search

[10] The parameter $\Delta_2 \equiv -\frac{\alpha(1-\beta-\beta\gamma)}{(1+\alpha)\Delta} > 1$ is used to simplify notation and is a function of the elasticity of substitution, the degree of diminishing returns to labour and the labour market situation.

cost effect. The effects of the transmission channel are cancelled out by the direct channel and a part of the effect of the labour market situation on the direct channel is left.

2.3 Data and Empirical Model

2.3.1 Data

To test the predictions derived in the theoretical model we use an empirical analysis based on micro data for Germany. As our theoretical model stresses the importance of differentiating between aggregate employment and firm-level employment, which is heterogeneously affected by trade liberalisation based on firms' productivity and export status, macro data would not allow us to test our predictions. Additionally, testing our theoretical predictions using country-level data would also make it very hard to distinguish between differences in regional labour markets and in other dimensions affecting a country's reaction to trade liberalisation, e.g. institutions, exchange rates, and fiscal policy. As our theoretical model requires specific information on heterogeneous firms, we use the linked employer-employee data set (LIAB) Cross-Section Model 2 1993-2010 provided by the Institute for Employment Research (IAB) in Nürnberg, Germany. The data contain a representative sample of German plants (the IAB Establishment Panel) with information on their employment level, exporting behaviour and other important firm characteristics. The IAB Establishment Panel is a representative sample of about 1% of German plants that is stratified over industries and firm size classes. Large plants are over-represented such that the data covers about 7% of all German employees. The survey is based on the population of all plants in Germany with at least one employee subject to social security. The survey is conducted in personal interviews with senior staff or personnel managers, has a high response rate, and low panel attrition. The questionnaire focuses on the plants' personnel structure, development and policy, and offers extensive information on firm characteristics. We can link information on all employees subject to social security in the plants surveyed to control for further employee-related factors, such as age, qualification or wages earned. We restrict our data to plants from manufacturing and service industries (excluding the public sector), and to plants with at least five employees. We access the data with the help of teleprocessing via the Research Data Centre (FDZ) of the IAB. For further information, see Klosterhuber et al. (2016).

The employment of the firms in our data can be observed in every wave of the survey. We use the yearly job growth rate, standardised by the proposed method of Davis and Haltiwanger (1992) as a dependent variable. It is defined as the difference in employment levels in one year and the last year, divided by the average of those years. The left panel of Figure 2.A.1 in the Appendix 2.A.3 shows the yearly average job growth rate. Job growth has been poor in the early 2000s in Germany and has increased since then.

From the literature we know that the decision to start (or stop) exporting, the extensive margin, is different to the decision to export one percent more (or less) given the firm exports, the intensive margin. We have precise yearly information on the exporting share of sales for all firms which express their business volume in terms of their sales (as opposed to their balance sheet or total budget).[11] We can divide firms into domestic firms, always exporters and marginal exporters. Marginal exporters export in some waves of the panel and do not export in others. The right panel of Figure 2.A.1 shows average job growth by export status over time. Domestic firms grow less on average, but the overall time pattern of job growth is similar between the three groups of firms. Exporting firms were hit more by the 2003 recession and by the 2009 financial crisis. They have grown more especially in the first half of the time interval.

A number of empirical studies have already made use of the LIAB data, for example in order to examine the relationship between export status and wage level or wage distribution (Baumgarten, 2013, 2015; Felbermayr et al., 2014; Schmillen, 2016). However, firms' trading behaviour is likely to be endogenous: More productive firms are more likely to be exporters, and firm productivity is also linked to employment growth. More recent studies analysing the effects of trade liberalisation use the fact that firms' changes in export behaviour are linked to their (endogenous) reaction to trade shocks. If trade costs decline for a certain industry, for example because of a reduction in tariffs, firms in that industry are ceteris paribus likely to increase their export share (or start exporting in the first place) (see for instance Amiti and Davis (2012); Autor et al. (2013)). We therefore use industry export shares imputed from the World Input-Output Database (WIOD) to each firm in our data depending on their 3-digit industry classification as an exogenous measure of trade liberalisation.[12]

The left panel of Figure 2.A.2 in the Appendix 2.A.3 shows the share of German exports to other countries on the sector level over time. The average sector-level export share in our sample is 18.2 %, increasing from 12.7 % in 1996 to 21.6 % in 2010. Domestic firms are concentrated in sectors with a low average export share of 10.7 %, while always exporters are concentrated in sectors with a high average export share of 41.6 %. Marginal exporters are in the middle with an average sector export share of 26.2 %. The right panel of Figure 2.A.2 in the Appendix 2.A.3 shows that average sector level exports have increased for all firms, but more so for exporters. Table 2.A.1 in the Appendix 2.A.3 shows the share of exporting firms over time for each of the industries in our data. The highest exporter share is in textiles (83 %), while the lowest exporter share is in education (0 %). Generally, higher export shares are found in manufacturing, excluding basic supplies and construction.

[11] We drop banks and insurances, public bodies, foundations, clubs and cooperatives and the like, which do not express their business volume in terms of sales.

[12] See Timmer et al. (2015) or https://www.wiod.org for further information on the WIOD data.

We append to this data information on regional labour market tightness, that is, the yearly local unemployment rate on the regional level (German Landkreise, NUTS-3).[13] An overview on the regional unemployment rates over the sample period are depicted in Table 2.A.2 in the Appendix 2.A.3. Under the assumption that firms and workers are mobile across industries, but not across regions, spill-over and cluster effects run in local commuting zones (Autor et al., 2013). Therefore, the regional unemployment rate is given for the firm and its workers. This argument continues to hold as long as the mobility of workers across sectors is larger than across regions and firms are not too large within a region.[14]

In our theoretical model, the search costs are modelled by $bh = x^{\alpha}h$. While labour market tightness $x = H/L$ per se is endogenous in the theoretical model, the parameter α is exogenous. We use the regional unemployment rate in addition to certain characteristics of a regional labour market, such as population density, mean wage level, workforce qualification, and federal states indicators. Therefore, the regional unemployment rate reflects the labour market situation inside a certain regional labour market, that is, how easy it is for a firm to fill a vacancy given a certain number of unemployed.[15]

Our sample contains up to 20,174 firms of which 3,206 are always exporters, 3,669 are marginal exporters and 13,299 are non-exporters with a total of 107,534 observations in 15 waves of the survey. Each firm has to be observed for at least three waves in a row. We present an overview of all variables in Table 2.A.3 in the Appendix 2.A.3.

2.3.2 Empirical Model

In the empirical analysis, we want to explain employment changes using trade shocks, the regional unemployment rate, their interaction, and other (control) variables. Using this model, we can expose heterogeneous effects of trade liberalisation on employment, given or depending on regional labour market tightness. To closely follow the predictions from the theoretical model, we look at firms which are exporters and firms which are non-exporters. The dependent variable is the job growth rate in firm i, industry j and regional labour market k between time t and $t-1$, which is firm- and year-specific, hence $y_{i(jk)t}$. We estimate all coefficients using a panel fixed-effects-model, which controls for time-invariant heterogeneity (e.g. Hijzen et al. 2011; Moser et al. 2015):

[13] The data is in fact based on a more detailed county level. We compute yearly averages from monthly data from the Federal Office of Employment, https://statistik.arbeitsagentur.de.

[14] This assumption is necessary, because when the unemployment rate is not exogenous for firms and workers, then they could choose a region with a suiting unemployment rate and this would result in a reverse causality problem.

[15] Kohlbrecher et al. (2016) analyse the relation between matches, unemployment and vacancies using administrative German labour market data and show that idiosyncratic productivity for new contacts is an important driver of the elasticity of the job-finding rate with respect to the market tightness.

$$y_{i(jk)t} = \alpha + \beta_1 \cdot EXPen_{(ik)jt} + \beta_2 \cdot LUR_{(ij)kt} + \beta_3 \cdot EXPen_{(ik)jt} \cdot LUR_{(ij)kt} \\ + \gamma \cdot X_{i(jk)t} + \alpha_i + \delta_j + \delta_t + \delta_k + \epsilon_{i(jk)t}. \tag{2.29}$$

The model contains time-varying firm-level controls $X_{i(jk)t}$, fixed effects on the firm level α_i, on the regional level δ_k, and for every year of the data δ_t.[16] Standard errors are clustered on the firm level.

The variables of interest are $LUR_{(ij)kt}$, which captures the effect of regional labour market tightness and varies on the regional and year-level, $EXPen_{(ik)jt}$, which captures changes in firm-level trading behaviour induced by exogenous changes in the trading behaviour of similar industries, which varies on the industry- and year-level, and their interaction. Using this specification, we can measure the heterogeneous effects of trade liberalisation on employment, depending on regional labour market tightness, and distinguishing between exporting and non-exporting firms. Note that directly measuring the exporting behaviour of firms would capture selection effects and these variables could not be interpreted causally.

The interaction effects capture the predictions of the cross derivatives in the theoretical specification: How is the effect of trade (liberalisation) on employment influenced by the regional labour market situation of the firm?

2.4 Empirical Analysis

2.4.1 Main Results

Table 2.1 presents the results for the variables of interest, while Table 2.A.4 in the Appendix 2.A.3 presents the full model. As we use within group estimators (firm-fixed-effects models), all coefficients are to be interpreted as changes of independent variables affecting changes in dependent variables. Interacting the predictions of the theoretical model with exporting behaviour would create triple interactions, which are hard to interpret. We, therefore, differentiate subsamples of domestic firms, always exporters, and marginal exporters. Specifications (1), (3) and (5) show the results explaining employment growth using the industry export share and the regional unemployment rate, while specifications (2), (4), and (6) add the interaction terms of these two variables to analyse the predictions of our theoretical model.

In the first line, we see the employment effects of a higher industry export share. More

[16] Fixed effects on the industry level, δ_j, are omitted when using firm-fixed effects. Alternative specifications have been employed using industry*year fixed effects, with no changes to the central results.

Table 2.1: *Employment Effects of Trade Liberalisation and Labour Market Tightness*

	Domestic Firms		Always Exporters		Marginal Exporters	
	(1)	(2)	(3)	(4)	(5)	(6)
Industry Export Share	-0.0637*	-0.0633*	0.0995**	0.0994**	0.1067***	0.1083***
	(0.0356)	(0.0356)	(0.0423)	(0.0423)	(0.0412)	(0.0411)
Regional Unemployment Rate	-0.3857***	-0.4271***	0.3006**	-0.0224	0.0226	-0.1038
	(0.0784)	(0.0933)	(0.1334)	(0.2770)	(0.1265)	(0.2068)
Interaction		0.3655		0.7387		0.4694
		(0.3998)		(0.5302)		(0.4803)
Firm-Level Control Variables	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm Size Classes FE	Yes	Yes	Yes	Yes	Yes	Yes
Number of Observations	69,493	69,493	16,253	16,253	21,788	21,788
Number of Clusters	13,299	13,299	3,206	3,206	3,669	3,669
F-Statistic	29.80	29.05	14.95	14.55	13.44	13.12
R Squared Within	0.06	0.06	0.07	0.07	0.05	0.05
ρ	0.69	0.69	0.71	0.71	0.59	0.59

*Note: dependent variable: job growth rate between June 30th each year; standard errors clustered at the plant level in parentheses; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Source: LIAB QM2 9310, Waves 1996 to 2010; own calculations (controlled remote data access via FDZ).*

specifically, we see what happens to employment growth when the firm's industry increases its exports. The results differ between domestic firms, where we see a negative effects, and exporters, where we see a positive effect for both always exporters and marginal exporters. A higher industry export share implicitly measures a positive trade shock, for example through falling tariffs or transport costs. Exporters profit from this change, implying a positive effect on employment growth. If the industry export share rises by one percentage point, the job growth rate of exporters rises by one tenth of a percentage point. The theory suggests that this effect runs through the better market access due to trade liberalisation. The resulting higher demand on the export market due to the decreased trade costs implies a higher labour demand for the exporting firms in the domestic country. The positive employment effect is within the scope of the theoretical prediction and suggests that the condition for a positive employment effect for exporters (2.25) is fulfilled.

The opposite effect can be seen for domestic firms, albeit smaller. If the industry export share rises by one percentage point, the job growth rate of domestic firms in that industry falls by one sixteenth of a percentage point. This is in line with Proposition 1 which suggest a negative employment effect for domestic firms. The theoretical model suggest two general equilibrium mechanisms at play. On the one hand the increased labour demand of the exporting firms and the additional entry implies a tougher competition for labour, therefore the wage level rises and domestic firms reduce their labour demand. On the other hand the resulting tighter labour market implies that search costs increase and thus also equilibrium labour demand by each firm will fall.

In the next line, we see the employment effects of regional labour market tightness. More specifically, we measure the effect of an increase in the regional unemployment rate, which we interpret as the labour market situation inside a regional labour market. Higher regional unemployment rates, hence a worse labour market situation, are negatively correlated with employment in domestic firms and positively correlated with employment for always exporters. The effects are positive but insignificant for marginal exporters. The results show that exporters have a higher or similar job growth rate in regional labour markets where unemployment is high, while domestic firms have a lower job growth rate. In our theoretical model, higher unemployment rates induce a reduction in search costs, which in turn leads to an increase in hiring by the firms. However, higher unemployment rates are also a sign of lower labour market efficiency and hence, for similar labour market tightness, higher search costs. Hence, the general equilibrium effect on firm employment is negative. We interpret the empirical results in such a way that domestic firms operate more in line with regional employment conditions and might suffer more from the general equilibrium effects than exporting firms, with marginal exporters in between.

The interactions effects between trade shocks and regional labour market tightness in specifications (2), (4) and (6) capture the predictions of the cross derivatives in the theoretical specification. They are positive but insignificant for all types of firms. We see, however, that the effect of the regional unemployment rate on exporting firms disappears when the interaction is introduced. This effect, therefore, might have been driven by counties with very high export exposure increases, while it is non-existent for counties with modest increases. The model suggests that any positive employment effect for exporting firms is reinforced while negative trade effects on domestic firms are dampened if the labour market situation is better.

2.4.2 Robustness Checks

A first robustness check concerns the exclusion of certain firms for which the theoretical model might not be applicable. This concerns large firms and especially multinational companies. These firms might exhibit endogenous location decisions, for example to expand a plant in a not so tight regional labour market as a response to a positive trade shock. Table 2.A.5 in the Appendix 2.A.3 presents the results for the variables of interest of two subsamples of the data. In the upper half, we have excluded all firms which are part of a multi-plant enterprise, be it as a subsidiary, central office or centre spot. This affects a total of 39,507 observations. In the lower half, we exclude all plants with more than 500 employees, which affects 10,191 observations.

For both subsamples we see very robust results for the employment effects of trade shocks and the regional labour market tightness per se. We also see some significant interaction effects. Domestic single firms are harmed less from a positive trade shock if the regional labour market is less tight. Small and medium-sized exporters profit more from a positive trade shock if the regional labour market is less tight. Both interaction effects are in line with the theoretical predictions.

A second robustness check analyses the use of different export measures, following the argumentation of Dauth et al. (2014). Industry export shares can also be affected by other determinants, which could be correlated with the outcome variables. Therefore, instrumented industry export shares or local export shares are used. We present the results for the subsample of always exporters in Table 2.A.6 in the Appendix 2.A.3. The results for the other two groups are similar in conclusion. We use different industry export share measures to estimate an effect from trade liberalisation on employment growth. As we restrict the sample to always exporters, the effect should be positive, as exporters should benefit from a positive trade shock.

We see positive effects on employment growth if the industry export share from Germany to the world rises and if the industry export share from the rest of the world (non-OECD countries) to OECD countries rises. We do not find significant effects if only the industry share of German exports to EU-10 or EU-12 countries rises or if the industry export share of non-EU OECD countries to the world or to the EU-12 countries rises.

When we use the proposed export measure by Dauth et al. (2014), the industry share of Non-EU OECD countries, we find mostly stable results, as Table 2.A.7 in the Appendix 2.A.3 shows. The industry share export itself has no significant effect on firm-level employment growth. The regional labour market tightness has the same effects as in our main results. The interaction between the two is positive, but not or only marginally significant, thereby suggesting some support for our theoretical predictions.

A final set of robustness checks has been performed regarding the estimation methodology. First, we have used industry-year fixed effects in addition to the firm-fixed effects without changes to the central results.[17] In addition, we have changed the independent sector-level export variables from the share of exports to the yearly changes in these shares, both explaining the job growth rate in a firm-fixed effects model and using OLS. The central results of the paper do not change.

2.5 Concluding Remarks

The analysis of the employment effects of trade liberalisation has, for a long time, not been in the focus of the international economics literature. Theoretical trade models have assumed perfect labour markets and most empirical analyses have lacked suitable

[17] As mentioned in the empirical model, only using industry-fixed effects does not work in addition to the firm-fixed effects as long as industries do not change over time within firms.

data which incorporates both trade and (un)employment information. Since employment effects play a major role in economic policy and other social sciences, the focus has shifted in the last 15 years and distributional aspects and imperfections on the labour market were integrated in the trade context. However, there is still little research highlighting regional differences in the labour market and thus heterogeneous labour market effects from trade across different regions. This is where our paper contributes to the literature.

This study builds on a simple theoretical model and uses linked employer-employee data for Germany to analyse the effects of trade liberalisation on German firms depending on the labour market situation in the local labour market or commuting zone. We identify the employment effects of trade liberalisation differentiated by heterogeneous firms and acknowledge their trading behaviour and the local labour market conditions they operate in, as well as general equilibrium effects which stem from labour market imperfections.

Our findings suggest that indeed the employment effects of trade liberalisation are heterogeneous with respect to the local labour market situation, in addition to firm productivity and the export status. In particular, a trade liberalisation effect will have more beneficial employment implications (which for exporting firms means a larger increase in employment and for domestic firms a smaller decline in employment) if they face a better labour market situation. This result might emphasises the importance of local labour markets already found in a number of (newer) empirical studies. However, it also implies that regional labour market heterogeneity widens after trade liberalisation. The emphasis on pre-existing differences is present in our theory and our estimates point in the same direction albeit not significant. The empirical results also indicate that exporters might react differently to the local labour market situation than domestic firms, depending on the regional unemployment rate. Our results can, therefore, help policy makers in understanding firms' reactions to changes in international trade flows on the regional level. It can also help the economic literature to expand its knowledge on heterogeneous effects of trade and the labour market.

Highlighting employment of firms and not only considering aggregate employment effects in a region or country indicates that trade liberalisation creates frictions and adjustments on the labour market. While some workers in the domestic firms will loose their jobs, other workers will be employed by exporters as a consequence of trade liberalisation even in a framework with a homogeneous workforce. Extending this argumentation further will be a goal of future research.

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2.A Appendix

2.A.1 Search Technology

Following Diamon-Mortensen-Pissarides, b can be interpreted as search costs which are derived from a constant returns to scale search technology together with costs of posting vacancies ψ_0. Assume the measure of matched workers is determined by a Cobb-Douglas function, including the number of vacancies V reported and the number of workers searching for employment L. One can set up the following relationship:

$$H = \psi_1 V^{\psi_2} L^{1-\psi_2},\ \psi_1 > 0,\ 0 < \psi_2 < 1.$$

Rearranging yields vacancies as a function of matched workers H and the labour market tightness $x = H/L$:

$$V = \psi_1^{-\frac{1}{\psi_2}} H^{\frac{1}{\psi_2}} L^{-\frac{1-\psi_2}{\psi_2}} = \psi_1^{-\frac{1}{\psi_2}} \left(\frac{H}{L}\right)^{\frac{1-\psi_2}{\psi_2}} H = \psi_1^{-\frac{1}{\psi_2}} x^{\frac{1-\psi_2}{\psi_2}} H.$$

This allows to solve for the search costs as a function of the labour market tightness:

$$\begin{aligned} bH &= \psi_0 V = \psi_0 \psi_1^{-\frac{1}{\psi_2}} x^{\frac{1-\psi_2}{\psi_2}} H \\ b &= \frac{x^\alpha}{\alpha_0} \\ b &= x^\alpha, \end{aligned}$$

where $\alpha \equiv \frac{1-\psi_2}{\psi_2} > 0$ is a inverse measure of the importance of vacancies in the search function. It thus can be interpret as an efficiency measure for the labour market indicating that less vacancies which are costly are needed to hire the same amount of workers. We will refer to it as the labour market situation. It is the elasticity of search costs with respect to the labour market tightness $\epsilon_{b,x} = \frac{\partial b}{\partial x}\frac{x}{b} = \alpha$. The second search technology parameter is defined as $\alpha_0 \equiv \frac{\psi_1^{\frac{1}{\psi_2}}}{\psi_0}$. For simplicity, costs for posting vacancies ψ_0 and the technology parameter ψ_1 are set to one ($\psi_0 = \psi_1 = 1 \rightarrow \alpha_0 = 1$).

2.A.2 Proof of Propositions

Effects of Trade Liberalisation

The employment of a firm is given by $h(\theta) = \frac{\beta\gamma f_d}{1-\beta\gamma}\theta^{\frac{\beta}{1-\beta\gamma}}\ \Upsilon_x(\theta)^{\frac{1-\beta}{1-\beta\gamma}}\theta_d^{-\frac{\beta}{1-\beta\gamma}} x^{-\alpha}$ (2.12). The employment effect of a bilateral change in the iceberg trade costs can be separated into the following three channels:

$$\begin{aligned}\frac{\partial h(\theta)}{\partial \tau} =& I_X(\theta)\frac{1-\beta}{1-\beta\gamma}\frac{h(\theta)}{\Upsilon_x}\frac{\partial \Upsilon_x}{\partial \tau} - \frac{\beta}{1-\beta\gamma}\frac{h(\theta)}{\theta_d}\frac{\partial \theta_d}{\partial \Upsilon_x}\frac{\partial \Upsilon_x}{\partial \tau} - \alpha\frac{h(\theta)}{x}\frac{\partial x}{\partial \Upsilon_x}\frac{\partial \Upsilon_x}{\partial \tau}\\ \frac{\partial h(\theta)}{\partial \tau}\Big/ h(\theta) =& \left[I_X(\theta)\frac{1-\beta}{1-\beta\gamma}\frac{1}{\Upsilon_X} - \frac{\beta}{1-\beta\gamma}\frac{1}{\theta_d}\frac{\partial \theta_d}{\partial \Upsilon_X} - \alpha\frac{1}{x}\frac{\partial x}{\partial \Upsilon_X}\right]\frac{\partial \Upsilon_X}{\partial \tau}\\ =& \left[I_X(\theta)\frac{1-\beta}{1-\beta\gamma}\frac{1}{\Upsilon_X} - \frac{\beta}{1-\beta\gamma}\frac{\beta+\beta\alpha-1}{(1+\alpha)\Delta}\frac{1}{\theta_d}\frac{\partial \theta_d}{\partial \Upsilon_X}\right]\frac{\partial \Upsilon_X}{\partial \tau},\end{aligned} \tag{2.24}$$

where we use that given the labour market tightness $x = \kappa_o^{-\frac{1}{(1+\alpha)\Delta}}\theta_d^{\frac{\beta}{(1+\alpha)\Delta}}L^{\frac{1-\beta}{(1+\alpha)\Delta}}$ (2.21). It is straight forward to show the relationship between the labour market tightness and the market access term $\frac{\partial x}{\partial \Upsilon_x} = \frac{\beta}{(1+\alpha)\Delta}\frac{x}{\theta_d}\frac{\partial \theta_d}{\partial \Upsilon_x}$. Using the following equality $\left(\frac{\beta}{1-\beta\gamma} + \frac{\alpha}{1+\alpha}\frac{\beta}{\Delta}\right) = \frac{\beta}{1-\beta\gamma}\frac{\beta+\beta\alpha-1}{(1+\alpha)\Delta}$ the derivation of the last equality above is straight forward. Using the domestic cutoff productivity $\theta_d = \left[\frac{\frac{\beta}{1-\beta\gamma}}{z-\frac{\beta}{1-\beta\gamma}}\left(f_d + f_x\left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right)\right)^{\frac{z(1-\beta\gamma)}{\beta}}\right)\Big/ f_E\right]^{\frac{1}{z}}\theta_{min}$ (2.15), one can show that the change in the cutoff productivity due to a change in the market access term is positive:

$$\frac{\partial \theta_d}{\partial \Upsilon_x} = \frac{(1-\beta)}{\beta}\frac{\left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right)\right)^{\frac{z(1-\beta\gamma)}{\beta}-1}\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}-1}}{f_d + f_x\left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right)\right)^{\frac{z(1-\beta\gamma)}{\beta}}} f_d\theta_d \qquad > 0. \tag{2.A.1}$$

Given the positive relationship between cutoff productivity and the market access term it is straight forward to show the positive relationship between the labour market tightness and the market access term $\frac{\partial x}{\partial \Upsilon_x} > 0$.

Sign Condition

In order to determine the sign of the derivative of employment with respect to the trade costs it is necessary to determine the sign of the square brackets. In order to differentiate between a situation with and without the search cost effect we will use $\nabla \equiv \frac{\beta+\beta\alpha-1}{(1+\alpha)\Delta}$ which is equal to one if their are no search cost effects. The condition for a positive employment effect of trade liberalisations for exporting firms is given by the following inequality:

$$\begin{aligned}\frac{\partial h(\theta)}{\partial \tau}\Big/ h(\theta)\Big|_{\theta>\theta_x} < 0 \text{ if } 0 <& \left[\frac{1-\beta}{1-\beta\gamma}\frac{1}{\Upsilon_X} - \frac{\beta}{1-\beta\gamma}\nabla\frac{1}{\theta_d}\frac{\partial \theta_d}{\partial \Upsilon_X}\right]\\ 1 >& \nabla\frac{\beta}{1-\beta}\frac{\Upsilon_X}{\theta_d}\frac{\partial \theta_d}{\partial \Upsilon_X}.\end{aligned} \tag{2.A.2}$$

The effect of a change in the market access on the domestic cutoff productivity (2.A.1) can be further simplified:

$$
\begin{aligned}
\frac{\Upsilon_X}{\theta_d}\frac{\partial\theta_d}{\partial\Upsilon_X} =& \frac{1-\beta}{\beta}\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} \frac{\left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}-1\right)\right)^{\frac{z(1-\beta\gamma)}{\beta}-1}}{1+\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}-1\right)\left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}-1\right)\right)^{\frac{z(1-\beta\gamma)}{\beta}-1}} \\
=& \frac{1-\beta}{\beta}\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}\left[\frac{1}{\left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}-1\right)\right)^{\frac{z(1-\beta\gamma)}{\beta}-1}}+\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}-1\right]^{-1} \\
=& \frac{1-\beta}{\beta}\frac{\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}}{\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}+\left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}-1\right)\right)^{1-\frac{z(1-\beta\gamma)}{\beta}}-1}. \qquad (2.A.3)
\end{aligned}
$$

This allows to rewrite the condition for a positive employment effect of trade liberalisation for an exporting firm. Using $\frac{1}{\nabla}=\frac{(1+\alpha)\Delta}{\beta+\beta\alpha-1}=\frac{-(1+\alpha)(1-\beta)+\alpha\beta\gamma}{\beta+\beta\alpha-1}=\frac{\beta+\beta\alpha-1-\alpha(1-\beta\gamma)}{\beta+\beta\alpha-1}=1-\frac{\alpha(1-\beta\gamma)}{\beta+\beta\alpha-1}$ it follows:

$$
\begin{aligned}
1>&\nabla\frac{\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}}{\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}+\left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}-1\right)\right)^{1-\frac{z(1-\beta\gamma)}{\beta}}-1} \qquad (2.A.4) \\
1<&\left(1-\frac{\alpha(1-\beta\gamma)}{\beta+\beta\alpha-1}\right)\left(1+\frac{\left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}-1\right)\right)^{1-\frac{z(1-\beta\gamma)}{\beta}}-1}{\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}}\right) \\
0<&\left(1-\frac{\alpha(1-\beta\gamma)}{\beta+\beta\alpha-1}\right)\frac{\left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}-1\right)\right)^{1-\frac{z(1-\beta\gamma)}{\beta}}-1}{\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}}-\frac{\alpha(1-\beta\gamma)}{\beta+\beta\alpha-1}. \qquad (2.A.5)
\end{aligned}
$$

Using $1-\frac{\alpha(1-\beta\gamma)}{\beta+\beta\alpha-1}=\frac{(1+\alpha)\Delta}{\beta+\beta\alpha-1}$ the condition for a positive employment effect for exporting firms due to trade liberalisation can be written in the following way:

$$
\left.\frac{\partial h(\theta)}{\partial\tau}\middle/h(\theta)\right|_{\theta>\theta_x}<0 \text{ if } \frac{\alpha(1-\beta\gamma)}{(1+\alpha)\Delta}<\frac{\left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}-1\right)\right)^{1-\frac{z(1-\beta\gamma)}{\beta}}-1}{\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}}. \qquad (2.25)
$$

α, β, γ, z, $\frac{f_d}{f_x}$ and τ will determine whether this inequality is true.

In a situation without the labour market effect ($\nabla=1$) the condition ensuring a positive employment effect of trade liberalisation for exporting firms (2.A.4) simplifies to:

$$\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} + \left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right)\right)^{1-\frac{z(1-\beta\gamma)}{\beta}} - 1 > \Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}$$
$$\left(\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right)\right)^{1-\frac{z(1-\beta\gamma)}{\beta}} > 1. \tag{2.A.6}$$

The assumption that all exporting firms also supply the domestic market (2.10) implies that $\frac{f_d}{f_x}\left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right) < 1$. A positive cutoff also implies $z > \frac{\beta}{1-\beta\gamma}$ which ensures a negative exponent. Thus, without search costs the inequality is fulfilled for all possible parameter constellations and thus trade liberalisation implies a positive employment effect for exporting firms.

Employment Effects of the Labour Market Situation

The domestic cutoff productivity (2.15) as well as the market access term (2.4) are both independent of the labour market situation in a symmetric world. Employment of a firm is given by $h(\theta) = \frac{\beta\gamma f_d}{1-\beta\gamma}\theta^{\frac{\beta}{1-\beta\gamma}}\ \Upsilon_x(\theta)^{\frac{1-\beta}{1-\beta\gamma}}\theta_d^{-\frac{\beta}{1-\beta\gamma}} x^{-\alpha}$ (2.12). The employment effect of a change in the labour market situation α can be separated into the following two channels:

$$\frac{\partial h(\theta)}{\partial \alpha} = h(\theta)\left[-\ln(x) - \frac{\alpha}{x}\frac{\partial x}{\partial \alpha}\right], \tag{2.A.7}$$

where $\ln(x) < 0$ as the labour market tightness has to lie below one ($0 < x < 1$). The following differentiation rule applies $\frac{\partial\left(a(x)^{f(x)}\right)}{\partial x} = a(x)^{f(x)}\left[f'(x)\ln(a(x)) + \frac{f(x)}{a(x)}\frac{\partial a(x)}{\partial x}\right]$. The direct positive employment effect due to an improvement in the labour market situation is captured by the $-\ln(x)$ term. The second effect due to an increase in the labour market tightness has negative employment effects for each firm as one can see in the following. In order to determine the effect of the labour market situation on the labour market tightness (2.21) we use the following definition to simplify notation: $o(\alpha) \equiv [(1+\alpha)\Delta]^{-1}$. Using $\frac{\partial a^{f(x)}}{\partial x} = a^{f(x)}\ln(a)f'(x)$, the derivative of the labour market tightens with respect to the labour market situation is given by:

$$\begin{aligned}\frac{\partial x}{\partial \alpha} &= x\left[\beta\ln(\theta_d) + (1-\beta)\ln(L) - \ln(\kappa_o)\right]\frac{\partial o(\alpha)}{\partial \alpha}\\ &= x(1+\alpha)\Delta\ln(x)\frac{(1-\beta-\beta\gamma)}{((1+\alpha)\Delta)^2}\\ &= \frac{1-\beta-\beta\gamma}{(1+\alpha)\Delta}\ \ln(x)x > 0,\end{aligned} \tag{2.A.8}$$

where $x = \kappa_o^{-o(\alpha)}\theta_d^{\beta o(\alpha)}L^{(1-\beta)o(\alpha)}$ (2.21) is the equilibrium labour market tightness and the log is given by $\ln(x) = \frac{1}{(1+\alpha)\Delta}\left[\beta\ln(\theta_d) + (1-\beta)\ln(L) - \ln(\kappa_o)\right]$. The change in the help variable $o(\alpha)$ is given by $\frac{\partial o(\alpha)}{\partial \alpha} = \frac{\partial(-[(1-\beta)(1+\alpha)-\beta\gamma\alpha])^{-1}}{\partial \alpha} = \frac{(1-\beta-\beta\gamma)}{((1+\alpha)\Delta)^2} < 0$. Thus, the overall effect is given by:

$$\frac{\partial h(\theta)}{\partial \alpha} \Big/ h(\theta) = -\ln(x) - \frac{\alpha}{x}\frac{\partial x}{\partial \alpha} = -\ln(x) + \frac{\alpha(-1+\beta+\beta\gamma)}{((1+\alpha)\Delta)}\ln(x)$$
$$= \left[\frac{1-\beta}{(1+\alpha)\Delta}\right]\ln(x) < 0. \tag{2.26}$$

The positive direct employment effect of an increase in the labour market effect is dominated by the negative general equilibrium effect through the increases labour market tightness. As such an improved labour market situation implies that overall more workers are hired but a given firm hires viewer workers.

Interaction of Trade Costs and Labour Market Situation

To determine the interaction effect the partial derivative of employment with respect to a bilateral trade cost change $\frac{\partial h(\theta)}{\partial \tau} \Big/ h(\theta) = \left[I_X(\theta)\frac{1-\beta}{1-\beta\gamma}\frac{1}{\Upsilon_X} - \frac{\beta}{1-\beta\gamma}\frac{1}{\theta_d}\frac{\partial \theta_d}{\partial \Upsilon_X} - \alpha\frac{1}{x}\frac{\partial x}{\partial \Upsilon_X}\right]\frac{\partial \Upsilon_X}{\partial \tau}$ (2.24) is used. Applying $\frac{\partial x}{\partial \Upsilon_x} = \frac{\beta}{(1+\alpha)\Delta}\frac{x}{\theta_d}\frac{\partial \theta_d}{\partial \Upsilon_x}$ and $\frac{\partial x}{\partial \alpha} = \frac{1-\beta-\beta\gamma}{(1+\alpha)\Delta}\ln(x)x$ (2.A.8) and using that Υ_X, θ_d, $\frac{\partial \theta_d}{\partial \Upsilon_X}$, $\frac{\partial \Upsilon_X}{\partial \tau}$, are all independent of the labour market situation the cross derivative with respect to the labour market situation is given by:

$$\begin{aligned}
\frac{\partial\left(\frac{\partial h(\theta)}{\partial \tau}/h(\theta)\right)}{\partial \alpha} &= -\frac{\partial\left(\frac{\alpha}{x}\frac{\partial x}{\partial \Upsilon_X}\right)}{\partial \alpha}\frac{\partial \Upsilon_X}{\partial \tau} \\
&= -\left[\frac{\partial\left(\frac{\alpha}{x}\right)}{\partial \alpha}\frac{\partial x}{\partial \Upsilon_X} + \frac{\alpha}{x}\frac{\partial\left(\frac{\partial x}{\partial \Upsilon_X}\right)}{\partial \alpha}\right]\frac{\partial \Upsilon_X}{\partial \tau} \\
&= -\left[\frac{\partial\left(\frac{\alpha}{x}\right)}{\partial \alpha}\frac{\beta}{(1+\alpha)\Delta}\frac{x}{\theta_d}\frac{\partial \theta_d}{\partial \Upsilon_x} + \frac{\alpha}{x}\frac{\partial\left(\frac{\beta}{(1+\alpha)\Delta}\frac{x}{\theta_d}\frac{\partial \theta_d}{\partial \Upsilon_x}\right)}{\partial \alpha}\right]\frac{\partial \Upsilon_X}{\partial \tau} \\
&= -\left[\frac{x-\alpha\frac{\partial x}{\partial \alpha}}{x^2}\frac{x}{(1+\alpha)\Delta} + \frac{\alpha}{x}\frac{\partial\left(\frac{x}{(1+\alpha)\Delta}\right)}{\partial \alpha}\right]\frac{\beta}{\theta_d}\frac{\partial \theta_d}{\partial \Upsilon_x}\frac{\partial \Upsilon_X}{\partial \tau} \\
&= -\left[\frac{x-\alpha\frac{\partial x}{\partial \alpha}}{(1+\alpha)\Delta\ x} + \frac{\alpha}{x}\frac{(1+\alpha)\Delta\frac{\partial x}{\partial \alpha} - x\frac{\partial((1+\alpha)\Delta)}{\partial \alpha}}{((1+\alpha)\Delta)^2}\right]\frac{\beta}{\theta_d}\frac{\partial \theta_d}{\partial \tau} \\
&= -\left[\frac{x-\alpha\frac{1-\beta-\beta\gamma}{(1+\alpha)\Delta}\ln(x)x}{(1+\alpha)\Delta\ x} + \frac{\alpha}{x}\frac{\frac{1-\beta-\beta\gamma}{(1+\alpha)\Delta}\ln(x)x - \frac{x\frac{\partial((1+\alpha)\Delta)}{\partial \alpha}}{(1+\alpha)\Delta}}{(1+\alpha)\Delta}\right]\frac{\beta}{\theta_d}\frac{\partial \theta_d}{\partial \tau}.
\end{aligned} \tag{2.A.9}$$

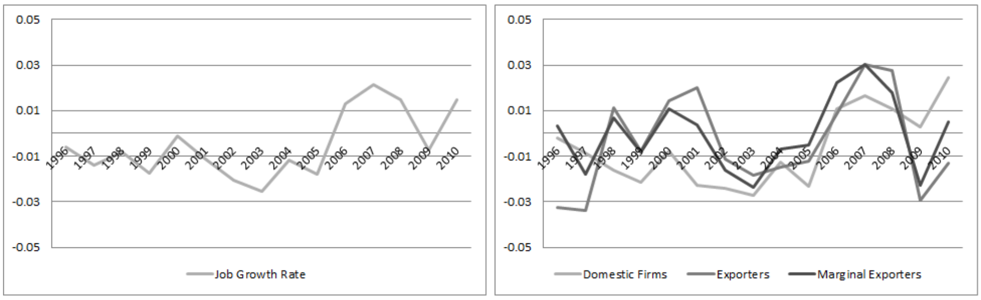
Using $-\alpha(1-\beta-\beta\gamma) = (1+\alpha)\Delta + 1 - \beta$ we can define $\Delta_2 \equiv -\frac{\alpha(1-\beta-\beta\gamma)}{(1+\alpha)\Delta} = \frac{(1+\alpha)\Delta+1-\beta}{(1+\alpha)\Delta} = 1 + \frac{1-\beta}{(1+\alpha)\Delta} > 1$ to simplify notation and solve for the cross derivative:

$$
\begin{aligned}
\frac{\partial\left(\frac{\partial h(\theta)}{\partial\tau}/h(\theta)\right)}{\partial\alpha} &= -\left[\frac{1+\Delta_2\ln(x)}{(1+\alpha)\Delta} + \frac{-\Delta_2\ln(x) - \frac{\alpha}{(1+\alpha)\Delta}\frac{\partial(\beta-1+\alpha(-1+\beta+\beta\gamma))}{\partial\alpha}}{(1+\alpha)\Delta}\right]\frac{\beta}{\theta_d}\frac{\partial\theta_d}{\partial\tau} \\
&= -\left[\frac{1+\Delta_2\ln(x)}{(1+\alpha)\Delta} - \frac{\Delta_2\ln(x) - \alpha\frac{1-\beta-\beta\gamma}{(1+\alpha)\Delta}}{(1+\alpha)\Delta}\right]\frac{\beta}{\theta_d}\frac{\partial\theta_d}{\partial\tau} \\
&= -\left[\frac{1+\Delta_2\ln(x)}{(1+\alpha)\Delta} - \frac{1+\frac{1-\beta}{(1+\alpha)\Delta}+\Delta_2\ln(x)}{(1+\alpha)\Delta}\right]\frac{\beta}{\theta_d}\frac{\partial\theta_d}{\partial\tau} \qquad (2.28) \\
&= -\left[\frac{1+\Delta_2\ln(x)}{(1+\alpha)\Delta} - \left(\frac{1+\Delta_2\ln(x)}{(1+\alpha)\Delta} + \frac{1-\beta}{((1+\alpha)\Delta)^2}\right)\right]\frac{\beta}{\theta_d}\frac{\partial\theta_d}{\partial\tau} \\
&= \frac{(1-\beta)}{((1+\alpha)\Delta)^2}\frac{\beta}{\theta_d}\frac{\partial\theta_d}{\partial\tau} < 0 \quad \textbf{as} \quad \frac{\partial\theta_d}{\partial\tau} < 0. \qquad (2.27)
\end{aligned}
$$

The cross derivative is negative independent of the parameter values.

2.A.3 Empirical Analyses

Figure 2.A.1: *Job Growth Rate by Year and Export Status*



Job growth rates calculated as differences in employment from last year to recent year divided by average employment; Source: LIAB QM2 9310, Waves 1996 to 2010; own calculations (controlled remote data access via FDZ).

Figure 2.A.2: *Sector-Level Export Share by Year and Export Status*

Share of exports from Germany to other countries the sector level; Source: World Input-output tables (WIOT), Release 2013; own calculations (controlled remote data access via FDZ).

Table 2.A.1: *Share of Exporting Firms by Economic Sector*

Economic Sector	Mean	Std. Dev.	Min	Median	Max	Number of Firm-Year Observations
Agriculture, Hunting, Forestry and Fishing	0.14	0.04	0.08	0.13	0.21	3,458
Mining and Quarrying	0.23	0.10	0.10	0.20	0.44	809
Food, Beverages and Tobacco	0.17	0.04	0.12	0.16	0.26	4,141
Textiles and Textile Products	0.83	0.09	0.57	0.87	0.93	1,211
Wood and Products of Wood and Cork, Leather, Leather and Footwear	0.31	0.07	0.13	0.31	0.39	1,691
Pulp, Paper , Printing and Publishing	0.40	0.09	0.25	0.41	0.56	2,054
Coke, Refined Petroleum and Nuclear Fuel, Chemicals and Chemical Products	0.75	0.12	0.21	0.75	0.90	2,201
Rubber and Plastics	0.48	0.07	0.33	0.49	0.58	2,096
Other Non-Metallic Mineral	0.31	0.07	0.17	0.33	0.41	2,134
Basic Metals and Fabricated Metal	0.45	0.05	0.36	0.45	0.53	7,092
Machinery, Nec	0.50	0.04	0.39	0.49	0.56	5,363
Electrical and Optical Equipment	0.58	0.07	0.43	0.58	0.70	5,146
Transport Equipment	0.55	0.02	0.52	0.55	0.58	2,791
Manufacturing, Nec; Recycling	0.40	0.02	0.35	0.41	0.43	1,823
Electricity, Gas and Water Supply	0.09	0.04	0.02	0.10	0.14	1,780
Construction	0.04	0.01	0.02	0.04	0.05	12,734
Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel	0.01	0.00	0.01	0.01	0.01	3,901
Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles	0.08	0.02	0.06	0.08	0.11	5,301
Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods	0.01	0.00	0.00	0.01	0.01	8,974
Hotels and Restaurants	0.42	0.15	0.15	0.46	0.60	3,709
Inland Transport, Water Transport, Air Transport	0.18	0.21	0.09	0.11	0.85	2,682
Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies	0.10	0.02	0.08	0.10	0.13	2,458
Post and Telecommunications	0.06	0.02	0.04	0.05	0.08	484
Real Estate Activities	0.01	0.00	0.00	0.01	0.01	2,002
Renting of M and Eq and Other Business Activities	0.08	0.02	0.05	0.09	0.10	12,777
Education	0.00	0.00	0.00	0.00	0.00	5,613
Health and Social Work	0.01	0.00	0.00	0.01	0.01	13,911
Other Community, Social and Personal Services	0.02	0.01	0.01	0.01	0.03	7,941
Total	0.18	0.22	0.00	0.08	0.93	126,277

Note: Yearly averages. Source: LIAB QM2 9310, Waves 1996 to 2010; own calculations (controlled remote data access via FDZ).

Table 2.A.2: *Regional Unemployment Rate*

Year	Mean	Std. Dev.	Observations	Min	Median	Max
1996	0.1058	0.03	289	0.04	0.09	0.22
1997	0.1157	0.03	284	0.04	0.10	0.24
1998	0.1101	0.03	297	0.04	0.10	0.24
1999	0.1037	0.03	286	0.03	0.09	0.24
2000	0.0984	0.04	322	0.03	0.08	0.24
2001	0.0952	0.04	328	0.03	0.07	0.25
2002	0.1015	0.04	336	0.03	0.08	0.25
2003	0.1093	0.04	342	0.04	0.09	0.27
2004	0.1097	0.04	341	0.04	0.09	0.27
2005	0.1087	0.04	337	0.04	0.09	0.24
2006	0.1013	0.04	341	0.03	0.09	0.23
2007	0.0846	0.04	341	0.02	0.07	0.22
2008	0.0726	0.03	345	0.01	0.06	0.19
2009	0.0774	0.03	346	0.02	0.06	0.17
2010	0.0730	0.03	342	0.01	0.06	0.16
Total	0.0973	0.04	4,877	0.01	0.08	0.27

Source: LIAB QM2 9310, Waves 1996 to 2010 and Federal Office of Employment, https://statistik.arbeitsagentur.de.; own calculations (controlled remote data access via FDZ).

Table 2.A.3: *Variables Used in the Analysis*

Variable	Mean	Std. Dev.	Min	Max
Log. Employment	3.65	1.64	0.00	10.83
Job Growth Rate	-0.0044	0.19	-1.98	1.98
Yearly Export Status	0.24	0.43	0.00	1.00
Industry Export Share	0.18	0.22	0.00	0.93
Job Growth Rate by Export Group				
No Exporter	-0.0062	0.2114	-1.97	1.97
Always Exporter	-0.0026	0.1397	-1.95	1.52
Marginal Exporter	0.0005	0.1864	-1.99	1.87
Regional Unemployment Rate	0.12	0.05	0.02	0.28
Sector				
Agriculture, Hunting, Forestry and Fishing	0.03	0.16	0.00	1.00
Mining and Quarrying	0.01	0.08	0.00	1.00
Food, Beverages and Tobacco	0.03	0.18	0.00	1.00
Textiles and Textile Products	0.01	0.10	0.00	1.00
Wood and Products of Wood and Cork, Leather, Leather and Footwear	0.01	0.12	0.00	1.00
Pulp, Paper , Printing and Publishing	0.02	0.13	0.00	1.00
Coke, Refined Petroleum and Nuclear Fuel, Chemicals and Chemical Products	0.02	0.13	0.00	1.00
Rubber and Plastics	0.02	0.13	0.00	1.00
Other Non-Metallic Mineral	0.02	0.13	0.00	1.00
Basic Metals and Fabricated Metal	0.06	0.23	0.00	1.00
Machinery, Nec	0.04	0.20	0.00	1.00
Electrical and Optical Equipment	0.04	0.20	0.00	1.00
Transport Equipment	0.02	0.14	0.00	1.00
Manufacturing, Nec; Recycling	0.01	0.12	0.00	1.00
Electricity, Gas and Water Supply	0.01	0.12	0.00	1.00
Construction	0.10	0.30	0.00	1.00
Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel	0.03	0.17	0.00	1.00
Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles	0.04	0.20	0.00	1.00
Retail Trade, Except of Motor Vehicles and Motorcycles; Repair of Household Goods	0.07	0.26	0.00	1.00
Hotels and Restaurants	0.03	0.17	0.00	1.00
Inland Transport, Water Transport, Air Transport	0.02	0.14	0.00	1.00
Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies	0.02	0.14	0.00	1.00
Post and Telecommunications	0.01	0.11	0.00	1.00
Real Estate Activities	0.02	0.12	0.00	1.00
Renting of M and Eq and Other Business Activities	0.11	0.31	0.00	1.00
Education	0.04	0.20	0.00	1.00
Health and Social Work	0.11	0.31	0.00	1.00
Other Community, Social and Personal Services	0.06	0.24	0.00	1.00

Table 2.A.3: *Variables, Continued*

Variable	Mean	Std. Dev.	Min	Max
Firm Size				
1-19 employees	0.40	0.49	0.00	1.00
20-49 employees	0.20	0.40	0.00	1.00
50-99 employees	0.12	0.33	0.00	1.00
100-199 employees	0.10	0.30	0.00	1.00
200-499 employees	0.10	0.30	0.00	1.00
500-999 employees	0.04	0.20	0.00	1.00
more than 1000 employees	0.03	0.18	0.00	1.00
German property	0.75	0.43	0.00	1.00
Foreign property	0.05	0.22	0.00	1.00
Public property	0.06	0.23	0.00	1.00
Unclear property	0.14	0.35	0.00	1.00
Personal firm	0.27	0.44	0.00	1.00
Limited company	0.54	0.50	0.00	1.00
Proprietary company	0.04	0.20	0.00	1.00
Public corporation	0.07	0.26	0.00	1.00
Other	0.08	0.27	0.00	1.00
Separate firm	0.72	0.45	0.00	1.00
Subsidiary	0.09	0.28	0.00	1.00
Central office	0.17	0.38	0.00	1.00
Centre spot	0.02	0.14	0.00	1.00
Founded after 1990	0.72	0.45	0.00	1.00
Up to date assets	0.19	0.39	0.00	1.00
Rather new assets	0.49	0.50	0.00	1.00
Partly new assets	0.29	0.45	0.00	1.00
Rather old assets	0.03	0.18	0.00	1.00
Increased imports	0.11	0.31	0.00	1.00
Offshoring plant	0.31	0.46	0.00	1.00

Note: observations; Source: LIAB QM2 9310, Waves 1996 to 2010; own calculations (controlled remote data access via FDZ).

Table 2.A.4: *Employment Effects of Trade Liberalisation and Labour Market Tightness*

	Domestic Firms		Always Exporters		Marginal Exporters	
	(1)	(2)	(3)	(4)	(5)	(6)
Industry Export Share	-0.0637*	-0.0633*	0.0995**	0.0994**	0.1067***	0.1083***
	(0.0356)	(0.0356)	(0.0423)	(0.0423)	(0.0412)	(0.0411)
Regional Unemployment Rate	-0.3857***	-0.4271***	0.3006**	-0.0224	0.0226	-0.1038
	(0.0784)	(0.0933)	(0.1334)	(0.2770)	(0.1265)	(0.2068)
Interaction		0.3655		0.7387		0.4694
		(0.3998)		(0.5302)		(0.4803)
Foreign Property	-0.0017	-0.0017	0.0136*	0.0138*	-0.0150	-0.0148
	(0.0143)	(0.0143)	(0.0075)	(0.0075)	(0.0110)	(0.0110)
Public Property	0.0071	0.0071	-0.0753*	-0.0760*	-0.0234	-0.0234
	(0.0051)	(0.0051)	(0.0408)	(0.0413)	(0.0175)	(0.0175)
No Principal Shareholder	-0.0022	-0.0022	0.0045	0.0049	0.0047	0.0047
	(0.0039)	(0.0039)	(0.0070)	(0.0070)	(0.0066)	(0.0066)
Limited Company	0.0075	0.0075	0.0108	0.0105	0.0137	0.0136
	(0.0067)	(0.0067)	(0.0106)	(0.0106)	(0.0097)	(0.0097)
Proprietary Company	0.0119	0.0119	0.0063	0.0060	-0.0044	-0.0041
	(0.0135)	(0.0135)	(0.0141)	(0.0141)	(0.0181)	(0.0181)
Public Corporation	0.0285**	0.0285**	-0.0015	-0.0017	0.0211	0.0209
	(0.0113)	(0.0113)	(0.0203)	(0.0204)	(0.0164)	(0.0164)
Other Firm	0.0113	0.0114	-0.0038	-0.0040	0.0118	0.0118
	(0.0102)	(0.0102)	(0.0142)	(0.0142)	(0.0184)	(0.0183)
Subsidiary	-0.0094**	-0.0093**	-0.0056	-0.0057	-0.0051	-0.0051
	(0.0047)	(0.0047)	(0.0051)	(0.0051)	(0.0078)	(0.0078)
Central Office	0.0051	0.0051	-0.0021	-0.0020	-0.0044	-0.0044
	(0.0048)	(0.0048)	(0.0059)	(0.0058)	(0.0076)	(0.0076)
Centre Spot	-0.0061	-0.0061	-0.0173	-0.0169	-0.0023	-0.0024
	(0.0067)	(0.0067)	(0.0122)	(0.0123)	(0.0161)	(0.0161)
Rather New Assets	-0.0069**	-0.0069**	-0.0152***	-0.0153***	-0.0165***	-0.0165***
	(0.0028)	(0.0028)	(0.0041)	(0.0041)	(0.0041)	(0.0041)
Partly New Assets	-0.0124***	-0.0124***	-0.0235***	-0.0236***	-0.0246***	-0.0246***
	(0.0034)	(0.0034)	(0.0052)	(0.0052)	(0.0055)	(0.0055)
Rather Old Assets	-0.0167**	-0.0167**	-0.0222**	-0.0221**	-0.0159	-0.0160
	(0.0073)	(0.0073)	(0.0100)	(0.0100)	(0.0124)	(0.0124)
Increased Imports	0.0079	0.0079	-0.0147	-0.0144	0.0095	0.0095
	(0.0066)	(0.0066)	(0.0123)	(0.0123)	(0.0104)	(0.0104)
Offshoring Plant	-0.0011	-0.0012	-0.0080	-0.0078	-0.0013	-0.0013
	(0.0062)	(0.0062)	(0.0117)	(0.0117)	(0.0098)	(0.0098)
Constant	-0.1705***	-0.1707***	-0.3241***	-0.3253***	-0.1452***	-0.1458***
	(0.0113)	(0.0113)	(0.0335)	(0.0335)	(0.0208)	(0.0208)
Number of Observations	69,493	69,493	16,253	16,253	21,788	21,788
Number of Clusters	13,299	13,299	3,206	3,206	3,669	3,669
F-Statistic	29.80	29.05	14.95	14.55	13.44	13.12
R Squared Within	0.06	0.06	0.07	0.07	0.05	0.05
ρ	0.69	0.69	0.71	0.71	0.59	0.59

*Note: dependent variable: job growth rate between June 30th each year; German Property, Personal Undertaking, Separate Firm and Up to Date Assets are chosen as reference category; standard errors clustered at the plant level in parentheses; Industry, Year and Firm Size Classes Fixed Effects included; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.*
Source: LIAB QM2 9310, Waves 1996 to 2010; own calculations (controlled remote data access via FDZ).

Table 2.A.5: *Robustness Checks: Employment Effects of Trade Liberalisation and Labour Market Tightness for Single Firms and Medium-Sized Companies*

Only Single Firms

	Domestic Firms		Always Exporters		Marginal Exporters	
	(1)	(2)	(3)	(4)	(5)	(6)
Industry Export Share	-0.0840**	-0.0839**	0.1055*	0.1072*	0.1057**	0.1081**
	(0.0400)	(0.0400)	(0.0574)	(0.0575)	(0.0478)	(0.0477)
Regional Unemployment Rate	-0.4548***	-0.5724***	0.3292**	-0.1177	0.0103	-0.1040
	(0.0922)	(0.1131)	(0.1578)	(0.3518)	(0.1547)	(0.2563)
Interaction		0.9255**		1.0533		0.4306
		(0.4542)		(0.6537)		(0.5846)
Number of Observations	51,023	51,023	10,124	10,124	15,691	15,691
Number of Clusters	10,641	10,641	2,373	2,373	2,959	2,959
F-Statistic	23.21	22.62	.	.	10.14	9.85
R Squared Within	0.05	0.05	0.07	0.07	0.05	0.05
ρ	0.67	0.68	0.69	0.69	0.59	0.59

Without Large Firms (<500 Employees

	Domestic Firms		Always Exporters		Marginal Exporters	
	(1)	(2)	(3)	(4)	(5)	(6)
Industry Export Share	-0.0590	-0.0585	0.1056**	0.1080**	0.1138**	0.1161***
	(0.0365)	(0.0365)	(0.0525)	(0.0526)	(0.0444)	(0.0443)
Regional Unemployment Rate	-0.4032***	-0.4556***	0.3828**	-0.0330	-0.0116	-0.1411
	(0.0819)	(0.0981)	(0.1528)	(0.3001)	(0.1339)	(0.2179)
Interaction		0.4528		0.9859*		0.4881
		(0.4127)		(0.5937)		(0.5168)
Number of Observations	65,660	65,660	13,025	13,025	20,240	20,240
Number of Clusters	12,660	12,660	2,693	2,693	3,458	3,458
F-Statistic	30.24	29.45	13.00	12.65	12.20	11.87
R Squared Within	0.06	0.06	0.07	0.07	0.05	0.05
ρ	0.64	0.64	0.66	0.66	0.55	0.55

*Note: dependent variable: job growth rate between June 30th each year; standard errors clustered at the plant level in parentheses; Firm-Level Control Variables as well as Industry, Year and Firm Size Classes Fixed Effects are included; * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Source: LIAB QM2 9310, Waves 1996 to 2010; own calculations (controlled remote data access via FDZ).*

Table 2.A.6: *Robustness Checks: Employment Effects of Trade Liberalisation for Different Export Measures*

Only Always Exporters						
Share of Exports ...	from Germany	only to EU-10	only to EU-12	from the Rest of the World	from OECD countries	only to EU-12
	(1)	(2)	(3)	(4)	(5)	(6)
Industry Export Share	0.0890**	0.3071	0.0737	0.0811*	-0.0760	-0.5149
	(0.0410)	(0.2350)	(0.1561)	(0.0464)	(0.0687)	(1.3276)
Firm-Level Control Variables	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm Size Classes FE	Yes	Yes	Yes	Yes	Yes	Yes
Number of Observations	17,058	17,058	17,058	17,058	17,058	17,058
Number of Clusters	3,309	3,309	3,309	3,309	3,309	3,309
F-Statistic	15.86	15.74	15.68	15.99	15.69	15.78
R Squared Within	0.08	0.08	0.08	0.08	0.08	0.08
ρ	0.71	0.70	0.70	0.71	0.69	0.70

*Note: dependent variable: job growth rate between June 30th each year; standard errors clustered at the plant level in parentheses; * p < 0.10, ** p < 0.05, *** p < 0.01. Source: LIAB QM2 9310, Waves 1996 to 2010; own calculations (controlled remote data access via FDZ).*

Table 2.A.7: *Employment Effects of Trade Liberalisation and Labour Market Tightness: Non-EU OECD Countries Export Share*

	Domestic Firms		Always Exporters		Marginal Exporters	
	(1)	(2)	(3)	(4)	(5)	(6)
Industry Export Share	-0.0584	-0.0557	-0.0804	-0.0740	-0.1225	-0.1187
	(0.0769)	(0.0764)	(0.0700)	(0.0697)	(0.0831)	(0.0830)
Regional Unemployment Rate	-0.3849***	-0.4227***	0.2825**	-0.0895	0.0105	-0.1323
	(0.0783)	(0.0980)	(0.1333)	(0.2504)	(0.1264)	(0.1979)
Interaction		0.5133		1.7604*		1.0425
		(0.7288)		(0.9682)		(0.9882)
Firm-Level Control Variables	Yes	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm Size Classes FE	Yes	Yes	Yes	Yes	Yes	Yes
Number of Observations	69,493	69,493	16,253	16,253	21,788	21,788
Number of Clusters	13,299	13,299	3,206	3,206	3,669	3,669
F-Statistic	29.77	28.99	14.75	14.35	13.44	13.09
R Squared Within	0.06	0.06	0.07	0.07	0.05	0.05
ρ	0.69	0.69	0.69	0.69	0.57	0.57

*Note: dependent variable: job growth rate between June 30th each year; standard errors clustered at the plant level in parentheses; * p < 0.10, ** p < 0.05, *** p < 0.01. Source: LIAB QM2 9310, Waves 1996 to 2010; own calculations (controlled remote data access via FDZ).*

2.B Supplementary Appendix

2.B.1 Preferences

Assuming constant elasticity of substitution between varieties ϑ the real consumption index Q is defined over the set of varieties M as:

$$Q = \left[\int_0^M q(\vartheta)^\beta \, d\vartheta\right]^{1/\beta} \qquad 0 < \beta < 1, \tag{2.1}$$

where β controls the elasticity of substitution between different varieties. In particular $\sigma = \frac{1}{1-\beta}$ is the elasticity of substitution.

A representative consumer will maximize consumption Q subject to its budget constraint. Given the ordinal utility concept one can also maximize Q^β.

$$\mathcal{L} = \int_0^M q(\vartheta)^\beta \, d\vartheta + \lambda \left(Y - \int_0^M p(\vartheta) q(\vartheta) \, d\vartheta\right)$$

$$\frac{\partial \mathcal{L}}{\partial q(\vartheta)} = \beta q(\vartheta)^{\beta-1} - \lambda p(\vartheta) = 0 \tag{2.B.10}$$

$$\frac{\partial \mathcal{L}}{\partial q(\nu)} = \beta q(\nu)^{\beta-1} - \lambda p(\nu) = 0 \tag{2.B.11}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = Y - \int_0^M p(\vartheta) q(\vartheta) \, d\vartheta = 0 \tag{2.B.12}$$

ν is used to index a second variety. Using (2.B.10) and (2.B.11) one can write:

$$q(\vartheta)^{\beta-1} = \frac{p(\vartheta)}{p(\nu)} q(\nu)^{\beta-1}$$

$$q(\vartheta) = \left(\frac{p(\vartheta)}{p(\nu)}\right)^{-\frac{1}{1-\beta}} q(\nu).$$

Using this in the budget constraint (2.B.12) yields:

$$Y = \int_0^M p(\vartheta) \left(\frac{p(\vartheta)}{p(\nu)}\right)^{-\frac{1}{1-\beta}} q(\nu) \, d\vartheta$$

$$= p(\nu)^{\frac{1}{1-\beta}} q(\nu) \int_0^M p(\vartheta)^{-\frac{\beta}{1-\beta}} \, d\vartheta$$

$$q(\nu) = p(\nu)^{-\frac{1}{1-\beta}} \left[\int_0^M p(\vartheta)^{-\frac{\beta}{1-\beta}} \, d\vartheta\right]^{-1} Y$$

$$q(\nu) = p(\nu)^{-\frac{1}{1-\beta}} P^{\frac{\beta}{1-\beta}} Y$$

$$q(\nu) = p(\nu)^{-\frac{1}{1-\beta}} \left(P^\beta Y^{1-\beta}\right)^{\frac{1}{1-\beta}}$$

$$q(\vartheta) = p(\vartheta)^{-\frac{1}{1-\beta}} A^{\frac{1}{1-\beta}}, \tag{2.B.13}$$

with the price index $P \equiv \left(\int_0^M p(\vartheta)^{-\frac{\beta}{1-\beta}}\right)^{-\frac{1-\beta}{\beta}}$ and the demand shifter $A \equiv P^\beta Y^{1-\beta}$. Using the demand for a variety (2.B.13) revenues can be written as:

$$r(\vartheta) = p(\vartheta)q(\vartheta) = q(\vartheta)^{-(1-\beta)}Aq(\vartheta) = Aq(\vartheta)^\beta. \tag{2.B.14}$$

2.B.2 Domestic and Export Production

In equilibrium supply has to equal demand, thus without trade costs demand for variety ϑ has to equal supply of this variety produced by a firm with productivity θ, $y(\theta) = q(\vartheta)$. For the export market τ-times the quantity has to be shipped $\tau q^*(\theta) = y_x(\theta)$. Revenues on both markets are then given by:

$$r_d(\theta) = Ay_D(\theta)^\beta, \tag{2.B.15}$$

$$r_x(\theta) = A^*\left(\frac{y_x(\theta)}{\tau}\right)^\beta, \tag{2.B.16}$$

where for the export market firms that choose to supply both markets have to allocate their output $y(\theta)$ between the two markets. They do so by choosing the amount produced for the domestic and export market ($y_d(\theta)$ and $y_x(\theta)$) such that the marginal revenue is the same in both markets:

$$\begin{aligned}
\beta A y_d(\theta)^{\beta-1} = \frac{\partial r_d(\theta)}{\partial y_d(\theta)} &\overset{!}{=} \frac{\partial r_x(\theta)}{\partial y_x(\theta)} = \beta A^* y_x(\theta)^{\beta-1}\tau^{-\beta} \\
\left(\frac{y_d(\theta)}{y_x(\theta)}\right)^{\beta-1} &= \frac{A^*}{A}\tau^{-\beta} \\
y_x(\theta) &= \left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}}\tau^{-\frac{\beta}{1-\beta}}y_d(\theta) \\
y_x(\theta) &= \left(\Upsilon_x(\theta) - 1\right)y_d(\theta),
\end{aligned} \tag{2.B.17}$$

where the market access term is defined as:

$$\Upsilon_x(\theta) \equiv 1 + I_x\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}}\tau^{-\frac{\beta}{1-\beta}} = 1 + I_x\tau^{-\frac{\beta}{1-\beta}}. \tag{2.4}$$

We use the symmetry assumption, $\frac{A^*}{A} = 1$. Overall production can be written as:

$$\begin{aligned}
y(\theta) &= y_d(\theta) + I_x y_x(\theta) = y_d(\theta) + I_x\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}}\tau^{-\frac{\beta}{1-\beta}}y_d(\theta) \\
y_d(\theta) &= y(\theta)\left[1 + I_x\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}}\tau^{-\frac{\beta}{1-\beta}}\right]^{-1} = \frac{y(\theta)}{\Upsilon_x(\theta)}.
\end{aligned} \tag{2.B.18}$$

Production for the export market thus can be written as:

$$y_x(\theta) = \frac{\Upsilon_x(\theta) - 1}{\Upsilon_x(\theta)} y(\theta). \tag{2.B.19}$$

Using (2.B.15, 2.B.16, 2.B.17) revenues of a firm are given by:

$$\begin{aligned}
r(\theta) = r_d(\theta) + I_x r_x(\theta) &= A y_D(\theta)^\beta + I_x A^* \left(\frac{y_x(\theta)}{\tau}\right)^\beta \\
&= A y_D(\theta)^\beta + I_x A^* \left(\frac{\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}} \tau^{-\frac{\beta}{1-\beta}} y_d(\theta)}{\tau}\right)^\beta \\
&= A y_D(\theta)^\beta \left[1 + I_x \left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}} \tau^{-\frac{\beta}{1-\beta}}\right].
\end{aligned} \tag{2.B.20}$$

Domestic revenues, using (2.B.18), are given by:

$$r_d(\theta) = A y_D(\theta)^\beta = A \left(\frac{y(\theta)}{\Upsilon_x(\theta)}\right)^\beta. \tag{2.B.21}$$

Revenues from exporting, using (2.B.19), are depicted by:

$$\begin{aligned}
r_x(\theta) = A y_D(\theta)^\beta I_x \left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}} \tau^{-\frac{\beta}{1-\beta}} &= A y_D(\theta)^\beta \left(\Upsilon_x(\theta) - 1\right)) \\
&= A \left(\frac{y(\theta)}{\Upsilon_x(\theta)}\right)^\beta \left(\Upsilon_x(\theta) - 1\right).
\end{aligned} \tag{2.B.22}$$

Using (2.B.18), the definition of the market access term (2.4) and the production technology $y(\theta) = \theta h^\gamma$ (2.2) one can write revenues (2.B.20) as:

$$\begin{aligned}
r(\theta) &= A \left(y(\theta) \left[1 + I_x \left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}} \tau^{-\frac{\beta}{1-\beta}}\right]^{-1} \right)^\beta \left[1 + I_x \left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}} \tau^{-\frac{\beta}{1-\beta}}\right] \\
&= A y(\theta)^\beta \left[1 + I_x \left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}} \tau^{-\frac{\beta}{1-\beta}}\right]^{1-\beta} \\
&= A y(\theta)^\beta \Upsilon_x(\theta)^{1-\beta} \\
&= A \theta^\beta h^{\beta\gamma} \Upsilon_x(\theta)^{1-\beta}.
\end{aligned} \tag{2.3}$$

Using this result in (2.B.21) and (2.B.22) yields a relationship between domestic and overall revenues as well as between export and overall revenues. Where using (2.B.21, 2.3) domestic revenues are given by:

$$r_d(\theta) = A\left(\frac{y(\theta)}{\Upsilon_x(\theta)}\right)^{\beta}\frac{\Upsilon_x(\theta)^{1-\beta}}{\Upsilon_x(\theta)^{1-\beta}} = \Upsilon_x(\theta)^{(-\beta-(1-\beta)}r(\theta) = \frac{r(\theta)}{\Upsilon_x(\theta)}. \tag{2.B.23}$$

Using (2.B.22, 2.3) revenues from exporting are depicted by:

$$r_x(\theta) = A\left(\frac{y(\theta)}{\Upsilon_x(\theta)}\right)^{\beta}(\Upsilon_x(\theta)-1) = \frac{\Upsilon_x(\theta)-1}{\Upsilon_x(\theta)}r(\theta). \tag{2.B.24}$$

2.B.3 Wage Bargaining

Firms choose the number of employees such that the costs of hiring that worker is equal to the marginal profits of hiring another worker. They thereby take into account that hiring another worker will change wages of all workers:

$$\begin{aligned} w(\theta,h) &= \frac{\partial\left[r(\theta,h)-w(\theta,h)h\right]}{\partial h} = \frac{\partial r(\theta,h)}{\partial h} - \frac{\partial w(\theta,h)}{\partial h}h - w(\theta,h) \\ w(\theta,h) &= \frac{1}{2}\left[\frac{\partial r(\theta,h)}{\partial h} - \frac{\partial w(\theta,h)}{\partial h}h\right]. \end{aligned}$$

It also holds that:

$$\begin{aligned} \frac{\partial w(\theta,h)}{\partial h} &= \frac{1}{2}\left[\frac{\partial^2 r(\theta,h)}{\partial h^2} - \frac{\partial^2 w(\theta,h)}{\partial h^2}h - \frac{\partial w(\theta,h)}{\partial h}\right] \\ \frac{\partial w(\theta,h)}{\partial h} &= \frac{1}{3}\left[\frac{\partial^2 r(\theta,h)}{\partial h^2} - \frac{\partial^2 w(\theta,h)}{\partial h^2}h\right] \end{aligned}$$

and:

$$\frac{\partial^n w(\theta,h)}{\partial h^n} = \frac{1}{n+2}\left[\frac{\partial^{n+1} r(\theta,h)}{\partial h^{n+1}} - \frac{\partial^{n+1} w(\theta,h)}{\partial h^{n+1}}h\right].$$

Using the revenue equation (2.3) one knows:

$$\begin{aligned} \frac{\partial r(\theta,h)}{\partial h} &= \frac{r(\theta,h)}{h}\gamma\beta \\ \frac{\partial^2 r(\theta,h)}{\partial h^2} &= \frac{r(\theta,h)}{h^2}\gamma\beta(\gamma\beta-1) \\ \frac{\partial^n r(\theta,h)}{\partial h^n} &= \frac{r(\theta,h)}{h^n}\prod_{j=1}^{n}\gamma\beta-(j-1). \end{aligned}$$

Solving this iterative system the differential equation can be solved in the following form:

$$w(\theta,h) = \frac{1}{2}\frac{r(\theta,h)}{h}\gamma\beta - \frac{1}{2}\frac{1}{3}\frac{r(\theta,h)}{h}\gamma\beta(\gamma\beta-1) + \frac{1}{2}\frac{1}{3}\frac{1}{4}\frac{r(\theta,h)}{h}\gamma\beta(\gamma\beta-1)(\gamma\beta-2) + ...,$$

with $0 < \gamma, \beta < 1$ it follows that $\gamma\beta - j < 0$ for $j > 1$. One thus can write the wage as:

$$w(\theta, h) = \frac{r(\theta, h)}{h} \gamma\beta \sum_{i=1}^{\infty} \frac{1}{(1+i)!} \prod_{j=1}^{i-1} (j - \gamma\beta) \equiv \frac{r(\theta, h)}{h} \frac{\gamma\beta}{1+\gamma\beta}. \tag{2.B.25}$$

The labour costs of the firm are given by the fraction $\kappa_w \equiv \beta\gamma/(1+\beta\gamma)$ of revenue, while it receives the fraction $1/(1+\beta\gamma)$ of revenue.

2.B.4 Profits, Employment and Revenue of a Firm

Anticipating the outcome of the bargaining game, firms maximize profits choosing whether to export or not as well as the number of workers to hire. Using the derived revenue equation (2.3) the profit maximization problem of a firm takes the following form:

$$\begin{aligned} \max_{\substack{h(\theta)\geq 0, \\ I_x(\theta)\in\{0,1\},}} \pi(\theta) =& \frac{1}{1+\beta\gamma} r(\theta) - bh(\theta) - f_d - I_x f_x \\ =& \frac{1}{1+\beta\gamma} A \left(\theta h(\theta)^{\gamma}\right)^{\beta} \Upsilon_x(\theta)^{1-\beta} - bh(\theta) - f_d - I_x f_x. \end{aligned} \tag{2.5}$$

The maximization problem yields the following optimality condition:

$$\frac{\partial \pi(\theta)}{\partial h(\theta)} = \kappa_w \frac{r(\theta)}{h(\theta)} - b = 0,$$

where $\kappa_w \equiv \frac{\beta\gamma}{1+\beta\gamma}$. Labour demand as a function of the revenues of a firm is given by:

$$h(\theta) = \kappa_w \frac{r(\theta)}{b} = \kappa_w \frac{r(\theta)}{x^{\alpha}}. \tag{2.6}$$

Using the relationship between the optimal number of workers in a firm (2.6) and its revenues one can rewrite profits (2.5) as follows:

$$\begin{aligned} \pi(\theta) &= \frac{1}{1+\beta\gamma} r(\theta) - b \frac{\beta\gamma}{1+\beta\gamma} \frac{r(\theta)}{b} - f_d - I_x f_x \\ &= \frac{1-\beta\gamma}{1+\beta\gamma} r(\theta) - f_d - I_x f_x = \kappa_f r(\theta) - f_d - I_x f_x, \end{aligned} \tag{2.7}$$

where $\kappa_f \equiv \frac{1-\beta\gamma}{1+\beta\gamma}$. Solving for the amount of workers hired using the optimal revenues (2.3) and (2.6) yields the number of workers and allows to rewrite revenues in the following way:

$$\begin{aligned} h(\theta) =& \kappa_w \frac{A\theta^{\beta} h^{\beta\gamma} \Upsilon_x(\theta)^{1-\beta}}{b} \\ h(\theta)^{1-\beta\gamma} =& A\kappa_w \Upsilon_x(\theta)^{1-\beta} \theta^{\beta} b^{-1} = \left[A\kappa_w \Upsilon_x(\theta)^{1-\beta} \theta^{\beta} b^{-1}\right]^{\frac{1}{1-\beta\gamma}}. \end{aligned} \tag{2.8}$$

Using (2.6) one can write revenues as:

$$
\begin{aligned}
r(\theta) =& \frac{b}{\kappa_w} h(\theta) = \left[A\kappa_w \kappa_w^{-(1-\beta\gamma)} \Upsilon_x(\theta)^{1-\beta} \theta^\beta b^{-1} b^{1-\beta\gamma} \right]^{\frac{1}{1-\beta\gamma}} \\
=& \left[A\kappa_w^{\beta\gamma} \Upsilon_x(\theta)^{1-\beta} \theta^\beta b^{-\beta\gamma} \right]^{\frac{1}{1-\beta\gamma}} . \qquad (2.9)
\end{aligned}
$$

Wages are the same across all firms within a region and can be determined using the result from the wage bargaining (2.B.25) and profit maximisation (2.6) which rearranged implies $\frac{r(\theta}{h(\theta)} = \frac{b}{\kappa_w}$:

$$
w(\theta) = \frac{r(\theta)}{h(\theta)} \frac{\gamma\beta}{1+\gamma\beta} = \frac{\gamma\beta}{1+\gamma\beta} \frac{b}{\kappa_w} = b. \qquad (2.B.26)
$$

2.B.5 Employment and Revenue as a Function of the Domestic Cutoff

Using the profit equation following from the firm optimisation (2.7) one can state that the marginal domestic firm, which earns zero profits ($\pi_d(\theta_d) = 0$) generates revenues of:

$$
\begin{aligned}
\pi(\theta) &= \kappa_f r(\theta) - f_d - I_x f_x \qquad (2.7) \\
\pi(\theta_d) &= \kappa_f r(\theta_d) - f_d = 0 \\
r(\theta_d) &= \frac{f_d}{\kappa_f}. \qquad (2.B.27)
\end{aligned}
$$

Using the relationship between employment and revenues of a firm (2.6) one can determine the employment of the marginal domestic firm as:

$$
\begin{aligned}
h(\theta) &= \kappa_w \frac{r(\theta)}{b} \qquad (2.6) \\
h(\theta_d) &= \kappa_w \frac{r(\theta_d)}{b} \\
&= \frac{\kappa_w f_d}{\kappa_f b} = \frac{\beta\gamma}{1+\beta\gamma} \frac{1+\beta\gamma}{1-\beta\gamma} \frac{f_d}{b} = \frac{\beta\gamma}{1-\beta\gamma} \frac{f_d}{b}. \qquad (2.B.28)
\end{aligned}
$$

Rewriting (2.B.27) using (2.9) yields:

$$
\begin{aligned}
f_d &= \kappa_f r(\theta_d) \\
&= \kappa_f \left[A\kappa_w^{\beta\gamma} \theta_d^\beta b^{-\beta\gamma} \right]^{\frac{1}{1-\beta\gamma}} . \qquad (2.B.29)
\end{aligned}
$$

The Export cutoff is determined by the firm which is indifferent between selling solely domestically and also supplying the export market $(\pi(\theta_x)|_{I_x=1} = \pi(\theta_x)|_{I_x=0}))$. We will use the following convention $\Upsilon_x \equiv \Upsilon_x(\theta)|_{I_x(\theta)=1}$ to indicate market access of an exporting firm:

$$
\begin{aligned}
\pi(\theta_x)|_{I_x=1} &= \pi(\theta_x)|_{I_x=0} \\
\kappa_f\ r(\theta_x)|_{I_x=1} - f_d - f_x &= \kappa_f\ r(\theta_x)|_{I_x=0} - f_d \\
\kappa_f \left[A\kappa_w^{\beta\gamma}\Upsilon_x^{1-\beta}\theta_x^{\beta}b^{-\beta\gamma}\right]^{\frac{1}{1-\beta\gamma}} - f_x &= \kappa_f \left[A\kappa_w^{\beta\gamma}\theta_x^{\beta}b^{-\beta\gamma}\right]^{\frac{1}{1-\beta\gamma}} \\
\kappa_f \left[A\kappa_w^{\beta\gamma}b^{-\beta\gamma}\theta_x^{\beta}\right]^{\frac{1}{1-\beta\gamma}} \left[\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right] &= f_x. \qquad (2.B.30)
\end{aligned}
$$

Using (2.B.29) and (2.B.30) yields:

$$
\begin{aligned}
\frac{f_d}{f_x} &= \frac{\kappa_f \left[A\kappa_w^{\beta\gamma}\theta_d^{\beta}b^{-\beta\gamma}\right]^{\frac{1}{1-\beta\gamma}}}{\kappa_f \left[A\kappa_w^{\beta\gamma}b^{-\beta\gamma}\theta_x^{\beta}\right]^{\frac{1}{1-\beta\gamma}} \left[\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right]} \\
\left(\frac{\theta_d}{\theta_x}\right)^{\frac{\beta}{1-\beta\gamma}} &= \left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right)\frac{f_d}{f_x} \\
\frac{\theta_d}{\theta_x} &= \left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right)^{\frac{1-\beta\gamma}{\beta}} \left(\frac{f_d}{f_x}\right)^{\frac{1-\beta\gamma}{\beta}} < 0. \qquad (2.10)
\end{aligned}
$$

Relating the number of workers hired by the domestic cutoff firm to the number of workers hired by any other firm (2.8) yields:

$$
\begin{aligned}
\frac{h(\theta)}{h(\theta_d)} &= \frac{\left[A\kappa_w\Upsilon_x(\theta)^{1-\beta}\theta^{\beta}b^{-1}\right]^{\frac{1}{1-\beta\gamma}}}{\left[A\kappa_w\theta_d^{\beta}b^{-1}\right]^{\frac{1}{1-\beta\gamma}}} = \frac{\left[\Upsilon_x(\theta)^{1-\beta}\theta^{\beta}\right]^{\frac{1}{1-\beta\gamma}}}{\left[\theta_d^{\beta}\right]^{\frac{1}{1-\beta\gamma}}} \\
h(\theta) &= \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} \Upsilon_x(\theta)^{\frac{1-\beta}{1-\beta\gamma}}\ h(\theta_d), \qquad (2.12)
\end{aligned}
$$

where the employment of the cutoff firm are given by (2.B.28). In similar fashion using (2.9) one can determine the amount of revenues as a function of the cutoff productivity:

$$
\begin{aligned}
\frac{r(\theta)}{r(\theta_d)} &= \frac{\left[A\kappa_w^{\beta\gamma}\Upsilon_x(\theta)^{1-\beta}\theta^{\beta}b^{-\beta\gamma}\right]^{\frac{1}{1-\beta\gamma}}}{\left[A\kappa_w^{\beta\gamma}\theta_d^{\beta}b^{-\beta\gamma}\right]^{\frac{1}{1-\beta\gamma}}} = \frac{\left[\Upsilon_x(\theta)^{1-\beta}\theta^{\beta}\right]^{\frac{1}{1-\beta\gamma}}}{\left[\theta_d^{\beta}\right]^{\frac{1}{1-\beta\gamma}}} \\
r(\theta) &= \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} \Upsilon_x(\theta)^{\frac{1-\beta}{1-\beta\gamma}}\ r(\theta_d), \qquad (2.11)
\end{aligned}
$$

where the revenues of the cutoff firm are given by (2.B.27).

2.B.6 Free Entry

Free Entry implies that new firms will enter the market as long as expected profits are larger than the entry costs. As such the sum of expected domestic and expected export profits must equal the entry costs in equilibrium. It is helpful to split profits into profits that a firm would realise if it only would supply the domestic market $\pi(\theta)|_{I_X=0}$ and into profits that a firm would make selling on possibly both markets while excluding profits this firm would make if it only sells on the domestic market $\pi(\theta)|_{I_X=1} - \pi(\theta)|_{I_X=0}$. The free entry condition then takes the following form:

$$\int_{\theta_d}^{\infty} [\pi(\theta)|_{I_X=0}] \; dG_\theta(\theta) + \int_{\theta_x}^{\infty} [\pi(\theta)|_{I_X=1} - \pi(\theta)|_{I_X=0}] \; dG_\theta(\theta) = f_E. \tag{2.13}$$

Using the profit equation (2.7) together with revenues as a function of the cutoff productivity (2.11) as well as the revenues of the cutoff firm (2.B.15), domestic profits with no exporting can be written as:

$$\begin{aligned}
\pi(\theta)|_{I_X=0} &= \kappa_f r(\theta)|_{I_X=0} - f_d - I_x|_{I_X=0} f_x \\
&= \kappa_f \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} \Upsilon_x(\theta)^{\frac{1-\beta}{1-\beta\gamma}}|_{I_X=0} \; r(\theta_d) - f_d = \kappa_f \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} \frac{f_d}{\kappa_f} - f_d \\
&= f_d \left[\left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} - 1\right].
\end{aligned} \tag{2.B.31}$$

It follows using the relationship between the domestic an export cutoff (2.10) that:

$$\begin{aligned}
\pi(\theta)|_{I_X=1} - \pi(\theta)|_{I_X=0} &= \kappa_f r(\theta)|_{I_X=1} - f_d - I_x|_{I_X=1} f_x - f_d \left[\left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} - 1\right] \\
&= \kappa_f \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} \Upsilon_x(\theta)^{\frac{1-\beta}{1-\beta\gamma}}|_{I_X=1} \; r(\theta_d) - f_x - f_d \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} \\
&= \left(\frac{\theta}{\theta_x}\right)^{\frac{\beta}{1-\beta\gamma}} \frac{\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}}}{\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1} f_x - f_x - f_x \frac{1}{\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1} \left(\frac{\theta}{\theta_x}\right)^{\frac{\beta}{1-\beta\gamma}} \\
&= \left[\left(\frac{\theta}{\theta_x}\right)^{\frac{\beta}{1-\beta\gamma}} - 1\right] f_x.
\end{aligned} \tag{2.B.32}$$

The free entry condition (2.13) can be rewritten using (2.B.31) and (2.B.32) as:

$$\begin{aligned}
\int_{\theta_d}^{\infty} [\pi(\theta)|_{I_X=0}] \; dG_\theta(\theta) + \int_{\theta_x}^{\infty} [\pi(\theta)|_{I_X=1} - \pi(\theta)|_{I_X=0}] \; dG_\theta(\theta) = f_E \\
f_d \int_{\theta_d}^{\infty} \left[\left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} - 1\right] \; dG_\theta(\theta) + f_x \int_{\theta_x}^{\infty} \left[\left(\frac{\theta}{\theta_x}\right)^{\frac{\beta}{1-\beta\gamma}} - 1\right] \; dG_\theta(\theta) = f_E.
\end{aligned} \tag{2.14}$$

Using the pareto distribution assumption it follows:

$$f_d \int_{\theta_d}^{\infty} \left[\left(\frac{\theta}{\theta_d} \right)^{\frac{\beta}{1-\beta\gamma}} - 1 \right] \theta^{-(z+1)} \, d\theta + f_x \int_{\theta_x}^{\infty} \left[\left(\frac{\theta}{\theta_x} \right)^{\frac{\beta}{1-\beta\gamma}} - 1 \right] \theta^{-(z+1)} \, d\theta = \frac{f_E}{z\theta_{min}^z}$$

$$f_d \left[\left(\frac{\beta}{1-\beta\gamma} - z \right)^{-1} \theta^{\frac{\beta}{1-\beta\gamma}-z} \theta_d^{-\frac{\beta}{1-\beta\gamma}} + \frac{1}{z}\theta^{-z} \right]_{\theta_d}^{\infty}$$

$$+ f_x \left[\left(\frac{\beta}{1-\beta\gamma} - z \right)^{-1} \theta^{\frac{\beta}{1-\beta\gamma}-z} \theta_x^{-\frac{\beta}{1-\beta\gamma}} + \frac{1}{z}\theta^{-z} \right]_{\theta_x}^{\infty} = \frac{f_E}{z\theta_{min}^z}.$$

With $\frac{\beta}{1-\beta\gamma} < z$ it follows that $(\infty)^{\frac{\beta}{1-\beta\gamma}-z} \rightarrow 0$. The above free entry condition reduces to:

$$-f_d \left(\left(\frac{\beta}{1-\beta\gamma} - z \right)^{-1} \theta_d^{\frac{\beta}{1-\beta\gamma}-z} \theta_d^{-\frac{\beta}{1-\beta\gamma}} + \frac{1}{z}\theta_d^{-z} \right)$$

$$-f_x \left(\left(\frac{\beta}{1-\beta\gamma} - z \right)^{-1} \theta_x^{\frac{\beta}{1-\beta\gamma}-z} \theta_x^{-\frac{\beta}{1-\beta\gamma}} + \frac{1}{z}\theta_x^{-z} \right) = \frac{f_E}{z\theta_{min}^z}$$

$$f_d\theta_d^{-z} \left(\left(\frac{z(1-\beta\gamma)-\beta}{1-\beta\gamma} \right)^{-1} - \frac{1}{z} \right) + f_x\theta_x^{-z} \left(\left(\frac{z(1-\beta\gamma)-\beta}{1-\beta\gamma} \right)^{-1} - \frac{1}{z} \right) = \frac{f_E}{z\theta_{min}^z}.$$

Using the relationship between the two cutoffs (2.10) yields:

$$\left(\frac{1-\beta\gamma}{z(1-\beta\gamma)-\beta} - \frac{1}{z} \right) \left[f_d\theta_d^{-z} + f_x \left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1 \right)^{z\frac{1-\beta\gamma}{\beta}} \left(\frac{f_d}{f_x} \right)^{z\frac{1-\beta\gamma}{\beta}} \theta_d^{-z} \right] = \frac{f_E}{z\theta_{min}^z}$$

$$\frac{\theta_{min}^z}{f_E} \left(\frac{z(1-\beta\gamma)}{z(1-\beta\gamma)-\beta} - 1 \right) \left[f_d + f_x \left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1 \right)^{z\frac{1-\beta\gamma}{\beta}} \left(\frac{f_d}{f_x} \right)^{z\frac{1-\beta\gamma}{\beta}} \right] = \theta_d^z$$

$$\left(\frac{\beta}{z(1-\beta\gamma)-\beta} \right) \frac{f_d + f_x \left(\frac{f_d}{f_x} \left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1 \right) \right)^{z\frac{1-\beta\gamma}{\beta}}}{f_E} \theta_{min}^z = \theta_d^z.$$

Using the productivity distribution assumption and the derived relationship between the domestic and export cutoff (2.10) one can solve for the domestic cutoff as a function of exogenous parameters:

$$\theta_d = \left[\frac{\frac{\beta}{1-\beta\gamma}}{z - \frac{\beta}{1-\beta\gamma}} \frac{f_d + f_x \left(\frac{f_d}{f_x} \left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1 \right) \right)^{\frac{z(1-\beta\gamma)}{\beta}}}{f_E} \right]^{\frac{1}{z}} \theta_{min}. \qquad (2.15)$$

In order to ensure that the productivity cutoff is positive it must hold that $z > \frac{\beta}{1-\beta\gamma}$.

2.B.7 General Equilibrium

In order to derive the general equilibrium conditions it is helpful to use the expected wage income which is given by the probability of being hired times the paid wage $\omega = wH/L = bx$ where we use that wages are equal to search costs $w = b$ (2.B.26). Using the definition of the search costs $b = x^{\alpha}$ allows to write labour market tightness (x) an search costs (b) as:

$$\begin{aligned} x &= \frac{\omega}{b} = \frac{\omega}{x^{\alpha}} & b &= x^{\alpha} = \left(\frac{\omega}{b}\right)^{\alpha} \\ x^{1+\alpha} &= \omega & b^{1+\alpha} &= \omega^{\alpha} \\ x &= \omega^{\frac{1}{1+\alpha}}\ , & b &= \omega^{\frac{\alpha}{1+\alpha}}. \end{aligned} \tag{2.16}$$

Total expenditure is given by Y where the real consumption index is depicted by Q. Using the normalisation of the price index it holds that $PQ = Q = Y$. The domestic demand shifter can be written as $A = Y^{1-\beta}P^{\beta} = Q^{1-\beta}P = Q^{1-\beta}$. For Foreign it holds that $P^*\ Q^* = Y^*$ and thus the demand shifter can be written as $A^* = {Y^*}^{1-\beta}{P^*}^{\beta} = {Q^*}^{1-\beta}P^*$. As both countries are symmetric the price index in foreign is also one and thus the demand shifter given by $A^* = {Q^*}^{1-\beta} = A$. Using the relationship between the consumption index and the demand shifter one can rewrite the zero profit condition of the cutoff firm (2.B.29) and solve for Q:

$$f_d = \kappa_f \left[A\kappa_w^{\beta\gamma}\theta_d^{\beta}b^{-\beta\gamma}\right]^{\frac{1}{1-\beta\gamma}} \tag{2.B.29}$$

$$A^{\frac{1}{1-\beta\gamma}} = \frac{f_d}{\kappa_f}\left[\kappa_w^{\beta\gamma}\theta_d^{\beta}b^{-\beta\gamma}\right]^{-\frac{1}{1-\beta\gamma}}$$

$$Q = A^{\frac{1}{1-\beta}} = \left(\frac{f_d}{\kappa_f}\right)^{\frac{1-\beta\gamma}{1-\beta}} \kappa_w^{-\frac{\beta\gamma}{1-\beta}}\theta_d^{-\frac{\beta}{1-\beta}}b^{\frac{\beta\gamma}{1-\beta}}. \tag{2.B.33}$$

Using (2.16) one can write the consumption index as:

$$Q = A^{\frac{1}{1-\beta}} = \left(\frac{f_d}{\kappa_f}\right)^{\frac{1-\beta\gamma}{1-\beta}} \kappa_w^{-\frac{\beta\gamma}{1-\beta}}\theta_d^{-\frac{\beta}{1-\beta}}\omega^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}}. \tag{2.17}$$

In equilibrium overall expected wage income in a country has to equal the sum of all wages paid by firm in that country:

$$\omega L = M\int_{\theta_d}^{\infty} w(\theta)h(\theta)\ dG_{\theta}(\theta) = \kappa_w M\int_{\theta_d}^{\infty} r(\theta)\ dG_{\theta}(\theta). \tag{2.B.34}$$

The consumption index can be written as $Q = QP = M\int_{\theta_d}^{\infty} r(\theta)\ dG_{\theta}(\theta)$. Thus, the labour market condition follows as:

$$\omega L = \kappa_w M\int_{\theta_d}^{\infty} r(\theta)\ dG_{\theta}(\theta) = \kappa_w Q. \tag{2.18}$$

Expected wage can be derived using the general equilibrium conditions (2.18) and (2.17):

$$\begin{aligned}
\kappa_w^{-1}\omega L &= \left(\frac{f_d}{\kappa_f}\right)^{\frac{1-\beta\gamma}{1-\beta}} \kappa_w^{-\frac{\beta\gamma}{1-\beta}} \theta_d^{-\frac{\beta}{1-\beta}} \omega^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}} \\
\omega^{\frac{1-\beta-\frac{\alpha}{1+\alpha}\beta\gamma}{1-\beta}} &= \left(\frac{f_d}{\kappa_f}\right)^{\frac{1-\beta\gamma}{1-\beta}} \kappa_w^{\frac{1-\beta-\beta\gamma}{1-\beta}} \theta_d^{-\frac{\beta}{1-\beta}} L^{-1} \\
\omega &= \left(\frac{f_d}{\kappa_f}\right)^{-\frac{1-\beta\gamma}{\Delta}} \kappa_w^{-\frac{1-\beta-\beta\gamma}{\Delta}} \theta_d^{\frac{\beta}{\Delta}} L^{\frac{1-\beta}{\Delta}} \\
\omega &= \kappa_o^{-\frac{1}{\Delta}} \theta_d^{\frac{\beta}{\Delta}} L^{\frac{1-\beta}{\Delta}}, \qquad (2.19)
\end{aligned}$$

with $\kappa_o \equiv \kappa_w^{1-\beta-\beta\gamma}\left(\frac{f_d}{\kappa_f}\right)^{1-\beta\gamma}$ and we assume $\Delta \equiv -\left(1-\beta-\frac{\alpha}{1+\alpha}\beta\gamma\right) > 0$ to be positive in order to have a stable equilibrium. Thus, the elasticity of substitution between varieties has to be sufficiently high (high β but less then one). It also implies that $-(1-\beta-\beta\gamma) > 0$ as $\frac{\alpha}{1+\alpha} < 1$. Additionally, for further derivation it is helpful to depict the following relationships:

$$\begin{aligned}
(1+\alpha)\Delta &= (1+\alpha)\left[-\left(1-\beta-\frac{\alpha}{1+\alpha}\beta\gamma\right)\right] \\
&= -\left[(1+\alpha)(1-\beta)-\beta\gamma\alpha\right] > 0 \qquad (2.B.35) \\
\frac{\alpha}{1+\alpha}\frac{1}{\Delta} &= \frac{\alpha}{1+\alpha}\frac{1}{-\left(1-\beta-\frac{\alpha}{1+\alpha}\beta\gamma\right)} \\
&= \frac{1}{-\left(\frac{1+\alpha}{\alpha}(1-\beta)-\beta\gamma\right)} > 0. \qquad (2.B.36)
\end{aligned}$$

Using the result from (2.17) and (2.19) the equilibrium consumption index is given by:

$$\begin{aligned}
Q &= \left(\frac{f_d}{\kappa_f}\right)^{\frac{1-\beta\gamma}{1-\beta}} \kappa_w^{-\frac{\beta\gamma}{1-\beta}} \theta_d^{-\frac{\beta}{1-\beta}} \omega^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}} \qquad (2.17) \\
&= \left(\left(\frac{f_d}{\kappa_f}\right)^{1-\beta\gamma} \kappa_w^{1-\beta-\beta\gamma}\kappa_w^{-(1-\beta)}\right)^{\frac{1}{1-\beta}} \theta_d^{-\frac{\beta}{1-\beta}} \left(\kappa_o^{-\frac{1}{\Delta}}\theta_d^{\frac{\beta}{\Delta}} L^{\frac{1-\beta}{\Delta}}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}} \\
&= \left(\left(\frac{f_d}{\kappa_f}\right)^{1-\beta\gamma} \kappa_w^{1-\beta-\beta\gamma}\right)^{\frac{1}{1-\beta}} \kappa_w^{-1}\kappa_o^{-\frac{\alpha}{1+\alpha}\frac{1}{\Delta}\frac{\beta\gamma}{1-\beta}} \theta_d^{-\frac{\beta}{1-\beta}} \theta_d^{\frac{\alpha}{1+\alpha}\frac{\beta}{\Delta}\frac{\beta\gamma}{1-\beta}} L^{\frac{\alpha}{1+\alpha}\frac{1-\beta}{\Delta}\frac{\beta\gamma}{1-\beta}} \\
&= \kappa_o^{\frac{1}{1-\beta}-\frac{\alpha}{1+\alpha}\frac{1}{\Delta}\frac{\beta\gamma}{1-\beta}} \kappa_w^{-1}\theta_d^{\frac{\beta}{1-\beta}\left(\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}-1\right)} L^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}} \\
&= \kappa_o^{\frac{1}{1-\beta}\left(1-\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}\right)} \kappa_w^{-1}\theta_d^{\frac{\beta}{1-\beta}\frac{1-\beta}{\Delta}} L^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}} \\
&= \kappa_o^{-\frac{1}{\Delta}}\kappa_w^{-1}\theta_d^{\frac{\beta}{\Delta}} L^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}}, \qquad (2.20)
\end{aligned}$$

where we use $\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}-1 = \frac{\frac{\alpha}{1+\alpha}\beta\gamma-\left(-\left(1-\beta-\frac{\alpha}{1+\alpha}\beta\gamma\right)\right)}{\Delta} = \frac{1-\beta}{\Delta}$. Using the result from (2.19) and (2.16) the equilibrium search costs as well as equilibrium labour market tightness can be determined:

$$x = \omega^{\frac{1}{1+\alpha}} \qquad = \kappa_o^{-\frac{1}{(1+\alpha)\Delta}} \theta_d^{\frac{\beta}{(1+\alpha)\Delta}} L^{\frac{1-\beta}{(1+\alpha)\Delta}} \tag{2.21}$$

$$b = \omega^{\frac{\alpha}{1+\alpha}} \qquad = \kappa_o^{-\frac{\alpha}{1+\alpha}\frac{1}{\Delta}} \theta_d^{\frac{\alpha}{1+\alpha}\frac{\beta}{\Delta}} L^{\frac{\alpha}{1+\alpha}\frac{1-\beta}{\Delta}}, \tag{2.22}$$

where $(1+\alpha)\Delta = (\beta + \beta\gamma - 1)\alpha - (1-\beta)$ and $\frac{1+\alpha}{\alpha}\Delta = \beta\gamma - (1-\beta)\frac{1+\alpha}{\alpha}$.

The number of firms can be determined using the equilibrium condition equating overall expected wage income with the sum of wages paid by firms:

$$\omega L = \kappa_w M \int_{\theta_d}^{\infty} r(\theta)\, dG_\theta(\theta) \tag{2.18}$$

$$\begin{aligned} M &= \frac{\omega L}{\kappa_w}\left[\int_{\theta_d}^{\infty} r(\theta)\, dG_\theta(\theta)\right]^{-1} \\ &= \frac{\omega L}{\kappa_w}\left[\int_{\theta_d}^{\infty} r(\theta)|_{I_x(\theta)=0}\, dG_\theta(\theta) + \int_{\theta_x}^{\infty} r(\theta)|_{I_x(\theta)=1} - r(\theta)|_{I_x(\theta)=0}\, dG_\theta(\theta)\right]^{-1}. \end{aligned} \tag{2.B.37}$$

Using revenues (2.11), revenues of a firm which is not exporting follow as:

$$r(\theta)|_{I_x(\theta)=0} = \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} \frac{f_d}{\kappa_f}. \tag{2.B.38}$$

Using the relationship between the two cutoffs (2.10) yields:

$$\begin{aligned} r(\theta)|_{I_x(\theta)=1} - r(\theta)|_{I_x(\theta)=0} &= \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} \frac{f_d}{\kappa_f}\left[\Upsilon_x(\theta)^{\frac{1-\beta}{1-\beta\gamma}} - 1\right] \\ &= \left(\frac{\theta}{\theta_x}\right)^{\frac{\beta}{1-\beta\gamma}} \left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1\right)^{-1} \left(\frac{f_d}{f_x}\right)^{-1} \frac{f_d}{\kappa_f}\left[\Upsilon_x(\theta)^{\frac{1-\beta}{1-\beta\gamma}} - 1\right] \\ &= \left(\frac{\theta}{\theta_x}\right)^{\frac{\beta}{1-\beta\gamma}} \frac{f_x}{\kappa_f}. \end{aligned} \tag{2.B.39}$$

Using (2.B.38) and (2.B.39) in (2.B.37) yields:

$$\begin{aligned} M =& \frac{\omega L}{\kappa_w}\left[\int_{\theta_d}^{\infty} \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{1-\beta\gamma}} \frac{f_d}{\kappa_f}\, dG_\theta(\theta) + \int_{\theta_x}^{\infty} \left(\frac{\theta}{\theta_x}\right)^{\frac{\beta}{1-\beta\gamma}} \frac{f_x}{\kappa_f}\, dG_\theta(\theta)\right]^{-1} \\ =& \frac{\omega L \kappa_f}{z\kappa_w \theta_{min}^z}\left[f_d \theta_d^{-\frac{\beta}{1-\beta\gamma}} \int_{\theta_d}^{\infty} \theta^{\frac{\beta}{1-\beta\gamma}-(z+1)}\, d\theta + f_x \theta_x^{-\frac{\beta}{1-\beta\gamma}} \int_{\theta_x}^{\infty} \theta^{\frac{\beta}{1-\beta\gamma}-(z+1)}\, d\theta\right]^{-1} \\ M =& \frac{\omega L \kappa_f}{z\kappa_w \theta_{min}^z}\left[f_d \theta_d^{-\frac{\beta}{1-\beta\gamma}} \left[\left(\frac{\beta}{1-\beta\gamma} - z\right)^{-1} \theta^{\frac{\beta}{1-\beta\gamma}-z}\right]_{\theta_d}^{\infty}\right. \\ &\left. + f_x \theta_x^{-\frac{\beta}{1-\beta\gamma}} \left[\left(\frac{\beta}{1-\beta\gamma} - z\right)^{-1} \theta^{\frac{\beta}{1-\beta\gamma}-z}\right]_{\theta_x}^{\infty}\right]^{-1}, \end{aligned}$$

with $\frac{\beta}{1-\beta\gamma} < z$ it follows that $(\infty)^{\frac{\beta}{1-\beta\gamma}-z} \rightarrow 0$. The number of firms is given by:

$$\begin{aligned} M =& \frac{\omega L \kappa_f}{z \kappa_w \theta_{min}^z} \left(\frac{\beta}{1-\beta\gamma} - z \right) \left[-f_d \theta_d^{-\frac{\beta}{1-\beta\gamma}} \theta_d^{\frac{\beta}{1-\beta\gamma}-z} - f_x \theta_x^{-\frac{\beta}{1-\beta\gamma}} \theta_x^{\frac{\beta}{1-\beta\gamma}-z} \right]^{-1} \\ =& \frac{\omega L \kappa_f}{z \kappa_w \theta_{min}^z} \left(\frac{z(1-\beta\gamma)-\beta}{1-\beta\gamma} \right) \left[f_d \theta_d^{-z} - f_x \theta_x^{-z} \right]^{-1}. \end{aligned}$$

Using the relationship between the two cutoffs (2.10) and the definition of κ_w and κ_f yields:

$$\begin{aligned} M =& \frac{\omega L}{z \theta_{min}^z} \frac{1-\beta\gamma}{\beta\gamma} \left(\frac{z(1-\beta\gamma)-\beta}{1-\beta\gamma} \right) \left[f_d \theta_d^{-z} + f_x \left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1 \right)^{z\frac{1-\beta\gamma}{\beta}} \left(\frac{f_d}{f_x} \right)^{z\frac{1-\beta\gamma}{\beta}} \theta_d^{-z} \right]^{-1} \\ =& \frac{\omega L}{\beta\gamma} \left(\frac{z(1-\beta\gamma)-\beta}{z} \right) \left(\frac{\theta_d}{\theta_{min}} \right)^z \left[f_d + f_x \left(\Upsilon_x^{\frac{1-\beta}{1-\beta\gamma}} - 1 \right)^{z\frac{1-\beta\gamma}{\beta}} \left(\frac{f_d}{f_x} \right)^{z\frac{1-\beta\gamma}{\beta}} \right]^{-1}. \end{aligned}$$

Using the domestic cutoff (2.15) allows to write the number of firms as follows:

$$M = \frac{\omega L}{\beta\gamma f_E} = \frac{1}{\beta\gamma f_E} \kappa_o^{-\frac{1}{\Delta}} \theta_d^{\frac{\beta}{\Delta}} L^{1+\frac{1-\beta}{\Delta}} = \frac{1}{\beta\gamma f_E} \kappa_o^{-\frac{1}{\Delta}} \theta_d^{\frac{\beta}{\Delta}} L^{\frac{\alpha\beta\gamma}{\Delta}}. \tag{2.23}$$

2.C Nomenclature

α search cost elasticity with respect to the labour market tightness

α_0 search technology parameter

β controls the elasticity of substitution where $\sigma \equiv \frac{1}{1-\beta}$ is the elasticity of substitution

Δ, Δ_2 help variables; $\Delta \equiv -\left(1 - \beta - \frac{\alpha}{1+\alpha}\beta\gamma\right) > 0$, $\Delta_2 \equiv -\frac{\alpha(1-\beta-\beta\gamma)}{(1+\alpha)\Delta} > 1$

γ degree of diminishing returns to labour ($0 < \gamma < 1$)

κ_f share of revenues belonging the firm after paying wages and search costs; $\kappa_f \equiv \frac{1-\beta\gamma}{1+\beta\gamma}$

κ_o help variable; $\kappa_o \equiv \kappa_w^{1-\beta-\beta\gamma} \left(\frac{f_d}{\kappa_f}\right)^{1-\beta\gamma}$

κ_w share of revenues belonging to the workforce; $\kappa_w \equiv \frac{\beta\gamma}{1+\beta\gamma}$

ω expected wage income

ψ_0 costs of posting vacancies

ψ_1 technology parameter in the search function

ψ_2 cobb douglas weight for vacancies in the search function

τ iceberg type trade costs ($\tau > 1$)

θ variety index and firm's productivity

θ_{min} lower bound of the pareto productivity distribution

∇ help paramete; $\nabla \equiv \frac{\beta+\beta\alpha-1}{(1+\alpha)\Delta}$

$\Upsilon_x(\theta)$ firm's export market access ($\Upsilon_x(\theta) = \Upsilon_x \geq 1$)

ϑ, ν indices for varieties

A demand shifter in a country; $A \equiv Y^{1-\beta} P^{\beta}$

b search costs per worker hired

f_d fixed costs of domestic production

f_E fixed costs of entry

f_x fixed costs of exporting

$G_\theta(\theta)$ pareto productivity distribution function

H number of individuals hired in a country

h measure of workers hired by a firm

$I_x(\theta)$ firm specific indicator variable being one if a firm decides to export ($I_x = \{0, 1\}$)

L potential work force in a country

M set of varieties within a country

P price index

$p(\vartheta)$ price of variety ϑ

Q real consumption index

$q(\vartheta)$ consumption/production of firm producing variety ϑ

$r(\vartheta)$ revenues of firm producing variety ϑ

$r_d(\theta)$ revenues of domestic sales

$r_x(\theta)$ revenues from exporting

u unemployment rate

V number of vacancies

x labour market tightness

Y total expenditure

$y(\theta)$ output of a firm with productivity θ

$y_d(\theta)$ output of firm θ sold on the domestic market

$y_x(\theta)$ output of firm θ shipped to the export market

z shape parameter of the pareto productivity distribution

Chapter 3

Wage Bargaining Improvements and the Export Decision

Abstract: *This paper emphasises the possibility of firms to influence their bargaining power in the intrafirm wage bargaining process and highlights the interplay of both the export and the bargaining power improvement decision of a firm. A model of international trade with labour market imperfection, ex ante heterogeneous workers and two occupation types differing in the way how firms can influence their bargaining position in the wage bargaining process is developed. The theoretical framework predicts that the possibility and the amount of a firm's improvement in their bargaining power can rise the share of exporting firms. This not only implies differences in labour market variables such as employment and wages within and across firms itself, but also affects those variables due to changes in the export decision.*

3.1 Introduction

The implications of international trade on labour market variables such as wages and employment are one of the most important topics in the field of international economics and are a prominent topic in the public debate. It is a common understanding that in wage negotiations both workers and firms are intersected in improving their bargaining position. The theoretical approach developed in this paper considers how the incentive of firms to improve profits by increasing their bargaining position in the wage bargaining stage alters other decisions made by firms such as the number of workers employed, the amount of wages paid or the ability of workers employed. The paper emphasise that the firms' extensive decision of exporting and improving the bargaining position can influences each other.

In the literature concerned with wage determination between workers and firms an important strand considers how workers and firms are able to influence the wages setting process. Those studies focusing on the way workers influence the wage setting process highlight the importance of unions. For instance, related to the findings of this paper, de Pinto and Lingens (2019) show that in a closed economy the cost that arises, if workers organise in unions can change both average productivity and wage inequality. In the international context de Pinto and Michaelis (2019) assume union heterogeneity across firms in a Melitz (2003) type model. They show that a symmetric increase in the bargaining power of unions has a smaller employment effect due to heterogeneity of union power across firms.

Versatile explanations and mechanisms how firms can influence the wage setting process in their favour are offered by the literature. They range from different payment schemes, such as an equity based compensation, over the possibility of firms strengthening their bargaining position by improving the job satisfaction and working environment, to firm restructuring and debt financing of firms (Brown, 1980; Matsa, 2010; Rosen, 1987; Yermack and Ofek, 2000). In the following, some exemplary works are highlighted which are related to the approach taken in this paper. Dossche et al. (2019) use an intra firm bargaining framework similar to the one used in this paper and show that if firms can choose the number of hours worked by each worker, they will reduce the number of hours worked per worker in order to reduce the bargaining wage paid. In the international context there are several papers that emphasise that firms have an incentive to invest abroad, and therby improve their bargaining power in the wage negotiations. For instance, Eckel and Egger (2009) develop a framework with collective bargaining between unions and heterogeneous firms. They show how economic integration and changes in the bargaining power of unions affect labour income and the unemployment rate. Carluccio and Bas (2015) use a union framework and find both theoretical and empirical evidence that fragmentation of the production process across boarders is especially profitable, if the worker bargaining power is high because the fragmentation process weakens the bargaining positions of workers. Adding to this literature strand, the focus of this study lies on the interaction between the export and the bargaining improvement decision of firms and their implications for the labour market.

The developed theoretical framework concentrates on the firms perspective and neglects the possibility of workers to improve their bargaining power. The framework abstracts from the internal mechanism how firms can improve their bargaining position and simply assumes that firms can pay a certain cost in order to improve their bargaining power. The theoretical framework developed in this paper builds on the work by Helpman and Itskhok (2010) and Helpman et al. (2010) who develop a traceable model of international trade, featuring labour market imperfection in form of search and matching friction as proposed by Mortensen and Pissarides (1994) and firm heterogeneity.[1] Like the paper by Helpman et al. (2010), the developed framework allows for ex ante heterogeneity in the worker's ability and thus firms have the possibility to screen the workforce and will differ not only in their productivity but also in the ability of their workers. Wages are determined by dividing revenues between workers and a firm, using a generalisation of the Nash bargaining to a multi worker case as proposed by Stole and Zwiebel (1996a,b). Hereby a firm bargains bilaterally with each worker. The developed framework contributes to the literature by introducing the endogenous possibility for firms to influence their bargaining power with respect to workers in the wage bargaining process. This implies that firms cannot only differ in their export decision but also in their decision whether to improve their bargaining power or not.

In particular it is shown that firms will sort into exporting, improving their bargaining power or both according to their productivity. As in a standard Melitz (2003) type framework and in line with a broad empirical evidence, assuming a respective cost structure, the most productive firms will export and in this case also improve their bargaining power. The framework also allows for the existence of an intermediate productivity range where firms either export or improve their bargaining power without doing the other. Which of the different sorting patterns applies depends on the costs and gains of both exporting and the improvement of bargaining power. Firms that export and or improve their bargaining power choose higher ability thresholds, sample and hire more workers, and generate higher revenues and profits.

[1]The literature in the field of international trade offers three main approaches that introduce labour market imperfections. The fair wage mechanism as proposed by Egger and Kreickemeier (2009a) Egger and Kreickemeier (2009b) and Amiti and Davis (2012), the efficiency wage aproach proposed by Davis and Harrigan (2011) and the search and matching approach chosen in this paper and introduced by (Helpman and Itskhok, 2010). As the search and matching approach directly assumes a certain bargaining power between workers and a firm the choice if this framework is straight forward as it allows a simple way o introducing the possibility of firms to improve their wage setting position.

While it is expected that the share of firms engaging in improving their bargaining power rises, the larger the possible gains from improving the bargaining power is, the paper shows that also the share of exporting firms rises, if the sorting pattern implies that the exporting firms also improve their bargaining power. Similarly, the share of firms that improve their bargaining power can rise, if the incentives to export rise and the exporting firms also improve their bargaining power. As such this introduces an new channel how trade liberalisations can rise within group wage inequalities, which is what different empirical studies find (Attanasio et al., 2004; Menezes-Filho et al., 2008). Song et al. (2018), for example, use US Data and show that wage differences of similar workers across different firms are substantial and contribute to a raising wage inequality.

In order to allow for heterogeneous bargaining powers across occupations two occupation specific worker types, referred to as type-1 and type-2, are introduced. Worker types can differ in the degree the bargaining power can be influenced by firms. While the framework generally allows that the bargaining power with respect to both worker types can be influenced, this paper aims at highlighting the implications of heterogeneity in terms of bargaining power across occupations and therefore a situation where only the bargaining power with respect to type-1 workers can be improved by firms is considered.[2] This implies that the possibility of improving the bargaining power creates heterogeneity between firms and within firms across worker types. The framework predicts that the intrafirm bargaining process between the firm and each worker implies that the share of revenues belonging to the type-1 workers falls, because of a larger bargaining power of the firm. Furthermore, also the share of revenues belonging to type-2 workers rises. Thus, the share of firm revenues belonging to type-2 workers relative to type-1 worker rises. This implies that wages of type-2 workers, which are not directly affected by the change in bargaining power, rise, if firms improve their bargaining power with respect to type-1 workers. The effect on wages for type-1 workers is ambiguous as not only the share of revenues belonging to type-1 workers declines, but also the overall revenues bargained over rise. Which of the two effects dominates depends on the extend of the bargaining power improvement. Given that not all firms improve their bargaining power those findings highlight different channels creating wage inequality within and across firms.

This paper also evaluates the effect of an increase in the possible bargaining power improvement by firms on average and aggregate variables in an economy. An increase in the possible bargaining power improvement in general equilibrium results in more active firms having a higher average productivity generating higher overall revenues and thus rising the consumption index. At the same time, the share of overall workers matched to firms and thus also the search cost for both types of workers rises. However, the share of work-

[2] In order to simplify terminology, improvement in the bargaining power will always refer to a situation where only the bargaining power with respect to type-1 workers is affected.

ers employed after being matched falls for both types of workers as firms choose a higher average worker ability. Thus, an increase in the possible bargaining power improvement has an ambiguous effect on the unemployment rate which can rise or fall depending on which of the two effects dominates. The paper shows that overall wage income of the worker type facing no bargaining power change rises due to the possibility of improving the bargaining power both in absolute terms and relative to overall wage income of the other worker type. At the same time, the share of overall revenues in a country belonging to the worker type facing the bargaining power improvement of the firm falls. The effect on overall wage income for this worker type depends on whether the falling wage share or the effect of the rising overall revenues dominates.

The remainder of this paper is structured in the following manner. Section 3.2 describes the model framework and discusses the wage bargaining, as well as the firm's decision. Section 3.3 discusses the sorting pattern based on the extensive decision of exporting and improving the bargaining power and evaluates the effect of the two decisions on firm variables. Closing the model in general equilibrium, Section 3.4 highlights the implications of the bargaining power improvement for country wide outcomes. Finally, Section 3.5 concludes and summarises the findings. The Appendix 3.A covers crucial proofs and additional illustrations, while a full derivation of the framework can be found in the Supplementary Appendix 3.B and the Nomenclature 3.C lists all variables and their definitions.

3.2 Model Framework

This section develops the theoretical model and derives the first consequences of the possibility of firms to improve their bargaining power for the different firm variables such as wages, employment and firm revenues. The model builds on the imperfect labour market trade model by Helpman et al. (2010) allowing for two types of workers each producing an occupation-specific task. Workers and firms engage in strategic wage bargaining and this paper contributes to the literature by introduces the possibility for firms to invest in improving their bargaining power in the wage negotiations. A world with two countries, home and foreign is considered, where an asterisk is used to depict the foreign variables and only the expressions for home are depicted when analogous relationships hold for foreign. This model equips us with a deeper understanding of the interaction between labour market imperfections, employment and wages in an international framework.

3.2.1 Preferences

Individuals are risk neutral and demand is defined over the consumption of a continuum of horizontally differentiated varieties. Assuming constant elasticity of substitution between a continuum of varieties and using $q(\vartheta)$ to depict demand of a specific variety ϑ, the real consumption index is given by:

$$Q = \left[\int_0^M q(\vartheta)^\beta \, d\vartheta\right]^{1/\beta} \qquad 0 < \beta < 1, \tag{3.1}$$

where β controls the elasticity of substitution between different varieties being part of the set M of varieties in a country. As a result of the utility maximisation of individuals, the revenue of a firm in equilibrium is given by:

$$r(\vartheta) = p(\vartheta)q(\vartheta) = Aq(\vartheta)^\beta, \tag{3.2}$$

where $p(\vartheta)$ denotes prices of variety ϑ, $q(\vartheta) = A^{\frac{1}{1-\beta}} p(\vartheta)^{\frac{-1}{1-\beta}}$ is the quantity sold by a firm, and $A = Y^{1-\beta}P^\beta$ is the demand shifter of a firm. Because each firm is supplying one of a continuum of varieties they take the demand shifter as given when making decisions. P denotes the price index dual to Q, and Y is the total expenditure on the set of varieties consumed.

3.2.2 Labour Market

A country is populated by L_i, L_j individuals of type $i, j = \{1, 2\}, i \neq j$. Each type of individual can work and produce an occupation-specific task.[3] One can think of the two types as high and low skilled or simply two different occupation fields requiring different skill sets. As each worker type is only able to produce its specific task the indexes i, j are used to depict both the type of worker and the task in the production process. The key differentiation between the two labour types will be in the wage bargaining between workers and the firms, which will be explained in Subsection 3.2.4. Apart from that, the labour market is modelled along the lines of Helpman et al. (2010). Workers of each labour type are heterogeneous in terms of their ability. It is assumed that information on the ability attained in a worker-firm relationship cannot be used by the worker when negotiating with another firm. One possible interpretation of this assumption is that ability is match specific and thus has no information for any future match of a worker with a firm. The ability a is assumed to be independently distributed across workers and drawn form a pareto distribution $G_a(a) = 1 - \left(\frac{a_{\min}}{a}\right)^k$; for $a \geq a_{\min} > 0$. The lower bound of the ability distribution is depicted by $a_{\min}$ and $k > 1$ is the shape parameter. For simplicity and in order to make this framework analytically traceable the ability distribution for producing either of the two labour types is assumed to be the same.[4]

[3] A generalisation to a multi task setup is possible, but comes at the cost of additional complexity and does not bring further valuable insights with respect to the raised questions in this paper.

[4] Alternatively, the labour market can be modelled by assuming only one labour type and workers are randomly matched to the two tasks in the search and matching process. This modelling would imply a single labour market and no differentiation in terms of expected wages, labour market tightness or the

Ability Screening. Worker's ability cannot be observed free of cost. In particular, a firm can engage in costly screening and in return receive an imprecise measure of the worker's ability. A firm has to pay a screening cost of $ca_{ci}(\theta)^{\delta}/\delta$ units of the numéraire, in order to learn whether a worker of type-i is below or above a specific ability threshold $a_{ci}(\theta)$ chosen by the firm θ. Where $c > 0$ scales the screening cost and $\delta > 0$ determines the degree to which a higher threshold implies higher screening costs.[5] The parameter constraints imply that the cost structure of screening is assumed to be increasing in the ability threshold. One intuition for this cost structure is that a higher ability threshold means more complex tests and thus higher costs.

The number of workers hired $h_i(\theta)$ for the production of task i of worker type i by firm θ is given by the number of workers screened $n_i(\theta)$ times the share of workers above the ability threshold $a_{ci}(\theta)$:

$$h_i(\theta) = n_i(\theta)\left(1 - G_a(a_{ci}(\theta))\ da\right) = n_i(\theta)\frac{a_{\min}^k}{a_{ci}(\theta)^k}. \tag{3.3}$$

While the number of workers hired is increasing in the number of workers sampled a higher ability threshold has a negative effect on the hiring rate $(h_i(\theta)/n_i(\theta))$. The pareto distribution assumption leads to a positive linear relationship between the ability threshold chosen by the firm and the average ability of workers hired:

$$\bar{a}_i(\theta) = \frac{\int_{a_{ci}(\theta)}^{\infty} a\ g_a(a)\ da}{1 - G_a(a_{ci}(\theta))} = \frac{k}{k-1}a_{ci}(\theta). \tag{3.4}$$

The firm's screening decision is a trade-off between two opposing effects. A higher ability threshold implies a lower hiring rate and thus ceteris paribus reduces output, revenues and profits. At the same time a higher ability threshold increases the average ability of workers hired $(\bar{a})$, which will ceteris paribus imply a larger output, as will be explained when discussing the production technology.

Search and Matching Frictions. Transition form one job to another is assumed to be not free of frictions. These search and matching frictions are modelled in the standard Diamond-Mortensen-Pissarides approach. A firm has to pay $b_i n_i(\theta)$ units of the numéraire in order to match with a measure $n_i(\theta)$ workers. The search cost of matching with a type i worker, b_i is endogenously determined by the labour market tightness:

$$b_i = \frac{x_i^{\alpha}}{\alpha_0}, \tag{3.5}$$

search cost between the labour types.

[5]For simplicity it is assumed that the screening cost are of the same type for both types of workers, but an extension to a situation where screening cost differ is possible.

where the tightness of the labour market $x_i = N_i/L_i$ is the ratio between the number of type i workers matched with firms N_i and the number of type i workers searching for a job L_i. The parameters ($0 < \alpha_0 < 1$, $\alpha > 0$) are exogenous search technology parameters. While α_0 scales the impact of the labour market tightness, α measures how elastic the search costs react to change in the labour market tightness. For both parameters a higher level relates to a more efficient labour market, because for a given labour market tightness search costs are lower.[6] The unemployment rate of type-i workers will fall the tighter the labour market (x) and the larger the hiring rate (σ_i):

$$u_i = 1 - \frac{H_i}{L_i} = 1 - \frac{H_i}{N_i}\frac{N_i}{L_i} = 1 - \sigma_i x_i \tag{3.6}$$

where H_i is the number of workers hired in a country.

3.2.3 Production Technology, Export Decision and Revenues

Similar to Melitz (2003) a continuum of potential market entrants can pay up front entry costs f_E in order to learn about their firm specific productivity θ. Following a large strand of literature, it is assumed that productivities are independently distributed and drawn from a pareto distribution $G_\theta(\theta) = 1 - (\theta_{\min}/\theta)^z \quad \text{for} \quad \theta \geq \theta_{min} > 0$, with $z > 1$ being the distribution shape parameter and $\theta_{\min}$ being the lower bound of the productivty distribution.[7] As firms are uniquely identified by their productivity and each firm produces a unique variety, θ can be used as an firm index. Domestic production involves fixed costs $f_D > 0$. Deviating from Melitz (2003) the production of the differentiated final variety $y(\theta)$ uses two tasks i, j where $y_i(\theta)$ is the quantity of task i used in the production of firm θ. Both tasks are complements. The introduction of a second task allows to introduce different wage bargaining scenarios for the two worker types. This formulation is closley related to the extension of Helpman et al. (2010) allowing for observable worker heterogeneity across workers. However, as stated before there will be no heterogeneity in terms of abilities or search costs between worker types in this framework. The final production technology is positively dependent on the productivity θ of a firm and combines

[6]This relationship between search cost and labour market tightness can be derived from a constant returns to scale Cobb-Douglas matching function as proposed by Blanchard and Galí (2010). The scaling parameter α_0 then is negatively affected by the vacancy posting cost and positively related to the productivity of the matching technology. The parameter α is a function of the importance of vacancies in the matching function. A derivation of the underlying matching framework can be found in the Supplementary Appendix of Chapter 2.

[7]Eventhough the pareto distribution is used mainly due to its tracable nature in the theoretical modelling, there is empirical evidence that it is a good approximation of productivity distribution across firms. It allows for reasonable prediction for both firm size distribution and the upper tail of the observe wage distribution (Axtell, 2001; Del Gatto et al., 2006; Saez, 2001).

the two tasks in a Cobb-Douglas fashion:

$$y(\theta) = \eta_0 \theta \prod_i y_i(\theta)^{\eta_i}, \tag{3.7}$$

where $0 < \eta_i < 1$, with $\sum_i \eta_i = 1$ is the Cobb-Douglas weight of task i and thus determines the importance of the respective task in the production process. Scaling the production η_0 is chosen to simplify notation.[8]

Production of Occupation-Specific Tasks. A firm can produce the two tasks i, j with help of heterogeneous workers. The production of each task features decreasing returns to labour $0 < \gamma < 1$ and dependence on the average worker ability which can be influenced by the choice of the screening threshold:

$$y_i(\theta) = h_i(\theta)^{\gamma} \bar{a}_i(\theta). \tag{3.8}$$

This technology can be rationalised by human capital externalities across workers within firms. For instance, Moretti (2004) shows empirical evidence on human capital externalities within plants. A possible underlying framework where a maneger allocates a share of his time to each worker is illustraed in the Online Appendix of Helpman et al. (2010). Using the production technology for each task (3.8) in the overall production function (3.7) the production of a firm with productivity θ can be written as a function of the chosen average abilities and the number of workers hired for the two tasks:

$$y(\theta) = \eta_0 \theta \prod_i \left(h_i(\theta)^{\gamma} \bar{a}_i(\theta) \right)^{\eta_i}. \tag{3.9}$$

A firm with a higher productivity, more workers of either type or on average more able workers will ceteris paribus produce a larger output. Using the average ability (3.4) and the relationship between the number of workers screened and the number of workers hired (3.3) in (3.9), the production of a firm with productivity θ is as a function of the chosen ability thresholds and the number of workers screened for the two tasks:

$$y(\theta) = \eta_0 \kappa_y \theta \prod_i \left(n_i(\theta)^{\gamma} a_{ci}(\theta)^{1-\gamma k} \right)^{\eta_i}, \tag{3.10}$$

where $\kappa_y \equiv \frac{k}{k-1} a_{\min}^{\gamma k}$ is used to simplify notation. There is a positive relationship between the firm's output and the number of workers sampled. It is assumed that $0 < \gamma k < 1$, ensuring that firms have an incentive to screen. Economically this condition implies that either the diminishing returns have to be sufficiently strong (sufficiently small γ) or there needs to be a sufficiently high dispersion in the ability distribution (sufficiently low k).

[8] $\eta_0 \equiv \left(\sum_i \eta_i^{\eta_i}\right)^{-(\gamma+(1-\gamma k)/\delta)}$ scales the production and is chosen to simplify notation where the production parameter γ, will be introduced in the following section.

Export Decision. In order to export a firm has to pay fixed cost of exporting f_X. For each unit exported variable iceberg type trade cost $\tau > 1$ arise. Efficiently high fixed cost of exporting ensure that no firm will serve the export market without also supplying the domestic market. When supplying both markets the output $y(\theta)$ needs to be split between the domestic $y_D(\theta)$ and the export market $y_X(\theta)$. Firms do so by choosing the two quantities such that the marginal revenue is the same in both markets. Thus, firms are indifferent between selling an additional unit domestically or exporting it. Equilibrium demand of a variety has to be equal to the supply of that variety in a country. In combination with the results from the utility maximisation of individuals (3.2), revenues of domestic sales are given by $r_D(\theta) = Ay_D(\theta)^\beta$. Due to the iceberg trade cost the units produced for the foreign market translate into the quantity sold on the export market in the following fashion $y_X(\theta) = \tau q(\theta)$. Revenues from export thus are given by $r_X(\theta) = A^*\tau^{-\beta}y_X^\beta$. Choosing quantities such that the marginal revenues both markets are the same, the following relationship between the quantity produced for the two markets can be derived: $y_X(\theta) = \left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}} \tau^{-\frac{\beta}{1-\beta}} y_D(\theta)$.

Revenues. When depicting the firm's export decision $I_X(\theta) = \{0, 1\}$ is used as an indicator variable, which is equal to one, if the firm exports, otherwise it is zero. Using the before mentioned relation between production for the domestic and the foreign market in combination with the condition ensuring that all quantities produced are either sold domestically or exported if a firm decides to export $y(\theta) = y_D(\theta) + I_X(\theta)y_X(\theta)$ one can solve for total revenues of a firm as a function of total production:

$$r(\theta) \equiv r_D(\theta) + I_X(\theta)r_X(\theta) = \Upsilon_X(\theta)^{1-\beta}Ay(\theta)^\beta, \tag{3.11}$$

$$\text{where} \quad \Upsilon_X(\theta) \equiv 1 + I_X(\theta)\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}} \tau^{-\frac{\beta}{1-\beta}} \geq 1. \tag{3.12}$$

The variable Υ_X can be interpreted as the firm's foreign market access, which is one if a firm does not export and larger than one if a firm exports. The market access term differs across firms only due to the extensive decision of exporting but is the same for all domestic and all exporting firms. The foreign market access term increases when trade costs are low and if the relative size of the foreign market is larger. As one would expect the larger the market access term and the larger the firm's production the larger the revenues of a firm.

3.2.4 Wage Bargaining, Bargaining Improvement

Workers can bargain over their wage because from a firm's perspective replacing a particular worker would involve further costs in form of search and matching costs, as well as screening costs. Following Helpman and Itskhok (2010) and Helpman et al. (2010)

workers are assumed to engage in strategic wage bargaining which is a natural extension of Nash bargaining to a multi worker case as proposed by Stole and Zwiebel (1996a,b). Hereby firms take into account the wage effect on all workers caused by one worker leaving the firm.

When firms engage in a wage bargaining process firms as well as workers might want to influence their bargaining power to alter the wage negotiations in their favour. The focus of this study lies on the interaction between the export and the bargaining decision of firms and their implications for the labour market. As such this framework neglects that workers can try to improve their bargaining power. A firm might influence its bargaining power by offering additional benefits or improving the working environment. Also the way a firm is perceived by the employees regarding different topics such as job security, workers satisfaction or even environmental sustainability of a firm might influence the workers job decision. A firm which is perceived as a "good place to work" is equipped with a higher bargaining power since workers do not only care about the wage they earn.At the same time all these measures increasing the bargaining power involve costs for the firm.

Following this motivation. different to Helpman and Itskhok (2010) and Helpman et al. (2010) a firm can decide to invest and pay a fixed cost f_B to improve the bargaining parameter to $\lambda > 1$.[9] By default there are equal bargaining weights between a firm and the workers employed $\lambda_i(\theta)|_{I_B=0} = 1$. In order to depict the firm's decision of investing in improving the bargaining power the indicator $I_B(\theta) = \{0, 1\}$ is used. The process of improving the bargaining power is only possible for type-1 workers. In particular the bargaining parameter is given by:

$$\lambda_i(\theta) = \begin{cases} \lambda > 1, & \text{if } I_B(\theta) = 1 \wedge i = 1 \\ 1, & \text{if } I_B(\theta) = 0 \vee i = 2. \end{cases} \tag{3.13}$$

These strongly simplifying assumptions of the bargaining improvement process are chosen to allow for a simple framework while at the same time featuring an extensive margin of improving the bargaining power as well as allowing to differentiate between workers who face different bargaining powers within a firm.[10] To simplify terminology the situation

[9] The upper case B index refers to the investment decision of improving the bargaining power and not to the search cost on the labour market, which are referred to by lower case b.

[10] This framework also allows to evaluate a situation in which a firm is able to change the bargaining power for both labour types. Key to this analysis is to indicate how the possibility to influence the bargaining power results in a trade off between the different types of labour and the export decisions. As long as the two types of labour face different bargaining parameters (e.g due to a different structure), the modified framework would yield a qualitatively comparable trade off. Thus, the simplifying assumption that only the bargaining power with respect to type-1 worker can be influenced is taken. Instead of a fixed possible bargaining power improvement one might also want to allow firms to choose the bargaining power level. The cost of improving the bargaining power might also be related to the level of bargaining improvement, the number of firms or the ability of workers employed. However, this paper concentrates on the extensive decision of a firm whether to invest in improving the bargaining power or not. In

where a firm invests in improving its bargaining power with respect to type-1 workers will be referred to as "a firm improves its bargaining power".

When a firm has observed its productivity, it has to choose whether or not to produce, export and whether or not to improve its bargaining power. It also has to decide on the measure of workers to sample, and the screening ability threshold which determines the number of worker hired. Once all these decisions have been taken the firm bargains bilaterally with every worker over the division of revenues. As there is no information asymmetry in terms of ability the only aspect known by both parties is that the workers ability is larger than a_{ci}. A firm treats each worker as if they had the average ability of the workers employed $\bar{a}$. The different fixed costs as well as search and matching and the screening costs are all sunk when entering the bargaining stage. As such, all other arguments of firm profits are fixed. Hired workers' outside option is given by unemployment which for simplicity is normalized to zero. Plugging the production function into the revenue equation firm revenue can be shown to be continuous, increasing, and concave in $h_i(\theta)$ and can be written as:

$$r\left(\theta\right) = \Upsilon_X(\theta)^{1-\beta} A \eta_0^{\beta} \theta^{\beta} \prod_i h_i(\theta)^{\beta\gamma\eta_i} \bar{a}_i(\theta)^{\beta\eta_i}. \tag{3.14}$$

The bargaining parameter $\lambda_i(\theta)$ determines how the net surplus is divided between the bargaining parties in a pairwise meeting between a worker and the firm:[11]

$$\frac{\partial\left[r(\theta, h_i, h_j) - w_i(\theta, h_i, h_j)h_i(\theta) - w_i(\theta, h_i, h_j)h_i(\theta)\right]}{\partial h_i(\theta)} = \lambda_i(\theta) w_i(\theta, h_i). \tag{3.15}$$

If the bargaining weight between the worker and the firm is the same $\lambda_i(\theta) = 1$ the gains in terms of operating profits (revenues minus labour cost) from employing an additional worker for the firm are equal to the wage paid to the worker. Using this relationship (3.15) one can solve the bargaining game.[12] Wages paid to workers of type i are given by:

$$w_i(\theta) = \Lambda_{wi}(\theta) \frac{r(\theta)}{h_i(\theta)}, \tag{3.16}$$

where the share of revenues belonging to a certain type of worker is given by:

$$\Lambda_{wi}(\theta) \equiv \frac{\beta\gamma\eta_i\lambda_j(\theta)}{\beta\gamma\eta_i\lambda_j(\theta) + \beta\gamma\eta_j\lambda_i(\theta) + \lambda_i(\theta)\lambda_j(\theta)}. \tag{3.17}$$

addition, the effect of a change in the exogenously given possible bargaining improvement on the firm and labour market variables is evaluated. As such, it is abstracted from a more extensive modelling of the bargaining improvement.

[11] To simplify notation the dependence of the two types of employment on the productivity is dropped and $w_i(\theta, h_i, h_j)$ is used to indicate that the wage for type-i workers depends on the number of both worker types hired which themselves depend on the productivity.

[12] The Supplementary Appendix 3.B.1 includes a more detailed derivation of the bargaining game.

It is important to highlight that the decision to improve the bargaining power with respect to type-1 workers also influences the share of revenues belonging to type-2 workers. The share of revenues belonging to the type-1 worker is declining and convex in the firms bargaining power of the firm with respect to the type-1 workers. At the same time the share of revenues of the type-2 workers is increasing and concave in the bargaining power of the firm with respect to type-1 workers. The share of revenues belonging to type-1 workers is smaller when a firm improves its bargaining power $\Lambda_{w1}(\theta)|_{I_B=1} < \Lambda_{w1}(\theta)|_{I_B=0}$. The share of revenues belonging to type-2 workers rises $\frac{\partial \Lambda_{w1}|_{I_B=1}}{\partial \lambda} < 0$ if a firm improves its bargaining power. This is because at a given revenue level, a higher bargaining power of the firm with respect to type-1 workers implies that a larger share of revenue is left for type-2 workers to bargain over with the firm. Thus, the share of revenues belonging to type-2 workers rises.

Proposition 1. *Firms that decide to improve their bargaining power pay a lower share of revenues to their type-1 workers while the share for type-2 workers rises.*

Using (3.16) and (3.17) a firm's wage cost share determining the relative cost of the two types of labour, is a function of the production weights of the two tasks and the chosen bargaining powers:

$$\frac{w_i(\theta)h_i(\theta)}{w_j(\theta)h_j(\theta)} = \frac{\Lambda_{wi}(\theta)}{\Lambda_{wj}(\theta)} = \frac{\eta_i \lambda_j(\theta)}{\eta_j \lambda_i(\theta)}. \quad (3.18)$$

Proposition 2. *If a firm is not investing in improving its bargaining power the wage share of the two worker types is given by the Cobb-Douglas weights. In case a firm is improving its bargaining power with respect to the type-1 workers the share paid to type-1 relative to type-2 workers declines.*

The share of revenues belonging to the firm after paying wages is given by:

$$\Lambda_{I_B}(\theta) \equiv 1 - \sum_i \Lambda_{wi}(\theta) = \frac{\lambda_i(\theta)\lambda_j(\theta)}{\lambda_i(\theta)\lambda_j(\theta) + \beta\gamma\eta_i\lambda_j(\theta) + \beta\gamma\eta_j\lambda_i(\theta)} = \Lambda_0 \Upsilon_B(\theta) < 1, \quad (3.19)$$

where $\Lambda_0 \equiv \Lambda_{I_B}(\theta)|_{I_B=0} = \frac{1}{1+\beta\gamma}$ depicts the share of revenues belonging to the firm after paying wages in case of no investment in improving the bargaining power and $\Lambda_1 \equiv \Lambda_{I_B}(\theta)|_{I_B=1}$ is used in case of investment. The last equality proofs useful in terms of interpretation and depicts that the share of revenues belonging to the firm after paying wages can be written as the share belonging to a firm which is not investing Λ_0 times the bargaining improvement term:

$$\Upsilon_B(\theta) \equiv \frac{\Lambda_{I_B}(\theta)}{\Lambda_0} = 1 + I_B(\theta)\frac{\Lambda_1 - \Lambda_0}{\Lambda_0} = 1 + I_B(\theta)\left(\frac{\lambda}{\lambda - 1}\frac{1+\beta\gamma}{\beta\gamma\eta_1} - 1\right)^{-1} \geq 1. \quad (3.20)$$

The bargaining improvement term is one if a firm does not invest in improving its bargaining power and larger than one if it is investing. The larger the improvement in terms of the bargaining power (λ) and the more important the labour type affected by the improvement in the production process (η_1), the larger the bargaining improvement term. The original Helpman et al. (2010) model is embedded in this framework in the limiting case of $\eta_1 = 0$. In this case only task two is necessary for the production and improving the bargaining power has no effect and thus the bargaining improvement term is one ($\Upsilon_B(\theta)|_{\eta_1=0} = 1$). For $\eta_1 = 1$ the framework collapses to a single task production technology where improving the bargaining power affects wages of all employed workers.

3.2.5 The Profit Maximisation

Firm's Optimisation Problem. Anticipating the outcome of the bargaining game, firms maximize profits choosing whether to export or not, whether to invest in an increase in the bargaining power with respect to type-1 workers or not, as well as the number of workers to screen for the two tasks and the respective ability thresholds. Using the production function in terms of the measure of workers matched ($n_i(\theta)$) and the screening ability thresholds ($a_{ci}(\theta)$) chosen (3.10) revenues (3.11) can be written as:

$$r(\theta) = \Upsilon_X(\theta)^{1-\beta} A\eta_0^\beta \kappa_y^\beta \theta^\beta \prod_i n_i(\theta)^{\beta\gamma\eta_i} a_{ci}(\theta)^{\beta(1-\gamma k)\eta_i}. \tag{3.21}$$

Given the revenue equation (3.21) the firm's profit maximization problem can be depicted by:

$$\pi(\theta) = \max_{\substack{a_{ci}(\theta)\geq a_{\min}, \\ n_i(\theta)\geq 0, \\ I_X(\theta)\in\{0,1\}, \\ I_B(\theta)\in\{0,1\}}} \quad \Lambda_0 \Upsilon_B(\theta)\ r\left(\theta, n_i(\theta), a_{ci}(\theta)\right) - \sum_i \left(b_i n_i(\theta) + \frac{c a_{ci}(\theta)^\delta}{\delta} \right) \\ - f_D - I_X(\theta) f_X - I_B(\theta) f_B. \tag{3.22}$$

The firm's decision results in the following system of optimality conditions determining the measure of workers hired and the ability thresholds chosen:[13]

$$\Lambda_0 \Upsilon_B(\theta)(1-\gamma k)\beta\eta_i r(\theta) = c a_{ci}(\theta)^\delta, \tag{3.23}$$

$$\Lambda_0 \Upsilon_B(\theta)\beta\gamma\eta_i r(\theta) = b_i n_i(\theta). \tag{3.24}$$

It follows that firms with larger revenues sample more workers, and choose a higher ability threshold. The more important the occupation-specific task (η_i) the higher the ability threshold chosen and the more workers of that type are sampled by a firm. Using optimality conditions (3.23,3.24) the following positive relationship between the number

[13] The extensive decision of a firm whether to start production, export and improve the bargaining power or not is considered when discussing the sorting pattern of firms.

of workers matched and the chosen ability threshold can be derived:

$$n_i(\theta) = \frac{\gamma}{1-\gamma k}\frac{c}{b_i}a_{ci}(\theta)^{\delta}. \tag{3.25}$$

Using the optimality condition (3.23), it is straight forward to show that the ability threshold chosen will be larger for the occupation with the larger Cobb-Douglas production share and its relative relationship is unaltered by the bargaining decision of a firm:

$$\frac{a_{ci}(\theta)}{a_{cj}(\theta)} = \left(\frac{\eta_i}{\eta_j}\right)^{\frac{1}{\delta}}. \tag{3.26}$$

As such, the reason for differences in employment between the two types of labour, due to different bargaining powers, exist because of a different choice on the workers matched and not due to a different screening thresholds.

Wages. Using this optimality conditions (3.23,3.24) together with the results from the wage bargaining (3.16, 3.17, 3.19) one can show that wages are determined by:

$$w_i(\theta) = \frac{b_i}{\lambda_i(\theta)}\frac{n_i(\theta)}{h_i(\theta)} = \frac{b_i}{\lambda_i(\theta)}\left(\frac{a_{ci}(\theta)}{a_{\min}}\right)^{k}. \tag{3.27}$$

Ceteris paribus, wages increase in the number of workers sampled ($n_i(\theta)$), as well as in the screening ability thresholds ($a_{ci}(\theta)$) chosen by the firm. The expected wage of type-i workers, conditional on being sampled by firm θ, is independent of the firm's particular productivity but depends on the firm's decision to improve its bargaining power and thus differs between firms investing and not investing in improving their bargaining power:

$$\frac{w_i(\theta)h_i(\theta)}{n_i(\theta)} = \frac{b_i}{\lambda_i(\theta)}. \tag{3.28}$$

This per se would imply an incentive for type-1 workers to direct their search to firms who do not invest in improving their bargaining power. However, while the expected wage is lower for type-1 workers in a firm which improves its bargaining power, the investment of the firm f_B makes the workers indifferent between working for a firm which improves its bargaining power and thus a smaller expected wage and a firm which is not improving its bargaining power.

Firm Variables. Before depicting the firm variables, it is helpful to introduce a set of help parameters, in order to simplify notation: $0 < \Gamma \equiv 1 - \beta\gamma - \beta/\delta(1-\gamma k) < 1$, $\phi_1 \equiv \left(\frac{\beta\gamma\kappa_y^{\beta}}{1+\beta\gamma}\right)^{\frac{1}{\Gamma}} > 0$, $\phi_2 \equiv \left(\frac{1-\gamma k}{\gamma}\right)^{\frac{1}{\delta\Gamma}} > 0$, $\kappa_r \equiv \phi_1\phi_2^{\beta(1-\gamma k)}$. In addition, $b \equiv (\prod_i b_i^{\eta_i})$ can be considered a combined search cost measure, weighting the endogenous search cost for both types of labour. Solving the system of optimality conditions (3.23, 3.24), using the relationship between employment the measure of workers sampled and the ability threshold (3.3) and the wage expression (3.27) yields the firm variables:

$$
\begin{aligned}
a_{ci}(\theta) &= \eta_i^{\frac{1}{\delta}} \phi_1^{\frac{1}{\delta}} \phi_2^{1-\beta\gamma} c^{-\frac{1-\beta\gamma}{\delta\Gamma}} b^{-\frac{\beta\gamma}{\delta\Gamma}} \Upsilon_X(\theta)^{\frac{1-\beta}{\delta\Gamma}} \Upsilon_B(\theta)^{\frac{1}{\delta\Gamma}} A^{\frac{1}{\delta\Gamma}} \theta^{\frac{\beta}{\delta\Gamma}}, \\
n_i(\theta) &= \eta_i b_i^{-1} \phi_1 \phi_2^{\beta(1-\gamma k)} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} b^{-\frac{\beta\gamma}{\Gamma}} \Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}} \Upsilon_B(\theta)^{\frac{1}{\Gamma}} A^{\frac{1}{\Gamma}} \theta^{\frac{\beta}{\Gamma}}, \\
h_i(\theta) &= \eta_i^{1-k/\delta} b_i^{-1} a_{\min}^k \phi_1^{1-k/\delta} \phi_2^{-(k-\beta)} c^{\frac{k-\beta}{\delta\Gamma}} b^{-\frac{\beta\gamma(1-k/\delta)}{\Gamma}} \left(\Upsilon_X(\theta)^{1-\beta} \Upsilon_B(\theta) A \theta^{\beta}\right)^{\frac{1-k/\delta}{\Gamma}}, \\
r(\theta) &= \frac{\phi_1 \phi_2^{\beta(1-\gamma k)}}{\beta\gamma\Lambda_0} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} b^{-\frac{\beta\gamma}{\Gamma}} \Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}} \Upsilon_B(\theta)^{\frac{1-\Gamma}{\Gamma}} A^{\frac{1}{\Gamma}} \theta^{\frac{\beta}{\Gamma}}, \\
w_i(\theta) &= \frac{b_i \eta_i^{k/\delta}}{\lambda_i(\theta)} a_{\min}^{-k} \phi_1^{k/\delta} \phi_2^{(1-\beta\gamma)k} c^{-\frac{(1-\beta\gamma)k}{\delta\Gamma}} b^{-\frac{\beta\gamma k}{\delta\Gamma}} \Upsilon_X(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}} \Upsilon_B(\theta)^{\frac{k}{\delta\Gamma}} A^{\frac{k}{\delta\Gamma}} \theta^{\frac{\beta k}{\delta\Gamma}}, \\
\pi(\theta) &= \frac{\Gamma}{\beta\gamma} \kappa_r \left[c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A\right]^{\frac{1}{\Gamma}} \theta^{\frac{\beta}{\Gamma}} \Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}} \Upsilon_B(\theta)^{\frac{1}{\Gamma}} - f_D - I_X(\theta) f_X - I_B(\theta) f_B.
\end{aligned}
\tag{3.29}
$$

Firms with a higher productivity generate larger revenues and higher profits. They also sample more workers, choose a higher ability threshold and pay higher wages. Assuming $\delta > k > 1$ more productive firms also hire more workers. The parameter assumption is assumed to hold in the following, because under this assumption the model captures the empirical findings of an employer-size wage premium.

The more important a specific task in the production process (η_i) is, the larger the measure of workers sampled and hired and the higher the wage paid to the particular type. The importance of a specific task in the production function also determines how strong the influence of the respective endogenous search cost measures (b_i) is on the firm variables. The larger the combined search cost measure b, which reflects tighter labour markets, the smaller the firm variables. The measure of type-i workers sampled and the number of workers hired is not only a negative function of the search cost for its type, but is also negatively influenced, even though to a smaller extend, by high search cost for the other type. This is because both types are complements and thus returns to each type depend positively on the number of workers of the other type.

Proposition 3. *The number of workers matched and hired, as well as the screening threshold of type-i workers not only depend negatively on the search cost for their own type-i workers, but also the search cost for type-j worker affect the firm variables negatively.*

Following the same reasoning, the wage of type-i workers is affected negatively by the search cost for the other labour type. The effect on type-i wage by its own search costs b_i includes two opposing channels. Higher search costs imply that revenues of a firm fall and thus ceteris paribus also revenues per worker fall. This effect implies a negative wage effect. At the same time, the number of workers hired due to higher search cost falls which ceteris paribus implies a higher revenue share per worker and thus higher wages for each employed worker. Search costs b_i have a combined positive wage impact for type-i workers if $\beta\gamma k\eta_j + \delta(1-\beta\gamma) - \beta > 0$. The higher the diminishing returns to labour (lower γ), the more likely that the wage rises.

Proposition 4. *The wages paid to type-i worker fall with higher search costs for type-j worker. The effect of their own search cost is ambiguous and depends on the parameter specification which determines whether the revenues or the number of workers hired fall to a stronger extend.*

3.3 Firms' Sorting Pattern

This section discusses the different extensive firm decisions. In the first part the decision of a firm whether to start production or exit the market is discussed and firm variables are expressed as a function of the domestic cutoff productivity determining the productivity of the firm, which is indifferent between exiting the market and producing for the domestic market. In a second step, the export decision and the bargaining improvement decision and the respective cutoff productivities are discussed. The existence of the different extensive decisions allows for different possible sorting patterns of firms along the firm productivity, which is discussed in an additional section. Finally, the relationship between the different cutoffs and thus the share of different firm types is discussed.

3.3.1 Cutoff Productivity

The empirical literature emphasizing that only the most productive firms export. Thus, it is assumed that parameters are such that the marginal active firm neither exports nor does it invest in improving its bargaining power nor does it both.[14] Using the first order conditions from the profit maximization (3.23, 3.24) one can rewrite profits (3.22) as follows:

$$\pi(\theta) = \Gamma\ \Lambda_0 \Upsilon_B(\theta)\ r\,(\theta) - f_D - I_X(\theta) f_X - I_B(\theta) f_B. \tag{3.30}$$

The marginal domestic firm with the cutoff productivity θ_D, which is neither exporting ($\Upsilon_X(\theta_D) = 1$) nor improving its bargaining power ($\Upsilon_B(\theta_D) = 1$) earns zero profits ($\pi_D(\theta_D) = 0$). Using this condition, the firms optimality conditions from the profit maximisation (3.23, 3.24) and the equations for the firm's wage (3.3) and the number of workers screened (3.16), the ability threshold, the number of workers hired as well as the

[14] The necessary conditions are derived in the Supplementary Appendix 3.B.2 and are given by (3.B.44, 3.B.45, 3.B.49).

wage of the cut off firm are given by:

$$
\begin{aligned}
r(\theta_D) &= \frac{f_D}{\Gamma\ \Lambda_0}, & h_i(\theta_D) &= \eta_i^{1-\frac{k}{\delta}} \frac{\beta\gamma}{\Gamma} \frac{f_D}{b_i} \left(\frac{\beta(1-\gamma k)}{\Gamma} \frac{f_D}{c a_{\min}^{\delta}} \right)^{-\frac{k}{\delta}}, \\
n_i(\theta_D) &= \frac{\beta\gamma\eta_i}{\Gamma} \frac{f_D}{b_i}, & w_i(\theta_D) &= b_i \left(\frac{\beta(1-\gamma k)\eta_i}{\Gamma} \frac{f_D}{c a_{\min}^{\delta}} \right)^{\frac{k}{\delta}}, \\
a_{ci}(\theta_D) &= \left(\frac{\beta(1-\gamma k)\eta_i}{\Gamma} \frac{f_D}{c} \right)^{\frac{1}{\delta}}.
\end{aligned}
\tag{3.31}
$$

Using the equations for the firm variables (3.29) and the results for the cutoff firms (3.31), the firm variables can be written as a function of the cutoff productivity:

$$
\begin{aligned}
a_{ci}(\theta) &= \Upsilon_X(\theta)^{\frac{1-\beta}{\delta\Gamma}} \Upsilon_B(\theta)^{\frac{1}{\delta\Gamma}} a_{ci}(\theta_D) \left(\frac{\theta}{\theta_D} \right)^{\frac{\beta}{\delta\Gamma}}, \\
n_i(\theta) &= \Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}} \Upsilon_B(\theta)^{\frac{1}{\Gamma}} n_i(\theta_D) \left(\frac{\theta}{\theta_D} \right)^{\frac{\beta}{\Gamma}}, \\
h_i(\theta) &= \Upsilon_X(\theta)^{(1-\beta)\frac{1-k/\delta}{\Gamma}} \Upsilon_B(\theta)^{\frac{1-k/\delta}{\Gamma}} h_i(\theta_D) \left(\frac{\theta}{\theta_D} \right)^{\frac{\beta(1-k/\delta)}{\Gamma}}, \\
r(\theta) &= \Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}} \Upsilon_B(\theta)^{\frac{1-\Gamma}{\Gamma}} r(\theta_D) \left(\frac{\theta}{\theta_D} \right)^{\frac{\beta}{\Gamma}}, \\
w_i(\theta) &= \frac{1}{\lambda_i(\theta)} \Upsilon_X(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}} \Upsilon_B(\theta)^{\frac{k}{\delta\Gamma}} w_i(\theta_D) \left(\frac{\theta}{\theta_D} \right)^{\frac{\beta k}{\delta\Gamma}}, \\
\pi(\theta) &= \Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}} \Upsilon_B(\theta)^{\frac{1}{\Gamma}} \left(\frac{\theta}{\theta_D} \right)^{\frac{\beta}{\Gamma}} f_D - f_D - I_X(\theta) f_X - I_B(\theta) f_B.
\end{aligned}
\tag{3.32}
$$

A higher domestic cutoff and thus a higher average productivity of active firms and a more competitive environment implies ceteris paribus a smaller screening threshold, fewer workers sampled and hired, lower wages paid and smaller revenues and profits of a given firm. Firms that decide to export or improve their bargaining power generate higher revenues and profits, sample more workers, choose a higher ability threshold and hire more workers. Comparing firm outcomes between firms, the ratio only depends on the relative productivities, the relative market access and the relative bargaining improvement term.

Proposition 5. *Exporting and, or improving the bargaining power implies that firms choose higher ability thresholds, sample and hire more workers and generate higher revenues and profits.*

While firms that decide to export pay higher wages than firms only supplying the domestic market, the effect of improving the bargaining power on the wages paid is more complex. As stated in Proposition 1, firms that decide to improve their bargaining power pay a lower share of revenues to their type-1 workers while the share for type-2 workers rises. However, the wage an individual worker earns also depends on the firm's revenues per worker employed. Proposition 5 states that revenues and employment rise when a firm improves its bargaining power. Whether the increase in revenues is larger than the one in the employment of a specific worker type is ambiguous. If $k/\delta > \Gamma$ is fulfilled, revenues per worker rise if the firm decides to improve its bargaining power. This is true for both types of workers and thus effects wages of both types positively.

Combining the two effects (rising share of revenues for type-2 workers and an ambiguous effect on revenues per type-2 worker), one can show that wages of type-2 workers are unambiguously higher, if a firm improves its bargaining power with respect to type-1 workers. This is true for both the extensive decision and a marginal increase in the possible bargaining power improvement. For type-1 workers the effect of the bargaining power improvement on the wages paid is ambiguous. The lower share of revenues belonging to type-1 workers indicates a decline in wages. However, revenues per type-1 worker can rise if the firm invests in improving their bargaining power and thus ceteris paribus imply higher wages. There are parameter constellations of $\beta, \gamma, \eta_i, k, \delta$ where a small market power improvement λ implies that wages of type-1 workers rise.[15] This is because for those parameter constellations there is an increase in the revenues per worker which is able to compensate for the drop in the share of revenues belonging to type-1 workers and thus type-1 wages rise. In contrast, the wage of type-1 workers falls as a consequence of a firm deciding to improve its bargaining power, if the bargaining power improvement λ is sufficiently large, or $k/\delta < \Gamma$ is satisfied and thus revenues per worker fall, or parameters are such that the rise in the bargaining power improvement term is small and less relevant in determining wages. A negative wage effect for type-1 workers is present, if $\lambda\left((1+\beta\gamma)\lambda^{-\frac{\Gamma\delta}{k}} - 1\right) - \beta\gamma(\eta_1 + \lambda\eta_2) < 0$ is satisfied.

Proposition 6. *Firms that improve their bargaining power pay a higher wage to type-2 workers. The influence on the wage of type-1 workers is ambiguous and depends on the parameter constellation.*

3.3.2 Export and Bargaining Improvement Decision

Apart from the exit decision which is determining the domestic cutoff productivity θ_D a firm has two extensive decisions to make, whether to export or not and whether to improve the bargaining power or not. The cutoff productivity at which point a firm is indifferent between domestic sales and starting to export, i.e. supplying both markets, is denoted by θ_X. In similar fashion θ_B is used to indicate the productivity at which firms are

[15] If $\eta_1 > \frac{(\beta\gamma+1)(\delta(1-\beta\gamma)+\beta(1-\gamma k))}{\beta\gamma k}$ is satisfied, there exist bargaining improvement levels $\lambda > 1$ which yield positive wage effects for type-1 workers.

indifferent between domestic sales and domestic sales in combination with improving their bargaining power. The cutoff θ_{XB} depicts the productivity at which firms are indifferent between exporting and exporting in combination with improving their bargaining power at the same time. Reversely, the cutoff θ_{BX} depicts the productivity at which firms are indifferent between improving their bargaining power and improving their bargaining power in combination with exporting at the same time. The productivity at which firms are indifferent between supplying domestically and supplying both markets in combination with improving their bargaining power at the same time is depicted by θ_{DXB}. The cutoff conditions for the six different cutoff productivities can be written in the following general manner where $\iota \in \{D, X, B, XB, BX, DXB\}$ is used to indicate the different possible cutoff productivities:[16]

$$\frac{\Gamma}{\beta\gamma}\kappa_r \left[c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A\right]^{\frac{1}{\Gamma}} \theta_\iota^{\frac{\beta}{\Gamma}} \Upsilon_\iota = f_\iota. \tag{3.33}$$

The relevant fixed costs f_ι and the relevant market and bargaining improvement terms Υ_ι for the respective case ι are given by:

$$f_\iota = \begin{cases} f_D, \\ f_X, \\ f_B, \\ f_B, \\ f_X, \\ f_X + f_B, \end{cases} \qquad \Upsilon_\iota = \begin{cases} 1, & \text{if } \iota = D \\ \Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1, & \text{if } \iota = X \\ \Upsilon_B^{\frac{1}{\Gamma}} - 1, & \text{if } \iota = B \\ \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right) \Upsilon_X^{\frac{1-\beta}{\Gamma}}, & \text{if } \iota = XB \\ \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \Upsilon_B^{\frac{1}{\Gamma}}, & \text{if } \iota = BX \\ \Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1, & \text{if } \iota = DXB. \end{cases} \tag{3.34}$$

3.3.3 Sorting Patterns

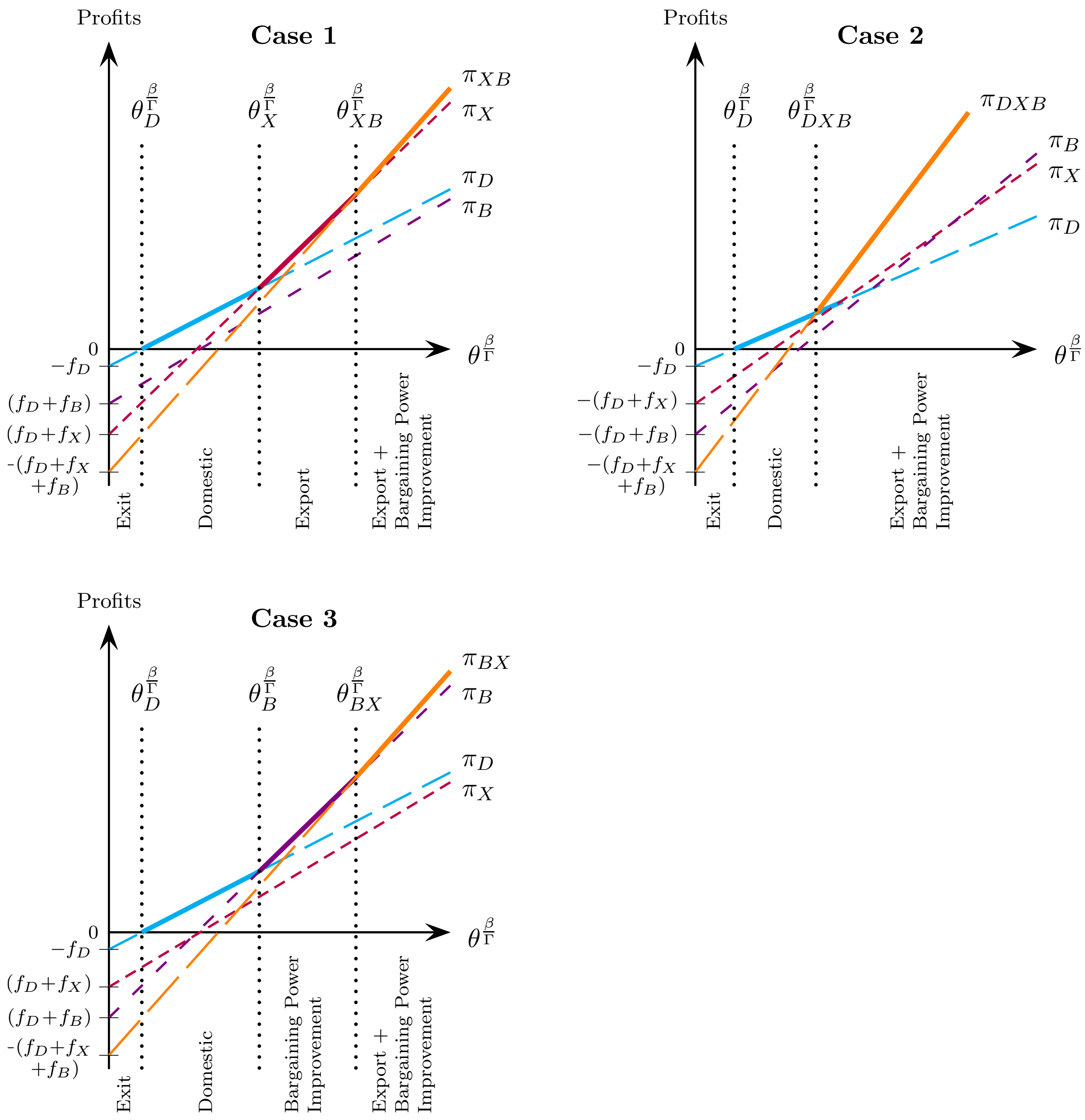
The existence of two extensive decisions, the export and the bargaining improvement decision, implies that profits for four different scenarios can be depicted: A firm selling only domestically π_D $(I_X = 0, I_B = 0)$, a firm exporting and selling to both markets π_X $(I_X = 1, I_B = 0)$, a firm selling only domestically and improving its bargaining power π_B $(I_X = 0, I_B = 1)$, and a firm selling domestically, exporting and improving its bargaining power $(I_X = 1, I_B = 1)$ depicted, dependent on the sorting pattern case, by $\pi_{XB}, \pi_{BX}, \pi_{DXB}$. The profits for those four scenarios as a function of a firms productivity are depicted in Figure 3.1.[17] The highlighted bold part depicts the relevant profits for

[16]The explicit condition for each cutoff productivity is depicted in the Supplementary Appendix 3.B.2 by equations (3.B.37-3.B.43). In principle, there is a seventh cutoff productivity, which determining the indifference between exporting and improving their bargaining power each without doing the other. In the Supplementary Appendix 3.B it is shown that this cutoff is never relevant for the sorting pattern, which is why this cutoff productivity is not considered in the main paper.

[17]Profits are depicted as a function of $\theta^{\frac{\beta}{\Gamma}}$ in order to linearise the graphical depiction.

a given productivity. The assumption that the least productive active firm is neither exporting nor improving its bargaining power implies that the equilibrium can feature three possible sorting patterns dependent on the size of the fixed costs and the potential gains form exporting and improving the bargaining power. The first plot in Figure 3.1

Figure 3.1: *Illustration of the Different Sorting Patterns*



illustrates parameter constellations where fixed cost of exporting are relatively low compared to the fixed cost if improving the bargaining power and the market access term is relatively large compared to the bargaining improvement term such that θ_X is sufficiently smaller than θ_B. The resulting sorting patter implies that the least productive active firms only supply domestically, the more productive firms also start exporting and the most productive firms export and improve their bargaining power. This sorting pattern will be referred to as *Case 1*. The second plot in Figure 3.1 depicts what will be referred to as sorting pattern of *Case 2*. Besides the domestic cutoff there is only one relevant productivity cutoff θ_{DXB}. Firms below this cutoff only supply domestically and above the cutoff firms additionally export and improve their bargaining power. *Case 2* is relevant when the fixed costs and market access and improvement term are such that θ_X is sufficiently close to θ_B. In contrast if θ_B is sufficiently smaller than θ_X the sorting pattern is given by the least productive active firms only supplying the domestic market, the more productive firms improve their bargaining power and the most productive firms do both, improve their bargaining power and export. This is referred to as *Case 3* and is depicted in the third plot in Figure 3.1. Using ι_c with $c = \{1, 2, 3\}$ to depict which of the three cases is relevant the three sorting patters can be defined as:

$$\theta_D < \theta_X < \theta_{XB} \rightarrow \iota_1 = \{D, X, XB\} \quad if \;\; \theta_X < \theta_B \;\wedge\; \theta_X < \theta_{DXB} \qquad (\textit{Case 1})$$

$$\theta_D < \theta_{DXB} \rightarrow \iota_2 = \{D, DXB\} \quad if \;\; \theta_{DXB} < \theta_X \;\wedge\; \theta_{DXB} < \theta_B \qquad (\textit{Case 2})$$

$$\theta_D < \theta_B < \theta_{BX} \rightarrow \iota_3 = \{D, B, BX\} \quad if \;\; \theta_B < \theta_X \;\wedge\; \theta_B < \theta_{DXB}. \qquad (\textit{Case 3})$$

Using the relationship between the productivity cutoffs, which is derived in the following section, one can rewrite the conditions for the respective sorting patterns and relate them to the possible bargaining improvement for type-1 workers (λ). A rather low possible bargaining improvement $\lambda < \lambda'$ implies that firms sort according to *Case 1*. Improving the bargaining power is only beneficial for really productive firms and thus with rising productivity firms first only supply domestically then start to export and only the most productive also improve their bargaining power. If the possible bargaining improvement is above this critical threshold λ', *Case 2* is relevant. As improving the bargaining power is now more attractive all exporting firms also improve their bargaining power. If the possible bargaining improvement is even higher $\lambda > \lambda''$, *Case 3* is the relevant sorting pattern. While the least productive firms only sell domestically more productive firms also improve their bargaining power and only the most productive firms also export.[18]

3.3.4 Cutoff Relationship

Using the equation determining the different cutoffs (3.33) the relationship between the domestic cutoff θ_D and any other cutoff θ_ι can be written as:

$$\frac{\theta_D}{\theta_\iota} = \Upsilon_\iota^{\frac{\Gamma}{\beta}} \left(\frac{f_D}{f_\iota}\right)^{\frac{\Gamma}{\beta}}. \tag{3.35}$$

It is helpful to define a general export cutoff as well a general bargaining improvement cutoff determining the productivity above which firms will export (θ_{X_c}) and above which

[18] The critical bargaining power levels are defined by $\theta_X\,|_{\lambda=\lambda'} \equiv \theta_{DXB}\,|_{\lambda=\lambda'}$ and $\theta_B\,|_{\lambda=\lambda''} \equiv \theta_{DXB}\,|_{\lambda=\lambda''}$.

firms will improve their bargaining power (θ_{B_c}). The relevant cutoff productivity will differ across sorting patterns. For example in *Case 1*, firms above θ_{XB} will improve their bargaining power while in *Case 3*, θ_B is the relevant cutoff:

$$\theta_{X_c} \equiv \begin{cases} \theta_X, \\ \theta_{DXB}, \\ \theta_{BX}, \end{cases} \qquad \theta_{B_c} \equiv \begin{cases} \theta_{XB}, & \text{if } c = 1 \\ \theta_{DXB}, & \text{if } c = 2 \\ \theta_B, & \text{if } c = 3. \end{cases} \tag{3.36}$$

The intensive margin of exporting is captured by the market access term (Υ_X) and the intensive margin of bargaining is captured by the bargaining improvement term (Υ_B). The ratio between the domestic and the relevant export (θ_{X_c}) or bargaining (θ_{B_c}) cutoff given the respective sorting pattern captures the extensive margins of exporting and improving the bargaining power. The share of firms exporting $\rho^z_{X_c}$ and the share of firms improving their bargaining power ρ^z_B is given by:

$$\rho_{X_c} = \left(\frac{1 - G_\theta\left(\theta_{X_c}\right)}{1 - G_\theta\left(\theta_{D_c}\right)} \right)^{\frac{1}{z}} = \frac{\theta_D}{\theta_{X_c}}, \tag{3.37}$$

$$\rho_{B_c} = \left(\frac{1 - G_\theta\left(\theta_{B_c}\right)}{1 - G_\theta\left(\theta_{D_c}\right)} \right)^{\frac{1}{z}} = \frac{\theta_D}{\theta_{B_c}}. \tag{3.38}$$

For the sorting patterns of *Case 2* and *3* the share of firms exporting rises with a higher possible bargaining improvement (larger bargaining improvement term). This is because in these cases all exporting firms also improve their bargaining power and thus there is a direct effect on the export cutoff. For *Case 1*, the share is not affected as the least productive exporters do not improve their bargaining power and thus there is no direct effect on the cutoff. The indirect effect on the export cutoff via the domestic cutoff alters the cutoff productivity but does not change the share of firms exporting.

The share of firms improving their bargaining power rises with a larger market access term, if *Cases 1* or *2* are relevant. This is because in those cases the firms, which improve their bargaining power also export. Exporting implies that firms generate higher revenues and thus improving the bargaining power is relatively more attractive. In *Case 3* the share is unaffected by the market access term as the least productive bargaining firms do not export and thus are not directly affected.[19]

Proposition 7. *If exporting firms improve their bargaining power (Case 2 or 3) the share of exporting firms rises with a higher possible bargaining improvement (larger bargaining improvement term). When the least productive exporting firms do not improve their bar-*

[19]The relationships are derived assuming that a change in neither the market access term nor the bargaining improvement term does change the relevant sorting pattern.

gaining power (Case 1) the share is not affected. The share of firms that improve their bargaining power rises with a larger market access term if firms that improve their bargaining power also export (Cases 1 or 2). If the least productive firms which improve their bargaining power do not export (Case 3) the share is unaffected by the market access term.

3.4 General Equilibrium

Applying the free entry condition, this section derives the domestic cutoff productivity and solves for the averages firm variables. Using these results the hiring rate and the general equilibrium conditions ensuring that the labour market and the goods market are in equilibrium are derived. Assuming a symmetric country case the general equilibrium conditions are solved and the expected wages, labour market tightness, search costs, unemployment rates, consumption index and the number of firms are derived. A focus of this section is to evaluate how the possibility to improve the bargaining power and the respective increase in the bargaining power effects the derived outcomes.

3.4.1 Free Entry

Free entry implies that new firms will enter the market until expected profits are equal to the entry costs. The generalised free entry condition for the three different cases take the following form:[20]

$$\sum_{\iota \in \iota_c} f_\iota \int_{\theta_\iota}^{\infty} \left[\left(\frac{\theta}{\theta_D} \right)^{\frac{\beta}{\Gamma}} - 1 \right] \, dG_\theta(\theta) = f_E. \tag{3.39}$$

Using the relationships between the cutoffs (3.35) and utilizing the productivity distribution assumption allows to solve for the domestic cutoff as:

$$\theta_{D_c} = \left[\frac{\beta}{z\Gamma - \beta} \frac{\sum_{\iota \in \iota_c} f_\iota \left(\frac{f_D}{f_\iota} \Upsilon_\iota \right)^{\frac{z\Gamma}{\beta}}}{f_E} \right]^{\frac{1}{z}} \theta_{min}. \tag{3.40}$$

In order to ensure a stable equilibrium and a positive domestic cutoff, parameters must satisfy $\frac{\beta}{\Gamma} - z < 0$. The index c in θ_{D_c} is used to indicate that the functional form differs across the different cases because the sorting pattern is different. However, there is no difference between the domestic cutoff referred to as θ_D and the derived cutoff productivity.

[20]The explicit free entry condition for all three cases as well as the derivation of the generalised free entry condition can be found in the Supplementary Appendix 3.B.3. To illustrate the generalised free entry condition the following depicts the condition for *Case 1*: $\int_{\theta_D}^{\infty} \left[\pi(\theta) \Big|_{\substack{I_X=0 \\ I_B=0}} \right] dG_\theta(\theta) + \int_{\theta_X}^{\infty} \left[\pi(\theta) \Big|_{\substack{I_X=1 \\ I_B=0}} - \pi(\theta) \Big|_{\substack{I_X=0 \\ I_B=0}} \right] dG_\theta(\theta) + \int_{\theta_{XB}}^{\infty} \left[\pi(\theta) \Big|_{\substack{I_X=1 \\ I_B=1}} - \pi(\theta) \Big|_{\substack{I_X=1 \\ I_B=0}} \right] dG_\theta(\theta) = f_E.$

The index is used in the following to highlight that the derivation of a variable will also differ in terms of the functional form across sorting patterns [21] It is straight forward to show that in a situation where bargaining improvement is not possible the domestic cutoff is smaller. Thus, the possibility to improve the bargaining power results in a higher cutoff productivity and therefore increases the average productivity of active firms. The larger the possible improvement in terms of the bargaining power and thus the larger the bargaining improvement term, the higher the domestic cutoff. As standard in Melitz (2003) type frameworks the possibility to trade, results in a larger domestic cutoff.

Proposition 8. *The domestic cutoff productivity is larger if firms have the possibility to improve their bargaining power and rises the larger the possible improvement in the bargaining power of the firm. Thus, the average productivity of firms rises with the possible bargaining power improvement.*

Export and Bargaining Decision. Combining the results of Proposition 7 and 8 it is possible to evaluate how the possibility to improve the bargaining power can alter the export cutoff θ_{X_c}. The effect can be disentangled into two effects. As stated in Proposition 8 the possibility of improving the bargaining power increases the domestic cutoff productivity and thus also has an increasing effect on the relevant export cutoff. At the same time, as stated in Proposition 7, the share of exporting firms rises for the sorting pattern of *Case 2* and *3* and thus, ceteris paribus, results in a smaller export cutoff. In *Case 1* the share of exporting firms is unaltered by the possibility that firms can improve their bargaining power. Thus, for parameter constellations resulting in the sorting pattern of case 1, the export cutoff rises the larger the possible bargaining improvement. For *Case 2* and *3* the two effects work in different directions and it is possible to show that the declining effect from the increase in the share of exporters dominates.[22] Thus, the larger the possible bargaining improvement the lower the export cutoff.

Proposition 9. *If the possible bargaining power improvement is sufficiently low (i.e. Case 1 is relevant), the productivity above which firms find it profitable to export rises, the higher the possible bargaining power improvement. If the bargaining power improvement is sufficiently large (i.e. firms sort according to Case 2 or 3), the export cutoff will decline, the higher the possible bargaining improvement .*

In similar fashion one can show that the bargaining cutoff θ_{B_c} rises when the market access term rises if parameters ensure that firms sort according to Case 3. For Case 1 and 2 the bargaining cutoff falls with a rising market access term.

[21] Each endogenous variable will differ depending on the sorting pattern. While adding the index c to all variable would complicate notation unnecessarily it is only used if the variables or parameter derivation is directly influenced by the sorting pattern.

[22] A detailed proof can be found in the Appendix 3.A.2.

3.4.2 Average Firm Variables and the Hiring Rate

Average Firm Variables. Average revenues and the average amount of the type-i workers matched and employed by all active firms in a country are denoted by $\bar{r}$, $\bar{n}_i$, $\bar{h}_i$ and are given by:

$$\begin{aligned} \bar{r}_c &= \int_{\theta_{D_c}}^{\infty} r(\theta)\ dG_\theta(\theta) = s_{r_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c})\ \bar{r}^A(\theta_{D_c}), \\ \bar{n}_{i_c} &= \int_{\theta_{D_c}}^{\infty} n_i(\theta)\ dG_\theta(\theta) = s_{n_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c})\ \bar{n}_i^A(\theta_{D_c}, b_i), \\ \bar{h}_{i_c} &= \int_{\theta_{D_c}}^{\infty} h_i(\theta)\ dG_\theta(\theta) = s_{h_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c})\ \bar{h}_i^A(\theta_{D_c}, b_i). \end{aligned} \tag{3.41}$$

The variables $\bar{r}^A(\theta_{D_c})$, $\bar{n}_i^A(\theta_{D_c}, b_i)$, $\bar{h}_i^A(\theta_{D_c}, b_i)$ denote average revenues, and the average amount of type-i workers matched and employed by all active firms in a situation where firms neither export nor improve their bargaining power.[23] The variables $s_{r_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c}) \geq 1$, $s_{n_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c}) \geq 1$, $s_{h_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c}) \geq 1$ denote the factor for the increase in the average variables due to the possibility of exporting and improving the bargaining power.

The average variables in a situation where firms neither export nor improve their bargaining power ($\bar{r}^A(\theta_{D_c})$, $\bar{n}_i^A(\theta_{D_c}, b_i)$, $\bar{h}_i^A(\theta_{D_c}, b_i)$), are a negative function of the endogenous domestic cutoff productivity and the average amount of workers matched and employed by firms in this situation are also a negative function of the endogenous worker-type specific search cost. [24] Average workers matched and employed differ between worker types dependent on the respective search cost level and the Cobb-Douglas weight η_i.

The factor determining the increase in average revenues, and the average amount of type-i workers matched and employed due to the possibility of exporting and improving the bargaining power is one, if neither exporting nor improving the bargaining power is possible. It depends positively on the intensive (Υ_X, Υ_B) and extensive (ρ_{X_c}, ρ_{B_c}) margin of exporting and bargaining power improvement and its functional form differs dependent on the sorting pattern case.[25]

Proposition 10. *The factors determining the increase in average revenues, as well as the average amount of type-i workers matched and employed, because firms can export*

[23] In terms of notation, the index A is used following the idea that in a framework where firms cannot improve their bargaining power this situation would be described as autarky.

[24] The average variables in a situation where firms neither export nor improve their bargaining power are given by $\bar{r}^A(\theta_{D_c}) \equiv \bar{r}\Big|_{\substack{I_X=0\\I_B=0}} = \frac{\theta_{min}^z f_D}{\Lambda_0} \frac{z}{z\Gamma-\beta} \theta_{D_c}^{-z}$ for average revenues, $\bar{n}_i^A(\theta_{D_c}, b_i) \equiv \bar{n}_i\Big|_{\substack{I_X=0\\I_B=0}} = \frac{\beta\gamma z \theta_{min}^z f_D}{z\Gamma-\beta} \frac{\eta_i}{b_i}$ depict average workers matched and $\bar{h}_i^A(\theta_{D_c}, b_i) \equiv \bar{h}_i\Big|_{\substack{I_X=0\\I_B=0}} = \frac{\beta\gamma z \theta_{min}^z f_D}{z\Gamma-\beta(1-k/\delta)} \left(\frac{\beta(1-\gamma k)}{\Gamma} \frac{f_D}{c a_{\min}^\delta}\right)^{-\frac{k}{\delta}} \frac{\eta_i^{1-\frac{k}{\delta}}}{b_i} \theta_{D_c}^{-z}$ are average workers employed in this situation.

[25] The explicit form is derived in the Appendix 3.A.2, equations 3.B.71, 3.B.75, 3.B.79).

and improve their bargaining power $s_{r_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c}) \geq 1$, $s_{n_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c}) \geq 1$, $s_{h_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c}) \geq 1$, *depends positively on the intensive* (Υ_X, Υ_B) *and extensive* (ρ_{X_c}, ρ_{B_c}) *margin of trade and bargaining improvement.*

Comparing average revenues and the average amount of type-i workers matched and employment between a situation where firms can improve their bargaining power and one where this is not possible, it can be shown that the average variables are smaller if firms can improve their bargaining power. This is shown by first solving for the average values explicitly and then incorporating that the parameters determining the extensive and intensive margin of trade and bargaining improvement must satisfy that the least productive firm is doing neither.[26] The possibility to improve the bargaining power has a negative effect on average firm variables. This is because the possibility to improve the bargaining power implies that before entry the expected share of revenues available to a firm to cover the fixed cost of entry is higher. Thus, free entry implies that in equilibrium expected revenues are smaller.

Using a similar approach, comparing the autarky situation with the open economy and assuming that firms can improve their bargaining power, one is able to show that average revenues and the amount of type-i workers matched and employed are smaller in the open economy than in a autarky situation.

Proposition 11. *Average revenues as well a the amount of type-i workers matched and employed are smaller if firms have the possibility to improve their bargaining power. Average revenues as well a the amount of type-i workers matched and employed under autarky are higher than in an open economy where trade is possible.*

Hiring Rate. The hiring rate, which is the share of type-i workers hired given a match with a firm, is given by:

$$\sigma_i = \frac{H_i}{N_i} = \frac{M\int_{\theta_D}^{\infty} h_i(\theta)dG_\theta(\theta)}{M\int_{\theta_D}^{\infty} n_i(\theta)dG_\theta(\theta)} = \frac{\bar{h}_i}{\bar{n}_i} = \frac{s_{h_c}\bar{h}_i^A}{s_{n_c}\bar{n}_i^A} = s_{\sigma_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c})\ \sigma_i^A. \tag{3.42}$$

The hiring rate in a situation where all firms neither export nor improve their bargaining power is given by:

$$\sigma_i^A = \sigma_i \Big|_{\substack{I_X=0 \\ I_B=0}} = \frac{z\Gamma - \beta}{z\Gamma - \beta(1-k/\delta)} \left(\frac{\Gamma}{\beta(1-\gamma k)\eta_i} \frac{c a_{\min}^{\delta}}{f_D} \right)^{\frac{k}{\delta}}, \tag{3.43}$$

[26] The average number of workers matched simplifies to $\bar{n}_i = z\gamma f_E \eta_i b_i^{-1}$. Thus, the effect of the possibility to improve the bargaining power is directly linked to the change in search cost in a negative manner. When determining the effect on average workers matched and employed the general equilibrium result that search cost rises when firms can export and improve their bargaining power is taken into account. This is derived in the following section.

where the term in brackets is the hiring rate by the domestic cutoff firm ($h(\theta_D)/n(\theta_D)$). The decline in the hiring rate due to the possibility to export and improve the bargaining power is depicted by:

$$s_{\sigma_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c}) \equiv \frac{s_{h_c}}{s_{n_c}}. \tag{3.44}$$

If no firm exports nor improves their bargaining power it holds that $s_{\sigma_c}(\Upsilon_X, \Upsilon_B, 0, 0) = 1$. As $0 < 1 - k/\delta < 1$ one can show that the hiring rate declines the more attractive either exporting or improving the bargaining power becomes $0 < s_{\sigma_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c}) < 1$. This is because in both cases the firm's distribution is shifted towards more productive firms that screen more intensively.

Proposition 12. *The hiring rate falls both with the market access and the bargaining improvement term.*

Importance of the Bargaining Improvement Firms. To determine the expected wages and the overall wage income of workers which are necessary to determine the general equilibrium it is necessary to know the share of revenues generated and the share of workers matched to firms improving the bargaining power $S_{rB_c} = \int_{\theta_{B_c}}^{\infty} r(\theta)\, dG_\theta(\theta) / \int_{\theta_{D_c}}^{\infty} r(\theta)\, dG_\theta(\theta)$, $S_{nB_c} = \int_{\theta_{B_c}}^{\infty} r(\theta)\, dG_\theta(\theta) / \int_{\theta_{D_c}}^{\infty} r(\theta)\, dG_\theta(\theta)$. Both shares can be derived using the results for average firm variables (3.41) and the relationship between the cutoffs (3.35).[27] The share of workers matched to firms that improve their bargaining power is the same for both types of workers. Both shares rise, the larger the possible improvement in the bargaining power. They also rise, the larger the market access term. If the sorting pattern of *Case 2* or *3* is relevant, this is because in those cases the positive effect of a larger market access term only affects firms which improve their bargaining power and thus the shares of those firms rises. In *Case 1*, where only the most productive firms improve their bargaining power, both the share of firms exporting while not improving their bargaining power $\rho_{X_1}^z$ and the share of firms exporting and improving their bargaining power $\rho_{B_1}^z$ rises. However, one is able to show that the increase in the share of firms, which improve their bargaining power dominates. Thus, also for *Case 1* the effect of an increase in the market access term on the share of revenues generated and the share of workers matched with firms, which improve their bargaining power is positive.

Proposition 13. *The share of revenues, as well as the share of both types of workers matched and employed by firms that improve their bargaining power rises, the larger the improvement in the bargaining power. The shares also rise if the country moves from autarky to an open economy or the market access term rises.*

[27] The detailed derivation and explicit term for the share of revenues generated by firms that improve their bargaining power can be found in the Appendix 3.A.2. The shares are given by (3.B.80,3.B.82).

3.4.3 General Equilibrium Variables

Expected Wage. When deriving the labour market equilibrium, it is helpful to use the expected wage income for the two types of labour. As depicted by equation (3.28), the expected wage, given that a worker is matched with a firm θ, only differs across firms if they differ in terms of their bargaining power improvement decision. This allows to write the expected wage given a worker is matched to any firm ($\bar{w}_i$), as the probability that the worker is matched to a firm which is not improving its bargaining power $(1 - S_{nB_c})$ times the expected wage in such a firm, given a match $((w_i(\theta)h_i(\theta)/n_i(\theta))\,|_{I_B=0})$, plus the probability that the worker is matched to a firm which is improving its bargaining power (S_{nB_c}), times the expected wage in such a firm, given a match $((w_i(\theta)h_i(\theta)/n_i(\theta))\,|_{I_B=1})$. As the probability for being hired by a firm that improves its bargaining power depends on the equilibrium sorting pattern, the expected wage given a match to a firm will depend on the sorting pattern case and is given by:

$$\bar{w}_i = (1 - S_{nB_c}) \left.\frac{w_i(\theta)h_i(\theta)}{n_i(\theta)}\right|_{I_B=0} + S_{nB_c} \left.\frac{w_i(\theta)h_i(\theta)}{n_i(\theta)}\right|_{I_B=1} = s_{\bar{w}_{i_c}} b_i, \tag{3.45}$$

where $s_{\bar{w}_{i_c}}$ is used to simplify notation and is a measure for the decline in the expected wage given a match ($\bar{w}_i$) due to the fact that some of the matched workers face lower expected wages because of the bargaining power improvement by the firms:

$$s_{\bar{w}_{i_c}} \equiv 1 - S_{nB_c} \left(1 - \frac{1}{\lambda_i}\right) \leq 1. \tag{3.46}$$

It is one for type-2 workers $s_{\bar{w}_{1_c}} = 1$. The expected wage income is given by the probability of being matched with a firm times the expected wage conditional on being matched:

$$\omega_i = \frac{N_i}{L_i}\bar{w}_i = x_i \bar{w}_i = s_{\bar{w}_{i_c}} x_i b_i. \tag{3.47}$$

Using the formulation of the search cost (3.5) allows to write labour market tightness and search cost as a function of the expected wage:

$$x_i = \left(\frac{\alpha_0\ \omega_i}{s_{\bar{w}_{i_c}}}\right)^{\frac{1}{1+\alpha}}, \qquad b_i = \alpha_0^{-\frac{1}{1+\alpha}} \left(\frac{\omega_i}{s_{\bar{w}_{i_c}}}\right)^{\frac{\alpha}{1+\alpha}}. \tag{3.48}$$

General Equilibrium Conditions. Normalising the price index $P = 1$, total expenditure Y equals the real consumption index Q $(PQ = Q = Y)$. The domestic demand shifter can be written as $A = Y^{1-\beta}P^{\beta} = Q^{1-\beta}P = Q^{1-\beta}$. For Foreign it holds that $P^*\ Q^* = Y^*$ and thus the demand shifter can be written as $A^* = {Y^*}^{1-\beta}{P^*}^{\beta} = {Q^*}^{1-\beta}P^*$. In order to solve for the general equilibrium in closed form, it is assumed in the following that countries are symmetric and therefore the price index in foreign is also one. The demand shifter in both countries are therefore the same $A^* = {Q^*}^{1-\beta} = A$. Using the relationship between the consumption index and the demand shifter in combination with the relationship between the search cost and the expected wages (3.48) one can rewrite the zero profit condition of the cutoff firm and solve for the consumption index:

$$Q = A^{\frac{1}{1-\beta}} = \left(\frac{f_D}{\kappa_r}\frac{\beta\gamma}{\Gamma}\right)^{\frac{\Gamma}{1-\beta}} \Lambda_0^{-\frac{1}{1-\beta}} \theta_{D_c}^{-\frac{\beta}{1-\beta}} c^{-\frac{\beta(1-\gamma k)}{(1-\beta)\delta}} \alpha_0^{-\frac{1}{1+\alpha}\frac{\beta\gamma}{1-\beta}} \left(\prod_i \left(\frac{\omega_i}{s_{\bar{w}_{ic}}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}}. \quad (3.49)$$

In equilibrium overall expected wage income by type-i workers in a country has to equal the sum of all wages paid by firms in that country to type-i workers. The equilibrium labour market condition follows as:[28]

$$\omega_i L_i = M\int_{\theta_D}^{\infty} w_i(\theta)h_i(\theta)\ dG_\theta(\theta) = \mu_{i_c} M\int_{\theta_D}^{\infty} r(\theta)\ dG_\theta(\theta) = \mu_{i_c} Q, \quad (3.50)$$

where the share of overall revenues belonging to workers of type-i is given by:

$$\mu_{i_c} \equiv \Lambda_{w0i}\left(1 - S_{rB_c}\right) + \Lambda_{w1i} S_{rB_c}. \quad (3.51)$$

The second equality of equation (3.50) reflects that the wages paid to all workers in a firm is a constant share of revenues and differs only between firms that do and do not improve their bargaining power. $\Lambda_{w1,i} \equiv \Lambda_{wi}(\theta)|_{I_B=1}$ is used to indicate the wage share earned by type-i workers in any firm θ which is investing in improving its bargaining power. In turn, $\Lambda_{w0,i} \equiv \Lambda_{wi}(\theta)|_{I_B=0}$ indicates the share in a situation where the same firm is not investing in improving its bargaining power. The overall revenue share belonging to type-1 workers (μ_{1_c}) falls the larger the bargaining improvement term while the share belonging to type-2 workers (μ_{2_c}) rises. This follows directly from utilizing Proposition 13 and Proposition 1. They state that the share of revenues generated by firms that improve their bargaining power (S_{rB_c}) rises the larger the bargaining improvement term. In addition, the share of revenues paid by firms improving their bargaining power (Λ_{w1i}) is smaller for type-1 workers and larger for type-2 workers the larger the bargaining improvement term.

Proposition 14. *The overall revenue share belonging to type-1 workers falls the larger the bargaining improvement term while the share belonging to type-2 workers rises.*

Equilibrium Variables. Using the two general equilibrium conditions (3.49) and (3.50) as well as the relationship between the expected wage and the labour market tightness and the search cost (3.48) one can solve for the the expected wage as well as the other

[28] The labour market condition is valid also for an asymmetric case. Only the last equality assumes symmetry between the two countries under which the consumption index can be written as $Q = QP = M\int_{\theta_D}^{\infty} r(\theta)\ dG_\theta(\theta)$.

endogenous equilibrium variables:

$$
\begin{aligned}
Q &= \kappa_o^{-\frac{1}{\Delta}} \theta_{D_c}^{\frac{\beta}{\Delta}} c^{\frac{\beta(1-\gamma k)}{\Delta\delta}} \alpha_0^{\frac{1}{1+\alpha}\frac{\beta\gamma}{\Delta}} \left(\prod_i \left(\frac{s_{\bar{w}_{i_c}} L_i}{\mu_{i_c}} \right)^{\eta_i} \right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}}, \\
M &= \frac{z\Gamma - \beta}{z} \frac{\Lambda_0}{f_D \theta_{min}^z} \kappa_o^{-\frac{1}{\Delta}} c^{\frac{\beta(1-\gamma k)}{\Delta\delta}} \alpha_0^{\frac{1}{1+\alpha}\frac{\beta\gamma}{\Delta}} \left(\prod_i \left(\frac{s_{\bar{w}_{i_c}} L_i}{\mu_{i_c}} \right)^{\eta_i} \right)^{\frac{\alpha}{1+\alpha}\frac{\beta\Gamma}{\Delta}} \frac{\theta_{D_c}^{z+\frac{\beta}{\Delta}}}{s_{r_c}}, \\
\omega_i &= \kappa_o^{-\frac{1}{\Delta}} \theta_{D_c}^{\frac{\beta}{\Delta}} c^{\frac{\beta(1-\gamma k)}{\delta\Delta}} \alpha_0^{\frac{1}{1+\alpha}\frac{\beta\gamma}{\Delta}} \left(\prod_i \left(\frac{s_{\bar{w}_{i_c}} L_i}{\mu_{i_c}} \right)^{\eta_i} \right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}} \left(\frac{L_i}{\mu_{i_c}} \right)^{-1}, \\
x_i &= \kappa_o^{-\frac{1}{(1+\alpha)\Delta}} \theta_{D_c}^{\frac{\beta}{(1+\alpha)\Delta}} c^{\frac{\beta(1-\gamma k)}{(1+\alpha)\Delta\delta}} \alpha_0^{\frac{\beta+\beta\gamma-1}{(1+\alpha)\Delta}} \left(\prod_i \left(\frac{s_{\bar{w}_{i_c}} L_i}{\mu_{i_c}} \right)^{\eta_i} \right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{(1+\alpha)\Delta}} \left(\frac{s_{\bar{w}_{i_c}} L_i}{\mu_{i_c}} \right)^{-\frac{1}{1+\alpha}}, \\
b_i &= \kappa_o^{-\frac{\alpha}{(1+\alpha)\Delta}} \theta_{D_c}^{\frac{\alpha\beta}{(1+\alpha)\Delta}} c^{\frac{\alpha\beta(1-\gamma k)}{(1+\alpha)\Delta\delta}} \alpha_0^{\frac{1-\beta}{(1-\alpha)\Delta}} \left(\prod_i \left(\frac{s_{\bar{w}_{i_c}} L_i}{\mu_{i_c}} \right)^{\eta_i} \right)^{\frac{\alpha}{1+\alpha}\frac{\alpha\beta\gamma}{(1+\alpha)\Delta}} \left(\frac{s_{\bar{w}_{i_c}} L_i}{\mu_{i_c}} \right)^{-\frac{\alpha}{1+\alpha}},
\end{aligned}
\tag{3.52}
$$

with $\kappa_o \equiv \left(\frac{f_D}{\kappa_r} \frac{\beta\gamma}{\Gamma} \right)^{\Gamma} \Lambda_0^{-1}$ and $\Delta \equiv -\left(1 - \beta - \frac{\alpha}{1+\alpha}\beta\gamma \right) > 0$ is assumed to be positive in order to have a stable equilibrium.

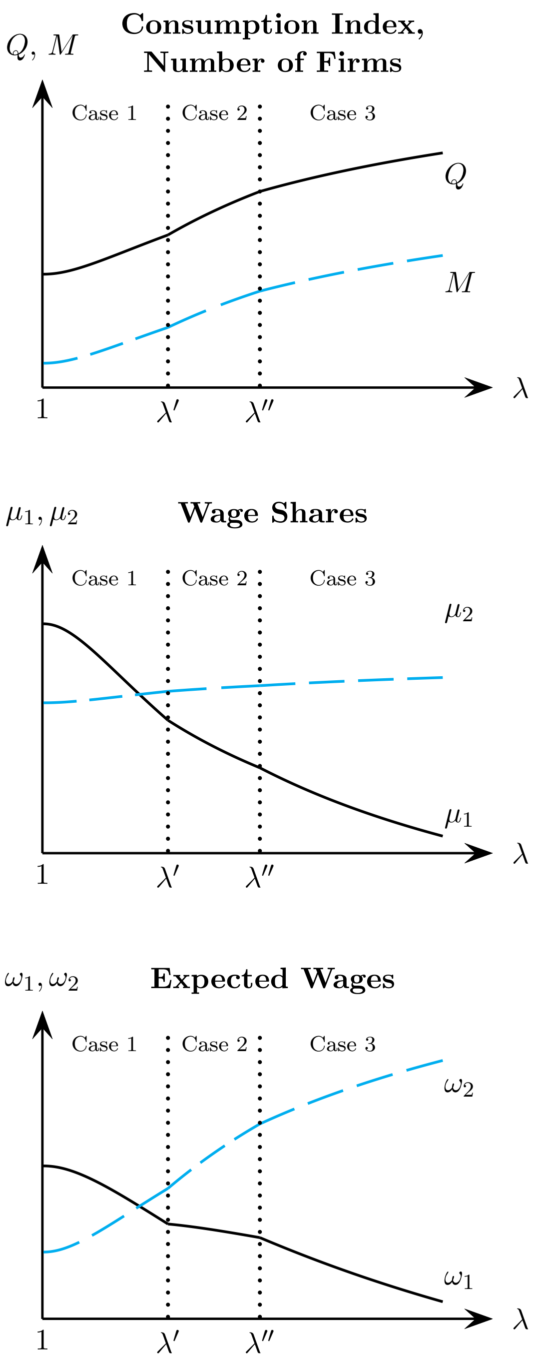
3.4.4 Implications of the Bargaining Power Improvement in General Equilibrium

In order to evaluate how the bargaining power improvement affects the different general equilibrium variables, it is helpful to consider how a change in the possible bargaining power improvement changes those variables. Figure 3.2 depicts the general equilibrium variables as a function of the possible bargaining power improvement with respect to type-1 workers. The graph is plotted such that the origin of the abscissa depicts a situation where there is no bargaining improvement $\lambda = 1$. With a rising possible bargaining improvement the sorting pattern changes from *Case 1* to *Case 2* and then to *Case 3*. The kink at the critical values is a result of the the change in sorting pattern. Dependent on the sorting pattern, the functional form determining how the bargaining power influences equilibrium variables differs. In order to depict Figure 3.2, parameters are chosen in line with Helpman et al. (2008). They use a framework where no bargaining improvement is possible, which is the limiting case $\lambda = 1$ and set the parameters such that key features of the data are matched.[29] It is important to mention that this is not a calibration of the model and the graphs are plotted for illustrative purposes. While the qualitative results for the wage share, labour market tightness, hiring rate, consumption index and number of firms are independent of the parameter choice, the graphs of the expected

[29] The Cobb-Douglas share of type-1 workers is set to $\eta_1 = 0.55$ and the two workforce types are assumed to be of equal size. In a case where the Cobb-Douglas share and the labour market size is the same for both types of worker, the labour market tightness, search cost, unemployment rate, as well as the hiring rate are the same for both types of labour and only the expected wage differs between the types. For more details also see the Appendix 3.A.1.

wages and the unemployment might also differ qualitatively depending on the parameter assumptions. Parameter constellations yielding different qualitative results are depicted in the Appendix 3.A.1 as Figure 3.A.1 and Figure 3.A.2. A detailed description of the

Figure 3.2: *General Equilibrium Variables and the Bargaining Power Improvement*



The different general equilibrium variables are depicted as a function of the possible bargaining improvement with respect to typ-1 workers (λ). The critical bargaining power improvement levels at which the sorting pattern changes are depicted by λ' and λ''. The illustration of the consumption index and the number of firms uses two different scales for the ordinate in order to depict both graphs in one chart.

different variables and the evaluation of how they are influenced by an improvement in the bargaining power will be conducted in the following. In the following the six different charts of Figure 3.2 will be discussed, using the derived general equilibrium variables (3.52). Starting with the upper left chart first the left column is discussed followed by the right column.

Consumption Index. The consumption index (3.52) is influenced by the size of the possible bargaining improvement via the domestic cutoff productivity (θ_D), the shares of overall revenues belonging to the two types of workers (μ_{i_c}) and finally by the measures for the importance of the share of workers facing a small bargaining power in the expected wage determination for both types of workers ($s_{\bar{w}_{i_c}}$).

Firstly, as stated in Proposition 8 a higher bargaining improvement term implies that the least productive firms exit the market and thus the domestic cutoff (θ_D) rises. This results in a higher average productivity which ceteris paribus increases overall revenues and the consumption index. Secondly, Proposition 14 states that a higher bargaining improvement term implies that the share of overall revenues in a country belonging to type-1 workers (μ_{1_c}) falls, while the share paid to type-2 workers (μ_{2_c}) rises. The changes in the share of overall revenues belonging to the two types of workers inversely affect overall revenues and the consumption index. This implies that due to the higher share of overall revenues spend for type-2 workers, overall revenues fall. In turn, the decline in the share of overall revenues spend for type-1 workers rises overall revenues as well as the consumption index. Thirdly and finally, the consumption index is influenced by the bargaining power via the measure for the importance of the share of workers facing a small bargaining power in the expected wage determination ($s_{\bar{w}_{i_c}}$). It is one for type-2 workers ($s_{\bar{w}_{2_c}} = 1$). The measure for type-1 workers is smaller than one ($s_{\bar{w}_{1_c}} < 1$) and falls the higher the bargaining power improvement term ($\frac{\partial s_{\bar{w}_{1_c}}}{\partial \lambda} < 0$) and thus influences overall revenues and the consumption index positively. Therefore, a higher bargaining improvement term has a negative effect on expected wages via this final channel.

Combining the different effects, one is able to show that overall revenues and the consumption index rise the higher the bargaining improvement term. Generally speaking, a larger possible bargaining improvement for firms and thus a higher bargaining improvement term, can be understood as a decline in average production cost across all firms. This decline in overall production cost results in more overall revenues generated and a larger consumption index.

Proposition 15. *Overall revenues and the consumption index rise the higher the bargaining improvement term.*

Number of Firms. The number of active firms is determined by overall revenues divided by average revenues. Proposition 15 states that overall revenues rise, while Proposition 11 states that average revenues fall, when the bargaining power improvement is larger. Therefore, the number of firms rises with the bargaining power. Putting it differently, the expected profits ceteris paribus rise with a higher bargaining improvement term. Thus, more firms enter the market pushing out some of the least productive firms. Overall this implies more active firms, which are on average more productive.

Proposition 16. *The number of active firms rises the higher the bargaining improvement term.*

Expected Wages. The effect of a higher bargaining improvement term on the expected wage can be separated into two components. Firstly, as stated by Proposition 15, an increased bargaining improvement term rises in the consumption index and overall revenues. This ceteris paribus indicates a rising expected wage for both types of worker. Secondly, a larger bargaining improvement implies that as stated by Proposition 14 the overall revenue share belonging to type-1 workers falls, while the share for type-2 workers rises. This is illustrated by the second chart in the left column of Figure 3.2. Combining the two findings, implies that expected wages of type-2 workers unambiguously rise with the bargaining improvement term. They profit from the higher overall revenues and the higher share of overall revenues belonging to them, while their bargaining power is unaltered. Type-1 workers also profit from the higher overall revenues, but they earn a smaller share of overall revenues. For small bargaining power improvements and a sufficiently large type-1 Cobb-Douglas production share parameters can be such, that the rise in overall revenues dominates the decline in the expected wage share for type-1 workers. In such a situation expected wages of type-1 workers rise, when the possible bargaining power improvement increases. However, if the bargaining power improvement is sufficiently larger or parameters are such that the rise in overall revenues is small compared to to change in the possible bargaining power improvement, expected wages of type-1 workers fall. Such a situation is depicted in the bottom chart in the left column of Figure 3.2.

Dependent on the the interpretation of the bargaining improvement process, it is important to mention that the bargaining improvement of the firm also involves an investment in form of fixed costs which compensates the type-1 workers. As such a declining expected wage does not imply that type-1 workers are actually worse off.

Proposition 17. *The expected wage of type-2 workers rises, the larger the possible bargaining improvement. The effect on the expected wage of type-1 workers is ambiguous. The direction of the effect depends on the size of the bargaining power improvement and the importance of the type-1 occupation in the production process.*

The relationship between the expected wage of the two labour types is given by:

$$\frac{\omega_i}{\omega_j} = \frac{\mu_{i_c}}{\mu_{j_c}} \frac{L_j}{L_i} \tag{3.53}$$

The expected wage of type-1 workers relative to type-2 workers depends on the relative labour force size and the relative shares of overall revenues belonging to the two types of workers. It falls the larger the possible bargaining improvement for type-1 workers.

Proposition 18. *The expected wage of type-1 workers relative to type-2 workers falls, the larger the possible bargaining improvement for type-1 workers.*

Labour Market Tightness and Search Cost. As depicted in equation (3.48) the labour market tightness is a positive function of the expected wage rate (ω_i) and a negative function of the measure for the importance of the share of workers facing a small bargaining power in the expected wage determination ($s_{\bar{w}_{i_c}}$). As argued before, the measure is unaffected for type-2 workers and for type-1 workers it is smaller than one and falls the higher the bargaining improvement term. This ceteris paribus would imply a tighter labour market for type-1 workers. As stated by Proposition 18, the expected wage of type-2 workers rises, the higher the possible bargaining improvement. This directly implies that also the labour market tightness for type-2 workers rises. A higher possible bargaining improvement has an ambiguous effect on the expected wage of type-1 workers (Proposition 18). However, one is able to show that the positive effect of ($s_{\bar{w}_{1_c}}$) dominates. Thus, as depicted by the upper right chart in Figure 3.2, the labour market tightness for type-1 workers rises with a higher bargaining improvement term and more workers of both types are matched to firms when the possible bargaining improvement is higher. Given the direct positive relationship between the labour market tightness and the search cost (3.5) a higher labour market tightness also implies that it is more expensive to match with workers and thus the search cost rises with the bargaining improvement term.

Proposition 19. *The labour market tightness as well as the search cost for both types of workers rises the higher the bargaining improvement term.*

Unemployment Rate. While the labour market tightness rises with a higher bargaining improvement term (Proposition 19) the hiring rate falls (Proposition 12), as depicted in the second chart in the right column. The combination of both effects determine the effect on the unemployment rate. The overall effect is ambiguous and depends on the parameter constellations, which determines whether the effect of the rising labour market tightness or the falling hiring rate dominates. The bottom right chart of Figure 3.2 depicts a situation where the unemployment rate rises with the possible bargaining power improvement level. In the Appendix 3.A.1 a situation where the search cost are less responsive to a rising labour market tightness is depicted. In this case the unemployment rate falls with higher possible bargaining improvements.

Proposition 20. *The effect of a higher bargaining improvement term on the unemployment rate is ambiguous and depends on the parameter assumptions.*

3.5 Concluding Remarks

The implications of international trade on different labour market variables such as wages and employment is one of the most important topics in the field of international economics and is a prominent topic in the public debate. Following the approach by Helpman et al. (2010) a theoretical framework which emphasises firm heterogeneity, ex ante heterogeneity in worker ability as well as Diamond-Mortensen-Pissarides search and matching frictions is used. Highlighting that firms have the possibility to influence the wage bargaining process, the possibility to improve the bargaining power in the bilateral bargaining stage between a firm and a worker is introduced. Two worker types are introduced each specific to their occupation. It is assumed that the improvement of the bargaining power comes at a cost for the firm and is only possible for one occupations. This introduces wage and employment differences within firms across occupations, as well as across firms, as only the more productive firms will find it profitable to improve the bargaining power.

Firms that decide to improve their bargaining power with respect to type-1 workers pay a lower share of revenues to those workers. Additionally the share for type-2 workers rises due to the rising bargaining power with respect to type-1 workers. The reason for this is that at a given revenue level a higher bargaining power of the firm with respect to type-1 workers implies that a larger share of revenue is left for type-2 workers to bargain over with the firm. The labour market of the two worker types are linked with each other. The number of workers matched and hired as well as the screening threshold chosen by a firm for each worker type not only depend negatively on the search cost for their own type, but also on the search cost for the other type. The wages paid to a worker fall with higher search costs for the other worker type, but the effect of their own search cost is ambiguous and depends on whether the revenues or the number of workers hired fall to a stronger extend.

Firms will decide on whether to export, improving their bargaining power or both based on their productivity. While the most productive firms will do both export and improve their bargaining power, the framework additionally allows for the existence of a intermediate productivity range where firms either export or improve their bargaining power without doing the other. The costs and gains of both exporting and improving the bargaining power decide which of the different sorting patterns is relevant. It is shown that firms that export and, or improve their bargaining power choose higher ability thresholds, sample and hire more workers, and generate higher revenues and profits. They also pay a higher wage to type-2 workers while the effect on wages for type-1 workers is ambiguous and depends on the size of the bargaining power improvement.

While it is rather obvious that the share of exporting firms rises with a larger market access term, and the share of firms improving their bargaining power rises with a larger bargaining improvement term, the two also can affect each other. It can be shown that a larger improvement in the bargaining power rises the share of exporting firms, if the exporting firms also improve their bargaining power. Similarly, if trade implies a larger market access term, the share of firms improving their bargaining power rises, if firms which improve their bargaining power also export. It is also shown that the actual productivity above which firms export can rise or fall with a higher possible bargaining improvement depending on the sorting pattern of firms. As such a change in the possible bargaining improvement can result in firms no longer exporting, if the initial level of the bargaining improvement was sufficiently low.

Considering a change in the possible bargaining power improvement it is shown that in general equilibrium a higher possible bargaining improvement results in more active firms having a higher average productivity, generating higher overall revenues and thus rising the consumption index. At the same time, the labour market tightness, as well as the search cost for both types of workers rises, while the hiring rate declines. The share of overall revenue in a country belonging to type-1 workers falls the larger the bargaining improvement term, while the share belonging to type-2 workers rises. In combination with larger overall revenues this implies that expected wages of type-2 workers rise the larger the possible bargaining improvement. The effect on the expected wage of type-1 workers is ambiguous and depends on the size of the bargaining power improvement and the importance of the type-1 occupation in the production process. Independent of the direction of that effect, the expected wage of type-1 workers relative to type-2 workers falls the larger the possible bargaining improvement for type-1 workers. In contrast, to the found unambiguous results of a falling hiring rate and a rising labour market tightness, the effect of a higher bargaining improvement term on the unemployment rate can be positive or negative.

Future research should develop a more explicit modelling of the bargaining improvement by firms allowing firms to choose the exact level of bargaining improvement and thus, adding an intensive margin to the bargaining decision. This approach should involve a more sophisticated cost structure which is based on an underlying framework modelling the bargaining improvement and the trade off from a worker's perspective explicitly. Such an extension would also allow for a more comprehensive welfare evaluation. Using the developed framework, further research could also evaluate inequality effects introduced due to the bargaining improvement and the export decision as well as their interplay in more detail using inequality measures like the Gini coefficient or the Theil index. Another interesting topic that can be evaluated using this framework is how the labour market conditions which determine the matching process between firms and workers influence the bargaining decision and the interaction between the export and the bargaining decision of firms.

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3.A Appendix

3.A.1 Illustration of General Equilibrium Variables

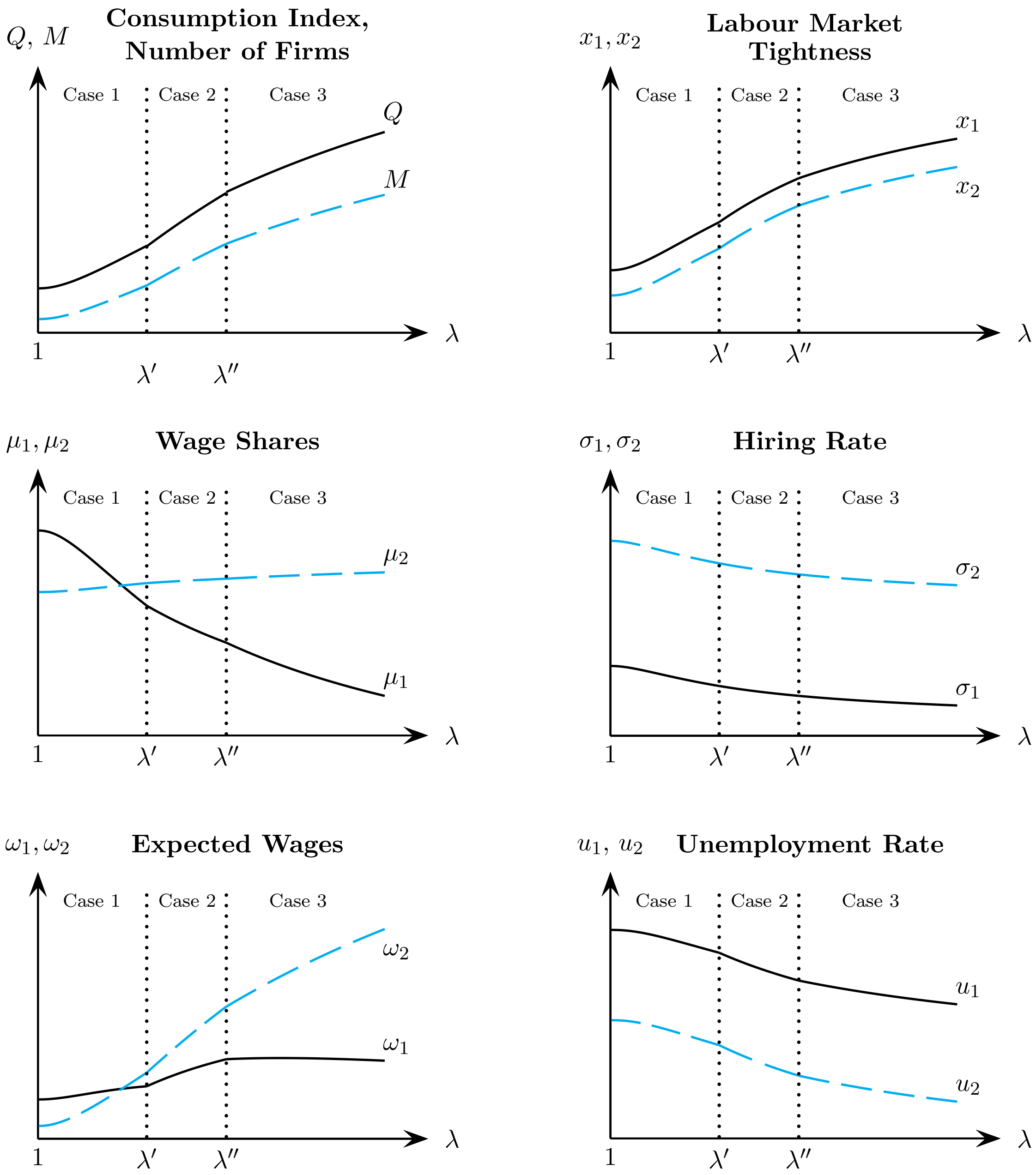
This section aims at illustrating how the general equilibrium variables might react differently to a change in the possible labour market improvement. In the first part the parameter choices for the illustrations are discussed and in the second part different interesting parameter constellations are illustrated.

Illustration Parameters. Whenever possible, parameters are chossen in line with Helpman et al. (2008). They use a similar framework without the possibility of firms to improve their bargaining power, an outside sector and only one worker type. The CES preference parameter is set to $\beta = 0.75$ which corresponds to a elasticity of substitution between varieties of 4. Choosing $\theta_{\min} = a_{\min} = 1$ is a simple normalisation of the ability and productivity distribution. Following Helpman et al. (2008), the shape parameter of the productivity distribution across firms is set to $z = 2.6$ and the shape parameter of the ability distribution is set to $k = 2$ which is consisten with the findings of Saez (2001) and Hsieh and Klenow (2009). In line with estimates of Anderson and van Wincoop (2004), iceberg trade cost are set to $\tau = 1.5$, indicating that variable trade costs are 50% of the production costs. Again following Helpman et al. (2008) the different fixed cost are set such that in a case where no bargaining power improvement is possible the share of firms exiting the market is 10% and the share of exporting firms is 19%, which is consistent with empirical evidence ($f_D = 5$, $\frac{f_E}{f_D} = 1/1.6$, $\frac{f_X}{f_D} = 0.2$). The fixed costs of improving the bargaining power are set to $\frac{f_B}{f_D} = 0.25$ indicating that the fixed costs of exporting are lower than the fixed costs of improving the bargaining power. As the possible bargaining power improvement is not set for the illustrations, this assumption on the fixed costs still allows for all the sorting patterns. Exemplary, in a situation where firms can improve their bargaining power with respect to type-1 workers to $\lambda = 2$, 15% of firms are improving their bargaining power. In line with Helpman et al. (2008), the measure for the decreasing returns to labour is set to $\gamma = 0.4$. The Cobb-Douglas share of type-1 workers is set to $\eta_1 = 0.55$ in order to evaluate a situation where both worker types are almost equally important, but differ to some extent. When the size of the two work forces is the same, an equal Cobb-Douglas share would imply that the hiring rate, labour market tightness and the unemployment rate for both types are the same. Search cost parameters ($\alpha_0 = 1300$, $\alpha = 20$) are chosen such that in equilibrium the labour market tightness is around $x_i \approx 0.95$ which indicates that 5% of the workers do not match with a firm, which is what Helpman et al. (2008) chose for their illustration. The following section also considers different parameter constellations of the search cost elasticity with respect to the labour market tightness. Screening cost parameters are set to $c = 0.28$ and $\delta = 7$ which results in a hiring rate of type-1 (type-2) workers of $\sigma_1^A = 0.78$ ($\sigma_2^A = 0.88$), if neither exporting nor improving the bargaining power is possible. In an open economy with no possibility to improve the bargaining power the hiring rate of type-1 (type-2) is $\sigma_1|_{I_B=0} = 0.77$ ($\sigma_2|_{I_B=0} = 0.86$ and $\sigma_1 = 0.76$ ($\sigma_2 = 0.85$), if firms can improve their bargaining power. This implies that under autarky 22% (12%) of the sampled type-1 (type-2) workers are not hired because of screening. Similarly, in the open economy, without the possibility to improve the bargaining power; 23% (14%) of the sampled type-1 (type-2) workers are not hired. If firms can improve their bargaining power to $\lambda = 2$, 24% (15%) of the sampled type-1 (type-2) workers are not hired. For type-2 workers those shares are closely comparable to Helpman et al. (2008). The measure of workers of both types is assumed to be of equal size and is set to $L_1 = L_2 = 120$, which results in a unemployment rate of around $u_i \approx= 0.2$.

Different Effects of the Bargaining Power Improvement. Figure 3.A.1 depicts different general equilibrium variables as a function of the possible improvement in the bargaining power (λ). The underlying parameters are chosen such that search cost react less elastic to a change in the labour market tightness ($\alpha = 7.5$) compared to the parameter choice for Figure 3.2. The consumption index and the number of firms rises, the wage share of type-1 workers falls, the share belonging to type-2 workers rises, the hiring rate falls and labour market tightness rises with λ. This is qualitatively the same a depicted in Figure 3.2 in the main paper where search cost react more elastic to changes in the labour market tightness. The effect on the expected wages of type-1 workers, as well as the unemployment rate of both worker types is affected differently by a change in λ. In particular due to the less elastic search cost function, the rising labour market tightness dominates the falling hiring rate and thus the unemployment rate for both worker types falls. The expected wage of type-1 workers rises with λ, if the initial λ is low enough. This may be surprising effect can be explained by a large increase in overall revenues generated by firms. Those larger overall revenues imply ceteris paribus higher expected wages for both types of workers and in the depicted parameter case also dominate the negative effect of the declining wage share of type-1 workers. Only if the possible bargaining power improvement is already rather large, a further increase results in lower expected wages.

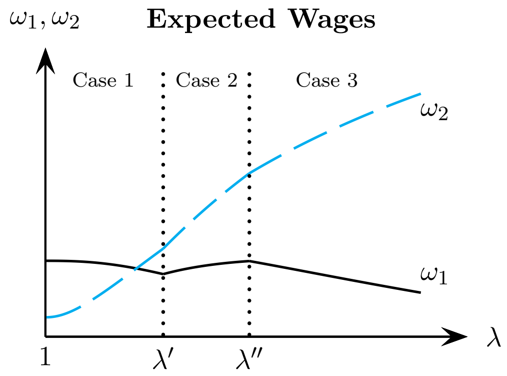
Figure 3.A.2 is based on a parameter choice where the elasticity of search cost with respect to the labour market tightness is in between the before depicted scenarios ($\alpha = 10$). As before the expected wage of type-2 workers rises with the possible bargaining power improvement λ. An increase in λ decreases expected wages of type-1 workers, if the possible bargaining power improvement is low and firms sort according to sorting pattern *Case 1*, which means that only the most productive firms, that export also improve their bargaining power. If the possible bargaining improvement is higher, so that firms sort according to *Case 2*, the effect of a further increase in the bargaining power reverses. Now a higher λ implies higher expected wages for type-1 workers. If λ is even higher and thus

Figure 3.A.1: *General Equilibrium Variables and the Bargaining Power Improvement - Low Search Cost Elasticity*



causes firms to sort according to *Case 3*, the expected wage of type-1 workers falls with higher λ.

Figure 3.A.2: *Expected Wages as a Function of the Bargaining Power Improvement*



3.A.2 Proof of Propositions

This sections includes some of the more advanced proofs of the derived propositions in the paper. A detailed derivation of the full model can be found in the Supplementary Appendix 3.B.

Proposition 1. The share of revenues belonging to a certain type of worker is given by:

$$\Lambda_{wi}(\theta) \equiv \frac{\beta\gamma\eta_i\lambda_j(\theta)}{\beta\gamma\eta_i\lambda_j(\theta) + \beta\gamma\eta_j\lambda_i(\theta) + \lambda_i(\theta)\lambda_j(\theta)}. \tag{3.17}$$

Only the bargaining power for type-1 workers can be influenced and for type-2 workers it is one ($\lambda_2(\theta) = 1$). This allows to write the share of revenues belonging to a certain type of worker as follows:

$$\Lambda_{wi}(\theta) \equiv \begin{cases} \frac{\beta\gamma\eta_1}{\beta\gamma\eta_1+\beta\gamma\eta_2\lambda+\lambda}, & \text{if } I_B(\theta) = 1 \wedge i = 1 \\ \frac{\beta\gamma\eta_2\lambda}{\beta\gamma\eta_1+\beta\gamma\eta_2\lambda+\lambda}, & \text{if } I_B(\theta) = 1 \wedge i = 2 \\ \frac{\beta\gamma\eta_i}{1+\beta\gamma}, & \text{if } I_B(\theta) = 0. \end{cases}$$

Given $\lambda > 1$ the share of revenues belonging to type-1 workers is declining when a firm invests in improving its bargaining power:

$$\begin{aligned} \Lambda_{w1}(\theta)|_{I_B=1} &< \Lambda_{w1}(\theta)|_{I_B=0} \\ \frac{\beta\gamma\eta_1}{\beta\gamma\eta_1 + \beta\gamma\eta_2\lambda + \lambda} &< \frac{\beta\gamma\eta_1}{1+\beta\gamma} \\ 1 + \beta\gamma &< \lambda + \beta\gamma\eta_1 + \beta\gamma\eta_2\lambda \\ 1 + \beta\gamma &< 1 + \beta\gamma + (1 + \beta\gamma\eta_2)(\lambda - 1) \\ 0 &< (1 + \beta\gamma\eta_2)(\lambda - 1). \end{aligned}$$

The share is the smaller the larger the increase in bargaining power is: $\frac{\partial \Lambda_{w1}|_{I_B=1}}{\partial \lambda} = -\frac{\beta\gamma\eta_1}{(\beta\gamma\eta_1+\beta\gamma\eta_2\lambda+\lambda)^2}(1+\beta\gamma\eta_2) < 0$. The share of type-2 workers rises if a firm is improving its bargaining power:

$$\begin{aligned}
\Lambda_{w2}(\theta)|_{I_B=1} &> \Lambda_{w2}(\theta)|_{I_B=0} \\
\frac{\beta\gamma\eta_2\lambda}{\beta\gamma\eta_1+\beta\gamma\eta_2\lambda+\lambda} &> \frac{\beta\gamma\eta_2}{1+\beta\gamma} \\
\frac{1}{1+\frac{\beta\gamma\eta_1}{\lambda}+\beta\gamma\eta_2} &> \frac{1}{1+\beta\gamma} \\
1+\beta\gamma &> 1+\frac{\beta\gamma\eta_1}{\lambda}+\beta\gamma\eta_2 \\
1 &> \frac{\eta_1}{\lambda}+\eta_2 = 1+\eta_1\left(\frac{1}{\lambda}-1\right) \\
1 &> 1-\eta_1\frac{\lambda-1}{\lambda}.
\end{aligned}$$

The share for type-2 workers in a firm that invest in improving the bargaining power is larger the higher the increase in bargaining power:

$$\begin{aligned}
\frac{\partial \Lambda_{w2}|_{I_B=1}}{\partial \lambda} &= \frac{\beta\gamma\eta_2\left(\beta\gamma\eta_1+\beta\gamma\eta_2\lambda+\lambda\right)-\beta\gamma\eta_2\lambda(1+\beta\gamma\eta_2)}{(\beta\gamma\eta_1+\beta\gamma\eta_2\lambda+\lambda)^2} \\
&= \frac{\beta^2\gamma^2\eta_2\eta_1}{(\beta\gamma\eta_1+\beta\gamma\eta_2\lambda+\lambda)^2} > 0.
\end{aligned}$$

Proposition 4. The wage paid by a firm (3.29) can be rewritten as:

$$w_i(\theta) = \frac{\eta_i^{k/\delta}}{\lambda_i(\theta)} b_i^{1-\frac{\eta_i\beta\gamma k}{\delta\Gamma}} b_j^{-\frac{\eta_j\beta\gamma k}{\delta\Gamma}} a_{\min}^{-k}\phi_1^{k/\delta}\phi_2^{(1-\beta\gamma)k} c^{-\frac{(1-\beta\gamma)k}{\delta\Gamma}} \Upsilon_X(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}} \Upsilon_B(\theta)^{\frac{k}{\delta\Gamma}} A^{\frac{k}{\delta\Gamma}} \theta^{\frac{\beta k}{\delta\Gamma}}.$$

The sign of the exponent of the search cost $1-\eta_i\frac{\beta\gamma k}{\delta\Gamma}$ determines how the wage of type i workers depends on the search cost for its type. A positive relationship between the search cost and the wage is given if:

$$\begin{aligned}
1-\eta_i\frac{\beta\gamma k}{\delta\Gamma} &= 1-\frac{\eta_i\beta\gamma k}{\delta(1-\beta\gamma-\beta/\delta(1-\gamma k))} = 1-\frac{\eta_i\beta\gamma k}{\beta\gamma k+\delta(1-\beta\gamma)-\beta} > 0 \\
1 &> \frac{\eta_i\beta\gamma k}{\beta\gamma k+\delta(1-\beta\gamma)-\beta} \\
\eta_i\beta\gamma k &< \beta\gamma k+\delta(1-\beta\gamma)-\beta \\
0 &< \beta\gamma k\eta_j+\delta(1-\beta\gamma)-\beta.
\end{aligned}$$

The wage of type i workers falls with higher search costs for type-j workers as $-\frac{\eta_j\beta\gamma k}{\delta\Gamma} < 0$.

Proposition 6. Revenues per worker are given by:

$$\frac{r(\theta)}{h_i(\theta)} = \frac{\Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}}\Upsilon_B(\theta)^{\frac{1-\Gamma}{\Gamma}} r(\theta_D)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}}}{\Upsilon_X(\theta)^{(1-\beta)\frac{1-k/\delta}{\Gamma}}\Upsilon_B(\theta)^{\frac{1-k/\delta}{\Gamma}} h_i(\theta_D)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta(1-k/\delta)}{\Gamma}}}$$
$$= \Upsilon_X(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}}\Upsilon_B(\theta)^{\frac{k/\delta-\Gamma}{\Gamma}}\frac{r(\theta_D)}{h_i(\theta_D)}\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta k}{\delta\Gamma}}.$$

The effect of the bargaining improvement term (Υ_B) on the share of revenues per worker depends on the parameter constellation. If the exponent is positive $k/\delta - \Gamma > 0$ firms that improve their bargaining power also have higher revenues per worker. Wages are given by (3.32):

$$w_i(\theta) = \frac{1}{\lambda_i(\theta)}\Upsilon_X(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}}\Upsilon_B(\theta)^{\frac{k}{\delta\Gamma}} w_i(\theta_D)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta k}{\delta\Gamma}}. \tag{3.32}$$

One can show that wages of type-2 workers are higher if a firm improves their bargaining power with respect to type-1 workers:

$$w_2(\theta)|_{I_B(\theta)=1} > w_2(\theta)|_{I_B(\theta)=0}$$
$$\Upsilon_X(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}}\Upsilon_B^{\frac{k}{\delta\Gamma}} w_i(\theta_D)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta k}{\delta\Gamma}} > \Upsilon_X(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}} w_i(\theta_D)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta k}{\delta\Gamma}}$$
$$\Upsilon_B^{\frac{k}{\delta\Gamma}} > 1,$$

and rise the larger the possible bargaining improvement:

$$\frac{\partial w_2(\theta)|_{I_B=1}}{\partial\lambda} = \frac{k}{\delta\Gamma}\frac{w_2(\theta)|_{I_B=1}}{\Upsilon_B}\frac{\partial\Upsilon_B}{\partial\lambda} > 1.$$

In order to derive conditions for the effect of wages for type-1 workers, one can rewrite wages for type-1 workers as follows:

$$w_1(\theta) = \frac{\Upsilon_B(\theta)^{\frac{k}{\delta\Gamma}}}{\lambda_1(\theta)}\Upsilon_X(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}} w_1(\theta_D)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta k}{\delta\Gamma}}.$$

Using the results from the derivation of the bargaining term (3.20), one can show that firms will pay higher wages to type-1 workers in a situation in which they improve their bargaining power compared to a situation in which they do not if the following parameter condition holds:

$$\begin{aligned}
w_1(\theta)|_{I_B(\theta)=1} &> w_1(\theta)|_{I_B(\theta)=0} \\
\frac{\Upsilon_B^{\frac{k}{\delta\Gamma}}}{\lambda} = \frac{\left(\frac{\lambda(1+\beta\gamma)}{\beta\gamma\eta_1+\beta\gamma\eta_2\lambda+\lambda}\right)^{\frac{k}{\delta\Gamma}}}{\lambda} = \frac{\left(\frac{\lambda(1+\beta\gamma)}{\beta\gamma\eta_1+\beta\gamma\eta_2\lambda+\lambda}\right)^{\frac{k}{\delta\Gamma}}}{\lambda} &> 1 \\
\frac{1+\beta\gamma\eta_1/\lambda+\beta\gamma\eta_2}{1+\beta\gamma} &< \lambda^{-\frac{\delta\Gamma}{k}} \\
\lambda\left((1+\beta\gamma)\lambda^{-\frac{\delta\Gamma}{k}}-1\right)-\beta\gamma(\lambda\eta_2+\eta_1) &> 0.
\end{aligned}$$

The condition above holds with equality for $\lambda = 1$, thus the condition is satisfied for small levels of $\lambda > 1$, if the derivative with respect to λ is positive around $\lambda = 1$:

$$\begin{aligned}
&\left.\frac{\partial\left(\lambda\left((1+\beta\gamma)\lambda^{-\frac{\delta\Gamma}{k}}-1\right)-\beta\gamma(\lambda\eta_2+\eta_1)\right)}{\partial\lambda}\right|_{\lambda=1} \\
&\qquad = (1+\beta\gamma)\lambda^{-\frac{\Gamma\delta}{k}}-1-\left.\frac{\Gamma\delta(1+\beta\gamma)\lambda^{-\frac{\Gamma\delta}{k}}}{k}-\beta\gamma\eta_2\right|_{\lambda=1} > 0 \\
&\qquad 0 < \beta\gamma(1-\eta_2)-\frac{\Gamma\delta(1+\beta\gamma)}{k} \\
&\qquad \eta_1 > \frac{(1+\beta\gamma)(\delta(1-\beta\gamma)+\beta(1-\gamma k))}{\beta\gamma k}.
\end{aligned}$$

The wage of type-1 workers changes with the level of possible bargaining power improvement in the following manner:

$$\begin{aligned}
\frac{\partial\, w_1(\theta)|_{I_B=1}}{\partial\lambda} &= \Upsilon_X(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}} w_1(\theta_D)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta k}{\delta\Gamma}} \frac{\partial\left(\frac{\Upsilon_B^{\frac{k}{\delta\Gamma}}}{\lambda}\right)}{\partial\lambda} \\
&= \Upsilon_X(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}} w_1(\theta_D)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta k}{\delta\Gamma}} \frac{\frac{k}{\delta\Gamma}\Upsilon_B^{\frac{k}{\delta\Gamma}-1}\frac{\partial\Upsilon_B}{\partial\lambda}\lambda-\Upsilon_B^{\frac{k}{\delta\Gamma}}}{\lambda^2} \\
&= w_1(\theta)|_{I_B=1}\frac{\frac{k}{\delta\Gamma}\frac{\lambda}{\Upsilon_B}\frac{\partial\Upsilon_B}{\partial\lambda}-1}{\lambda}.
\end{aligned}$$

If parameters satisfy the following condition, the wages for type-1 workers rise the higher the possible bargaining improvement ($\frac{\partial\, w_1(\theta)|_{I_B=1}}{\partial\lambda} > 0$):

$$\begin{aligned}
\frac{k}{\delta\Gamma}\frac{\lambda}{\Upsilon_B}\frac{\partial\Upsilon_B}{\partial\lambda}-1 = \frac{k}{\delta\Gamma}\frac{\beta\gamma\eta_1}{\beta\gamma\eta_1+\beta\gamma\eta_2\lambda+\lambda}-1 &> 0 \\
\frac{k}{\delta\Gamma}\Lambda_{w1}|_{I_B=1}-1 &> 0 \\
\Lambda_{w1}|_{I_B=1} &> \frac{\delta\Gamma}{k}.
\end{aligned}$$

In the above derivation it is used that the change in the bargaining improvement term due to a change in λ is given by equation (3.B.18) in the Supplementary Appendix 3.B.1.

Proposition 7. Using the measure for the share of firms exporting (3.37) in combination with the relevant export cutoffs given by (3.36) and the relationship between the cutoff productivities given by (3.35), which are explicitly derived for the three cases in the Supplementary Appendix 3.B.2 (3.B.44, 3.B.47, 3.B.49), yields:

$$\rho_{X_c} = \left(\frac{1-G_\theta\left(\theta_{X_c}\right)}{1-G_\theta\left(\theta_{D_c}\right)}\right)^{\frac{1}{z}} = \begin{cases} \dfrac{\theta_D}{\theta_X} & = \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\dfrac{f_D}{f_X}\right)^{\frac{\Gamma}{\beta}}, \qquad \text{if } c = 1 \\ \dfrac{\theta_D}{\theta_{DXB}} & = \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\dfrac{f_D}{f_X + f_B}\right)^{\frac{\Gamma}{\beta}}, \quad \text{if } c = 2 \\ \dfrac{\theta_D}{\theta_{BX}} & = \Upsilon_B^{\frac{1}{\beta}} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\dfrac{f_D}{f_X}\right)^{\frac{\Gamma}{\beta}}, \qquad \text{if } c = 3. \end{cases} \tag{3.37}$$

With $\frac{\partial \Upsilon_X}{\partial \lambda} = 0$ and $\frac{\partial \Upsilon_B}{\partial \lambda} > 0$ it is straight forward to conclude that $\left.\frac{\partial \rho_{X_c}}{\partial \lambda}\right|_{c\in\{2,3\}} > 0$ and $\frac{\partial \rho_{X_1}}{\partial \lambda} = 0$. Which means that if exporting firms also improve their bargaining power (*Case* 2 or *3*), the share of exporting firms rises with a higher possible bargaining improvement. If the least productive exporting firms do not improve their bargaining power (*Case 1*), the share is not affected. It is also straight forward to conclude that a larger market access term increases the share of exporting firms ($\frac{\partial \rho_{X_c}}{\partial \Upsilon_X} > 0$).

Using the measure for the share of firms improving their bargaining power (3.38) in combination with the relevant bargaining cutoffs given by (3.36) and the relationship between the cutoff productivities given by (3.35), which are explicitly derived for the three cases in the Supplementary Appendix 3.B.2 (3.B.45, 3.B.46, 3.B.49) yields:

$$\rho_{B_c} = \left(\frac{1-G_\theta\left(\theta_{B_c}\right)}{1-G_\theta\left(\theta_{D_c}\right)}\right)^{\frac{1}{z}} = \begin{cases} \dfrac{\theta_D}{\theta_{XB}} & = \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \Upsilon_X^{\frac{1-\beta}{\beta}} \left(\dfrac{f_D}{f_B}\right)^{\frac{\Gamma}{\beta}}, \qquad \text{if } c = 1 \\ \dfrac{\theta_D}{\theta_{DXB}} & = \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\dfrac{f_D}{f_X + f_B}\right)^{\frac{\Gamma}{\beta}}, \quad \text{if } c = 2 \\ \dfrac{\theta_D}{\theta_B} & = \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\dfrac{f_D}{f_B}\right)^{\frac{\Gamma}{\beta}}, \qquad \text{if } c = 3. \end{cases} \tag{3.38}$$

It is straight forward to conclude that $\left.\frac{\partial \rho_{B_c}}{\partial \Upsilon_X}\right|_{c\in\{1,2\}} > 0$ and $\frac{\partial \rho_{X_3}}{\partial \Upsilon_X} = 0$. Which means that if firms that improve their bargaining power also export (*Case 1 or 2*), the share of firms that improve their bargaining power rises with the market access term. If the least productive firms improving their bargaining power do not export (*Case 3*), the share is not affected. One can also conclude that a larger possible bargaining improvement increases the share of firms improving their bargaining power ($\frac{\partial \rho_{B_c}}{\partial \lambda} > 0$).

Proposition 8. Using the domestic cutoffs (3.40) and writing the cutoff explicitly for the three different cases yields:

$$\theta_{D_c} = \begin{cases} \left(f_D + f_X \left(\frac{f_D}{f_X} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1 \right) \right)^{\frac{z\Gamma}{\beta}} + f_B \left(\frac{f_D}{f_B} \Upsilon_X^{\frac{1-\beta}{\Gamma}} \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1 \right) \right)^{\frac{z\Gamma}{\beta}} \right)^{\frac{1}{z}} \kappa_\theta, & \text{if } c = 1 \\ \left(f_D + (f_X + f_B) \left(\frac{f_D}{f_X + f_B} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1 \right) \right)^{\frac{z\Gamma}{\beta}} \right)^{\frac{1}{z}} \kappa_\theta, & \text{if } c = 2 \\ \left(f_D + f_X \left(\frac{f_D}{f_X} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1 \right) \Upsilon_B^{\frac{1}{\Gamma}} \right)^{\frac{z\Gamma}{\beta}} + f_B \left(\frac{f_D}{f_B} \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1 \right) \right)^{\frac{z\Gamma}{\beta}} \right)^{\frac{1}{z}} \kappa_\theta, & \text{if } c = 3. \end{cases}$$

where $\kappa_\theta \equiv \left(\frac{\beta}{(z\Gamma - \beta) f_E} \right)^{\frac{1}{z}} \theta_{min}$ is used to simplify notation. It is easy to see that a larger possible bargaining power improvement and thus a larger bargaining improvement term ($\frac{\partial \Upsilon_B}{\partial \lambda} > 0$) results in a higher domestic cutoff productivity $\frac{\partial \theta_{D_c}}{\partial \lambda} > 0$. Form the above equations, one can also draw the conclusion that the domestic cutoff productivity is larger if firms have the possibility to improve their bargaining power.

Proposition 9. Using the relationship between the cutoff productivities (3.35), the export cutoff in *Case 1* is given by:

$$\theta_{X_1} = \theta_X = \left(\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1 \right)^{\frac{\Gamma}{\beta}} \left(\frac{f_D}{f_X} \right)^{\frac{\Gamma}{\beta}} \right)^{-1} \theta_{D_1},$$

where the relevant ratio is depicted in the Supplementary Appendix 3.B.2 by equation (3.B.44). The relevant domestic cutoff is depicted in the proof of Proposition 8 and is derived in equation (3.B.60) in the Supplementary Appendix 3.B.3. As shown in the previous section, the domestic cutoff productivity rises, if firms have the possibility to improve their bargaining power. The relationship between θ_{X_1} and θ_{D_1} is unaffected by such a change. Therefore, the cutoff above which firms export, rises if firms have the possibility to improve their bargaining power and the higher the bargaining term $\frac{\partial \theta_X}{\partial \lambda} > 0$.

If parameters are such that the sorting pattern of *Case 2* occurs, the productivity above which firms decide to export is given by:

$$\theta_{X_2} = \theta_{DXB} = \left(\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1 \right)^{\frac{\Gamma}{\beta}} \left(\frac{f_D}{f_X + f_B} \right)^{\frac{\Gamma}{\beta}} \right)^{-1} \theta_{D_2},$$

where the relevant ratio is also depicted in the Supplementary Appendix 3.B.2 by equation (3.B.49). The relevant domestic cutoff is depicted in the proof of Proposition 8 and is derived by equation (3.B.61) in the Supplementary Appendix 3.B.3.

Writing the export cutoff in *Case 2* explicitly the bargaining power yields:

$$\theta_{DXB} = \left(\frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1}{\Gamma}}-1}{f_X+f_B}\right)^{-\frac{\Gamma}{\beta}} \left(f_D + (f_X+f_B)\left(\frac{f_D}{f_X+f_B}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1}{\Gamma}}-1\right)\right)^{\frac{z\Gamma}{\beta}}\right)^{\frac{1}{z}} \frac{\kappa_\theta}{(f_x+f_B)^{\frac{\Gamma}{\beta}}}$$

$$= \left(\left(\frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1}{\Gamma}}-1}{f_X+f_B}\right)^{-\frac{z\Gamma}{\beta}} f_D + (f_X+f_B) f_D^{\frac{z\Gamma}{\beta}}\right)^{\frac{1}{z}} \frac{\kappa_\theta}{(f_x+f_B)^{\frac{\Gamma}{\beta}}}.$$

It follows that $\frac{\partial \theta_{X_c}}{\partial \Upsilon_B} = \frac{\partial \theta_{DXB}}{\partial \Upsilon_B} < 0$. In combination with $\frac{\partial \Upsilon_B}{\partial \lambda} < 0$ this implies that the export cutoff falls, if the possible bargaining improvement rises $\frac{\partial \theta_{DXB}}{\partial \lambda} < 0$.

In *Case 3* the productivity above which firms decide to export is given by:

$$\theta_{X_3} = \theta_{BX} = \left(\Upsilon_B^{\frac{1}{\beta}}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}-1\right)^{\frac{\Gamma}{\beta}}\left(\frac{f_D}{f_X}\right)^{\frac{\Gamma}{\beta}}\right)^{-1}\theta_{D_3},$$

where the relevant ratio is depicted in the Supplementary Appendix 3.B.2 by equation (3.B.47). The relevant domestic cutoff is depicted in the proof of Proposition 8 and is given by equation (3.B.62) in the Supplementary Appendix 3.B.3. Writing the export cutoff in *Case 3* explicitly yields:

$$\theta_{BX} = \frac{\kappa_\theta}{\left(\frac{f_D}{f_X}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}-1\right)\right)^{\frac{\Gamma}{\beta}}} \left(f_D \Upsilon_B^{-\frac{z}{\beta}} + f_X\left(\frac{f_D}{f_X}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}-1\right)\right)^{\frac{z\Gamma}{\beta}} + f_B\left(\frac{f_D}{f_B}\left(1-\Upsilon_B^{-\frac{1}{\Gamma}}\right)\right)^{\frac{z\Gamma}{\beta}}\right)^{\frac{1}{z}}.$$

Taking the derivative with respect to the bargaining improvement term, one can show that the cutoff above which firms' export falls, the larger the possible bargaining power improvement:

$$\frac{\partial \theta_{BX}}{\partial \lambda} = \left(\frac{z f_D \Upsilon_B^{-\frac{1}{\Gamma}-1}}{\beta}\left(\frac{f_D\left(1-\Upsilon_B^{-1/\Gamma}\right)}{f_B}\right)^{\frac{z\Gamma}{\beta}-1} - \frac{z f_D \Upsilon_B^{-\frac{z}{\beta}-1}}{\beta}\right)\frac{\partial \Upsilon_B}{\partial \lambda}\cdot \text{constant} < 0$$

$$\left(\frac{f_D\left(1-\Upsilon_B^{-1/\Gamma}\right)}{f_B}\right)^{\frac{z\Gamma}{\beta}-1} - 1 < 0$$

$$\frac{f_D}{f_B}\left(1-\Upsilon_B^{-1/\Gamma}\right) < 1.$$

The assumption that the least productive active firm is not improving its bargaining power ($\theta_D/\theta_B < 1$) ensures that the above inequality is fulfilled. This can be shown using equation (3.35) which for the relevant cutoff relationship is explicitly given in the

Supplementary Appendix 3.B.2 by equation (3.B.45):

$$\left(\frac{\theta_D}{\theta_B}\right)^{\frac{\beta}{\Gamma}} = \frac{f_D}{f_B}\left(\Upsilon_B^{1/\Gamma}-1\right) < 1$$
$$\frac{f_D}{f_B}\left(1-\Upsilon_B^{-1/\Gamma}\right) < \Upsilon_B^{-1/\Gamma} < 1.$$

Thus, for *Case 3*, the cutoff above which firms' export falls the larger the possible improvement in the bargaining power $\frac{\partial \theta_{X_c}}{\partial \Upsilon_B} = \frac{\partial \theta_{BX}}{\partial \Upsilon_B} < 0$.

Proposition 10. The measure for the increase in average revenues due to the possibility of exporting and improving the bargaining power in the respective cases are derived in the Supplementary Appendix 3.B.2 and are given by:

$$s_{r_c} \equiv \begin{cases} 1+\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}} + \Upsilon_X^{\frac{1-\beta}{\Gamma}}\left(\Upsilon_B^{\frac{1-\Gamma}{\Gamma}}-1\right)\rho_{B_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c=1 \\ 1+\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1-\Gamma}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c=2 \\ 1+\left(\Upsilon_B^{\frac{1-\Gamma}{\Gamma}}-1\right)\rho_{B_c}^{z-\frac{\beta}{\Gamma}} + \Upsilon_B^{\frac{1-\Gamma}{\Gamma}}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c=3. \end{cases} \tag{3.B.71}$$

With $z-\frac{\beta}{\Gamma}>0$ one can confirm, that $s_{r_c}(\Upsilon_X,\Upsilon_B,\rho_{X_c},\rho_{B_c})$ depends positively on the intensive (Υ_X,Υ_B) and extensive (ρ_{X_c},ρ_{B_c}) margin of trade and bargaining improvement. Similarly, the measure for the increase in overall employment due to the possibility of exporting and improving the bargaining power in the respective cases are given by:

$$s_{h_c} \equiv \begin{cases} 1+\left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}} + \Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}}\left(\Upsilon_B^{\frac{\kappa_h}{\Gamma}}-1\right)\rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}}, & \text{if } c=1 \\ 1+\left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}}\Upsilon_B^{\frac{\kappa_h}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}}, & \text{if } c=2 \\ 1+\left(\Upsilon_B^{\frac{\kappa_h}{\Gamma}}-1\right)\rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}} + \Upsilon_B^{\frac{\kappa_h}{\Gamma}}\left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}}, & \text{if } c=3. \end{cases} \tag{3.B.75}$$

One can confirm, that the factor determining the increase in average employment due to the possibility of exporting and improving the bargaining power (s_{h_c}), depends positively on the intensive (Υ_X,Υ_B) and extensive (ρ_{X_c},ρ_{B_c}) margin of trade and bargaining improvement. The measure for the increase in overall matched workers due to the possibility of exporting and improving the bargaining power in the respective cases are given by:

$$s_{n_c} \equiv \begin{cases} 1+\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}} + \Upsilon_X^{\frac{1-\beta}{\Gamma}}\left(\Upsilon_B^{\frac{1}{\Gamma}}-1\right)\rho_{B_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c=1 \\ 1+\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c=2 \\ 1+\left(\Upsilon_B^{\frac{1}{\Gamma}}-1\right)\rho_{B_c}^{z-\frac{\beta}{\Gamma}} + \Upsilon_B^{\frac{1}{\Gamma}}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c=3. \end{cases} \tag{3.B.79}$$

The factor determining the increase in the average number of workers matched to firms due to the possibility of exporting and improving the bargaining power (s_{n_c}), depends positively on the intensive (Υ_X, Υ_B) and extensive (ρ_{X_c}, ρ_{B_c}) margining of trade and bargaining improvement.

Proposition 13. The shares of revenues produced by firms that improve their bargaining power in the three sorting pattern cases is derived in the Supplementary Appendix 3.B.2 and is given by:

$$S_{rB_c} = \begin{cases} \left(1 + \dfrac{1 + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}} - \Upsilon_X^{\frac{1-\beta}{\Gamma}}\rho_{B_c}^{z-\frac{\beta}{\Gamma}}}{\Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1-\Gamma}{\Gamma}}\rho_{B_c}^{z-\frac{\beta}{\Gamma}}}\right)^{-1}, & \text{if } c = 1 \\ \left(1 + \dfrac{1 - \rho_{X_c}^{z-\frac{\beta}{\Gamma}}}{\Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1-\Gamma}{\Gamma}}\rho_{X_c}^{z-\frac{\beta}{\Gamma}}}\right)^{-1}, & \text{if } c = 2 \\ \left(1 + \dfrac{1 - \rho_{B_c}^{z-\frac{\beta}{\Gamma}}}{\Upsilon_B^{\frac{1-\Gamma}{\Gamma}}\left(\rho_{B_c}^{z-\frac{\beta}{\Gamma}} + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}}\right)}\right)^{-1}, & \text{if } c = 3. \end{cases} \quad (3.B.80)$$

As shown before, a higher possible bargaining improvement λ implies that the share of firms that improve their bargaining power rises $\frac{\partial \rho_{B_c}}{\partial \lambda} > 0$. In *Case 2* and *3* also the share of exporting firms rises $\left.\frac{\partial \rho_{X_c}}{\partial \lambda}\right|_{c\in\{2,3\}} > 0$, while for *Case 1* it is unaffected $\frac{\partial \rho_{X_1}}{\partial \lambda} = 0$. Finally, also the bargaining improvement term rises $\frac{\partial \Upsilon_B}{\partial \lambda} > 0$. All those effects have a positive impact on the share of revenues generated by firms improving their bargaining power. Consequently, $\frac{\partial S_{rB_c}}{\partial \lambda} > 0$ holds.

The shares of workers employed by firms that improve their bargaining power in the three sorting pattern cases is derived in the Supplementary Appendix 3.B.2 and is given by:

$$S_{hB_c} = \begin{cases} \left(1 + \dfrac{1 + \Upsilon_X^{\frac{(1-\beta)\kappa_k}{\Gamma}}\rho_{B_c}^{z-\frac{\beta\kappa_k}{\Gamma}} + \left(\Upsilon_X^{\frac{(1-\beta)\kappa_k}{\Gamma}} - 1\right)\rho_{X_c}^{z-\frac{\beta\kappa_k}{\Gamma}}}{\Upsilon_X^{\frac{(1-\beta)\kappa_k}{\Gamma}}\Upsilon_B^{\frac{\kappa_h}{\Gamma}}\rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}}}\right)^{-1}, & \text{if } c = 1 \\ \left(1 + \dfrac{1 - \rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}}}{\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}}\Upsilon_B^{\frac{\kappa_h}{\Gamma}}\rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}}}\right)^{-1}, & \text{if } c = 2 \\ \left(1 + \dfrac{1 - \rho_{B_c}^{z-\frac{\beta\kappa_k}{\Gamma}}}{\Upsilon_B^{\frac{\kappa_h}{\Gamma}}\left(\rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}} + \left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} - 1\right)\rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}}\right)}\right)^{-1}, & \text{if } c = 3. \end{cases} \quad (3.B.81)$$

A higher possible bargaining improvement λ has the following effects, $\frac{\partial \rho_{B_c}}{\partial \lambda} > 0$, $\frac{\partial \rho_{X_1}}{\partial \lambda} > 0$,

$\frac{\partial \rho_{X_c}}{\partial \lambda}\Big|_{c\in\{2,3\}} > 0$, $\frac{\partial \Upsilon_B}{\partial \lambda} > 0$. All those effects have an positive impact on the share of workers employed by firms improving their bargaining power. Consequently, $\frac{\partial S_{hB_c}}{\partial \lambda} > 0$ holds.

The shares of workers matched with firms that improve their bargaining power in the three sorting pattern cases is derived in the Supplementary Appendix 3.B.2 and is given by:

$$S_{nB_c} = \begin{cases} \left(1 + \frac{1 + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}} - \Upsilon_X^{\frac{1-\beta}{\Gamma}}\rho_{B_c}^{z-\frac{\beta}{\Gamma}}}{\Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1}{\Gamma}}\rho_{B_c}^{z-\frac{\beta}{\Gamma}}}\right)^{-1}, & \text{if } c = 1 \\ \left(1 + \frac{1 - \rho_{X_c}^{z-\frac{\beta}{\Gamma}}}{\Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1}{\Gamma}}\rho_{X_c}^{z-\frac{\beta}{\Gamma}}}\right)^{-1}, & \text{if } c = 2 \\ \left(1 + \frac{1 - \rho_{B_c}^{z-\frac{\beta}{\Gamma}}}{\Upsilon_B^{\frac{1}{\Gamma}}\left(\rho_{B_c}^{z-\frac{\beta}{\Gamma}} + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}}\right)}\right)^{-1}, & \text{if } c = 3. \end{cases} \tag{3.B.82}$$

A higher possible bargaining improvement λ has the following effects, $\frac{\partial \rho_{B_c}}{\partial \lambda} > 0$, $\frac{\partial \rho_{X_1}}{\partial \lambda} > 0$, $\frac{\partial \rho_{X_c}}{\partial \lambda}\Big|_{c\in\{2,3\}} > 0$, $\frac{\partial \Upsilon_B}{\partial \lambda} > 0$. All those effects have a positive impact on the share of workers matched with firms improving their bargaining power. Consequently, $\frac{\partial S_{nB_c}}{\partial \lambda} > 0$ holds.

3.B Supplementary Appendix

This supplementary appendix contains the full derivation of the framework in the paper. Equations are numbered with the section number 3.B in case they are not also represented in the main paper. Sections 3.B.1 to 3.B.3 correspond to sections 3.2 to 3.4 in the paper.

3.B.1 Model Framework

Preferences. Assuming constant elasticity of substitution between varieties ϑ and depicting the consumed quantity of a variety by $q(\vartheta)$, the real consumption index Q is defined over the set of varieties M as:

$$Q = \left(\int_0^M q(\vartheta)^\beta \; d\vartheta \right)^{1/\beta} \qquad 0 < \beta < 1, \tag{3.1}$$

where β controls the elasticity of substitution between different varieties. In particular $\frac{1}{1-\beta}$ is the elasticity of substitution.

A representative consumer will maximize consumption Q subject to its budget Y. Given the ordinal utility concept one can also maximize Q^β. The optimisation problem is given by:

$$\begin{aligned}
\mathcal{L} &= \int_0^M q(\vartheta)^\beta \; d\vartheta + \lambda \left(Y - \int_0^M p(\vartheta) q(\vartheta) \; d\vartheta \right), \\
\frac{\partial \mathcal{L}}{\partial q(\vartheta)} &= \beta q(\vartheta)^{\beta-1} - \lambda p(\vartheta) = 0, && (3.B.1) \\
\frac{\partial \mathcal{L}}{\partial q(\nu)} &= \beta q(\nu)^{\beta-1} - \lambda p(\nu) = 0, && (3.B.2) \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= Y - \int_0^M p(\vartheta) q(\vartheta) \; d\vartheta = 0. && (3.B.3)
\end{aligned}$$

ν is used to index a second variety. Using (3.B.1) and (3.B.2) one can write:

$$\begin{aligned}
q(\vartheta)^{\beta-1} &= \frac{p(\vartheta)}{p(\nu)} q(\nu)^{\beta-1} \\
q(\vartheta) &= \left(\frac{p(\vartheta)}{p(\nu)} \right)^{-\frac{1}{1-\beta}} q(\nu).
\end{aligned}$$

Using this in the budget constraint (3.B.3) yields:

$$\begin{aligned}
Y &= \int_0^M p(\vartheta) \left(\frac{p(\vartheta)}{p(\nu)} \right)^{-\frac{1}{1-\beta}} q(\nu) \; d\vartheta \\
&= p(\nu)^{\frac{1}{1-\beta}} q(\nu) \int_0^M p(\vartheta)^{-\frac{\beta}{1-\beta}} \; d\vartheta. && (3.B.4)
\end{aligned}$$

The demand function follows as:

$$
\begin{aligned}
q(\nu) &= p(\nu)^{-\frac{1}{1-\beta}} \left(\int_0^M p(\vartheta)^{-\frac{\beta}{1-\beta}} \, d\vartheta \right)^{-1} Y \\
&= p(\nu)^{-\frac{1}{1-\beta}} P^{\frac{\beta}{1-\beta}} Y \\
&= p(\vartheta)^{-\frac{1}{1-\beta}} A^{\frac{1}{1-\beta}}, \qquad (3.B.5)
\end{aligned}
$$

with the price index $P \equiv \left(\int_0^M p(\vartheta)^{-\frac{\beta}{1-\beta}} \right)^{-\frac{1-\beta}{\beta}}$ and the demand shifter $A \equiv P^{\beta} Y^{1-\beta}$. Using the demand for a variety (3.B.5) and the equilibrium condition stating that demand for a variety is equal to supply, revenues from selling variety ϑ can be written as:

$$
r(\vartheta) = p(\vartheta) q(\vartheta) = q(\vartheta)^{-(1-\beta)} A q(\vartheta) = A q(\vartheta)^{\beta}. \qquad (3.2)
$$

Labour Market. There are two types of workers $i, j \in \{1, 2\}, i \neq j$ each specific to their occupation. In particular there are L_i, L_j workers in each country who can work in occupation i, j and produce $y_i(\theta), y_j(\theta)$. Within each labour type a heterogeneous workforce is considered where workers have match specific abilities a. Abilities are assumed to be independently pareto distributed:

$$
G_a(a) = 1 - \left(\frac{a_{\min}}{a} \right)^k; \quad \text{for} \quad a \geq a_{\min} > 0, \qquad (3.B.6)
$$

with the shape parameter $k > 1$ and the lower bound of the ability distribution $a_{\min}$. The ability distribution for producing either of the two labour types is the same. While the match specific ability is not known by firms and workers, firms have the possibility to screen workers. In particular, they can determine whether a worker's ability is above or below a chosen ability threshold $a_{ci}(\theta)$, but cannot determine the exact match specific ability of the worker. This process is costly and screening costs are denoted by $c a_c^{\delta}/\delta$. Where $c > 0$ scales the screening cost and $\delta > 0$ determines the degree to which a higher threshold implies higher screening costs.[30] The number of workers hired $h_i(\theta)$ for the production of task-i of type-i by firm θ is given by the number of workers screened $n_i(\theta)$ times the share of workers above the ability threshold $a_{ci}(\theta)$. Using the ability distribution (3.B.6) it follows:

$$
h_i(\theta) = n_i(\theta) \left(1 - G_a(a_{ci}(\theta)) \, da \right) = n_i(\theta) \frac{a_{\min}^k}{a_{ci}(\theta)^k}. \qquad (3.3)
$$

[30]For simplicity it is assumed that the screening cost are of the same type for both types of workers.

The average ability of employed workers by a firm given the chosen ability threshold is depicted by:

$$\begin{aligned}\bar{a}_i(\theta) &= \frac{\int_{a_{ci}(\theta)}^{\infty} a\ g_a(a)\ da}{1 - G_a(a_{ci}(\theta))} \\ &= \frac{a_{ci}(\theta)^k}{a_{\min}^k} \int_{a_{ci}(\theta)}^{\infty} a \frac{k a_{\min}^k}{a^{k+1}}\ da \\ &= a_{ci}(\theta)^k \left[\frac{k}{1-k} a^{1-k} \right]_{a_{ci}(\theta)}^{\infty} = \frac{k}{k-1} a_{ci}(\theta). \end{aligned} \tag{3.4}$$

In addition to the screening cost, it is also assumed that firms have to match with workers, which evolves search and matching costs. In order to match with $n_i(\theta)$ workers a firm has to pay search and matching cost of $b_i n_i(\theta)$ where b_i is an endogenous measure of the labour market tightness for type i workers. In particular the matching cost b_i are given by:

$$b_i = \frac{x_i^{\alpha}}{\alpha_0}, \tag{3.5}$$

where $0 < \alpha_0 < 1$, $\alpha > 0$ and $x_i = \frac{N_i}{L_i}$ is the labour market tightness and is the ratio between the number of type i workers matched with a firm N_i and the number of type i workers searching for a job L_i.

Unemployment Rate. The unemployment rate can be calculated as one minus the share of workers employed relative to the overall number of workers of a specific worker type. The share of workers employed is given by the share of workers matched (x_i)times the share of workers hired (σ_i) given a match. Thus, the unemployment rate is given by:

$$u_i = 1 - \frac{H_i}{L_i} = 1 - \frac{H_i}{N_i}\frac{N_i}{L_i} = 1 - \sigma_i x_i. \tag{3.6}$$

It is straight forward to determine that the unemployment rate falls the tighter the labour market ($\frac{\partial u_i}{\partial x_i} < 0$) and the larger the share of workers hired ($\frac{\partial u_i}{\partial \sigma_i} < 0$).

Production Technology. A continuum of potential market entrants can pay up front entry costs f_E in order to learn about their firm specific productivity θ. Productivities are independently distributed and drawn from a pareto distribution:

$$G_\theta(\theta) = 1 - (\theta_{\min}/\theta)^z \quad \text{for} \ \ \theta \geq \theta_{min} > 0, \tag{3.B.7}$$

with $z > 1$ being the distribution shape parameter. As firms are uniquely identified by their productivity, θ is used as a firm index. Domestic production involves fixed costs $f_D > 0$. The final variety $y(\theta)$ is produced by two occupations $i = \{1, 2\}$, where $y_i(\theta)$ is the quantity produced by occupation i in the production of firm θ. The final production technology is positively dependent on the productivity θ of a firm and combines the two tasks in a Cobb-Douglas fashion:

$$y(\theta) = \theta \left(\frac{y_1(\theta)}{\eta_1^{1-\Gamma}}\right)^{\eta_1} \left(\frac{y_2(\theta)}{\eta_2^{1-\Gamma}}\right)^{\eta_2} = \eta_0 \theta \prod_i y_i(\theta)^{\eta_i}; \quad \text{where } 0 < \eta_i < 1, \tag{3.7}$$

where $0 < \eta_i < 1$ is the Cobb-Douglas weight of task i and $\sum_i \eta_i = 1$. and η_0 scales the production and is choose such that it simplifies notation:

$$\eta_0 \equiv \left(\prod_i \eta_i^{\eta_i}\right)^{-(\gamma+(1-\gamma k)/\delta)} = \left(\prod_i \eta_i^{\eta_i}\right)^{-\frac{1-\Gamma}{\beta}},$$

where Γ is a function of preference, production technology, worker ability and search cost parameters:

$$\Gamma \equiv 1 - \beta\gamma - \beta/\delta(1 - \gamma k).$$

The production parameter γ, worker ability parameter k and the search cost parameter δ will be introduced in the respective sections.

Production of Occupation-Specific Tasks. A firm can produce the two tasks with help of heterogeneous workers. The production of the task features decreasing returns to labour $0 < \gamma < 1$ and depends on the average worker ability $\bar{a}(\theta)$ which can be influenced by the choice of the screening threshold:

$$y_i(\theta) = h_i(\theta)^\gamma \bar{a}_i(\theta). \tag{3.8}$$

Using the production technology of the task (3.8) in the production function (3.7) the production of a firm with productivity θ can be written as a function of the chosen average abilities and the number of workers hired for the two tasks.

$$y(\theta) = \eta_0 \theta \prod_i \left(h_i(\theta)^\gamma \bar{a}_i(\theta)\right)^{\eta_i}. \tag{3.9}$$

Using the average ability (3.4) and the relationship between the number of workers screened and the number of workers hired (3.3) in (3.9), the production of a firm with productivity θ can be written as a function of the chosen ability thresholds and the number of workers screened for the two tasks:

$$\begin{aligned} y(\theta) &= \eta_0 \theta \prod_i \left(\left(n_i(\theta) \frac{a_{min}^k}{a_{ci}(\theta)^k} \right)^\gamma \frac{k}{k-1} a_{ci}(\theta) \right)^{\eta_i} \\ &= \eta_0 \kappa_y \theta \prod_i \left(n_i(\theta)^\gamma a_{ci}(\theta)^{1-\gamma k} \right)^{\eta_i}, \end{aligned} \tag{3.10}$$

where:

$$\kappa_y \equiv \frac{k}{k-1} a_{\min}^{\gamma k}.$$

In order to assure that choosing a higher ability threshold increases the output, it is assumed that parameters satisfy $0 < \gamma k < 1$, otherwise no firm would invest in screening.

Export Decision. In equilibrium supply has to equal demand, thus without trade cost demand for variety ϑ has to equal supply of this variety produced by a firm with productivity θ, $y(\theta) = q(\vartheta)$. For the export market τ-times the quantity have to be shipped $\tau q^*(\theta) = y_X(\theta)$. Using the results from the preference side (3.2) revenues on the both markets are then given by:

$$r_D(\theta) = A y_D(\theta)^\beta, \tag{3.B.8}$$

$$r_X(\theta) = A^* \left(\frac{y_X(\theta)}{\tau}\right)^\beta. \tag{3.B.9}$$

Firms that choose to supply both markets have to allocate their output $y(\theta)$ between the two markets. They do so by choosing the amount produced for the domestic and export market ($y_D(\theta)$ and $y_X(\theta)$) such that the marginal revenue is the same in both markets:

$$\begin{aligned}
\beta A y_D(\theta)^{\beta-1} = \frac{\partial r_D(\theta)}{\partial y_D(\theta)} &\stackrel{!}{=} \frac{\partial r_X(\theta)}{\partial y_X(\theta)} = \beta A^* y_X(\theta)^{\beta-1} \tau^{-\beta} \\
\left(\frac{y_D(\theta)}{y_X(\theta)}\right)^{\beta-1} &= \frac{A^*}{A}\tau^{-\beta} \\
y_X(\theta) &= \left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}} \tau^{-\frac{\beta}{1-\beta}} y_D(\theta) \\
y_X(\theta) &= \left(\Upsilon_X(\theta) - 1\right) y_D(\theta),
\end{aligned} \tag{3.B.10}$$

where the market access term is defined as:

$$\Upsilon_X(\theta) \equiv 1 + I_X(\theta) \left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}} \tau^{-\frac{\beta}{1-\beta}}. \tag{3.12}$$

and $I_X(\theta) = \{0, 1\}$ is an indicator variable indicating whether a firm decides to export or not. Using the export supply (3.B.10) overall production thus can be written as:

$$y(\theta) = y_D(\theta) + I_X y_X(\theta) = y_D(\theta) + \left(\Upsilon_X(\theta) - 1\right) y_D(\theta) = \Upsilon_X(\theta) y_D(\theta). \tag{3.B.11}$$

Using (3.B.10) and (3.B.11) production for the domestic and the export market thus can be written as:

$$y_D(\theta) = \frac{y(\theta)}{\Upsilon_X(\theta)}, \qquad\qquad y_X(\theta) = \frac{\Upsilon_X(\theta) - 1}{\Upsilon_X(\theta)} y(\theta). \tag{3.B.12}$$

Domestic revenues, using (3.B.8) and (3.B.12), are given by:

$$r_D(\theta) = Ay_D(\theta)^\beta = A\left(\frac{y(\theta)}{\Upsilon_X(\theta)}\right)^\beta . \tag{3.B.13}$$

Revenues from exporting, using (3.B.9) and (3.B.12), are depicted by:

$$\begin{aligned} r_X(\theta) &= Ay_D(\theta)^\beta I_X\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}}\tau^{-\frac{\beta}{1-\beta}} = Ay_D(\theta)^\beta\left(\Upsilon_X(\theta)-1\right)\right) \\ &= A\left(\frac{y(\theta)}{\Upsilon_X(\theta)}\right)^\beta\left(\Upsilon_X(\theta)-1\right). \end{aligned} \tag{3.B.14}$$

Using (3.B.13) and (3.B.14) overall revenues of a firm are given by:

$$\begin{aligned} r(\theta) &= r_D(\theta) + r_X(\theta) \\ &= A\left(\frac{y(\theta)}{\Upsilon_X(\theta)}\right)^\beta + A\left(\frac{y(\theta)}{\Upsilon_X(\theta)}\right)^\beta\left(\Upsilon_X(\theta)-1\right) = \Upsilon_X(\theta)^{1-\beta}Ay(\theta)^\beta. \end{aligned} \tag{3.11}$$

Using this result in (3.B.13) and (3.B.14) yields a relationship between domestic and overall revenues as well as between export and overall revenues. Where using (3.B.13, 3.11) domestic revenues are given by:

$$r_D(\theta) = A\left(\frac{y(\theta)}{\Upsilon_X(\theta)}\right)^\beta \frac{\Upsilon_X(\theta)^{1-\beta}}{\Upsilon_X(\theta)^{1-\beta}} = \Upsilon_X(\theta)^{(-\beta-(1-\beta))}r(\theta) = \frac{r(\theta)}{\Upsilon_X(\theta)}.$$

Using (3.B.14, 3.11) revenues from exporting are depicted by:

$$r_X(\theta) = A\left(\frac{y(\theta)}{\Upsilon_X(\theta)}\right)^\beta\left(\Upsilon_X(\theta)-1\right) = \frac{\Upsilon_X(\theta)-1}{\Upsilon_X(\theta)}r(\theta).$$

Using 3.11 and the production technology (3.9) and (3.10) one can rewrite revenues as:

$$r(\theta) = \Upsilon_X(\theta)^{1-\beta}A\eta_0^\beta\theta^\beta\prod_i h_i(\theta)^{\beta\gamma\eta_i}\bar{a}_i(\theta)^{\beta\eta_i} \tag{3.14}$$

$$= \Upsilon_X(\theta)^{1-\beta}A\eta_0^\beta\kappa_y^\beta\theta^\beta\prod_i n_i(\theta)^{\beta\gamma\eta_i}a_{ci}(\theta)^{\beta(1-\gamma k)\eta_i}. \tag{3.21}$$

Wage Bargaining, Bargaining Improvement. Following the bargaining approach by Stole and Zwiebel (1996a,b) at the stage of wage bargaining all fixed costs (f_D, f_X, f_B), as well as the search and matching and the screening cost ($b_i n_i$, ca_c^δ/δ) are sunk. Thus, all other arguments besides the number of workers hired is fixed in the revenue equation. The outside option for workers is unemployment, whose value is set to zero $\underline{w} = 0$. The bargaining parameter $\lambda_i(\theta)$ determines how the net surplus is divided between the bargaining parties in the pairwise meeting between an employee and the firm:

$$\begin{aligned} \Delta\tilde{\pi}(\theta, h_i, h_j) &= \lambda_i(\theta)\left(w_i(\theta, h_i, h_j) - \underline{\mathrm{w}}\right) \\ \frac{\partial\left(r(\theta, h_i, h_j) - w_i(\theta, h_i, h_j)h_i(\theta) - w_j(\theta, h_i, h_j)h_j(\theta)\right)}{\partial h_i(\theta)} &= \lambda_i(\theta)\left(w_i(\theta, h_i, h_j)\right), \end{aligned} \tag{3.15}$$

where a higher bargaining parameter indicates a larger bargaining power of the firm. A firm can decide to invest and pay a fixed cost f_B to improve its bargaining parameter to $\lambda > 1$.[31] $I_B(\theta) = \{0, 1\}$ is used as an indicator depicting the firm's decision of investing in improving the bargaining power. By default there are equal bargaining weights between a firm and the workers employed $\lambda_i(\theta)|_{I_B=0} = 1$. The bargaining improvement process is only possible with type-1 workers. In particular, the bargaining parameter is given by:

$$\lambda_i(\theta) = \begin{cases} \lambda > 1, & I_B(\theta) = 1 \wedge i = 1 \\ 1, & I_B(\theta) = 0 \vee i = 2. \end{cases} \tag{3.13}$$

Firms choose the number of employees such that the cost of hiring that worker is equal to the marginal profits of hiring another worker for the production of task i. They thereby take into account that hiring another worker will change wages of all workers employed in the production of task i:[32]

$$\begin{aligned} \lambda_i(\theta)w_i(\theta, h_i, h_j) &= \frac{\partial\left(r(\theta, h_i, h_j) - w_i(\theta, h_i, h_j)h_i(\theta) - w_j(\theta, h_i, h_j)h_j(\theta)\right)}{\partial h_i(\theta)} \\ &= \frac{\partial r(\theta, h_i, h_j)}{\partial h_i(\theta)} - \frac{\partial w_i(\theta, h_i, h_j)}{\partial h_i(\theta)}h_i(\theta) - w_i(\theta, h_i, h_j) - \frac{\partial w_j(\theta, h_i, h_j)}{\partial h_i(\theta)}h_j(\theta) \\ w_i(\theta, h_i, h_j) &= \frac{1}{1+\lambda_i(\theta)}\left(\frac{\partial r(\theta, h_i, h_j)}{\partial h_i(\theta)} - \frac{\partial w_i(\theta, h_i, h_j)}{\partial h_i(\theta)}h_i(\theta) - \frac{\partial w_j(\theta, h_i, h_j)}{\partial h_i(\theta)}h_j(\theta)\right). \end{aligned} \tag{3.B.15}$$

It also holds that:

$$\begin{aligned} \frac{\partial w_i(\theta, h_i, h_j)}{\partial h_i(\theta)} &= \frac{1}{1+\lambda_i(\theta)}\left(\frac{\partial^2 r(\theta, h_i, h_j)}{\partial h_i(\theta)^2} - \frac{\partial^2 w_i(\theta, h_i, h_j)}{\partial h_i(\theta)^2}h_i(\theta)\right. \\ &\qquad \left. - \frac{\partial w_i(\theta, h_i, h_j)}{\partial h_i(\theta)} - \frac{\partial^2 w_j(\theta, h_i, h_j)}{\partial h_i(\theta)^2}h_j(\theta)\right) \\ \frac{\partial w_i(\theta, h_i, h_j)}{\partial h_i(\theta)} &= \frac{1}{2+\lambda_i(\theta)}\left(\frac{\partial^2 r(\theta, h_i, h_j)}{\partial h_i(\theta)^2} - \frac{\partial^2 w_i(\theta, h_i, h_j)}{\partial h_i(\theta)^2}h_i(\theta) - \frac{\partial^2 w_j(\theta, h_i, h_j)}{\partial h_i(\theta)^2}h_j(\theta)\right) \end{aligned}$$

[31] Upper case B refers to the investment decision of improving the bargaining power and not to the search cost on the labour market which are referred to by lower case b.

[32] In the following section, $r(\theta, h_i, h_j)$ and $w(\theta, h_i, h_j)$ is used to emphasise the dependence between revenues or wages and the number of workers employed. However, h_i is still in itself a function of the firms productivity.

and:

$$\frac{\partial^n w_i(\theta,h_i,h_j)}{\partial h_i(\theta)^n} = \frac{1}{n+1+\lambda_i(\theta)}\left(\frac{\partial^{n+1} r(\theta,h_i,h_j)}{\partial h_i(\theta)^{n+1}} - \frac{\partial^{n+1} w_i(\theta,h_i,h_j)}{\partial h_i(\theta)^{n+1}}h_i(\theta) - \frac{\partial^{n+1} w_j(\theta,h_i,h_j)}{\partial h_i(\theta)^{n+1}}h_j(\theta)\right). \tag{3.B.16}$$

Using the revenue equation (3.14) one knows:

$$\begin{aligned}\frac{\partial r(\theta,h_i,h_j)}{\partial h_i(\theta)} &= \frac{r(\theta,h_i,h_j)}{h_i(\theta)}\beta\gamma\eta_i\\ \frac{\partial^2 r(\theta,h_i,h_j)}{\partial h_i(\theta)^2} &= \frac{r(\theta,h_i,h_j)}{h_i(\theta)^2}\beta\gamma\eta_i(\beta\gamma\eta_i-1)\\ \frac{\partial^n r(\theta,h_i,h_j)}{\partial h_i(\theta)^n} &= \frac{r(\theta,h_i,h_j)}{h_i(\theta)^n}\prod_{j=1}^{n}\beta\gamma\eta_i-(j-1).\end{aligned} \tag{3.B.17}$$

Solving this iterative system by plugging (3.B.16) and (3.B.17) in (3.B.15) the bargaining game can be solved in the following form:

$$w_i(\theta) = \frac{\beta\gamma\eta_i\lambda_j(\theta)}{\beta\gamma\eta_i\lambda_j(\theta)+\beta\gamma\eta_j\lambda_i(\theta)+\lambda_i(\theta)\lambda_j(\theta)}\frac{r(\theta)}{h_i(\theta)} = \Lambda_{wi}(\theta)\frac{r(\theta)}{h_i(\theta)}. \tag{3.16}$$

where the share of revenues belonging to a certain type of worker is given by:

$$\Lambda_{wi}(\theta) \equiv \frac{\beta\gamma\eta_i\lambda_j(\theta)}{\beta\gamma\eta_i\lambda_j(\theta)+\beta\gamma\eta_j\lambda_i(\theta)+\lambda_i(\theta)\lambda_j(\theta)}. \tag{3.17}$$

Only the bargaining power for type-1 workers can be influenced. For type-2 workers it is one. Thus, the share of revenues belonging to a certain type of worker can be written as:

$$\Lambda_{wi}(\theta) \equiv \begin{cases}\dfrac{\beta\gamma\eta_1}{\beta\gamma\eta_1+\beta\gamma\eta_2\lambda+\lambda}, & \text{if } I_B(\theta)=1\wedge i=1\\ \dfrac{\beta\gamma\eta_2\lambda}{\beta\gamma\eta_1+\beta\gamma\eta_2\lambda+\lambda}, & \text{if } I_B(\theta)=1\wedge i=2\\ \dfrac{\beta\gamma\eta_i}{1+\beta\gamma}, & \text{if } I_B(\theta)=0.\end{cases}$$

Rewriting 3.16 overall wage costs for the respective occupation are given by:

$$w_i(\theta)h_i(\theta) = \Lambda_{wi}(\theta)r(\theta).$$

Comparing the wage share of revenues paid to the two worker types, it follows:

$$\frac{w_i(\theta)h_i(\theta)}{w_j(\theta)h_ji(\theta)} = \frac{\Lambda_{wi}(\theta)r(\theta)}{\Lambda_{wj}(\theta)r(\theta)} = \frac{\Lambda_{wi}(\theta)}{\Lambda_{wj}(\theta)} = \frac{\frac{\beta\gamma\eta_i\lambda_j(\theta)}{\beta\gamma\eta_i\lambda_j(\theta)+\beta\gamma\eta_j\lambda_i(\theta)+\lambda_i(\theta)\lambda_j(\theta)}}{\frac{\beta\gamma\eta_j\lambda_i(\theta)}{\beta\gamma\eta_i\lambda_j(\theta)+\beta\gamma\eta_j\lambda_i(\theta)+\lambda_i(\theta)\lambda_j(\theta)}} = \frac{\eta_i\lambda_j(\theta)}{\eta_j\lambda_i(\theta)}. \tag{3.18}$$

The share of revenues belonging to the firm after paying wages is given by:

$$\Lambda_{I_B}(\theta) \equiv 1 - \sum_i \Lambda_{wi}(\theta) = 1 - \frac{\beta\gamma\eta_i\lambda_j(\theta) + \beta\gamma\eta_j\lambda_i(\theta)}{\beta\gamma\eta_i\lambda_j(\theta) + \beta\gamma\eta_j\lambda_i(\theta) + \lambda_i(\theta)\lambda_j(\theta)}$$
$$= \frac{\lambda_i(\theta)\lambda_j(\theta)}{\beta\gamma\eta_i\lambda_j(\theta) + \beta\gamma\eta_j\lambda_i(\theta) + \lambda_i(\theta)\lambda_j(\theta)} < 1. \tag{3.19}$$

The share of revenues belonging to the firm after paying wages in case of no investment/ investment into improving the bargaining power is denoted by:

$$\Lambda_0 \equiv \Lambda_{I_B}(\theta)|_{I_B=0} = \frac{1}{1+\beta\gamma}$$
$$\Lambda_1 \equiv \Lambda_{I_B}(\theta)|_{I_B=1} = \frac{\lambda}{\beta\gamma\eta_1 + \beta\gamma\eta_2\lambda + \lambda}.$$

It proofs useful to depict the share of revenues belonging to the firm after paying wages as the share belonging to a firm which is not investing Λ_0 times the bargaining improvement term $\Upsilon_B(\theta)$,

$$\Lambda_{I_B}(\theta) \equiv \Lambda_0 \Upsilon_B(\theta).$$

The bargaining improvement term then is given by:

$$\Upsilon_B(\theta) \equiv \frac{\Lambda_{I_B}(\theta)}{\Lambda_0} = 1 + \frac{\Lambda_{I_B}(\theta) - \Lambda_0}{\Lambda_0} = 1 + I_B(\theta)\frac{\Lambda_1 - \Lambda_0}{\Lambda_0} = 1 + I_B(\theta)\left(\frac{\Lambda_1}{\Lambda_0} - 1\right)$$
$$= 1 + I_B(\theta)\left(\frac{\lambda(1+\beta\gamma)}{\beta\gamma\eta_1 + \beta\gamma\eta_2\lambda + \lambda} - 1\right)$$
$$= 1 + I_B(\theta)\left(\frac{\lambda + \beta\gamma\lambda - \beta\gamma\eta_1 - \beta\gamma\eta_2\lambda - \lambda}{\beta\gamma\eta_1 + \beta\gamma\eta_2\lambda + \lambda}\right)$$
$$= 1 + I_B(\theta)\left(\frac{\beta\gamma\eta_1(\lambda - 1)}{\beta\gamma\eta_1 + \beta\gamma\eta_2\lambda + \lambda}\right)$$
$$= 1 + I_B(\theta)\left(\frac{\beta\gamma\eta_1 + \beta\gamma\eta_2\lambda + \lambda}{\beta\gamma\eta_1(\lambda - 1)}\right)^{-1}$$
$$= 1 + I_B(\theta)\left(\frac{\beta\gamma\eta_1 + \beta\gamma\eta_2\lambda + \lambda + \beta\gamma\eta_1(\lambda-1)}{\beta\gamma\eta_1(\lambda - 1)} - 1\right)^{-1}$$
$$= 1 + I_B(\theta)\left(\frac{\lambda}{\lambda - 1}\frac{1+\beta\gamma}{\beta\gamma\eta_1} - 1\right)^{-1} \geq 1, \tag{3.20}$$

which is equal to one, if a firm does not invest into improving its bargaining power and larger than one otherwise. This implies that when a firm invests into improving its bargaining power, a larger share of revenues belongs to the firm after paying the workers. To simplify notation:

$$\Upsilon_B \equiv \Upsilon_B(\theta)|_{I_B=1} = \frac{\Lambda_1}{\Lambda_0} = \frac{\lambda(1+\beta\gamma)}{\beta\gamma\eta_1 + \beta\gamma\eta_2\lambda + \lambda} = \frac{1+\beta\gamma}{1 + \beta\gamma\eta_1/\lambda + \beta\gamma\eta_2} > 1,$$

is used to depict the ration between the revenue shares belonging to the firm in case of investment into improving the bargaining power and not. It rises the larger the possible bargaining power of the firm, if the firm invests in improving the bargaining power:

$$\frac{\partial \Upsilon_B}{\partial \lambda} = -\frac{1+\beta\gamma}{(1+\beta\gamma\eta_1/\lambda+\beta\gamma\eta_2)^2}\left(-\frac{\beta\gamma\eta_1}{\lambda^2}\right)$$
$$= \frac{\beta\gamma\eta_1(1+\beta\gamma)}{(\lambda+\beta\gamma\eta_1+\beta\gamma\eta_2\lambda)^2} = \frac{\beta\gamma\eta_1}{\beta\gamma\eta_1+\beta\gamma\eta_2\lambda+\lambda}\frac{\Upsilon_B}{\lambda} = \Lambda_{w1}\frac{\Upsilon_B}{\lambda} > 0. \tag{3.B.18}$$

The wage cost of a firm can be written as:

$$\sum_i w_i(\theta)h_i(\theta) = (1-\Lambda_0\Upsilon_B(\theta))r(\theta). \tag{3.B.19}$$

If $\eta_1 = 0$ only task-2 is used in the production and improving the bargaining power has no effect. Thus, the framework reducing to the original Helpman et al. (2010) framework and the bargaining improvement term is one ($\Upsilon_B(\theta)|_{\eta_1=0} = 1$). For $\eta = 1$ the framework collapses to a single task production technology where improving the bargaining power affects wages of all employed workers. The share of revenues belonging to the firm after paying wages (3.19) in those two cases is given by:

$$\Lambda_{I_B}(\theta) = \begin{cases} \dfrac{1}{1+\beta\gamma}, \\ \dfrac{\lambda(\theta)}{\lambda(\theta)+\beta\gamma}, \end{cases} \qquad \Lambda_0 = \begin{cases} \dfrac{1}{1+\beta\gamma}, & \eta_1 = 0 \\ \dfrac{1}{1+\beta\gamma}, & \eta_1 = 1. \end{cases}$$

Firm's Optimisation Problem. Anticipating the outcome of the bargaining game, firms maximize profits by choosing whether to export or not, whether to invest in an increase in the bargaining power with respect to type-1 workers or not, as well as the number of workers to screen for the two tasks and the respective ability thresholds. The profit maximization problem of a firm takes the following form:

$$\pi(\theta) = \max_{\substack{a_{ci}(\theta)\geq a_{\min},\\ n_i(\theta)\geq 0,\\ I_X(\theta)\in\{0,1\},\\ I_B(\theta)\in\{0,1\}}} \Lambda_0\Upsilon_B(\theta)\ r\left(\theta, n_i(\theta), a_{ci}(\theta)\right) - \sum_i \left(b_i n_i(\theta) + \frac{c a_{ci}(\theta)^\delta}{\delta}\right)$$
$$- f_D - I_X(\theta)f_X - I_B(\theta)f_B. \tag{3.22}$$

Using revenues (3.21) the maximisation problem yields the following conditions:

$$\Lambda_0\Upsilon_B(\theta)(1-\gamma k)\beta\eta_i r(\theta) = c a_{ci}(\theta)^\delta, \tag{3.23}$$
$$\Lambda_0\Upsilon_B(\theta)\beta\gamma\eta_i r(\theta) = b_i n_i(\theta). \tag{3.24}$$

Using the optimality condition, it follows that:

$$\frac{\Lambda_0 \Upsilon_B(\theta)(1-\gamma k)\beta\eta_i r(\theta)}{\Lambda_0 \Upsilon_B(\theta)\beta\gamma\eta_i r(\theta)} = \frac{c a_{ci}(\theta)^\delta}{b_i n_i(\theta)}$$
$$n_i(\theta) = \frac{\gamma}{1-\gamma k}\frac{c}{b_i} a_{ci}(\theta)^\delta. \tag{3.25}$$

Using $i, j = \{1, 2\}, i \neq j$ and thus $1 - \eta_j = \eta_i$ one can determine a relationship between the two ability thresholds using the optimality condition (3.23) as:

$$\frac{a_{ci}(\theta)^\delta}{a_{cj}(\theta)^\delta} = \frac{\Lambda_0 \Upsilon_B(\theta)(1-\gamma k)\beta\eta_i r(\theta)/c}{\Lambda_0 \Upsilon_B(\theta)(1-\gamma k)\beta\eta_j r(\theta)/c}$$
$$\frac{a_{ci}(\theta)}{a_{cj}(\theta)} = \left(\frac{\eta_i}{\eta_j}\right)^{\frac{1}{\delta}}. \tag{3.26}$$

The ability threshold will be larger in the production of the task with the larger Cobb-Douglas production share.

Wages. Using the wage formulation (3.16) in combination with the optimality condition (3.24) and the relationship between employment, the ability threshold and the number of workers sampled (3.3), yields the following relationship:

$$\begin{aligned} w_i(\theta) = \Lambda_{wi}(\theta)\frac{r(\theta)}{h_i(\theta)} &= \frac{\Lambda_{wi}(\theta)}{\Lambda_0 \Upsilon_B(\theta)\beta\gamma\eta_i} b_i \frac{n_i(\theta)}{h_i(\theta)} \\ &= \frac{\beta\gamma\eta_i\lambda_j(\theta)}{\lambda_i(\theta)\lambda_j(\theta)}\frac{1}{\beta\gamma\eta_i} b_i \frac{n_i(\theta)}{h_i(\theta)} = \frac{b_i}{\lambda_i(\theta)}\frac{n_i(\theta)}{h_i(\theta)} \\ &= \frac{b_i}{\lambda_i(\theta)}\left(\frac{a_{ci}(\theta)}{a_{\min}}\right)^k. \end{aligned} \tag{3.27}$$

The expected wage producing task i in firm θ, given a match with the firm can be written using (3.16,3.24) as:

$$\begin{aligned} \frac{w_i(\theta)h_i(\theta)}{n_i(\theta)} &= \frac{\Lambda_{wi}(\theta)r(\theta)}{\Lambda_0 \Upsilon_B(\theta)\beta\gamma\eta_i r(\theta) b_i^{-1} r(\theta)} \\ &= \frac{\Lambda_{wi}(\theta)}{\Lambda_0 \Upsilon_B(\theta)}\frac{b_i}{\beta\gamma\eta_i}. \end{aligned} \tag{3.28}$$

Firm Variables. Using (3.23) in combination with the revenues (3.21), the ability threshold can be written as:

$$\begin{aligned} a_{ci}(\theta)^\delta =& \Lambda_0 \Upsilon_B(\theta)(1-\gamma k)\beta\eta_i r(\theta)/c \\ a_{ci}(\theta)^\delta =& \Lambda_0 \Upsilon_B(\theta)(1-\gamma k)\beta\eta_i c^{-1}\eta_0^\beta \Upsilon_X(\theta)^{1-\beta} A\kappa_y^\beta \theta^\beta \prod_i n_i(\theta)^{\beta\gamma\eta_i} a_{ci}(\theta)^{\beta(1-\gamma k)\eta_i}. \end{aligned}$$

Using (3.25) one can rewrite the above:

$$a_{ci}(\theta)^{\delta} = \Lambda_0 \Upsilon_B(\theta)\beta\gamma \frac{1-\gamma k}{\gamma}\eta_i c^{-1}\eta_0^{\beta}\Upsilon_X(\theta)^{1-\beta}A\kappa_y^{\beta}\theta^{\beta}\prod_i\left(\frac{\gamma}{1-\gamma k}\frac{c}{b_i}a_{ci}(\theta)^{\delta}\right)^{\beta\gamma\eta_i}a_{ci}(\theta)^{\beta(1-\gamma k)\eta_i}$$

$$a_{ci}(\theta)^{\delta} = \Lambda_0 \Upsilon_B(\theta)\beta\gamma\eta_i\eta_0^{\beta}\Upsilon_X(\theta)^{1-\beta}A\kappa_y^{\beta}\theta^{\beta}\left(\frac{1-\gamma k}{\gamma}\right)^{1-\beta\gamma}c^{-(1-\beta\gamma)}\prod_i a_{ci}(\theta)^{\beta\gamma\delta\eta_i+\beta(1-\gamma k)\eta_i}b_i^{\beta\gamma\eta_i}.$$

Using the relationship between the two ability thresholds (3.26) allows to solve for the ability threshold in the following manner:

$$a_{ci}(\theta)^{\delta-\beta\gamma\delta\eta_i-\beta(1-\gamma k)\eta_i} = \Lambda_0\Upsilon_B(\theta)\beta\gamma\kappa_y^{\beta}\eta_0^{\beta}\Upsilon_X(\theta)^{1-\beta}A\theta^{\beta}\left(\frac{1-\gamma k}{\gamma}\right)^{1-\beta\gamma}c^{-(1-\beta\gamma)}\left(\prod_i b_i^{\eta_i}\right)^{-\beta\gamma}$$
$$\eta_i\left(a_{ci}(\theta)\left(\frac{\eta_j}{\eta_i}\right)^{\frac{1}{\delta}}\right)^{\beta\gamma\delta\eta_j+\beta(1-\gamma k)\eta_j}$$

$$a_{ci}(\theta)^{\delta(1-\beta\gamma-\beta/\delta(1-\gamma k))} = \Lambda_0\Upsilon_B(\theta)\beta\gamma\kappa_y^{\beta}\Upsilon_X(\theta)^{1-\beta}A\theta^{\beta}\left(\frac{1-\gamma k}{\gamma}\right)^{1-\beta\gamma}c^{-(1-\beta\gamma)}\left(\prod_i b_i^{\eta_i}\right)^{-\beta\gamma}$$
$$\eta_0^{\beta}\eta_i^{1-\beta\gamma-\beta/\delta(1-\gamma k)}\left(\prod_i \eta_i^{\eta_i}\right)^{\beta\gamma+\beta/\delta(1-\gamma k)}.$$

To simplify notation the following help parameters are used:

$$\eta_0 \equiv \left(\prod_i \eta_i^{\eta_i}\right)^{-\frac{1-\Gamma}{\beta}}, \qquad \phi_1 \equiv \left(\beta\gamma\kappa_y^{\beta}\Lambda_0\right)^{\frac{1}{\Gamma}},$$

$$\Gamma \equiv 1-\beta\gamma-\beta/\delta(1-\gamma k), \qquad \phi_2 \equiv \left(\frac{1-\gamma k}{\gamma}\right)^{\frac{1}{\delta\Gamma}},$$

$$b \equiv \left(\prod_i b_i^{\eta_i}\right),$$

where b can be considered as a combined search cost measure weighting the search cost for the both types of labour. Thus, the ability thresholds are given by:

$$a_{ci}(\theta)^{\delta\Gamma} = \Lambda_0\Upsilon_B(\theta)\beta\gamma\kappa_y^{\beta}\Upsilon_X(\theta)^{1-\beta}A\theta^{\beta}\left(\frac{1-\gamma k}{\gamma}\right)^{1-\beta\gamma}c^{-(1-\beta\gamma)}b^{-\beta\gamma}\eta_i^{\Gamma}$$
$$a_{ci}(\theta) = \eta_i^{\frac{1}{\delta}}\phi_1^{\frac{1}{\delta}}\phi_2^{1-\beta\gamma}c^{-\frac{1-\beta\gamma}{\delta\Gamma}}b^{-\frac{\beta\gamma}{\delta\Gamma}}\Upsilon_X(\theta)^{\frac{1-\beta}{\delta\Gamma}}\Upsilon_B(\theta)^{\frac{1}{\delta\Gamma}}A^{\frac{1}{\delta\Gamma}}\theta^{\frac{\beta}{\delta\Gamma}}. \tag{3.B.20}$$

Using this in combination with (3.25) yields the number of workers screened by a firm:

$$n_i(\theta) = \frac{\gamma}{1-\gamma k}\frac{c}{b_i}\eta_i\phi_1\phi_2^{\delta(1-\beta\gamma)}c^{-\frac{1-\beta\gamma}{\Gamma}}b^{-\frac{\beta\gamma}{\Gamma}}\Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}}\Lambda_0\Upsilon_B(\theta)^{\frac{1}{\Gamma}}A^{\frac{1}{\Gamma}}\theta^{\frac{\beta}{\Gamma}}$$
$$n_i(\theta) = \eta_i b_i^{-1}\phi_1\phi_2^{\beta(1-\gamma k)}c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}}b^{-\frac{\beta\gamma}{\Gamma}}\Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}}\Upsilon_B(\theta)^{\frac{1}{\Gamma}}A^{\frac{1}{\Gamma}}\theta^{\frac{\beta}{\Gamma}} \tag{3.B.21}$$
$$n_i(\theta) = \eta_i b_i^{-\frac{\Gamma+\beta\gamma\eta_i}{\Gamma}}b_j^{-\frac{\beta\gamma\eta_j}{\Gamma}}\phi_1\phi_2^{\beta(1-\gamma k)}c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}}\Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}}\Upsilon_B(\theta)^{\frac{1}{\Gamma}}A^{\frac{1}{\Gamma}}\theta^{\frac{\beta}{\Gamma}}.$$

Employment of a firm can be derived using (3.3):

$$h_i(\theta) = n_i(\theta)\frac{a_{\min}^k}{a_{ci}(\theta)^k}. \tag{3.3}$$

$$h_i(\theta) = \eta_i^{1-k/\delta} b_i^{-1} a_{\min}^k \phi_1^{1-k/\delta} \phi_2^{-(k-\beta)} c^{\frac{k-\beta}{\delta\Gamma}} b^{-\frac{\beta\gamma(1-k/\delta)}{\Gamma}} \left(\Upsilon_X(\theta)^{1-\beta}\Upsilon_B(\theta)A\theta^\beta\right)^{\frac{1-k/\delta}{\Gamma}} \tag{3.B.22}$$

$$h_i(\theta) = \eta_i^{1-k/\delta} b_i^{-\frac{\Gamma+\beta\gamma\eta_i(1-k/\delta)}{\Gamma}} b_j^{-\frac{\beta\gamma\eta_j(1-k/\delta)}{\Gamma}} a_{\min}^k \phi_1^{1-k/\delta} \phi_2^{-(k-\beta)} c^{\frac{k-\beta}{\delta\Gamma}} \left(\Upsilon_X(\theta)^{1-\beta}\Upsilon_B(\theta)A\theta^\beta\right)^{\frac{1-k/\delta}{\Gamma}}.$$

Firms with a higher productivity not only sample more workers and choose a higher ability threshold, but also hire more workers if $\delta > k$. Revenues of a firm can be derived using (3.24):

$$r(\theta) = \frac{bn_i}{\Lambda_0\Upsilon_B(\theta)\beta\gamma\eta_i}$$

$$r(\theta) = \frac{\phi_1\phi_2^{\beta(1-\gamma k)}}{\beta\gamma\Lambda_0} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} b^{-\frac{\beta\gamma}{\Gamma}} \Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}} \Upsilon_B(\theta)^{\frac{1-\Gamma}{\Gamma}} A^{\frac{1}{\Gamma}} \theta^{\frac{\beta}{\Gamma}}. \tag{3.B.23}$$

Using the wage formulation (3.16) in combination with the optimality condition (3.24) and the result for the ability threshold (3.B.20) yields the wage paid by a firm:

$$w_i(\theta) = \frac{b_i}{\lambda_i(\theta)}\left(\frac{a_{ci}(\theta)}{a_{\min}}\right)^k \tag{3.27}$$

$$= \frac{b_i\eta_i^{k/\delta}}{\lambda_i(\theta)} a_{\min}^{-k} \phi_1^{k/\delta} \phi_2^{(1-\beta\gamma)k} c^{-\frac{(1-\beta\gamma)k}{\delta\Gamma}} b^{-\frac{\beta\gamma k}{\delta\Gamma}} \Upsilon_X(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}} \Upsilon_B(\theta)^{\frac{k}{\delta\Gamma}} A^{\frac{k}{\delta\Gamma}} \theta^{\frac{\beta k}{\delta\Gamma}} \tag{3.B.24}$$

$$= \frac{\eta_i^{k/\delta}}{\lambda_i(\theta)} b_i^{1-\frac{\eta_i\beta\gamma k}{\delta\Gamma}} b_j^{-\frac{\eta_j\beta\gamma k}{\delta\Gamma}} a_{\min}^{-k} \phi_1^{k/\delta} \phi_2^{(1-\beta\gamma)k} c^{-\frac{(1-\beta\gamma)k}{\delta\Gamma}} \Upsilon_X(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}} \Upsilon_B(\theta)^{\frac{k}{\delta\Gamma}} A^{\frac{k}{\delta\Gamma}} \theta^{\frac{\beta k}{\delta\Gamma}}.$$

Using the first order conditions from the profit maximization (3.23, 3.24) one can rewrite profits (3.22) as follows:

$$\begin{aligned}
\pi(\theta) &= \Lambda_0\Upsilon_B(\theta)r(\theta) - \sum_i\left(b_in_i + \frac{ca_{ci}^\delta}{\delta}\right) - f_D - I_X(\theta)f_X - I_B(\theta)f_B \\
&= \Lambda_0\Upsilon_B(\theta)r(\theta) - \sum_i\left(\Lambda_0\Upsilon_B(\theta)\beta\gamma\eta_ir(\theta) + \frac{\Lambda_0\Upsilon_B(\theta)(1-\gamma k)\beta\eta_ir(\theta)}{\delta}\right) \\
&\qquad - f_D - I_X(\theta)f_X - I_B(\theta)f_B \\
&= \left(\Lambda_0\Upsilon_B(\theta)\left(1 - \sum_i(\beta\gamma\eta_i + \beta/\delta(1-\gamma k)\eta_i)\right)\right) r(\theta) - f_D - I_X(\theta)f_X - I_B(\theta)f_B \\
&= \left(\Lambda_0\Upsilon_B(\theta)\left(1 - (\beta\gamma + \beta/\delta(1-\gamma k))\sum_i\eta_i\right)\right) r(\theta) - f_D - I_X(\theta)f_X - I_B(\theta)f_B \\
&= \Gamma\ \Lambda_0\Upsilon_B(\theta)\ r(\theta) - f_D - I_X(\theta)f_X - I_B(\theta)f_B, \qquad (3.30)
\end{aligned}$$

where $\Gamma\ \Lambda_0\Upsilon_B(\theta)$ is the share of revenues belonging to the firm after paying wages, as well as search and matching and screening costs. Using the revenue function (3.B.23) in the profit function (3.30) profits also can be written as:

$$\begin{aligned}\pi(\theta) &= \Gamma\Lambda_0\Upsilon_B(\theta)r(\theta) - f_D - I_X(\theta)f_X - I_B(\theta)f_B \\ &= \Gamma\Lambda_0\Upsilon_B(\theta)\frac{\phi_1\phi_2^{\beta(1-\gamma k)}}{\beta\gamma\Lambda_0}\left(c^{-\frac{\beta(1-\gamma k)}{\delta}}b^{-\beta\gamma}A\right)^{\frac{1}{\Gamma}}\theta^{\frac{\beta}{\Gamma}}\Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}}\Upsilon_B(\theta)^{\frac{1-\Gamma}{\Gamma}} \\ &\qquad - f_D - I_X(\theta)f_X - I_B(\theta)f_B \\ &= \frac{\Gamma}{\beta\gamma}\kappa_r\left(c^{-\frac{\beta(1-\gamma k)}{\delta}}b^{-\beta\gamma}A\right)^{\frac{1}{\Gamma}}\theta^{\frac{\beta}{\Gamma}}\Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}}\Upsilon_B(\theta)^{\frac{1}{\Gamma}} - f_D - I_X(\theta)f_X - I_B(\theta)f_B,\end{aligned} \tag{3.B.25}$$

where: $\kappa_r \equiv \phi_1\phi_2^{\beta(1-\gamma k)}$ is used. Equations (3.B.20, 3.B.21, 3.B.22, 3.B.23, 3.B.24, 3.B.25) are all combined in equation (3.29) in the paper.

3.B.2 Firms' Sorting Pattern

Domestic Cutoff Productivity. Concentrating on the most inserting sorting patterns it is assumed that parameters are such that the marginal active firm neither exports, nor does it invest in improving its bargaining power, nor does it both.[33] Using the profit equation following the firm's optimisation (3.30) one can state that the marginal domestic firm, which earns zero profits ($\pi_D(\theta_D) = 0$) generates revenues of:

$$\begin{aligned}\pi(\theta_D) &= \Gamma\ \Lambda_0\ r\left(\theta\right) - f_D = 0 \\ r(\theta_D) &= \frac{f_D}{\Gamma\ \Lambda_0}.\end{aligned} \tag{3.B.26}$$

In similar fashion using the first order conditions from the profit maximisation (3.23, 3.24) as well as (3.3) and (3.16) the number of workers screened, the ability threshold, the number of workers hired as well as the wage of the cutoff firm can be determined:

$$\begin{aligned}a_{ci}(\theta_D) &= \left(\Lambda_0\beta(1-\gamma k)\eta_i c^{-1}r(\theta_D)\right)^{\frac{1}{\delta}} \\ &= \left(\frac{\beta(1-\gamma k)\eta_i}{\Gamma}\frac{f_D}{c}\right)^{\frac{1}{\delta}},\end{aligned} \tag{3.B.27}$$

$$\begin{aligned}n_i(\theta_D) &= \Lambda_0\beta\gamma\eta_i b_i^{-1}r(\theta_D) \\ &= \frac{\beta\gamma\eta_i}{\Gamma}\frac{f_D}{b_i},\end{aligned} \tag{3.B.28}$$

[33] The necessary conditions are derived in the following section and are given by (3.B.44, 3.B.45, 3.B.49).

$$\begin{aligned} h_i(\theta_D) &= n_i(\theta_D)\frac{a_{\min}^k}{a_{ci}(\theta_D)^k} \\ &= \frac{\beta\gamma\eta_i}{\Gamma}\frac{f_D}{b_i}\frac{a_{\min}^k}{\left(\left(\frac{\beta(1-\gamma k)\eta_i}{\Gamma}\frac{f_D}{c}\right)^{\frac{1}{\delta}}\right)^k} \\ &= \eta_i^{1-\frac{k}{\delta}}\frac{\beta\gamma}{\Gamma}\frac{f_D}{b_i}\left(\frac{\beta(1-\gamma k)}{\Gamma}\frac{f_D}{ca_{\min}^\delta}\right)^{-\frac{k}{\delta}}, \end{aligned} \tag{3.B.29}$$

$$\begin{aligned} w_i(\theta_D) =& \Lambda_{wi}(\theta_D)\frac{r(\theta_D)}{h_i(\theta_D)} \\ =& \frac{\beta\gamma\eta_i}{1+\beta\gamma\eta_i+\beta\gamma\eta_j}\frac{\frac{f_D}{\Gamma\ \Lambda_0}}{h_i(\theta_D)} \\ =& b_i\left(\frac{\beta(1-\gamma k)\eta_i}{\Gamma}\frac{f_D}{ca_{\min}^\delta}\right)^{\frac{k}{\delta}}. \end{aligned} \tag{3.B.30}$$

Equations (3.B.26, 3.B.27, 3.B.28, 3.B.29, 3.B.30) are all combined in equation (3.31) in the paper. Using (3.B.20), the ability threshold of a firm can be written as a function of the cutoff productivity and the ability threshold of the cutoff firm (3.B.27):

$$a_{ci}(\theta) = \Upsilon_X(\theta)^{\frac{1-\beta}{\delta\Gamma}}\Upsilon_B(\theta)^{\frac{1}{\delta\Gamma}}a_{ci}(\theta_D)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\delta\Gamma}}. \tag{3.B.31}$$

Using (3.B.21), the number of workers screened by a firm can be written as a function of the cutoff productivity and the number of worker screened by the cutoff firm (3.B.28):

$$n_i(\theta) = \Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}}\Upsilon_B(\theta)^{\frac{1}{\Gamma}}n_i(\theta_D)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}}. \tag{3.B.32}$$

Using (3.B.22), the number of workers hired by a firm for the production of task i can be written as a function of the cutoff productivity and the number of worker hired by the cutoff firm (3.B.29):

$$h_i(\theta) = \Upsilon_X(\theta)^{(1-\beta)\frac{1-k/\delta}{\Gamma}}\Upsilon_B(\theta)^{\frac{1-k/\delta}{\Gamma}}h_i(\theta_D)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta(1-k/\delta)}{\Gamma}}. \tag{3.B.33}$$

Using (3.B.23), the revenues of a firm can be written as a function of the cutoff productivity and the revenues of the cutoff firm (3.B.26):

$$r(\theta) = \Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}}\Upsilon_B(\theta)^{\frac{1-\Gamma}{\Gamma}}r(\theta_D)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}}. \tag{3.B.34}$$

Using (3.B.24) the wages paid by a firm for the production of task i can be written as a function of the cutoff productivity and the wage paid by the cutoff firm (3.B.30):

$$w_i(\theta) = \frac{1}{\lambda_i(\theta)} \Upsilon_X(\theta)^{\frac{(1-\beta)k}{\delta\Gamma}} \Upsilon_B(\theta)^{\frac{k}{\delta\Gamma}} w_i(\theta_D) \left(\frac{\theta}{\theta_D}\right)^{\frac{\beta k}{\delta\Gamma}}. \tag{3.B.35}$$

Using the profit equation (3.30) and plugging in (3.B.34) and (3.B.26) profits can be rewritten as:

$$\begin{aligned}
\pi(\theta) &= \Gamma\ \Lambda_0\ \Upsilon_B(\theta)\ r(\theta) - f_D - I_X(\theta) f_X - I_B(\theta) f_B \\
&= \Gamma\ \Lambda_0\ \Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}} \Upsilon_B(\theta)^{1+\frac{1-\Gamma}{\Gamma}} r(\theta_D) \left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}} - f_D - I_X(\theta) f_X - I_B(\theta) f_B \\
&= \Upsilon_X(\theta)^{\frac{1-\beta}{\Gamma}} \Upsilon_B(\theta)^{\frac{1}{\Gamma}} \left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}} f_D - f_D - I_X(\theta) f_X - I_B(\theta) f_B.
\end{aligned} \tag{3.B.36}$$

Equations (3.B.31, 3.B.32, 3.B.33, 3.B.34, 3.B.35, 3.B.36) are all combined in equation (3.32) in the paper.

Export and Bargaining Improvement Decision. With the possibility to export and improve a firm's bargaining power the model features several cutoff productivities and allows for different sorting patterns, depending on the parameter assumptions. In particular 7 different cutoffs can be identified. Firstly, there is the already introduced domestic cutoff θ_D determining the firm which earns zero profits, while neither export nor investing into improving their bargaining power. If the firm is not improving its bargaining power, the export cutoff θ_X determines the firm which is indifferent between supplying solely the domestic market and starting to export $(\pi(\theta_X)\,|_{I_X=1,I_B=0} = \pi(\theta_X)\,|_{I_X=0,I_B=0})$. If the firm is not exporting, the bargaining cutoff θ_B determines the firm, which is indifferent between improving its bargaining power and not $(\pi(\theta_B)\,|_{I_X=0,I_B=1} = \pi(\theta_B)\,|_{I_X=0,I_B=0})$. If the firm is exporting, the combined export bargaining cutoff θ_{XB} determines the firm which is indifferent between improving its bargaining power and not $(\pi(\theta_{XB})\,|_{I_X=1,I_B=1} = \pi(\theta_{XB})\,|_{I_X=1,I_B=0})$. If the firm is improving its bargaining power, the combined bargaining export cutoff θ_{BX} determines the firm which is indifferent between supplying solely the domestic market and starting to export $(\pi(\theta_{BX})\,|_{I_X=1,I_B=1} = \pi(\theta_{BX})\,|_{I_X=0,I_B=1})$. The switch cutoff determines the firm which is indifferent between improving its bargaining power while not exporting and exporting while not improving the bargaining power $(\pi(\theta_{X/B})\,|_{I_X=1,I_B=0} = \pi(\theta_{X/B})\,|_{I_X=0,I_B=1})$. Where $\theta_{X/B}$ is used to depict a parameter situation where the more productive firms invest in improving their bargaining power, while the less productive firm export and $\theta_{B/X}$ depicts the opposite case, in which the more productive firms export while the less productive firms invest in improving their bargaining power. However, the cutoff is independent of the case the same $\theta_{B/X} = \theta_{X/B}$.[34] The final cutoff θ_{DXB} determines the firm, which is indifferent between neither exporting nor investing in improving the bargaining power and doing both $(\pi(\theta_{DXB})\,|_{I_X=1,I_B=1} = \pi(\theta_{DXB})\,|_{I_X=0,I_B=0})$.

The following convention $\Upsilon_X \equiv \Upsilon_X(\theta)|_{I_X(\theta)=1}$ is used to indicate market access of an exporting firm. The seven different cutoff conditions can be rewritten using the profit function (3.B.25) in the following manner:

θ_D:

$$\pi(\theta_D)|_{I_X=0,I_B=0} = 0$$

$$\frac{\Gamma}{\beta\gamma}\kappa_r \left(c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A\right)^{\frac{1}{\Gamma}} \theta_D^{\frac{\beta}{\Gamma}} = f_D, \tag{3.B.37}$$

θ_X:

$$\pi(\theta_X)|_{I_X=1,I_B=0} = \pi(\theta_X)|_{I_X=0,I_B=0}$$

$$\frac{\Gamma}{\beta\gamma}\kappa_r \left(c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A\right)^{\frac{1}{\Gamma}} \theta_X^{\frac{\beta}{\Gamma}} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) = f_X, \tag{3.B.38}$$

θ_B:

$$\pi(\theta_B)|_{I_X=0,I_B=1} = \pi(\theta_B)|_{I_X=0,I_B=0}$$

$$\frac{\Gamma}{\beta\gamma}\kappa_r \left(c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A\right)^{\frac{1}{\Gamma}} \theta_B^{\frac{\beta}{\Gamma}} \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right) = f_B, \tag{3.B.39}$$

θ_{XB}:

$$\pi(\theta_{XB})|_{I_X=1,I_B=1} = \pi(\theta_{XB})|_{I_X=1,I_B=0}$$

$$\frac{\Gamma}{\beta\gamma}\kappa_r \left(c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A\right)^{\frac{1}{\Gamma}} \theta_{XB}^{\frac{\beta}{\Gamma}} \Upsilon_X^{\frac{1-\beta}{\Gamma}} \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right) = f_B, \tag{3.B.40}$$

θ_{BX}:

$$\pi(\theta_{BX})|_{I_X=1,I_B=1} = \pi(\theta_{BX})|_{I_X=0,I_B=1}$$

$$\frac{\Gamma}{\beta\gamma}\kappa_r \left(c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A\right)^{\frac{1}{\Gamma}} \theta_{BX}^{\frac{\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) = f_X, \tag{3.B.41}$$

$\theta_{X/B}$:

$$\pi(\theta_{X/B})|_{I_X=1,I_B=0} = \pi(\theta_{X/B})|_{I_X=0,I_B=1}$$

$$\frac{\Gamma}{\beta\gamma}\kappa_r \left(c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A\right)^{\frac{1}{\Gamma}} \theta_{X/B}^{\frac{\beta}{\Gamma}} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - \Upsilon_B^{\frac{1}{\Gamma}}\right) = f_X - f_B, \tag{3.B.42}$$

θ_{DXB}:

$$\pi(\theta_{X/B})|_{I_X=1,I_B=1} = \pi(\theta_{X/B})|_{I_X=0,I_B=0}$$

$$\frac{\Gamma}{\beta\gamma}\kappa_r \left(c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A\theta_{DXB}^{\beta}\right)^{\frac{1}{\Gamma}} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1\right) = f_X + f_B. \tag{3.B.43}$$

[34] As shown in Supplementary Appendix 3.B.2 this cutoff will not be relevant. Firms always will prefer exporting and improving their bargaining power than to switch from one to the other.

Using

$$f_\iota = \begin{cases} f_D, \\ f_X, \\ f_B, \\ f_B, \\ f_X, \\ f_X + f_B, \end{cases} \qquad \Upsilon_\iota = \begin{cases} 1, & \text{if } \iota = D \\ \Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1, & \text{if } \iota = X \\ \Upsilon_B^{\frac{1}{\Gamma}} - 1, & \text{if } \iota = B \\ \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\Upsilon_X^{\frac{1-\beta}{\Gamma}}, & \text{if } \iota = XB \\ \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\Upsilon_B^{\frac{1}{\Gamma}}, & \text{if } \iota = BX \\ \Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1}{\Gamma}} - 1, & \text{if } \iota = DXB. \end{cases} \tag{3.34}$$

the cutoff conditions (3.B.37, 3.B.38, 3.B.39, 3.B.40, 3.B.41, 3.B.42, 3.B.43) can be generally written in the following manner where $\iota \in \{D, X, B, XB, BX, DXB\}$ is used to indicate the different possible cutoffs:

$$\frac{\Gamma}{\beta\gamma}\kappa_r \left(c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A\right)^{\frac{1}{\Gamma}} \theta_\iota^{\frac{\beta}{\Gamma}} \Upsilon_\iota = f_\iota. \tag{3.33}$$

Cutoff Relationship. The relationship between the domestic cutoff θ_D and any other cutoff θ_ι can be written as:

$$\frac{f_D}{f_\iota} = \frac{\frac{\Gamma}{\beta\gamma}\kappa_r \left(c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A\right)^{\frac{1}{\Gamma}} \theta_D^{\frac{\beta}{\Gamma}}}{\frac{\Gamma}{\beta\gamma}\kappa_r \left(c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A\right)^{\frac{1}{\Gamma}} \theta_\iota^{\frac{\beta}{\Gamma}} \Upsilon_\iota}$$

$$\frac{\theta_D}{\theta_\iota} = \Upsilon_\iota^{\frac{\Gamma}{\beta}} \left(\frac{f_D}{f_\iota}\right)^{\frac{\Gamma}{\beta}}. \tag{3.35}$$

Rewriting (3.35) for the different cutoffs implies:

$$\frac{\theta_D}{\theta_X} = \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\frac{f_D}{f_X}\right)^{\frac{\Gamma}{\beta}} < 1, \tag{3.B.44}$$

$$\frac{\theta_D}{\theta_B} = \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\frac{f_D}{f_B}\right)^{\frac{\Gamma}{\beta}} < 1, \tag{3.B.45}$$

$$\frac{\theta_D}{\theta_{XB}} = \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \Upsilon_X^{\frac{1-\beta}{\beta}} \left(\frac{f_D}{f_B}\right)^{\frac{\Gamma}{\beta}}, \tag{3.B.46}$$

$$\frac{\theta_D}{\theta_{BX}} = \Upsilon_B^{\frac{1}{\beta}} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\frac{f_D}{f_X}\right)^{\frac{\Gamma}{\beta}}, \tag{3.B.47}$$

$$\frac{\theta_D}{\theta_{X/B}} = \frac{\theta_D}{\theta_{B/X}} = \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - \Upsilon_B^{\frac{1}{\Gamma}}\right)^{\frac{\Gamma}{\beta}} \left(\frac{f_D}{f_X - f_B}\right)^{\frac{\Gamma}{\beta}}, \tag{3.B.48}$$

$$\frac{\theta_D}{\theta_{DXB}} = \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\frac{f_D}{f_X + f_B}\right)^{\frac{\Gamma}{\beta}} < 1. \tag{3.B.49}$$

The inequality assumption of (3.B.44) ensures that trade parameters (f_D, f_X, Υ_X) are such that not all active firms engage in exporting and thus the export cutoff is larger than the domestic cutoff. The inequality assumption of (3.B.45) ensures that not all firms will invest in improving their bargaining power. The inequality assumption of (3.B.49) ensures that not all firms will invest in improving their bargaining power and export. Economically this means that the fixed costs (of exporting and improving the bargaining power) are high enough or the improvement in the market access term and the bargaining improvement term is low enough, so that the domestic cutoff firm which is indifferent between supplying the market or not is not willing to invest in either of the options.

Sorting Pattern. The assumption that the least productive active firm is neither exporting nor investing in improving its bargaining power implies that there are three possible sorting patterns $(c = \{1, 2, 3\})$ left, depending on the size of the fixed costs and the potential gains from exporting and improving the bargaining power. In particular the following three patterns exist:

$$\theta_D < \theta_X < \theta_{XB} \rightarrow \iota_1 = \{D, X, XB\} \quad if \;\; \theta_X < \theta_B \;\wedge\; \theta_X < \theta_{DXB} \qquad \text{(Case 1)}$$
$$\theta_D < \theta_{DXB} \rightarrow \iota_2 = \{D, DXB\} \quad if \;\; \theta_{DXB} < \theta_X \;\wedge\; \theta_{DXB} < \theta_B \qquad \text{(Case 2)}$$
$$\theta_D < \theta_B < \theta_{BX} \rightarrow \iota_3 = \{D, B, BX\} \quad if \;\; \theta_B < \theta_X \;\wedge\; \theta_B < \theta_{DXB} \qquad \text{(Case 3)}$$

In all three cases the least productive active firms neither improve their bargaining power nor export while the most productive firms do both. Case 1 (ι_1) describes a situation, in which $\theta_X < \theta_B$ and also $\theta_X < \theta_{DXB}$. In this case there exists a range of intermediate productive firms which sell domestically and export to the foreign market, but do not improve their bargaining power. Using equations (3.B.44, 3.B.45, 3.B.46) it is straight forward to see that the necessary conditions for *Case 1* are fulfilled, if the fixed costs of improving the bargaining power are high and the possible improvement in the bargaining power is low.

If fixed costs of improving the bargaining power are lower and or the possible improvement in the bargaining power is larger there exists a parameter range where equations (3.B.44, 3.B.45, 3.B.46) satisfy $\theta_{DXB} < \theta_X$ and also $\theta_{DXB} < \theta_B$, which are the parameter conditions for *Case 2* (ι_2). It describes a situation, where there exists no range of intermediate productive firms where firms do only one of the two.

If fixed costs of improving the bargaining power are sufficiently low and or the possible improvement in the bargaining power is sufficiently large, equations (3.B.44, 3.B.45, 3.B.46) satisfy $\theta_B < \theta_X$ and also $\theta_B < \theta_{DXB}$, which are the parameter conditions for *Case 3* (ι_3). It describes a situation, where there exists a range of intermediate productive firms where firms do improve their bargaining power but do not export.

Extensive Margins. The general export and bargaining improvement cutoffs which will differ across sorting patterns $c \in \{1, 2, 3\}$, will be denoted by:

$$\theta_{X_c} \equiv \begin{cases} \theta_X, \\ \theta_{DXB}, \\ \theta_{BX}, \end{cases} \qquad \theta_{B_c} \equiv \begin{cases} \theta_{XB}, & \text{if } c = 1 \\ \theta_{DXB}, & \text{if } c = 2 \\ \theta_B, & \text{if } c = 3. \end{cases} \tag{3.36}$$

The productivity cutoffs depend on both, an extensive and an intensive margin, which relate to the trade openness and bargaining possibilities. The intensive margin is captured by the market access term (Υ_X) and the bargaining improvement term (Υ_B). The ratio between the domestic and the relevant export (θ_{X_c}) or bargaining (θ_{B_c}) cutoff for the respective sorting pattern captures the extensive margin. The share of firms exporting ($\rho^z_{X_c}$) can be written as:

$$\rho_{X_c} = \left(\frac{1 - G_\theta(\theta_{X_c})}{1 - G_\theta(\theta_{D_c})}\right)^{\frac{1}{z}} = \begin{cases} \dfrac{\theta_D}{\theta_X} & = \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\dfrac{f_D}{f_X}\right)^{\frac{\Gamma}{\beta}}, & \text{if } c = 1 \\ \dfrac{\theta_D}{\theta_{DXB}} & = \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\dfrac{f_D}{f_X + f_B}\right)^{\frac{\Gamma}{\beta}}, & \text{if } c = 2 \\ \dfrac{\theta_D}{\theta_{BX}} & = \Upsilon_B^{\frac{1}{\beta}} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\dfrac{f_D}{f_X}\right)^{\frac{\Gamma}{\beta}}, & \text{if } c = 3, \end{cases} \tag{3.37}$$

where the relationship between the cutoff productivities is given by (3.B.44, 3.B.47, 3.B.49). The share of firms improving their bargaining power ($\rho^z_{B_c}$) is given by:

$$\rho_{B_c} = \left(\frac{1 - G_\theta(\theta_{B_c})}{1 - G_\theta(\theta_{D_c})}\right)^{\frac{1}{z}} = \begin{cases} \dfrac{\theta_D}{\theta_{XB}} & = \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \Upsilon_X^{\frac{1-\beta}{\beta}} \left(\dfrac{f_D}{f_B}\right)^{\frac{\Gamma}{\beta}}, & \text{if } c = 1 \\ \dfrac{\theta_D}{\theta_{DXB}} & = \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\dfrac{f_D}{f_X + f_B}\right)^{\frac{\Gamma}{\beta}}, & \text{if } c = 2 \\ \dfrac{\theta_D}{\theta_B} & = \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\dfrac{f_D}{f_B}\right)^{\frac{\Gamma}{\beta}}, & \text{if } c = 3, \end{cases} \tag{3.38}$$

where the relationship between the cutoff productivities is given by (3.B.45, 3.B.46, 3.B.49). For *Cases 2* and *3* the share of firms exporting rises with a higher possible bargaining improvement (larger bargaining improvement term) while for *Case 1* the share is not affected. The share of firms improving their bargaining power rises with a larger market access term if *Cases 1* or *2* are relevant. In *Case 3* the share is unaffected by the market access term.[35]

[35] The relationships are derived assuming that a change in neither the market access term nor the bargaining improvement term does change the relevant sorting pattern.

3.B.3 General Equilibrium

Free Entry Condition. Free Entry implies that new firms will enter the market as long as expected profits are larger than the entry costs. The free entry condition for the three different sorting pattern cases take the following form:

Case 1 ($\theta_D < \theta_X < \theta_{XB}$):

$$\int_{\theta_D}^{\infty} \pi(\theta)\Big|_{\substack{I_X=0\\I_B=0}} dG_\theta(\theta) + \int_{\theta_X}^{\infty} \left(\pi(\theta)\Big|_{\substack{I_X=1\\I_B=0}} - \pi(\theta)\Big|_{\substack{I_X=0\\I_B=0}}\right) dG_\theta(\theta) + \int_{\theta_{XB}}^{\infty} \left(\pi(\theta)\Big|_{\substack{I_X=1\\I_B=1}} - \pi(\theta)\Big|_{\substack{I_X=1\\I_B=0}}\right) dG_\theta(\theta) = f_E, \tag{3.B.50}$$

Case 2 ($\theta_D < \theta_{XB}$):

$$\int_{\theta_D}^{\infty} \pi(\theta)\Big|_{\substack{I_X=0\\I_B=0}} dG_\theta(\theta) + \int_{\theta_{XB}}^{\infty} \left(\pi(\theta)\Big|_{\substack{I_X=1\\I_B=1}} - \pi(\theta)\Big|_{\substack{I_X=0\\I_B=0}}\right) dG_\theta(\theta) = f_E, \tag{3.B.51}$$

Case 3 ($\theta_D < \theta_B < \theta_{XB}$):

$$\int_{\theta_D}^{\infty} \pi(\theta)\Big|_{\substack{I_X=0\\I_B=0}} dG_\theta(\theta) + \int_{\theta_B}^{\infty} \left(\pi(\theta)\Big|_{\substack{I_X=0\\I_B=1}} - \pi(\theta)\Big|_{\substack{I_X=0\\I_B=0}}\right) dG_\theta(\theta) + \int_{\theta_{XB}}^{\infty} \left(\pi(\theta)\Big|_{\substack{I_X=1\\I_B=1}} - \pi(\theta)\Big|_{\substack{I_X=0\\I_B=1}}\right) dG_\theta(\theta) = f_E. \tag{3.B.52}$$

Generalizing the Free Entry Condition. The following section depicts how the separate parts of the free entry condition are derived. Using the profit equation (3.B.36) domestic profits with no exporting and no investment into improving the bargaining power can be written as:

$$\pi(\theta)\Big|_{\substack{I_X=0\\I_B=0}} = \left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}} f_D - f_D = f_D\left(\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}} - 1\right). \tag{3.B.53}$$

It follows from the profit equation (3.B.36) and using the relationship between the domestic and export cutoff θ_X (3.B.44) that:

$$\pi(\theta)\Big|_{\substack{I_X=1\\I_B=0}} - \pi(\theta)\Big|_{\substack{I_X=0\\I_B=0}} = \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}} f_D - f_X = \left(\left(\frac{\theta}{\theta_X}\right)^{\frac{\beta}{\Gamma}} - 1\right) f_X. \tag{3.B.54}$$

It follows from the profit equation (3.B.36) and using the relationship between the domestic and bargaining cutoff θ_B (3.B.45) that:

$$\pi(\theta)\Big|_{\substack{I_X=0\\I_B=1}} - \pi(\theta)\Big|_{\substack{I_X=0\\I_B=0}} = \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}} f_D - f_B = \left(\left(\frac{\theta}{\theta_B}\right)^{\frac{\beta}{\Gamma}} - 1\right) f_B. \tag{3.B.55}$$

It follows from the profit equation (3.B.36) and using the relationship between the domestic and export bargaining cutoff θ_{XB} (3.B.46) that:

$$\pi(\theta)\Big|_{\substack{I_X=1\\I_B=1}} - \pi(\theta)\Big|_{\substack{I_X=1\\I_B=0}} = \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\Upsilon_X^{\frac{1-\beta}{\Gamma}}\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}} f_D - f_B \quad = \left(\left(\frac{\theta}{\theta_{XB}}\right)^{\frac{\beta}{\Gamma}} - 1\right) f_B. \tag{3.B.56}$$

It follows from the profit equation (3.B.36) and using the relationship between the domestic and export bargaining cutoff θ_{BX} (3.B.47) that:

$$\pi(\theta)\Big|_{\substack{I_X=1\\I_B=1}} - \pi(\theta)\Big|_{\substack{I_X=0\\I_B=1}} = \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\Upsilon_B^{\frac{1}{\Gamma}}\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}} f_D - f_X = \left(\left(\frac{\theta}{\theta_{BX}}\right)^{\frac{\beta}{\Gamma}} - 1\right) f_X. \tag{3.B.57}$$

It follows from the profit equation (3.B.36) and using the relationship between the domestic and export bargaining cutoff $\theta_{X/B}$ (3.B.49) that:

$$\begin{aligned}\pi(\theta)\Big|_{\substack{I_X=1\\I_B=1}} - \pi(\theta)\Big|_{\substack{I_X=0\\I_B=0}} &= \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}} f_D - f_X - f_B \\ &= \left(\left(\frac{\theta}{\theta_{DXB}}\right)^{\frac{\beta}{\Gamma}} - 1\right)(f_X + f_B).\end{aligned} \tag{3.B.58}$$

The different parts of the free entry conditions (3.B.53-3.B.58) can be generalised as:

$$FE_\iota = \left(\left(\frac{\theta}{\theta_\iota}\right)^{\frac{\beta}{\Gamma}} - 1\right) f_\iota. \tag{3.B.59}$$

Using the derived generalised form (3.B.59), the free entry conditions in the three cases (3.B.50, 3.B.51, 3.B.52) also can be written in a generalised manner:

$$\sum_{\iota\in\iota_c} f_\iota \int_{\theta_\iota}^{\infty}\left(\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}} - 1\right) dG_\theta(\theta) = f_E. \tag{3.39}$$

Determining the Domestic Cutoffs. Using the assumption of pareto distributed firm productivity, the free entry (3.39) can be written as:

$$\sum_{\iota\in\iota_c} f_\iota \int_{\theta_\iota}^{\infty}\left(\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}} - 1\right)\theta^{-(z+1)}\, d\theta = \frac{f_E}{z\theta_{min}^z}$$

$$\sum_{\iota\in\iota_c} f_\iota \left[\left(\frac{\beta}{\Gamma} - z\right)^{-1}\theta^{\frac{\beta}{\Gamma}-z}\theta_D^{-\frac{\beta}{\Gamma}} + \frac{1}{z}\theta^{-z}\right] :_{\theta_\iota}^{\infty} = \frac{f_E}{z\theta_{min}^z}.$$

With $\frac{\beta}{\Gamma} < z$ it follows that $(\infty)^{\frac{\beta}{\Gamma}-z} \to 0$. Thus, the above free entry condition reduces to:

$$
\begin{aligned}
-\sum_{\iota\in\iota_c} f_\iota \left(\left(\frac{\beta}{\Gamma}-z\right)^{-1}\theta_\iota^{\frac{\beta}{\Gamma}-z}\theta_\iota^{-\frac{\beta}{\Gamma}}+\frac{1}{z}\theta_\iota^{-z}\right) &= \frac{f_E}{z\theta_{min}^z}\\
\left(\frac{z\Gamma}{z\Gamma-\beta}-1\right)\sum_{\iota\in\iota_c} f_\iota\theta_\iota^{-z} &= \frac{f_E}{\theta_{min}^z}\\
\frac{\beta}{z\Gamma-\beta}\sum_{\iota\in\iota_c} f_\iota\theta_\iota^{-z} &= \frac{f_E}{\theta_{min}^z}.
\end{aligned}
$$

Using the relationships between the cutoffs (3.35) yields the domestic cutoff as::

$$
\begin{aligned}
\frac{f_E}{\theta_{min}^z} &= \frac{\beta}{z\Gamma-\beta}\sum_{\iota\in\iota_c} f_\iota\left(\frac{f_D}{f_\iota}\right)^{\frac{z\Gamma}{\beta}}\Upsilon_i^{\frac{z\Gamma}{\beta}}\theta_D^{-z}\\
\theta_D^z &= \frac{\beta}{z\Gamma-\beta}\frac{\sum_{\iota\in\iota_c} f_\iota\left(\frac{f_D}{f_\iota}\right)^{\frac{z\Gamma}{\beta}}\Upsilon_i^{\frac{z\Gamma}{\beta}}}{f_E}\theta_{min}^z\\
\theta_{D_c} &= \left(\frac{\beta}{z\Gamma-\beta}\frac{\sum_{\iota\in\iota_c} f_\iota\left(\frac{f_D}{f_\iota}\Upsilon_i\right)^{\frac{z\Gamma}{\beta}}}{f_E}\right)^{\frac{1}{z}}\theta_{min}. \qquad (3.40)
\end{aligned}
$$

In order to indicate that the functional form determining the domestic cutoff productivity θ_{D_c} differs across sorting patterns the index $c \in \{1,2,3\}$ is used. In order to ensure that the productivity cutoff is positive it must hold that $z > \frac{\beta}{\Gamma}$. Depicting the domestic cutoffs explicitly for the three different cases yields:

Case 1 ($\theta_D < \theta_X < \theta_{XB}$):

$$
\theta_{D_1} = \left(f_D + f_X\left(\frac{f_D}{f_X}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}-1\right)\right)^{\frac{z\Gamma}{\beta}} + f_B\left(\frac{f_D}{f_B}\Upsilon_X^{\frac{1-\beta}{\Gamma}}\left(\Upsilon_B^{\frac{1}{\Gamma}}-1\right)\right)^{\frac{z\Gamma}{\beta}}\right)^{\frac{1}{z}}\kappa_\theta. \qquad (3.B.60)
$$

Case 2 ($\theta_D < \theta_{DXB}$):

$$
\theta_{D_2} = \left(f_D + (f_X+f_B)\left(\frac{f_D}{f_X+f_B}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1}{\Gamma}}-1\right)\right)^{\frac{z\Gamma}{\beta}}\right)^{\frac{1}{z}}\kappa_\theta. \qquad (3.B.61)
$$

Case 3 ($\theta_D < \theta_B < \theta_{XB}$):

$$
\theta_{D_3} = \left(f_D + f_X\left(\frac{f_D}{f_X}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}-1\right)\Upsilon_B^{\frac{1}{\Gamma}}\right)^{\frac{z\Gamma}{\beta}} + f_B\left(\frac{f_D}{f_B}\left(\Upsilon_B^{\frac{1}{\Gamma}}-1\right)\right)^{\frac{z\Gamma}{\beta}}\right)^{\frac{1}{z}}\kappa_\theta. \qquad (3.B.62)
$$

where $\kappa_\theta \equiv \left(\frac{\beta}{(z\Gamma-\beta)f_E}\right)^{\frac{1}{z}}\theta_{min}$ is used to simplify notation.

The Effect of Bargaining Power Improvement on the Domestic Cutoff Productivity. The domestic cutoff productivity is higher than in a situation where firms cannot invest in improving their bargaining power. Indicating the introduction of the possibility to improve the bargaining power of firms leads to a more productive firm distribution in the market. In order to proof that the domestic cutoff productivity is higher when firms have the possibility to improve their bargaining power $\lambda > 1$, the following condition needs to be fulfilled $\theta_{D_c}|_{\lambda=1} < \theta_{D_c}|_{\lambda>1}$. The cutoff productivity when firms cannot improve their bargaining power is derived using (3.40) and is given by:

$$\theta_D|_{\lambda=1} = \left(\frac{\frac{\beta}{\Gamma}}{z - \frac{\beta}{\Gamma}} \frac{f_D + f_X \left(\frac{f_D}{f_X} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1 \right) \right)^{\frac{z\Gamma}{\beta}}}{f_E} \right)^{\frac{1}{z}} \theta_{min}. \tag{3.B.63}$$

As the domestic cutoff differs dependent on the parameter constellation and the resulting sorting pattern the inequality condition needs to hold for all three cases. For the sorting pattern of *Case 1* this implies that using (3.B.60) and (3.B.63) the following condition needs to hold:

$$\begin{gathered}
\theta_D|_{\lambda=1} < \theta_{D_1}|_{\lambda>1} \\
f_X \left(\frac{f_D}{f_X} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1 \right) \right)^{\frac{z\Gamma}{\beta}} < f_X \left(\frac{f_D}{f_X} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1 \right) \right)^{\frac{z\Gamma}{\beta}} + f_B \left(\frac{f_D}{f_B} \Upsilon_X^{\frac{1-\beta}{\Gamma}} \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1 \right) \right)^{\frac{z\Gamma}{\beta}} \\
0 < f_B \left(\frac{f_D}{f_B} \Upsilon_X^{\frac{1-\beta}{\Gamma}} \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1 \right) \right)^{\frac{z\Gamma}{\beta}}.
\end{gathered}$$

This condition always holds true as $\Upsilon_B > 1$ and all other parameters are positive. Using (3.B.61) and (3.B.63) for the sorting pattern of *Case 2* the following condition needs to hold:

$$\begin{gathered}
\theta_D|_{\lambda=1} < \theta_{D_2}|_{\lambda>1} \\
f_X \left(\frac{f_D}{f_X} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1 \right) \right)^{\frac{z\Gamma}{\beta}} < (f_X + f_B) \left(\frac{f_D}{f_X + f_B} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1 \right) \right)^{\frac{z\Gamma}{\beta}}.
\end{gathered}$$

As $f_X < f_X + f_B$ a sufficient condition that ensure the above inequality is:

$$\begin{gathered}
\left(\frac{f_D}{f_X} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1 \right) \right)^{\frac{z\Gamma}{\beta}} < \left(\frac{f_D}{f_X + f_B} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1 \right) \right)^{\frac{z\Gamma}{\beta}} \\
\left(\frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1}{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1} \right)^{\frac{z\Gamma}{\beta}} < \left(\frac{f_X}{f_X + f_B} \right)^{\frac{z\Gamma}{\beta}}.
\end{gathered}$$

This condition is fulfilled as it is one of the necessary condition for *Case 2* to be the active case. More specifically, using (3.B.49) and (3.B.44), the condition $\theta_{DXB} < \theta_X$ implies exactly the above inequality:

$$\frac{\theta_{DXB}}{\theta_X} = \frac{\frac{\theta_D}{\theta_X}}{\frac{\theta_D}{\theta_{DXB}}} < 1$$

$$\frac{\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\frac{f_D}{f_X}\right)^{\frac{\Gamma}{\beta}}}{\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1\right)^{\frac{\Gamma}{\beta}} \left(\frac{f_D}{f_X+f_B}\right)^{\frac{\Gamma}{\beta}}} < 1$$

$$\left(\frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1}{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1}\right)^{\frac{\Gamma}{\beta}} < \left(\frac{f_X + f_B}{f_X}\right)^{\frac{\Gamma}{\beta}}.$$

In *Case 3*, using (3.B.62) and (3.B.63), the following condition needs to hold:

$$\theta_D|_{\lambda=1} < \theta_{D_3}|_{\lambda>1}$$

$$f_X\left(\frac{f_D}{f_X}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\right)^{\frac{z\Gamma}{\beta}} < f_X\left(\frac{f_D}{f_X}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\Upsilon_B^{\frac{1}{\Gamma}}\right)^{\frac{z\Gamma}{\beta}} + f_B\left(\frac{f_D}{f_B}\left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\right)^{\frac{z\Gamma}{\beta}}$$

$$f_X\left(\frac{f_D}{f_X}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\right)^{\frac{z\Gamma}{\beta}}\left(1 - \Upsilon_B^{\frac{1}{\Gamma}}\right) < f_B\left(\frac{f_D}{f_B}\left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\right)^{\frac{z\Gamma}{\beta}}.$$

This condition always holds true as $\Upsilon_B > 1$, $\Upsilon_X > 1$ and all other parameters are positive and thus the left hand side is negative while the right hand side is positive.

Why the Switch Cutoff $\theta_{X/B}$ Is Not Relevant in Equilibrium. In general one could think of a situation, in which firms switch from exporting and not improving their bargaining power to improving their bargaining power and not exporting with increasing productivity or the other way around. This would imply the following two sorting patterns $\theta_D < \theta_X < \theta_{X/B} < \theta_{BX}$ and $\theta_D < \theta_B < \theta_{B/X} < \theta_{XB}$. In the following it is shown that $\theta_X < \theta_{X/B}$ and $\theta_{X/B} < \theta_{BX}$ or $\theta_B < \theta_{B/X}$ and $\theta_{B/X} < \theta_{XB}$ is not possible at the same time and thus only the above mentioned three sorting patterns are possible. The relationship between the domestic and the switch cutoff (3.B.48) indicates that the switch cutoff is only positive $\theta_{B/X} > 0$ if $sign(f_X - f_B) = sign(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - \Upsilon_B^{\frac{1}{\Gamma}})$. More productive firms will only switch from exporting to improving the bargaining power or the other way around, if the choice with larger fixed costs implies a stronger increase of profits the larger the firms productivity (has a steeper slope). Otherwise, firms will prefer the option with lower fixed costs and a higher increase in profits with rising productivity. Using this the sorting patterns and the corresponding inequality conditions imply the following parameter conditions:

$\theta_X < \theta_{X/B}$:

$$\frac{\theta_X}{\theta_{X/B}} = \frac{\frac{\theta_D}{\theta_{X/B}}}{\frac{\theta_D}{\theta_X}} < 1$$

$$\frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} - \Upsilon_B^{\frac{1}{\Gamma}}}{\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1} \frac{f_X}{f_X - f_B} < 1$$

$$\frac{|\Upsilon_X^{\frac{1-\beta}{\Gamma}} - \Upsilon_B^{\frac{1}{\Gamma}}|}{\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1} \frac{f_X}{|f_X - f_B|} < 1, \tag{3.B.64}$$

$\theta_B < \theta_{B/X}$:

$$\frac{\theta_B}{\theta_{B/X}} = \frac{\frac{\theta_D}{\theta_{B/X}}}{\frac{\theta_D}{\theta_B}} < 1$$

$$\frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} - \Upsilon_B^{\frac{1}{\Gamma}}}{\Upsilon_B^{\frac{1}{\Gamma}} - 1} \frac{f_B}{f_X - f_B} < 1$$

$$\frac{|\Upsilon_X^{\frac{1-\beta}{\Gamma}} - \Upsilon_B^{\frac{1}{\Gamma}}|}{\Upsilon_B^{\frac{1}{\Gamma}} - 1} \frac{f_B}{|f_X - f_B|} < 1, \tag{3.B.65}$$

$\theta_{X/B} < \theta_{BX}$:

$$\frac{\theta_{X/B}}{\theta_{BX}} = \frac{\frac{\theta_D}{\theta_{BX}}}{\frac{\theta_D}{\theta_{X/B}}} < 1$$

$$\frac{\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\Upsilon_B^{\frac{1}{\Gamma}}}{\Upsilon_X^{\frac{1-\beta}{\Gamma}} - \Upsilon_B^{\frac{1}{\Gamma}}} \frac{f_X - f_B}{f_X} < 1$$

$$\frac{\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\Upsilon_B^{\frac{1}{\Gamma}}}{|\Upsilon_X^{\frac{1-\beta}{\Gamma}} - \Upsilon_B^{\frac{1}{\Gamma}}|} \frac{|f_X - f_B|}{f_X} < 1$$

$$\frac{|\Upsilon_X^{\frac{1-\beta}{\Gamma}} - \Upsilon_B^{\frac{1}{\Gamma}}|}{\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1} \frac{f_X}{|f_X - f_B|} > \Upsilon_B^{\frac{1}{\Gamma}}, \tag{3.B.66}$$

$\theta_{B/X} < \theta_{XB}$:

$$\frac{\theta_{B/X}}{\theta_{XB}} = \frac{\frac{\theta_D}{\theta_{XB}}}{\frac{\theta_D}{\theta_{B/X}}} < 1$$

$$\frac{\left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\Upsilon_X^{\frac{1-\beta}{\Gamma}}}{\Upsilon_X^{\frac{1-\beta}{\Gamma}} - \Upsilon_B^{\frac{1}{\Gamma}}} \frac{f_X - f_B}{f_B} < 1$$

$$\frac{\left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\Upsilon_X^{\frac{1-\beta}{\Gamma}}}{|\Upsilon_X^{\frac{1-\beta}{\Gamma}} - \Upsilon_B^{\frac{1}{\Gamma}}|} \frac{|f_X - f_B|}{f_B} < 1$$

$$\frac{|\Upsilon_X^{\frac{1-\beta}{\Gamma}} - \Upsilon_B^{\frac{1}{\Gamma}}|}{\Upsilon_B^{\frac{1}{\Gamma}} - 1} \frac{f_B}{|f_X - f_B|} > \Upsilon_X^{\frac{1-\beta}{\Gamma}}. \tag{3.B.67}$$

As $\Upsilon_B > 1$ the conditions (3.B.64) and (3.B.66) cannot be fulfilled at the same time and thus $\theta_D < \theta_X < \theta_{X/B} < \theta_{BX}$ cannot be an equilibrium sorting pattern. Similarly, $\Upsilon_X > 1$ implies that the conditions (3.B.65) and (3.B.67) cannot be fulfilled at the same time and thus $\theta_D < \theta_B < \theta_{BX} < \theta_{XB}$ cannot be an equilibrium sorting pattern.

Average Revenues. Using the revenue equation (3.B.34), average revenues of firms with a productivity above θ_ι in a situation where firms neither export nor improve their bargaining power are given by:

$$\begin{aligned}
\int_{\theta_\iota}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\ I_B=0}} dG_\theta(\theta) &= r(\theta_D)\int_{\theta_\iota}^{\infty}\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}} dG_\theta(\theta)\\
&= r(\theta_D)\int_{\theta_\iota}^{\infty}\left(\frac{\theta}{\theta_D}\right)^{\frac{\beta}{\Gamma}} \theta^{-(z+1)} z\theta_{min}^z \, d\theta\\
&= r(\theta_D) z\theta_{min}^z\left[\left(\frac{\beta}{\Gamma} - z\right)^{-1} \theta_D^{-\frac{\beta}{\Gamma}} \theta^{\frac{\beta}{\Gamma}-z}\right]_{\theta_\iota}^{\infty}\\
&= r(\theta_D) z\theta_{min}^z \frac{\Gamma}{z\Gamma - \beta}\left(\frac{\theta_\iota}{\theta_D}\right)^{\frac{\beta}{\Gamma}} \theta_\iota^{-z}.
\end{aligned}$$

This can be further simplified and written as:

$$\int_{\theta_\iota}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \; dG_\theta(\theta) = \frac{\theta_{min}^z f_D}{\Lambda_0}\frac{z}{z\Gamma-\beta}\left(\frac{\theta_D}{\theta_\iota}\right)^{z-\frac{\beta}{\Gamma}}\theta_{D_c}^{-z}$$
$$= \left(\frac{\theta_D}{\theta_\iota}\right)^{z-\frac{\beta}{\Gamma}}\bar{r}^A(\theta_{D_c}). \quad (3.B.68)$$

The ratio between the cutoffs is given by equation (3.35). Average revenues of all active firms in a situation where firms can neither export nor improve their bargaining power are denoted by $\bar{r}^A(\theta_{D_c})$. The index A is used following the idea that when bargaining would not be possible this situation would be described as autarky. From (3.B.68) it follows:

$$\bar{r}^A(\theta_{D_c}) \equiv \bar{r}_c\Big|_{\substack{I_X=0\\I_B=0}} = \int_{\theta_{D_c}}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \; dG_\theta(\theta) = \frac{\theta_{min}^z f_D}{\Lambda_0}\frac{z}{z\Gamma-\beta}\theta_{D_c}^{-z}. \quad (3.B.69)$$

Even though the above depicts average revenues if firms neither export nor improve their bargaining power the term depends on the endogenous domestic cutoff productivity which can differ across sorting pattern cases. Thus, $\bar{r}^A(\theta_{D_c})$ is used to indicate this relationship. The derivation of average revenues depends on the sorting pattern of the three cases. Generally one can write average revenues of case c as:

$$\bar{r}_c = \int_{\theta_{D_c}}^{\infty} r(\theta)\; dG_\theta(\theta) = s_{r_c}\; \bar{r}^A(\theta_{D_c}), \quad (3.B.70)$$

where $s_{r_c} > 1$ is the factor by which average revenues are larger due to the possibility of exporting and improving the bargaining power. The functional form differs across sorting pattern cases. For the sorting pattern of *Case 1* average revenues are given by:

$$\begin{aligned}
\bar{r}_1 &= \int_{\theta_{D_1}}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \; dG_\theta(\theta) + \int_{\theta_X}^{\infty} r(\theta)\Big|_{\substack{I_X=1\\I_B=0}} - r(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \; dG_\theta(\theta)\\
&\qquad + \int_{\theta_{XB}}^{\infty} r(\theta)\Big|_{\substack{I_X=1\\I_B=1}} - r(\theta)\Big|_{\substack{I_X=1\\I_B=0}} \; dG_\theta(\theta)\\
&= \int_{\theta_{D_1}}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \; dG_\theta(\theta) + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}-1\right)\int_{\theta_X}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \; dG_\theta(\theta)\\
&\qquad + \Upsilon_X^{\frac{1-\beta}{\Gamma}}\left(\Upsilon_B^{\frac{1-\Gamma}{\Gamma}}-1\right)\int_{\theta_{XB}}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \; dG_\theta(\theta)\\
&= \left(1+\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}-1\right)\left(\frac{\theta_D}{\theta_X}\right)^{z-\frac{\beta}{\Gamma}} + \Upsilon_X^{\frac{1-\beta}{\Gamma}}\left(\Upsilon_B^{\frac{1-\Gamma}{\Gamma}}-1\right)\left(\frac{\theta_D}{\theta_{XB}}\right)^{z-\frac{\beta}{\Gamma}}\right)\bar{r}^A(\theta_{D_c})\\
&= s_{r_1}\bar{r}^A(\theta_{D_1}),
\end{aligned}$$

where s_{r_1} is a measure for the increase in average revenues due to the possibility of exporting and improving the bargaining power. Using the relationship between the cutoffs

(3.35) it follows that:

$$s_{r_1} \equiv 1 + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \rho_{X_c}^{z-\frac{\beta}{\Gamma}} + \Upsilon_X^{\frac{1-\beta}{\Gamma}} \left(\Upsilon_B^{\frac{1-\Gamma}{\Gamma}} - 1\right) \rho_{B_c}^{z-\frac{\beta}{\Gamma}}.$$

Average revenues in *Case 2* and *3* can be determined similarly as:

$$\begin{aligned}
\bar{r}_2 &= \int_{\theta_D}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) + \int_{\theta_{DXB}}^{\infty} r(\theta)\Big|_{\substack{I_X=1\\I_B=1}} - r(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) \\
&= s_{r_2} \, \bar{r}^A(\theta_{D_2}), \\
\bar{r}_3 &= \int_{\theta_{D_3}}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) + \int_{\theta_B}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\I_B=1}} - r(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) \\
&\qquad + \int_{\theta_{BX}}^{\infty} r(\theta)\Big|_{\substack{I_X=1\\I_B=1}} - r(\theta)\Big|_{\substack{I_X=0\\I_B=1}} \, dG_\theta(\theta) \\
&= s_{r_3} \, \bar{r}^A(\theta_{D_3}).
\end{aligned}$$

The measure for the increase in average revenues due to the possibility of exporting and improving the bargaining power in the respective cases are given by:

$$s_{r_c} \equiv \begin{cases} 1 + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \rho_{X_c}^{z-\frac{\beta}{\Gamma}} + \Upsilon_X^{\frac{1-\beta}{\Gamma}} \left(\Upsilon_B^{\frac{1-\Gamma}{\Gamma}} - 1\right) \rho_{B_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c = 1 \\ 1 + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1-\Gamma}{\Gamma}} - 1\right) \rho_{X_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c = 2 \\ 1 + \left(\Upsilon_B^{\frac{1-\Gamma}{\Gamma}} - 1\right) \rho_{B_c}^{z-\frac{\beta}{\Gamma}} + \Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \rho_{X_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c = 3. \end{cases} \tag{3.B.71}$$

In order to indicate that the factor determining the increase in average revenues due to the possibility of exporting and improving the bargaining power is a function of the intensive (Υ_X, Υ_B) and extensive (ρ_{X_c}, ρ_{B_c}) margining of trade and bargaining improvement, the following notation is used in the paper $s_{r_c} = s_{r_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c})$. It is straight forward to confirm that $s_{r_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c})$ depends positively on the intensive (Υ_X, Υ_B) and extensive (ρ_{X_c}, ρ_{B_c}) margining of trade and bargaining improvement.

Average Employment. Using (3.B.33) average employment of type i workers by firms with a productivity above θ_ι in a situation where firms do neither export nor improve their bargaining power is given by:

$$\begin{aligned}
\int_{\theta_\iota}^{\infty} h_i(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) &= h_i(\theta_D) \int_{\theta_\iota}^{\infty} \left(\frac{\theta}{\theta_D}\right)^{\frac{\beta(1-k/\delta)}{\Gamma}} dG_\theta(\theta) \\
&= h_i(\theta_D) \int_{\theta_\iota}^{\infty} \left(\frac{\theta}{\theta_D}\right)^{\frac{\beta(1-k/\delta)}{\Gamma}} \theta^{-(z+1)} z \theta_{min}^z \, d\theta \\
&= h_i(\theta_D) z \theta_{min}^z \left[\left(\frac{\beta(1-k/\delta)}{\Gamma} - z\right)^{-1} \theta_D^{-\frac{\beta(1-k/\delta)}{\Gamma}} \theta^{\frac{\beta(1-k/\delta)}{\Gamma} - z} \right]_{\theta_\iota}^{\infty}.
\end{aligned}$$

This can be further simplified and written as:

$$\int_{\theta_\iota}^{\infty} h_i(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) = \frac{z\Gamma\theta_{min}^z h_i(\theta_D)}{z\Gamma - \beta(1-k/\delta)} \left(\frac{\theta_\iota}{\theta_D}\right)^{\frac{\beta(1-k/\delta)}{\Gamma}-z} \theta_D^{-z}$$
$$= \left(\frac{\theta_D}{\theta_\iota}\right)^{z-\frac{\beta(1-k/\delta)}{\Gamma}} \bar{h}_i^A(\theta_{D_c}, b_i), \tag{3.B.72}$$

where the ratio between the cutoffs is given by (3.35). The average employment of type i workers by all active firms in a situation where firms neither export nor improve their bargaining power is denoted by $\bar{h}_i^A(\theta_{D_c}, b_i)$. Using equations (3.B.72, 3.B.29) it follows:

$$\bar{h}_i^A(\theta_{D_c}, b_i) \equiv \bar{h}_{i_c}\Big|_{\substack{I_X=0\\I_B=0}} \equiv \int_{\theta_{D_c}}^{\infty} h_i(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) = \frac{z\Gamma\theta_{min}^z h_i(\theta_D)}{z\Gamma - \beta(1-k/\delta)} \theta_{D_c}^{-z}$$
$$= \frac{z\beta\gamma\eta_i^{1-\frac{k}{\delta}}}{z\Gamma - \beta(1-k/\delta)} \frac{f_D}{b_i} \left(\frac{\beta(1-\gamma k)}{\Gamma} \frac{f_D}{c a_{\min}^\delta}\right)^{-\frac{k}{\delta}} \left(\frac{\theta_{min}}{\theta_{D_c}}\right)^{-z}, \tag{3.B.73}$$

where even though the above depicts average employment, if firms neither export nor improve their bargaining power, the term depends on the domestic cutoff productivity which can be related to the export and the bargaining decision and thus differs across sorting pattern cases. The derivation of average employment also depends on the sorting pattern of the three cases. Generally, one can write average employment of case c as:

$$\bar{h}_{i_c} = \int_{\theta_{D_c}}^{\infty} h(\theta) \, dG_\theta(\theta) = s_{h_c} \, \bar{h}_i^A(\theta_{D_c}, b_i), \tag{3.B.74}$$

where $s_{h_c} > 1$ is the factor by which average employment is larger due to the possibility of exporting and improving the bargaining power. The functional form differs across sorting pattern cases. For the sorting pattern of *Case 1* average employment is given by:

$$\bar{h}_{i_1} = \int_{\theta_{D_1}}^{\infty} h_i(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) + \int_{\theta_X}^{\infty} h_i(\theta)\Big|_{\substack{I_X=1\\I_B=0}} - h_i(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta)$$
$$+ \int_{\theta_{XB}}^{\infty} h_i(\theta)\Big|_{\substack{I_X=1\\I_B=1}} - h_i(\theta)\Big|_{\substack{I_X=1\\I_B=0}} \, dG_\theta(\theta)$$
$$= \int_{\theta_{D_1}}^{\infty} h_i(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) + \left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} - 1\right) \int_{\theta_X}^{\infty} h_i(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta)$$
$$+ \Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} \left(\Upsilon_B^{\frac{\kappa_h}{\Gamma}} - 1\right) \int_{\theta_{XB}}^{\infty} h_i(\theta)\Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta)$$
$$= \left(1 + \left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} - 1\right)\left(\frac{\theta_D}{\theta_X}\right)^{z-\frac{\beta\kappa_h}{\Gamma}} + \Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} \left(\Upsilon_B^{\frac{\kappa_h}{\Gamma}} - 1\right)\left(\frac{\theta_D}{\theta_{XB}}\right)^{z-\frac{\beta\kappa_h}{\Gamma}}\right) \bar{h}_i^A(\theta_{D_1}, b_i)$$
$$= s_{h_1} \, \bar{h}_i^A(\theta_{D_1}, b_i),$$

where $\kappa_h \equiv 1 - k/\delta > 0$ allows to simplify the notation and s_{h_1} measures the increase in average employment of either worker type due to the possibility of exporting and improving the bargaining power in *Case 1*:

$$s_{h_1} \equiv 1 + \left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} - 1 \right) \rho_{X_c}^{z-\frac{\beta\kappa_h)}{\Gamma}} + \Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} \left(\Upsilon_B^{\frac{\kappa_h}{\Gamma}} - 1 \right) \rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}}.$$

Average employment in *Case 2* and *3* can be determined similarly as:

$$\begin{aligned}
\bar{h}_{i_2} &= \int_{\theta_D}^{\infty} h_i(\theta) \Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) + \int_{\theta_{DXB}}^{\infty} h_i(\theta) \Big|_{\substack{I_X=1\\I_B=1}} - h_i(\theta) \Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) \\
&= s_{h_2} \; \bar{h}_i^A(\theta_{D_2}, b_i) \\
\bar{h}_{i_3} &= \int_{\theta_D}^{\infty} h_i(\theta) \Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) + \int_{\theta_B}^{\infty} h_i(\theta) \Big|_{\substack{I_X=0\\I_B=1}} - h_i(\theta) \Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) \\
&\qquad + \int_{\theta_{BX}}^{\infty} h_i(\theta) \Big|_{\substack{I_X=1\\I_B=1}} - h_i(\theta) \Big|_{\substack{I_X=0\\I_B=1}} \, dG_\theta(\theta) \\
&= s_{h_3} \; \bar{h}_i^A(\theta_{D_3}, b_i).
\end{aligned}$$

The measure for the increase in overall employment due to the possibility of exporting and improving the bargaining power in the respective cases are given by:

$$s_{h_c} \equiv \begin{cases} 1 + \left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} - 1 \right) \rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}} + \Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} \left(\Upsilon_B^{\frac{\kappa_h}{\Gamma}} - 1 \right) \rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}}, & \text{if } c = 1 \\ 1 + \left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} \Upsilon_B^{\frac{\kappa_h}{\Gamma}} - 1 \right) \rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}}, & \text{if } c = 2 \\ 1 + \left(\Upsilon_B^{\frac{\kappa_h}{\Gamma}} - 1 \right) \rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}} + \Upsilon_B^{\frac{\kappa_h}{\Gamma}} \left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} - 1 \right) \rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}}, & \text{if } c = 3. \end{cases} \tag{3.B.75}$$

One can confirm, that $s_{h_c} = s_{h_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c})$, the factor determining the increase in average employment due to the possibility of exporting and improving the bargaining power depends positively on the intensive (Υ_X, Υ_B) and extensive (ρ_{X_c}, ρ_{B_c}) margining of trade and bargaining improvement.

Average Measure of Workers Matched. Using (3.B.32), the average measure of type i workers matched with a firm with a productivity above θ_ι in a situation where firms neither export nor improve their bargaining power is given by:

$$\begin{aligned}
\int_{\theta_\iota}^{\infty} n_i(\theta) \Big|_{\substack{I_X=0\\I_B=0}} \, dG_\theta(\theta) &= n_i(\theta_D) \int_{\theta_\iota}^{\infty} \left(\frac{\theta}{\theta_D} \right)^{\frac{\beta}{\Gamma}} \, dG_\theta(\theta) \\
&= n_i(\theta_D) \int_{\theta_\iota}^{\infty} \left(\frac{\theta}{\theta_D} \right)^{\frac{\beta}{\Gamma}} \theta^{-(z+1)} z \theta_{min}^z \, d\theta \\
&= n_i(\theta_D) z \theta_{min}^z \left[\left(\frac{\beta}{\Gamma} - z \right)^{-1} \theta_D^{-\frac{\beta}{\Gamma}} \theta^{\frac{\beta}{\Gamma}-z} \right]_{\theta_\iota}^{\infty}.
\end{aligned}$$

This can be further simplified and written as:

$$\begin{aligned}\int_{\theta_\iota}^{\infty} n_i(\theta)\Big|_{\substack{I_X=0\\ I_B=0}} \ dG_\theta(\theta) &= \frac{z\Gamma\theta_{min}^z n_i(\theta_D)}{z\Gamma-\beta}\left(\frac{\theta_\iota}{\theta_D}\right)^{\frac{\beta}{\Gamma}-z}\theta_D^{-z} \\ &= \frac{z\beta\gamma}{z\Gamma-\beta}\frac{\eta_i}{b_i} f_D\left(\frac{\theta_D}{\theta_\iota}\right)^{z-\frac{\beta}{\Gamma}}\left(\frac{\theta_{min}}{\theta_D}\right)^{z} \\ &= \left(\frac{\theta_D}{\theta_\iota}\right)^{z-\frac{\beta}{\Gamma}} \bar{n}_i^A(\theta_{D_c}, b_i). \end{aligned} \tag{3.B.76}$$

The ratio between the cutoffs is again given by (3.35). Average measure of type i workers matched by all active firms in a situation where firms neither export nor improve their bargaining power is denoted by $(\bar{n}_i^A(\theta_{D_c}, b_i))$ and using (3.B.76) is give as:

$$\bar{n}_i^A(\theta_{D_c}, b_i) \equiv \bar{n}_{i_c}\Big|_{\substack{I_X=0\\ I_B=0}} \equiv \int_{\theta_{D_c}}^{\infty} n_i(\theta)\Big|_{\substack{I_X=0\\ I_B=0}} \ dG_\theta(\theta) = \frac{z\beta\gamma}{z\Gamma-\beta}\frac{\eta_i}{b_i} f_D\left(\frac{\theta_{min}}{\theta_{D_c}}\right)^{z}, \tag{3.B.77}$$

where even though the above depicts the average measure of workers matched, if firms neither export nor improve their bargaining power the term depends on the domestic cutoff productivity which can be related to the export and the bargaining decision and thus differs across sorting pattern cases. The derivation of the average measure of workers matched also depends on the sorting pattern of the three cases. Generally, one can write average employment of case c as:

$$\bar{n}_{i_c} = \int_{\theta_{D_c}}^{\infty} n_i(\theta) \ dG_\theta(\theta) = s_{n_c} \ \bar{n}_i^A(\theta_{D_c}, b_i), \tag{3.B.78}$$

where $s_{n_c} > 1$ is the factor by which the average measure of workers matched is larger due to the possibility of exporting and improving the bargaining power. The functional form differs across sorting pattern cases. Average revenues (3.B.70), average employment (3.B.74) and the average number of workers matched (3.B.78) are combined in the main paper in equation (3.41). For the sorting pattern of *Case 1* the average measure of workers matched is given by:

$$\begin{aligned}\bar{n}_{i_1} &= \int_{\theta_{D_1}}^{\infty} n_i(\theta)\Big|_{\substack{I_X=0\\ I_B=0}} \ dG_\theta(\theta) + \int_{\theta_X}^{\infty} n_i(\theta)\Big|_{\substack{I_X=1\\ I_B=0}} - n_i(\theta)\Big|_{\substack{I_X=0\\ I_B=0}} \ dG_\theta(\theta) \\ &\qquad + \int_{\theta_{XB}}^{\infty} n_i(\theta)\Big|_{\substack{I_X=1\\ I_B=1}} - n_i(\theta)\Big|_{\substack{I_X=1\\ I_B=0}} \ dG_\theta(\theta) \\ &= \int_{\theta_{D_1}}^{\infty} n_i(\theta)\Big|_{\substack{I_X=0\\ I_B=0}} \ dG_\theta(\theta) + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\int_{\theta_X}^{\infty} n_i(\theta)\Big|_{\substack{I_X=0\\ I_B=0}} \ dG_\theta(\theta) \\ &\qquad + \Upsilon_X^{\frac{1-\beta}{\Gamma}}\left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\int_{\theta_{XB}}^{\infty} n_i(\theta)\Big|_{\substack{I_X=0\\ I_B=0}} \ dG_\theta(\theta). \end{aligned}$$

This can be further simplified and written as:

$$\begin{aligned}\bar{n}_{i_1} &= \left(1 + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\left(\frac{\theta_D}{\theta_X}\right)^{z-\frac{\beta}{\Gamma}} + \Upsilon_X^{\frac{1-\beta}{\Gamma}}\left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\left(\frac{\theta_D}{\theta_{XB}}\right)^{z-\frac{\beta}{\Gamma}}\right)\bar{n}_i^A(\theta_{D_1}, b_i) \\ &= s_{n_1}\ \bar{n}_i^A(\theta_{D_1}, b_i),\end{aligned}$$

where $s_{n_1}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c})$ is a measure for the increase in the average measure of workers matched due to the possibility of exporting and improving the bargaining power compared to a situation where neither is possible:

$$s_{n_1} \equiv 1 + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\left(\frac{\theta_D}{\theta_X}\right)^{z-\frac{\beta}{\Gamma}} + \Upsilon_X^{\frac{1-\beta}{\Gamma}}\left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\left(\frac{\theta_D}{\theta_{XB}}\right)^{z-\frac{\beta}{\Gamma}}.$$

Average employment in *Case 2* and *3* can be determined similarly as:

$$\begin{aligned}\bar{n}_{i_2} &= \int_{\theta_D}^{\infty} n_i(\theta)\Big|_{\substack{I_X=0\\ I_B=0}}\ dG_\theta(\theta) + \int_{\theta_{DXB}}^{\infty} n_i(\theta)\Big|_{\substack{I_X=1\\ I_B=1}} - n_i(\theta)\Big|_{\substack{I_X=0\\ I_B=0}}\ dG_\theta(\theta) \\ &= s_{n_2}\ \bar{n}_i^A(\theta_{D_2}, b_i) \\ \bar{n}_{i_3} &= \int_{\theta_D}^{\infty} n_i(\theta)\Big|_{\substack{I_X=0\\ I_B=0}}\ dG_\theta(\theta) + \int_{\theta_B}^{\infty} n_i(\theta)\Big|_{\substack{I_X=0\\ I_B=1}} - n_i(\theta)\Big|_{\substack{I_X=0\\ I_B=0}}\ dG_\theta(\theta) \\ &\qquad + \int_{\theta_{BX}}^{\infty} n_i(\theta)\Big|_{\substack{I_X=1\\ I_B=1}} - n_i(\theta)\Big|_{\substack{I_X=0\\ I_B=1}}\ dG_\theta(\theta) \\ &= s_{n_3}\ \bar{n}_i^A(\theta_{D_3}, b_i).\end{aligned}$$

The measure for the increase in overall matched workers due to the possibility of exporting and improving the bargaining power in the respective cases are given by:

$$s_{n_c} \equiv \begin{cases} 1 + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}} + \Upsilon_X^{\frac{1-\beta}{\Gamma}}\left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\rho_{B_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c = 1 \\ 1 + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c = 2 \\ 1 + \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right)\rho_{B_c}^{z-\frac{\beta}{\Gamma}} + \Upsilon_B^{\frac{1}{\Gamma}}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c = 3. \end{cases} \tag{3.B.79}$$

One can confirm, that $s_{n_c} = s_{n_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c})$, the factor determining the increase in the average number of workers matched to firms due to the possibility of exporting and improving the bargaining power, depends positively on the intensive (Υ_X, Υ_B) and extensive (ρ_{X_c}, ρ_{B_c}) margining of trade and bargaining improvement.

Hiring Rate. Using equations (3.B.74) and (3.B.78), the hiring rate, which is the share of type-i workers hired given a match with a firm, can be written as:

$$\sigma_i = \frac{H_i}{N_i} = \frac{M\int_{\theta_D}^{\infty} h_i(\theta)dG_\theta(\theta)}{M\int_{\theta_D}^{\infty} n_i(\theta)dG_\theta(\theta)} = \frac{\bar{h}_{i_c}}{\bar{n}_{i_c}} = \frac{s_{h_c}\bar{h}_i^A}{s_{n_c}\bar{n}_i^A} = s_{\sigma_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c})\ \sigma_i^A. \tag{3.42}$$

Using (3.B.73) and (3.B.77) the hiring rate in a situation where all firms neither export nor improve their bargaining power is given by:

$$\begin{aligned}\sigma_i^A = \sigma_i\Big|_{\substack{I_X=0\\ I_B=0}} = \frac{\bar{h}_i^A}{\bar{n}_i^A} &= \frac{\frac{z\beta\gamma\eta_i^{1-\frac{k}{\delta}}}{z\Gamma-\beta(1-k/\delta)}\frac{f_D}{b_i}\left(\frac{\beta(1-\gamma k)}{\Gamma}\frac{f_D}{ca_{\min}^{\delta}}\right)^{-\frac{k}{\delta}}\left(\frac{\theta_{min}}{\theta_{D_c}}\right)^{-z}}{\frac{z\beta\gamma\eta_i}{z\Gamma-\beta}\frac{f_D}{b_i}\left(\frac{\theta_{min}}{\theta_{D_c}}\right)^{z}},\\ &= \frac{z\Gamma-\beta}{z\Gamma-\beta(1-k/\delta)}\left(\frac{\Gamma}{\beta(1-\gamma k)\eta_i}\frac{ca_{\min}^{\delta}}{f_D}\right)^{\frac{k}{\delta}},\end{aligned} \tag{3.43}$$

where the term in brackets is the hiring rate by the cutoff firm $(h(\theta_D)/n(\theta_D))$. The decline in the hiring rates due to the possibility to improve the bargaining power is depicted by:

$$0 < s_{\sigma_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c}) \equiv \frac{s_{h_c}}{s_{n_c}} < 1. \tag{3.44}$$

This can be written explicitly for the three cases:

$$\frac{s_{h_c}}{s_{n_c}} = \begin{cases} \dfrac{1+\left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}}+\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}}\left(\Upsilon_B^{\frac{\kappa_h}{\Gamma}}-1\right)\rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}}}{1+\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}}+\Upsilon_X^{\frac{1-\beta}{\Gamma}}\left(\Upsilon_B^{\frac{1}{\Gamma}}-1\right)\rho_{B_c}^{z-\frac{\beta}{\Gamma}}}, & \text{if } c=1\\[2ex] \dfrac{1+\left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}}\Upsilon_B^{\frac{\kappa_h}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}}}{1+\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}\Upsilon_B^{\frac{1}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}}}, & \text{if } c=2\\[2ex] \dfrac{1+\left(\Upsilon_B^{\frac{\kappa_h}{\Gamma}}-1\right)\rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}}+\Upsilon_B^{\frac{\kappa_h}{\Gamma}}\left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}}}{1+\left(\Upsilon_B^{\frac{1}{\Gamma}}-1\right)\rho_{B_c}^{z-\frac{\beta}{\Gamma}}+\Upsilon_B^{\frac{1}{\Gamma}}\left(\Upsilon_X^{\frac{1-\beta}{\Gamma}}-1\right)\rho_{X_c}^{z-\frac{\beta}{\Gamma}}}, & \text{if } c=3. \end{cases}$$

Revenue Shares. When determining the general equilibrium and more explicitly the expected wages paid, it is helpful to know the share of revenues produced by firms improving the bargaining power S_{rB_c}. Using the cutoff productivity above which firms decide to improve their bargaining power θ_{B_c}, which differs across cases (c), allows to write the revenue share produced by firms that improve their bargaining power as:

$$S_{rB_c} = \frac{\int_{\theta_{B_c}}^{\infty} r(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_c}}^{\infty} r(\theta)\ dG_\theta(\theta)} = \begin{cases} \frac{\int_{\theta_{XB}}^{\infty} r(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_1}}^{\infty} r(\theta)\ dG_\theta(\theta)}, & \text{if } c = 1 \\ \frac{\int_{\theta_{DXB}}^{\infty} r(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_2}}^{\infty} r(\theta)\ dG_\theta(\theta)}, & \text{if } c = 2 \\ \frac{\int_{\theta_{B}}^{\infty} r(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_3}}^{\infty} r(\theta)\ dG_\theta(\theta)}, & \text{if } c = 3. \end{cases} \tag{3.B.80}$$

Using the relationship between the cutoffs (3.35) and the above results the share of revenues generated by firms improving their bargaining power can be derived for the three sorting pattern cases in the following manner:

Case 1:

$$S_{rB_1} = \frac{\int_{\theta_{XB}}^{\infty} r(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_1}}^{\infty} r(\theta)\ dG_\theta(\theta)} = \frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \int_{\theta_{XB}}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\ I_B=0}}\ dG_\theta(\theta)}{\bar{r}_1}$$
$$= \frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \left(\frac{\theta_D}{\theta_{XB}}\right)^{z-\frac{\beta}{\Gamma}} \bar{r}_1^A}{\bar{r}_1} = \frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1-\Gamma}{\Gamma}}}{s_{r_1}} \left(\frac{\theta_D}{\theta_{XB}}\right)^{\frac{z\Gamma-\beta}{\Gamma}},$$

Case 2:

$$S_{rB_2} = \frac{\int_{\theta_{DXB}}^{\infty} r(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_2}}^{\infty} r(\theta)\ dG_\theta(\theta)} = \frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \int_{\theta_{DXB}}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\ I_B=0}}\ dG_\theta(\theta)}{\bar{r}_2}$$
$$= \frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \left(\frac{\theta_D}{\theta_{DXB}}\right)^{z-\frac{\beta}{\Gamma}} \bar{r}_2^A}{\bar{r}_2} = \frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1-\Gamma}{\Gamma}}}{s_{r_2}} \left(\frac{\theta_D}{\theta_{DXB}}\right)^{\frac{z\Gamma-\beta}{\Gamma}},$$

Case 3:

$$S_{rB_3} = \frac{\int_{\theta_{B}}^{\infty} r(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_3}}^{\infty} r(\theta)\ dG_\theta(\theta)}$$
$$= \frac{\Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \int_{\theta_B}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\ I_B=0}}\ dG_\theta(\theta) + \Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \int_{\theta_{BX}}^{\infty} r(\theta)\Big|_{\substack{I_X=0\\ I_B=0}}\ dG_\theta(\theta)}{\bar{r}_3}$$
$$= \frac{\Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \left(\frac{\theta_D}{\theta_B}\right)^{z-\frac{\beta}{\Gamma}} \bar{r}_3^A + \Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \left(\frac{\theta_D}{\theta_{BX}}\right)^{z-\frac{\beta}{\Gamma}} \bar{r}_3^A}{\bar{r}_3}$$
$$= \frac{\Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \left(\frac{\theta_D}{\theta_B}\right)^{z-\frac{\beta}{\Gamma}} + \Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \left(\frac{\theta_D}{\theta_{BX}}\right)^{z-\frac{\beta}{\Gamma}}}{s_{r_3}}.$$

Thus, one can write the share of revenues produced by firms that improve their bargaining power for all three cases as:

$$S_{rB_c} = \begin{cases} \dfrac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1-\Gamma}{\Gamma}}}{s_{r_1}} \rho_{B_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c = 1 \\ \dfrac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1-\Gamma}{\Gamma}}}{s_{r_2}} \rho_{X_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c = 2 \\ \dfrac{\Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \left(\rho_{B_c}^{z-\frac{\beta}{\Gamma}} + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1 \right) \rho_{X_c}^{z-\frac{\beta}{\Gamma}} \right)}{s_{r_3}}, & \text{if } c = 3. \end{cases}$$

Using (3.B.71) one can rewrite the shares as follows:

$$S_{rB_c} = \begin{cases} \left(1 + \dfrac{1 + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1 \right) \rho_{X_c}^{z-\frac{\beta}{\Gamma}} - \Upsilon_X^{\frac{1-\beta}{\Gamma}} \rho_{B_c}^{z-\frac{\beta}{\Gamma}}}{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \rho_{B_c}^{z-\frac{\beta}{\Gamma}}} \right)^{-1}, & \text{if } c = 1 \\ \left(1 + \dfrac{1 - \rho_{X_c}^{z-\frac{\beta}{\Gamma}}}{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \rho_{X_c}^{z-\frac{\beta}{\Gamma}}} \right)^{-1}, & \text{if } c = 2 \\ \left(1 + \dfrac{1 - \rho_{B_c}^{z-\frac{\beta}{\Gamma}}}{\Upsilon_B^{\frac{1-\Gamma}{\Gamma}} \left(\rho_{B_c}^{z-\frac{\beta}{\Gamma}} + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1 \right) \rho_{X_c}^{z-\frac{\beta}{\Gamma}} \right)} \right)^{-1}, & \text{if } c = 3. \end{cases} \tag{3.B.80}$$

A higher possible bargaining improvement λ implies that the share of firms that improve their bargaining power rises $\frac{\partial \rho_{B_c}}{\partial \lambda} > 0$. In *Case 2* and *3* also the share of exporting firms rises $\frac{\partial \rho_{X_c}}{\partial \lambda}\Big|_{c \in \{2,3\}} > 0$ while for *Case 1* it is unaffected $\frac{\partial \rho_{X_1}}{\partial \lambda} = 0$. Finally, also the bargaining improvement term rises $\frac{\partial \Upsilon_B}{\partial \lambda} > 0$. All those effects have a positive impact on the share of revenues generated by firms improving their bargaining power. Consequently, $\frac{\partial S_{rB_c}}{\partial \lambda} > 0$ holds.

Employment Shares. Using cutoff productivity above which firms decide to improve their bargaining power which differs across cases (c) and is denoted by θ_{B_c}, allows to write the employment share by firms that improve their bargaining power as:

$$S_{hB_c} \equiv \frac{\int_{\theta_{B_c}}^{\infty} h(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_c}}^{\infty} h(\theta)\ dG_\theta(\theta)} = \begin{cases} \dfrac{\int_{\theta_{XB}}^{\infty} h_i(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_1}}^{\infty} h_i(\theta)\ dG_\theta(\theta)}, & \text{if } c = 1 \\ \dfrac{\int_{\theta_{DXB}}^{\infty} h_i(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_2}}^{\infty} h_i(\theta)\ dG_\theta(\theta)}, & \text{if } c = 2 \\ \dfrac{\int_{\theta_{B}}^{\infty} h_i(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_3}}^{\infty} h_i(\theta)\ dG_\theta(\theta)}, & \text{if } c = 3. \end{cases} \tag{3.B.81}$$

Using the relationship between the cutoffs (3.35) and the results above, the share of employment by firms improving their bargaining power can be derived for the three sorting pattern cases in the following manner:
Case 1:

$$\begin{aligned}
S_{hB_1} &= \frac{\int_{\theta_{XB}}^{\infty} h(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_1}}^{\infty} h(\theta)\ dG_\theta(\theta)} = \frac{\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} \Upsilon_B^{\frac{\kappa_h}{\Gamma}} \left(\frac{\theta_D}{\theta_{XB}}\right)^{z-\frac{\beta\kappa_h}{\Gamma}} \bar{h}_{i_1}^A}{s_{h_1}\bar{h}_{i_1}^A} \\
&= \frac{\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} \Upsilon_B^{\frac{\kappa_h}{\Gamma}} \left(\frac{\theta_D}{\theta_{XB}}\right)^{z-\frac{\beta\kappa_h}{\Gamma}}}{s_{h_1}} \\
&= \frac{\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} \Upsilon_B^{\frac{\kappa_h}{\Gamma}} \left(\frac{\theta_D}{\theta_{XB}}\right)^{z-\frac{\beta\kappa_h}{\Gamma}}}{1+\left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}}-1\right)\left(\frac{\theta_D}{\theta_X}\right)^{z-\frac{\beta\kappa_h}{\Gamma}} + \Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}}\left(\Upsilon_B^{\frac{\kappa_h}{\Gamma}}-1\right)\left(\frac{\theta_D}{\theta_{XB}}\right)^{z-\frac{\beta\kappa_h}{\Gamma}}},
\end{aligned}$$

Case 2:

$$\begin{aligned}
S_{hB_2} &= \frac{\int_{\theta_{DXB}}^{\infty} h(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_2}}^{\infty} h(\theta)\ dG_\theta(\theta)} = \frac{\Upsilon_B^{\frac{\kappa_h}{\Gamma}} \Upsilon_X^{(1-\beta)\frac{\kappa_h}{\Gamma}} \left(\frac{\theta_D}{\theta_{DXB}}\right)^{z-\frac{\beta\kappa_h}{\Gamma}} \bar{h}_{i_2}^A}{s_{h_2}\bar{h}_{i_2}^A} \\
&= \frac{\Upsilon_B^{\frac{\kappa_h}{\Gamma}} \Upsilon_X^{(1-\beta)\frac{\kappa_h}{\Gamma}} \left(\frac{\theta_D}{\theta_{DXB}}\right)^{z-\frac{\beta\kappa_h}{\Gamma}}}{s_{h_2}} = \frac{\Upsilon_B^{\frac{\kappa_h}{\Gamma}} \Upsilon_X^{(1-\beta)\frac{\kappa_h}{\Gamma}} \left(\frac{\theta_D}{\theta_{DXB}}\right)^{z-\frac{\beta\kappa_h}{\Gamma}}}{1+\left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}}\Upsilon_B^{\frac{\kappa_h}{\Gamma}}-1\right)\left(\frac{\theta_D}{\theta_{DXB}}\right)^{z-\frac{\beta\kappa_h}{\Gamma}}},
\end{aligned}$$

Case 3:

$$\begin{aligned}
S_{hB_3} &= \frac{\int_{\theta_B}^{\infty} h(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_3}}^{\infty} h(\theta)\ dG_\theta(\theta)} = \frac{\Upsilon_B^{\frac{\kappa_h}{\Gamma}}\left(\left(\frac{\theta_D}{\theta_B}\right)^{z-\frac{\beta\kappa_h}{\Gamma}} + \left(\Upsilon_X^{(1-\beta)\frac{\kappa_h}{\Gamma}}-1\right)\left(\frac{\theta_D}{\theta_{BX}}\right)^{z-\frac{\beta\kappa_h}{\Gamma}}\right)\bar{h}_{i_3}^A}{s_{h_3}\bar{h}_{i_3}^A} \\
&= \frac{\Upsilon_B^{\frac{\kappa_h}{\Gamma}}\left(\left(\frac{\theta_D}{\theta_B}\right)^{z-\frac{\beta\kappa_h}{\Gamma}} + \left(\Upsilon_X^{(1-\beta)\frac{\kappa_h}{\Gamma}}-1\right)\left(\frac{\theta_D}{\theta_{BX}}\right)^{z-\frac{\beta\kappa_h}{\Gamma}}\right)}{s_{h_3}} \\
&= \frac{\Upsilon_B^{\frac{\kappa_h}{\Gamma}}\left(\left(\frac{\theta_D}{\theta_B}\right)^{z-\frac{\beta\kappa_h}{\Gamma}} + \left(\Upsilon_X^{(1-\beta)\frac{\kappa_h}{\Gamma}}-1\right)\left(\frac{\theta_D}{\theta_{BX}}\right)^{z-\frac{\beta\kappa_h}{\Gamma}}\right)}{1+\left(\Upsilon_B^{\frac{\kappa_h}{\Gamma}}-1\right)\left(\frac{\theta_D}{\theta_B}\right)^{z-\frac{\beta\kappa_h}{\Gamma}} + \Upsilon_B^{\frac{\kappa_h}{\Gamma}}\left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}}-1\right)\left(\frac{\theta_D}{\theta_{BX}}\right)^{z-\frac{\beta\kappa_h}{\Gamma}}}.
\end{aligned}$$

The share of workers employed by firms improving their bargaining powers is the same for both types of workers. One can write the share of workers employed by firms that improve their bargaining power for all three cases as:

$$S_{hB_c} = \begin{cases} \dfrac{\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} \Upsilon_B^{\frac{\kappa_h}{\Gamma}}}{s_{h_1}} \rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}}, & \text{if } c=1 \\ \dfrac{\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} \Upsilon_B^{\frac{\kappa_h}{\Gamma}}}{s_{h_2}} \rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}}, & \text{if } c=2 \\ \dfrac{\Upsilon_B^{\frac{\kappa_h}{\Gamma}} \left(\rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}} + \left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} - 1 \right) \rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}} \right)}{s_{h_3}}, & \text{if } c=3. \end{cases}$$

Using (3.B.75) one can rewrite the shares as follows:

$$S_{hB_c} = \begin{cases} \left(1 + \dfrac{1 + \Upsilon_X^{\frac{(1-\beta)\kappa_k}{\Gamma}} \rho_{B_c}^{z-\frac{\beta\kappa_k}{\Gamma}} + \left(\Upsilon_X^{\frac{(1-\beta)\kappa_k}{\Gamma}} - 1 \right) \rho_{X_c}^{z-\frac{\beta\kappa_k}{\Gamma}}}{\Upsilon_X^{\frac{(1-\beta)\kappa_k}{\Gamma}} \Upsilon_B^{\frac{\kappa_h}{\Gamma}} \rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}}} \right)^{-1}, & \text{if } c=1 \\ \left(1 + \dfrac{1 - \rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}}}{\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} \Upsilon_B^{\frac{\kappa_h}{\Gamma}} \rho_{X_c}^{z-\frac{\beta\kappa_h}{\Gamma}}} \right)^{-1}, & \text{if } c=2 \\ \left(1 + \dfrac{1 - \rho_{B_c}^{z-\frac{\beta\kappa_k}{\Gamma}}}{\Upsilon_B^{\frac{\kappa_h}{\Gamma}} \left(\rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}} + \left(\Upsilon_X^{\frac{(1-\beta)\kappa_h}{\Gamma}} - 1 \right) \rho_{B_c}^{z-\frac{\beta\kappa_h}{\Gamma}} \right)} \right)^{-1}, & \text{if } c=3. \end{cases} \tag{3.B.81}$$

As derived before a higher possible bargaining improvement λ has the following effects, $\frac{\partial \rho_{B_c}}{\partial \lambda} > 0$, $\frac{\partial \rho_{X_c}}{\partial \lambda}\Big|_{c \in \{2,3\}} > 0$, $\frac{\partial \rho_{X_1}}{\partial \lambda} > 0$, $\frac{\partial \Upsilon_B}{\partial \lambda} > 0$. All those effects have a positive impact on the share of workers employed by firms improving their bargaining power. Consequently, $\frac{\partial S_{hB_c}}{\partial \lambda} > 0$ holds.

Share of Workers Matched. Using cutoff productivity above which firms decide to improve their bargaining power, which differs across cases (c) and is denoted by θ_{B_c}, allows to write the share of matched workers by firms that improve their bargaining power as:

$$S_{nB_c} = \frac{\int_{\theta_{B_c}}^{\infty} n(\theta)\, dG_\theta(\theta)}{\int_{\theta_{D_c}}^{\infty} n(\theta)\, dG_\theta(\theta)} \begin{cases} \dfrac{\int_{\theta_{XB}}^{\infty} n_i(\theta)\, dG_\theta(\theta)}{\int_{\theta_D}^{\infty} n_i(\theta)\, dG_\theta(\theta)}, & \text{if } c=1 \\ \dfrac{\int_{\theta_{DXB}}^{\infty} n_i(\theta)\, dG_\theta(\theta)}{\int_{\theta_D}^{\infty} n_i(\theta)\, dG_\theta(\theta)}, & \text{if } c=2 \\ \dfrac{\int_{\theta_B}^{\infty} n_i(\theta)\, dG_\theta(\theta)}{\int_{\theta_D}^{\infty} n_i(\theta)\, dG_\theta(\theta)}, & \text{if } c=3. \end{cases} \tag{3.B.82}$$

Using the relationship between the cutoffs (3.35) and the above results the share of matched workers by firms improving their bargaining power can be derived for the three

sorting pattern cases in the following manner:
Case 1:

$$
\begin{aligned}
S_{nB_1} &= \frac{\int_{\theta_{XB}}^{\infty} n(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_1}}^{\infty} n(\theta)\ dG_\theta(\theta)} = \frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} \left(\frac{\theta_D}{\theta_{XB}}\right)^{z-\frac{\beta}{\Gamma}} \bar{n}_{i_1}^A}{s_{n_1} \bar{n}_{i_1}^A} \\
&= \frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} \left(\frac{\theta_D}{\theta_{XB}}\right)^{z-\frac{\beta}{\Gamma}}}{s_{n_1}} = \frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} \left(\frac{\theta_D}{\theta_{XB}}\right)^{z-\frac{\beta}{\Gamma}}}{1 + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \left(\frac{\theta_D}{\theta_X}\right)^{z-\frac{\beta}{\Gamma}} + \Upsilon_X^{\frac{1-\beta}{\Gamma}} \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right) \left(\frac{\theta_D}{\theta_{XB}}\right)^{z-\frac{\beta}{\Gamma}}},
\end{aligned}
$$

Case 2:

$$
\begin{aligned}
S_{nB_2} &= \frac{\int_{\theta_{DXB}}^{\infty} n(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_2}}^{\infty} n(\theta)\ dG_\theta(\theta)} = \frac{\Upsilon_B^{\frac{1}{\Gamma}} \Upsilon_X^{\frac{1-\beta}{\Gamma}} \left(\frac{\theta_D}{\theta_{DXB}}\right)^{z-\frac{\beta}{\Gamma}} \bar{n}_{i_2}^A}{s_{n_2} \bar{n}_{i_2}^A} \\
&= \frac{\Upsilon_B^{\frac{1}{\Gamma}} \Upsilon_X^{\frac{1-\beta}{\Gamma}} \left(\frac{\theta_D}{\theta_{DXB}}\right)^{z-\frac{\beta}{\Gamma}}}{s_{n_2}} = \frac{\Upsilon_B^{\frac{1}{\Gamma}} \Upsilon_X^{\frac{1-\beta}{\Gamma}} \left(\frac{\theta_D}{\theta_{DXB}}\right)^{z-\frac{\beta}{\Gamma}}}{1 + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} - 1\right) \left(\frac{\theta_D}{\theta_{DXB}}\right)^{z-\frac{\beta}{\Gamma}}},
\end{aligned}
$$

Case 3:

$$
\begin{aligned}
S_{nB_3} &= \frac{\int_{\theta_B}^{\infty} n(\theta)\ dG_\theta(\theta)}{\int_{\theta_{D_3}}^{\infty} n(\theta)\ dG_\theta(\theta)} = \frac{\Upsilon_B^{\frac{1}{\Gamma}} \left(\left(\frac{\theta_D}{\theta_B}\right)^{z-\frac{\beta}{\Gamma}} + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \left(\frac{\theta_D}{\theta_{BX}}\right)^{z-\frac{\beta}{\Gamma}}\right) \bar{n}_{i_3}^A}{s_{n_3} \bar{n}_{i_3}^A} \\
&= \frac{\Upsilon_B^{\frac{1}{\Gamma}} \left(\left(\frac{\theta_D}{\theta_B}\right)^{z-\frac{\beta}{\Gamma}} + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \left(\frac{\theta_D}{\theta_{BX}}\right)^{z-\frac{\beta}{\Gamma}}\right)}{s_{n_3}} \\
&= \frac{\Upsilon_B^{\frac{1}{\Gamma}} \left(\left(\frac{\theta_D}{\theta_B}\right)^{z-\frac{\beta}{\Gamma}} + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \left(\frac{\theta_D}{\theta_{BX}}\right)^{z-\frac{\beta}{\Gamma}}\right)}{1 + \left(\Upsilon_B^{\frac{1}{\Gamma}} - 1\right) \left(\frac{\theta_D}{\theta_B}\right)^{z-\frac{\beta}{\Gamma}} + \Upsilon_B^{\frac{1}{\Gamma}} \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \left(\frac{\theta_D}{\theta_{BX}}\right)^{z-\frac{\beta}{\Gamma}}}.
\end{aligned}
$$

The share of matched workers by firms improving their bargaining powers is the same for both types of workers. One can write the share of workers matched with firms that improve their bargaining power for all three cases as:

$$
S_{nB_c} = \begin{cases}
\frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}}}{s_{n_1}} \rho_{B_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c = 1 \\
\frac{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}}}{s_{n_2}} \rho_{X_c}^{z-\frac{\beta}{\Gamma}}, & \text{if } c = 2 \\
\frac{\Upsilon_B^{\frac{1}{\Gamma}} \left(\rho_{B_c}^{z-\frac{\beta}{\Gamma}} + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \rho_{X_c}^{z-\frac{\beta}{\Gamma}}\right)}{s_{n_3}}, & \text{if } c = 3.
\end{cases}
$$

Using (3.B.79) one can rewrite the shares as follows:

$$S_{nB_c} = \begin{cases} \left(1 + \dfrac{1 + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \rho_{X_c}^{z-\frac{\beta}{\Gamma}} - \Upsilon_X^{\frac{1-\beta}{\Gamma}} \rho_{B_c}^{z-\frac{\beta}{\Gamma}}}{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} \rho_{B_c}^{z-\frac{\beta}{\Gamma}}}\right)^{-1}, & \text{if } c = 1 \\ \left(1 + \dfrac{1 - \rho_{X_c}^{z-\frac{\beta}{\Gamma}}}{\Upsilon_X^{\frac{1-\beta}{\Gamma}} \Upsilon_B^{\frac{1}{\Gamma}} \rho_{X_c}^{z-\frac{\beta}{\Gamma}}}\right)^{-1}, & \text{if } c = 2 \\ \left(1 + \dfrac{1 - \rho_{B_c}^{z-\frac{\beta}{\Gamma}}}{\Upsilon_B^{\frac{1}{\Gamma}} \left(\rho_{B_c}^{z-\frac{\beta}{\Gamma}} + \left(\Upsilon_X^{\frac{1-\beta}{\Gamma}} - 1\right) \rho_{X_c}^{z-\frac{\beta}{\Gamma}}\right)}\right)^{-1}, & \text{if } c = 3. \end{cases} \tag{3.B.82}$$

A higher possible bargaining improvement λ has the following effects, $\frac{\partial \rho_{B_c}}{\partial \lambda} > 0$, $\frac{\partial \rho_{X_c}}{\partial \lambda}\Big|_{c \in \{2,3\}} > 0$, $\frac{\partial \rho_{X_1}}{\partial \lambda} > 0$, $\frac{\partial \Upsilon_B}{\partial \lambda} > 0$. All those effects have a positive impact on the share of workers matched with firms improving their bargaining power. Consequently, $\frac{\partial S_{nB_c}}{\partial \lambda} > 0$ holds.

General Equilibrium Variables In order to derive the general equilibrium conditions, it is helpful to use the expected wage income for the two types of labour. In a first step it is important to recap that from equation (3.28) one knows that the expected wage in a firm θ, given that a worker is matched with that firm, only differs across firms if they differ in terms of their bargaining power improvement decision. This allows to write the expected wage, given a worker is matched to any firm by the probability that the worker is matched to a firm, which is not improving its bargaining power times the expected wage in that firm, given a match plus the probability that the worker is matched to a firm, which is improving its bargaining power, times the expected wage in that firm:

$$\bar{w}_i = (1 - S_{nB_c}) \left.\frac{w_i(\theta) h_i(\theta)}{n_i(\theta)}\right|_{I_B=0} + S_{nB_c} \left.\frac{w_i(\theta) h_i(\theta)}{n_i(\theta)}\right|_{I_B=1}. \tag{3.45}$$

This allows to rewrite the expected wage given a worker is matched with a firm (3.45) using (3.28) and the share of workers matched with firms that improve their bargaining power (S_{nB_c}) as:

$$\bar{w}_i = (1 - S_{nB_c}) b_i + S_{nB_c} \frac{b_i}{\lambda_i} = \left(1 - S_{nB_c}\left(1 - \frac{1}{\lambda_i}\right)\right) b_i = s_{\bar{w}_{ic}} b_i, \tag{3.45}$$

where $s_{\bar{w}_{ic}}$ is a help parameter and is a measure for the importance of the share of workers facing a smaller bargaining power which is one for type-2 workers:

$$s_{\bar{w}_{ic}} \equiv 1 - S_{nB_c}\left(1 - \frac{1}{\lambda_i}\right) \leq 1. \tag{3.46}$$

The expected wage income is given by the probability of being matched with a firm times the expected wage conditional on being matched ($\bar{w}_i$):

$$\omega_i = \frac{N_i}{L_i}\bar{w}_i = x_i\bar{w}_i = s_{\bar{w}_{i_c}} x_i b_i. \tag{3.47}$$

Using the formulation of the search cost (3.5) allows to write labour market tightness and search cost as a function of the expected wage:

$$\begin{aligned} x_i &= \frac{\omega_i}{s_{\bar{w}_{i_c}} b_i} = \frac{\alpha_0 \omega_i}{s_{\bar{w}_{i_c}} x_i^{\alpha}} & b_i &= \frac{x_{i_c}^{\alpha}}{\alpha_0} = \frac{1}{\alpha_0}\left(\frac{\omega_i}{s_{\bar{w}_{i_c}} b_i}\right)^{\alpha} \\ x_i^{1+\alpha} &= \frac{\alpha_0\ \omega_i}{s_{\bar{w}_{i_c}}} & b_i^{1+\alpha} &= \frac{1}{\alpha_0}\left(\frac{\omega_i}{s_{\bar{w}_{i_c}}}\right)^{\alpha} \\ x_i &= \left(\frac{\alpha_0\ \omega_i}{s_{\bar{w}_{i_c}}}\right)^{\frac{1}{1+\alpha}}, & b_i &= \alpha_0^{-\frac{1}{1+\alpha}}\left(\frac{\omega_i}{s_{\bar{w}_{i_c}}}\right)^{\frac{\alpha}{1+\alpha}}. \end{aligned} \tag{3.48}$$

Total expenditure is given by Y, where the real consumption index is depicted by Q. Normalising the price index, it holds that $PQ = Q = Y$. The domestic demand shifter can be written as $A = Y^{1-\beta}P^{\beta} = Q^{1-\beta}P = Q^{1-\beta}$. For Foreign it holds that $P^*\ Q^* = Y^*$ and thus the demand shifter can be written as $A^* = {Y^*}^{1-\beta}{P^*}^{\beta} = {Q^*}^{1-\beta}P^*$. Assuming that both countries are symmetric, the price index in foreign is also one and thus the demand shifter given by $A^* = {Q^*}^{1-\beta} = A$. Using the relationship between the consumption index and the demand shifter one can rewrite the zero profit condition of the cutoff firm (3.B.37) and solve for Q:

$$f_D = \frac{\Gamma}{\beta\gamma}\Lambda_0^{\frac{1}{\Gamma}}\kappa_r\left(c^{-\frac{\beta(1-\gamma k)}{\delta}}b^{-\beta\gamma}A\theta_{D_c}^{\beta}\right)^{\frac{1}{\Gamma}} \tag{3.B.37}$$

$$A^{\frac{1}{\Gamma}} = \frac{f_D}{\kappa_r}\frac{\beta\gamma}{\Gamma}\Lambda_0^{-\frac{1}{\Gamma}}\left(c^{-\frac{\beta(1-\gamma k)}{\delta}}b^{-\beta\gamma}\theta_{D_c}^{\beta}\right)^{-\frac{1}{\Gamma}}$$

$$Q = A^{\frac{1}{1-\beta}} = \left(\frac{f_D}{\kappa_r}\frac{\beta\gamma}{\Gamma}\right)^{\frac{\Gamma}{1-\beta}}\Lambda_0^{-\frac{1}{1-\beta}}\theta_{D_c}^{-\frac{\beta}{1-\beta}}c^{-\frac{\beta(1-\gamma k)}{(1-\beta)\delta}}b^{\frac{\beta\gamma}{1-\beta}}.$$

Using (3.48) and $b \equiv b_i^{\eta_i}b_j^{\eta_j}$ the consumption index can be rewritten as a function of the expected wage:

$$\begin{aligned} Q = A^{\frac{1}{1-\beta}} &= \left(\frac{f_D}{\kappa_r}\frac{\beta\gamma}{\Gamma}\right)^{\frac{\Gamma}{1-\beta}}\Lambda_0^{-\frac{1}{1-\beta}}\theta_{D_c}^{-\frac{\beta}{1-\beta}}c^{-\frac{\beta(1-\gamma k)}{(1-\beta)\delta}}\alpha_0^{-\frac{1}{1+\alpha}\frac{\beta\gamma}{1-\beta}}\left(\prod_i\left(\frac{\omega_i}{s_{\bar{w}_{i_c}}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}} \\ &= \kappa_o^{\frac{1}{1-\beta}}\theta_{D_c}^{-\frac{\beta}{1-\beta}}c^{-\frac{\beta(1-\gamma k)}{(1-\beta)\delta}}\alpha_0^{-\frac{1}{1+\alpha}\frac{\beta\gamma}{1-\beta}}\left(\prod_i\left(\frac{\omega_i}{s_{\bar{w}_{i_c}}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}}, \end{aligned} \tag{3.49}$$

with $\kappa_o \equiv \left(\frac{f_D}{\kappa_r}\frac{\beta\gamma}{\Gamma}\right)^{\Gamma}\Lambda_0^{-1}$. In equilibrium overall expected wage income by type i workers in a country has to equal the sum of all wages paid by firms in that country to type i

workers. Using (3.B.19) it follows:

$$\begin{aligned}\omega_i L_i &= M\int_{\theta_D}^{\infty} w_i(\theta)h_i(\theta)\ dG_\theta(\theta) = M\int_{\theta_D}^{\infty} \Lambda_{wi}(\theta)\ r(\theta)\ dG_\theta(\theta)\\ &= M\left(\int_{\theta_D}^{\theta_{B_c}} \Lambda_{wi}(\theta)\ r(\theta)|_{I_B=0}\ dG_\theta(\theta) + \int_{\theta_{B_c}}^{\infty} \Lambda_{wi}(\theta)\ r(\theta)|_{I_B=1}\ dG_\theta(\theta)\right). \end{aligned} \tag{3.B.83}$$

The revenue share earned by type i workers in any firm θ which is investing in improving its bargaining power is denoted by $\Lambda_{w1i} \equiv \Lambda_{wi}(\theta)|_{I_B=1}$, while $\Lambda_{w0i} \equiv \Lambda_{wi}(\theta)|_{I_B=0}$ indicates the share in a situation, in which the same firm is not investing in improving its bargaining power. Where the share of revenues belonging to workers of type i is given by (3.17). The share of revenues generated by firms which invest in improving the bargaining power is given by (3.B.80). This allows to rewrite the above condition (3.B.83) as:

$$\begin{aligned}\omega_i L_i &= M\left(\Lambda_{w0i}\int_{\theta_D}^{\theta_{B_c}} r(\theta)\ dG_\theta(\theta) + \Lambda_{w1i}\int_{\theta_{B_c}}^{\infty} r(\theta)\ dG_\theta(\theta)\right)\\ &= M\left(\Lambda_{w0i}\int_{\theta_D}^{\infty} r(\theta)\ dG_\theta(\theta) + (\Lambda_{w1i}-\Lambda_{w0i})\int_{\theta_{B_c}}^{\infty} r(\theta)\ dG_\theta(\theta)\right)\\ &= \Lambda_{w0i} M\int_{\theta_D}^{\infty} r(\theta)\ dG_\theta(\theta) + (\Lambda_{w1i}-\Lambda_{w0i})\, M\frac{\int_{\theta_{B_c}}^{\infty} r(\theta)\ dG_\theta(\theta)}{\int_{\theta_D}^{\infty} r(\theta)\ dG_\theta(\theta)}\int_{\theta_D}^{\infty} r(\theta)\ dG_\theta(\theta)\\ &= \left(\Lambda_{w0i} + (\Lambda_{w1i}-\Lambda_{w0i})\, S_{rB_c}\right) M\int_{\theta_D}^{\infty} r(\theta)\ dG_\theta(\theta)\\ &= \mu_{i_c} M\int_{\theta_D}^{\infty} r(\theta)\ dG_\theta(\theta), \end{aligned} \tag{3.B.84}$$

where:

$$\mu_{i_c} \equiv \Lambda_{w0i} + (\Lambda_{w1i}-\Lambda_{w0i})\, S_{rB_c}. \tag{3.51}$$

The consumption index can be written as $Q = QP = M\int_{\theta_{D_c}}^{\infty} r(\theta)\ dG_\theta(\theta)$. Thus, using (3.B.84) the labour market condition follows as:

$$\omega_i L_i = \mu_{i_c} M\int_{\theta_{D_c}}^{\infty} r(\theta)\ dG_\theta(\theta) = \mu_{i_c} Q. \tag{3.50}$$

Expected wage can be derived using the general equilibrium conditions (3.50) and (3.49):

$$\begin{aligned}\mu_{i_c}^{-1}\omega_i L_i &= \kappa_o^{\frac{1}{1-\beta}}\theta_{D_c}^{-\frac{\beta}{1-\beta}} c^{-\frac{\beta(1-\gamma k)}{(1-\beta)\delta}} \alpha_0^{-\frac{1}{1+\alpha}\frac{\beta\gamma}{1-\beta}} \left(\prod_i \left(\frac{\omega_i}{s_{\bar{w}_{i_c}}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}}\\ \frac{\mu_{i_c}^{-1}\omega_i L_i}{\mu_{j_c}^{-1}\omega_{j_c} L_j} &= \frac{\kappa_o^{\frac{1}{1-\beta}}\theta_{D_c}^{-\frac{\beta}{1-\beta}} c^{-\frac{\beta(1-\gamma k)}{(1-\beta)\delta}} \alpha_0^{-\frac{1}{1+\alpha}\frac{\beta\gamma}{1-\beta}} \left(\prod_i \left(\frac{\omega_i}{s_{\bar{w}_{i_c}}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}}}{\kappa_o^{\frac{1}{1-\beta}}\theta_{D_c}^{-\frac{\beta}{1-\beta}} c^{-\frac{\beta(1-\gamma k)}{(1-\beta)\delta}} \alpha_0^{-\frac{1}{1+\alpha}\frac{\beta\gamma}{1-\beta}} \left(\prod_i \left(\frac{\omega_i}{s_{\bar{w}_{i_c}}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}}}\\ \omega_{j_c} &= \frac{L_i}{L_j}\frac{\mu_{j_c}}{\mu_{i_c}}\omega_i. \end{aligned} \tag{3.53}$$

This allows to rewrite the initial equation and solve for the expected wage as:

$$\begin{aligned}
\mu_{i_c}^{-1}\omega_i L_i &= \kappa_o^{\frac{1}{1-\beta}}\theta_{D_c}^{-\frac{\beta}{1-\beta}}c^{-\frac{\beta(1-\gamma k)}{(1-\beta)\delta}}\alpha_0^{-\frac{1}{1+\alpha}\frac{\beta\gamma}{1-\beta}}\left(\prod_i s_{\bar{w}_{i_c}}^{-\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}}\omega_i^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma\eta_i}{1-\beta}}\left(\frac{L_i}{L_j}\frac{\mu_{j_c}}{\mu_{i_c}}\omega_i\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma\eta_j}{1-\beta}}\\
\omega_i^{1-\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{(1-\beta)}} &= \kappa_o^{\frac{1}{1-\beta}}\theta_{D_c}^{-\frac{\beta}{1-\beta}}c^{-\frac{\beta(1-\gamma k)}{(1-\beta)\delta}}\alpha_0^{-\frac{1}{1+\alpha}\frac{\beta\gamma}{1-\beta}}\left(\prod_i s_{\bar{w}_{i_c}}^{-\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}}\left(\frac{L_i}{L_j}\frac{\mu_{j_c}}{\mu_{i_c}}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma\eta_j}{1-\beta}}\frac{\mu_{i_c}}{L_i}\\
\omega_i^{-\frac{\Delta}{1-\beta}} &= \kappa_o^{\frac{1}{1-\beta}}\theta_{D_c}^{-\frac{\beta}{1-\beta}}c^{-\frac{\beta(1-\gamma k)}{(1-\beta)\delta}}\alpha_0^{-\frac{1}{1+\alpha}\frac{\beta\gamma}{1-\beta}}\left(\prod_i\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\eta_i}\right)^{-\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}}\left(\frac{L_i}{\mu_{i_c}}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}(\eta_i+\eta_j)-1}\\
\omega_i &= \kappa_o^{-\frac{1}{\Delta}}\theta_{D_c}^{\frac{\beta}{\Delta}}c^{\frac{\beta(1-\gamma k)}{\delta\Delta}}\alpha_0^{\frac{1}{1+\alpha}\frac{\beta\gamma}{\Delta}}\left(\prod_i\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}}\left(\frac{L_i}{\mu_{i_c}}\right)^{\frac{1-\beta-\frac{\alpha}{1+\alpha}\beta\gamma}{\Delta}}\\
\omega_i &= \kappa_o^{-\frac{1}{\Delta}}\theta_{D_c}^{\frac{\beta}{\Delta}}c^{\frac{\beta(1-\gamma k)}{\delta\Delta}}\alpha_0^{\frac{1}{1+\alpha}\frac{\beta\gamma}{\Delta}}\left(\prod_i\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}}\left(\frac{L_i}{\mu_{i_c}}\right)^{-1}. \qquad (3.B.85)
\end{aligned}$$

The help parameter $\Delta \equiv -\left(1-\beta-\frac{\alpha}{1+\alpha}\beta\gamma\right) > 0$ needs to be positive in order to ensure a stable equilibrium. Thus, the elasticity of substitution between varieties has to be sufficiently high (high β, but less than one). With $\frac{\alpha}{1+\alpha} < 1$ it follows that $1-\beta-\beta\gamma < 0$. Using the result from (3.49) and (3.B.85) the equilibrium consumption index is given by:

$$\begin{aligned}
Q &= \kappa_o^{\frac{1}{1-\beta}}\theta_{D_c}^{-\frac{\beta}{1-\beta}}c^{-\frac{\beta(1-\gamma k)}{(1-\beta)\delta}}\alpha_0^{-\frac{1}{1+\alpha}\frac{\beta\gamma}{1-\beta}}\left(\prod_i\left(\frac{\omega_i}{s_{\bar{w}_{i_c}}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}}\\
&= \kappa_o^{\frac{1}{1-\beta}}\theta_{D_c}^{-\frac{\beta}{1-\beta}}c^{-\frac{\beta(1-\gamma k)}{(1-\beta)\delta}}\alpha_0^{-\frac{1}{1+\alpha}\frac{\beta\gamma}{1-\beta}}\left(\kappa_o^{-\frac{1}{\Delta}}\theta_{D_c}^{\frac{\beta}{\Delta}}c^{\frac{\beta(1-\gamma k)}{\delta\Delta}}\alpha_0^{\frac{1}{1+\alpha}\frac{\beta\gamma}{\Delta}}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}(\eta_i+\eta_j)}\\
&\cdot\left(\left(\prod_i\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}(\eta_i+\eta_j)}\prod_i\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{-\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{1-\beta}}\\
&= \kappa_o^{-\frac{1}{\Delta}}\theta_D^{\frac{\beta}{\Delta}}c^{\frac{\beta(1-\gamma k)}{\Delta\delta}}\alpha_0^{\frac{1}{1+\alpha}\frac{\beta\gamma}{\Delta}}\left(\prod_i\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}}, \qquad (3.B.86)
\end{aligned}$$

where $\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta} - 1 = \frac{\frac{\alpha}{1+\alpha}\beta\gamma-\left(-\left(1-\beta-\frac{\alpha}{1+\alpha}\beta\gamma\right)\right)}{\Delta} = \frac{1-\beta}{\Delta}$ is used. Using the result from (3.B.85) and (3.48) the equilibrium labour market tightness is given by:

$$\begin{aligned}
x_i &= \left(\frac{\alpha_0\ \omega_i}{s_{\bar{w}_{i_c}}}\right)^{\frac{1}{1+\alpha}} = \left(\alpha_0\kappa_o^{-\frac{1}{\Delta}}\theta_{D_c}^{\frac{\beta}{\Delta}}c^{\frac{\beta(1-\gamma k)}{\delta\Delta}}\alpha_0^{\frac{1}{1+\alpha}\frac{\beta\gamma}{\Delta}}\left(\prod_i\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}}\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{-1}\right)^{\frac{1}{1+\alpha}}\\
&= \kappa_o^{-\frac{1}{(1+\alpha)\Delta}}\theta_D^{\frac{\beta}{(1+\alpha)\Delta}}c^{\frac{\beta(1-\gamma k)}{(1+\alpha)\Delta\delta}}\alpha_0^{\frac{\beta+\beta\gamma-1}{(1+\alpha)\Delta}}\left(\prod_i\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{(1+\alpha)\Delta}}\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{-\frac{1}{1+\alpha}} \qquad (3.B.87)\\
&= \kappa_o^{-\frac{1}{(1+\alpha)\Delta}}\theta_D^{\frac{\beta}{(1+\alpha)\Delta}}c^{\frac{\beta(1-\gamma k)}{(1+\alpha)\Delta\delta}}\alpha_0^{\frac{\beta+\beta\gamma-1}{(1+\alpha)\Delta}}\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\frac{1-\beta-\frac{\alpha}{1+\alpha}\beta\gamma\eta_i}{(1+\alpha)\Delta}}\left(\frac{s_{\bar{w}_{j_c}}L_j}{\mu_{j_c}}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma\eta_j}{(1+\alpha)\Delta}},
\end{aligned}$$

where $\frac{1}{1+\alpha}\left(\frac{1}{1+\alpha}\frac{\beta\gamma}{\Delta}+1\right)=\frac{\beta+\beta\gamma-1}{(1+\alpha)\Delta}$ and $\frac{1}{1+\alpha}\left(\frac{\alpha}{1+\alpha}\frac{\beta\gamma\eta_i}{\Delta}-1\right)=\frac{1-\beta-\frac{\alpha}{1+\alpha}\beta\gamma\eta_j}{(1+\alpha)\Delta}$ is applied. Similarly search cost are given, using (3.B.85) and (3.48) by:

$$\begin{aligned}
b_i &= \alpha_0^{-\frac{1}{1+\alpha}}\left(\frac{\omega_i}{s_{\bar{w}_{i_c}}}\right)^{\frac{\alpha}{1+\alpha}} \\
&= \alpha_0^{-\frac{1}{1+\alpha}}\left(\kappa_o^{-\frac{1}{\Delta}}\theta_{D_c}^{\frac{\beta}{\Delta}}c^{\frac{\beta(1-\gamma k)}{\delta\Delta}}\alpha_0^{\frac{1}{1+\alpha}\frac{\beta\gamma}{\Delta}}\left(\prod_i\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\gamma}{\Delta}}\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{-1}\right)^{\frac{\alpha}{1+\alpha}} \\
&= \kappa_o^{-\frac{\alpha}{(1+\alpha)\Delta}}\theta_{D_c}^{\frac{\alpha\beta}{(1+\alpha)\Delta}}c^{\frac{\alpha\beta(1-\gamma k)}{(1+\alpha)\Delta\delta}}\alpha_0^{\frac{1-\beta}{(1-\alpha)\Delta}}\left(\prod_i\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\alpha\beta\gamma}{(1+\alpha)\Delta}}\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{-\frac{\alpha}{1+\alpha}} \qquad (3.B.88)\\
&= \kappa_o^{-\frac{\alpha}{(1+\alpha)\Delta}}\theta_{D_c}^{\frac{\alpha\beta}{(1+\alpha)\Delta}}c^{\frac{\alpha\beta(1-\gamma k)}{(1+\alpha)\Delta\delta}}\alpha_0^{\frac{1-\beta}{(1-\alpha)\Delta}}\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\frac{\alpha\left(1-\beta-\frac{\alpha}{1+\alpha}\beta\gamma\eta_i\right)}{(1+\alpha)\Delta}}\left(\frac{s_{\bar{w}_{j_c}}L_j}{\mu_{j_c}}\right)^{\frac{\alpha}{1+\alpha}\frac{\alpha\beta\gamma\eta_j}{(1+\alpha)\Delta}},
\end{aligned}$$

where $\frac{1}{1+\alpha}\left(\frac{\beta\gamma}{\Delta}\frac{\alpha}{1+\alpha}-1\right)=\frac{1-\beta}{(1-\alpha)\Delta}$ and $\frac{\alpha}{1+\alpha}\left(\frac{\alpha}{1+\alpha}\frac{\beta\gamma\eta_i}{\Delta}-1\right)=\frac{\alpha\left(1-\beta-\frac{\alpha}{1+\alpha}\beta\gamma\eta_j\right)}{(1+\alpha)\Delta}$ is applied.

The number of firms can be determined using the equilibrium condition, equating overall expected wage income of each type of work force with the sum of wages paid by firms to this type of workers:

$$\begin{aligned}
\omega_i L_i &= \mu_{i_c} M_c \int_{\theta_{D_c}}^{\infty} r(\theta)\ dG_\theta(\theta) \qquad (3.50)\\
M &= \frac{\omega_i L_i}{\mu_{i_c}}\left(\int_{\theta_{D_c}}^{\infty} r(\theta)\ dG_\theta(\theta)\right)^{-1}.
\end{aligned}$$

Using the result for the average revenues of firms (3.B.70) in combination with average revenues in case of no exporting or improving the bargaining power (3.B.69) and the scaler for exporting and improving the bargaining power (3.B.71), as well as the equation for expected wages (3.B.85), allows to rewrite the number of firms as:

$$\begin{aligned}
M &= \kappa_o^{-\frac{1}{\Delta}}\theta_{D_c}^{\frac{\beta}{\Delta}}c^{\frac{\beta(1-\gamma k)}{\Delta\delta}}\alpha_0^{\frac{1}{1+\alpha}\frac{\beta\gamma}{\Delta}}\left(\prod_i\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\Gamma}{\Delta}}\left(s_{r_c}\bar{r}\Big|_{\substack{I_X=0\\I_B=0}}\right)^{-1} \\
&= \kappa_o^{-\frac{1}{\Delta}}\theta_{D_c}^{\frac{\beta}{\Delta}}c^{\frac{\beta(1-\gamma k)}{\Delta\delta}}\alpha_0^{\frac{1}{1+\alpha}\frac{\beta\gamma}{\Delta}}\left(\prod_i\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\Gamma}{\Delta}}\frac{\Lambda_0}{\theta_{min}^z f_D}\frac{z\Gamma-\beta}{z}\frac{\theta_{D_c}^z}{s_{r_c}} \\
&= \frac{\Lambda_0}{\theta_{min}^z f_D}\frac{z\Gamma-\beta}{z}\kappa_o^{-\frac{1}{\Delta}}c^{\frac{\beta(1-\gamma k)}{\Delta\delta}}\alpha_0^{\frac{1}{1+\alpha}\frac{\beta\gamma}{\Delta}}\left(\prod_i\left(\frac{s_{\bar{w}_{i_c}}L_i}{\mu_{i_c}}\right)^{\eta_i}\right)^{\frac{\alpha}{1+\alpha}\frac{\beta\Gamma}{\Delta}}\frac{\theta_{D_c}^{z+\frac{\beta}{\Delta}}}{s_{r_c}}. \qquad (3.B.89)
\end{aligned}$$

The general equilibrium variables depicted in equation (3.52) are given by (3.B.85 - 3.B.89).

3.C Nomenclature

α search cost elasticity with respect to the labour market tightness x ($\alpha > 0$)

α_0 scaling parameter of the search technology ($0 < \alpha_0 < 1$)

$\bar{a}_i(\theta)$ average ability of workers of type-i employed by firm θ

$\bar{h}_i$ average number of workers of type-i employed by a firm

$\bar{h}_i^A$ average number of workers of type-i employed by a firm, if firms can neither export nor improve their bargaining power ($\bar{h}_i^A = \bar{h}_i^A(\theta_{D_c}, b_i)$)

$\bar{n}_i$ average number of workers of type-i matched to a firm

$\bar{n}_i^A$ average number of workers of type-i matched to a firm, if firms can neither export nor improve their bargaining power ($\bar{n}_i^A = \bar{n}_i^A(\theta_{D_c}, b_i)$)

$\bar{r}$ average revenues of all active firms

$\bar{r}^A$ average revenues of all active firms, if firms can neither export nor improve their bargaining power ($\bar{r}^A = \bar{r}^A(\theta_{D_c})$)

$\bar{w}_i$ expected wage income of a type-i worker given a match with a firm

β controls the elasticity of substitution where $\frac{1}{1-\beta}$ is the elasticity of substitution ($0 < \beta < 1$)

Δ help parameter ($\Delta \equiv -\left(1 - \beta - \frac{\alpha}{1+\alpha}\beta\gamma\right) > 0$)

δ screening cost parameter determining the degree to which a higher ability threshold implies higher screening costs ($\delta > 0$)

η_0 production scalar ($\eta_0 \equiv (\sum_i \eta_i^{\eta_i})^{-(\gamma+(1-\gamma k)/\delta)}$)

η_i Cobb-Douglas weight of occupation i ($0 < \eta_i < 1,\ \sum_i \eta_i = 1$)

Γ help parameter ($1 > \Gamma \equiv 1 - \beta\gamma - \beta/\delta(1-\gamma k) > 0$)

γ measure of the decreasing returns to labour ($0 < \gamma < 1$)

ι index used to indicate the different cutoff productivities $\iota \in \{D, X, B, XB, BX, DXB\}$

ι_c indicates which of the cutoffs productivities θ_ι are relevant for the sorting pattern $c = \{1, 2, 3\}$

κ_h help parameter ($1 > \kappa_h \equiv 1 - k/\delta > 0$)

κ_o help parameter ($\kappa_o \equiv \left(\frac{f_D}{\kappa_r}\frac{\beta\gamma}{\Gamma}\right)^\Gamma \Lambda_0^{-1} > 0$)

κ_r help parameter ($\kappa_r \equiv \phi_1\phi_2^{\beta(1-\gamma k)}$)

κ_y help parameter ($\kappa_y \equiv \frac{k}{k-1} a_{\min}^{\gamma k} \equiv \frac{k}{k-1} a_{\min}^{\gamma k}$)

κ_θ help parameter ($\kappa_\theta \equiv \left(\frac{\beta}{(z\Gamma-\beta) f_E}\right)^{\frac{1}{z}} \theta_{min}$)

λ a firm's bargaining power with respect to type-1 workers if it decides to improve the bargaining power ($\lambda > 1$)

λ'' level of bargaining power improvement above which sorting pattern 3 is relevant

λ' level of bargaining power improvement below which sorting pattern 1 is relevant

Λ_0 share of revenues belonging to the firm after paying wages when $I_B = 0$ ($\Lambda_0 < 1$)

Λ_1 share of revenues belonging to the firm after paying wages when $I_B = 1$ ($\Lambda_1 < 1$)

$\lambda_i(\theta)$ bargaining power of firm θ with respect to type-i workers ($\lambda_i(\theta) \geq 1$)

$\Lambda_{I_B}(\theta)$ share of revenues belonging to the firm after paying wages ($\Lambda_{I_B}(\theta) < 1$)

$\Lambda_{wi}(\theta)$ share of revenues belonging to type-i workers employed by firm θ ($\Lambda_{wi}(\theta) < 1$)

μ_{i_c} share of overall revenues belonging to workers of type-i ($\mu_{i_c} < 1$)

ω_i expected wage income of a type-i worker

ϕ_1 help parameter ($\phi_1 \equiv \left(\frac{\beta\gamma\kappa_y^\beta}{1+\beta\gamma}\right)^{\frac{1}{\Gamma}} > 0$)

ϕ_2 help parameter ($\phi_2 \equiv \left(\frac{1-\gamma k}{\gamma}\right)^{\frac{1}{\delta\Gamma}} > 0$)

$\pi(\theta)$ profits of firm θ

π_ι profits in case a firm only sells domestically (π_D), sells domestically and improves its bargaining power but does not export (π_B), sells domestically and exports but does not improve its bargaining power (π_X), sells domestically, exports and improves its bargaining power ($\pi_{XB}, \pi_{BX}, \pi_{DXB}$)

ρ_{B_c} share of firms improving their bargaining power ($0 \leq \rho_{B_c} \leq 1$)

ρ_{X_c} share of firms exporting ($0 \leq \rho_{X_c} \leq 1$)

σ_i hiring rate of type-i workers ($0 < \sigma_i < 1$)

σ_i^A hiring rate of type-i workers in a situation, in which neither exporting nor improving the bargaining power is possible ($0 < \sigma_i^A < 1$)

τ iceberg type trade costs ($\tau > 1$)

θ variety index and a firm's productivity

θ_D domestic cutoff productivity ($\theta_D > \theta_{min}$)

θ_ι cutoff productivity ι

$\theta_{\min}$ lower bound of the pareto productivity distribution ($\theta_{\min} > 0$)

θ_{B_c} relevant cutoff productivity above which firms improve their bargaining power, if sorting pattern c applies

θ_{X_c} relevant cutoff productivity above which firms export, if sorting pattern c applies

$\Upsilon_B(\theta)$ bargaining improvement term ($\Upsilon_B(\theta) \equiv 1 + I_B(\theta)\left(\frac{\lambda}{\lambda-1}\frac{1+\beta\gamma}{\beta\gamma\eta_1} - 1\right)^{-1} \geq 1.$)

$\Upsilon_X(\theta)$ market access term ($\Upsilon_X(\theta) \equiv 1 + I_X(\theta)\left(\frac{A^*}{A}\right)^{\frac{1}{1-\beta}} \tau^{-\frac{\beta}{1-\beta}} \geq 1$)

Υ_ι relevant market access and bargaining improvement term for the respective case ι

ϑ, ν indices for varieties

A demand shifter in a country ($A \equiv Y^{1-\beta}P^{\beta}$)

a ability of a worker

$a_{\min}$ minimum ability in the pareto ability distribution of a worker ($a_{\min} > 0$)

$a_{ci}(\theta)$ screening threshold chosen by a firm θ for the workers producing task i

b weighted combined search cost measure ($b \equiv (\prod_i b_i^{\eta_i})$)

b_i search cost per worker matched of type i

c parameter scaling the screening cost ($c > 0$)

f_B fixed cost of improving the bargaining power ($f_B > 0$)

f_D fixed costs of domestic production ($f_D > 0$)

f_E fixed costs of entry ($f_E > 0$)

f_X fixed costs of export ($f_X > 0$)

f_ι relevant fixed costs when determining cutoff productivity θ_ι

$G_a(a)$ pareto ability distribution function ($G_a(a) = 1 - \left(\frac{a_{\min}}{a}\right)^k$)

$G_\theta(\theta)$ pareto productivity distribution function ($G_\theta(\theta) = 1 - \left(\frac{\theta_{\min}}{\theta}\right)^z$)

H_i overall number of type-i workers hired by firms in a country

$h_i(\theta)$ employment of firm θ for the production of task i

i, j index for the worker type and the respective occupation in the production technology ($i = \{1, 2\}$, $i \neq j$)

$I_B(\theta)$ index indicating whether a firm invests in improving is bargaining parameter $I_B = 1$ or not $I_B = 0$.

$I_X(\theta)$ indicator variable indicating whether a firm decides to export or not $I_X(\theta) = \{1, 0\}$

k pareto shape parameter of the ability distribution ($k > 1$)

L_i potential work force in a country of type i

M set of varieties within a country/ number of firms

N_i overall number of type-i workers matched to firms in a country

$n_i(\theta)$ number of workers screened by a firm θ for the production of task i

P price index

$p(\vartheta)$ price of variety ϑ

Q real consumption index

$q(\vartheta)$ consumption of variety ϑ

$r(\vartheta)$ revenues of a firm producing variety ϑ

$r_D(\theta)$ revenues of a firm with productivity θ generated on the domestic market

$r_X(\theta)$ revenues of a firm with productivity θ generated on the export market

$s_{\bar{w}_{ic}}$ measure for the decline in the expected wage given a match ($\bar{w}_i$), because some of the matched workers face lower expected wages because of bargaining power improvement by the firms ($s_{\bar{w}_{1c}} < 1$, $s_{\bar{w}_{2c}} = 1$)

s_{σ_c} measure for the decline in the hiring rate, because firms can export and improve their bargaining power ($0 < s_{\sigma_c = } s_{\sigma_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c}) < 1$)

s_{h_c} measures the increase in the average number of workers employed, because firms can export and improve their bargaining power ($s_{h_c} = s_{h_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c}) \geq 1$)

S_{hB_c} share of overall workers employed in a country, who are employed by firms that improve their bargaining power ($0 < S_{hB_c} < 1$)

s_{n_c} measures the increase in the average number of workers matched, because firms can export and improve their bargaining power ($s_{n_c} = s_{n_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c}) \geq 1$)

S_{nB_c} share of overall workers matched in a country, who are matched to firms that improve their bargaining power ($0 < S_{nB_c} < 1$)

s_{r_c} factor for the increase in the average revenues, because firms can export and improve their bargaining power ($s_{r_c} = s_{r_c}(\Upsilon_X, \Upsilon_B, \rho_{X_c}, \rho_{B_c}) \geq 1$)

S_{rB_c} share of overall revenues in a country, generated by firms that improve their bargaining power ($0 < S_{rB_c} < 1$)

u_i unemployment rate for type-i workers ($0 < u_i < 1$)

$w_i(\theta)$ wage paid to workers of type-i, employed by firm θ

x_i labour market tightness for type-i workers ($0 < x_i < 1$)

Y total expenditure

$y(\theta)$ output of a firm with productivity θ/ production technology of a variety θ

$y_D(\theta)$ output of a firm with productivity θ produced for the domestic market

$y_i(\theta)$ quantity produced by occupation i, used in the production of firm θ

$y_X(\theta)$ output of a firm with productivity θ produced for the export market

z shape parameter of the pareto productivity distribution ($z > 1$)

Chapter 4

Regional Implications of National Tax Policy*

Abstract: *In this paper we study the region and sector specific effects of changes in national corporate tax policies. We are the first to do so in a new, general equilibrium, spatial quantitative model that features heterogeneous responses to national policies due to region and sector specific production structures, including varying usage of deductible capital asset types, and spillovers through a full set of input-output relations and mobility. Calibrating the model for 13 sectors across 1306 European NUTS3 regions we derive three key results: Firstly, real consumption responses to a one percentage point increase in national corporate taxes are vastly heterogeneous, ranging from -0.08% to 0.06%. Secondly, moving to an EU wide common corporate tax at the current mean increases welfare by 0.005%. Finally, adopting a cash-flow taxation with full immediate deductibility of all capital assets decreases average welfare by -0.007%.*

*This chapter is based on joint work with Peter Egger, Oliver Krebs, Valeria Merlo and Georg Wamser.

4.1 Introduction

Most policy instruments are set uniformly at the national level, regardless of whether countries have a unitary or federal system of government. Examples include tariffs, taxes on income and sales or other non-tax provisions or regulations. While the extent to which policies are implemented at a more regional level depends on the degree of decentralisation of countries, policy rules almost always condition on the specific characteristics of economic units to be regulated or taxed. For this reason, homogeneous policies may cause very heterogeneous responses at the level of workers and firms, which raises questions about distributional consequences of ex-ante non-discriminatory policy actions.

This paper contributes to the literature by analysing the regional and sectoral implications of national tax policies. We provide structural estimates and new quantifications on the effects of national corporate tax policy on regional economic outcomes. The consequences of profit taxes seem to be particularly interesting in this context as most countries levy uniform corporate tax rates. At the same time, however, the respective tax base of firms depends on firm specific determinants such as type, extent, financing, or the timing of investment. Differences in endowments and regional specialization patterns suggest that the latter determinants strongly vary across regions. For example, asset-specific deduction rules govern to what extent changes in national tax policy affect effective tax rates across sectors and regions.

We develop a theoretical model which acknowledges regional and sectoral differences in economic activity, as well as their interdependence through trade and mobility. In particular, workers are allowed to move across locations depending on real incomes and individual specific local amenities (Redding, 2016; Tabuchi and Thisse, 2002). Firms in each region and sector use varying shares of labour, land, intermediates and capital goods to produce output with interregional linkages emerging from the trade of final and intermediate goods, subject to trade cost, as in the seminal contribution by Caliendo and Parro (2015).

We finally add two tax instruments. Firstly, a profit tax on operating profits, which is set at the national (country) level and, secondly, national deductibility rates of different capital asset types. National shocks to both translate into heterogeneous region and sector specific shocks through differences in location specific fundamentals. Specifically, the model acknowledges that regional production differs in sectoral composition as well as input and asset structure. This is important when examining the effects of national tax policy: our approach results in local, region-specific responses to national tax policy, as well as tax policy spillovers across regions implied by interregional trade and mobility.

The most important theoretical findings can be summarised as follows. First, in partial equilibrium, a higher country-wide profit tax leads to redistribution of income to regions with a high ratio of labour to land and capital. Second, in general equilibrium, the tax redistribution makes these regions more attractive and implies a population inflow as households relocate. Locations more closely linked to regions with favourable conditions also gain as there is trade between regions.

The empirical part of the paper calibrates the model for 13 sectors and 1306 European NUTS3 regions using interregional shipment data from the ETISplus project as well as structural estimations to recover model parameters.[1] This allows to derive quantifications of tax policy effects, including general equilibrium repercussions, at an unprecedented depth. Very few previous contributions to the effects of corporate taxes have accounted for geographical heterogeneity. An exception is the paper by Suárez Serrato and Zidar (2016) which studies tax incidence in a one sector monopolistic competition spatial equilibrium model with imperfectly mobile firms and workers.

In line with our argument we find a substantial level of heterogeneity in local responses to national tax policy shocks. Across the EU the regional (NUTS3) real consumption response to a one percentage point increase of the respective country's national tax rate ranges from -0.08% to 0.06%. Geographically, the most adverse effects are felt in regions that are the nations' manufacturing centres, such as the north of Italy, the north of Spain, German car manufacturing regions or the areas around Rotterdam and Amsterdam. Less productive regions benefit from higher redistribution of national tax income.

We also simulate two prominently discussed tax policies. Firstly, the adoption of a common EU corporate tax and capital asset deduction scheme and, secondly, the introduction of a cash-flow taxation in which capital assets are fully deductible.

Regarding the former we find that adopting a common EU tax policy at the current mean leads to an average welfare gain of 0.0054% in spatial general equilibrium. In our class of quantitative spatial models the existence of a common expected utility across regions is driven by amenity differences and thus hides a significant amount of underlying heterogeneity in real consumption possibilities.[2] Specifically, we find that real consumption effects of the common tax policy range from -1.37% to 1.07%. Surprisingly, both the strongest winners and losers can be found among eastern European regions.

With respect to the introduction of a cash-flow taxation our simulations show a welfare reduction of -0.067% with a similar range of regional real consumption effects from -1.33% to 1.98%. In this case, the most negatively affected regions would be found in eastern Europe and southern Italy. The strongest winners would again be the production centres in northern Italy and Spain, the car manufacturing regions in Germany as well as the areas around the major ports of Rotterdam and Amsterdam.

Overall, the findings we provide are at an unprecedented level of sectoral and regional

[1]See Appendix 4.4.1 for a description of the data sources.

[2]See Redding and Rossi-Hansberg (2017) for a comprehensive survey of this literature.

disaggregation and serve as vital information for policy makers.

The remainder of the paper is structured as follows. Section 4.3 develops our underlying theoretical model and derives its equilibrium in changes. Section 4.4 discusses the various employed data sets as well as our calibration strategy and structural estimation of fundamental model parameters. Section 4.5 presents and discusses our simulations' results. Section 4.6 concludes and discusses what future work should improve upon. The Appendix 4.A covers additional illustrations and a description of the data used. Further derivation and possible extensions of the theoretical framework can be found in the Supplementary Appendix 4.B and the Nomenclature 4.C lists all variables and their definitions.

4.2 Literature Review

Our contribution relates to two strands of literature. Firstly, the literature on heterogeneous investment responses to taxation in public economics, which focuses for the most part on identifying the channels which lead to heterogeneous tax responses by firms.

The second strand is the literature in quantitative regional economics studying policy effects in general equilibrium models with multiple regions and mobile factors/households. Only few papers so far analyse the effects of taxation in a quantitative spatial framework. To our knowledge we are the first to incorporate corporate taxation and deductibility of different asset types in a quantitative regional economy framework, thus allowing for regional differences and regional spillover effects.

4.2.1 Corporate Taxes and Firm-Level Investment

Understanding the consequences of corporate taxes for investment and other economic outcomes is central to the design of optimal policy measures. An excellent summary on the effects of taxes in the neoclassical investment model is provided by Auerbach (2002). Empirical evidence suggests that on average, firms' investments are quite responsive to tax incentives, but that there is large heterogeneity in tax responses. Egger et al. (2014) show that while most firms react significantly to tax incentives, investment by the largest firms is not responsive to taxes at all. Differences in firms' asset and financing structures lead to differences in effective tax rates and hence to heterogeneous responses to statutory tax rate changes.[3] Zwick and Mahon (2017) find heterogeneity in investment responses to depreciation rules and show that particularly small firms respond to taxes.

While many empirical contributions suggest heterogeneity in the tax responsiveness of firms, they all provide point estimates on the effect of taxes on firms' investments and are silent about the regional implications of such heterogeneity.

[3] Devereux and Griffith (1998, 2003) develop a widely used framework to compute effective tax levels.

4.2.2 Taxes in Quantitative Spatial Models

Allowing for regional implications of tax policy this paper builds on a branch of literature relying on spatial quantitative trade models that connect theory with data to quantify policy effects in general equilibrium. An extensive survey of this literature is provided by Redding and Rossi-Hansberg (2017). Our paper builds on the seminal work by Caliendo and Parro (2015), who develop a multisector framework for quantitative models, and Redding (2016) who introduces (imperfect) worker mobility. In addition, we add a set of capital assets to the production technology and governments collecting corporate taxes using a deduction scheme and distributing tax revenues across individuals.

A few papers introduce government spending and transfer policy in quantitative spatial models. Henkel et al. (2018) calibrate a multi-region general equilibrium model for Germany to study how fiscal equalization schemes shift tax revenues across regions. Fajgelbaum and Gaubert (2018) develop a spatial quantitative trade model to determine the spatial transfers necessary to reach an efficient allocation of heterogeneous workers across regions and calibrate their model for the US economy. Suárez Serrato and Wingender (2014) show that local economic activity responds to public spending. They show that when workers value publicly-provided goods, a change in government spending at the local level will affect equilibrium wages through shifts in labour demand and supply. Using data for the US they show that workers value government services as amenities. While all those papers have a spatial dimension and allow for different regions, they all consider only a singular country and focus on public spending.

Only few papers consider taxation in a quantitative spatial framework. Fajgelbaum et al. (2019) study state taxes (sales taxes) in a spatial general equilibrium framework, which features fixed (land and structures) and mobile factors (workers and firms). They determine how worker and firm location respond to changes in state taxes and find welfare gains from state tax harmonization using US data. Brülhart et al. (2019) and Eeckhout and Guner (2017) consider income taxation in a spatial quantitative framework. While the latter uses US data to determine optimal income taxation depending on city size, Brülhart et al. (2019) consider the incidence of local income taxes. Using data for Switzerland they uncover heterogeneous preferences towards public services. Suárez Serrato and Zidar (2016) study the incidence of state corporate taxes in the US in a one sector monopolistic competition spatial equilibrium model with imperfectly mobile firms and workers. They do not differentiate between asset types with different deduction rules.

Our contribution is to identify heterogeneous effects of national tax policies due to spatial and sectoral heterogeneity. We also consider the spillovers not only across regions within a country but also across countries.

4.3 Model Framework

4.3.1 Endowments

In the calibration of this paper, we think of the European Union (EU) as a closed economy of N countries and R Nomenclature des Unités Territoriales Statistiques 3-digit (NUTS3) regions. We index the regions by r and, where necessary, countries by n. It will turn out useful to further introduce $\Re_n$ to denote the set of regions in country n.

The world is endowed with a capital stock K and an aggregate mass of worker-consumers $L = \sum_{r=1}^{R} L_r$, with each individual inelastically supplying one unit of homogeneous labour. Workers are mobile across regions (and countries) subject to individual region-specific amenity draws as in Tabuchi and Thisse (2002) and Redding (2016). The number L_r of worker-consumers settling in region r is thus endogenous in equilibrium. Similarly, mobile capital can be transformed into regional stocks K_{rk} of one of $\mathfrak{K}$ different capital asset types indexed by $k \in \mathfrak{K}$ and used in goods production. For each individual unit of capital, however, this transformation process, described in detail below, entails a loss drawn from a region and capital asset type specific random distribution. Moreover, each region is also endowed with a fixed (quality adjusted) amount of land, $H_r > 0$ used in production.

4.3.2 Preferences and Residential Choice

Preferences. Conditional on chosing location r all consumers have equal income, face equal prices and have the same consumption preferences. Thus, they will consume the same amounts C_{rj} of each of the final outputs from sectors $j \in \{1, ..., J\}$, combining them into a Cobb-Douglas aggregate C_r according to:

$$C_r = \prod_{j=1}^{J} C_{rj}^{\alpha_{rj}}, \quad \text{where} \quad \sum_{j=1}^{J} \alpha_{rj} = 1 \tag{4.1}$$

and α_{rj} denote sectoral expenditure shares. Apart from consumption, individuals indexed by Ω derive utility from a stochastic, individual and region specific amenity draw $a_r(\Omega)$. The latter is isomorphic to a stochastic, real residence cost of individual Ω measured in units of utility. With these definitions, we may write the utility function of individual Ω in region r as:

$$u_r(\Omega) = a_r(\Omega)\, C_r. \tag{4.2}$$

In expectation, the latter is the same for all worker-consumers in r. Following Redding (2016), we assume that the individual and region specific taste shock $a_r(\Omega)$ is identically and independently distributed Fréchet with the cumulative distribution function (CDF) given by:

$$\Pr\left[a_r(\Omega) \leq a\right] = e^{-A_r a^{-1/\varepsilon}}, \tag{4.3}$$

where the scale parameter A_r sets the average amenity level for region r across all consumers and the shape parameter $\varepsilon < 1$ measures the taste heterogeneity across individuals.

Mobility. Individuals are mobile across regions and will locate in the region yielding the highest utility. Given the consumption levels C_r and the properties of the Fréchet distribution the share of individuals who prefer region r over all other regions can be expressed as:

$$\Pr\left[u_r(\Omega) \geq \max_{s \neq r} u_s(\Omega)\right] = \frac{A_r C_r^{\frac{1}{\varepsilon}}}{\sum_{s \in R} A_s C_s^{\frac{1}{\varepsilon}}} = \frac{L_r}{L}, \tag{4.4}$$

where the last equality follows from the fact that with an infinitely divisible mass of workers the probability to obtain the highest utility in region r is also the share of workers for which this will be true in equilibrium. Moreover, the average or ex-ante expected utility of workers is the same conditioning on the chosen residence as well as for all workers as a whole and can be derived as:

$$\mathbb{E}(u(\Omega)) = \left(\sum_{r=1}^{R} A_r C_r^{\frac{1}{\varepsilon}}\right)^{\varepsilon} \Gamma(1-\varepsilon) \ , \tag{4.5}$$

where $\Gamma(\cdot)$ denotes the Gamma function.

4.3.3 Capital

In economic models, capital is often considered to be immobile (in the short-run) or perfectly mobile (in the long-run). In reality, of course, certain assets (e.g. cash, software) are regionally much more mobile than others (e.g. machinery, buildings). Moreover, even in the long-run path dependencies can imply that capital can not be costlessly transferred between regions and asset types, i.e. independent of what an ex-ante optimal capital allocation would look like it might be perpetually cheaper to renovate an existing traditional production plant over rebuilding in a different location or investing in robotiziation (machinery). For this reason, while we do consider scenarios with immobile and perfectly mobile capital below, our preferred model specification features imperfectly mobile heterogeneous capital.

Of course, we can not model the specific bilateral region-asset-type to region-asset-type transfer costs due to the enormous data requirements this would entail at the level of disaggregation we consider. Therefore, we instead assume that the world capital endowment of mass K has to be transformed into regional stocks of the $\mathfrak{K}$ different capital asset types before it can be used in production and that this transformation process entails a cost that is heterogeneous across (infinitesimal) units of capital. Specifically, if unit i of the world capital stock is employed as capital asset type k in region r the respective regional stock K_{rk} increases by $\delta_{rk}(i)$, where the latter is drawn from a random distribution with CDF:[4]

$$\Pr\left[\delta_{rk}(i) \leq \delta\right] = e^{-\bar{\delta}_{rk}\delta^{-\frac{1}{\varepsilon_\delta}}}.$$

In equivalence to the case of labour mobility, $\bar{\delta}_{rk}$ and ε_δ are the scale and shape parameter respectively. Given ι_{rk}, the equilibrium compensation per unit of capital asset stock K_{rk}, the distribution of potential per unit compensation of world capital K if invested in region r as asset type k inherits the Fréchet distribution. The CDF becomes:

$$\mathbb{G}_{\iota,rk}(\iota) = \Pr\left[\iota_{rk}(i) \leq \iota\right] = \Pr\left[\delta_{rk}(i)\,\iota_{rk} \leq \iota\right] = e^{-\bar{\delta}_{rk}\left(\frac{\iota}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}},$$

with $\iota_{rk}(i)$ denoting the compensation of capital unit i if invested as asset type k in region r. We assume that the world capital endowment is allocated to regions and asset types based on where it obtains the highest compensation. Therefore, in analogy to the case of labour mobility, the average or ex-ante expected compensation per unit of base capital can be derived as $\left(\sum_r \sum_k \bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}\right)^{\varepsilon_\delta}\Gamma(1-\varepsilon_\delta)$ and the equilibrium capital asset stock K_{rk} becomes:

$$K_{rk} = K\frac{\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}}{\sum_r \sum_k \bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}}\frac{\left(\sum_r \sum_k \bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}\right)^{\varepsilon_\delta}\Gamma(1-\varepsilon_\delta)}{\iota_{rk}}. \tag{4.6}$$

The first fraction represents the probability of a particular unit of base capital obtaining the highest possible compensation in region r and asset class k which is - by the law of large numbers - also equal to the share of base capital for which this will be true in equilibrium. The second fraction represents the losses in converting base capital to the specific capital asset type k in region r. In particular, the higher the compensation offered to units invested in r, k relative to the average world wide compensation, the more capital flows there, leading to increasing conversion costs.

This new framework has a number of desirable features. Firstly, the ex-ante compensation per unit of capital, i.e. prior to drawing transformation costs, is equal for the entire world endowment. Secondly, and despite the previous feature, per unit equilibrium capital asset compensation varies across types and, thirdly, also across regions. Fourthly, increasing any particular region-asset-type stock comes at an increasing cost as less and less 'similar' asset types from more and more distant (in terms of transferability) regions have to be converted.

[4] Obviously, the transformation cost can take values larger than 1. However, as we do not take a stance on the units of measurement for our asset classes and solve the model, as explained below, in changes this is simply a question of normalisation.

4.3.4 Technology

On the production side we assume that in each region r each of the J sectors, indexed by j or g, potentially produces a continuum of intermediate varieties $\omega \in [0,1]$ under perfect competition and with constant returns to scale. A second set of perfectly competetive firms in each region and sector sources these sectoral intermediate varieties from the lowest cost suppliers (gross of transaction costs) all over the world and costlessly bundles them into a non-traded aggregate Q_{rj} using the following CES technology:

$$Q_{rj} = \left[\int_0^1 x_{rj}(\omega)^{\frac{\sigma_j - 1}{\sigma_j}} \, d\omega \right]^{\frac{\sigma_j}{\sigma - 1}}, \tag{4.7}$$

where $x_{rj}(\omega)$ is region r's demand for intermediate variety ω from sector j and $\sigma_j > 1$ is the constant elasticity of substitution between varieties in sector j. These non-traded sectoral variety bundles are then used in region r for final consumption or as inputs by intermediate variety producers.[5]

Intermediate producers of variety ω in region r and sector j combine labour $l_{rj}(\omega)$, land $h_{rj}(\omega)$, intermediate goods $m_{rjg}(\omega)$ from all sectors $g \in \{1, ..., J\}$, and capital goods $\kappa_{rjk}(\omega)$ of all types $k \in \mathfrak{K}$ in a Cobb-Douglas fashion. Following Eaton and Kortum (2002) exogenous productivities $z_{rj}(\omega)$ for producing variety ω in region r and sector j are drawn from region and sector specific Fréchet distributions with the CDFs $\Pr[z_{rj}(\omega) \leq z] = e^{-T_{rj} z^{-\theta_j}}$, where θ_j measures the dispersion of the productivities in region r and sector j and T_{rj} determines their average.

The production technology for output ω in region r and sector j can be written as:

$$q_{rj}(\omega) = z_{rj}(\omega) \, l_{rj}(\omega)^{\gamma_{rj}} \, h_{rj}(\omega)^{\eta_{rj}} \left(\prod_{g=1}^{J} m_{rjg}(\omega)^{\mu_{rjg}} \right) \left(\prod_{k=1}^{\mathfrak{K}} \kappa_{rjk}(\omega)^{\beta_{rjk}} \right), \tag{4.8}$$

where $\gamma_{rj},\ \eta_{rj},\ \mu_{rjg},\ \beta_{rjk} \in [0,1]$ respectively set the weight of labour, land, composite intermediates from each sector, and each capital good in production and where $\gamma_{rj} + \eta_{rj} + \sum_{g=1}^{J} \mu_{rjg} + \sum_{k=1}^{\mathfrak{K}} \beta_{rjk} = 1$. As will become clear below, the introduction of taxes and differences in the deductibility of costs of different factors imply that these parameters no longer necessarily equal the respective cost shares in the production process.

4.3.5 Prices, Taxes and Factor Demand

Prices. Production technologies of all varieties within sector j and region r differ only with respect to productivities. Perfect competition, therefore, implies that all producers

[5] This setting is chosen for ease of notation. It is isomorph to a model with consumers having a CES subutility function in each sector and intermediate variety producers employing the same CES function to combine their sectoral inputs.

in sector j and region r face the same marginal production costs per efficiency unit c_{rj} and set mill prices of $p_{rj}(\omega) = \frac{c_{rj}}{z_{rj}(\omega)}$.[6] Intermediate varieties can be traded subject to sector specific iceberg transaction costs between regions r and s such that $\tau_{rsj} \geq 1$ units have to be shipped from region r for one unit to arrive in region s. Adopting the customary assumption that $\tau_{rrj} = 1$, the price at which variety ω from sector j in region r is offered to sectoral compound producers in region s can be expressed as:

$$p_{rsj}(\omega) \equiv p_{rj}(\omega)\,\tau_{rsj} = \frac{c_{rj}\tau_{rsj}}{z_{rj}(\omega)}. \tag{4.9}$$

As prices depend on productivities they inherit their stochastic nature. Producers will buy each variety from the cheapest source at price $\min\{p_{rsj}; s \in R\}$, using the properties of the Fréchet distribution and following Eaton and Kortum (2002) we can derive both the price P_{rj} of sectoral compound goods from sector j in region r as well as the share π_{rsj} of region s's expenditure in sector j on varieties from r. Specifically,

$$P_{rj} = \Gamma\left(\frac{\theta_j + 1 - \sigma_j}{\theta_j}\right)^{\frac{1}{1-\sigma_j}} \left[\sum_{s=1}^{R} T_{sj}\left(c_{sj}\tau_{srj}\right)^{-\theta_j}\right]^{-1/\theta_j} \tag{4.10}$$

and

$$\pi_{rsj} = \frac{T_{rj}\left[\tau_{rsj}c_{rj}\right]^{-\theta_j}}{\sum_{t=1}^{R} T_{tj}\left[\tau_{tsj}c_{tj}\right]^{-\theta_j}}\ , \tag{4.11}$$

where $\Gamma(\cdot)$ denotes the gamma function.

Taxes. Let us use t_r to denote the tax rate charged in country r on profits before land and capital costs and after deducting the depreciation allowances d_{rh} and d_{rk} for these factors respectively. Notice that both t_r and the depreciation allowances will be the same for all regions in the same country $r \in \Re_n$ and that d_{rk} differs by k as certain capital inputs (e.g., machines, buildings, etc.) can be deducted at specific rates.

Given our assumptions on taxes the after tax profit maximisation problem for the production of variety ω in sector j and region r can be written as:

$$\max_{\substack{l_{rj}(\omega),\\ h_{rj}(\omega),\\ m_{rjg}(\omega)\forall g,\\ \kappa_{rjk}(\omega)\forall k}} \left\{ v_{rj}(\omega) - w_r l_{rj}(\omega) - \sum_{g=1}^{J} P_{rg} m_{rjg}(\omega) - s_r h_{rj}(\omega) - \sum_{k=1}^{\mathfrak{K}} \iota_{rk}\kappa_{rjk}(\omega) \right.$$

$$\left. -t_r\left[v_{rj}(\omega) - w_r l_{rj}(\omega) - \sum_{g=1}^{J} P_{rg} m_{rjg}(\omega) - d_{rh} s_r h_{rj}(\omega) - \sum_{k=1}^{\mathfrak{K}} d_{rk}\iota_{rk}\kappa_{rjk}(\omega)\right]\right\}, \tag{4.12}$$

[6] We consider c_{rj} to be marginal production costs per efficiency unit after taxes and deductions, as will become clear below.

where $v_{rj}(\omega)$ denotes the revenue generated by variety ω in sector j and region r, w_r the region's wage, s_r is the rental rate of land and structures, and ι_{rk} the rental rate of capital asset k.

Factor Demand. Integrating the resulting first order conditions over all varieties in the sector and region yields factor usages in the sector-region in terms of revenues and factor prices. Denoting sector-region aggregates by dropping the dependence on ω we obtain:

$$\begin{aligned} l_{rj} &= \frac{\gamma_{rj} v_{rj}}{w_r}, & m_{rjg} &= \frac{\mu_{rjg} v_{rj}}{P_{rg}}, \\ h_{rj} &= \frac{\eta_{rj} v_{rj}}{\tilde{s}_r}, & \kappa_{rjk} &= \frac{\beta_{rjk} v_{rj}}{\tilde{\iota}_{rk}}, \end{aligned} \tag{4.13}$$

where $\tilde{s}_r \equiv \frac{1-d_{rh}t_r}{1-t_r} s_r$ and $\tilde{\iota}_{rk} \equiv \frac{1-d_{rk}t_r}{1-t_r} \iota_{rk}$ are the user costs of land and capital respectively. Given the Cobb-Douglas production structure in (4.8) the implied tax inclusive marginal production costs per efficiency unit of output become:

$$c_{rj} = \chi_{rj} w_r^{\gamma_{rj}} \tilde{s}_r^{\eta_{rj}} \prod_{g=1}^{J} P_{rg}^{\mu_{rjg}} \prod_{k=1}^{\mathfrak{K}} \tilde{\iota}_{rk}^{\beta_{rjk}}, \tag{4.14}$$

where $\chi_{rj} = \left(\gamma_{rj}^{\gamma_{rj}} \eta_{rj}^{\eta_{rj}} \prod_{g=1}^{J} \mu_{rjg}^{\mu_{rjg}} \prod_{k=1}^{\mathfrak{K}} \beta_{rjk}^{\beta_{rjk}} \right)^{-1}$ is a region and sector specific constant.

4.3.6 Expenditure and Consumption

Tax Transfers. National governments in each country n collect tax payments from all sectors j in all regions $r \in \Re_n$ and redistribute them by equal shares to all workers in the country, thus ensuring a balanced national government budget. Using (4.13) and the definitions of $\tilde{s}_r$ and $\tilde{\iota}_{rk}$ to express tax payments as a share of revenue the budget G_n of the government in country n can be written as:

$$G_n = \sum_{r \in \Re_n} t_r \sum_{j=1}^{J} v_{rj} \left(\frac{1-d_{rh}}{1-d_{rh}t_r} \eta_{rj} + \sum_{k=1}^{\mathfrak{K}} \frac{1-d_{rk}}{1-d_{rk}t_r} \beta_{rjk} \right) \tag{4.15}$$

and aggregate transfers to region r as $\frac{L_r}{\sum_{r \in \Re_n} L_r} G_n$.

Land and Capital Returns. Following Caliendo et al. (2014) we allow for a further source of endogenous trade imbalances besides tax transfers. Specifically, we assume that a share $(1-\phi_r)$ of all non-labour returns (after taxes) is redistributed to consumers within the region where the income is generated and the remaining share ϕ_r is paid into an international portfolio that is redistributed among consumers of all regions. In the simulations below the share ϕ_r is then calibrated to match each region's observed trade imbalance.

Specifically, denoting the non-labour income in region r and sector j as I_{rj} and using (4.13) we have:

$$I_{rj} = \sum_{j=1}^{J} \left(\frac{1-t_r}{1-d_{rh}t_r} \eta_{rj} + \sum_{k=1}^{\mathfrak{K}} \frac{1-t_r}{1-d_{rk}t_r} \beta_{rjk} \right) v_{rj} \ , \tag{4.16}$$

which, together with government transfers (4.15), allows to write each region's aggregate trade imbalance D_r as:

$$D_r = \frac{L_r}{\sum_{r=1}^{R} L_r} \sum_{r=1}^{R} \phi_r \left(1-t_r\right) \sum_{j=1}^{J} I_{rj} - \phi_r \left(1-t_r\right) \sum_{j=1}^{J} I_{rj} + \frac{L_r}{\sum_{r \in \Re_n} L_r} G_n - t_r \sum_{j=1}^{J} v_{rj} \left(\frac{1-d_{rh}}{1-d_{rh}t_r} \eta_{rj} + \sum_{k=1}^{\mathfrak{K}} \frac{1-d_{rk}}{1-d_{rk}t_r} \beta_{rjk} \right) \ . \tag{4.17}$$

The first two terms capture differences in payments from and returns to the international capital asset portfolio and the latter two terms such differences from tax payments and received national government transfers.

Importantly, while similar at first glance, the redistribution through taxes and through the international portfolio have starkly different consequences. In particular, note that taxes and deductions change local capital asset returns and, therefore, the supply of these assets. In contrast, payments to and from the international portfolio - implicitly capturing the unequal real world distribution of capital owners - have no direct influence on regional capital asset returns and therefore do not influence their supply.

Expenditure. Finally, combining(4.13) with endogenous deficits (4.17) the aggregate consumer expenditure E_r in any region r in country n including all transfers can be written as:

$$E_r = \sum_{j=1}^{J} \left(\gamma_{rj} v_{rj} + I_{rj} \right) + D_r \ . \tag{4.18}$$

The total expenditure X_{rj} on sector j non-traded compound goods in region r then consists of the share α_{rj} of consumer expenditure and of demand for sector j intermediates from firms from all sectors in r. Hence,

$$X_{rj} = \alpha_{rj} E_r + \sum_{g=1}^{J} \mu_{rgj} v_{rg} \ . \tag{4.19}$$

Consumption. Given aggregate income (4.18) and the price indices (4.10) the non-stochastic part of utility (4.1) can be rewritten as:

$$C_r = \frac{E_r / L_r}{\prod_{j=1}^{J} P_{rj}^{\alpha_{rj}}} \ . \tag{4.20}$$

4.3.7 Equilibrium

Goods Market Clearing. In equilibrium goods markets must clear. For final goods this implies that supply Q_{rj} must equal demand X_{rj}. Consequently the demand for intermediate goods from region r and sector j from final goods producer in location s must be $\pi_{rsj}X_{sj}$. In equilibrium the production value v_{rj} of intermediate goods in sector j and region r must equal demand for these goods from final good producers from all regions:

$$v_{rj} = \sum_{s=1}^{R} \pi_{rsj} X_{sj} = \sum_{s=1}^{R} \pi_{rsj} \left(\alpha_{sj} E_s + \sum_{g=1}^{J} \mu_{sgj} v_{sg} \right) \ , \tag{4.21}$$

where the second equality follows from equation (4.19).

Factor Market Clearing. Given region-sector revenues and the subsequent factor demands according to equation (4.13) wages, land rents and the rental rates of capital adjust such that factor markets clear in equilibrium. Specifically, dividing factor demands of all sectors in r by the fixed supply of housing, the supply of capital goods, and by the number of workers determined by equation (4.4) respectively, we can derive:

$$w_r = \frac{\sum_{j=1}^{J} \gamma_{rj} v_{rj}}{L_r} \tag{4.22}$$

$$s_r = \frac{\frac{1-t_r}{1-d_{rh}t_r} \sum_{j=1}^{J} \eta_{rj} v_{rj}}{H_r} \tag{4.23}$$

$$\iota_{rk} = \frac{\frac{1-t_r}{1-d_{rk}t_r} \sum_{j=1}^{J} \beta_{rjk} v_{rj}}{K_{rk}} \ . \tag{4.24}$$

Capital market clearing further requires that the value of total world wide demand for the fixed base capital stock must equal its compensation. Mathematically,

$$K \left(\sum_{r} \sum_{k} \bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}} \right)^{\varepsilon_\delta} \Gamma\left(1-\varepsilon_\delta\right) = \sum_{r=1}^{R} \sum_{k=1}^{\mathfrak{K}} \frac{1-t_r}{1-d_{rk}t_r} \sum_{j=1}^{J} \beta_{rjk} v_{rj} \ . \tag{4.25}$$

Combining this condition with the capital mobility equation (4.6) and capital asset stock demand (4.24) determines the equilibrium level of capital asset stocks in each region as:

$$K_{rk} = K\Gamma\left(1-\varepsilon_\delta\right) \bar{\delta}_{rk}^{\varepsilon_\delta} \left(\frac{\frac{1-t_r}{1-d_{rk}t_r} \sum_{j=1}^{J} \beta_{rjk} v_{rj}}{\sum_{r=1}^{R} \sum_{k=1}^{\mathfrak{K}} \frac{1-t_r}{1-d_{rk}t_r} \sum_{j=1}^{J} \beta_{rjk} v_{rj}} \right)^{1-\varepsilon_\delta} \ . \tag{4.26}$$

Equilibrium Conditions. Given our previous results the model's equilibrium can be reduced to solving three sets of equations as stated in the following proposition.

Proposition 4.1. *An equilibrium of the model is defined by values of L_r, P_{rj}, and v_{rj} for all r and j that satisfy the following equilibrium equations given all preference parameters α_{rj} and σ_j, all cost shares γ_{rj}, β_{rjk}, η_{rj}, and μ_{rjg}, all tax parameters t_r, d_{rh}, and d_{rk}, all amenity, productivity and capital asset distribution parameters T_{rj}, A_r, $\bar{\delta}_{rk}$, θ_j, ε and ε_δ, all land endowments H_r, total population L and capital endowment K.*

1. *labour mobility conditions (4.4) after replacing consumption using (4.20) and plugging in income (4.18) and government transfers (4.15)*
2. *price index equations (4.10) after replacing marginal costs using (4.14), factor prices using (4.22) - (4.24) and capital asset stocks using (4.26)*
3. *goods market clearing (4.21) after plugging in expenditure (4.18), deficits (4.17), government transfers (4.15), and trade shares (4.11) combined with marginal costs (4.14) and factor prices (4.22) - (4.24)*

Equilibrium in Changes. We rely on the popular method by Dekle et al. (2007) to solve counterfactual model equilibria in response to a shock in terms of changes. Denoting variables after the shock with a prime and their relative changes with a hat we can state the following proposition.

Proposition 4.2. *Given a shock defined by counterfactual tax parameters t'_r, d'_{rh}, and d'_{rk} as well as relative changes in average amenities $\hat{A}_r$, average productivities $\hat{T}_{rj}$, and trade costs $\hat{\tau}_{rsj}$ for all regions r, sectors j and capital asset types k the equilibrium of the model in changes consists of values of $\hat{L}_r$, $\hat{P}_{rj}$, and $\hat{v}_{rj}$ for all r and j that satisfy the following equilibrium equations given all α_{rj}, all cost shares γ_{rj}, β_{rjk}, η_{rj}, and μ_{rjg}, the amenity, capital asset transformation cost and all productivity distribution shape parameters ε, ε_δ and θ_j, as well as all initial population numbers L_r, trade shares π_{rsj}, and revenues v_{rj} in the ex-ante equilibrium:*

$$\hat{L}_r = \frac{\hat{A}_r \hat{E}_r^{\frac{1}{\varepsilon}} \hat{L}_r^{-\frac{1}{\varepsilon}} \left(\prod_{j=1}^{J} \hat{P}_{rj}^{\alpha_{rj}} \right)^{-\frac{1}{\varepsilon}}}{\sum_{s \in R} \frac{L_s}{L} \hat{A}_s \hat{E}_s^{\frac{1}{\varepsilon}} \hat{L}_s^{-\frac{1}{\varepsilon}} \left(\prod_{j=1}^{J} \hat{P}_{sj}^{\alpha_{sj}} \right)^{-\frac{1}{\varepsilon}}} \tag{4.27}$$

$$\hat{P}_{rj} = \hat{\pi}_{rrj}^{\frac{1}{\theta_j}} \hat{T}_{rj}^{-\frac{1}{\theta_j}} \hat{c}_{rj} \tag{4.28}$$

$$v_{rj}\hat{v}_{rj} = \sum_{s=1}^{R} \pi_{rsj}\hat{\pi}_{rsj} \left(\alpha_{sj} E'_s + \sum_{g=1}^{J} \mu_{sgj} v_{sg} \hat{v}_{sg} \right). \tag{4.29}$$

4.4 Data and Calibration

To calibrate the model our analysis relies on four main data sources. Firstly, sectoral production data, international trade data, input-output relations and the consumption structure at the country level are taken from the World Input- Output Database (WIOD). Secondly, regional trade data is derived using bilateral freight data by product on a NUTS3 level from the ETISPlus database. Thirdly, missing trade flows are imputed relying on travel times between NUTS3 regions based on open source OSRM map data.[7] Finally, national tax rates and depreciation allowances of different assets are from Steinmüller et al. (2019) where we use asset shares on a sectoral level from Fabling et al. (2014). We calibrate our model for the year 2010 as the regional trade data is only available until 2010. Details of all four data sources and the final calibration of the model are discussed in the following. Subsection 4.4.4 highlights how future research can improve in terms of calibration and emphasises what kind of data would be necessary.

4.4.1 Trade and I-O Data

World Input-Output Database. Our country level data stems mainly from the World Input-Output Database (WIOD).[8] It provides a time-series of world input-output tables compiled on the basis of officially published input-output tables in combination with national accounts and international trade statistics. The tables cover data from 56 industries in 44 countries, including all current members of the European Union and one artificial "rest of the world" (ROW) country. Following Costinot and Rodríguez-Clare (2014) positive inventory changes in the WIOD are included in final demand and negative inventory changes are treated as if they had been produced in 2010 as well. The details of this process are laid out in Krebs and Pflüger (2018) and are summarised in Appendix 4.A.3. The resulting input-output table is used to derive sectoral consumption and intermediate good shares, the share of value added, and the bilateral industry trade shares at the country level.

European Transport Information System. Bilateral regional transport data on a product level stems from the ETISPlus project. ETISPlus is the continuation of the European transport information system project (ETIS) funded by the European Commission (Breemersch et al., 2013). One of the aims of the ETISPlus project is to construct an origin-destination road freight matrix using Eurostat data. For the year 2010, it contains bilateral tonnage flows between NUTS3 regions covering the EU 27, candidate countries and EFTA as well as neighboring countries. The harmonised data differentiates between

[7]For these calculations we rely on the user written stata command 'osrmtime' by Huber and Rust (2016).

[8]See Timmer et al. (2015) for an introduction to the WIOD.

20 different commodities and harmonises several country specific data sets with tables from Eurostat to generate an origin-destination road freight matrix.[9]

We restrict our analysis to road freight data only. For regions with good rail or waterway access this might distort our analysis. However, based on Eurostat data, transport within Europe is mainly operated by trucks, with the respective modal shares for total tonne-kilometers for all EU inland freight transport given by 76% for road, and only 17% rail and 7% inland waterways in 2010.[10] Moreover, and as explained in the following, the use of exporter and importer fixed effects in our estimations below can at least partially capture regional differences in the road transport share.

For 17 islands including the EU member Malta no or only little ferry based, road freight data is available and therefore those regions are dropped in the analysis.[11] This leaves us with a bilateral transport data set for 1306 NUTS3 regions in 27 EU countries, and 20 different product categories. We match the 20 product categories to 12 sectors.[12] For the remaining 12 sectors about half of the bilateral sector observations have freight volume information.

Travel Times. Given the incomplete bilateral freight data matrix we impute missing bilateral flows relying on a gravity approach. To this end we use, among other qualifiers, travel times between NUTS3 regions as a proxy for bilateral barriers. To calculate travel times between all bilateral regional pairs we rely on OpenStreeMap data and software.[13] Specifically, for each region pair we take the closest points to the regions' centroids on the street network and calculate the travel time between those two points.[14] We are using the shape files of the regions and the travel times between the boarders of a region to create

[9]Given that the freight data uses the NUTS3 classification of 2006 we use this classification throughout.

[10]Rail and waterway transport matrices from ETISplus are only available at the NUTS2 level with NST 1-digit sector classification and would thus necessitate considerable imputations and potentially introduce another type of bias.

[11]The Finnish island Åland, the two NUTS3 regions of Malta, the two autonomous regions of Portugal, Região Autónoma dos Açores and Região Autónoma da Madeira, and the three overseas regions of France, Guadeloupe, Martinique, and Guyane are dropped because no freight information is available in the ETISPlus database. The Greece islands Ikaria, Samos and Chios as well as the seven Canary islands, El Hierro, Fuerteventura, Gran Canaria, La Gomera, La Palma, Lanzarote , Tenerife are dropped because there is insufficient freight data available and no appropriate travel times can be calculated.

[12]The product category "Coal and lignite" and "Metal ores and other mining and quarrying products" are combined in the industry "Mining and quarrying". Four product groups are dropped because there is no or no sufficient match to our final sectors and are dropped from the data 8"Secondary raw materials", "Mail, parcels", "Equipment and material utilized in the transport of goods", "Goods moved in the course of household and office removals"). Three further categories ("Grouped goods" "Unidentifiable goods" "Other goods") can not be matched directly and are instead used to scale bilateral flows in all other sectors. A table with the concordance between the product groups and the final sectors is in the Appendix 4.A.3.

[13]We rely on the open source routing machine (OSRM) project together with map data from the OpenStreeMap project and the user written stata command osrmtime by Huber and Rust (2016).

[14]For a small set of regions the software only allows to calculate the travel times in one direction. For those regions we assume that travel times are the same in both directions.

an approximation for the travel times within a region.[15]

Imputation of Bilateral Barriers. Multiplying the import share equation (4.11) with region-sector demands X_{sj} and dividing by region-sector prices the theoretical framework allows to separate the imported quantity of region s in sector j from r into an exporter, importer and bilateral component:

$$\log(\frac{X_{sj}\pi_{rsj}}{P_{sj}}) = \log\left(T_{rj}c_{rj}^{-\theta_j}\right) + \log\left(\frac{X_{sj}}{P_{sj}\sum_{t=1}^{R} T_{tj}\left[\tau_{tsj}c_{tj}\right]^{-\theta_j}}\right) + \log\left(\tau_{rsj}^{-\theta_j}\right).$$

Rewriting this equation in its stochastic form yields:

$$\log(\frac{X_{sj}\pi_{rsj}}{P_{sj}}) = S_{rj} + M_{sj} + D_{rsj} + \epsilon_{rsj} \quad , \tag{4.30}$$

where $S_{rj} \equiv \log\left(T_{rj}c_{rj}^{-\theta_j}\right)$, $M_{sj} = \log\left(\frac{X_{sj}}{P_{sj}\sum_{t=1}^{R} T_{tj}[\tau_{tsj}c_{tj}]^{-\theta_j}}\right)$, $D_{rsj} = \log\left(\tau_{rsj}^{-\theta_j}\right)$ and ϵ_{rsj} is an error term. We rely on the observed transport data using a CANOVA approach (Egger and Nigai, 2015) to estimate the bilateral fixed effects D_{rsj} under the assumption of symmetric trade barriers ($\tau_{rsj} = \tau_{srj}$).

In a second step, in order to impute bilateral effects for observations where no freight data is available we regress the obtained bilateral components on gravity qualifiers and travel times between the regions. Specifically, we estimate:

$$D_{rsj} = \delta Duration_{rs} + \delta_{1j}B_{rs} + \delta_{2j}Lang_{rs} + \delta_{3j}Country_{rs} + \tilde{S}_{rj} + \tilde{M}_{sj} + \tilde{\epsilon}_{rsj}, \tag{4.31}$$

where B_{rs}, $Lang_{rs}$, and $Country_{rs}$ are dummies taking a value of 1, if regions have a common border, common language, or belong to the same country respectively, $\tilde{S}_{rj}$ and $\tilde{M}_{sj}$ are exporter and importer fixed effects and $\tilde{\epsilon}_{rsj}$ is an error term.[16] The estimated point coefficients for the 12 sectors are depicted in Table 4.1.

In a third step, we predict bilateral barriers, including those region-sector pairs with missing trade flows, using the estimated coefficients together with observed travel times and dummy controls. Given the full set of bilateral barriers and importer and exporter fixed effects M_{sj} and S_{rj} we can calculate predicted trade flows between all NUTS3 regions in all sectors. The final result is thus a data set of transport flows for 1306x1306 region pairs in 12 sectors, implying a total of 20,467,632 bilateral sector observations for the year 2010.

[15] We randomly select 8 pairs of points on the boarder of the region and calculate travel times between those points. Our final approximation for the within region travel time is then the average of the eight travel times divided by two to account for the fact that within region transport is not only from boarder to boarder but instead within the region.

[16] The common language control is taken from the CEPII gravity data base.

Table 4.1: *OLS Gravity Estimation*

Sector	log duration	common border	common language	common country	R^2	Obs.
Agriculture	-2.38	0.84	0.65	0.50	0.83	912797
Mining	-3.11	0.34	0.71	1.10	0.90	945892
Food, Beverages, Tabacco	-2.42	0.93	0.52	0.22	0.79	1074800
Textiles, Leather	-1.52	0.91	0.07	1.00	0.85	486721
Wood, Paper, Printing	-2.46	0.64	0.63	-0.07	0.83	938468
Coke, Petroleum	-2.58	0.38	0.40	1.12	0.90	482454
Chemicals, Pharmaceuticals	-2.27	0.77	0.63	0.02	0.82	929181
Non-Metallic Minerals	-2.74	0.81	0.57	0.52	0.88	919642
Metals	-2.23	0.86	0.71	0.23	0.83	885395
Machinery, Electrical Equipment	-1.73	1.35	0.32	0.69	0.83	877312
Transport Equipment	-1.73	1.12	0.29	0.50	0.81	698100
Furniture and other Manufacturing	-1.58	1.02	0.73	0.82	0.84	616338

The estimation uses exporter and importer region fixed effects.

4.4.2 Taxes and Deduction Rates

Both corporate tax rates and net present values of depreciation allowances per asset type are taken from Steinmüller et al. (2019). They collect, on a national level, a set of tax rates, depreciation allowances and depreciation regimes for the years 1996 to 2018 with a full country coverage for the years 2004 to 2019. Subsequently, they calculate the net present values of tax depreciation allowances based on the depreciation regime in each country. Seven asset types are differentiated (buildings, machinery, intangible fixed assets, inventory, office equipment, vehicles, computers). We abstract from inventory in our framework such that we are left with 6 different asset types. Table 4.2 illustrates the distribution of both the tax rate and the depreciation allowances across the different countries.

Table 4.2: *Taxes and Deductions, Percentiles Across the EU27*

Taxes across EU27	1%	25%	50%	75%	99%
	0.100	0.190	0.245	0.276	0.344
Deduction across EU27	**1%**	**25%**	**50%**	**75%**	**99%**
Buildings	0.049	0.119	0.141	0.167	0.225
Office	0.081	0.171	0.223	0.245	0.313
Machinery	0.081	0.167	0.214	0.249	0.309
Computers	0.090	0.174	0.233	0.256	0.313
Vehicles	0.090	0.171	0.215	0.245	0.323
IFAS	0.090	0.163	0.204	0.237	0.309

4.4.3 Further Calibrations

In order to allow for different asset structures in different sectors we use the work by Fabling et al. (2014) who determine the asset composition by industry for new Zealand in the year 2010.[17] In combination with the WIOD data, from which the share of overall asset inputs in a sector can be derived as a residual, we compute sector specific Cobb-Douglas shares for the six asset types $\beta_{rjk} \in [0, 0.072]$. The capital efficiencies shape parameter is set to $\varepsilon_\delta = 0.3$. Following the work by Redding (2016) we set the amenities shape parameter to $\epsilon = 3$ and the productivity shape parameters for the different sectors are taken from Krebs and Pflüger (2018) $\theta_j \in [1.02, 22.68]$.[18]

4.4.4 Alternative Approach

This section discusses some shortcomings of the current calibration and sketches possible solutions we are currently working on. It also stresses where future research is needed and where additional data is necessary.

Transport Mode. The current analysis uses only truck transport data. While a little more then 75% of ton-fright transport is by road it is still important to include the other modes: rail, waterway, sea and air transport. This is especially important, because the main transport mode most likely differs across sectors. Before discussing a possible solution to this problem we will discuss another issue in the current approach first, which will also be addressed by the suggested solution.

Gravity Estimation. So far we impute the bilateral barriers using a three step procedure. In this approach we perform a simple OLS gravity estimation. The OLS approach has well known shortcomings with respect to zero trade flows. Therfore, using a Pseudo Poisson Maximum Likelihood estimator (PPML) instead of the OLS approach would be preferable.[19] The regional dimension with more than 1300 trading partners and thus the large number of fixed effects imposes some computational problems. However, instead of re-estimating we suggest a different approach in order to improve the calibration of our framework.

[17] A Table of their results can be found in Supplementary Appendix 4.B.3. We acknowledge that the sectoral asset composition data is originally derived for New Zealand. Applying these shares to EU members can thus only be an approximation of the true shares, despite the countries' similar development status. Furthermore, the scrotal asset composition data does not allow for a disaggregation of the manufacturing sector. Thus, the asset structure is the same for all manufacturing sectors.

[18] Our theoretical framework allows for a structural estimation of the amenities shape parameter. As discussed in the Subsection 4.4.4 future work should structurally estimate the Fréchet distribution parameter of amenities, relying on real GDP and observed population shares.

[19] Larch and Yotov (2016) describe the current state of the art gravity estimation approach.

Regional Trade Data. In order to calibrate our framework it is essential to have interregional trade data. To the best of our knowledge the ETISPlus shipment data is the only cross country data source that supplies transport data for the EU on a substantial regional (NUTS3) and sectoral disaggregation. The ETISPlus project collects observed data on truck, rail, waterway, sea and air transport on different regional and sectoral disaggregation levels. Besides the used "harmonised" data ETISPlus also offers a "modelled" freight data matrix where missing flows and different national collection methods are accounted for using non-linear programming and a range of further observed NUTS3-level characteristics. The "modelled" freight data matrix is further disaggregated and allows to differentiate between 51 different NST/R 2-digit product groups. However a match with the WIOD Data as depicted in Table 4.A.2 in Appendix 4.A.3 only allows to differentiate between 12 sectors. As for the used "harmonised" transport data, 10 out of the 12 sectors are manufacturing sectors. In order to include other sectors besides the manufacturing sector and agriculture and mining such as construction, utilities or different service sectors additional trade data on a regional level would be necessary which to our knowledge does not exist on a comparable level across EU countries.

One advantage of the imputed modelled transport data is the disaggregation on the NUTS3 level for all transport modes. Therefore, it is a simple mean of aggregation to incorporate all modes of transportation. It is important to mention that while this dataset is available at the NUTS3 NST 2-digit level the underlying raw data for all transport modes apart from road transport is sill only on a NUTS2 or NUTS1 level and no or only little product group disaggregation.The ETISPlus project augments this data with different additional country level data and transport network information in order to achieve the additional disaggregation. Furthermore, the fact that the "modelled" freight data matrix already accounts for missing trade flows implies that no additional imputation of trade flows using travel times between different regions is necessary.

While using the "modelled" freight data matrix comes at the cost of relying on the imputations performed by the ETISPlus project, which are based on a different structural framework, the data has several advantages, which is why we intend to re-calibrate our framework in the following manner. When using the "modelled" freight data matrix one still has to overcome two potential problems. First of all, the data depicts transport between regions and thus while a certain good might be shipped between two regions this transport might be only one leg of a whole transport chain. Therefore, first one needs to convert the transport data into trade flows. Secondly, the transport data is in tons and in order to calibrate the model, trade flows in values are necessary. In order to move from transport data to trade flows we intend to apply the following process. We interpret the share of goods flowing from region r to region s in sector j as the probability that any unit from j in r will take that route. Further we assume that only the first and last leg of a transport chain can be a truck shipment or a non-EU flow and restrict the overall number of legs to 3. We then calculate the aggregate share of r's production sent to s in sector j via 0, 1, 2, or 3 transshipment locations. This allows to calculate the aggregate share of r's production in sector j that flows between any two regions s-s'. Given these shares we find the production vector that minimizes the squared difference of implied and observed transport flows on any leg. Using production data and the derived shares flowing from r to s, we can impute trade flows between r and s. In order to translate tonnage flows into values we use the derived trade flows as initial values in an RAS-matrix-updating approach. In this Approach we constrain national sectoral production and trade to values derived from the WIOD. In addition, we constrain aggregate regional production to regional GDP from Eurostat, scaled to match national WIOD values. In a final step we use national sectoral I-O linkages from the WIOD to construct an initial full inter-regional IO table and then fit I-O and consumption coefficients to agree with the imputed trade flows using another RAS-updating. An alternative to this approach is to come up and use prices per tonnage on a sectoral and if possible regional level. A feasible approach and a first approximation might be to calculate prices per tonnage on a sector and country level using country level data. However, especially if the price per tonnage differs substantially across products within a sector this might imply that prices per tonnage are also substantially different across regions.

Asset Structure. In order to allow for different asset structures in different sectors we use the work by Fabling et al. (2014), who determine the asset composition by industry for New Zealand in the year 2010. This approach has several shortcomings. Firstly, the sectoral asset composition data is derived for New Zealand. Applying these shares to EU members can thus only be an approximation of the true shares. Secondly, the used data does not allow for disaggregation of the manufacturing sector. Therefore, we have to assume the same asset structure for 10 out of the 12 sectors in our analysis, because those belong to the manufacturing sector. Estimations based on European firm samples and a disaggregation of the manufacturing sector would be preferable but are currently unavailable. Future work including data collection is needed to address this issue.

Amenities. So far the amenities shape parameter is taken from the literature. The theoretical framework in general allows for a structural estimation relying on real GDP and observed population shares. Taking logs of the mobility condition (4.4) implies the following relationship:

$$\log\left(\frac{L_r}{L}\right) = \log A_r - \log\left(\sum_{s\in R} A_s C_s^{\frac{1}{\varepsilon}}\right) + \frac{1}{\varepsilon}\log C_r. \tag{4.32}$$

Using the population share and real income as measure for the aggregate consumption in a region we can use the following regression to determine the shape parameter of the

Fréchet distribution of amenities:

$$\log pop\_shr_r = c_0 + \frac{1}{\varepsilon} \log real\_income_r + e_r, \tag{4.33}$$

where $c_0 = -\log\left(\sum_{s\in R} A_s C_s^{\frac{1}{\varepsilon}}\right)$ and e_r is an error term that includes A_r. Using population data as well as GDP per inhabitant and purchasing power per inhabitant on a NUTS3 level from Eurostat we run a first simple OLS regression. The estimated coefficient of 0.476 implying a shape parameter $\varepsilon = 2,1$. The unobserved characteristics in the error term are potentially correlated with both real income and population shares. Thus, one needs to instrument for real income. Centrality might be a suitable instrument. In order to construct a centrality measure for region r the size of all possible trading partners in terms of GDP can be weighted with their distance or travel time.

4.5 Simulation Results

Given the calibration of our model in changes, described in Subsection 4.3.7, we can simulate the heterogeneous effects of national tax policies across regions. Specifically, we will first turn to a general analysis of the strength of heterogeneity as well as the geographic and sectoral distribution of responses. The following two subsections then turn to two specific, actively discussed policy experiments: setting a common tax and deduction scheme across all European Union members as well as moving to a cash-flow taxation.

4.5.1 Regional Heterogeneities

As regions differ in terms of their sectoral production structure, usage of capital assets and intermediates as well as connections through trade and worker mobility we expect them to react differently to a national tax shock. To test this prediction we perform 27 separate simulations. Specifically, in each case we increase the tax rate in a single EU member country and derive the response in terms of utility and real consumption ($\hat{C}_r$) in all regions across Europe.

To this end it is important to note that in the spatial general equilibrium implied by our model, the relative change in expected or average utility, that is workers expected utility prior to learning about their amenity draw, is the same conditional on choosing a specific location as well as across the European Union as a whole. Intuitively, regions with relatively strong real consumption gains compared to other locations attract consumers that on average have lower amenity draws for the region as the population already living there. Vice versa, regions with relative real consumption losses will first be left by consumers with relatively low amenity draws for the specific region. In equilibrium this implies equal levels and relative changes of average utility including amenity draws.

Figure 4.1: *Regional Effects of 1 Percentage Point National Tax Increase*

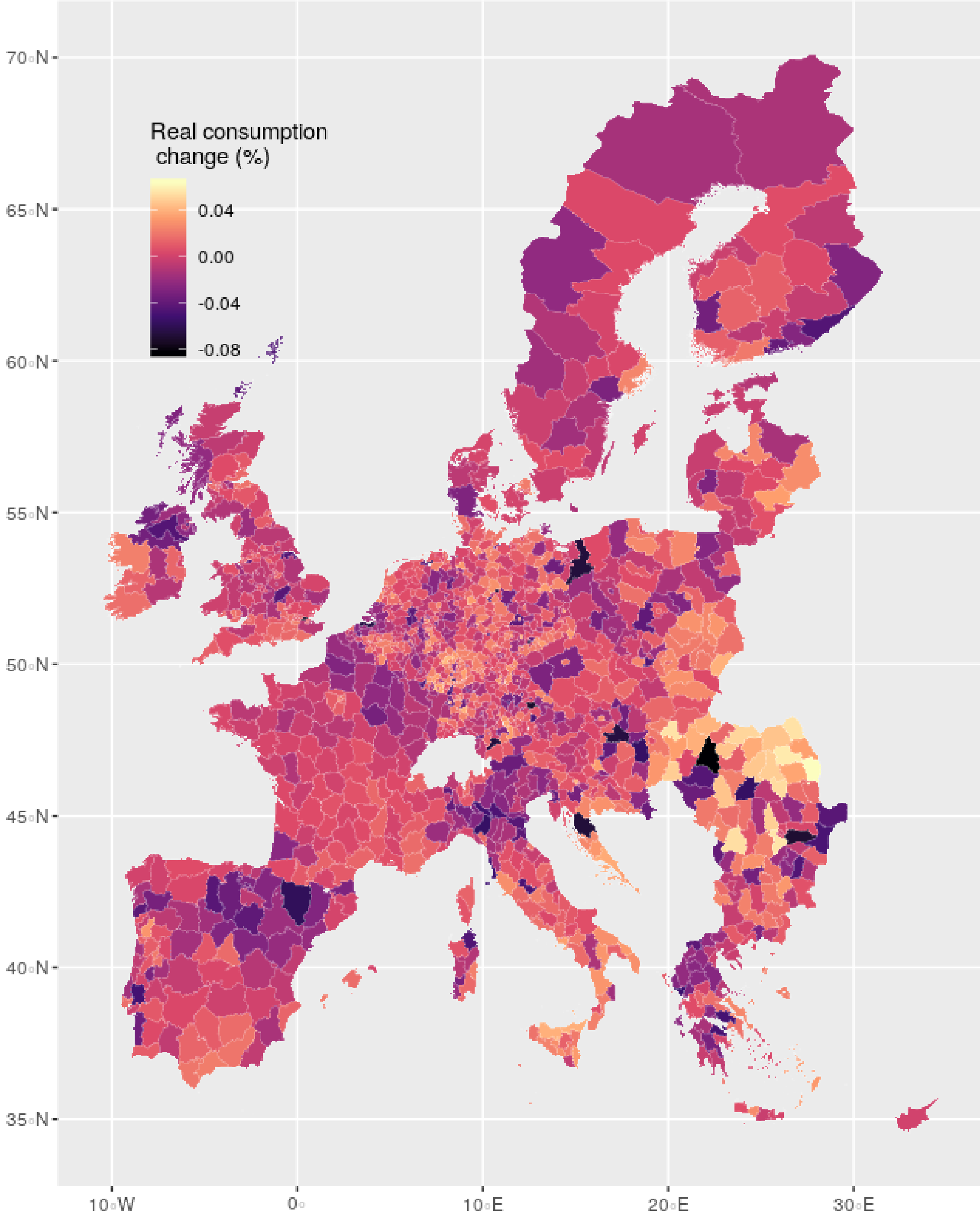


Figure 4.1 summarises the results of all 27 separate simulations. Each region is coloured according to the elasticity of real consumption with respect to a 1 percentage *point* increase in the respective nation's corporate tax rate.[20] Thus, Figure 4.1 only depicts the domestic

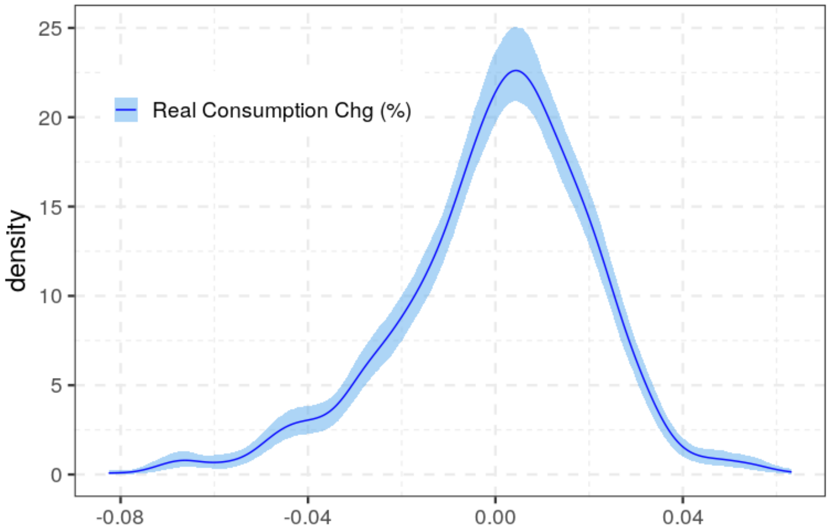
[20]To derive these values we calculate the effects of 5 percentage points increases in national tax rates and divide the results by 5. General equilibrium responses with respect to tax rate changes need of course not be linear but simulations with 1 and 10 percentage point shocks show that the resulting elasticities vary only slightly.

effects for each of the 27 simulations, depicting the heterogeneity of domestic regions' responses to national tax policy shocks. As can be seen these heterogeneities are strong ranging from -0.08% to 0.04% real consumption change.

The resulting heterogeneities are strongest within Romania and Slovakia as well as within Germany. For the latter this is potentially affected by the much smaller geographic area of NUTS3 regions compared to most other countries. Overall, the most adverse effects are felt in regions that are the nations' manufacturing centres, such as the north of Italy, the north of Spain, German car manufacturing regions or the areas around Rotterdam and Amsterdam. Less productive regions, in turn, benefit from higher redistribution of national tax income.

Figure 4.2 depicts the density distribution of the same values shown on the map. Clearly strong negative real consumption changes are less common with the largest mass of responses lying between -0.04% and 0.04%.

Figure 4.2: *Regional Effects of 1 Percentage Point National Tax Increase (Densities)*

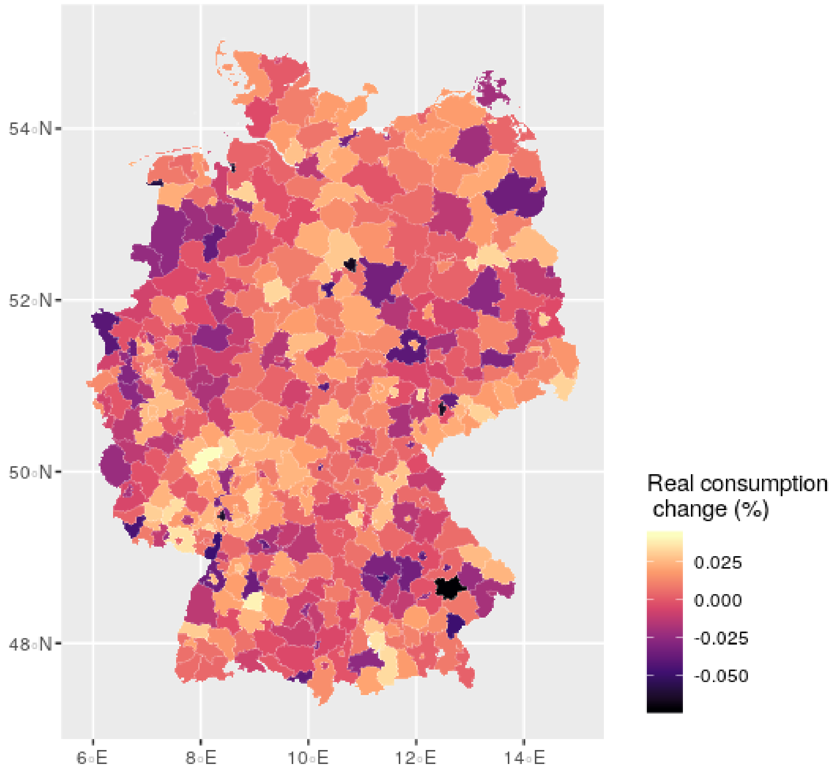


Zooming in on Germany. Figure 4.3 depicts the same results for Germany only. Here also heterogeneities are substantial. As mentioned regions that feel the strongest adverse effects are home to important car manufacturers (Wolfsburg with VW in the North, Dingolfingen with BMW in the south east) as well as important producers of automobile components. Among the benefiters, in contrast, are the cities of Berlin and Munich that have strong service sector shares.

4.5.2 Common EU Taxation Scheme

We next turn to the effects of the introduction of a common taxation and deductibility scheme across the European Union. We simulate a shock that sets the corporate tax as well as capital asset deductibilities to the respective EU wide means for all countries.

Figure 4.3: *German Regional Effects of 1 Percentage Point National Tax Increase*



Importantly, we find that such a common policy would increase the average welfare in spatial general equilibrium by 0.0054%. Again Eastern European states, especially, Romania, Bulgaria, Hungary and Estonia experience the strongest effects both in negative and positive terms, with real consumption changes ranging from -0.79% to 0.59%.[21] This is partially driven by these countries also experiencing the by far strongest adjustments compared to their current taxation schemes. The relatively strong changes in these countries also hide substantial heterogeneity among regions of other nations, where the major production centres and - in the case of Germany - car manufacturing locations profit the strongest. Geographically, the resulting changes in population size mirror real consumption effects as in spatial general equilibrium those countries with the strongest real consumption gains also experience the largest population inflows. In terms of magnitude we project that the strongest regional population size changes reach -0.42% and 0.32% respectively. In most regions, however, effects are much milder with 80% of values in the range of -0.02% to 0.03%.

[21]Figure 4.A.1 in the appendix depicts the full results.

4.5.3 Cash-flow Taxation Scheme

Finally, we simulate the effects of moving to a cash-flow taxation scheme within the EU. Specifically, we allow for the full immediate deductibility of all asset types including land. Overall such a policy implies an EU wide negative welfare effect of -0.067%.

Figure 4.4: *Regional Effects of Full Asset Deductibility*

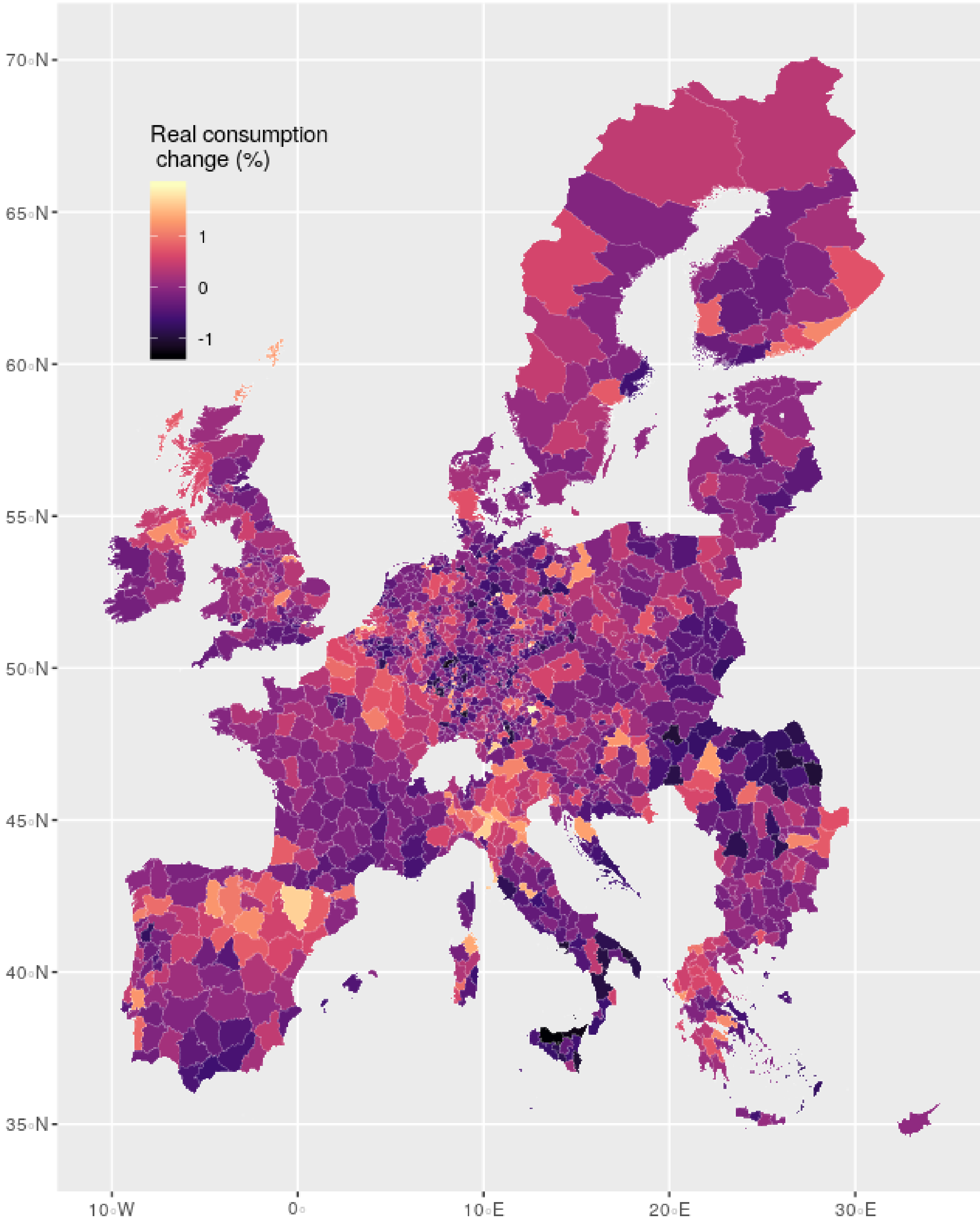


Figure 4.4 depicts the very heterogeneous regional real consumption effects of this policy across EU members. Negative effects are again strongest in Romania, but also particularly

pronounced in southern Italy and select rural regions of Germany. The strongest benefiters are the countries' production centres as well as German car manufacturing regions as these locations substantially reduce their tax base. Both negative as well as positive effects are much more pronounced than in previous simulations, ranging from real consumption losses of -1.33% to gains of 1.98%. The now particularly pronounced difference of German car manufacturing locations shows that heterogeneity is not only driven by a geographical but also by a sectoral component.

In contrast to the much stronger real consumption effects, population share changes are in a similar range to the previous policy experiment. Specifically, some rural regions experience population losses of up to -0.38% while manufacturing centres see population inflows of up to 0.61%.

4.6 Conclusion

This paper develops a theoretical model that acknowledges regional and sectoral differences in economic activity as well as their interdependence through trade and mobility to study regional general equilibrium effects of national trade policy. By calibrating this model based on a unique collection of data sets, we find that there is substantial heterogeneity in local responses to national tax policy, which is driven by the different production structures and linkages.

Specifically, across the EU the regional (NUTS3) real consumption response to a one percentage point increase of the respective country's national tax rate ranges from -0.08% to 0.06%. Geographically, the most adverse effects are felt in regions that are the nations' manufacturing centres, such as the north of Italy, the north of Spain, German car manufacturing regions or the areas around Rotterdam and Amsterdam. Less productive regions benefit from higher redistribution of national tax income. Varying dependence on endowment with different capital asset types as well as differences in their deductibility also have a large influence on the derived heterogeneity.

With respect to two prominently discussed tax policies, the adoption of a common EU corporate tax and capital asset deduction scheme as well as the introduction of a cash-flow taxation in which capital assets are fully deductible, we find that the former has a slight welfare increasing effect, whereas the latter leads to welfare losses. In both cases, however, heterogeneities across regions are very strong.

Overall our results thus clearly point to the importance of considering regions when evaluating the effects of national tax policy. The derived strong heterogeneities are of vital importance for policy makers and understanding the underlying mechanisms for the varying responses is crucial for economists trying to project the effects of tax policies.

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4.A Appendix

4.A.1 Equilibrium in Changes

Using the three sets of equations derived in Proposition 4.2 and plugging in the respective equations yields the following results for the post shock variables:

$$E'_r = \sum_{j=1}^{J} \left(\gamma_{rj} \upsilon_{rj} \hat{\upsilon}_{rj} + I'_{rj} \right) + D'_r,$$

$$\begin{aligned} D'_r = & \frac{L_r \hat{L}_r}{\sum_{r=1}^{R} L_r \hat{L}_r} \sum_{r=1}^{R} \phi_r \left(1 - t'_r\right) \sum_{j=1}^{J} I'_{rj} - \phi_r \left(1 - t'_r\right) \sum_{j=1}^{J} I'_{rj} \\ & + \frac{L_r \hat{L}_r}{\sum_{r \in \Re_n} L_r \hat{L}_r} G'_n - t'_r \sum_{j=1}^{J} \upsilon_{rj} \hat{\upsilon}_{rj} \left(\frac{1 - d'_{rh}}{1 - d'_{rh} t'_r} \eta_{rj} + \sum_{k=1}^{\mathfrak{K}} \frac{1 - d'_{rk}}{1 - d'_{rk} t'_r} \beta_{rjk} \right), \end{aligned}$$

$$I'_{rj} = \sum_{j=1}^{J} \left(\frac{1 - t'_r}{1 - d'_{rh} t'_r} \eta_{rj} + \sum_{k=1}^{\mathfrak{K}} \frac{1 - t'_r}{1 - d'_{rk} t'_r} \beta_{rjk} \right) \upsilon_{rj} \hat{\upsilon}_{rj},$$

$$G'_n = \sum_{r \in \Re_n} \sum_{j=1}^{J} t'_r \upsilon_{rj} \hat{\upsilon}_{rj} \left(\frac{1 - d'_{rh}}{1 - d'_{rh} t'_r} \eta_{rj} + \sum_{k=1}^{\mathfrak{K}} \frac{1 - d'_{rk}}{1 - d'_{rk} t'_r} \beta_{rjk} \right),$$

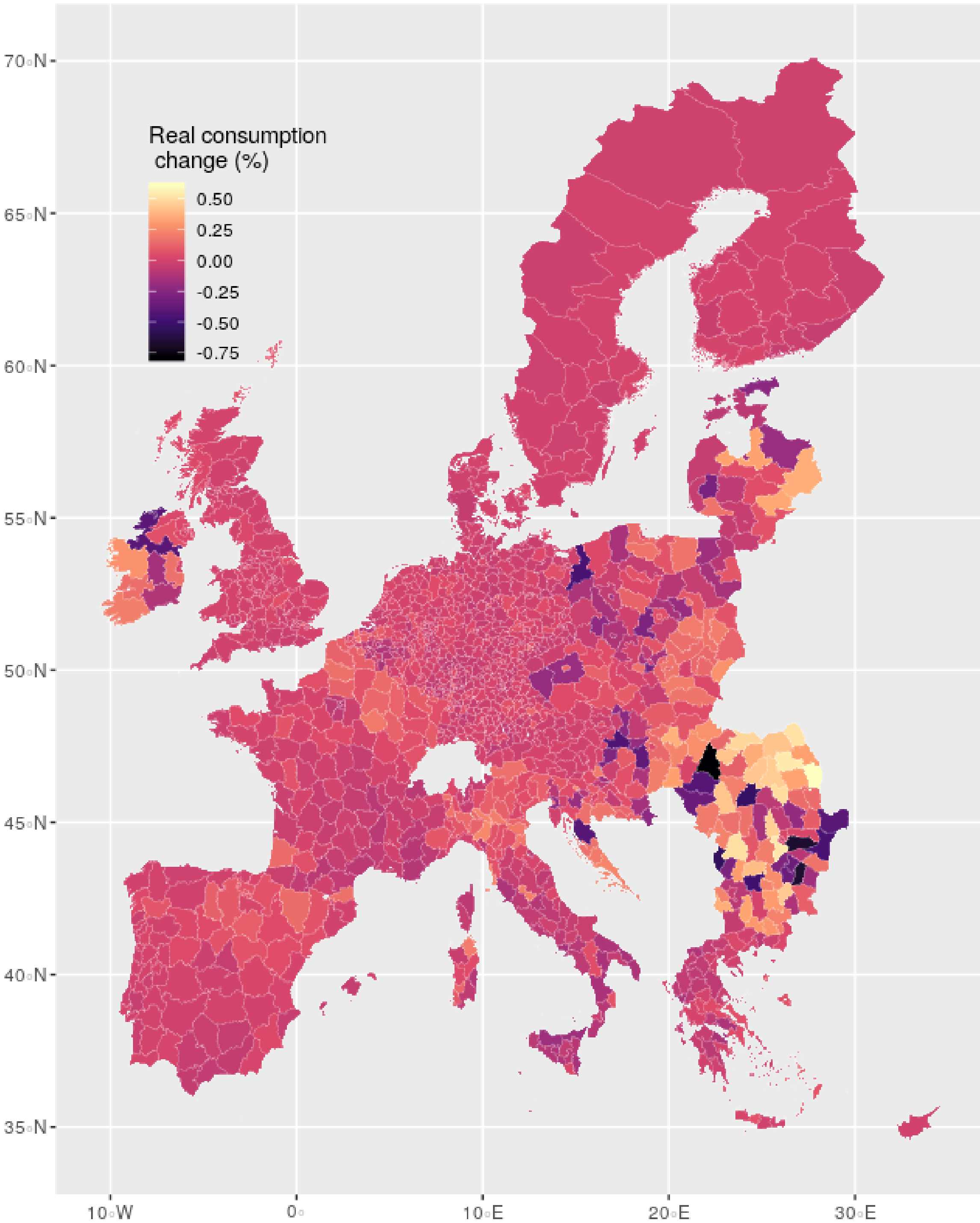
$$\hat{\pi}_{rsj} = \frac{\hat{T}_{rj} \left[\hat{\tau}_{rsj} \hat{c}_{rj} \right]^{-\theta_j}}{\sum_{t=1}^{R} \pi_{tsj} \hat{T}_{tj} \left[\hat{\tau}_{tsj} \hat{c}_{tj} \right]^{-\theta_j}},$$

$$\hat{c}_{rj} = \left(\frac{1}{\hat{L}_r} \frac{\sum_{g=1}^{J} \gamma_{rg} \upsilon_{rg} \hat{\upsilon}_{rg}}{\sum_{g=1}^{J} \gamma_{rg} \upsilon_{rg}} \right)^{\gamma_{rj}} \left(\frac{\sum_{g=1}^{J} \eta_{rg} \upsilon_{rg} \hat{\upsilon}_{rg}}{\sum_{g=1}^{J} \eta_{rg} \upsilon_{rg}} \right)^{\eta_{rj}} \prod_{g=1}^{J} \hat{P}_{rg}^{\ \mu_{rjg}} \prod_{k=1}^{\mathfrak{K}} \hat{\iota}_{rk}^{\beta_{rjk}},$$

$$\hat{\iota}_{rk} = \frac{\sum_{j=1}^{J} \beta_{rjk} \upsilon_{rj} \hat{\upsilon}_{rj}}{\sum_{j=1}^{J} \beta_{rjk} \upsilon_{rj}} \left(\frac{\frac{1-t'_r}{1-d'_{rk} t'_r} \sum_{j=1}^{J} \beta_{rjk} \upsilon_{rj} \hat{\upsilon}_{rj}}{\frac{1-t_r}{1-d_{rk} t_r} \sum_{j=1}^{J} \beta_{rjk} \upsilon_{rj} \hat{\upsilon}_{rj}} \right)^{\varepsilon_\delta - 1} \left(\frac{\sum_{s=1}^{R} \sum_{m=1}^{\mathfrak{K}} \frac{1-t'_s}{1-d'_{sm} t'_s} \sum_{j=1}^{J} \beta_{sjm} \upsilon_{sj} \hat{\upsilon}_{sj}}{\sum_{s=1}^{R} \sum_{m=1}^{\mathfrak{K}} \frac{1-t_s}{1-d_{sm} t_s} \sum_{j=1}^{J} \beta_{sjm} \upsilon_{sj}} \right)^{1-\varepsilon_\delta}.$$

4.A.2 Additional Figures

Figure 4.A.1: *Regional Effects of a Common EU Tax Policy*



4.A.3 Data

WIOD Preparation

For each combination of countries and sectors the WIOT contains an entry $X_{ni,jk}$ for the value of flows from industry k in supplier country i to industry j in destination country n, including within-country flows $X_{ii,jk}$. It also provides the values of flows from industry k in country i to country n that end up as final consumption by households $X_{ni,Ck}$, final consumption by non-profit organizations $X_{ni,Pk}$, government spending $X_{ni,Gk}$, investments $X_{ni,Ik}$ and inventory changes $X_{ni,Qk}$.

Of course, inventory changes can be negative and sometimes they are significantly large. If we were to calculate final demand by simply summing over consumption, investment, government spending and inventory changes we would end up with a negative final demand in some cases. To reconcile the real world data with our static model that has no room for inventories we follow Costinot and Rodríguez-Clare (2014) and split the vector of inventory changes into a vector with all positive changes $X_{ni,Qk+}$ and one with all negative changes $X_{ni,Qk-}$ and treat them as follows.

Positive inventory changes are directly included in final demand as are final consumption, government spending and investments. Therefore, we treat the build-up of inventory as if it was consumed in the current period. Formally, final demand in country n for goods from industry k in country i, $X_{ni,Fk}$, is thus defined as $X_{ni,Fk} = X_{ni,Ck} + X_{ni,Pk} + X_{ni,Gk} + X_{ni,Ik} + X_{ni,Qk+}$.

In contrast, negative inventory changes are treated as if they were produced (and consumed) in the current period. To do this, we cannot simply increase our output vector by the respective (absolute) value of inventory changes because the production of the inventory in the last period also requires intermediates and, thus, has a larger overall effect. To see how to calculate the necessary changes consider N countries and K sectors in matrix notation. X is the original $(N \cdot K) \times 1$-vector of total outputs, A the $(N \cdot K) \times (N \cdot K)$ matrix of input coefficients, F the $(N \cdot K) \times 1$ vector of final demand including positive inventory changes and Inv the $(N \cdot K) \times 1$ vector of negative inventory changes. Then the total output can be calculated as the sum of intermediate flows, final demand, and inventory changes as $X = AX + F + Inv$. We want to calculate the new level X_{new} for which the final demand vector is unchanged but inventory changes Inv are set to 0, which is the total output, if the negative inventory changes have been produced in the current period. Rearranging terms we get $X_{new} = (E - A)^{-1}F$, where E is the unit matrix. We then obtain the new input-output matrix by combining intermediate good flows AX_{new} and the unchanged final demand vector F.

Concordance Table

Table 4.A.1: *Sector Correspondence*

This Paper		Shipment Data ETISPlus		WIOD	
#	Label	#	Label	#	Label
1	Agriculture	GT01	Products of agriculture, hunting, and forestry; fish and and other fishing product	A01 A02 A03	Crop and animal production, hunting and related service activities Forestry and logging Fishing and aquaculture
2	Mining	GT02 GT03	Coal and lignite; crude petroleum and natural gas Metal ores and other mining and quarrying products; peat; uranium and thorium	B	Mining and quarrying
3	Food, Beverages, Tobacco	GT04	Food products, beverages and tobacco	C10 - C12	Manufacture of food products, beverages and tobacco products
4	Textiles, Leather	GT05	Textiles and textile products; leather and leather product	C13 - C15	Manufacture of textiles, wearing apparel and leather products
5	Wood, Paper, Printing	GT06	Wood and products of wood and cork (except furniture); articles	C16 C17 C18	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials Manufacture of paper and paper products Printing and reproduction of recorded media
6	Petroleum, Coke	GT07	Coke and refined petroleum products	C19	Manufacture of coke and refined petroleum products
7	Chemicals, Pharmaceuticals	GT08	Chemicals, chemical products, and man-made fibers; rubber and plastic products, nuclear fuel	C20 C21 C22	Manufacture of chemicals and chemical products Manufacture of basic pharmaceutical products and preparations Manufacture of rubber and plastic products
8	Non-Metallic Minerals	GT09	Other non metallic mineral products	C23	Manufacture of other non-metallic mineral products
9	Metal	GT10	Basic metals; fabricated metal products, except machinery and equipment	C24 C25	Manufacture of basic metals Manufacture of fabricated metal products, except machinery and equipment

This Paper		Shipment Data ETISPlus		WIOD	
#	Label	#	Label	#	Label
10	Machinery, Electrical Equipment	GT11	Machinery and equipment n.e.c.; office machinery and computers	C26	Manufacture of computer, electronic and optical products
				C27	Manufacture of electrical equipment
				C28	Manufacture of machinery and equipment n.e.c.
11	Transport Equipment	GT12	Transport equipment	C29	Manufacture of motor vehicles, trailers and semi-trailers
				C30	Manufacture of other transport equipment
12	Furniture and other Manufacturing	GT13	Furniture; other manufactured goods n.e.c.	C31 - C32	Manufacture of furniture; other manufacturing
				C33	Repair and installation of machinery and equipment

The used shipment data from ETISPlus is based on the NST/R 2-digit goods classification. The used harmonised data is grouped into 20 product groups (GT01-GT20). Four product groups are dropped because there is no or no sufficient match to our final sectors ("Secondary raw materials", "Mail, parcels", "Equipment and material utilized in the transport of goods", "Goods moved in the course of household and office removals"). Three further categories ("Grouped goods", "Unidentifiable goods", "Other goods") can not be matched directly and are instead used to scale bilateral flows in all other sectors. The WIOD Sectors D35-U which cover construction, utilities and services can not be matched to our final sectors and therefore are dropped.

Table 4.A.2: *Sector Correspondence for Modelled Shipment Data*

This Paper		Shipment Data ETISPlus (NST/R 2-digit)			WIOD	
#	Label	#	weight	Label	#	Label
1	Agriculture	0	1	Live animals	A01	Crop and animal production, hunting and related service activities
		1	1	Cereals		
		2	1	Potatoes		
		3	1	Other fresh or frozen fruit and vegetables	A02	Forestry and logging
					A03	Fishing and aquaculture
		4	1	Textiles textile articles and man-made fibres		
		5	1	Wood and cork		
		6	1	Sugar-beet		
		9	1	Other raw animal and vegetable materials		
		13	0.16	Stimulants and spices		
2	Mining	21	1	Coal	B	Mining and quarrying
		22	0.86	Lignite and peat		
		31	1	Crude petroleum		
		41	1	Iron-ore		
		45	0.24	Non-ferrous ores and waste		
		61	1	Sand gravel clay and slag		
		62	0.37	Salt iron pyrites sulphur		
		63	1	Other stone earths and minerals		
		71	1	Natural fertilisers		
3	Food, Beverages, Tobacco	11	1	Sugars	C10-C12	Manufacture of food products, beverages and tobacco products
		12	1	Beverages		
		13	0.84	Stimulants and spices		
		14	1	Perishable foodstuffs		
		16	1	Other non-perishable foodstuffs and hops		
		17	1	Animal food and foodstuff waste		
		18	1	Oil seeds and oleaginous fruit and fats		
		62	0.23	Salt iron pyrites sulphur		
4	Textiles, Leather	96	1	Leather textiles and clothing	C13-C15	Manufacture of textiles, wearing apparel and leather products
5	Wood, Paper, Printing	97	1	Paper pulp and waste paper	C16	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
		62	0.84	Other manufactured articles		
					C17	Manufacture of paper and paper products
					C18	Printing and reproduction of recorded media

This Paper		Shipment Data ETISPlus (NST/R 2-digit)			WIOD	
#	Label	#	weight	Label	#	Label
6	Petroleum, Coke	22	0.14	Lignite and peat	C19	Manufacture of coke and refined petroleum products
		23	1	Coke		
		32	1	Fuel derivatives		
		33	1	Gaseous hydrocarbons liquid or compressed		
		34	1	Non-fuel derivatives		
		83	1	Coal chemicals		
7	Chemicals, Pharma-ceuticals	62	0.40	Salt iron pyrites sulphur	C20	Manufacture of chemicals and chemical products
		72	1	Chemical fertilisers		
		81	1	Basic chemicals	C21	Manufacture of basic pharmaceutical products and preparations
		82	1	Aluminium oxide and hydroxide		
		89	1	Other chemical products	C22	Manufacture of rubber and plastic products
		97	0.09	Other manufactured articles		
8	Non-Metallic Minerals	64	1	Cement lime	C23	Manufacture of other non-metallic mineral products
		65	1	Plasters		
		69	1	Other manufactured building materials		
		95	1	Glass, glassware ceramic products		
9	Metal	45	0.16	Non-ferrous ores and waste	C24	Manufacture of basic metals
		51	1	Pig iron and crude steel	C25	Manufacture of fabricated metal products, except machinery and equipment
		52	1	Semi-finished rolled steel products		
		53	1	Bars sections wire rod railway and tramway track construction material of iron or steel		
		54	1	Steel sheets plates hoop and strip		
		55	1	Tubes pipes iron and steel castings and forgings		
		56	1	Non-ferrous metals		
		94	1	Manufactures of material		
		99	1	Miscellaneous articles		
10	Machinery, Electrical Equipment	92	1	Tractors	C26	Manufacture of computer, electronic and optical products
		93	1	Other machinery apparatus and appliances engines parts thereof	C27	Manufacture of electrical equipment
					C28	Manufacture of machinery and equipment n.e.c.
11	Transport Equipment	91	1	Transport equipment	C29	Manufacture of motor vehicles, trailers and semi-trailers
					C30	Manufacture of other transport equipment

This Paper		Shipment Data ETISPlus (NST/R 2-digit)			WIOD	
#	Label	#	weight	Label	#	Label
12	Furniture and other Manufacturing	97	0.07	Other manufactured articles	C31, C32	Manufacture of furniture; other manufacturing
					C33	Repair and installation of machinery and equipment
	Dropped	45	0.61	Non-ferrous ores and waste		
		46	1	Iron and steel waste and blast furnace dust		

We match the NST/R 2-digit product groups to 12 sectors. Most of the product groups fit quite closely into one of the sectors. Five of the product groups cannot be uniquely matched to one sector and a substantial share (more than 5%) belongs to a second or third sector ("stimulants and spices", "lignite and peat", "non-ferrous ores and waste", "salt iron pyrites sulphur" and "other manufacturing articles"). In those cases, we calculate weights using the converting key between NST/R and NST 2007 of the Statistische Bundesamt (Einheitliches Güterverzeichnis für die Verkehrsstatistik – 2007, Umsteigeschlüssel zwischen NST/R und NST-2007) in combination with NST/R 3-digit sector weights which we calculate from the EXTRA-EU trade data from Eurostat for the year 2010 from the EU to rest of the world. Those sectors for which a specific product group share is below five percentage points are not considered and the shares are proportionally adjusted such that they add up to one. We drop the product group "iron and steel waste and blast furnace dust" as we exclude the waste sector in our approach. Using the same argumentation, we also drop the share of 60.1% for the product group "non-ferrous ores and waste as it captures the part belonging to the waste sector.

4.B Supplementary Appendix

4.B.1 Theoretical Framework

Mobility Equation. We assume that the individual-region-specific amenity draw $a_r(\Omega)$ is identically and independently distributed Fréchet, where the cumulative distribution function of taste shocks is given by:

$$Pr[a_r(\Omega) \leq a] = e^{-A_r a^{-1/\varepsilon}}. \tag{4.3}$$

This functional form implies that the utility of individual Ω in region r will also be distributed Fréchet with the CDF $\mathbb{G}_r(u)$ inherited from (4.3):

$$\mathbb{G}_r(u) = \Pr\left[u_r(\Omega) \leq u\right] = \Pr\left[a_r(\Omega) C_r \leq u\right] = \Pr\left[a_r(\Omega) \leq \frac{u}{C_r}\right] = e^{-A_r\left(\frac{u}{C_r}\right)^{-\frac{1}{\varepsilon}}}.$$

The probability that any individual Ω will prefer region r over all other regions can then be derived by multiplying the probability of drawing a value smaller u in all regions $s \in R \setminus r$ with the probability of obtaining exactly utility level u in r (i.e. $d\mathbb{G}_r(u)/du$) integrated for all possible utility levels from 0 to infinity. Given a continuum of consumers this probability must also equal the share of consumers that choose to live in location r in equilibrium:

$$\begin{aligned}
\frac{L_r}{L} &= \Pr\left[u_r(\Omega) \geq \max_{s \in R \setminus r} u_s(\Omega)\right] = \int_0^\infty \Pr\left[u \geq \max_{s \in R \setminus r} u_s(\Omega)\right] \frac{d\mathbb{G}_r(u)}{du} du \\
&= \int_0^\infty \prod_{s \in R \setminus r} \Pr\left[u_s(\Omega) \leq u\right] \frac{d\mathbb{G}_r(u)}{du} du \\
&= \int_0^\infty \prod_{s \in R \setminus r} \left(e^{-A_s\left(\frac{u}{C_s}\right)^{-\frac{1}{\varepsilon}}}\right) e^{-A_r\left(\frac{u}{C_r}\right)^{-\frac{1}{\varepsilon}}} A_r \frac{1}{\varepsilon} C_r^{\frac{1}{\varepsilon}} u^{-\frac{1}{\varepsilon}-1} du \\
&= A_r C_r^{\frac{1}{\varepsilon}} \int_0^\infty e^{-u^{-\frac{1}{\varepsilon}} \sum_{s \in R} A_s C_s^{\frac{1}{\varepsilon}}} \frac{1}{\varepsilon} u^{-\frac{1}{\varepsilon}-1} du = A_r C_r^{\frac{1}{\varepsilon}} \left[\frac{e^{-u^{-\frac{1}{\varepsilon}} \sum_{s \in R} A_s C_s^{\frac{1}{\varepsilon}}}}{\sum_{s \in R} A_s C_s^{\frac{1}{\varepsilon}}}\right]_0^\infty \\
&= \frac{A_r C_r^{\frac{1}{\varepsilon}}}{\sum_{s \in R} A_s C_s^{\frac{1}{\varepsilon}}} \left[e^{-\sum_{s \in R} A_s\left(\frac{u}{C_s}\right)^{-\frac{1}{\varepsilon}}}\right]_0^\infty = \frac{A_r C_r^{\frac{1}{\varepsilon}}}{\sum_{s \in R} A_s C_s^{\frac{1}{\varepsilon}}}.
\end{aligned} \tag{4.4}$$

Prices Indices. Productivity is identically and independently distributed Fréchet on a sector region level. The CDF of productivities is given by:

$$\Pr[z_{rj}(\omega) \leq z] = e^{-T_{rj} z^{-\theta_j}}. \tag{4.B.1}$$

This functional form implies that the prices that region r offers to region s in sector j are also distributed Fréchet with the CDF $\mathbb{F}_{rsj}(p)$ given by:

$$\mathbb{F}_{rsj}(p) = \Pr\left[p_{rsj}(\omega) \leq p\right] = \Pr\left[\frac{c_{rj}\tau_{rsj}}{z_{rj}(\omega)} \leq p\right] = \Pr\left[\frac{c_{rj}\tau_{rsj}}{p} \leq z_{rj}(\omega)\right]$$
$$= 1 - \Pr\left[z_{rj}(\omega) \leq \frac{c_{rj}\tau_{rsj}}{p}\right] = 1 - e^{-T_{rj}\left(\frac{c_{rj}\tau_{rsj}}{p}\right)^{-\theta_j}}.$$

The equilibrium price in region s for variety ω in sector j is given by $p_{sj}(\omega) \equiv \min_r p_{rsj}(\omega)$. Let us denote the probability $\mathbb{F}_{sj}(p)$ that this lowest price is below some price p as follows:

$$\mathbb{F}_{sj}(p) = \Pr\left[\min_r p_{rsj}(\omega) \leq p\right] = 1 - \Pr\left[\min_r p_{rsj}(\omega) > p\right] = 1 - \prod_{r=1}^{R} \Pr\left[p_{rsj}(\omega) > p\right]$$
$$= 1 - \prod_{r=1}^{R}\left(1 - \mathbb{F}_{rsj}(p)\right) = 1 - \prod_{r=1}^{R}\left(1 - \left(1 - e^{-T_{rj}\left(\frac{c_{rj}\tau_{rsj}}{p}\right)^{-\theta_j}}\right)\right) = 1 - \prod_{r=1}^{R} e^{-T_{rj}\left(\frac{c_{rj}\tau_{rsj}}{p}\right)^{-\theta_j}}$$
$$= 1 - e^{-\sum_{r=1}^{R} T_{rj}\left(\frac{c_{rj}\tau_{rsj}}{p}\right)^{-\theta_j}} = 1 - e^{-p^{\theta_j}\sum_{r=1}^{R} T_{rj}(c_{rj}\tau_{rsj})^{-\theta_j}} = 1 - e^{-p^{\theta_j}\Phi_{sj}},$$

with $\Phi_{sj} \equiv \sum_{r=1}^{R} T_{rj}\left(c_{rj}\tau_{rsj}\right)^{-\theta_j}$. The CES price index in sector j in region s can then be derived in the following way:

$$P_{sj} = \left(\int_0^1 p_{sj}\left(\omega\right)^{1-\sigma_j} d\omega\right)^{\frac{1}{1-\sigma_j}}$$
$$P_{sj}^{1-\sigma_j} = \int_0^1 p_{sj}\left(\omega\right)^{1-\sigma_j} d\omega = \int_0^\infty p^{1-\sigma_j}\frac{d\mathbb{F}_{sj}(p)}{dp}dp$$
$$= \int_0^\infty p^{1-\sigma_j}\theta_j\Phi_{sj}p^{\theta_j-1}e^{-p^{\theta_j}\Phi_{sj}}dp.$$

Defining $x \equiv p^{\theta_j}\Phi_{sj}$ we get:

$$P_{sj}^{1-\sigma_j} = \int_0^\infty \left(\frac{x}{\Phi_{sj}}\right)^{\frac{1-\sigma_j}{\theta_j}} \frac{dx}{dp}e^{-x}dp = \int_0^\infty \left(\frac{x}{\Phi_{sj}}\right)^{\frac{1-\sigma_j}{\theta_j}} e^{-x}dx$$
$$= \Phi_{sj}^{-\frac{1-\sigma_j}{\theta_j}} \int_0^\infty x^{\frac{1-\sigma_j}{\theta_j}} e^{-x}dx = \Phi_{sj}^{-\frac{1-\sigma_j}{\theta_j}}\,\Gamma\left(\frac{\theta_j + 1 - \sigma_j}{\theta_j}\right),$$

where $\Gamma(t) \equiv \int_0^\infty x^{t-1}e^{-x}\,dx$ is the gamma function. Consequently:

$$P_{sj} = \Phi_{sj}^{-\frac{1}{\theta_j}}\,\Gamma\left(\frac{\theta_j + 1 - \sigma_j}{\theta_j}\right)^{\frac{1}{1-\sigma_j}}$$
$$= \Gamma\left(\frac{\theta_j + 1 - \sigma_j}{\theta_j}\right)^{\frac{1}{1-\sigma_j}} \left(\sum_{r=1}^{R} T_{rj}\left(c_{rj}\tau_{rsj}\right)^{-\theta_j}\right)^{-\frac{1}{\theta_j}}. \tag{4.10}$$

Expenditure Shares. By the law of large numbers the share π_{rsj} of all varieties from sector j bought by region s that is produced in region r is given by the probability that no other region offers one particular variety to region s at a lower price than r:

$$
\begin{aligned}
\pi_{rsj} &= \Pr\left[p_{rsj}(\omega) < \min_{t \in R \setminus r} p_{tsj}(\omega)\right] = \int_0^\infty \Pr\left[\min_{t \in R \setminus r} p_{tsj}(\omega) > p\right] \frac{d\mathbb{F}_{rsj}(p)}{dp} dp \\
&= \int_0^\infty \prod_{t \in R \setminus r} \left(1 - \Pr\left[p_{tsj}(\omega) \leq p\right]\right) \frac{d\mathbb{F}_{rsj}(p)}{dp} dp = \int_0^\infty \prod_{t \in R \setminus r} \left(1 - \mathbb{F}_{tsj}(p)\right) \frac{d\mathbb{F}_{rsj}(p)}{dp} dp \\
&= \int_0^\infty \prod_{t \in R \setminus r} \left(e^{-T_{tj}\left(\frac{c_{tj}\tau_{tsj}}{p}\right)^{-\theta_j}}\right) \frac{d\mathbb{F}_{rsj}(p)}{dp} dp = \int_0^\infty e^{-\sum_{t \in R \setminus r} T_{tj}\left(\frac{c_{tj}\tau_{tsj}}{p}\right)^{-\theta_j}} \frac{d\mathbb{F}_{rsj}(p)}{dp} dp \\
&= \int_0^\infty e^{-\sum_{t=1}^R T_{tj}\left(\frac{c_{tj}\tau_{tsj}}{p}\right)^{-\theta_j}} \left(\theta_j T_{rj} \left(\frac{c_{rj}\tau_{rsj}}{p}\right)^{-\theta_j} \frac{1}{p}\right) dp \\
&= T_{rj}\left(c_{rj}\tau_{rsj}\right)^{-\theta_j} \int_0^\infty \theta_j p^{\theta_j - 1} e^{-p^{\theta_j} \sum_{t=1}^R T_{tj}(c_{tj}\tau_{tsj})^{-\theta_j}} dp \\
&= T_{rj}\left(c_{rj}\tau_{rsj}\right)^{-\theta_j} \left[\frac{1 - e^{-p^{\theta_j} \sum_{t=1}^R T_{tj}(c_{tj}\tau_{tsj})^{-\theta_j}}}{\sum_{t=1}^R T_{tj}\left(c_{tj}\tau_{tsj}\right)^{-\theta_j}}\right]_0^\infty = \frac{T_{rj}\left(\tau_{rsj}c_{rj}\right)^{-\theta_j}}{\sum_{t=1}^R T_{tj}\left(\tau_{tsj}c_{tj}\right)^{-\theta_j}}. \qquad (4.11)
\end{aligned}
$$

The distribution of prices of what region s actually buys from any region r is independent from conditioning on the region. To see this we derive the respective conditional CDF:

$$
\begin{aligned}
&\Pr\left[p_{rsj} \leq p | p_{rsj} \leq \min_{t \in R \setminus r} p_{tsj}\right] = \frac{1}{\pi_{rsj}} \int_0^p \Pr\left[\min_{t \in R \setminus r} p_{tsj} > \rho\right] \frac{d\mathbb{F}_{rsj}(\rho)}{d\rho} d\rho \\
&\quad = \frac{1}{\pi_{rsj}} \int_0^p \prod_{t \in R \setminus r} \Pr\left[p_{tsj} > \rho\right] \frac{d\mathbb{F}_{rsj}(\rho)}{d\rho} d\rho = \frac{1}{\pi_{rsj}} \int_0^p \prod_{t \in R \setminus r} \left(1 - \mathbb{F}_{tsj}(\rho)\right) \frac{d\mathbb{F}_{rsj}(\rho)}{d\rho} d\rho.
\end{aligned}
$$

Using the derivation of the trade shares above this is equal to:

$$
\frac{1}{\pi_{rsj}} \pi_{rsj} \left[1 - e^{-p^{\theta_j} \sum_{t=1}^R T_{tj}(c_{tj}\tau_{tsj})^{-\theta_j}}\right]_0^p = 1 - e^{-p^{\theta_j} \sum_{t=1}^R T_{tj}(c_{tj}\tau_{tsj})^{-\theta_j}} = \mathbb{F}_{sj}(p).
$$

As the distribution of prices of what s buys from any region r in sector j is the same for all r, it must be true that the share of varieties that region s buys from region r in sector j, i.e. π_{rsj}, is also the share of expenditure of region s in sector j on varieties from region r.

Factor Demand. Firms' profit maximisation (4.12) yields in the following set of optimality conditions:

$$(1-t_r)\left(\frac{\partial v_{rj}(\omega)}{\partial l_{rj}(\omega)} - w_r\right) \stackrel{!}{=} 0, \qquad (1-t_r)\frac{\partial v_{rj}(\omega)}{\partial \kappa_{rjk}(\omega)} - (1-t_r d_{rk})\,\iota_{rk} \stackrel{!}{=} 0,$$
$$(1-t_r)\frac{\partial v_{rj}(\omega)}{\partial h_{rj}(\omega)} - (1-t_r d_{rh})\,s_r \stackrel{!}{=} 0, \qquad (1-t_r)\left(\frac{\partial v_{rj}(\omega)}{\partial m_{rjg}(\omega)} - P_{rg}\right) \stackrel{!}{=} 0.$$

These optimality conditions can be rewritten indicating the marginal costs of the input factors in the following fashion:

$$\frac{\partial v_{rj}(\omega)}{\partial l_{rj}(\omega)} = w_r, \qquad \frac{\partial v_{rj}(\omega)}{\partial m_{rjg}(\omega)} = P_{rg},$$
$$\frac{\partial v_{rj}(\omega)}{\partial h_{rj}(\omega)} = \frac{1-d_{rh}t_r}{1-t_r}s_r \equiv \tilde{s}_r, \qquad \frac{\partial v_{rj}(\omega)}{\partial \kappa_{rjk}(\omega)} = \frac{1-d_{rk}t_r}{1-t_r}\iota_{rk} \equiv \tilde{\iota}_{rk}.$$

Using the production technology (4.8) together with the fact that revenues generated from a single variety are given by $v_{rj}(\omega) = p_{rj}(\omega)q_{rj}(\omega)$ yields marginal revenues as:

$$\frac{\partial v_{rj}(\omega)}{\partial l_{rj}(\omega)} = \gamma_{rj}\frac{v_{rj}(\omega)}{l_{rj}(\omega)}, \qquad \frac{\partial v_{rj}(\omega)}{\partial h_{rj}(\omega)} = \mu_{rjg}\frac{v_{rj}(\omega)}{m_{rjg}(\omega)},$$
$$\frac{\partial v_{rj}(\omega)}{\partial m_{rjg}(\omega)} = \eta_{rj}\frac{v_{rj}(\omega)}{h_{rj}(\omega)}, \qquad \frac{\partial v_{rj}(\omega)}{\partial \kappa_{rjk}(\omega)} = \beta_{rjk}\frac{v_{rj}(\omega)}{\kappa_{rjk}(\omega)}.$$

This allows to solve for the factor demands:

$$l_{rj}(\omega) = \frac{\gamma_{rj}v_{rj}(\omega)}{w_r}, \qquad m_{rjg}(\omega) = \frac{\mu_{rjg}v_{rj}(\omega)}{P_{rg}},$$
$$h_{rj}(\omega) = \frac{\eta_{rj}v_{rj}(\omega)}{\tilde{s}_r}, \qquad \kappa_{rjk}(\omega) = \frac{\beta_{rjk}v_{rj}(\omega)}{\tilde{\iota}_{rk}}. \tag{4.B.2}$$

Denoting $\int_0^1 x(\omega)d\omega \equiv x$, sector factor demand follows as:

$$l_{rj} = \frac{\gamma_{rj}v_{rj}}{w_r}, \qquad m_{rjg} = \frac{\mu_{rjg}v_{rj}}{P_{rg}},$$
$$h_{rj} = \frac{\eta_{rj}v_{rj}}{\tilde{s}_r}, \qquad \kappa_{rjk} = \frac{\beta_{rjk}v_{rj}}{\tilde{\iota}_{rk}}. \tag{4.13}$$

Marginal Production Costs per Efficiency Unit of Output. Using (4.B.2) to express the demand for other factors in terms of labour demand yields:

$$h_{rj}(\omega) = l_{rj}(\omega)\frac{\eta_{rj}}{\gamma_{rj}}\frac{w_r}{\tilde{s}_r}, \qquad m_{rjg}(\omega) = l_{rj}(\omega)\frac{\mu_{rjg}}{\gamma_{rj}}\frac{w_r}{P_{rg}}, \qquad \kappa_{rjk}(\omega) = l_{rj}(\omega)\frac{\beta_{rjk}}{\gamma_{rj}}\frac{w_r}{\tilde{\iota}_{rk}}.$$

Plugging those results into the production function allows to solve for the optimal labour input in terms of quantity:

$$q_{rj}(\omega) = z_{rj}(\omega)\, l_{rj}(\omega) \left(\frac{w_r}{\tilde{s}_r}\frac{\eta_{rj}}{\gamma_{rj}}\right)^{\eta_{rj}} \left(\prod_{g=1}^{J}\left(\frac{w_r}{P_{rg}}\frac{\mu_{rjg}}{\gamma_{rj}}\right)^{\mu_{rjg}}\right) \left(\prod_{k=1}^{\mathfrak{K}}\left(\frac{w_r}{\tilde{\iota}_{rk}}\frac{\beta_{rjk}}{\gamma_{rj}}\right)^{\beta_{rjk}}\right)$$

$$= z_{rj}(\omega)\, l_{rj}(\omega) \left(\frac{\gamma_{rj}}{w_r}\right)^{\gamma_{rj}-1} \left(\frac{\eta_{rj}}{\tilde{s}_r}\right)^{\eta_{rj}} \left(\prod_{g=1}^{J}\left(\frac{\mu_{rjg}}{P_{rg}}\right)^{\mu_{rjg}}\right) \left(\prod_{k=1}^{\mathfrak{K}}\left(\frac{\beta_{rjk}}{\tilde{\iota}_{rk}}\right)^{\beta_{rjk}}\right)$$

$$\Rightarrow l_{rj}(\omega) = \frac{\gamma_{rj}}{w_r}\frac{q_{rj}(\omega)}{z_{rj}(\omega)}\chi_{rj} w_r^{\gamma_{rj}} \tilde{s}_r^{\eta_{rj}} \left(\prod_{g=1}^{J} P_{rg}^{\mu_{rjg}}\right)\left(\prod_{k=1}^{\mathfrak{K}} \tilde{\iota}_{rk}^{\beta_{rjk}}\right).$$

Where, for notational purposes, we have defined the region and sector specific constant $\chi_{rj} = \left(\gamma_{rj}^{\gamma_{rj}} \eta_{rj}^{\eta_{rj}} \prod_{g=1}^{J} \mu_{rjg}^{\mu_{rjg}} \prod_{k=1}^{\mathfrak{K}} \beta_{rjk}^{\beta_{rjk}}\right)^{-1}$. Since perfect competition and constant returns to sclae imply that revenue must equal total costs we can use (4.B.2) to derive the tax inclusive marginal costs per efficiency unit of output as:

$$c_{rj} = \frac{dv_{rj}(\omega)}{d\left(\frac{q_{rj}(\omega)}{z_{rj}(\omega)}\right)} = \frac{d\left(\frac{l_{rj}(\omega)w_{rj}}{\gamma_{rj}}\right)}{d\left(\frac{q_{rj}(\omega)}{z_{rj}(\omega)}\right)} = \chi_{rj} w_r^{\gamma_{rj}} \tilde{s}_r^{\,\eta_{rj}} \prod_{g=1}^{J} P_{rg}^{\,\mu_{rjg}} \prod_{k=1}^{\mathfrak{K}} \tilde{\iota}_{rk}^{\,\beta_{rjk}}. \tag{4.14}$$

Government Budget. Plugging optimal factor demands (4.13) into the tax base of a firm, summing over all sectors and all regions and integrating over all varieties within a sector, using the constant returns to scale property of the production function and thus $1 - \gamma_{rj} - \sum_{g=1}^{J} \mu_{rjg} = \eta_{rj} + \sum_{k=1}^{\mathfrak{K}} \iota_{rk}$ as well as the definitions of $\tilde{s}_r$ and $\tilde{\iota}_{rk}$ the budget G_n of the government in country n can be written as:

$$G_n = \sum_{r\in\Re_n}\sum_{j=1}^{J}\int_0^\infty t_r \left[v_{rj}(\omega) - w_r l_{rj}(\omega) - \sum_{g=1}^{J} P_{rg} m_{rjg}(\omega) \right.$$
$$\left. - d_{rh} s_r h_{rj}(\omega) - \sum_{k=1}^{\mathfrak{K}} d_{rk}\iota_{rk}\kappa_{rjk}(\omega)\right] d\omega$$
$$= \sum_{r\in\Re_n}\sum_{j=1}^{J} t_r v_{rj}\left[1 - \gamma_{rj} - \sum_{g=1}^{J}\mu_{rjg} - d_{rh}s_r\frac{\eta_{rj}}{\tilde{s}_r} - \sum_{k=1}^{\mathfrak{K}} d_{rk}\iota_{rk}\frac{\beta_{rjk}}{\tilde{\iota}_{rk}}\right]$$
$$= \sum_{r\in\Re_n}\sum_{j=1}^{J} t_r v_{rj}\left[\eta_{rj} + \sum_{k=1}^{\mathfrak{K}}\beta_{rk} - d_{rh}s_r\frac{\eta_{rj}}{\frac{1-d_{rh}t_r}{1-t_r}s_r} - \sum_{k=1}^{\mathfrak{K}} d_{rk}\iota_{rk}\frac{\beta_{rjk}}{\frac{1-d_{rk}t_r}{1-t_r}\iota_{rk}}\right]$$
$$= \sum_{r\in\Re_n}\sum_{j=1}^{J} t_r v_{rj}\left[\eta_{rj} + \sum_{k=1}^{\mathfrak{K}}\beta_{rk} - \frac{(1-t_r)d_{rh}}{1-d_{rh}t_r}\eta_{rj} - \sum_{k=1}^{\mathfrak{K}}\frac{(1-t_r)d_{rk}}{1-d_{rk}t_r}\beta_{rjk}\right]$$
$$= \sum_{r\in\Re_n}\sum_{j=1}^{J} t_r v_{rj}\left(\frac{1-d_{rh}}{1-d_{rh}t_r}\eta_{rj} + \sum_{k=1}^{\mathfrak{K}}\frac{1-d_{rk}}{1-d_{rk}t_r}\beta_{rjk}\right). \tag{4.15}$$

User Cost of Capital Assets. Let K_{rk} denote the capital asset stock of type k in region r that can be used in production. Per unit compensation ι_{rk} of this stock can be derived using the optimality condition from the firm maximisation (4.13). Aggregating across all sectors yields the per unit compensation where we use $\tilde{\iota}_{rk} \equiv \iota_{rk} \frac{1-d_{rk}t_r}{1-t_r}$ to denote the user cost of capital assets:

$$\kappa_{rjk} = \frac{\beta_{rjk} \upsilon_{rj}}{\tilde{\iota}_{rk}} \tag{4.13}$$

$$\iota_{rk} = \frac{1-t_r}{1-d_{rk}t_r} \frac{\beta_{rjk}\upsilon_{rj}}{\kappa_{rjk}}$$

$$\sum_{j=1}^{J} \iota_{rk}\kappa_{rjk} = \sum_{j=1}^{J} \frac{1-t_r}{1-d_{rk}t_r} \beta_{rjk}\upsilon_{rj}$$

$$\iota_{rk} = \frac{\frac{1-t_r}{1-d_{rk}t_r} \sum_{j=1}^{J} \beta_{rjk}\upsilon_{rj}}{\sum_{j=1}^{J} \kappa_{rjk}} = \frac{\frac{1-t_r}{1-d_{rk}t_r} \sum_{j=1}^{J} \beta_{rjk}\upsilon_{rj}}{K_{rk}}. \tag{4.24}$$

The user cost of capital assets and the change in user cost thus are given by:

$$\tilde{\iota}_{rk} = \frac{\sum_{j=1}^{J} \beta_{rjk}\upsilon_{rj}}{K_{rk}}$$

$$\hat{\tilde{\iota}}_{rk} = \frac{\sum_{j=1}^{J} \beta_{rjk}\upsilon_{rj}\hat{\upsilon}_{rj}}{\sum_{j=1}^{J} \beta_{rjk}\upsilon_{rj}} \hat{K}_{rk}^{-1}. \tag{4.B.3}$$

Setting and Compensation. We assume that a worldwide stock of base capital can be transformed into any asset type in any region subject to a transformation cost. Specifically, we assume that this transformation cost varies for each marginal unit of the heterogeneous world endowment with base capital K and implies that if capital unit i is employed in region r and asset type k only $\delta_{rk}(i)$ units will be added to K_{rk}. For each marginal unit of capital the transformation cost for use in each possible region and asset type is drawn from region and asset type specific Fréchet distributions:

$$\Pr\left[\delta_{rk}(i) \le \delta\right] = e^{-\bar{\delta}_{rk}\delta^{-\frac{1}{\varepsilon_\delta}}}.$$

The distribution of potential compensation of capital in region r and asset class k is:

$$\mathbb{G}_{\iota,rk}(\iota) = \Pr\left[\iota_{rk}(i) \le \iota\right] = \Pr\left[\delta_{rk}(i)\,\iota_{rk} \le \iota\right] = e^{-\bar{\delta}_{rk}\left(\frac{\iota}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}}$$

$$\frac{d\mathbb{G}_{\iota,rk}(\iota)}{d\iota} = \bar{\delta}_{rk} e^{-\bar{\delta}_{rk}\left(\frac{\iota}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}} \iota_{rk}^{\frac{1}{\varepsilon_\delta}} \iota^{-\frac{1}{\varepsilon_\delta}-1} \frac{1}{\varepsilon_\delta}.$$

One might argue that the transformation cost factor should take values between 0 and 1, while here it is between 0 and infinity, but as we solve the model in changes we neither need to take a stand on nominal values of capital asset stocks nor on the world base capital endowment. Therefore, this is just a matter of normalisation.

Capital Shares by Region and Asset Class. We use r, s to index regions, j, g to index sectors and k, m to index capital asset class. The probability of any base capital unit having the highest capital compensation in region r and asset class k is:

$$
\begin{aligned}
&\Pr\left[\iota_{rk}(i) \geq \max_{s,m\neq r,k} \iota_{sm}(i)\right] \\
&\quad = \int_0^\infty \Pr\left[\iota \geq \max_{s,m\neq r,k} \iota_{sm}(i)\right] \frac{d\mathbb{G}_{\iota,rk}(\iota)}{d\iota} d\iota \\
&\quad = \int_0^\infty \prod_{s,m\neq r,k} \Pr\left[\iota_{sm}(i) \leq \iota\right] \bar{\delta}_{rk} e^{-\bar{\delta}_{rk}\left(\frac{\iota}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}} \iota_{rk}^{\frac{1}{\varepsilon_\delta}} \iota^{-\frac{1}{\varepsilon_\delta}-1} \frac{1}{\varepsilon_\delta} d\iota \\
&\quad = \int_0^\infty \prod_{s,m\neq r,k} e^{-\bar{\delta}_{sm}\left(\frac{\iota}{\iota_{sm}}\right)^{-\frac{1}{\varepsilon_\delta}}} \bar{\delta}_{rk} e^{-\bar{\delta}_{rk}\left(\frac{\iota}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}} \iota_{rk}^{\frac{1}{\varepsilon_\delta}} \iota^{-\frac{1}{\varepsilon_\delta}-1} \frac{1}{\varepsilon_\delta} d\iota \\
&\quad = \bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}} \int_0^\infty e^{-\sum_r \sum_k \bar{\delta}_{rk}\left(\frac{\iota}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}} \iota^{-\frac{1}{\varepsilon_\delta}-1} \frac{1}{\varepsilon_\delta} d\iota \\
&\quad = \frac{\bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}}}{\sum_r \sum_k \bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}}} \int_0^\infty e^{-\sum_r \sum_k \bar{\delta}_{rk}\left(\frac{\iota}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}} \left(\sum_r \sum_k \bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}}\right) \iota^{-\frac{1}{\varepsilon_\delta}-1} \frac{1}{\varepsilon_\delta} d\iota \\
&\quad = \frac{\bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}}}{\sum_r \sum_k \bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}}} \left[e^{-\sum_r \sum_k \bar{\delta}_{rk}\left(\frac{\iota}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}}\right]_0^\infty = \frac{\bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}}}{\sum_r \sum_k \bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}}}. \qquad (4.B.4)
\end{aligned}
$$

Distribution of Returns. Then the CDF of the compensation of capital units that actually flow to r, k is given by:

$$
\begin{aligned}
&\Pr\left[\iota_{rk}(i) < \iota | \iota_{rk}(i) \geq \max_{s,m\neq r,k} \iota_{sm}(i)\right] \\
&\quad = \frac{1}{\frac{\bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}}}{\sum_r \sum_k \bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}}}} \int_0^\iota \Pr\left[\max_{s,m\neq r,k} \iota_{sm}(i) < x\right] \frac{d\mathbb{G}_{\iota,rk}(x)}{dx} dx \\
&\quad = \frac{1}{\frac{\bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}}}{\sum_r \sum_k \bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}}}} \int_0^\iota \prod_{s,m\neq r,k} \Pr\left[\iota_{sm}(i) \leq x\right] \bar{\delta}_{rk} e^{-\bar{\delta}_{rk}\left(\frac{x}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}} \iota_{rk}^{\frac{1}{\varepsilon_\delta}} x^{-\frac{1}{\varepsilon_\delta}-1} \frac{1}{\varepsilon_\delta} dx \\
&\quad = \int_0^\iota e^{-\sum_r \sum_k \bar{\delta}_{rk}\left(\frac{x}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}} \sum_r \sum_k \bar{\delta}_{rk} \iota_{rk}^{\frac{1}{\varepsilon_\delta}} x^{-\frac{1}{\varepsilon_\delta}-1} \frac{1}{\varepsilon_\delta} dx \\
&\quad = \left[e^{-\sum_r \sum_k \bar{\delta}_{rk}\left(\frac{x}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}}\right]_0^\iota = e^{-\sum_r \sum_k \bar{\delta}_{rk}\left(\frac{\iota}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}}.
\end{aligned}
$$

The compensation of capital units that actually flow to location r and capital asset class k is thus the same across all regions and asset classes and therefore also for total assets.

The PDF is:

$$\frac{d\left(e^{-\sum_r\sum_k\bar{\delta}_{rk}\left(\frac{\iota}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}}\right)}{d\iota}=\frac{1}{\varepsilon_\delta}e^{-\sum_r\sum_k\bar{\delta}_{rk}\left(\frac{\iota}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}}\iota^{-\frac{1}{\varepsilon_\delta}-1}\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}.$$

Average Returns. These results allow to derive the average or expected return of a capital unit flowing to any location and asset class:

$$\int_0^\infty\iota\frac{d(e^{-\sum_r\sum_k\bar{\delta}_{rk}\left(\frac{\iota}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}})}{d\iota}d\iota=\int_0^\infty\frac{1}{\varepsilon_\delta}e^{-\sum_r\sum_k\bar{\delta}_{rk}\left(\frac{\iota}{\iota_{rk}}\right)^{-\frac{1}{\varepsilon_\delta}}}\iota^{-\frac{1}{\varepsilon_\delta}}\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}d\iota.$$

Defining $x\left(\iota\right)=\iota^{-\frac{1}{\varepsilon_\delta}}\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}$ and thus $dx/d\iota=-\frac{1}{\varepsilon_\delta}\iota^{-\frac{1}{\varepsilon_\delta}-1}\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}=-\frac{1}{\varepsilon_\delta}\frac{x}{\iota}$ and $\iota=\left(\frac{x}{\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}}\right)^{-\varepsilon_\delta}$ yielding $\frac{d\iota}{dx}x=-\varepsilon_\delta\left(\frac{x}{\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}}\right)^{-\varepsilon_\delta}$ allows to transform the above into the following specification of average or expected return of a capital unit flowing to any location and asset class:

$$\begin{aligned}\frac{1}{\varepsilon_\delta}\int_{x(0)}^{x(\infty)}e^{-x}x\frac{d\iota}{dx}dx&=-\int_\infty^0e^{-x}\left(\frac{x}{\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}}\right)^{-\varepsilon_\delta}dx\\&=-\left(\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}\right)^{\varepsilon_\delta}\int_\infty^0e^{-x}x^{-\varepsilon_\delta}dx\\&=\left(\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}\right)^{\varepsilon_\delta}\int_0^\infty e^{-x}x^{-\varepsilon_\delta}dx\\&=\left(\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}\right)^{\varepsilon_\delta}\Gamma\left(1-\varepsilon_\delta\right).\end{aligned}\quad(4.B.5)$$

Equilibrium. In equilibrium within each region and asset class the respective capital stock compensated with the respective compensation level must equal the amount of base capital that flows there compensated at the average compensation. Using (4.B.4) and (4.B.5) yields:

$$\begin{aligned}K_{rk}\iota_{rk}&=K\frac{\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}}{\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}}\left(\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}\right)^{\varepsilon_\delta}\Gamma\left(1-\varepsilon_\delta\right)\\K_{rk}&=K\frac{\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}}{\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}}\frac{\left(\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}\right)^{\varepsilon_\delta}\Gamma\left(1-\varepsilon_\delta\right)}{\iota_{rk}}.\end{aligned}\quad(4.B.6)$$

Average (or ex-ante expected) capital compensation is the same everywhere. In equilibrium the total worldwide capital compensation, which is the average returns (4.B.5) multiplied by the base capital stock, must equal total capital demand in all regions (using the optimality condition (4.13) allows to derive total capital demand in values as: $\sum_{r=1}^{R}\sum_{k=1}^{\mathfrak{K}}\sum_{j=1}^{J}\iota_{rk}\kappa_{rjk} = \sum_{r=1}^{R}\sum_{k=1}^{\mathfrak{K}}\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}v_{rj}$):

$$K\left(\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}\right)^{\varepsilon_\delta}\Gamma\left(1-\varepsilon_\delta\right) = \sum_{r=1}^{R}\sum_{k=1}^{\mathfrak{K}}\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}v_{rj}$$

$$\sum_r\sum_k\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}} = \left(\frac{\sum_{r=1}^{R}\sum_{k=1}^{\mathfrak{K}}\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}v_{rj}}{K\Gamma\left(1-\varepsilon_\delta\right)}\right)^{\frac{1}{\varepsilon_\delta}}. \tag{4.B.7}$$

Using this to rewrite equation (4.B.6) yields:

$$K_{rk} = \frac{\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}}}{\left(\frac{\sum_{r=1}^{R}\sum_{k=1}^{\mathfrak{K}}\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}v_{rj}}{K\Gamma(1-\varepsilon_\delta)}\right)^{\frac{1}{\varepsilon_\delta}}}\frac{\sum_{r=1}^{R}\sum_{k=1}^{\mathfrak{K}}\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}v_{rj}}{\iota_{rk}}$$

$$K_{rk} = \left(K\Gamma\left(1-\varepsilon_\delta\right)\right)^{\frac{1}{\varepsilon_\delta}}\bar{\delta}_{rk}\iota_{rk}^{\frac{1}{\varepsilon_\delta}-1}\left(\sum_{r=1}^{R}\sum_{k=1}^{\mathfrak{K}}\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}v_{rj}\right)^{1-\frac{1}{\varepsilon_\delta}}.$$

Plugging in the definition of ι_{rk} (4.24) we get an explicit solution for the capital asset stock:

$$K_{rk} = \left(K\Gamma\left(1-\varepsilon_\delta\right)\right)^{\frac{1}{\varepsilon_\delta}}\bar{\delta}_{rk}\left(\frac{\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}v_{rj}}{K_{rk}}\right)^{\frac{1}{\varepsilon_\delta}-1}\left(\sum_{r=1}^{R}\sum_{k=1}^{\mathfrak{K}}\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}v_{rj}\right)^{1-\frac{1}{\varepsilon_\delta}}$$

$$K_{rk} = K\Gamma\left(1-\varepsilon_\delta\right)\bar{\delta}_{rk}^{\varepsilon_\delta}\left(\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}v_{rj}\right)^{1-\varepsilon_\delta}\left(\sum_{r=1}^{R}\sum_{k=1}^{\mathfrak{K}}\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}v_{rj}\right)^{\varepsilon_\delta-1}.$$

Equilibrium in Changes. The change in the capital stock of assets type k in region r is given by:

$$\hat{K}_{rk} = \left(\frac{\frac{1-t_r'}{1-d_{rk}'t_r'}\sum_{j=1}^{J}\beta_{rjk}v_{rj}\hat{v}_{rj}}{\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}v_{rj}}\right)^{1-\varepsilon_\delta}\left(\frac{\sum_{s=1}^{R}\sum_{m=1}^{\mathfrak{K}}\frac{1-t_s'}{1-d_{sm}'t_s'}\sum_{j=1}^{J}\beta_{sjm}v_{sj}\hat{v}_{sj}}{\sum_{s=1}^{R}\sum_{m=1}^{\mathfrak{K}}\frac{1-t_s}{1-d_{sm}t_s}\sum_{j=1}^{J}\beta_{sjm}v_{sj}}\right)^{\varepsilon_\delta-1}. \tag{4.B.8}$$

The following change in user cost of asset k in region r follows:

$$\hat{\iota}_{rk} = \frac{\sum_{j=1}^{J}\beta_{rjk}v_{rj}\hat{v}_{rj}}{\sum_{j=1}^{J}\beta_{rjk}v_{rj}}\hat{K}_{rk}^{-1} \tag{4.B.3}$$

$$= \frac{\sum_{j=1}^{J}\beta_{rjk}v_{rj}\hat{v}_{rj}}{\sum_{j=1}^{J}\beta_{rjk}v_{rj}}\left(\frac{\frac{1-t_r'}{1-d_{rk}'t_r'}\sum_{j=1}^{J}\beta_{rjk}v_{rj}\hat{v}_{rj}}{\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}v_{rj}}\right)^{\varepsilon_\delta-1}\left(\frac{\sum_{s=1}^{R}\sum_{m=1}^{\mathfrak{K}}\frac{1-t_s'}{1-d_{sk}'t_s'}\sum_{j=1}^{J}\beta_{sjm}v_{sj}\hat{v}_{sj}}{\sum_{s=1}^{R}\sum_{m=1}^{\mathfrak{K}}\frac{1-t_s}{1-d_{sm}t_s}\sum_{j=1}^{J}\beta_{sjm}v_{sj}}\right)^{1-\varepsilon_\delta}.$$

4.B.2 Extensions

This subsection considers different possible alternative capital asset stock specifications and their implications for the framework. First we will consider immobile asset stocks. Where each capital asset type has a specific stock in each region. Secondly, we will consider a situation where the different capital asset type stocks are given in the world but are perfectly mobile across regions. In the third specification, one single type of worldwide capital stock K can perfectly flow to any region and asset class.

Immobile Capital Asset Stocks. In this extension we consider a situation where the stocks K_{rk} of each capital asset type are regionally immobile. The rental rate of capital asset k in region r, the user cost of capital assets and the change in user cost in this case can be denoted by:

$$\begin{aligned} \iota_{rk} &= \frac{\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}\upsilon_{rj}}{K_{rk}}, \\ \tilde{\iota}_{rk} &= \iota_{rk}\frac{1-d_{rk}t_r}{1-t_r} = \frac{\sum_{j=1}^{J}\beta_{rjk}\upsilon_{rj}}{K_{rk}}, \\ \hat{\tilde{\iota}}_{rk} &= \frac{\sum_{j=1}^{J}\beta_{rjk}\upsilon_{rj}\hat{\upsilon}_{rj}}{\sum_{j=1}^{J}\beta_{rjk}\upsilon_{rj}}\hat{K}_{rk}^{-1}. \end{aligned} \tag{4.24}$$

This is equivalent to to our preferred specification where we assume imperfect mobility of a worldwide capital stock. Different to the preferred modelling approach the immobility assumption $\hat{K}_{rk}^{-1} = 1$ implies that the change in user cost and the change in the marginal cost per efficiency unit can be denoted by:

$$\begin{aligned} \hat{\tilde{\iota}}_{rk} &= \frac{\sum_{j=1}^{J}\beta_{rjk}\upsilon_{rj}\hat{\upsilon}_{rj}}{\sum_{j=1}^{J}\beta_{rjk}\upsilon_{rj}}, \\ \hat{c}_{rj} &= \left(\frac{1}{\hat{L}_r}\frac{\sum_{g=1}^{J}\gamma_{rg}\upsilon_{rg}\hat{\upsilon}_{rg}}{\sum_{g=1}^{J}\gamma_{rg}\upsilon_{rg}}\right)^{\gamma_{rj}}\left(\frac{\sum_{g=1}^{J}\eta_{rg}\upsilon_{rg}\hat{\upsilon}_{rg}}{\sum_{g=1}^{J}\eta_{rg}\upsilon_{rg}}\right)^{\eta_{rj}}\prod_{g=1}^{J}\hat{P}_{rg}^{\mu_{rjg}}\prod_{k=1}^{\mathfrak{K}}\left(\frac{\sum_{g=1}^{J}\beta_{rgk}\upsilon_{rg}\hat{\upsilon}_{rg}}{\sum_{g=1}^{J}\beta_{rgk}\upsilon_{rg}}\right)^{\beta_{rjk}}. \end{aligned}$$

Perfectly Mobile Worldwide Capital Asset Stocks. In this specification we assume a worldwide stock K_k for each capital asset class which can flow perfectly to any region. The rental rate of capital asset k in region r, the user cost of capital assets and the change in user cost in this case can be denoted by:

$$\begin{aligned} \iota_{rk} = \iota_k &= \frac{\sum_{r=1}^{R}\left(\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}\upsilon_{rj}\right)}{K_k}, \\ \tilde{\iota}_{rk} &= \frac{1-d_{rk}t_r}{1-t_r}\iota_k = \frac{1-d_{rk}t_r}{1-t_r}\frac{\sum_{r=1}^{R}\left(\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}\upsilon_{rj}\right)}{K_k}, \\ \hat{\tilde{\iota}}_{rk} &= \frac{\frac{1-d'_{rk}t'_r}{1-t'_r}}{\frac{1-d_{rk}t_r}{1-t_r}}\frac{\sum_{r=1}^{R}\left(\frac{1-t'_r}{1-d'_{rk}t'_r}\sum_{j=1}^{J}\beta_{rjk}\upsilon_{rj}\hat{\upsilon}_{rj}\right)}{\sum_{r=1}^{R}\left(\frac{1-t_r}{1-d_{rk}t_r}\sum_{j=1}^{J}\beta_{rjk}\upsilon_{rj}\right)}\hat{K}_k^{-1}. \end{aligned}$$

The capital stock of assets type k in region r is given by:

$$K_{rk} = \frac{\sum_{j=1}^{J} \beta_{rjk} v_{rj}}{\tilde{\iota}_{rk}} = \frac{\sum_{j=1}^{J} \beta_{rjk} v_{rj}}{\frac{1-d_{rk}t_r}{1-t_r} \frac{\sum_{r=1}^{R} \left(\frac{1-t_r}{1-d_{rk}t_r} \sum_{j=1}^{J} \beta_{rjk} v_{rj} \right)}{K_k}}.$$

Given the worldwide fixed endowment of capital asset stocks $\hat{K}_k^{-1} = 1$, this implies the following change in the stock of capital asset k in a region r, change in user cost and the change in the marginal cost per efficiency unit:

$$\begin{aligned}
\hat{K}_{rk} =& \frac{\sum_{j=1}^{J} \beta_{rjk} v_{rj} \hat{v}_{rj}}{\sum_{j=1}^{J} \beta_{rjk} v_{rj}} \cdot \frac{\frac{1-d_{rk}t_r}{1-t_r} \sum_{s=1}^{R} \sum_{g=1}^{J} \frac{1-d_{sk}t_s}{1-t_s} \beta_{sgk} v_{sg}}{\frac{1-d'_{rk}t'_r}{1-t'_r} \sum_{s=1}^{R} \sum_{g=1}^{J} \frac{1-d'_{sk}t'_s}{1-t'_s} \beta_{sgk} v_{sg} \hat{v}_{sg}}, \\
\hat{\tilde{\iota}}_{rk} =& \frac{\frac{1-d'_{rk}t'_r}{1-t'_r}}{\frac{1-d_{rk}t_r}{1-t_r}} \frac{\sum_{r=1}^{R} \sum_{j=1}^{J} \frac{1-d'_{rk}t'_r}{1-t'_r} \beta_{rjk} v_{rj} \hat{v}_{rj}}{\sum_{r=1}^{R} \sum_{j=1}^{J} \frac{1-d_{rk}t_r}{1-t_r} \beta_{rjk} v_{rj}}, \\
\hat{c}_{rj} =& \left(\frac{1}{\hat{L}_r} \frac{\sum_{g=1}^{J} \gamma_{rg} v_{rg} \hat{v}_{rg}}{\sum_{g=1}^{J} \gamma_{rg} v_{rg}} \right)^{\gamma_{rj}} \left(\frac{\sum_{g=1}^{J} \eta_{rg} v_{rg} \hat{v}_{rg}}{\sum_{g=1}^{J} \eta_{rg} v_{rg}} \right)^{\eta_{rj}} \prod_{g=1}^{J} \hat{P}_{rg}^{\mu_{rjg}} \\
& \cdot \prod_{k=1}^{\mathfrak{K}} \left(\frac{\frac{1-d'_{rk}t'_r}{1-t'_r}}{\frac{1-d_{rk}t_r}{1-t_r}} \frac{\sum_{s=1}^{R} \sum_{g=1}^{J} \frac{1-d'_{sk}t'_s}{1-t'_s} \beta_{sgk} v_{sg} \hat{v}_{sg}}{\sum_{s=1}^{R} \sum_{g=1}^{J} \frac{1-d_{sk}t_s}{1-t_s} \beta_{sgk} v_{sg}} \right)^{\beta_{rjk}}.
\end{aligned}$$

Perfectly Mobile Worldwide Capital Stock. For this specification we consider one single type of worldwide capital stock K that can flow perfectly to any region and asset class. The rental rate of capital asset k in region r, the user cost of capital assets and the change in user cost in this case can be denoted by:

$$\begin{aligned}
\iota_{rk} &= \iota = \frac{\sum_{r=1}^{R} \sum_{k=1}^{\mathfrak{K}} \left(\frac{1-t_r}{1-d_{rk}t_r} \sum_{j=1}^{J} \beta_{rjk} v_{rj} \right)}{K}, \\
\tilde{\iota}_{rk} &= \frac{1-d_{rk}t_r}{1-t_r} \iota = \frac{1-d_{rk}t_r}{1-t_r} \frac{\sum_{r=1}^{R} \sum_{k=1}^{\mathfrak{K}} \left(\frac{1-t_r}{1-d_{rk}t_r} \sum_{j=1}^{J} \beta_{rjk} v_{rj} \right)}{K}, \\
\hat{\tilde{\iota}}_{rk} &= \frac{\frac{1-d'_{rk}t'_r}{1-t'_r}}{\frac{1-d_{rk}t_r}{1-t_r}} \frac{\sum_{r=1}^{R} \sum_{k=1}^{\mathfrak{K}} \left(\frac{1-t'_r}{1-d'_{rk}t'_r} \sum_{j=1}^{J} \beta_{rjk} v_{rj} \hat{v}_{rj} \right)}{\sum_{r=1}^{R} \sum_{k=1}^{\mathfrak{K}} \left(\frac{1-t_r}{1-d_{rk}t_r} \sum_{j=1}^{J} \beta_{rjk} v_{rj} \right)} \hat{K}^{-1}.
\end{aligned}$$

The capital stock of assets type k in region r is given by:

$$K_{rk} = \frac{\sum_{j=1}^{J} \beta_{rjk} v_{rj}}{\tilde{\iota}_{rk}} = \frac{\sum_{j=1}^{J} \beta_{rjk} v_{rj}}{\frac{1-d_{rk}t_r}{1-t_r} \frac{\sum_{r=1}^{R} \sum_{k=1}^{\mathfrak{K}} \left(\frac{1-t_r}{1-d_{rk}t_r} \sum_{j=1}^{J} \beta_{rjk} v_{rj} \right)}{K}}.$$

Given the worldwide fixed endowment of overall capital $\hat{K}^{-1} = 1$ this implies the following change in the stock of capital asset, change in user cost and the change in the marginal cost per efficiency unit:

$$\hat{K}_{rk} = \frac{\sum_{j=1}^{J} \beta_{rjk} v_{rj} \hat{v}_{rj}}{\sum_{j=1}^{J} \beta_{rjk} v_{rj}} \cdot \frac{\frac{1-d_{rk}t_r}{1-t_r} \sum_{s=1}^{R} \sum_{k=1}^{\mathfrak{K}} \sum_{g=1}^{J} \frac{1-d_{sk}t_s}{1-t_s} \beta_{sgk} v_{sg}}{\frac{1-d'_{rk}t'_r}{1-t'_r} \sum_{s=1}^{R} \sum_{k=1}^{\mathfrak{K}} \sum_{g=1}^{J} \frac{1-d'_{sk}t'_s}{1-t'_s} \beta_{sgk} v_{sg} \hat{v}_{sg}},$$

$$\hat{\tilde{\iota}}_{rk} = \frac{\frac{1-d'_{rk}t'_r}{1-t'_r} \sum_{s=1}^{R} \sum_{k=1}^{\mathfrak{K}} \sum_{g=1}^{J} \frac{1-d'_{sk}t'_s}{1-t'_s} \beta_{sgk} v_{sg} \hat{v}_{sg}}{\frac{1-d_{rk}t_r}{1-t_r} \sum_{s=1}^{R} \sum_{k=1}^{\mathfrak{K}} \sum_{g=1}^{J} \frac{1-d_{sk}t_s}{1-t_s} \beta_{sgk} v_{sg}},$$

$$\hat{c}_{rj} = \left(\frac{1}{\hat{L}_r} \frac{\sum_{g=1}^{J} \gamma_{rg} v_{rg} \hat{v}_{rg}}{\sum_{g=1}^{J} \gamma_{rg} v_{rg}} \right)^{\gamma_{rj}} \left(\frac{\sum_{g=1}^{J} \eta_{rg} v_{rg} \hat{v}_{rg}}{\sum_{g=1}^{J} \eta_{rg} v_{rg}} \right)^{\eta_{rj}}$$

$$\cdot \prod_{g=1}^{J} \hat{P}_{rg}^{\mu_{rjg}} \prod_{k=1}^{\mathfrak{K}} \left(\frac{\frac{1-d'_{rk}t'_r}{1-t'_r}}{\frac{1-d_{rk}t_r}{1-t_r}} \frac{\sum_{s=1}^{R} \sum_{g=1}^{J} \frac{1-d'_{sk}t'_s}{1-t'_s} \beta_{sgk} v_{sg} \hat{v}_{sg}}{\sum_{s=1}^{R} \sum_{g=1}^{J} \frac{1-d_{sk}t_s}{1-t_s} \beta_{sgk} v_{sg}} \right)^{\beta_{rjk}}.$$

4.B.3 Empirics

Table 4.B.1: *Assets Composition by Industry in New Zealand*

Sector\Asset	Land	Buildings	Furniture	Plant, Machinery and Equipment	Computers	Vehicles	Intangibles
Services to Agriculture, and Fishing	0,137	0,127	0,016	0,285	0,005	0,377	0,053
Mining	0,070	0,162	0,016	0,429	0,026	0,236	0,060
Manufacturing	0,041	0,099	0,049	0,430	0,019	0,235	0,128
Electricity, Gas and Water Supply	0,032	0,290	0,051	0,295	0,083	0,185	0,064
Construction	0,027	0,066	0,027	0,251	0,014	0,536	0,078
Wholsale Trade	0,040	0,078	0,117	0,242	0,052	0,319	0,151
Retail Trade	0,049	0,092	0,095	0,291	0,032	0,203	0,238
Accomodation, Cafes and Restaurants	0,093	0,172	0,111	0,308	0,010	0,089	0,219
Transport and Storage	0,030	0,057	0,047	0,143	0,014	0,601	0,108
Communication Services	0,007	0,049	0,026	0,110	0,012	0,636	0,160
Finance and Insurance	0,040	0,083	0,167	0,147	0,104	0,229	0,231
Property and Business Services	0,072	0,147	0,139	0,195	0,071	0,275	0,101
Education	0,032	0,142	0,144	0,217	0,095	0,241	0,128
Healt and Community Services	0,033	0,100	0,138	0,257	0,040	0,244	0,189
Cultural and Recreational Services	0,039	0,125	0,100	0,332	0,079	0,260	0,065
Personal and Other Services	0,034	0,082	0,099	0,321	0,032	0,241	0,190

Source: Fabling et al. (2014)

4.C Nomenclature

α_j consumer expenditure share on sector j final goods in location r

$\bar{\delta}_{rk}$ scale parameter of region r's and asset type k's Fréchet distribution of the base capital transformation parameter

β_{rjk} Cobb-Douglas weight of capital asset k in the production of varieties from sector j in region r

χ_{rj} constant in the marginal cost term

δ_{rk} base capital transformation parameter

η_{rj} Cobb-Douglas weight of land in the production of varieties from sector j in region r

$\Gamma(\cdot)$ the gamma function

γ_{rj} Cobb-Douglas weight of labor in the production of varieties from sector j in region r

ι_{rk} rental rate of capital asset k in region r

κ_{rjk} total amount of type k capital goods used in the production of sector j and region r

$\kappa_{rjk}(\omega)$ total amount of type k capital goods used in the production of variety ω in sector j and region r

$\mathbb{F}_s j(p)$ Distribution of prices of what region s actually buys in sector j overall, as well as when conditioning on the source region r

$\mathbb{F}_{rsj}(p)$ Distribution of prices that region r offers to region s in sector j

$\mathbb{G}_r(u)$ Fréchet cumulative distribution function of potential utility levels in region r

$\mathfrak{K}$ number of different asset/capital good types

μ_{rjg} Cobb-Douglas weight of composite intermediate from sector j in the production of varieties from sector j in region r

Ω index for individual worker-consumers

ω index for a specific variety

ϕ_r share of region r's non-labor income paid into the international portfolio

Φ_{sj} Price parameter of the CES price index in sector j in region s

π_{rsj} share of region s's spending in sector j attributed to region r

$\mathfrak{R}_n$ set of regions in country n

σ_j CES between varieties in sector j

τ_{rsj} iceberg transport costs for shipping sector j varieties from region r to region s

θ_j shape parameter of the Fréchet distributions of productivities for varieties in sector j

ε shape parameter of the Fréchet distributions of amenities

ε_δ shape parameter of the Fréchet distribution of the base capital transformation parameter

A_r scale parameter of region r's Fréchet distribution of amenities

$a_r(\Omega)$ amenity draw of consumer Ω for region r

C_r individual consumption of the aggregate Cobb-Douglas bundle in region r

C_{rj} individual consumption of sector-j final output in region r

c_{rj} marginal costs per efficiency unit for producing a variety from sector j in region r

d_{rh} depreciation allowance rate for housing in region r

d_{rk} depreciation allowance rate for kapital type k in region r

D_r aggregate trade imbalance of region r, positive for trade deficit

E_r aggregate expenditure - including government and capital asset transfers - of consumers in region r

G_n government budget in country n

H_r fixed stock of (quality adjusted) land in region r

h_{rj} total land used in the production of sector j in region r

$h_{rj}(\omega)$ total land used in the production of variety ω in sector j and region r

i unit of the world capital stock

I_{rj} non-labor income in region r and sector j

J number of sectors

j, g indices for sectors

K fixed world endowment with capital

k, m indices for the asset/capital good type

K_{rk} stock of capital asset k in region r

L fixed world endowment mass of worker-consumers

L_r endogenous mass of workers in region r

l_{rj} total labor used in the production of sector j in region r

$l_{rj}(\omega)$ total labor used in the production of variety ω in sector j and region r

m_{rjg} total amount of sector g CES bundle used in the production of sector j in region r

$m_{rjg}(\omega)$ total amount of sector g CES bundle used in the production of variety ω in sector j and region r

N number of countries

n index for countries

P_{rj} price of the non-traded CES compound of sector j in region r

$p_{rj}(\omega)$ the mill price at which region r offers variety ω from sector j

$p_{rsj}(\omega)$ the price at which region r offers variety ω from sector j to region s

Q_{rj} amount of sector j's non-traded CES aggregate produced in region r

R number of regions

r,s indices for regions

s_r rental rate of land and structures in region r

t_r tax rate in region r

T_{rj} scale parameter of the Fréchet distribution of productivities for varieties in sector j and region r

$u_r(\Omega)$ utility of consumer Ω in region r

$v_{rj}(\omega)$ revenue from the production of variety ω in region r and sector j

w_r wage rate in region r

$x_{rj,\omega}$ demand of region r's sectoral CES aggregate producers for intermediate variety ω from sector j

X_{rj} region r's total expenditure on sector j goods

$z_{rj}(\omega)$ productivity for producing variety ω in sector j in region r