Unbounded number line estimation:
A purer measure of numerical estimation?

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Abstract

Recently, a new unbounded version of the number line estimation task has been introduced by Cohen and Blanc-Goldhammer (2011). The authors suggested the task to provide a purer measure of the representation of numerical magnitude than the traditional bounded number line estimation task. The present dissertation considers various factors influencing estimation strategies to solve the unbounded number line estimation task with the aim at evaluating the claims associated with its validity more appropriately. This question was pursued in three studies that (1) systematically evaluate similarities and differences between symbolic and non-symbolic estimation with the bounded and unbounded number line estimation task with a closer association of non-symbolic with unbounded than bounded number line estimation, (2) examine eye-fixation behaviour in these two task versions by an expected decrease of the numbers of fixations with increasing target size on the unbounded number line, and (3) investigate sex differences in the aforementioned tasks with solution strategies applied respectively.

The first study drew on the conceptual similarity between unbounded number line estimation and the renowned analogue non-symbolic numerosity estimation task to generalize systematic biases of under- and overestimation (for the perception vs. production version of the task, respectively) observed in the latter to the unbounded number line estimation task. The same pattern of systematic biases of under- and overestimation in numerosity estimation was also found in the unbounded but not in the bounded number line estimation task.

The second study was conducted to investigate solution strategies in bounded and unbounded number line estimation by contrasting participants´ estimation performance with their corresponding eye-fixation behaviour. Results substantiated the use of reference points in the bounded version of the task and suggested the location of the very first fixation on the number line to be a reliable predictor of the final estimation.

The third study addressed sex differences in number line estimation focussing on differences in solution strategies in terms of differences between males and females in approaching unconventional problems. As women tend more strongly to use (classroom-)learnt procedures as compared to estimation, they were found to be at a disadvantage in unbounded number line estimation as it does not allow to apply leant strategies such as proportion judgement in bounded number line estimation, but requires numerical estimations.

In summary, the results of all three experiments support the claim that unbounded number line estimation might indeed provide a more pure and valid measure of number magnitude representation. This conclusion was particularly supported by i) comparable estimation biases as observed in non-symbolical numerosity estimation, ii) fewer fixations on reference points, and iii) sex differences associated with applying specific solution strategies. Nevertheless, the findings of the current dissertation also suggest that unbounded number line estimation task is not independent of specific estimation strategies.

Zusammenfassung

Kürzlich präsentierten Cohen und Blanc-Goldhammer (2011) eine neue, unbounded Version der Zahlenstrahl-Schätzaufgabe. Die Autoren gehen davon aus, dass diese Aufgabe ein reineres Maß als die traditionelle bounded Zahlenstrahl-Schätzaufgabe darstellt, um die zugrunde liegende Zahlenrepräsentation zu messen. In der vorliegenden Dissertation werden verschiedene Einflussfaktoren auf Schätzstrategien betrachtet, die beim Lösen angewendet werden, mit dem Ziel, deren Validität genauer zu beurteilen. Diese Fragestellung wurde in drei Studien näher überprüft, welche (1) die Ähnlichkeit sowie Unterschiede zwischen symbolischen und nicht-symbolischen Schätzungen mit der bounded wie auch unbounded Zahlenstrahl-Schätzaufgabe systematisch untersuchten, wobei eine stärkere Assoziation zwischen der nicht-symbolischen und unbounded im Vergleich zur bounded Aufgabe bestehen soll, (2) Augenbewegungen in beiden Aufgabenversionen untersuchen und eine Abnahme an Fixationen mit grösser werdenden Targets bei der unbounded Version erwartet wurde und (3) Geschlechterunterschiede in beiden genannten Aufgaben erforschen sowie die jeweils angewandten Lösungsstrategien.

Die erste Studie stützte sich auf die konzeptionelle Ähnlichkeit zwischen der unbounded Zahlenstrahl-Schätzaufgabe und der bewährten analogen nicht-symbolischen Numerositäts-Aufgabe, um systematische Verzerrungen von Unter- und Überschätzung (in der Wahrnehmungs- und Produktionsversion der Aufgabe), welche in letzterer beobachtet wurden, auf die unbounded Aufgabe zu verallgemeinern. Dasselbe Muster systematischer Unter- und Überschätzung bei Numerositäts-Schätzungen wurde auch bei der unbounded, jedoch nicht bei der bounded Zahlenstrahl-Schätzaufgabe gefunden.

Die zweite Studie wurde durchgeführt, um Lösungsstrategien in der bounded und unbounded Zahlenstrahl-Schätzaufgabe zu erforschen, indem die Schätzleistung der Teilnehmer den dazugehörigen Augenbewegungsmustern gegenübergestellt wurde. Die Ergebnisse untermauern die Verwendung von Referenzpunkten in der bounded Aufgabe und deuten darauf hin, dass diejenige Stelle, welche zuerst auf dem Zahlenstrahl fixiert wurde, ein zuverlässiger Prädiktor der endgültigen Schätzung darstellt.

Die dritte Studie befasste sich mit Geschlechterunterschieden bei Zahlenstrahl-Schätzungen, wobei Unterschiede zwischen Männern und Frauen beim Anwenden unkonventioneller Lösungsstrategien im Fokus standen. Da Frauen eher dazu tendieren, Methoden anzuwenden, welche sie (in der Schule) gelernt haben, wurde festgestellt, dass sie bei der unbounded Aufgabe im Nachteil sind, da in dieser keine spezifisch bekannten Strategien anwendbar sind, sondern numerisches Größenschätzen notwendig ist.

Insgesamt stützen die Ergebnisse aller drei Experimente die Annahme, dass die unbounded Zahlenstrahl-Schätzaufgabe tatsächlich ein reineres und valideres Maß zur Erfassung der Zahlenrepräsentation darstellt. Diese Schlussfolgerung wurde insbesondere bekräftigt i) durch ähnliche Schätzverzerrungen, wie sie bei der nicht-symbolischen Numerositäts-Aufgabe beobachtet wurden, ii) weniger Fixierungen auf Referenzpunkten und iii) Geschlechterunterschieden bei der Anwendung spezifischer Lösungsstrategien. Allerdings legen die Ergebnisse der vorliegenden Dissertation nahe, dass auch die unbounded Zahlenstrahl-Schätzaufgabe nicht frei von spezifischen Schätzstrategien ist.

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GENERAL DISCUSSION

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LIST OF ABBREVIATIONS

AEE	Absolute Estimation Error
AIC	Akaike Information Criterion
ANOVA	Analysis of Variance
cf.	Compare
CRR	Cued Retrospective Reporting
EBSCOhost	Elton B. Stephens Company
e.g.	Exempli Gratia/For Example
ER	Error Rate
et al.	Et Alii
GG	Greenhouse-Geisser coefficient
i.e.	Id Est/That Is
LMM	Linear Mixed Models
ms	Milliseconds
NA	Not Available
NLE	Number Line Estimation
OM	Operational Momentum
px	Pixels
PAE	Percentage Absolute Error
PRISMA	Preferred Reporting Items for Systematic Reviews and Meta-Analyses
PubMed	Medical Publications
R	The R Project for Statistical Computing
REE	Relative Estimation Error
SE	Standard Error
SEM	Standard Error of the Mean
SD	Standard Deviation
SNARC	Spatial-Numerical Association of Response Codes

Bibliography of the studies incorporated in the present thesis

Study 1: Reinert, R. M., Hartmann, M., Huber, S., & Moeller, K. (2019). Unbounded number line estimation as a measure of numerical estimation**.** *PLoS ONE 14*(3): e0213102. doi:10.1371/journal.pone.0213102

Study 2: Reinert, R. M., Huber, S., Nuerk, H.-C., & Moeller, K. (2015). Strategies in unbounded number line estimation? Evidence from eye-tracking. *Cognitive Processing, 16(1),* 359-363. doi:10.1007/s10339-015-0675-z

Study 3: Reinert, R. M., Huber, S., Nuerk, H.-C., & Moeller, K. (2017). Sex differences in number line estimation: The role of numerical estimation. *British Journal of Psychology, 108(2),* 334-350. doi:10.1111/bjop.12203

I. GENERAL INTRODUCTION

1. Introduction

In the past two decades, a wide range of research on numerical cognition using various types of tasks has been carried out to investigate the mental representation of number magnitude. In order to measure the characteristics of children´s psychological representation of numbers, one of the most commonly applied tools is the so called number line estimation (NLE) task (see, e.g., Ashcraft & Moore, 2012; Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Friso-van den Bos et al., 2015; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Laski & Siegler, 2007; LeFevre, Jimenez Lira, Sowinski, Cankaya, Kamawar, & Skwarchuk, 2013; Link, Huber, Nuerk, & Moeller, 2014; Moeller, Pixner, Kaufmann, & Nuerk, 2009; Opfer & Martens, 2012; Peeters, Sekeris, Verschaffel, & Luwel, 2017; Ramani & Siegler, 2008; Sasanguie, De Smedt, Defever, & Reynvoet, 2012; Schneider, Grabner, & Paetsch, 2009; Siegler & Opfer, 2003; Thompson & Opfer, 2008; Thompson & Siegler, 2010; White & Szűcs, 2012; Yuan, Prather, Mix, & Smith, 2019). There are different versions of this task. In the most popular bounded production version (*number-to-position*, after Siegler & Opfer, 2003) of the NLE task, a horizontal, empty number line with only start and end point labeled is presented showing the minimum and maximum numerical values of the range (e.g., 0 and 100). Additionally, participants are displayed a target number on every trial – usually in the form of Arabic numerals (e.g., 72) – and requested to mark the spatial position corresponding to this probed number on the respective number line. In the original paper-pencil version, the estimated position is indicated with a pen and on the other hand, in the computer-based version of the task, participants make their estimates by a mouse click.

This ´standard´ bounded version of the NLE task rests on the idea of the metaphor of a mental number line (Buckley & Gilman, 1974; Dehaene, Bossini, & Gireaux, 1993; Moyer & Landauer, 1967; Restle, 1970), which supposes that individuals represent magnitudes along an internal continuum being referred to mental number line. This horizontal number line is assumed to be organized spatially with numbers in ascending order from left to right (Dehaene, 2011; Dehaene, Piazza, Pinel, & Cohen, 2003; Galton, 1880; Hubbard, Piazza, Pinel, & Dehaene, 2005). Correspondingly, smaller magnitudes are systematically related to the left-hand side of space, while larger ones are spatially associated with the right (e.g., Dehaene et al., 1993; Dehaene, 2011; Zorzi, Priftis, Meneghello, Marenzi, & Umiltà, 2006; see also de Hevia, Girelli, & Macchi-Cassia, 2012, for an overview). The particular direction of this representation "seems to be determined by the direction of writing" (Dehaene et al., 1993, p. 394) and reading in one's native language and is therefore right to left in Hebrew, Arabic, Urdu, and Farsi (see Göbel, Shaki, & Fischer, 2011, for a review on cultural and linguistic influences on the development of number processing).

This mental number line hypothesis is corroborated by distinct types of evidence such as for example the SNARC (Spatial-Numerical Association of Response Codes) or the OM (operational momentum) effect (see Fischer, 2001, 2003, for other effects; Fischer & Shaki, 2014): the former effect describes that individuals tend to respond faster with a left button-press when presented with relatively smaller numbers (e.g., 1 or 2). Conversely, when they are given larger numbers (e.g., 8 or 9), right side responses are instead consistently facilitated (e.g., Dehaene et al., 1993; Dehaene, Dupoux, & Mehler, 1990; Fischer, Warlop, Hill, & Fias, 2004; Nuerk, Wood, & Willmes, 2005; Wood, Willmes, Nuerk, & Fischer, 2008, for a meta-analysis). On the other hand, the OM effect indicates a spatial bias found for mental arithmetic when participants have to solve additions and/or subtractions (e.g., McCrink, Dehaene, & Dehaene-Lambertz, 2007). In the original experiment, participants were presented moving dot patterns being added or subtracted from one another and they had to indicate whether the final set of dots was the correct outcome. The operational momentum effect indicates that addition problems tended to result in overestimation whereas subtraction problems rather led to underestimation of the correct result. McCrink and colleagues (2007) interpreted this error pattern as an overshoot of the intended magnitude while moving towards the target position on the mental number line. Accordingly, mental calculations were speculated to be equivalent to movements along the spatial-numerical continuum of the mental number line and reflecting participants´ tendency to move “too far” (see also Lindemann & Tira, 2011; Pinhas & Fischer, 2008). In sum, both effects provide convincing evidence supporting the mental number line hypothesis and are possibly resulting from cultural immersion as well as spatially directional habits such as reading or finger counting (see Fischer & Shaki, 2014, for a review).

In a recent review on the relevance of number magnitude understanding, Siegler (2016) stressed that numerical magnitude can be regarded “the common core of numerical development” (p. 341) as well as the most relevant measure of the mental representation of numbers. In particular, there is compelling evidence showing fairly strong associations between arithmetic proficiency and estimation performance in NLE (see e.g., Ashcraft & Moore, 2012; Booth & Siegler, 2008; Cowan & Powell, 2014; Gunderson, Ramirez, Beilock, & Levin, 2012; LeFevre, Jiménez Lira, Sowinski, Cankaya, Kamawar et al., 2013). Furthermore, performance on the NLE task also represents a powerful predictor of broader mathematical outcomes with high correlations between estimation accuracy and scores of math achievement tests that are measured concurrently or in the future (see also Booth & Siegler, 2006; Geary et al., 2008; Laski & Siegler, 2007; Sasanguie et al., 2012; Sella, Berteletti, Brazzolotto, Luncageli, & Zorzi, 2013). For instance, Torbeyns and colleagues (2015) found consistent strong correlations between fraction magnitude understanding of sixth and eighth graders from different cultures measured by the bounded NLE task and their mathematical achievement scores. Additionally, these observations seems even more remarkable as the associations remain significant even when controlling for influences of more general cognitive measures such as working memory, reading achievement, intelligence, but also family education and income, gender, race, as well as non-symbolic numerical knowledge and so on (e.g., Bailey, Siegler, & Geary, 2014; Cowan & Powell, 2014; Fazio, Bailey, Thompson, & Siegler, 2014).

The observed associations of NLE with math achievement but also more general cognitive constructs further indicate that research on basic numerical abilities such as number magnitude understanding may also have practical significance. For instance, Ritchie and Bates (2013) found in a large, population-representative, longitudinal sample collected in the United Kingdom that number magnitude understanding at age 7 significantly determines attained socio-economic status by age 42 (p. 1301). Moreover, a meta-analysis of six large longitudinal studies in the U.K., U.S. as well as Canada conducted by Duncan et al. (2007) confirmed this fact showing that early math skills have the greatest predictive power of later academic performance – followed by reading ability and attention skills – even after controlling for various relevant variables (see also Siegler & Braithwaite, 2017). Moreover, it was found that the associations between basic numerical abilities and later mathematical achievement were of particular importance as they were stronger than for other relevant measures such as socioemotional behavior and social skills (Duncan et al., 2007). In turn, Parson and Bynner (2005) found that considerable negative consequences of poor numeracy were even more detrimental than those of low levels of literacy skills. Additionally, poor numeracy was also found to lead to considerable difficulties for life prospects in different areas (e.g., higher likelihood to be unemployed or have experienced depression, lower hourly rates of pay and economic well-being) and represents a specific problem in our modern knowledge societies (Parsons & Bynner, 2005, p. 36). Therefore, it seems crucial to have a valid and reliable measure capturing the mental representation of numerical magnitude such as the NLE task.

In recent years, however, there is growing evidence suggesting that it may not be warranted to draw conclusions about the mental representation of number magnitude from individuals' estimation performance on the ´standard´ bounded task version as it may not only capture pure numerical estimation (e.g., Barth & Paladino, 2011; Chesney & Matthews, 2013; Cohen & Blanc-Goldhammer, 2011; Cohen & Quinlan, 2018; Cohen & Sarnecka, 2014; Link, Nuerk, & Moeller, 2014; Rouder & Geary, 2014; Slusser, Santiago, & Barth, 2013; White & Szűcs, 2012).

Initially, it was the prevailing view that children's (as well as adults') mental representations of number magnitude develop with experience and age from an originally less accurate logarithmic representation to a more precise and linear one (see Barth & Paladino, 2011; Booth & Siegler, 2006, 2008; Laski & Siegler, 2007; Opfer & Siegler, 2012; Opfer, Thompson, & Kim, 2016; Siegler & Opfer, 2003). This shift from logarithmic to linear representations has been reflected in participants' estimation performance when they had to indicate the spatial position of a given target number on the line. Younger children usually display less accurate estimation patterns better fitted by logarithmic functions whereas older children as well as adults produce more and more linear estimation patterns.

Even though this result pattern was quite robust across numerous studies, it remains unclear whether the explanation of a logarithmic-to-linear representational shift is the most appropriate (see Barth & Paladino, 2011). Presently, there is increasing evidence indicating that individuals may apply specific proportion-judgement strategies while completing the traditional bounded task version (Ashcraft & Moore, 2012; Barth & Paladino, 2011; Dackermann, Kroemer, Nuerk, Moeller, & Huber, 2018; Hollands & Dyre, 2000; Spence, 1990; Sullivan, Juhasz, Slattery, & Barth, 2011; Zax, Slusser, & Barth, 2019). As such, individuals seem to convert target numbers into proportions of the whole using reference marks on the number line like the start-, mid- or end point. For example, to estimate the spatial position of the target ´72´ on a 0 to 100 number line, they may first consider the reference point ´50´ which is halfway between the start point 0 and the end point 100 (rather than directly estimate 72, which is 72 units to the right, see Cohen & Blanc-Goldhammer, 2011) and continue to quarter the line. In a next step, they decide whether the probed number is smaller or larger than a quarter and compute the distance from this chosen reference point and then put their estimate for 72 somewhat to the left (see also Dackermann, Huber, Bahnmueller, Nuerk, & Moeller, 2015). Such a proportion-based solution strategy results in a specific M-shaped error pattern with smaller and less variant estimation errors at and around reference marks.

Respective evidence against the representational shift hypothesis (see, for instance, Opfer & Siegler, 2007; Siegler & Ramani, 2006; Siegler, Thompson, & Opfer, 2009) – suggesting the change from a logarithmically to a more linearly spaced underlying representation of number magnitude described above – was argued by Barth and Paladino (2011) who identified that the systematically biased estimation patterns arising in bounded NLE cannot be accounted for by the log-to-linear shift hypothesis. In particular, the authors argue that above described result pattern mirrors consideration of specific reference points such as the origin, mid and end point of a given number line. In doing so, participants cannot just ignore the two boundaries, they have to make a rough estimate of a part relative to the size of the whole number line, instead. This argument is corroborated by results indicating that estimates in the bounded task version follow cyclical power functions (see Hollands & Dyre, 2000; Slusser et al., 2013) being characteristic for proportion judgement. In this generalized form of a power function, the pattern of over- and underestimation is repeating between every pair of reference points used and is thereby similar to predictions by Spence (1990) of S-shaped or reverse S-shaped curves. Consequently, individuals have to consider part-whole relations and thus estimate two magnitudes: first the whole (100) and afterwards the part (72). Hence, Barth and Paladino (2011) inferred that the result patterns found in bounded NLE cannot be construed as a direct measure of individuals' underlying mental representation of number magnitude.

Moreover, Slusser and colleagues (2013) as well as Rouder and Geary (2014) also shared this critical view regarding the validity of conclusions drawn from NLE on the underlying number magnitude representation. In addition, this is further emphasized by the study by Cohen and Quinlan (2018) running computer simulations. Based on the assumption that participant´s response function to the bounded NLE task serves as a "direct window on their psychological understanding of quantities" (p. 448) which they termed the *direct response strategy*, the authors simulated an underlying quantity representation for number magnitudes within the range of the upper and lower boundaries of the bounded number line. For each target number, they sampled an estimate from the corresponding quantity distribution. Applying this procedure of computer simulations, they affirmed that the constraints of this standard bounded NLE task produce logarithmic-linear response functions when individuals use the direct response strategy. Thus, the authors showed that this task version does not provide a transparent window onto the underlying representations of number magnitude.

Taken together, effective performance on the bounded NLE task seems to involve the deployment of particular cognitive strategies and is hence contaminated "by the effects of truncation" (see Cohen &

Quinlan, 2018, p. 453). This constraint is reflected by the fact that typical bounded NLE task is largely similar to proportion estimation tasks (e.g., Slusser et al., 2013). As a result of this, the direction and extent of the resulting estimation errors are limited by the lower as well as upper bounds: under- and overestimation of small and large magnitudes is constrained by the two end points respectively (see Barth & Paladino, 2011; Cohen & Blanc-Goldhammer, 2011). These proportion judgements, in turn, result in typical estimation patterns (for an overview of proportion estimation see also Cohen, Ferrell, & Johnson, 2002) which can lead to misinterpretations and therefore remain contested.

In an effort to reduce this response bias inherent in the bounded NLE task, Cohen and Blanc-Goldhammer (2011) introduced a new task version that does not require an upper end point and termed it *unbounded NLE task*. In this task version, individuals were given just a standard line segment as a start point which denotes a measurement unit of usually 1 (e.g., two vertical lines that represent the distance of one unit) but no labeled end point was presented to prevent them using systematic reference marks and proportion-judgement strategies. They were then requested to reproduce a target magnitude based on the respective unit length. The observed estimation patterns were considerably different as compared to those found in the standard bounded version of the task. For example, estimation errors linearly increased with the magnitude of the target number in contrast to the M-shaped error pattern which is obtained in the bounded task version. Considering the distribution of error patterns in the unbounded NLE task – but also from additional evidence such as the accelerating perceived distance function –, the authors inferred that this task version constitutes a purer and more unbiased measure of the underlying representation of number magnitude (Cohen & Blanc-Goldhammer, 2011). Hence, this newly introduced task may overcome the limitations of the standard bounded task version outlined previously (see also Cohen & Quinlan, 2018) when implemented correctly (Cohen & Ray, 2020).

Nonetheless, the authors observed that participants seem to use other strategies such as the *dead-reckoning* strategy to complete this task. Applying this strategy, "participants first moved a unit on the number line, then estimated the position of the next unit based on this current position, and so on" (Cohen & Blanc-Goldhammer, p. 335). The use of multiples of a quantity then results in estimation patterns of repetitive scallops. A subsequent study conducted by Reinert, Huber, Nuerk, and Moeller (2015a) further investigated the processes that drive the estimation performance. Therefore, they adapted the task varying unit sizes ranging from 1 to 10 to assess influences of the unit size as well as multiples of it on individuals' estimation performance. Interestingly, they found that participants' estimates were more accurate the larger the unit size – probably because fewer steps had to be taken on the number line. Besides, the *working window of numbers* – a term also introduced by Cohen and Blanc-Goldhammer (2011) describing the range of "multiples of a small quantity (about ten)" (p. 335) that individuals use to estimate target numbers and that emerges from the use of the dead-reckoning strategy – was observed to be fix at about 10 for every unit size, participants did not adapt it to the respective unit size and was therefore not influenced by the manipulation of unit size. These results show influences different than those found in bounded NLE. However, little is known about the use of specific strategies in bounded NLE task so far.

Therefore, in the current doctoral thesis, I further examined the newly introduced unbounded NLE task in three empirical studies with the aim to evaluate the claim that unbounded NLE provides a purer measure of number magnitude representation. In particular, I was interested in identifying and quantifying influences of aspects beyond number magnitude representation, such as strategy use, sex, and estimation biases in a conceptually similar numerosity estimation task. Furthermore, the present dissertation was intended to provide new insights with regard to the general validity of this new task version as well as the interpretation of the estimation patterns observed in it. In the next section, more specific research questions will be identified breaking down this overall goal of this thesis and the main research aims will be derived thereof.

2. Research Aims

Altogether, the findings of the studies outlined in the general introduction provide support for the claim that the unbounded NLE task indeed provides a purer measure of the underlying number magnitude representation (but see Kim & Opfer, 2017, 2020, for a different view). Nonetheless, for a better understanding of (i) the general validity of this measure of number magnitude representation and (ii) potential solution strategies used to complete this task, it requires additional research on various aspects of unbounded NLE. To close this gap in the literature, the two overarching research aims that I pursued in the present dissertation will be specified in the next section and related to the three empirical studies that were carried out therefore:

Research aim 1: Evaluating the general validity of the newly introduced unbounded number line estimation task.

Study 1 of the current dissertation aimed to assess the validity of the unbounded NLE task in more detail. As recommended by Ebersbach, Luwel, and Verschaffel (2013), I primarily intended "to systematically manipulate the methodological aspects […] in magnitude estimation tasks" (p. 3) to gain new insights. In particular, following this suggestion might provide additional evidence on the mental representation of number magnitude as well as on the generalizability of estimation patterns observed in unbounded NLE. Therefore, I compared the performance in unbounded as well as bounded number line with that shown in a well-established non-symbolic numerosity estimation task. On the one hand, employing non-symbolic numerosity estimation as a tool that is conceptually quite similar to unbounded NLE as well as evaluating its differential associations with both unbounded as well as bounded NLE, may improve our knowledge on the general validity of this new instrument. When the same estimation patterns can be observed in both non-symbolic numerosity as well as unbounded NLE, the latter might also be a reliable and valid measure of number magnitude representation.

Following the *bi-directional mapping hypothesis* postulated by Crollen, Castronovo, and Seron (2011), I aimed at analyzing systematic patterns of under- and overestimation applying both perception and production versions of the respective tasks (see also Castronovo & Seron, 2007; Crollen et al., 2011; Crollen, Grade, Pesenti, & Dormal, 2013; Crollen & Seron, 2012; Mandler & Shebo, 1982). In the perception version of the task, participants are given non-symbolic stimuli (e.g., collections of dots) and are asked to estimate their numerosity by producing symbolic outputs such as Arabic or verbal numerals. According to the bi-directional mapping hypothesis, the numerical estimation process is assumed to go from a position on a representation of non-symbolic magnitudes that is logarithmically compressed to its corresponding linearly spaced representation of symbolic number magnitude which leads to systematic underestimation. In contrast, in the production version of the task, participants have to produce non-symbolic numerosities such as collections of dots that correspond to a symbolic stimuli they were presented with (e.g., Arabic digits). The mapping process here starts from a position on the linear symbolic representation of numbers and goes to its associated analogue, logarithmically compressed numerical representation on the subjective non-symbolic number line. As the objective magnitude is smaller here, the transcoding process results in systematic overestimation. In line with the literature claiming unbounded NLE to be a purer measure of number magnitude representation, I expected that estimation performance in the unbounded NLE task should show the same systematic biases of overestimation in the production and underestimation in the perception version as in the non-symbolic numerosity estimation task which is agreed to represent a reliable measure of the mental representation of number magnitude.

Research aim 2: Identifying various factors that affect solution strategies to complete the new unbounded number line estimation task.

In addition to examining the validity of the newly introduced unbounded task version as a valid measure of number magnitude representation, Studies 2 and 3 of the current thesis mainly explored factors which may affect solution strategies applied in both unbounded as well as bounded NLE by considering participants' eye-fixation behaviour (Study 2) and sex differences (Study 3) regarding the performance shown in this task. Furthermore, I will take into account results of a prior study by Reinert and colleagues (2015a) to further examine factors that affect estimation performance in unbounded NLE.

In particular, Study 2 of the present dissertation was supposed to provide new insights into solution strategies – like for instance the consideration of possible reference points – deployed by participants in the unbounded NLE task. To do so, I analyzed individuals´ estimation accuracy as well as their corresponding eye-fixation behaviour similar to the procedure used by Sullivan, Juhasz, Slattery, and Barth´s (2011) study on bounded NLE. As previously discovered by Cohen and Blanc-Goldhammer (2011), I evaluated processes that underlie the *dead-reckoning strategy* in unbounded NLE more closely in Study 2 which follows up on the recent study by Reinert and colleagues (2015a). Systematically varying unit size from 1 to 10, Reinert et al. (2015a) confirmed that the size of participants' *working window of numbers* actually seemed to be about 10 and independent of the actual unit size. In an attempt to gain deeper insights into solution strategies possibly applied by individuals or additional predictors of estimation performance observed in these tasks, Study 2 of this thesis was conducted.

On the one hand, I assumed that eye-fixation behaviour in the unbounded task version would not reflect consideration of typical reference points as reflected by more frequent fixations at specific regions of the number line (see Sullivan et al., 2011, for findings on bounded NLE). In contrast, I hypothesized a constant decrease of fixations along the unbounded number line as the numerical size of the target numbers increases. On the other hand, for unbounded NLE I expected increased numbers of fixations at and around the start, mid and end point which implies the use of proportion-based estimation strategies specifically considering these reference points. Obtaining such a result pattern would provide further evidence for the assumption that estimation performance in the unbounded NLE task is not driven by proportion judgement strategies.

Finally, in Study 3 of this dissertation I investigated whether estimation performance in both the bounded and unbounded NLE task is influenced by participants' sex as an additional contributing variable. In particular, I intended to evaluate whether differences in the use of solution strategies between females and males – as known for mathematical/arithmetical tasks – generalize to these two basic numerical tasks in a similar manner. Usually, women tend to be more prone to use procedures learnt in school while men are assumed to be more inclined to apply individually developed strategies (including estimation) when solving numerical problems (e.g., Gallagher, 1998; Kessel & Linn, 1996). Lately, a further study indicated that spatial numerical associations might differ between sexes and showed several sex disparities in a series of experiments (such as for example equity, color as well as magnitude decisions, and bounded NLE; see Bull, Cleland, & Mitchell, 2013). Hence, I aimed at extending our understanding on how sex differences effect solution strategies applied in numerical estimation in Study 3 of this thesis.

In particular, as previous studies indicated that men indeed employ estimation strategies more flexibly in numerical problem solving, I expected them to outperform women – who rather adhere to well-known learnt strategies – in the new unbounded task version. In contrast, there should be no sex-related disparity in the bounded task version because this can be completed by specific strategies such as proportion judgement. Obtaining such a result pattern would constitute additional evidence suggesting that unbounded NLE is less confounded by the application of proportion-based solution strategies (see Cohen & Blanc-Goldhammer, 2011). Moreover, it would point to the fact that this new task version represents indeed a purer measure of number magnitude representation whereas individuals` estimation patterns in the bounded NLE task version may also reflect specifically applied solution strategies used to complete this task version.

To sum up these two overarching research aims, an overall outline of the research questions of the empirical studies of this doctoral thesis is displayed in Table 1. It also indicates the specific topics, the respective research subjects and the related methods applied to answer the particular research questions. Furthermore, the last column of this summary table depicts preceding studies identifying important research gaps in the literature that are addressed in the present dissertation[1]. Whereas the first research question follows a methodological approach, the second research question focuses more on application related aspects influencing task performance.

Table 1. Overall overview of the topics, research subjects, and methods of this dissertation.

	Topics	Research subject	Methods	Preceding studies
1. Is the new unbounded number line estimation task indeed a purer measure of numerical estimation?				
Study 1	Generalizability of estimation biases in perception and production tasks to number line estimation	Conceptual similarity of the unbounded number line with the non-symbolic numerosity estimation task	Systematic comparison of numerosity with number line estimation tasks / Bi-directional mapping hypothesis	Crollen, Castronovo, & Seron (2011)
2. Which factors influence the solution strategies applied to solve the unbounded number line estimation task?				
Study 2	Solution strategies in number line estimation tasks	Possible use of reference points in unbounded number line estimation	Eye-tracking	Sullivan, Juhasz, Slattery, & Barth (2011)
Study 3	Solution strategies applied by males and females in number line estimation tasks	Sex differences in strategies and procedures used to solve these tasks	Mathematic fitting models, AICc	Bull, Cleland, & Mitchell (2013)

[1] Please note that the studies in the following chapters are written as separately readable manuscripts (see bibliography of the studies that are incorporated in this thesis, p. VIII). This results in overlapping contents to this introduction and between the empirical chapters.

II. EMPIRICAL STUDIES

STUDY 1:
Unbounded number line estimation as a measure of numerical estimation

Authors:	Regina M. Reinert, Matthias Hartmann, Stefan Huber & Korbinian Moeller
Individual contribution:	I was mainly responsible for the conceptualization, planning, data generation, analyses as well as paper writing.
Contribution of co-authors:	Matthias Hartmann was involved in the conceptualization, data generation as well as in the software development and paper review. Stefan Huber created the stimulus sets and prepared the data for the analyses. Korbinian Moeller was also responsible for the conceptualization and planning, supervision and paper review & editing.
Current stage:	Accepted for publication in *PLoS ONE*
Citation:	Reinert RM, Hartmann M, Huber S, Moeller K (2019) Unbounded number line estimation as a measure of numerical estimation. PLoS ONE 14(3): e0213102. https://doi.org/10.1371/journal.pone.0213102

1.1 Abstract

Number magnitude estimation has been investigated over the last decades using different tasks including non-symbolic numerosity but also number line estimation tasks. Recently, a bi-directional mapping process was suggested for numerosity estimation accounting for underestimation in a perception version of the task (i.e., indicating the number of non-symbolic dots in a set) and overestimation in the corresponding production task (i.e., produce the number of dots indicated by a symbolic number). In the present study, we evaluated the generalizability of these estimation biases in perception and production tasks to bounded and unbounded number line estimation. Importantly, target numbers were underestimated/overestimated by participants in the perception/production version of numerosity estimation as well as unbounded number line estimation. However, this pattern was reversed for bounded number line estimation. Thereby, the present data indicate a conceptual similarity of unbounded number line estimation and the established non-symbolic numerosity estimation task as a measure of numerical estimation. Accordingly, this corroborates the notion that unbounded number line estimation may reflect a purer measure of number magnitude representation than the bounded task version. Furthermore, our findings strengthen the bi-directional mapping hypothesis for numerical estimation by providing evidence for its generalizability to unbounded number line estimation for the first time.

Keywords: numerical estimation biases, underestimation, overestimation, numerical mapping process, symbolic and non-symbolic magnitudes, number line estimation tasks.

1.2 Introduction

Magnitude estimation tasks are typically employed to investigate numerical cognition. On the one hand, non-symbolic stimuli like collections of dots or sequences of sounds are used to assess the underlying representation of number magnitude. Usually, participants have to estimate their numerosity producing symbolic outputs such as Arabic or oral verbal numerals (e.g., Barth, Kanwisher, & Spelke, 2003; Mandler & Shebo, 1982). On the other hand, the (spatial) representation of number magnitude is often investigated using symbolic stimuli in tasks such as number line estimation (e.g., Siegler & Opfer, 2003). So far, however, similarities and differences in performance patterns for non-symbolic numerosity estimation and number line estimation have hardly been investigated. Therefore, the current study set off to evaluate whether results on non-symbolic numerosity estimation (e.g., Crollen, Castronovo, & Seron, 2011) can be generalized to bounded (e.g., Siegler & Opfer, 2003) as well as unbounded number line estimation (e.g., Cohen & Blanc-Goldhammer, 2011). Generalizable patterns of results would provide further evidence that bounded and/or unbounded number line estimation indeed rely on number magnitude estimation processes. In the following, we will first elaborate on the specifics of number line estimation and non-symbolic numerosity estimation before outlining the details of the present study.

1.2.1 Number line estimation tasks

The traditional version of this task is the bounded number line estimation task in which participants have to indicate the spatial position of a target number (e.g., 45) on a number line with a given start and endpoint (e.g., 0 to 100; e.g., Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Booth & Siegler, 2006; Moeller, Pixner, Kaufmann, & Nuerk, 2009; Nuerk, Moeller, Klein, Willmes, & Fischer, 2011; Siegler & Booth, 2004; Siegler & Opfer, 2003). The observed estimation pattern is then used to infer on the nature of the underlying representation of number magnitude.

However, it has been controversially debated in recent years whether this task indeed allows for inferences on the spatial layout of the underlying mental number line representation (e.g., Siegler & Opfer, 2003) or rather task-specific strategies that participants apply while they solve the task (Hollands & Dyre, 2000; Slusser, Santiago, & Barth, 2013; Sullivan, Juhasz, Slattery, & Barth, 2011). Barth and Paladino (2011) were the first to argue that the observed estimation pattern in this task may not reflect pure numerical estimation but the use of proportion-judgement strategies (see also Ashcraft & Moore, 2012; Cohen & Blanc-Goldhammer, 2011; Sullivan et al., 2011). Support for the assumption of the use of such task-specific strategies comes from the observation that participants' estimates are biased towards specific reference points (e.g., start and endpoint of the scale as well as the middle) with estimations being relatively more accurate near these reference points.

Recently, Cohen and Blanc-Goldhammer (2011) presented a new unbounded version of the number line estimation task and argued that this version provides a purer measurement of numerical estimation than the original bounded version. In this task version, only the start point and a scaling unit, but no endpoint of the number line is given. The observed estimation patterns led the authors to the conclusion that the bounded number line estimation task may be an invalid measure of number magnitude representation. Importantly, the results observed in this unbounded number line estimation task differed from the typical error pattern found in the bounded version of the task: Instead of estimations being more accurate around reference points, the authors found an error pattern that was consistent with scalar variance (see Gibbon, 1977; Gibbon & Church, 1981; Meck & Church, 1983; Whalen, Gallistel, & Gelman, 1999), this means that estimation errors increased linearly with the size of the target number (see also Reinert, Huber, Nuerk, & Moeller, 2015). Interestingly, this nicely reflects the pattern of results usually observed in estimation tasks in which, for instance, the number of dots in a given set has to be indicated (e.g., Crollen et al., 2011; Castronovo & Seron, 2007; Crollen & Seron, 2012).

So far, however, this potential conceptual similarity between unbounded number line estimation and the estimation of the magnitude of non-symbolic stimuli has not been investigated yet. Instead, the argument that unbounded number line estimation reflects a purer measure of numerical estimation has primarily been made based on comparisons of estimation performance in unbounded and bounded number line estimation. This seems surprising because non-symbolic numerosity estimation tasks are generally agreed to reflect a reliable measure of numerical estimation. Therefore, the current study set out to systematically evaluate similarities and differences between non-symbolic estimation and unbounded as well as bounded number line estimation. We expected a closer association of non-symbolic estimation with unbounded than with bounded number line estimation.

1.2.2 Numerosity estimation

To do so, we followed the suggestion of Crollen, Castronovo, and Seron (2011; see also Ebersbach, Luwel, & Verschaffel, 2013) for a taxonomy of paradigms of studies on magnitude estimation) and specifically investigated differences between production and perception type of the respective estimation tasks. For tasks using numbers of dots as non-symbolic targets, Crollen et al. (2011) showed opposing biases for the different types of the tasks (i.e., production and perception version). In their numerosity perception task, participants estimated the numerosity of collections of dots and expressed their estimates via symbolic Arabic numerals. In contrast, in their numerosity production task, symbolic stimuli in terms of an Arabic number were presented to participants who then had to estimate the denoted magnitude by non-symbolic output, this means, by adjusting the numerosity of a dot pattern so that it reflected the magnitude of the presented Arabic numeral. Depending on task type, the authors observed opposing patterns of performance: The numerosity of a dot set was systematically underestimated in the perception task, whereas the numerosity produced in the production task was overestimated significantly (see also Castronovo & Seron, 2007). The authors accounted for these systematic biases of under- and overestimation by the bi-directional mapping-hypothesis (see also Brooke & MacRae, 1977; Cohen, Ferrell, & Johnson, 2002; Shepard, 1981; Slusser & Barth, 2017) that is based on three assumptions: (1)

There are different numerical representations. In the present context, the first assumption refers to the differentiation between symbolic (e.g., 3) and non-symbolic (e.g., •••) representations of numerical magnitude. (2) Transcoding routes between these representations are assumed that allow for translating magnitude information from one representation to the other (i.e., symbolic to non-symbolic and non-symbolic to symbolic). Moreover, it is supposed that (3) precision differs between different numerical representations. The non-symbolic representation is assumed to be less precise as it is logarithmically compressed resulting in decreasing differences between adjacent numbers with increasing magnitude (e.g., Buckley & Gillman, 1974; Holloway & Ansari, 2009; Piazza, Pinel, Le Bihan, & Dehaene, 2007).

Based on these three assumptions, the opposing biases observed in numerosity perception and production in terms of under- and overestimation are assumed to stem from biases in transcoding between symbolic representations and their corresponding analogue (non-symbolic) magnitude representations. In the perception task, the numerical estimation process goes from a position on the logarithmically compressed representation of non-symbolic magnitudes to its associated linear symbolic numerical representation. Accordingly, the subjective magnitude is constantly larger compared to the objective one, and the perceived numerosity is therefore underestimated by participants. In contrast, in the production task, the mapping process starts from a position on the linear symbolic numerical representation and goes to its corresponding analogue but logarithmically compressed magnitude representation on the subjective non-symbolic number line. In this case, the objective magnitude is smaller than the subjective one. Hence, this leads to an overestimation of the target magnitude (see Figure 1a).

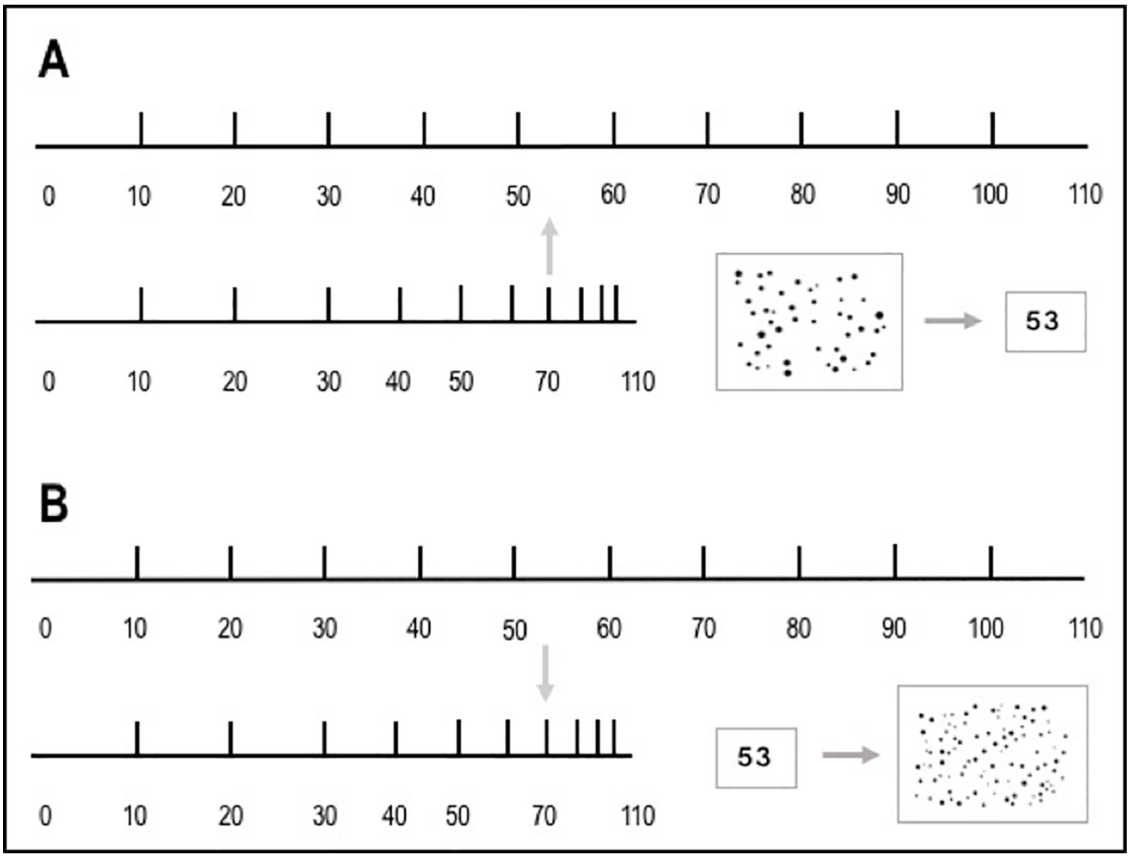


Figure 1. Schematic illustration of Crollen et al.'s (2011, p. 41) bi-directional mapping processes (in gray arrows): Panel (A) shows the assumed estimation process in a perception task requiring estimation from the logarithmically compressed representation of non-symbolic numerosity onto the linearly spaced representation of symbolic number magnitude which leads to underestimation. Panel (B) depicts the estimation process in the production task, in which linearly spaced symbolic number magnitude representations have to be mapped onto the compressed representation of non-symbolic numerosity, hence, leading to overestimation.

Interestingly, even though number line estimation is argued to capture processes of magnitude estimation, there is only very few research on number line estimation considering both directions of the mapping. To the best of our knowledge, there are so far only two studies directly comparing a perception version (position-to-number) with the more commonly used production version (number-to-position) of the bounded number line estimation task. First, Siegler and Opfer (2003) observed that estimates of second graders in US produced linear estimation patterns on a 0 to 100 scale in the production version of the task, but logarithmic one on the 0 to 1,000 scale. For the perception version of the task, the authors found that children's estimation patterns were exponential in nature. In contrast, in older students as well as adults, results followed a linear pattern with no clear indication of under- or overestimation.

Additionally, Slusser and Barth (2017) found relatively consistent estimation accuracy across both task versions as well as patterns of under- and overestimation specific task version. Additionally, proportion estimation models offered a more appropriate explanation of participants' performance than linear, logarithmic or exponential models and consistent results of estimation bias remain on both bounded task versions over the course of development. These findings indicate that the different task versions of the bounded number line estimation task induce people to apply different solution strategies calling into question whether this task is a valid measure of mental number representation. Besides, only little work has been done applying the perception version of a number line estimation task (cf. Ashcraft & Moore, 2012; Iuculano & Butterworth, 2011).

For unbounded number line estimation, there is currently no study investigating similarities and differences between the production and perception version of the task. In fact, we are not aware that the latter was ever used in research before. Furthermore, there is currently no study evaluating the conceptual similarity of bounded and unbounded number line estimation to with analogue (non-symbolic) numerosity estimation.

1.3 The present study

In the present study, we therefore aimed at investigating the conceptual similarity of unbounded and bounded number line estimation with non-symbolic numerosity estimation. In line with previous results, we generally expected to replicate the previous findings of Crollen and colleagues (2011) in terms of underestimation in our numerosity perception task and overestimation in our numerosity production task.

Furthermore, in line with the argument that unbounded number line estimation reflects a measure of numerical estimation (e.g., Cohen & Blanc-Goldhammer, 2011), we hypothesized that estimation performance in the two versions of the unbounded number line estimation task should follow the pattern of under- and overestimation observed in the perception and production version of the numerosity estimation task. In the perception version of unbounded number line estimation, participants should systematically underestimate numerical magnitudes whereas we expected them to overestimate numerical magnitude in the production version. So far, a perception version of this task has never been described in the literature, only the production version.

The single unit distance at the origin of the number line as well as the hatch mark at the position to be estimated in the perception task of the unbounded number line estimation task are assumed to activate the non-symbolic, analogue representation of numbers. When estimating the corresponding Arabic number reflecting the spatial position on the number line, participants have to transcode from the logarithmically compressed analogue to the symbolic linear Arabic representation. Accordingly, corresponding values of the target number are always smaller and participants are expected to underestimate the respective numerical magnitude. The other way round, in the production task, we expect that the magnitude of the spatial position of the target number on the number line is overestimated by participants as the numerical estimation process is based on symbolic to non-symbolic mapping. In particular, in this version of the task, the target number is presented as an Arabic number that has to be transcoded into an analogue representation of numerosity. Therefore, the estimation process starts from a position on the symbolic linear numerical representation to its associated magnitude on the logarithmically compressed subjective number line. As such, objective magnitude seems smaller than its subjective counterpart and participants should overestimate the respective magnitude. Observation of such a result pattern would provide additional evidence for the assumption that the unbounded number line estimation task reflects a more unbiased measure of the mental number line representation as compared to the bounded number line estimation task.

Because bounded number line estimation was supposed to entail proportion-based judgments, we expected a more task-specific result pattern. With regard to over- and underestimation in the production and perception task version Siegler and Opfer (2003, see also Slusser & Barth, 2017) found inconsistent result patterns for their children and adult participants. Therefore, we had no specific expectations with respect to the pattern of over- and underestimation in bounded number line estimation.

In sum, considering the bi-directional mapping hypothesis, we put forth the following hypotheses concerning the estimation patterns of performance in the different tasks (see Figure 2 for the expected error pattern): (1) a replication of the result pattern of Crollen and colleagues (2011) in non-symbolic numerosity estimation with underestimation in the perception and overestimation in the production version of the task. (2) Because unbounded number line estimation has been argued to reflect a purer measure of numerical estimation, we analogously expected underestimation in the perception and overestimation in the production version of the task. In contrast, for (3) bounded number line estimation we did not have any specific expectation in both versions of the task.

Our study addressed this issue by a systematic comparison between estimation patterns from both bounded and unbounded number line estimation with analogue numerosity estimation. We expected that the systematic pattern of under- and overestimation in the numerosity estimation task should generalize to unbounded but not bounded number line estimation. In this case, our data would provide converging evidence for the claim that unbounded number line estimation is indeed a purer measure of number magnitude estimation from a new, complementary perspective never been taken before.

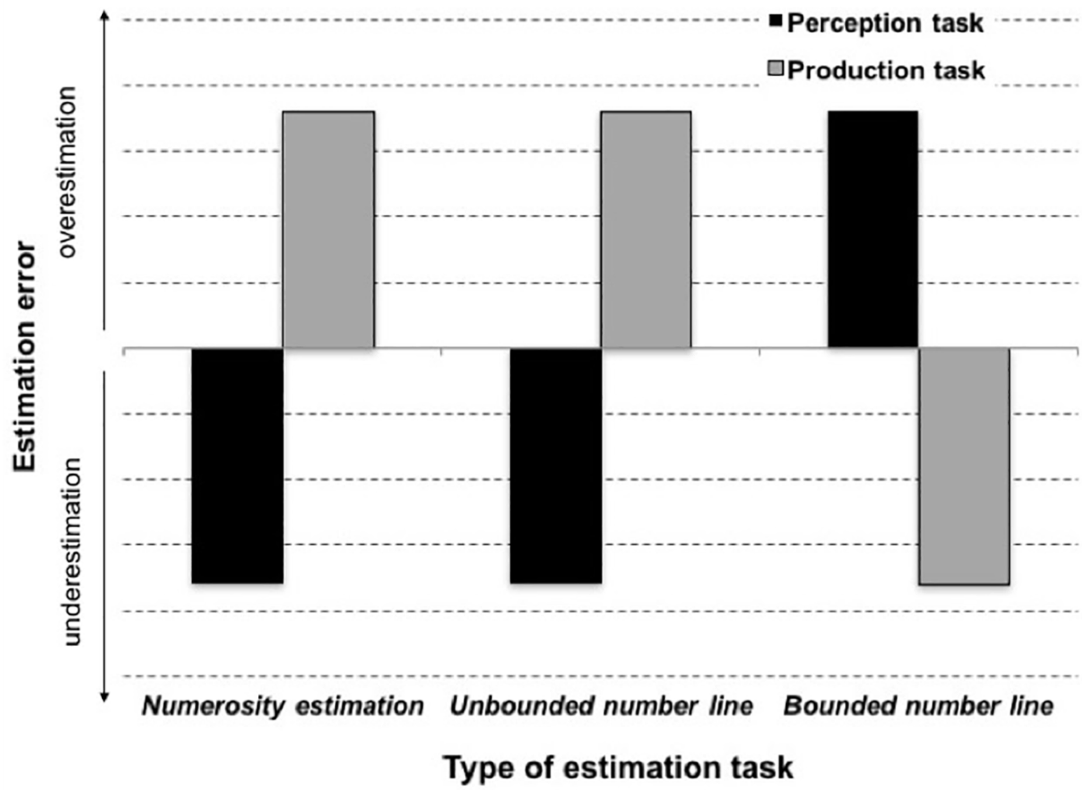


Figure 2. Expected error patterns in the three different types of estimation tasks. We hypothesized that error patterns of numerosity estimation and unbounded number line estimation should be identical, whereas the error pattern for bounded number line estimation should be reversed.

1.4 Methods

1.4.1 Participants

A total of 75 German-speaking students of the University of Bern (21 males; 10 left-handers) participated voluntarily in the experiment for course credits. The average age was 23.6 years (*SD* = 4.4 years; *range* = 19-39 years). All participants reported normal or corrected-to-normal vision. Additionally, all participants signed an informed consent form prior to the study, which was approved by the local ethics committee of the University of Bern (Nr. 2017-02-00008).

1.4.2 Stimuli and procedure

Participants were assessed individually with a battery of tasks which were instructed separately. For all tasks, stimuli were presented as pictures on a laptop with a 15´´ screen with a resolution of 1,024 x 768 pixels. Each task consisted of two types of estimation: perception and production of numerosities. The perception version of each task was always administered first to avoid that the maximal number of the numerosity presented in the production task made participants anticipate the maximum magnitude in the production task. Moreover, participants received no information about the range of numerosities being presented and no feedback was provided. Items were presented in randomized order for each participant individually.

Tasks were administered in the following order: First, two different numerosity perception tasks were presented to participants whereas the order of both versions was counterbalanced. Half of the participants started with a version in which all presented dots had the same size whereas the other half began with the version in which overall area covered by the presented dots was matched (see next section for details). These tasks were directly followed by the numerosity production task. Second, perception and production versions of the unbounded number line estimation task were administered prior to the two respective versions of the bounded number line estimation tasks. Thereby, the task which explicitly defines a number range (i.e., bounded number line estimation) was administered last to avoid participants building up expectations about the number ranges used in the tasks. For both, numerosity estimation as well as unbounded number line estimation the upper bound of the number range covered by the tasks is not specified to participants, and thus they may not use a given upper bound as an additional reference point. As the bounded number line estimation task explicitly defines a number range by its upper bound, it was conducted last to prevent participants to assume a similar number range for numerosity and unbounded number line estimation.

Two additional control tasks were given last: one task in which participants had to estimate proportions of areas like triangles, circles, squares and rectangles and the Berlin Numeracy Task (Cokely, Galesic, Schulz, Ghazal, & Garcia-Retamero, 2012). These were administered to investigate a research question different from the one described in the present study. Therefore, results for these two tasks will not be considered in the present study. In sum, the experiment took approximately 45 min.

Numerosity estimation:

Twenty-four different numerosities were used to create the target sets of dots that ranged from 30 to 100. Two different target sets were developed: one for the two *perception tasks* (30, 34, 39, 41, 43, 46, 48, 52, 55, 57, 60, 64, 65, 69, 70, 75, 78, 82, 83, 86, 87, 91, 94, 99), and an additional one for the *production task* (31, 35, 38, 40, 42, 47, 49, 53, 54, 56, 62, 63, 66, 67, 71, 76, 77, 80, 84, 85, 89, 92, 93, 98). Target numbers were chosen to have the same mean problem size for the overall number range as well as numbers within each decade (i.e. 31-39, 41-49, etc.). Furthermore, we developed two different versions of stimuli for the perception task to control for perceptive parameters such as total occupied area and dot size. In one stimuli set, all dots had the same size and therefore the area covered by the array increased with increasing numerosity. In the other set, the sum of the area of all the dots on the screen was kept constant.

Each trial in the *numerosity perception task* started with a centrally presented black fixation cross presented for 1,000 ms against a white background followed by a pattern of dots that was flashed on the screen for 250 ms. Afterwards, the sign "=" was displayed and replaced after 500 ms by the Arabic numeral "1". Participants' task was to indicate as quickly and accurately as possible the numerosity of presented dots as an Arabic number. To give their answers, they had to scroll the mouse wheel up through the sequence of Arabic numbers (or down to go back to correct the answer) and press the "Enter" button to finish their estimation. No information about the number range and feedback concerning their performance were provided.

In the *numerosity production task*, participants were required to produce a dot pattern that was equivalent to the presented Arabic number magnitude. Each trial started with the presentation of a black fixation cross (for 1,000 ms) in the middle of the screen against a white background, followed by the two-digit target Arabic numeral for 250 ms. Then, the sign "=" was displayed (500 ms) and the production phase began with a single dot on the screen at a randomly determined position. Participants then had to start dots production by scrolling the mouse wheel up to increase the number of dots that appear on the screen and down to decrease the number of dots. When they had the impression that the numerosity of dots corresponded to the requested number, they pressed the "Enter" button. The maximum number of dots that participants could produce was limited to 254 (cf. Experiment 2 of Crollen et al., 2011).

Number line estimation:

In the production version of both unbounded and bounded number line estimation tasks, participants were instructed to indicate as accurately and as fast as possible the spatial position of the given target number on the number line using the mouse to click at the estimated position (upper panels of Figure 3a and b). In the perception version of the tasks, participants had to insert the Arabic number specifying the spatial location already indicated on the number line by using the keyboard. The number lines as well as target numbers were presented in black colour against a white background (see Figure 3).

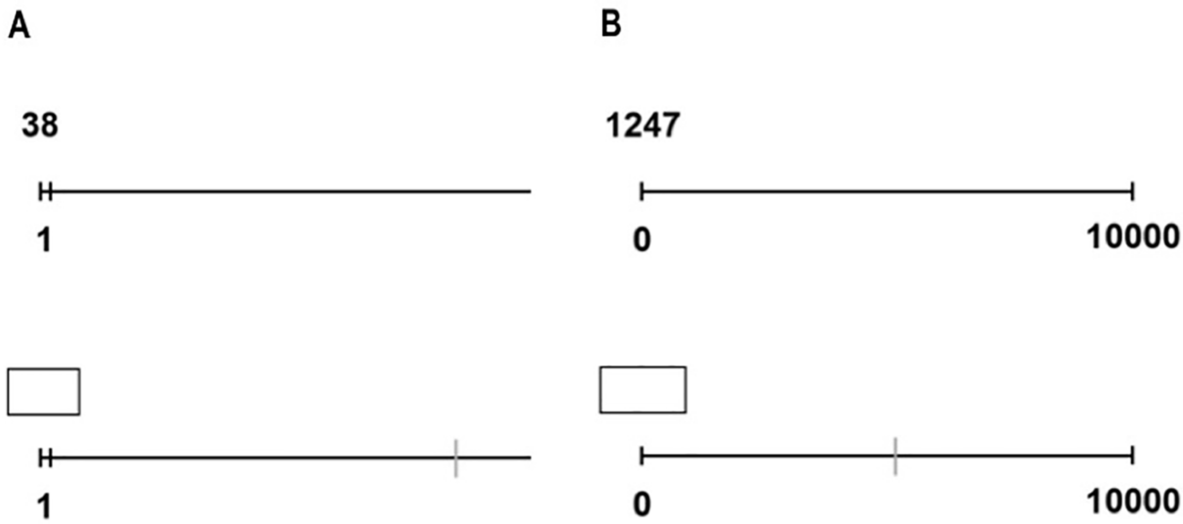


Figure 3. Schematic illustration of an example of the (A) unbounded and (B) bounded number line estimation. The production tasks are displayed in the upper row, the perception tasks in the lower row.

Unbounded number line estimation task

A total of twenty-four items were used as target sets ranging from 2 to 49, one for the *perception task* (3, 4, 7, 8, 11, 12,15, 16, 19, 20, 23, 24, 27, 28, 31, 32, 35, 36, 39, 40, 43, 44, 47, 48) and another for the *production task* (2, 5, 6, 9, 10, 13, 14, 17, 18, 21, 22, 25, 26, 29, 30, 33, 34, 37, 38, 41, 42, 45, 46, 49). Here, too, target numbers were chosen to have the same mean problem size regarding both numbers within each decade as well as the overall number range covered. Each number line was a horizontal line with a physical length of 18 cm. Only the start-point together with the predefined unit of 1 having a length of 0.3 cm was presented (see Figure 3, Panel A). We used a smaller number range for unbounded number line estimation as there is no evidence that number range influences participants' estimation patterns (i.e., increasing error variability with increasing target numbers). In fact, estimation patterns were virtually identical across smaller and larger ranges (e.g., 25 in Cohen & Blanc-Goldhammer, 2011; 49 in Reinert et al., 2015; 400 in van der Weijden, Kamphorst, Willemsen, Kroesbergen, & van Hoog-moed, 2018; 1,000 in Kim & Opfer, 2017).

In the *perception task*, a response box was displayed above the start-point and the position to be estimated was marked with a blue vertical line. Participants had to insert the corresponding Arabic number reflected by the spatial position on the number line using the number keys on the keyboard. To complete the estimation, they had to press the "Enter" button and then, the next trial appeared at a random position on the screen to prevent participants from using external reference points.

In the *production task*, target numbers were displayed above the start point of the number line (see Figure 3, lower chart) at a position on the screen randomly varying from trial to trial. The blue vertical line which reflected the mouse curser always appeared in the centre of the screen with a vertical length of 1.4 cm. To give their responses, participants had to mark the estimated spatial position of the respective target number by moving the blue line mouse curser there and click on the felt button of the mouse.

Bounded number line estimation task

The bounded number line estimation task covered the number range from 0 to 10.000 using the target numbers (162, 453, 820, 1027, 1644, 2341, 2660, 3221, 3786, 4259, 4575, 4808, 5128, 5420, 5827, 6390, 6915, 7237, 7682, 8406, 8753, 9049, 9561, 9876) for the *perception task* and an additional target set (124, 439, 951, 1247, 1594, 2318, 2763, 3085, 3610, 4173, 4580, 4872, 5192, 5425, 5741, 6214, 6770, 7340, 7659, 8356, 8973, 9180, 9547, 9838) for the *production task*. These were selected with a slight oversampling at the midpoint 5.000 as well as the start- and endpoint (0 and 10,000) as reference marks. Start- and endpoint were always displayed at the same position on the screen and the physical length of the number line was 18 cm.

In the *perception task*, a response box was presented above the start-point and the spatial location of the target number was marked with a blue vertical line. Participants had to insert the Arabic numbers reflecting the position on the number line using the number keys of the keyboard (see Figure 3, Panel B). To give finalize their answers, they had to press the "Enter" button. Then, the next trial appeared.

In the *production task*, participants were required to indicate the spatial position of a given number on an empty number line with specified start- and endpoint. Therefore, they were instructed to mark their estimated position of the respective target number with the blue vertical line always appearing in the left lower corner on the number line.

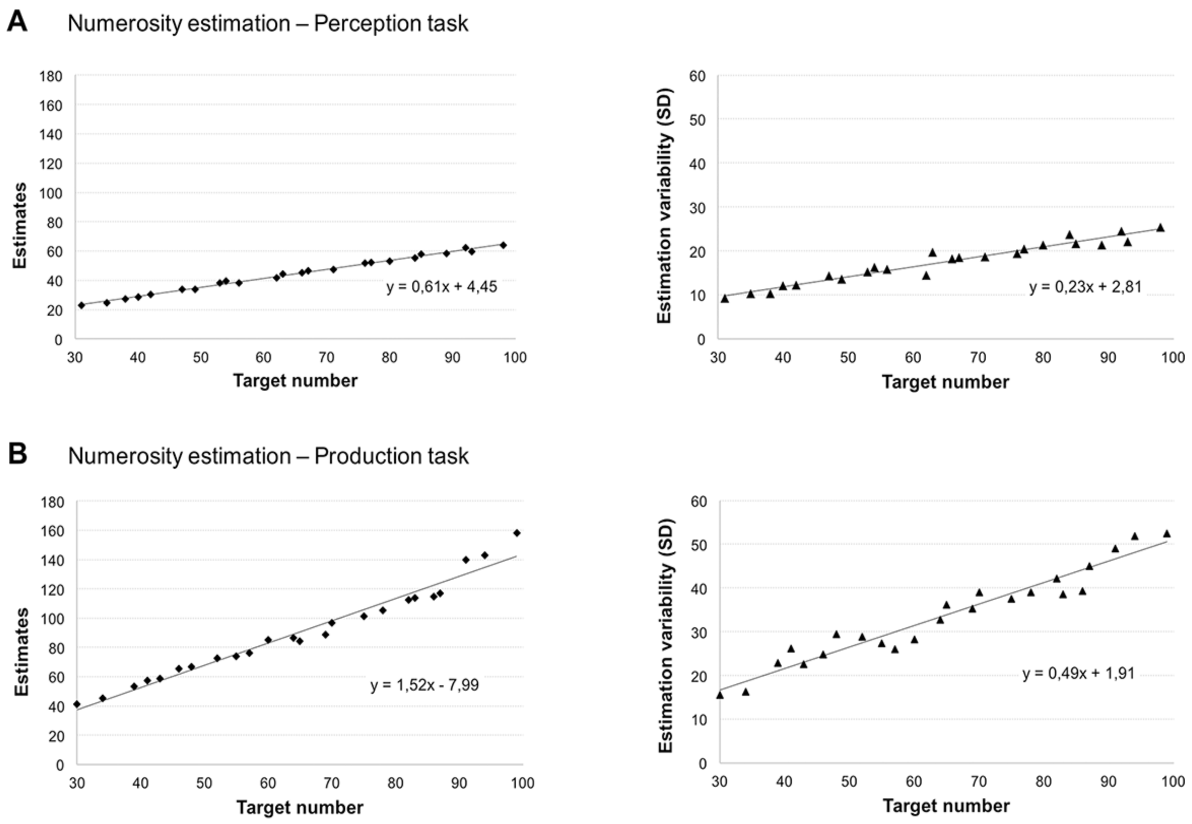
1.4.3 Analyses

As preliminary analyses indicated that mean estimates as well as their standard deviations were not normally distributed in all tasks, we used the median as a measure of central tendency of the relative estimation errors. For all magnitude estimation tasks, the mean percent relative estimation error [REE = (estimation number – target number)/number range of the task * 100] served as dependent variable, this means that we standardized participants' estimation errors on the number range of the respective task to increase comparability of results. Please note that we were not interested in comparing the unstandardized magnitude of estimation errors across the different task versions. Instead, we were specifically interested in the pattern of over- and underestimation for perception and production versions of the three tasks, which should be reflected in REE.

Number ranges considered were 1 to 10,000 for bounded and 2 to 49 in the unbounded number line estimation task as well as 30 to 99 in the numerosity estimation task. Please note that to evaluate patterns of over- and underestimation we used the *relative* estimation error as dependent variable and not the more often used absolute estimation error (e.g., Siegler & Opfer, 2003). Accordingly, a REE of zero would reflect accurate estimations, whereas negative REE indicate underestimation and positive REE overestimation of the target numbers. Furthermore, in case the sphericity assumption of the ANOVA (analysis of variance) was violated, the Greenhouse-Geisser coefficient (GG) is given to adjust the degrees of freedom. An overview of participants' estimation patterns (left charts) and error variability (right charts) is given in Figure 4.

1.5 Results

In a first step, individual trials that differed more than ± 4 standard deviations from the overall mean estimates were excluded from the analysis (0.25% of the data). Moreover, we used the average mean of both versions of the perception numerosity estimation task as dependent variable for the ANOVA because the correlation between REE in both versions was $r(75) = .89$, $p < .001$, and therefore sufficiently high to pool the respective means.



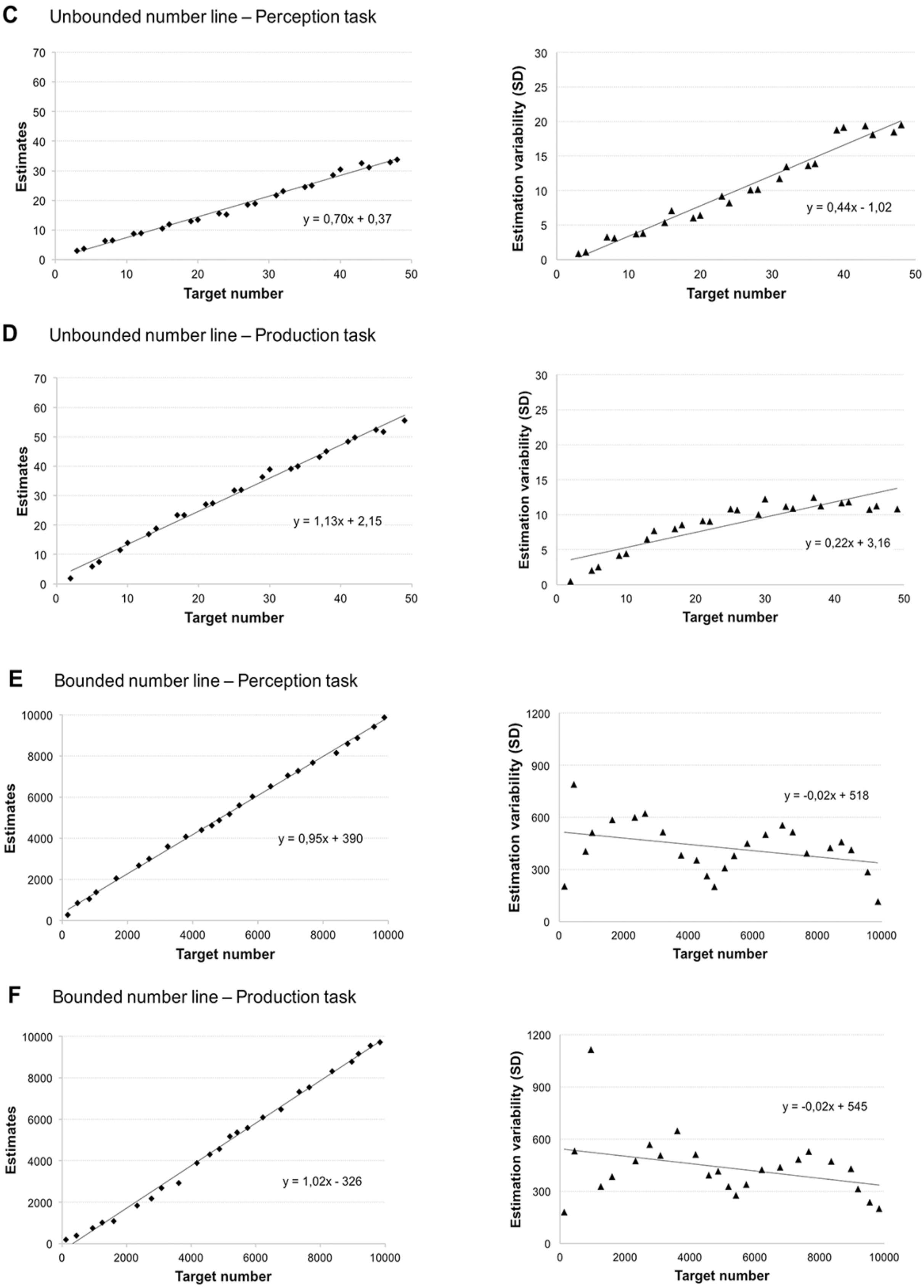


Figure 4. Estimation patterns (mean estimates across all participants, left charts) and estimation error variability (SD of REE, right charts) for numerosity estimation (Panels A + B), unbounded number line estimation (Panels C + D) and bounded number line estimation (Panels E + F).

To examine the performance patterns in the different estimation tasks, we ran a 3 x 2 repeated-measures ANOVA with the factors *type of estimation* (numerosity estimation, unbounded vs. bounded number line estimation task) and the *task type* (perception vs. production; see Figure 5). This analysis revealed a significant main effect of task type, $F(1, 74) = 167.80$, $p < .001$, $\eta^2_p = .694$, indicating that participants showed systematic underestimation in perception and overestimation in production tasks ($M_{perception}$ = -11.44% vs. $M_{production}$ = 10.04% REE). The main effect for type of estimation $F(2, 148) = 2.44$, $p = .104$, $\eta^2 p = .032$, $GG = .783$, was not significant, reflecting that there were no significant differences in accuracy of participants' estimation patterns between the three estimation tasks ($M_{numerosity\ estimation}$ = .61 vs. $M_{bounded}$= -.40 vs. $M_{unbounded}$ = -2.30).

Most importantly, we observed that the main effect of task type was qualified by the significant interaction between the factors type of estimation and task type $F(2, 148) = 59.03$, $p < .001$, $\eta^2 p = .444$, $GG = .782$. The interaction indicated that, in line with our expectations, target numbers were underestimated in the perception version of the numerosity estimation task ($M_{perception}$ = -20.01% vs. $M_{production}$ = 21.24% REE) as well as the unbounded number line estimation task ($M_{perception}$ = -15.32% vs. $M_{production}$ = 10.72% REE) whereas participants overestimated target numbers in the production version of these two tasks. However, this pattern was reversed for bounded number line estimation. Here, participants overestimated target numbers in the perception version and underestimated then in the production version of this task ($M_{perception}$ = 1.02% vs. $M_{production}$ = -1.83% REE). Testing simple effects using Bonferroni-Holm corrected *t*-tests revealed that estimation accuracy differed significantly between perception and production versions of all tasks (numerosity estimation: $t(74) = 10.35$, $p < .001$; bounded number line estimation: $t(74) = 9.66$, $p < .001$; and unbounded number line estimation: $t(74) = 8.55$, $p < .001$.

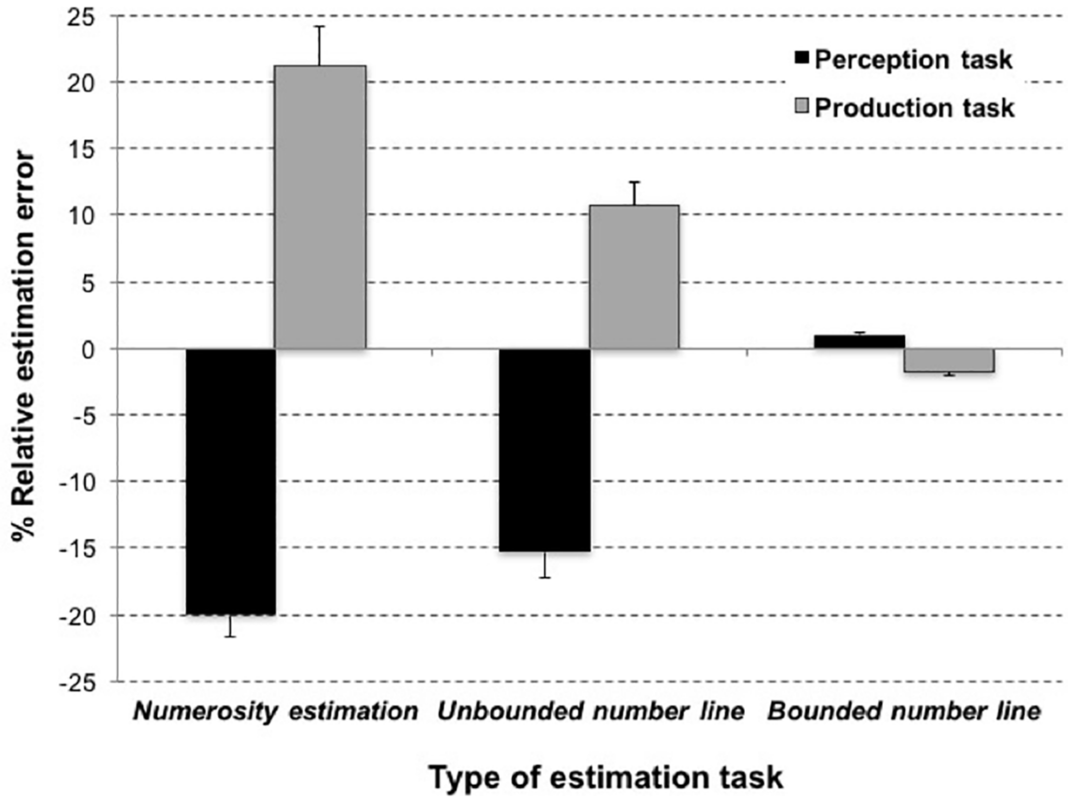


Figure 5. Mean percent REE for the three magnitude estimation tasks separated for perception and production task type. Error bars reflect 1 Standard Error of the Mean (SEM).

1.6 Discussion

The present study set off to systematically investigate similarities and differences between non-symbolic numerosity estimation (e.g., Crollen et al., 2011) and bounded (e.g., Siegler & Opfer, 2003) as well as unbounded number line estimation (e.g., Cohen & Blanc-Goldhammer, 2011). In particular, the aim of our study was to examine the generalizability of patterns of underestimation in the perception and over-estimation in the production version of non-symbolic numerosity estimation to bounded and unbounded number line estimation. This approach seems promising to evaluate whether unbounded number line estimation is more similar to non-symbolic estimation and thus reflects a purer measure of magnitude representation. In the following, we will first discuss the results with respect to unbounded number line estimation as a measure of number magnitude representation before elaborating on the broader implications of these findings for the bi-directional mapping hypothesis.

Unbounded number line estimation as a purer measure of number magnitude representation

We expected to replicate the result pattern of underestimation in the perception and overestimation in the production version non-symbolic numerosity estimation (Crollen et al., 2011). Furthermore, we analogously hypothesized the same pattern of estimation errors for the perception and production version of unbounded number line estimation. The current data corroborated these two hypotheses by revealing that systematic estimation biases were identical for non-symbolic numerosity and unbounded number line estimation. For both estimation tasks, we observed that participants systematically underestimated target numbers in the perception, and overestimated them in the production version.

Importantly, our data provide additional evidence for the argument of Cohen and Blanc-Goldhammer (2011) that unbounded number line estimation might be a purer measure of the (spatial) representation of number magnitude than bounded number line estimation (see also Barth & Paladino, 2011; Slusser et al., 2013). So far, the argument that unbounded number line estimation represents a more valid measure of number magnitude representation originally came from comparisons of estimation performance in bounded and unbounded number line estimation. Here, we showed its similarity with non-symbolic numerosity estimation that is commonly agreed on to constitute a reliable measure of number magnitude representation.

Additionally, the observation of a reversed estimation pattern of overestimation in the perception and under estimation in the production version for bounded number line estimation further corroborates this interpretation. These findings are in line with our expectations and seems to reflect previous findings of Siegler and Opfer (2003) who observed a systematic pattern of overestimation in the perception task, at least in children. Additionally, for the production version of the task Booth and Siegler (2006) observed evidence for underestimation in the production version of the task. This pattern of under- and overestimation clearly differed for that found for numerosity estimation and unbounded number line estimation. Therefore, these data provide converging evidence for the notion that bounded number line estimation may not only measure numerical estimation but also task-specific strategies (see Barth & Paladino, 2011; Slusser et al., 2013; Sullivan et al., 2011).

In sum, our results seem to substantiate claims that unbounded number line estimation may be more suitable to draw inferences on adults´ (spatial) representation of number magnitude (see also Link, Huber, Nuerk, & Moeller, 2014; Siegler & Booth, 2004). Importantly, however, these data are not only relevant for our understanding of unbounded number line estimation but also for the bi-directional mapping hypothesis on non-symbolic magnitude estimation.

1.6.1 Further evidence for the bi-directional mapping hypothesis

For the first time, we investigated perception and production versions for all non-symbolic, unbounded as well as bounded estimation tasks. As expected, we observed that the characteristic pattern of overestimation in the production and underestimation in the perception version of numerosity estimation (Crollen et al., 2011) as well as the unbounded number line estimation task. Importantly, this is in line with the postulates of the model on the mapping between symbolic and non-symbolic representations proposed by Izard and Dehaene (2008). They suggested different mapping processes from non-symbolic input to symbolic output and symbolic input to non-symbolic output, leading participants to systematically under- and overestimate the respective target magnitudes, respectively.

In the perception version of numerosity estimation as well as unbounded number line estimation, the estimation process goes from the logarithmically compressed representation of non-symbolic numerosity to the linearly spaced representation of symbolic number magnitude resulting in the observed underestimation. In the perception version of unbounded number line estimation, the unit distance given at the origin of the number line as well as the hatch mark at the position to be estimated activated a non-symbolic, analogue representation of the respective target numbers. When estimating the corresponding Arabic number based on predefined spatial position on the number line, participants had to transcode from the non-symbolic analogue into the symbolic Arabic number form. Consequently, the corresponding values of the target numbers are always smaller and participants underestimate numerical magnitude. In contrast, in the production version of unbounded number line estimation, a linearly spaced representations of symbolic number magnitude had to be mapped onto the logarithmically compressed representation of non-symbolic numerosity, which, in turn, led to the observed pattern of overestimation (see Figure 1). These findings for unbounded number line estimation further strengthen the bi-directional mapping hypothesis (Castronovo & Seron, 2007) by providing convergent evidence from perception and production versions for the first time.

1.6.2 Limitations and perspectives

There are aspects to keep in mind when interpreting the current results. On a methodological level it should be acknowledged that to get an accurate measure of estimation errors in unbounded number line estimation, it is critical to allow enough room for participants to express errors. In the current task, the number line had a length of 50 units and target numbers up to 49 had to be estimated. Therefore, the end of the number line may have acted as a boundary that participants will likely not extend their answer past. This might have influenced estimation errors for target numbers approaching 49. However, the overall pattern of estimation errors we observed in the unbounded task was more or less identical to the patterns observed in previous studies (e.g., Cohen & Blanc-Goldhammer, 2011; Link et al., 2014). Therefore, we are confident that this should not have biased results. Nevertheless, it may be desirable for future studies to allow more space between the largest target number and the end of the number line so that implicit boundaries are so far beyond the participant's likely response that they will not influence it.

On the theoretical level, it is important to note that the bi-directional mapping hypothesis implicitly assumes a logarithmically compressed non-symbolic representation of number magnitude representation in comparison to a linear one for symbolic magnitudes. In this context, it should be considered that there is a long-lasting scientific debate about the layout of number magnitude representations (i.e., logarithmically compressed vs. linear, e.g., Cantlon, Cordes, Libertus, & Brannon, 2009; see also Beran, Johnson-Pynn, & Ready, 2008; Gibbon & Church, 1981). However, it was not at the heart of this study to evaluate this. Instead, we only considered the predictions of the bi-directional mapping hypothesis, which reflect specific assumptions on logarithmic non-symbolic and linear symbolic magnitude representations. As such, the observation of identical patterns of over and underestimation in the perception vs. production version of numerosity and unbounded number line estimation might imply a similar logarithmic layout of the representation of non-symbolic magnitude in unbounded number line estimation. However, further research is needed addressing this question more specifically.

Finally, as also noted by Crollen and colleagues (2011), the investigation of the mapping process between symbolic and non-symbolic magnitude representations is only in its early stages with very few research directly addressing this question. More precise and general conclusions will become possible when taking a closer look at the development of the numerical mapping abilities in children (see also Link et al., 2014; Lipton & Spelke, 2006; Mundy & Gilmore, 2009). Future studies should therefore further investigate the model of bi-directional mapping by exploring data of children. It would be interesting whether the same pattern of under- and overestimation would be replicated in such a sample.

1.7 Conclusions

Taken together, to the best of our knowledge, the present study is the first to systematically assess different types of estimation tasks (i.e., numerosity, as well as bounded and unbounded number line estimation) in both their perception and production version in adults to evaluate similarities and differences between these tasks. We replicated the pattern of systematic biases of under- and overestimation for numerosity estimation and also found that this pattern generalized to unbounded but not bounded number line estimation. Therefore, our results indicated conceptual similarity of unbounded number line and non-symbolic numerosity estimation. As such, these findings provide converging evidence from results of an established magnitude estimation task that unbounded number line estimation might be a purer and more valid measure of (spatial) number magnitude representation as compared to bounded number line estimation.

STUDY 2:

Strategies in unbounded number line estimation? - Evidence from eye-tracking

Authors:	Regina M. Reinert, Stefan Huber, Hans-Christoph Nuerk & Korbinian Moeller
Individual contribution:	I was mainly responsible for the scientific ideas, conceptualization, data generation, introduction, methods and discussion part of the paper.
Contribution of co-authors:	Stefan Huber run the analyses and wrote the results part of the paper. Hans-Christoph Nuerk was involved in the conceptualization of the study. Korbinian Moeller was also responsible for the conceptualization, data analyses, supervision and paper review and editing.
Presentation:	This paper was presented at the 6th International Conference on Spatial Cognition: "Space and Situated Cognition" in September 2015 in Rome, Italy.
Current stage:	Accepted for publication in *Cognitive Processing*
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2.1 Abstract

For bounded number line estimation recent studies indicated influences of proportion-based strategies as documented by eye-tracking data. In the current study, we investigated solution strategies in bounded and unbounded number line estimation by directly comparing participants' estimation performance as well as their corresponding eye-fixation behaviour. For bounded number line estimation increased numbers of fixations at and around reference points (i.e., start-, middle, and endpoint) confirmed the prominent use of proportion-based strategies. In contrast, in unbounded number line estimation the number of fixations on the number line decreased continuously of with increasing magnitude of the target number. Additionally, we observed that in bounded and unbounded number line estimation participants' first fixation on the number line was a valid predictor of the location of the target number. In sum, these data corroborate the idea that unbounded number line estimation is not influenced by proportion-based estimation strategies not directly related to numerical estimations.

Keywords: unbounded number line estimations, eye-fixation behaviour, solution strategies.

2.2 Introduction

The representation of number magnitude is often described as a mental number line upon which magnitudes are arranged spatially in ascending order. A common task proposed to assess this spatial representation of number magnitude is the number line estimation task. It requires participants to indicate the spatial position of a target number on a given number line (e.g. the position of 68 on a 0-to-100 number line; e.g., Siegler & Opfer, 2003).

However, there is a controversial debate on the question of what is actually assessed by this task: the spatial layout of the underlying mental number line representation (e.g., Berteletti et al., 2010; Siegler & Opfer, 2003) or rather strategies applied by participants to solve the task. In particular, Barth and Paladino (2011; see also Slusser, Santiago, & Barth, 2013) argued that estimation performance in the bounded number line task reflects application of proportion judgement strategies as indicated by the systematic use of reference points. This claim was supported by superior model fits of proportion judgment models as well as the evaluation of participants' eye-fixation patterns. For instance, Sullivan, Juhasz, Slattery and Barth (2011) evaluated eye-fixation behaviour of children while solving a bounded number line estimation task on the 0-to-1,000 scale. The participants' eye-fixation pattern clearly indicated an explicit preference to fixate on reference points (i.e., the start, mid, and endpoint of the number line). Thereby, these findings discourage the claim that the bounded number line estimation task provides a measure of the underlying spatial representation of number magnitude.

However, the eye-tracking results of Sullivan et al. (2011) also revealed another interesting aspect. The authors observed that the magnitude of the target number influenced the location of the very first fixation on the number line – with larger target numbers being associated with first fixations further to the left. This indicated some kind of internal scaling of participants' first saccade onto the number line by the magnitude of the target number. Furthermore, the location of the first fixation was predictive of participants` final estimate indicating that there are early processes of numerical estimation in the bounded number line estimation task before proportion judgement strategies become dominant.

Recently, Cohen and Blanc-Goldhammer (2011) introduced a new *unbounded* number line estimation task, which they claimed to provide a "purer" measure of numerical estimation. In the unbounded version of the task, only the start point and a scaling unit, but no end point is given. Therefore, Cohen and Blanc-Goldhammer (2011) argued that participants cannot use reference points and thus, proportion judgement strategies, but have to actually estimate the spatial location of the target numbers. This argument was supported by the fact that estimation errors increased linearly with the magnitude of the target numbers. Synced with model fitting evidence, this argues against the reliance on reference points and thus, the application of proportion judgement strategies in unbounded number line estimation. However, different from bounded number line estimation, there is currently no independent evidence from – for instance eye-tracking – corroborating this claim.

Therefore, the present study aimed at investigating the possible use of reference points in the unbounded number line estimation by evaluating participants' eye-fixation behaviour similar to what was done by Sullivan et al. (2011) for bounded number line estimation. In line with recent findings for estimation errors, we hypothesized that (i) eye-fixation patterns should also not indicate the use of specific reference points (reflected by specific peaks in fixation frequency). Instead, we anticipated a continuous decrease of fixations on the number line with increasing magnitude of the target number. Furthermore, (ii) we expected to observe a similar scaling of the location of the first fixation as previously observed for bounded number line estimation (Sullivan et al., 2011) reflecting early processes of numerical estimation.

2.3 Method

2.3.1 Participants

Twenty-seven students of the University of Tuebingen (seven male) participated in the study for course credits. Average age was 23.6 years (*SD* = 4.1 years; range = 19-37 years). All participants reported normal or corrected-to-normal vision.

2.3.2 Stimuli and Design

Participants had to complete an unbounded followed by bounded number line estimation task. For both tasks stimuli were presented on a 26" monitor as pictures with a resolution of 1,920 x 1,200 pixels and with number lines and target numbers in black against a white background (see Reinert, Huber, Nuerk, & Moeller, 2015 for further details).

In the *unbounded* number line estimation task, only the start point and the unit 1 was given. Number lines were 54, 58, 62 or 66 units long with physical length varying from 930 to 1,276 pixels with physical and numerical length uncorrelated. We used 20 target numbers ranging from 11 to 49, which were shown above the unit on the left side of the number line.

For the *bounded* number line estimation tasks, number lines were 50 units long with two labeled endpoints (0 and 50) while physical length of the number line varied between 930 to 1,276 pixels. The stimulus set consisted of all numbers from 1 to 49 without multiples of ten and ties leaving 41 critical items.

2.3.3 Apparatus

Eye movements were recorded using an EyeLink 1000 tracking device (SR-Research, Kanata, Ontario, Canada) providing a spatial resolution of less than 0.5 degree of visual angle at a sampling rate of 1000 Hz. Participants` eyes were about 60 cm from the screen.

2.3.4 Procedure

Participants were assessed individually. After the system had been calibrated, they were instructed to indicate the spatial position of the target number on the number line by a mouse click as accurately and fast as possible. Before each trial, a fixation mark primed the position, where the origin of the number line would appear together with the mouse cursor. Thereafter, the number line and the target number were presented together and remained visible on the screen until the participant's response.

2.3.5 Analysis

We evaluated differences between bounded and unbounded number line estimation in absolute estimation errors (AEE = absolute difference between actual and estimated position of target number) and participants' fixation pattern. For the analyses of the fixation pattern, the number line was subdivided into 50 equal-sized areas (height: 40 pixels, width depending on physical line length). All fixations that fell into these interest areas were considered for analyses (i.e., 36% fixations in bounded, 40% in unbounded number line estimation). The mean number of fixations in the 50 interest areas served as the variable of interest.

Statistical analyses comprised linear mixed models (LMM) run using the R packages lme4 (Bates, Maechler, Bolker, & Walker, 2014). *P*-values were calculated using the Satterthwaite approximation for degrees of freedom available via the R package lmerTest (Kuznetsova, Brockhoff, & Christensen, 2014).

2.4 Results

With respect to estimation errors, proportion judgement strategies are associated with a characteristic M-shaped pattern indicating estimation errors to be smaller at and around reference points (e.g., Barth & Palladino, 2011). This is accompanied by a W-shaped fixation pattern with more fixations at reference points (e.g., Sullivan et al., 2011). For both variables, we evaluated this pattern conducting a LMM including task version (bounded vs. unbounded) and reference point (yes vs. no) as fixed effects and participants as random effect. Because target numbers and the mouse curser were presented at the start point of the number line in both task versions, we excluded this reference point from further analyses. To increase reliability, we considered the mean of three point estimates at and around reference points (i.e., 24-26 and 47-49) and farthest away from reference points (i.e., 12-14 and 36-38). Fixed effects were effect-coded prior to data analysis.

2.4.1 Absolute Estimation Errors

The main effect of task version, $F(1, 81.00) = 303.21$, $p < .001$ and the interaction of task version and reference point, $F(1, 81.00) = 32.53$, $p < .001$, were significant, whereas the main effect of reference point was not, $F(1, 81.99) < 1$. The interaction indicated that AEE were smaller around the reference points for bounded number line estimation (reference: 0.82, *SE* = 0.50 vs. not reference: 2.53, *SE* = 0.50). For unbounded number line estimation this was reversed: AEE were larger around the reference points (reference: 9.83, *SE* = 0.50 vs. not reference: 8.10, *SE* = 0.50). The latter was driven by the continuous increase of AEE with the magnitude of the target number in unbounded number line estimation, estimate = 0.21 AEE/size target number, $t(27.01) = 8.74$, $p < .001$ (see Figure 6).

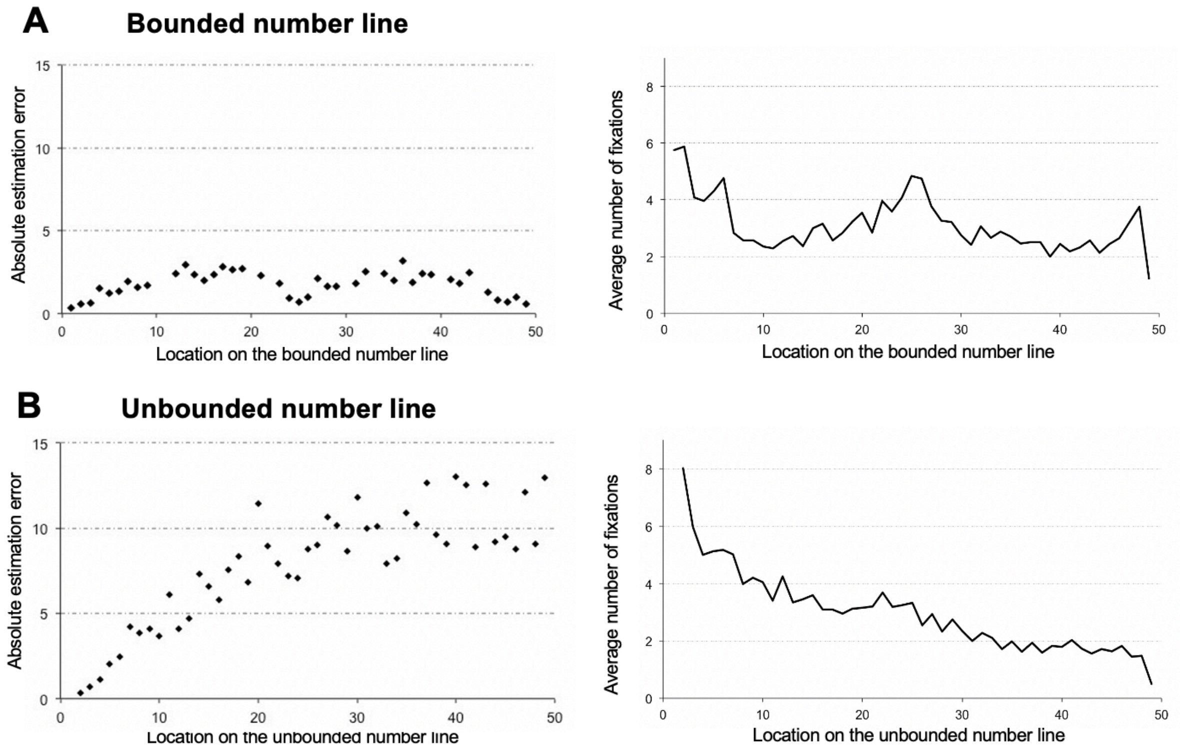

Figure 6. Estimation patterns in the (A) bounded and (B) unbounded number line estimation task. Absolute estimation error is depicted in the left and average number of fixations in the right column.

2.4.2 Fixation Pattern

The LMM revealed a main effect of reference point, $F(1, 79.12) = 3.98$, $p = .049$, and a significant interaction of target number and task version, $F(1, 79.12) = 14.75$, $p < .001$. The main effect task version was not significant, $F(1, 1.30) = 1.30$, $p = .260$. The interaction indicated that the fixation patterns differed reliably between task versions. For bounded number line estimation, participants fixated at or around reference points more often than at or around target numbers farthest away from the reference points (reference: 3.81, *SE* = 0.32 vs. not reference: 2.41, *SE* = 0.32). Again, this was reversed for unbounded number line estimation (reference: 2.61, *SE* = 0.33 vs. not reference: 3.06, *SE* = 0.32). Comparable to the case of AEE the latter was driven by the continuous decrease of the number of fixations on target numbers as their magnitude increased, estimate = -0.11 number of fixations/size target number, $t(26.53) = -6.48$, $p < .001$ (see Figure 6).

2.4.3 First fixation location

We additionally examined whether first fixations on the number line were reliable predictors of participants' final estimations. Therefore, we ran regression analyses with mean location of first fixations as predictor and mean estimated numbers as dependent variable. We observed that in both tasks participants' first fixation on the number line was a valid predictor of the location of the target number, bounded: $B = 2.56$ ($SE = 0.33$), $t(49) = 7.72$, $p < .001$, and unbounded: $B = 2.88$ ($SE = 0.16$), $t(49) = 17.97$, $p < .001$.

2.5 Discussion

In the current study, we evaluated participants' eye-fixation behaviour to investigate the use of reference points in unbounded number line estimation. We expected proportion-based estimation strategies in bounded number line estimation to be indicated by reduced variance of estimation error at and around reference points which should be associated with a specific increase of fixations at and around these reference points. In contrast, we hypothesized a continuous decrease of the number of fixations on the number line in unbounded number line estimation with increasing magnitude of the target number, because previous research did not indicate proportion-based strategies to be applied in this task (e.g., Cohen & Blanc-Goldhammer, 2011).

Replicating the results of Sullivan et al. (2011) for bounded number line estimation, absolute estimation errors were indeed reduced at and around reference points (i.e., 0, 25 and 50) resulting in the typical *M*-shaped pattern indicating the application of proportion-based strategies. Additionally, this was accompanied by a *W*-shaped distribution of fixations across the number line with higher numbers of fixations at and around reference points. However, this was different for unbounded number line estimation: absolute estimation errors continuously increased as the magnitude of the target number increased. Moreover, this was accompanied by a decreasing number of fixations on the number line with increasing target number. Taken together, these eye-tracking data provide convergent evidence on the assumption that no proportion-based estimation strategies are applied in unbounded number line estimation – corroborating the claim of Cohen and Blanc-Goldhammer (2011) that unbounded number line estimation might reflect a "purer" measure of numerical estimation than the bounded number line estimation task.

This conclusion is also backed by the finding that the location of participants' first fixation on the number line was a valid predictor of the location of the target number in unbounded number line estimation. This indicates that the location of the first fixation is calibrated by the magnitude of the target number and therefore, seems to be a direct reflection of underlying processes of numerical estimation. Interestingly, Sullivan and colleagues (2011) as well as ourselves observed this association for bounded number line estimation. Importantly, this indicates a similar calibration of participants' first fixation on the number line and thus, initial processes of numerical estimations in both task versions. However, in bounded number line estimation these initial processes of numerical estimation are then overridden by proportion-based estimation strategies which might be more precise and less resource demanding than numerical estimation.

2.6 Conclusions

The present study evaluated participants' eye-fixation behaviour to investigate strategies in unbounded number line estimation. We observed that the number of fixations on the number line decreased linearly with the magnitude of the target number in unbounded number line estimation. Additionally, the location of participants' first fixation on the number line was associated with their later estimated position of the target number. These findings corroborate the claim that unbounded compared to bounded number line estimation is not influenced by proportion-based estimation strategies which are not directly related to numerical estimations. Therefore, the unbounded number line estimation task might indeed measure numerical estimation more "purely".

STUDY 3:

Sex differences in number line estimation: The role of numerical estimation

Authors:	Regina M. Reinert, Stefan Huber, Hans-Christoph Nuerk & Korbinian Moeller
Individual contribution:	I was mainly responsible for conceptualization, data generation, analyses and paper writing.
Contribution of co-authors:	Stefan Huber developed the stimulus sets and ran the formal analyses. Hans-Christoph Nuerk was involved in the conceptualization and reviewed the paper draft. Korbinian Moeller was also responsible for the conceptualization, supervision and paper review & editing.
Presentation:	A prior version of this paper was presented as a poster at the Workshop Educational Neuroscience of Mathematics 2014 in Tübingen, Germany. The title was “Sex differences in number line estimation depend on solution strategies applied”.
Current stage:	Accepted for publication in the *British Journal of Psychology*
Cite this article:	Reinert, R.M., Huber, S., Nuerk, H.-C. and Moeller, K. (2017), Sex differences in number line estimation: The role of numerical estimation. Br J Psychol, 108: 334-350. https://doi.org/10.1111/bjop.12203

3.1 Abstract

Sex differences in mathematical performance have frequently been examined over the last decades indicating an advantage for males especially when numerical problems cannot be solved by (classroom-)learnt strategies and/or estimation. Even in basic numerical tasks such as number line estimation, males were found to outperform females – with sex differences argued to emerge from different solution strategies applied by males and females. We evaluated the latter using two versions of the number line estimation task: a bounded and an unbounded task version. Assuming that women tend more strongly to apply known procedures, we expected them to be at a particular disadvantage in the unbounded number line estimation task which is less prone to be solved by specific strategies such as proportion judgement but requires numerical estimation. Results confirmed more pronounced sex differences for unbounded number line estimation with males performing significantly more accurately in this task version. This further adds to recent evidence suggesting that estimation performance in the bounded task version may reflect solution strategies rather than numerical estimation. Additionally, it indicates that sex differences regarding the spatial representation of number magnitude may not be universal, but associated with spatial–numerical estimations in particular.

Keywords: unbounded number line estimation task, sex-related differences, efficient estimating.

3.2 Introduction

Sex differences have been reported for many content domains like verbal and mathematical abilities (Benbow, 1988; Geary, 1996, 2000; Halpern, 1986; Hyde, 2014; Hyde, Fennema, & Lamon, 1990; Hyde & Linn, 1988; Linn & Hyde, 1989; see also Lindberg, Hyde, Petersen, & Linn, 2010 for a review; Linn & Hyde, 1989) as well as in technical aptitude (Pereira & Miller, 2012; Schmidt, 2011), spatial abilities (Casey, Nuttall, & Pezaris, 2001; Cutmore, Hine, Maberly, Langford, & Hawgood, 2000; Levine, Vasilyeva, Lourenco, Newcombe, & Huttenlocher, 2005; Linn & Petersen, 1985; Maeda & Yoon, 2013; Voyer, 2011), and many other skills (Hyde, 1981; Johnson, 1996). Usually, girls and women tend to score higher on tests of verbal ability (Backman, 1972; Coie & Dorval, 1973; Hyde, 2014; Reilly, 2012), whereas boys and men generally achieve higher test scores in physical science and in many aspects of mathematics (Ceci & Williams, 2010; Halpern et al., 2007; Johnson, 1996; Spelke, 2005).

However, recent evidence indicates that sex-related differences in mathematics performance may have declined over the years and are at most small to moderate in size favouring males on average, but not in every content domain (Beller & Gafni, 1996; Else-Quest, Hyde, & Linn, 2010; Hedges & Nowell, 1995; Hyde, 2014; Zhu, 2007). In one of the first meta-analysis on this topic, Hyde, Fennema and Lamon (1990) indicated that men did not outperform women in understanding of mathematics concepts or computational ability, but did excel them in advanced problem-solving (Kimura, 2000) at high school and college levels (cf. Spencer, Steele, & Quinn, 1999). More recent studies also reported no consistent sex differences across grade levels (Hyde, 2014; Hyde, Lindberg, Linn, Ellis, & Williams, 2008) and nations (Else-Quest et al., 2010). Interestingly, current data of the Programme for International Student Assessment (PISA) showed that sex differences in favour of boys for mathematics are three times smaller than sex differences in favour of girls for reading. However, the former were observed consistently across nations. Additionally, boys were found to excel girls at the highest levels of mathematics ability in particular (Reilly, Neumann, & Andrews, 2015; Stoet & Geary, 2013). In contrast, Lindberg and colleagues observed similar mathematical performance in males and females (Hyde, 2014; Lindberg et al., 2010). Consequently, it can be stated that sex differences in mathematics are not consistent across age, culture, content domains, etc. (Linn & Hyde, 1989). However, when sex differences are found, males tend to outperform females in quickly solving complex mathematical tasks under unfamiliar conditions (Gallagher et al., 2000). Note that these studies might even underestimate actual sex differences in mathematics, as the performance in solving mathematical tasks not only depends on mathematical competency but also on general cognitive ability. Importantly, Brunner, Krauss, and Martignon (2011) observed even larger sex differences in mathematics favouring males when considering influences of general cognitive ability. The origin of this male advantage has seen diverse explanations ranging from computational fluency to solution strategies applied (Casey, Nuttall, & Pezaris, 1997; Geary, Saults, Liu, & Hoard, 2000; Royer, Tronsky, Chan, Jackson, & Marchant, 1999; see Zhu, 2007 for a recent review). On the one hand, Royer et al. (1999) found that male participants were in general faster than female participants on math-fact retrieval tasks suggesting that speed of fact retrieval in mathematics contributed to the sex disparity favouring males (Zhu, 2007). On the other hand, Geary et al. (2000) summarized that the pattern across studies suggests that sex differences might be due to differences in strategic approaches to solve arithmetic problems and in the speed of performing several component processes excepting retrieval (Geary, 1999). That is, males apply a larger repertoire of strategies for solving unknown mathematical problems and are more self-confident in trying out new strategies (Hyde, Fennema, Ryan, Frost, & Hopp, 1990). In line with this, it can be stated that men in general use more skilful approaches for arithmetic problems and are faster in arithmetical reasoning and retrieving arithmetic facts than are females. In addition to that, cultural stereotypes about females' lower math abilities were found to reduce their mathematics performance by interfering with their ability to generate problem-solving strategies (Quinn & Spencer, 2001). Furthermore, Gallagher et al. (2000) observed that male participants had the tendency to apply solution strategies more flexibly than female participants. In particular, males were more successful in matching strategies to problem characteristics in multiple solution path problems compared to women when completing the Scholastic Assessment Test – Mathematics (SAT-M). Furthermore, men excelled women in items of the Graduate Record Examination – Quantitative (GRE-Q), which assesses the development of procedural shortcuts, in particular so for items posing high demands on spatial skills. Additionally, these authors found that 'female students were more likely than male students to correctly solve "conventional" problems using algorithmic strategies [whereas] male students were more likely than female students to correctly solve "unconventional" problems using logical estimation and insight' (Gallagher et al., 2000, p. 167). In this context, conventional problems are mainly textbook problems, which can be solved by familiar algorithms. In accordance with this finding, males were also observed to score higher on average in mathematical tests not being related closely to curricular procedures (see Halpern et al., 2007 for a review) and preferred solving mathematical tasks individually by developing own strategies (see Zhu, 2007 for a review). Women, on the contrary, rather than men adhered to classroom-learnt methods and procedures for solving numerical/mathematical problems (Gallagher, 1998; Kessel & Linn, 1996).[2]

Taken together, men seem to be more prone than females to use individually developed strategies or even simply rely on estimation approaches for solving numerical/mathematical problems quickly (cf. Zhu, 2007). Thus, females should have a disadvantage when faced with numerical/mathematical problems for which the solution strategy is unknown or which explicitly require numerical estimation because standard procedures learnt at school cannot be applied or are at least less efficient in this context.

In this study, we aimed at evaluating this hypothesis for the basic numerical task of number line estimation. Generally, the number line estimation task requires participants to indicate the spatial position of a given target number on an otherwise empty number line. Recently, Bull, Cleland, and Mitchell (2013,

[2] Note that sex differences have also been reported in affect and attitude towards mathematics with more negative attitudes in females (Else-Quest et al., 2010; Hyde, Fennema, Ryan et al., 1990), which have also been considered to correlate with sex differences in performance (Caplan & Caplan, 2005; Krinzinger, Wood, & Willmes, 2012; Zhu, 2007). However, in the current manuscript we focused on possible differences in strategy use and related sex differences in number line estimation performance.

see also Thompson & Opfer, 2008 for sex differences in children) observed a male advantage in number line estimation for adults and attributed it to differences in the solution strategies applied to solve the task. In particular, they argued 'that men are […] more likely to use a spatial representation of number as a strategy for performing tasks' (p. 188). However, as they only used the traditional bounded version of the number line estimation task (with the start point and end point of the number range within which the respective number has to be located are given; e.g., 0–100, see Figure 7a, e.g., Siegler & Opfer, 2003), this conclusion may be premature. Originally, estimation performance in the bounded number line estimation task was regarded to allow for a direct assessment of the underlying mental number line representation (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Booth & Siegler, 2006, 2008; Siegler & Opfer, 2003). Based on changes in children's estimation patterns, Siegler and colleagues argued for a shift from a logarithmically compressed to a linearly spaced mental number line with age and experience (Booth & Siegler, 2006; Siegler & Booth, 2004). However, in recent years evidence accumulated that estimation performance in the bounded number line estimation task may not reflect pure number line estimation but is susceptible to influences of specific solution strategies. Barth and Paladino (2011) were the first to argue that the observed estimation pattern may reflect the use of proportion judgement strategies (see also Ashcraft & Moore, 2012; Cohen & Blanc-Goldhammer, 2011; Sullivan, Juhasz, Slattery, & Barth, 2011). Thereby, participants make use of specific reference points (e.g., start point and end point of the scale but also its middle) and orient their positioning of the target numbers at these reference points. Importantly, no indications for such proportion-based strategies were observed in a recently proposed unbounded number line estimation task (with only the start point and a unit are given, e.g., 0 as the start point and the length of the unit 1, see Figure 7b, e.g., Cohen & Blanc-Goldhammer, 2011; Cohen & Sarnecka, 2014; Link, Huber, Nuerk, & Moeller, 2014; Link, Nuerk, & Moeller, 2014). Based on this criticism, Cohen and Blanc-Goldhammer (2011) considered the unbounded number line estimation task to be a more pure measure of number line estimation.

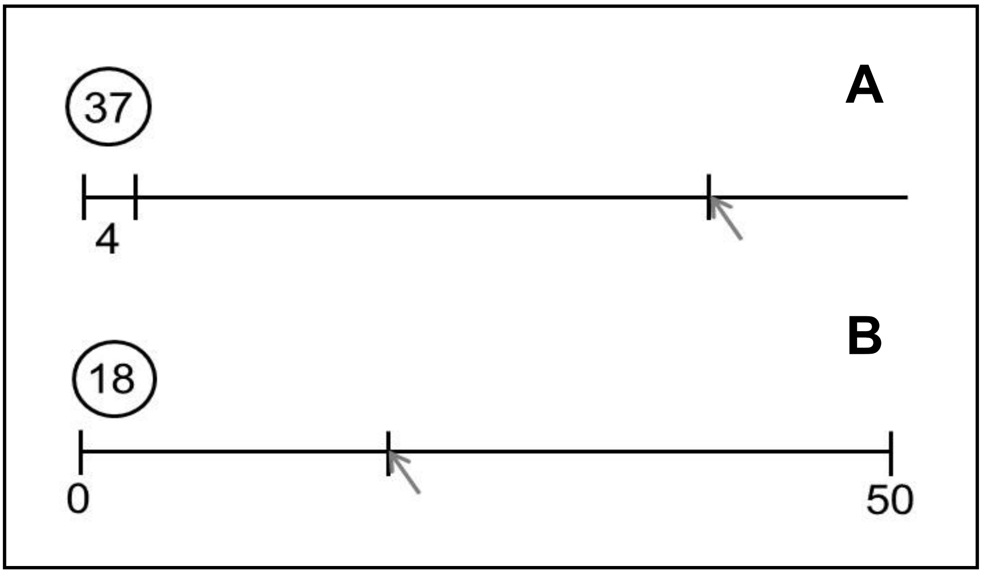

Figure 7. Panel A depicts an example of an unbounded number line estimation with the unit 4 whereas Panel B shows an example of a bounded number line estimation.

Because of this differential association of bounded and unbounded number line estimation with the (non-)application of specific solution strategies, one might expect sex differences for the unbounded but not the bounded number line estimation task. In the latter, the two end points and the middle of the scale should be considered as reference points with participants applying algorithmic proportion-judgement strategies (e.g., halving to identify the middle of the scale as a reference point, deciding whether the target number is smaller or larger than the reference point, adding to or subtracting from the reference point to get to the location of the target number, etc.; Link, Huber et al., 2014; Link, Nuerk et al., 2014). In contrast, there are no commonly known algorithmic strategies to solve the unbounded number line estimation task with only the start point of the scale as well as a unit size given. Therefore, the unbounded number line estimation task rather represents an unconventional task to be solved by flexibly developing appropriate strategies or even (logical) numerical estimation for which significant sex differences favouring males have been observed repeatedly (e.g., Gallagher et al., 2000; Halpern et al., 2007).

Against this background, we expected that - if there is a sex difference in number line estimation performance (cf. Bull et al., 2013) – it should be more pronounced for the unbounded as compared to the bounded number line estimation task. In particular, we expected that absolute differences in estimation accuracy should be smaller for both males and females for the traditional bounded number line estimation task because proportion-based solution strategies should be commonly applied by men and women. Therefore, no significant sex differences should be observed in this task version. On the other hand, sex differences for unbounded number line estimation should be more pronounced, with male participants' estimates being more accurate than female participants' estimates, because males were found to be better when flexibly developing appropriate strategies and relying on numerical estimation.

3.3 Method

3.3.1 Participants

Thirty eight students (18 males and 20 females) of a University in Germany participated in the study in exchange for course credit. Mean age was 22.8 years with a standard deviation (*SD*) of 3.7 years (range: 18–37 years). Average age of male (21.9 years) and female participants (23.6 years) did not differ significantly, $t(1, 36) = 1.42$, $p = .165$, $d_{if} = 1.66$. All participants reported normal or corrected-to-normal vision.

3.3.2 Stimuli and Design

The study comprised two distinct number line estimation tasks which were separated into two consecutive runs both requiring the participants to indicate the correct position of a presented target number on a number line. All stimuli were shown on a 26″ monitor as pictures with a resolution of 1,920 x 1,200 pixels displaying the number lines in both versions of the number line estimation task as well as target numbers in black against a white background (see Figure 7a, b). To prevent the use of external reference points (e.g., the position of the left edge or the centre of the screen), the position of the number line on the screen was varied randomly across trials with the constraint that number lines did not reach into an area 200 pixels wide at all four edges of the monitor. Additionally, white tape at the edges of the screen should conceal potential landmarks.

First, participants had to complete an *unbounded* number line estimation task with only the start point and a variable unit ranging from 1 to 10 given (cf. Cohen & Blanc-Goldhammer, 2011; see Figure 7a). Number lines had lengths of 54, 58, 62 and 66 units with physical length varying from 930 to 1,276 pixels. Thereby, numerical and physical lengths were uncorrelated. Target numbers were presented above the unit on the left side of the number line. Target numbers ranged from 11 to 49 and were chosen with the constraint of matching problem size over all unit sizes for both targets within each decade (i.e., 11 - 19, 31 - 39) as well as overall. For each unit, twenty target numbers were given (i.e., five targets per decade) summing up to a total of 200 critical trials. The stimulus set used in the present study was identical to the one used by Reinert, Huber, Nuerk, and Moeller (2015, see appendix of their article for details on the stimuli set).

Second, participants had to indicate the position of target numbers in a *bounded* number line estimation task with the start point and end point given (e.g., Siegler & Opfer, 2003). Number lines had a constant length of 50 units long with two end points labelled 0 (left end point) and 50 (right end point; see Figure 7b). Comparable to the unbounded number line estimation task physical length of the number line varied from 930 to 1,276 pixels. The stimulus set included all numbers from 1 to 49 with multiples of ten as well as ties excluded leaving 41 critical items.

3.3.3 Procedure

Participants were tested individually in a dimly-lit room sitting approximately 60 cm away from the monitor. They all started with the unbounded number line estimation task followed by the bounded number line estimation task to avoid that the end point '50' given in the bounded number line estimation task biased estimations in the unbounded number line estimation task. In particular, we wanted to avoid that the end point '50' given in the bounded number line estimation task made participants anticipate that the unbounded number line might also cover the range from 0 to 50. Participants were not informed about the number range covered by the unbounded task prior to performing it. This procedure is in line with two recent studies employing the same order of the two tasks for exactly the same reasons (Cohen & Sarnecka, 2014; Link, Huber et al., 2014; Link, Nuerk et al., 2014). Comparable to Cohen and Sarnecka (2014), we also found that estimation performance in the two tasks was not correlated reliably ($r = .16$, $p = .34$) indicating no transfer effects from the bounded to the unbounded task version. Participants were instructed to indicate as fast and as accurately as possible the spatial position of the target number on the number line by using the mouse to click at the estimated position.

Each trial started with a fixation mark priming the position on the monitor where the origin of the number line would be displayed together with the mouse cursor. Then, the number line together with the target number was presented and remained visible on the monitor until the participant gave her/his answer with a mouse click, immediately followed by the fixation mark of the consecutive trial.

In the unbounded number line task items were presented in ten blocks with unit size (i.e., 1-10) held constant within the same block to avoid the necessity of recalibration processes by each trial. Block order was randomized across participants. The 41 target numbers of the bounded number line estimation task were separated into two blocks of 20 and 21 items with the procedure being identical to that of the unbounded number line estimation task. After finishing the two tasks, participants were asked to indicate their last mathematics grade. Unfortunately, it was not possible to obtain the mathematics grades from three participants. In total, the study took approximately 30 min.

3.4 Results

The data of one female participant were excluded from the analysis as she did not adhere to task instructions and solved the unbounded number line estimation task using a counting strategy. This was also reflected by her mean reaction time (14,207 ms) being more than three SD longer than the mean reaction time over all participants (4,933 ms; $SD = 2{,}516$ ms). Subsequent analyses will focus on absolute estimation error reflecting the absolute deviation of the estimated position of a target number from the actual position of the respective target number on the number line. While the absolute estimation error may be considered a crude measure of task performance, it is nevertheless the standard dependent variable used in studies on number line estimation (Ashcraft & Moore, 2012; Barth & Paladino, 2011; Berteletti et al., 2010; Booth & Siegler, 2006; Cohen & Blanc-Goldhammer, 2011; Geary et al., 2000; Laski & Siegler, 2007; Link, Moeller, Huber, Fischer, & Nuerk, 2013; Moeller, Pixner, Kaufmann, & Nuerk, 2009; Reinert et al., 2015; Siegler & Booth, 2004; Siegler & Opfer, 2003; Slusser, Santiago, & Barth, 2013).

Sex differences in bounded and unbounded number line estimation

Please note, to evaluate number line estimation performance participants' estimation patterns are often fitted by mathematic models associated with specific estimation strategies. For bounded number line estimation, the most common ones are a linear model indicating direct estimation (Siegler & Opfer, 2003) as well as cyclic-power models indicating proportion-judgement strategies (Barth & Paladino, 2011). For unbounded number line estimation, the linear model indicating direct estimation is differentiated from so-called scallop models reflecting the application of scallop strategies (Cohen & Blanc-Goldhammer, 2011). Thus, the distribution of best-fitting models may also indicate possible sex differences in *how* males and females perform bounded and unbounded number line estimation. Therefore, we also fitted these models to the individual participants' estimation data. The individually best-fitting model was chosen on the basis of the respective AICc values of the models. Afterwards, the distribution of males and females with respect to a best-fitting linear versus cyclic-power model for bounded and a best-fitting linear versus scallop model for unbounded number line estimation was evaluated using chi-square tests. The chi-square tests indicated that there were no significant differences in the distribution of males and females with regard to best-fitting models for both task versions, bounded: $\chi^2(1) = 0.084$, $p = .96$; unbounded: $\chi^2(1) = 0.26$, $p = .61$, applying Yates correction for frequencies < 5. For bounded number line estimation, the linear model provided the best fit to the data of five females and four males, the one-cycle power model to nine females and eight males, and the two-cycle power model to six females and six males. For unbounded number line estimation, the linear model provided the best fit to seven females and four males and the one-scallop model to 13 females and 14 males on average across all units.

With respect to the absolute estimation error, a 2 x 2 repeated-measures ANOVA with the factors *task* (bounded vs. unbounded number line task) and *sex* (males vs. females) was conducted to evaluate sex differences in the two number line tasks (Figure 8). Mean estimates were calculated for each participant, separately for males and females. For the unbounded number line estimation task, mean estimation error over all units was computed. Evaluating whether the accuracy differed in the two number line tasks revealed that the mean estimation error was significantly smaller for the bounded as compared to the unbounded number line task $M_{\text{bounded}} = 1.71$ versus $M_{\text{unbounded}} = 5.51$; $F(1, 35) = 204.32$, $p < .001$, $\eta^2_p = .854$. Furthermore, the main effect of sex was significant, $F(1, 35) = 15.42$, $p < .001$, $\eta^2_p = .306$, indicating that male participants' estimates were significantly more accurate than female participants' estimates, $M_{\text{male}} = 3.17$ versus $M_{\text{female}} = 4.03$. At last, the ANOVA revealed a significant interaction of *task* and *sex*, $F(1, 35) = 7.04$, $p = .012$, $\eta^2_p = .167$, indicating that the sex difference in estimation error between males and females was significantly smaller for the bounded as compared to the unbounded number line estimation task (0.17 vs. 1.57, respectively).[3]

Inspection of the simple effects indicated that estimation accuracy did not differ between males and females for the bounded number line task, $M_{male} = 1.63$ versus $M_{female} = 1.80$; $t(35) < 1$, $p = .396$, $\eta^2_p = .021$. A Bayesian analysis substantiated this result as the posterior probability for the null hypothesis was $P(H_0|D) = .81$, which – according to Raftery (1995) – indicates positive evidence in favour of the null hypothesis that there is no difference between males and females. In contrast, male participants' estimates were significantly more accurate in the unbounded number line task as compared to female participants' estimates, $M_{male} = 4.70$ versus $M_{female} = 6.27$; $t(35) = 3.51$, $p = .001$, $\eta^2_p = .261$.

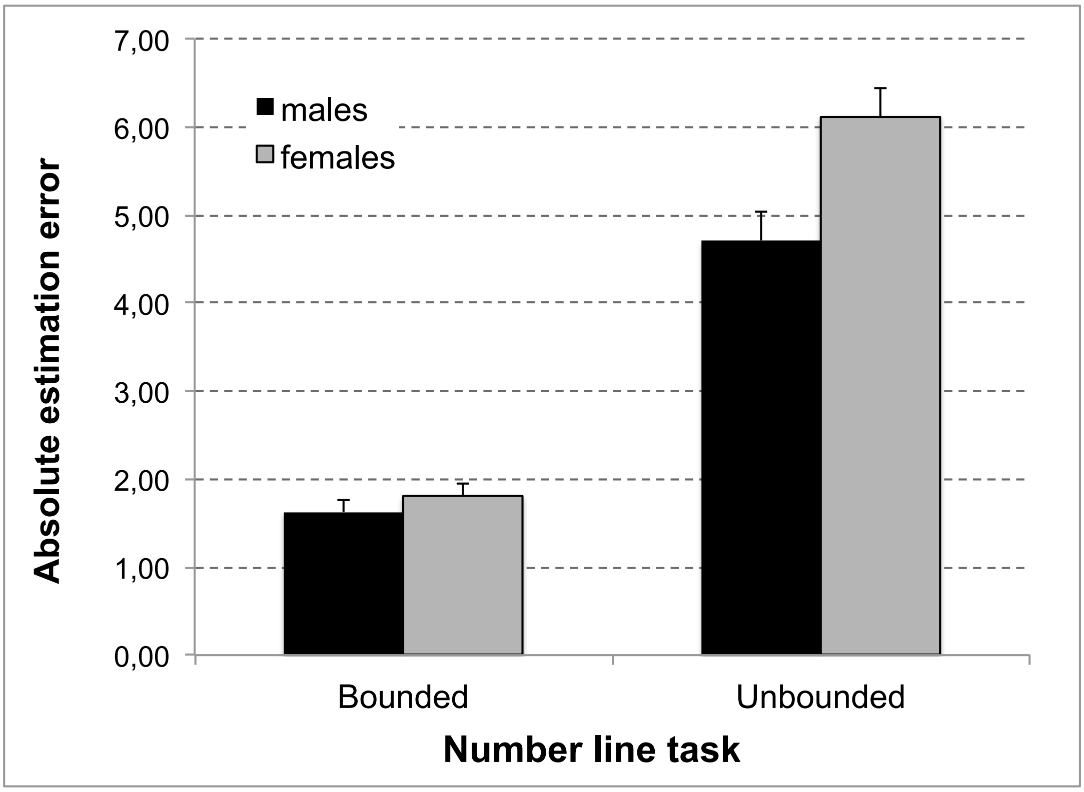

Figure 8. Marginal means of absolute estimation error for bounded and unbounded number line estimation separated for male and female participants. Error bars reflect 1 Standard Error of the Mean (SEM).

[3] *Note that the observed interaction between sex (female vs. male) and task version (bounded vs. unbounded) prevailed significant even in case a trimming procedure was applied which excluded individual estimates deviating more than three, F(1,36) = 5.16, p < .05, or two standard deviations from the individual's mean estimation error, F(1, 36) = 5.16, p < .05. This indicates that the interaction is not driven by potential outliers.*

Specific analyses for bounded and unbounded number line estimation

Bounded number line task: contour analysis

Additionally, we conducted a contour analysis in line with the procedure suggested by Ashcraft and Moore (2012). Therein, we directly contrasted mean estimation errors of target numbers at and around reference points (i.e., start point: target numbers 1, 2, 3; midpoint: target numbers 24, 25, 26; end point: target numbers 47, 48, 49) with those farthest away from reference points (i.e., target numbers 12, 13, 14, 36, 37, 38, and 39, reflect numbers ±1.5 around 12.5 and 37.5 please note, however, that 11 was not included in the stimulus set as it is a tie number).

A 2 x 2 ANOVA discerning the within-subject factor *target at reference point* (yes vs. no) and the between-subject factor *sex* (female vs. male) indicated a significant main effect of target at reference point, $F(1, 35) = 75.06$, $p < .001$, $\eta^2_p = .682$. In line with our expectation, target numbers at or around reference points were estimated more accurately than target numbers farthest away from reference points (0.70 vs. 2.40, respectively). The non-significant main effect of sex, $F(1, 35) < 1$, $\eta^2_p =.005$, replicated the lack of sex differences in bounded number line estimation. Furthermore, the nonsignificant interaction between sex and target at reference point, $F(1, 35) < 1$, $\eta^2_p = .021$, indicates that the increase in estimation error for target numbers farthest away from reference points was comparable across sexes. This implies that both males and females relied on proportion-based solution strategies considering reference points when solving the bounded number line estimation task. A Bayesian analysis substantiated this claim as the posterior probability for the null hypothesis was $P(H_0|D) = .84$. According to Raftery (1995), this indicates positive evidence in favour of the null hypothesis that there is no difference between males and females.

Unbounded number line task: influence of unit size

Because the overall estimation error in the unbounded number line estimation score is a composite score averaging across different units, we also had a closer look on whether sex differences in estimation accuracy between male and female participants were moderated by unit size. Therefore, we first computed the mean absolute estimation error separately for all units and for male and female participants (Figure 9). A repeated measures ANOVA with the factors *unit size* (1, 2, 3, 4, 5, 6, 7, 8, 9 vs. 10) and sex (male vs. female) was conducted to evaluate sex differences. As expected, a significant main effect of sex was observed $F(1, 35) = 12.34$, $p = .001$, $\eta^2_p =.261$, with male participants' estimates being more accurate than female participants' estimates ($M_{\text{male}} = 4.70$ vs. $M_{\text{female}} = 6.27$). Moreover, the main effect of *units* was significant, $F(9, 315) = 32.13$, $p < .001$, $\eta^2_p = .479$, Greenhouse–Geisser coefficient (GG) to correct df for violation of sphericity: .474, indicating that the absolute estimation errors differed between units (see Table 2 for significant differences between units). Importantly, a significant linear trend for the estimation error over the units, $F(1, 35) = 103.05$, $p < .001$, $\eta^2_p = .746$, indicated that the estimation error decreased with increasing unit size (cf. Reinert et al., 2015). Moreover, the interaction of *units* and *sex* was significant, $F(9, 315) = 3.52$, $p = .008$, $\eta^2_p = .091$. The significant linear trend for the differences between males' and females' estimation error, $F(1, 35) = 4.68$, $p = .037$, $\eta^2_p = .118$, suggested that the differences between males' and females' estimation error decreased with increasing unit size. Additionally, the simple effects indicated at least marginally significant sex differences in favour of male participants (smaller mean estimation error) for all units except 2, 5, 9, and 10 (see Table 3 for statistical details).

In sum, these results corroborated our hypotheses as we observed that male participants' estimates were more accurate as compared to females' estimates in the unbounded number line task, whereas no sex differences were observed in the standard bounded number line task. This overall pattern of results was further substantiated by the outcome of a contour analysis for bounded number line estimation (cf. Ashcraft & Moore, 2012) as well as an analysis considering unit size for unbounded number line estimation.

Table 2. Significant differences between the units.

Compared units	*p*	*mean difference*
unit 1 - unit 5	< .001	3.15
unit 1 - unit 6	< .001	3.19
unit 1 - unit 7	< .001	3.16
unit 1 - unit 8	< .001	4.04
unit 1 - unit 9	< .001	4.72
unit 1 - unit 10	< .001	4.79
unit 2 - unit 5	< .001	3.28
unit 2 - unit 6	< .001	3.33
unit 2 - unit 7	< .001	3.30
unit 2 - unit 8	< .001	4.18
unit 2 - unit 9	< .001	4.86
unit 2 - unit 10	< .001	4.93
unit 3 - unit 5	< .001	2.49
unit 3 - unit 6	< .001	2.54
unit 3 - unit 7	< .001	2.50
unit 3 - unit 8	< .001	3.39
unit 3 - unit 9	< .001	4.06
unit 3 - unit 10	< .001	4.14
unit 4 - unit 6	0.043	1.18
unit 4 - unit 8	< .001	2.03
unit 4 - unit 9	< .001	2.71
unit 4 - unit 10	< .001	2.78
unit 5 - unit 9	< .001	1.57
unit 5 - unit 10	< .001	1.65
unit 6 - unit 9	0.001	1.53
unit 6 - unit 10	0.005	1.60
unit 7 - unit 9	< .001	1.56
unit 7 - unit 10	< .001	1.63

*** < .001; ** < .01; * < .05

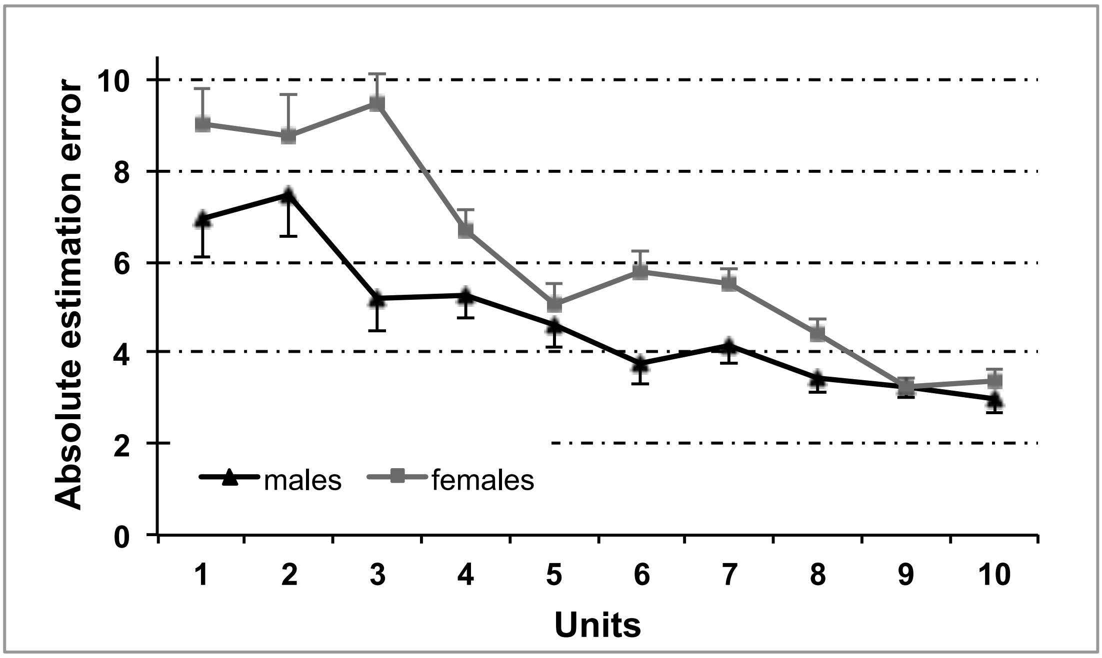

Figure 9. Absolute estimation error for each unit separated for male and female participants. Error bars indicate 1 SEM.

Table 3. Sex differences separated for the units.

Mean males – mean females:	*t*	*p*	*mean difference*
unit 1	1.86	.070	2.11
unit 2	1.08	.289	1.35
unit 3	4.59[#]	< .001	4.28
unit 4	2.10	.043	1.41
unit 5	.62	.541	.42
unit 6	3.15[#]	.004	1.98
unit 7	2.65	.012	1.35
unit 8	1.96	.057	.92
unit 9	< .01	.995	< .01
unit 10	1.01	.321	.41

[#] contrast corrected for heteroscedastity

3.5 Discussion

In the present study, we were interested in sex differences in number line estimation. More specifically, we hypothesized differential effects for two different versions of the number line estimation task (i.e., the standard bounded, Siegler & Opfer, 2003; and a new unbounded number line estimation task, Cohen & Blanc-Goldhammer, 2011): Sex differences in number line estimation should be more pronounced in the unbounded as compared to the bounded number line estimation task. Only in the latter overlearnt strategies such as proportion judgement should generally be applied by both men and women. On the other hand, unbounded number line estimation has been argued to be a more pure measure of numerical estimation and thus should not be subject to the application of specific solution strategies. Therefore, males should be at an advantage in this task version because they were observed to outperform women in unconventional numerical tasks relying on numerical estimation more strongly (Gallagher et al., 2000; Halpern et al., 2007). Generally, the present data corroborated these hypotheses. As expected, sex differences were more pronounced for the unbounded as compared to the bounded number line estimation task. This was further corroborated by the specific results of a contour analysis for bounded number line estimation (cf. Ashcraft & Moore, 2012) and an analysis considering unit size for unbounded number line estimation. In the following, we will first discuss these results with respect to sex differences in strategy use in number processing before elaborating on broader implications of these findings.

Strategy use as a cause for sex differences

The present data indicated a male advantage in unbounded but not in bounded number line estimation. Considering the broad literature on differences in strategy use in numerical tasks between the sexes, we argue that these sex difference favouring males in the unbounded number line estimation task may stem from differences in strategy use to estimate the respective position on the number line and in their differential applicability to the two task versions. While participants can successfully apply overlearnt strategies such as proportion-based judgments using halving, quartering, etc. in the bounded number line estimation task, this is not possible in the unbounded number line estimation task. The latter was argued to reflect a more pure measure of numerical estimation (Cohen & Blanc-Goldhammer, 2011) and thus seems to reflect an unconventional problem, which requires (1) numerical estimation and (2) more flexibility in the development and application of solution strategies. On both of these aspects, males have been argued to outperform females (e.g., Zhu, 2007 for a review). This theoretical argument is in line with our empirical data. As expected, males showed no advantage over females in the bounded number line estimation task. In contrast, and corroborating our hypothesis, males performed better compared to females (i.e., their estimates were more accurate) in the unbounded number line task.

In accordance with earlier findings, this corroborates the assumption that males may be more successful in (1) developing their own strategies in unfamiliar circumstances and (2) numerical estimations in particular (Gallagher et al., 2000; Zhu, 2007). This interpretation is further substantiated by the contour analysis for the bounded number line estimation task as well as a closer inspection of the sex difference in the unbounded number line estimation task. When discerning the different unit sizes, we found that the observed sex differences decreased with increasing unit sizes. This seems plausible as larger units may reduce the demands on spatial–numerical estimations because fewer steps in terms of multiples of the unit are needed to reach the position of the target number on the number line. In turn, this should reduce the accumulation of estimation error: The more steps are needed, the higher the overall estimation error for the sum of these steps should be because it reflects the accumulated error of individual steps. Thus, estimating the physical distance corresponding to a relatively large unit and then multiplying this estimate followed by more fine-grained adjustments to locate the final position of the respective target number should be less error prone. A similar argument also applies here. Using steps of approximately 10 to get to the target number may be specifically advantageous because multiples of 10 (e.g., 10, 20, and 30) help to efficiently structure the range of two-digit numbers (Fuson, 1988). In this sense, Reinert et al. (2015) observed a specific advantage for locating multiples of a given unit for units larger than 5. This seemed to help women to locate the respective target numbers more accurately. In contrast, for smaller unit sizes it might be more efficient to simply estimate the position of the target numbers directly or first develop one's own larger unit of about 10 and then go on with estimating the position of the target number based on this. Therefore, women might have been at a disadvantage for these smaller units. The question why there are no sex differences for particular units (i.e., 2, 5, 9, and 10) may be answered by considering known specificities of simple multiplication (Campbell & Graham, 1985; Miller, Perlmutter, & Keating, 1984; Siegler, 1988). These and other studies found that multiplications involving the operands 2, 5, and 10 are significantly easier than others (e.g., 3, 7, or 8). To a lesser degree, this also holds for multiples of 9 for which there is the work-around strategy of multiplying by 10 and then subtracting the respective non-nine operand (e.g., 6 x 9 = 6 x 10 – 6). In this case, one makes use of the known rule that multiplication by 10 simply means adding a 0 to the other operand. As women should be particularly susceptible to apply known strategies and arithmetic knowledge, they should benefit from these unit sizes (i.e., 2, 5, 9, and 10) in particular, resulting in the observed non-significant sex differences. In case of both small unit sizes and those associated with less automated multiplication tables, (numerical) estimations may also pose higher demands on visuospatial abilities for which a male advantage is reported in the literature as well (Hugdahl, Thomsen, & Ersland, 2006; Kaufman, 2007; Weiss et al., 2003). Therefore, the male advantage observed here might also be explained by the more elaborate use of visuo-spatially based numerical estimation as required to complete the unbounded number line task. Such a more elaborate use of numerical estimation strategies is also in line with the results of the model fitting analyses. These did not indicate a qualitative difference between males and females with regard to the strategies applied (i.e., direct estimation vs. proportion judgement in bounded, direct estimation vs. scallop strategies in unbounded number line estimation). Moreover, in their recent review Levine, Foley, Lourenco, Ehrlich, and Ratliff (2016) concluded that sex differences have primarily been observed for rather complex spatial tasks such as mental rotation. As both versions of the number line estimation task only require spatial processing on the left-to-right dimension and no active manipulation of information (such as rotations), sex differences should be small. Therefore, we suggest that the less accurate estimation performance of females in unbounded number line estimation might be due to a specific female disadvantage in numerical estimation – which is the primary solution strategy in unbounded number line estimation.

This argument is also in line with a recent study of Bull et al. (2013) on sex differences in number line estimation. The authors also suggest that men and women may apply different strategies and resources to solve math tasks. In particular, they claim that males generally use spatial representations of numbers (aka the mental number line) more often as a successful strategy to perform numerical tasks – even when it is not required. In contrast to our data, however, Bull et al. (2013) observed small sex differences in bounded number line estimation. However, it is important to note that they used the number range from 0 to 1,000 in their experiment. Thus, the stimuli used in the present study differ considerably in the number line length from only 0 to 50, that is a much smaller and more familiar number range. Therefore, proportion-based solution strategies might be even more prominent in our study than in that of Bull et al. (2013) explaining why we did not observe a significant sex difference in bounded number line estimation. Future studies evaluating the dependency of sex differences on the number range used in a particular bounded number line estimation would be desirable. Following the argument of the present study, we would predict sex differences to increase when the range covered is more difficult. However, as proportion-judgement strategies can be applied quite universally to basically every number range (e.g., 0 to 10 but also 0 to 100,000) sex differences in bounded number line estimation should be most pronounced in case the end points are not 0 and multiples of 10 (e.g., 173 to 829), for instance.

Importantly, our interpretation of sex differences being due to differences in strategy use and application are also consistent with the results of Brunner et al. (2011) discussed above. These authors observed that considering the influence of general intelligence on sex differences in mathematics led to even larger differences favouring males. Thus, one might argue that potential differences in the bounded number line estimation task due to differences in spatial–numerical cognition may even be masked by efficient strategies applied such as proportion-judgement strategies in the case of bounded number line estimation. In turn, it might be that sex differences in number line estimation as observed in the present study even underestimate actual sex differences in spatial–numerical cognition.

Finally, one might speculate that the present results are influenced by rather general differences in math achievement found between males and females. However, this seems unlikely due to at least two reasons. First, a recent review on this by Reilly et al. (2015) indicates stable but only small sex differences in math achievement (d = .1). In contrast, the interaction effect in the present study indicating sex differences in unbounded number line estimation to be significantly more pronounced than in bounded number line estimation was much larger with a d of .89. Moreover, to address this point more directly, we asked participants to report their last mathematics grade on a voluntary basis. Thirty-five (18 females) of the 38 participants did so. We then reran the analyses with last reported math grade as a covariate to control for possible general differences in math achievement between males and females in our sample. Even though the reported math grades of males were significantly better than that of females, $t(33) = 2.41$, $p = .021$, the interaction between sex and task version did not change substantially and was still significant, $F(1, 32) = 5.12$, $p = .031$. This indicates that even after controlling for general differences in math achievement, sex differences were more pronounced for unbounded than for bounded number line estimation. These results are hard to reconcile with the claim that the present findings are influenced by general sex differences in math achievement. Instead, they corroborate our interpretation of specific differences in solution strategies applied seem to drive the observed sex differences.

3.6 Conclusion

In the present study, we hypothesized sex differences in number line estimation to be more pronounced in the unbounded as compared to the bounded number line estimation task. A male advantage was expected because males were argued to outperform females when it comes to the processing of unconventional problems requiring (1) numerical estimation and (2) more flexibility in the development and application of solution strategies (Gallagher et al., 2000; Halpern et al., 2007). The observed male advantage in unbounded but not bounded number line estimation corroborated our hypothesis and – in line with recent data – indicates that males seem to be at an advantage when there are no learnt solution strategies to be applied, but responses require spatial–numerical estimations in particular.

III. GENERAL DISCUSSION

General Discussion

The last part of this thesis first discusses the key research findings of the three empirical studies in general and summarizes its main results along the two overarching research aims. Following up on this, I will integrate my new insights on unbounded NLE with the existing literature in terms of a systematic literature review considering all empirical studies carried out so far employing an unbounded NLE task. Subsequently, I will propose an updated taxonomy of magnitude estimation tasks on this basis that enables a simplified and more comprehensive classification of present as well as future approaches measuring magnitude estimation. Afterwards, I will outline potential limitations of studies conducted in this dissertation and possible ways to address these constraints in future research. Finally, the last paragraph will close with an overall conclusion concerning the validity of unbounded NLE reiterating the most relevant findings.

1. Research Aims

At the end of the general introduction, I identified two overarching research aims to-be-pursued in the present dissertation: (i) Evaluating the validity of the unbounded NLE task as a measure of number magnitude representation in general as well as (ii) identifying possible factors which may affect performance shown in this task. The obtained findings of all three empirical studies presented in this thesis were meaningful to both research questions and I will discuss these with respect to the two research aims in the next paragraphs.

1.1 The validity of the newly introduced unbounded number line estimation task

As described in the general introduction, an increasing body of evidence over a number of studies carried out in the last decade indicated that the newly introduced unbounded NLE task might reflect a purer measure of the mental representation of number magnitude as compared to the standard bounded task version. Altogether, all three empirical studies included in the present dissertation substantiated this claim first made by Cohen and Blanc-Goldhammer (2011). In particular, findings of Study 1 provided additional evidence for this assumption using a non-symbolic numerosity estimation task. As this task is widely agreed on to represent a reliable measure of the underlying number magnitude representation, it was chosen to appraise its conceptual similarity with the unbounded NLE task. As expected, estimation patterns of overestimation of target numbers in the production as well as underestimation in the perception version were found for both non-symbolic numerosity estimation as well as unbounded but not bounded NLE. These generalizable patterns of systematic biases from non-symbolic numerosity estimation to symbolic unbounded – but not bounded – NLE provides converging evidence for the claim that the new unbounded task version might indeed reflect a purer measure of number magnitude representation (see Barth & Paladino, 2011; Slusser et al., 2013).

Moreover, data of eye-fixation behavior in Study 2 further corroborated this interpretation: In the unbounded task version, the number of fixations on the number line declined constantly as the magnitude of the target number increased. In contrast, a typical pattern of increased numbers of fixations in regions around possible reference marks – such as the start-, mid- and end point – for the bounded counterpart indicated that this task version "is fundamentally a proportion-judgement task" (see Barth & Paladino, 2011, p. 134). These results replicated those from Sullivan et al. (2011), as expected, with a lower estimation error at and around reference points. Additionally, I did not observe the specific M-shaped error pattern which is characteristic for proportion judgement strategies in the unbounded but, as expected, in the traditional bounded NLE task version.
Hence, the present thesis provides further evidence that unbounded NLE recently proposed by Cohen and Blanc-Goldhammer (2011) might in fact reflect a more valid picture of the internal representation of numbers. Nevertheless, there are still factors affecting estimation performance in this unbounded task version, which will be discussed in the following paragraph.

1.2 Divers factors that affect estimation strategies to complete the unbounded number line estimation task

With respect to the aim of determining influencing factors, which may affect solution strategies when participants solve the new unbounded NLE task version, Studies 2 and 3 of this thesis revealed that individuals did not use proportion judgement strategies (see also Link et al., 2014b), but that there are nonetheless factors beyond number magnitude processing influencing estimation performance in this task version. Regarding my second research objective, I aimed at addressing some promising factors such as eye-fixation behaviour or the sex of participants affecting estimation patterns.

First, I showed that eye-tracking data in Study 2 of the current dissertation did also not indicate the application of specific reference marks such as the origin-, mid- and end point in the unbounded NLE task reflected by more frequent peaks in fixation. As described in the section above, this finding provides further evidence that unbounded NLE seems to be less affected by proportional judgement. Importantly, prior research by Reinert et al. (2015a) allows for further insights into potential solution strategies used in this new task version. Their findings suggest that multiples of a given unit benefited from multiplication fact knowledge in terms of faster (but not more accurate) estimations compared to non-multiples. This implicates that the location of the target is reached faster as there seems to be no need to adjust the initial estimate to either the left or right which is the case of non-multiples of the units. Accordingly, a possible strategy that participants adopt to complete an unbounded NLE task is using multiples of the unit which Cohen and Blanc-Goldhammer (2011) termed the *dead-reckoning strategy*. More specifically, the authors found that individuals had a fixed working window of numbers which did not depend on the unit size they had manipulated. Making their estimates, participants generally chose scallops of about 10. Hence, the data suggest that a possible solution strategy in the unbounded task version might be to proceed in bigger junks such as steps of the unit given and/or overarchingly in steps of 10 to make their estimates on the number line most efficiently.

Second, I expected that women would be excelled by men in unbounded but not in bounded NLE because males are supposed to have a special advantage in solving numerical problems without (classroom-)learnt strategies or pure estimation. In fact, findings of Study 3 substantiated this assumption: No sex difference was found in traditional bounded NLE. However, men actually performed more accurately in the unbounded task version as compared to women, as expected. In the former, both sexes were assumed to use overlearnt solution strategies (e.g., proportion judgement like halving, quartering, etc.) in a similar way. In contrast, I expected that no commonly applicable solution strategies are involved in the unbounded NLE task version that has been suggested to provide a more valid measure of the underlying magnitude representation of numbers. Consequently, men outperformed women in Study 3 as they are supposed to find solutions and develop strategies (incl. estimation) for unconventional numerical problems more flexibly (see Gallagher et al., 2000; Halpern et al., 2007). In turn, these findings further corroborate the assumption that unbounded NLE is not completed by the use of particular solution strategies drawing on overlearnt procedures.

Taken together, data of all three empirical studies of the present thesis suggest that participants performing the unbounded NLE task (see Cohen & Blanc-Goldhammer, 2011) indeed did not seem to use specific solution strategies such as proportion judgements as it is the case in its bounded counterpart. They rather seemed to refer to "rough" estimation (see also Huber, Bloechle, Dackermann, Scholl, Sassenberg, & Moeller, 2017). Nevertheless, I also identified factors such as the sex of participants or multiples of the unit given as well as the working window of numbers influencing estimation performance shown here (see also Reinert et al., 2015a). These results add to prior findings and shed light on further factors that affect performance in unbounded NLE and provide compelling evidence for the claim that it represents a purer and more valid measure of the underlying number magnitude representation. Altogether, the present dissertation suggests that this task is a valuable assessment tool being at least less biased than its standard bounded counterpart.

The next section provides a summative review of all empirical studies employing unbounded NLE that have been published so far to gain an overview of the current state of research. The purpose of this systematic literature review is to locate the findings of the present dissertation within the context of existing prior literature and discuss how my studies contribute to better understand processes and representations underlying performance in unbounded NLE. After systematically integrating my findings in a chronological order, I will propose a new taxonomy of magnitude estimation tasks in the subsequent section based on this review of existing studies. Finally, I will identify possible future perspectives in the research on unbounded NLE.

2. Literature Search and Definition of Inclusion and Exclusion Criteria

Even though the limitations of the ´standard´ bounded NLE task (cf. Siegler & Opfer, 2003) discussed in the general introduction are known, it seems to persist within research into numerical cognition as a popular measure allowing insights into how children understand the "relationship of integers with each other" (see Honour, 2020, p. 7). Compared to the bounded task version, remarkably few studies have been conducted employing unbounded NLE to explore the underlying mental representation of numerical magnitude (see also Schneider et al., 2018a). This seems surprising as this field of research is fairly young but quickly expanding (Schneider et al., 2018a) and this new version has gained more and more popularity (see also Cohen & Ray, 2020). To the best of my knowledge, there is currently no systematic review that integrates results of all studies that employed unbounded NLE so far. Lately, Cohen and Ray (2020) commented on the only study reporting uncommon findings (Kim & Opfer, 2017) that are opposite of all other results found generally. A first systematic summary of certain features across ten studies employing unbounded NLE is provided in Kim and Opfer's (2020) reply to Cohen and Ray's (2020) comment. However, this table overview just focusses on some of the major methodological and analytic differences among these studies, but there is no comprehensive outline of all research articles yet.

For a complete overview of the present state of research on unbounded NLE, and in order to evaluate other study results and how they used that new measure, I conducted a systematic literature review. This will form the basis for the integration of the empirical results of my dissertation as well as the subsequent proposition of a taxonomy of magnitude estimation tasks. Searches for publications employing unbounded NLE were performed in two different electronical databases, (1) PubMed as well as (2) EBSCOhost (Elton B. Stephens Company). Therefore, I followed the procedure of the most recent meta-analysis performed by Schneider and colleagues (2018a) that systematically reviews the literature on associations of NLE performance with mathematical competence. In July 2020, I used the search terms and keywords "unbounded number line", "unbounded number line estimation", "unbounded* numerical cognition", "unbounded* numerical magnitude", "unbounded* numerical representation", "unbounded and bounded* numerical estimation" as well as "unbounded number to position". In total, this initial search returned 57 hits, including papers of other disciplines (e.g., physics, mathematics, biology or pharmacology). Articles were selected in accordance with the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) guidelines (see Moher et al., 2015; Shamseer et al., 2015). After a first screening process based on abstract and method section, I identified 12 publications from these two databases that met my inclusion criteria. Moreover, I screened the reference lists of these papers to identify further articles.

In a second step, I additionally scanned the 125 titles on Google Scholar in a separate search that cite the pioneering article published by Cohen and Blanc-Goldhammer (2011) introducing the unbounded NLE task. For this systematic review, I examined title, abstract, keywords as well as the method section of these 125 studies and just took into consideration articles that fulfilled all of the following selection criteria:

1. The study was published in English language in a peer-reviewed journal (publications in any language other than English were excluded).
2. It referenced Cohen and Blanc-Goldhammers' (2011) pioneering study and the manuscript was hence published after 2011.
3. The authors employed an unbounded NLE task and collected empirical data in their experiment (e.g., neither reviews or comments, nor re-analysis of findings – such as computer simulations – that have already been reported).

Altogether, this search procedure in three different electronic databases yielded 15 relevant studies using the unbounded task version that met the eligible criteria containing original data [1. Cohen & Blanc-Goldhammer, 2011; 2. Cohen, Blanc-Goldhammer, & Quinlan, 2018; 3. Cohen & Sarnecka, 2014; 4. Dackermann, Fischer, Huber, & Moeller, 2016; 5. Ebersbach, Luwel, & Verschaffel, 2015; 6. Jung, Roesch, Klein, Dackermann, Heller, & Moeller, 2020; 7. Kim & Opfer, 2017; 8. Kim & Opfer, 2020; 9. Link, Huber, Nuerk, & Moeller, 2014; 10. Link, Nuerk, & Moeller, 2014; 11. Reinert, Hartmann, Huber, & Moeller, 2019; 12. Reinert, Huber, Nuerk, & Moeller, 2015a; 13. Reinert, Huber, Nuerk, & Moeller, 2015b; 14. Reinert, Huber, Nuerk, & Moeller, 2017; 15. van der Weijden, Kamphorst, Willemsen, Kroesbergen, & van Hoogmoed, 2018].

The meta-analysis by Schneider and colleagues reported 10 articles in 2018 which roughly confirms this number of pertinent studies published in the literature (2018a, p. 1473). Since then, a further five studies on the subject have been published. Besides, some unpublished reports – such as proceeding papers or master theses – employing the unbounded NLE task were not publicly available and therefore excluded from this literature review (see, e.g., Olver, 2013; Qin, Kim, & Opfer, 2017; van Wijk, 2017). Moreover, duplicates, citations and articles that did not apply an unbounded NLE task were also excluded (e.g., Cohen & Quinlan, 2018; Cohen & Ray, 2020; Huber et al., 2017). See Figure 10 for the PRISMA flow diagram summarizing my search procedure. Brief summaries of the included 15 studies are outlined in chronological order in the following (see also Table 4 for an overview).

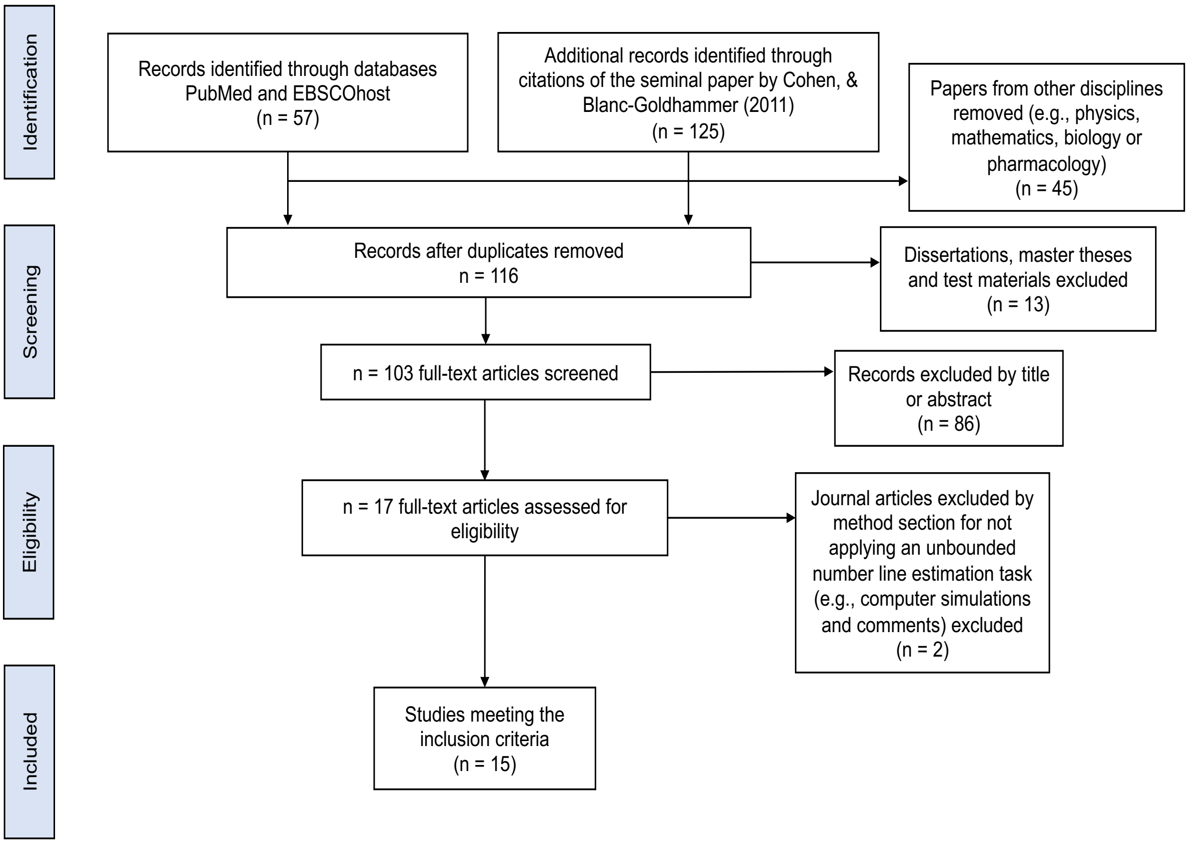

Figure 10. PRISMA flow diagram.

2.1 Studies using the unbounded number line estimation task

As described in the general introduction section, Cohen and Blanc-Goldhammer were the pioneering researchers in the year 2011 presenting their adult participants with the new unbounded NLE task without a predefined fixed end point. Both the standard bounded and unbounded NLE task ranged from 0 to 26. As expected, the used scaling unit of '1' evoked counting-based strategies such as the dead-reckoning strategy on this unbounded number line. Moreover, there were no indications of the use of reference points that are usually reflected by systematic estimation biases such as reduced error variability around reference points. In contrast, the bounded version triggered proportional reasoning strategies to complete this task. From this specific result pattern, the authors inferred then that albeit "both tasks are tapping into the same underlying numerical cognition structure, […] the error data suggest that the unbounded NLE task is a more pure measure of integer representation" (Cohen & Blanc-Goldhammer, 2011, p. 336).

Against this background, Link, Huber, Nuerk, and Moeller (2014a) carried out the first cross-sectional study on unbounded NLE in primary school children. Their results provided evidence for systematic strategic influences like the use of reference points for older children in the traditional bounded NLE task. However, the comparison between estimation performance in both the unbounded and bounded task versions showed neither evidence of such strategic influences nor a decrease of error variability at and around specific reference points for the unbounded NLE task. As such, these data support Cohen and Blanc-Goldhammers' (2011) conclusion that the unbounded task version actually constitutes a more valid measure of number magnitude representation as compared to the traditional bounded task version. Furthermore, their results revealed a qualitative change in performance for bounded but not unbounded NLE as age of students increased. This clearly indicated the application of proportion judgement in the traditional bounded version in older children whereas younger ones did not employ this strategy yet. Contrary to the findings of Schneider and colleagues (2009) as well as Slusser et al. (2013) indicating an application of proportion judgement strategies in children from the age of seven, the study of Link and colleagues (2014) showed that from third grade on pupils seemed to start using external (such as the start or end point) and internal (e.g., mid-point) benchmarks on the number line. Nevertheless, for first- and second-graders, the authors found similar estimation patterns for unbounded and bounded NLE. Hence, they speculated that both task versions might assess the same mental representation of number magnitude in younger children.

In another study, Cohen and Sarnecka (2014) compared unbounded with bounded NLE in 3.5- to 8-year-old children and argued that age-related changes in estimation performance on the latter may not reflect developments to their internal magnitude representation but rather improvements in young children's mensuration skills particularly the ability to scaling numbers to the line length by using complex calculation-based strategies involving subtraction or division. In addition, the negatively accelerating error pattern only in bounded NLE looking like a logarithmic curve was interpreted to be the result of lacking mensuration skills. Therefore, the authors inferred that unbounded NLE requires less sophisticated measuring skills (e.g., repeated addition or counting) and measures the representation of number magnitude more precisely, especially in those children with weaker subtraction competencies. Bounded NLE, in contrast, was interpreted to reflect the development of advanced mensuration skills requiring mastery of subtraction and/or division rather than changes in representations of number magnitude.

In a further study, Link, Nuerk, and Moeller (2014b) investigated the association between performance in unbounded as well as bounded NLE and various basic numerical as well as arithmetic competencies (such as addition/subtraction and number magnitude comparison). However, neither did they find correlations between either arithmetic or numerical abilities and estimation performance in the unbounded NLE task, nor evidence for the use of proportion-based strategies in this task version (see also Cohen & Sarnecka, 2014). Significant correlations were only observed between other numerical and arithmetic competencies and performance in the traditional bounded NLE task. The authors interpreted this differential result for these two distinct NLE tasks as additional evidence for the claim that they do not capture the mental representation of numerical magnitude in exactly the same way. Importantly, this finding suggests that arithmetic and numerical processes are required to apply proportion-based strategies in the bounded NLE task (e.g., calculate reference marks, evaluate through number comparison whether the probed number is smaller or larger than the benchmark, then compute the difference from this chosen reference point and the target number and so on). This, again, seems to be driven by the fact that using proportion judgement strategies needs other basic numerical operations such as adding or subtracting magnitudes from reference points. Hence, the purer estimation performance in unbounded NLE may not be associated to arithmetic skills such as addition or subtraction.

Extending the original study design, Reinert, Huber, Nuerk, and Moeller (2015a) adapted the unbounded NLE task by varying the size of the scaling unit using all magnitudes from 1 to 10 to examine influences of (1) the size of a predefined unit, on the one hand, as well as (2) multiples of the units as target numbers on the estimation pattern of individuals, on the other. As expected, they found that estimation accuracy improved as unit size increased, probably as a result of fewer steps needed to make on the number line reducing the accumulation of estimation error. In addition, varied unit sizes did not affect individuals´ 'working window of numbers', it rather seemed fix at about a size of 10 irrespective of the given unit size. This might indicate that the more the unit size approached the working window of participants, the more accurate and faster their estimations got. Moreover, as hypothesized, multiples of a predefined unit benefited from multiplication fact knowledge with regard to faster but not more accurate estimations which argues for a task inclusive recruitment of different numerical representations (see Reinert et al., 2015a).

Ebersbach, Luwel, and Verschaffel (2015) examined whether the variability and accuracy of kindergartners, first- as well as second-graders' NLEs were affected by their familiarity with numbers, their age group as well as the presence or absence of an upper endpoint of the number line (i.e., bounded vs. unbounded). All three age groups were given numbers on number lines being represented by a three-dimensional bar with a length of 100 cm. In the unbounded NLE task, only two reference points were marked in the form of a given scaling unit from 1 to 10 – but not the position of the upper end point 100. However, in the bounded condition, this additional third reference point at the position of 100 was given. The authors found that estimations were less variable as well as more accurate in older children as compared to younger ones as well as in those who were familiar with a larger number range. However, the presence of an upper endpoint in the bounded condition did neither affect the variability nor the accuracy of children's estimations. This rather unexpected finding is consistent with the results found by Link and colleagues (2014a) in which only third-graders performed differently in both task conditions. The authors concluded that familiarity with larger numbers is associated with a more precise mental number line representation (see also Dehaene et al., 2008) and hence facilitates more effective estimation strategies that give individuals a general advantage in both the unbounded as well as the bounded task version.

In a follow-up study, Reinert and colleagues (2015b) carried out the first eye-tracking experiment to explore solution strategies employed in unbounded in direct comparison with bounded NLE. Using the eye-tracking methodology permitted to investigate the possible consideration of reference points in both task versions by examining individuals´ eye-fixation behavior – in a similar way as in the study by Sullivan and colleagues (2011) for bounded NLE. Interestingly, Reinert and colleagues (2015b) found that numbers of fixations at and around reference marks were increased (e.g., origin, end- and mid-point) in the bounded NLE task, supporting the idea of the prominent use of proportion-based strategies. In contrast, they observed for the unbounded task version that the number of fixations on the number line continuously decreased as target number magnitude increased. As such, these results indicated that unbounded NLE might be less influenced by proportion judgement strategies as compared to bounded NLE.

Following an embodied training approach, Dackermann, Fischer, Huber, Nuerk, and Moeller (2016) aimed to train second-grade children to walk a given distance with equally spaced steps on a number line that was taped to the floor. This served to train the principle of equidistant spacing of adjacent numbers with a full-body response. Compared to the control training where children had to separate a given line into equally sized segments on a tablet PC, specific training gains were more pronounced after the embodied training. However, training effects were quite inconsistent for unbounded and bounded NLE: Children only improved their performance in unbounded – but not bounded – NLE after the embodied training increasing their understanding of equidistance. However, without whole-body movements, more favorable performance improvements were only observed in subtraction and bounded NLE after the control training. Thus, these data suggest that the training effects of the embodied training might originate from different cognitive mechanisms that are involved in the respective training conditions. These processes include children's flexible adjustment to the spatial orientation or the change of their perspectives on the training task (walking on the line) combined with bodily experiences. The full-body movement might have helped them realizing its congruency with the spatial dimension of the trained concept.

A further study by Reinert, Huber, Nuerk, and Moeller (2017) evaluated potential sex differences in performing unbounded and bounded NLE. In the latter, no sex differences were observed whereas in unbounded NLE women were outperformed by men who achieved more accurate estimates. Observing this male advantage in unbounded, but not bounded NLE, suggests that unbounded NLE may not be solved applying specific strategies learnt at school like proportional judgement. Instead, it may rely more heavily on processes of numerical estimation for which an advantage for men has been reported. Moreover, this finding shows that the advantage for males emerges from solution strategies employed while completing this task.

However, the only inconsistent findings reporting result patterns opposite to those found in the initial studies by Cohen and colleagues (Cohen & Blanc-Goldhammer, 2011; Cohen & Sarnecka, 2014) and more generally were reported by Kim and Opfer (2017, see also 2020): Investigating children's unbounded and bounded NLE as well as addition and subtraction, the authors found even more logarithmic estimates in the unbounded than bounded task version. Moreover, the results for both the bounded as well as the unbounded NLE task showed that the more logarithmic children's estimates were, the worse was their performance in arithmetic tests. This result was interpreted as evidence for a unified framework for numerical estimation supported by the logarithmic-to-linear shift theory. According to this, Kim and Opfer suggested that both tasks might induce similar estimation strategies and offer the same reflection of children´s representation of number magnitude. Nevertheless, this observation was uncommon and not in line with previous data because usually the bounded number line correlates with measures such as addition and subtraction problems but the new unbounded task version does not (see Link et al., 2014a; 2014b). However, the findings of this study were not consistent with the assumption that bounded NLE requires better arithmetic skills compared to unbounded NLE that is thought to be solved by less advanced mensuration skills. It rather showed that logarithmic index values predicted arithmetic scores reliably.

Van der Weijden and colleagues (2018) were the first to conduct a qualitative study combining eye-tracking recordings with Cued Retrospective Reporting (CRR) to explore strategy use and accuracy in adults with and without dyscalculia while completing both unbounded as well as bounded NLE. The authors found several newly described strategies such as the *use of the previous target number* as well as additional tools and steps (e.g., *rounding off* the target such as 88 to 90, *estimation of small units,* or *orientation on the line* looking at the start and then to the end point trying to estimate the length of the line) applied to complete the two NLE tasks. Their findings suggest that typically developing and dyscalculic adults use fairly similar strategies in unbounded and bounded NLE. One of the most interesting observations obtained with the combined method of eye-tracking and CRR was that even when participants did not use the two end points and the midpoint as reference points in bounded NLE they may still have used the location of the previous target number as functional reference point. Functionality, thus, seems not only to be defined on the basis of known reference points but can also be considered *adaptive* (see also van 't Noordende, van Hoogmoed, Schot, & Kroesbergen, 2016) including reference points that are most proximal to the target.

Recently, Cohen, Blanc-Goldhammer, and Quinlan (2018) presented a computational model of NLE and specified a unified mathematical theory that links the underlying magnitude representation and the associated solution strategies in the four variations of the number line tasks used here: In two experiments, the authors presented participants with a production as well as a perception version of both the unbounded and bounded NLE task. It was observed that each task produced a distinct result pattern arising more or less from the same underlying representation of number magnitude and created equivalent biases in all variations. Only the perception version of the bounded NLE task showed systematic biases different from all three other task versions that did not fit with the model. The authors concluded that performance in this task version may reflect a complex interaction between number processing in general as well as constraints being an immanent part of the task itself but are not related to number processing. These data provide additional evidence that qualitatively different strategies are invoked in both NLE tasks and that their proposed model "captures the underlying processes driving completion of the unbounded number line task" (p. 2641). Hence, these findings, once more, support the claim that the new unbounded NLE task reveals more transparent answers on how number magnitude is processed (see also Cohen & Blanc-Goldhammer, 2011; Cohen & Quinlan, 2018).

Lately, Reinert, Hartmann, Huber, and Moeller (2019) carried out a systematic comparison between estimation performance in a non-symbolic estimation task and both the unbounded and bounded NLE task by using perception and production versions of each task, respectively. They aimed at evaluating the generalizability of systematic estimation biases observed in non-symbolic numerosity estimation. As expected, their data showed a systematically biased pattern of under- and overestimation that was replicated for the non-symbolic numerosity estimation task and generalized to unbounded though not to bounded NLE. The authors interpreted this closer association of non-symbolic numerosity estimation with unbounded than with bounded NLE as further substantiating the claim that unbounded NLE constitutes a more valid measure of the underlying representation of numbers.

In a further study, Jung, Roesch, Klein, Dackermann, Heller, and Moeller (2020) investigated both the traditional bounded as well as the unbounded NLE task in secondary school children to assess strategy use as well as estimation accuracy and the association of NLE and basic arithmetic. The authors observed significantly better performance of children in bounded NLE – also improving with age – as compared to unbounded NLE. Furthermore, estimation performance of bounded but not unbounded NLE was found to be associated with basic arithmetic operations (i.e., addition, subtraction and so on). Interestingly, these associations increased in size with age and therefore indicated developmental change. As expected, these findings also confirmed the use of proportion-based estimation strategies in the bounded task version and estimation-based strategies in the unbounded task version corroborating Cohen and Blanc-Goldhammer's (2011) assumption on the validity of unbounded NLE.

In the most recent investigation by Kim and Opfer (2020), the authors reply to a comment by Cohen and Ray (2020) arguing that the result pattern in their preceding study (Kim & Opfer, 2017) may be driven by methodological weaknesses. In particular, they were criticized as not having provided sufficient space for overestimates of participants' responses in unbounded NLE on the right side of the display leading to biased results. In order to address this issue, Kim and Opfer (2020) conducted an additional study employing the methods as suggested by Cohen and Ray (2020). Children were presented four estimation tasks – three unbounded versions with a small, medium as well as large number range and with enough space left on the right side of the monitor as well as one bounded NLE task version with a large number range. However, the estimation pattern observed for unbounded NLE was again logarithmically compressed as in Kim and Opfer (2017) even when following Cohen and Ray (2020)'s suggestions (unbounded-small condition). The authors still explain the appearance of compression in both the bounded as well as the unbounded NLE task by a general developmental shift rather than methodological differences in various settings.

In sum, these 15 empirical articles described above investigated the unbounded task version in various settings and almost all of them suggested that this new task constitutes a purer and more valid measure of the underlying representation of number magnitude (however see Kim & Opfer, 2017, 2020, for inconsistent results). Nevertheless, there are still factors which may affect solution strategies and thus need to be examined in future studies. Furthermore, many research questions still await a conclusive answer. In the next paragraph, a systematic integration of these 15 studies employing unbounded NLE was therefore undertaken along nine key variables to identify commonalities and differences as well as significant research gaps in this young research area.

2.2 Systematic overview of the unbounded number line estimation task

In this systematic overview and evaluation of the current state of research and literature on unbounded NLE, I aimed at identifying commonalities and differences in studies on unbounded NLE. Therefore, I selected the most relevant key variables according to the moderators that were chosen in the meta-analysis by Schneider and colleagues (2018a; see also Kim & Opfer, 2020, Appendix B, p. 859, for a further comparison of different methodological and analytic features among unbounded number line studies). These are listed in the top line of Table 4 for the 15 research articles identified by my systematic literature search in the chapter above. In their meta-analysis, Schneider et al. (2018a) inferred that all study results show consistent evidence for the unbounded task version to be regarded a more reliable measure of the representation of number magnitude (see also Cohen & Ray, 2020). Besides, they further debated that proportional reasoning is nearly impossible or at least quite difficult in this task version (see Link et al., 2014b). Overall, and in line with the results of the three empirical studies of this dissertation, the findings of this review confirm this proposition. 13 out of 15 articles (87%) substantiated the notion that unbounded NLE provides a more valid measure and it is probably unlikely that both NLE tasks assess the underlying representation of number magnitude in the same way (but see Kim & Opfer, 2017, 2020).

In the following paragraph, a closer examination of unbounded NLE only was undertaken considering different variables distinguishing the unbounded number line task version based on key criteria and compare them systematically. However, some of the variables investigated by Schneider et al. (2018a) as well as features in Kim and Opfer (2020) were not relevant for the current literature overview – amongst them for instance the number type (fractions vs. whole numbers) as fractions for example have not yet been employed in the unbounded NLE task. The same applies to the temporal order of the number line task in the assessments (before or after another mathematical competence measure). In the following overview, I evaluate the identified studies on the basis of the following nine variables: (a) the *country* in which the study was carried out, (b) the *age group* investigated in the experiment, (c) the *number range* of the line presented to participants, (d) the *task type* distinguishing between position-to-number and number-to-position tasks, (e) the *presentation medium* on which the task was administered, (f) the *display width* of the monitor screen, (g) the *maximum response line length*, (h) the *unit size* indicating the physical distance between 0 and 1, as well as (i) the *measure of NLE proficiency*.

All in all, a large overlap between the main variables in all empirical experiments in respect of the examined age groups, the presentation medium and the pattern of estimation errors can be noticed. Overall, 8 studies drew on an adult sample (50%), and 8 (50%) administered the task to children (both a children and adult sample were used in Link et al., 2014a).

In the vast majority of articles researchers presented the stimuli on a computer screen (80%), only 3 studies carried out the experiment with primary school children using a paper-pencil version of the unbounded NLE task (20%) or more specifically, one of these studies used a 100 cm long, three-dimensional bar and target numbers were printed on small cards (Ebersbach et al., 2015). Remarkably, however, Schneider and colleagues´ meta-analysis (2018a) revealed that the presentation medium did not affect the correlation between NLE and distinct mathematical competencies for bounded NLE.

With respect to the measure of estimation error indicating proficiency in unbounded NLE performance, I found six different types that were calculated, most frequently (33% of studies) the percent absolute error [PAE = |estimated – target number|/scale; cf. Booth & Siegler, 2008, see also Schneider, Thompson, & Rittle-Johnson, 2018b, for a comparative review on magnitude comparison tasks], followed by the absolute estimation error (22%), and the mean estimate of the target numbers (17%). Also, two studies (11%) used the relative estimation error [REE = (estimation number – target number)/number range of the task * 100], and the beta parameter (β) that reflects the slope of fitted linear models (*line/quantity bias*, 11%). One study calculated what they called error rate [ER = (estimation – target number)/target number] (6%). In addition, further variables such as, for instance, eye-fixation data (28%) or reaction times (11%) were inspected in some experiments.

Furthermore, all 15 studies employed the production version (number-to-position) of this task (88%), with the exception of two more recent investigations (12%) carried out by Cohen et al. (2018) and Reinert et al. (2019) also using the perception version (position-to-number) to compare performance across both versions. Hence, the latter has not been sufficiently examined so far and should thus be explored in more detail in future research projects.

Finally, it is worth noting that more than half of studies (*N* = 8) were carried out in Germany (53%), followed by the United States (33%). Only one study each was conducted in another European country (the Netherlands and Switzerland).

In contrast, a crucial and main difference between these 15 empirical studies relates to the numerical ranges of the number lines that were chosen, which varied widely from 0 to 20 up to 0 to 1,000. Researchers used smaller number ranges (up to about 20) for children, while larger number ranges were usually presented to adult participants (e.g., up to approximately 50). More specifically, the ranges in the 15 relevant studies were 0 to 20 (20%), 0 to 22 (7%), 0 to 25 (7%), 0 to 29 (13%), 0 to 30 (7%), 0 to 40 (7%), 0 to 50 (27%), 0 to 58 (7%), 0 to 100 (13%), 0 to 132 (7%), 0 to 400 (7%), 0 to 448 (7%) and 0 to 1,000 (7%). Three experiments performed subtasks with different numerical ranges on the unbounded number line (*2 different ranges: 0-40 and 0-400*, van der Weijden et al., 2018; *3 different ranges: 0-30, 0-100 and 0-1,000*, Kim & Opfer, 2017; *0-58, 0-132 and 0-448,* Kim & Opfer, 2020). However, these large target ranges may lead to problems in implementing the unbounded NLE task correctly to allow for the expected overestimation as such big computer monitors do not exist (see Cohen & Ray, 2020).

Furthermore, these 15 studies differed considerably with regard to specific methodological features such as display width, maximum number line length as well as unit size reflecting the physical distance between 0 and 1. Stimuli were presented on tablet PCs (6%), and on computer monitors with horizontal screen resolutions varying from 1,024 (13%), 1,280 (19%), 1,440 (6%) to 1,920 pixels (px, 38%). Three studies did not use a computerized form (19%) and carried out the experiment in paper-pencil format (13%) or using a three-dimensional bar as the number line (6%). Moreover, the maximum response line length also differed considerably depending on the display width. Six studies randomly varied the length of the number line (40%) to distract participants from generating expectations on the maximum response line length, whereas 60% did not. However, this seems to be "the critical inhibitory feature" (see Cohen & Ray, 2020) when the unbounded number line is limited to the maximum target number and researchers do not leave enough space to the right to allow participants to overestimate numbers. However, there is a controversial debate about how much these features do matter or not (see Kim & Opfer, 2020). Furthermore, some studies (47%) did not provide explicit details about the physical size of the scaling unit but give some information on the number of units of a given line length in pixel. It rather seems to be relevant whether the 0–1 number line changes its location and length on every trial (see Kim & Opfer, 2020). This may probably be one of the reasons why Kim and Opfer (2020) observed findings inconsistent with most previously found results. In their experiment participants might have started building expectations about the actual upper bound of the number line and/or using landmarks on the screen as external reference points because length and location of the number line stayed the same on every trial.

Taken together, this systematic overview showed that studies employing the recently introduced unbounded NLE task have several commonalities, but also some considerable differences. This new task version has been examined in various settings with different age groups including both children and adult samples, most commonly employing the production version (number-to-position) of the task (see also Huber et al., 2017, p. 148; Reinert et al., 2019) with computerized presentation. Furthermore, the most often used dependent variable to calculate the accuracy of individuals´ estimations is percent absolute error (PAE, see also Schneider et al., 2018b). Most frequently, numerical ranges up to 50 were chosen for adult samples, whereas children were tested on ranges up to 20 most often, only few studies used ranges up to numbers larger than 100. Finally, the majority of experiments were performed in Germany and the United States. Based upon these findings, future directions and recommendations will be discussed in the limitations and future perspectives sections below. In the next paragraph, I will propose an updated taxonomy of magnitude estimation tasks enabling a more comprehensive and simplified classification of approaches that measure magnitude estimation in general and number line estimation in particular.

Table 4. Overview of empirical studies investigating the unbounded NLE task.

Study	Country	Age group	Number range	Task type	Presentation medium	Display width	Max. response line length	Unit size	Measure of NLE proficiency
Cohen & Blanc-Goldhammer (2011)	USA	Adults / Undergraduates	*Bounded & unbounded:* 0 – 25	Production	Computer	1,920 px	52 – 832 px	2 – 32 px	Mean estimate of each target number (+ Model fittings)
Link, Huber, Nuerk, & Moeller (2014a)	Germany	Children in primary school and adults (students)	*Bounded:* - first-graders: 0 – 10 - second-graders: 0 – 20 - third-graders: 0 – 100 - forth-graders: 0 – 1,000 - adults: 0 – 10´000 *Unbounded:* 0 – 20	Production	Paper	DIN A4 sheet: 29.7 cm	20 cm	NA	Standard deviation of percent absolute error & mean estimates
Cohen & Sarnecka (2014)	USA	Children aged 3.5 – 8 years	*Bounded & unbounded:* 0 – 20	Production	Computer	1,920 px	10 – 30 px	1 px	Mean estimate of each target number
Link, Nuerk, & Moeller (2014b)	Germany	Children (forth-graders)	*Bounded:* 0 – 1,000 *Unbounded:* 0 – 20	Production	Paper	DIN A4 sheet: 29.7 cm	20 cm	NA	Percent absolute error
Reinert, Huber, Nuerk, & Moeller (2015a)	Germany	Adults (students)	*Unbounded:* 0 – 50	Production	Computer	1,920 px	930 – 1,276 px	17 – 19 px	Absolute estimation errors & reaction times
Ebersbach, Luwel, & Verschaffel (2015)	Germany	Children (kindergartners, first- and second-graders)	*Bounded & unbounded:* 0 – 100	Production	Paper/bar	–	100 cm	1 cm	Error rate, absolute error rate & standard deviation (+ Model fittings)
Reinert, Huber, Nuerk, & Moeller (2015b)	Germany	Adults (students)	*Bounded & unbounded:* 0 – 50	Production	Computer	1,920 px	930 – 1,276 px	17 – 19 px	Absolute estimation errors & first fixation location
Dackermann, Fischer, Huber, Nuerk, & Moeller (2016)	Germany	Children (second-graders)	*Bounded:* 0 – 100 *Unbounded:* 0 – 29	Production	Computer	Tablet PC	*Embodied:* 1.5 – 2 m *Control:* 464 – 782 px	NA	Percent absolute error

Reinert, Huber, Nuerk, & Moeller (2017)	Germany	Adults (students)	*Bounded & unbounded:* 0 – 50	Production	Computer	1,920 px	930 – 1,276 px	17 – 19 px	Absolute estimation error Reaction times
Kim & Opfer (2017)	USA	Children aged 5 – 9 years	*Bounded & unbounded:* - 5 – 6-years-olds: 0 – 30 - first-graders: 0 – 100 - second-graders: 0 – 1,000	Production	Computer	1,280 px	1,100 px	- *5–6-years olds:* 36 px - *1st*: 11 px - *2nd*: 1 px	Percent absolute error & power models
Van der Weijden, Kamphorst, et al. (2018)	Nether-lands	Adults with dyscalculia and typically developed	*Bounded:* 0 – 100 & 0 – 1,000 *Unbounded:* 0 – 40 & 0 – 400	Production	Computer	1,280 px	757 px (20 cm)	19 px	Percentage of absolute error
Cohen, Blanc-Goldhammer, & Quinlan (2018)	USA	Adults (undergradu-ates)	*Bounded & unbounded:* 0 – 22	Production and percep-tion (= esti-mation) variations	Computer	1,920 px	1,720 px*	2 – 32 px	Beta parameter (β) estimating the line/quantity bias (+ Model fittings)
Reinert, Hartmann, Huber, & Moeller (2019)	Switzer-land	Adults (students)	*Bounded:* 0 – 10,000 *Unbounded:* 0 – 50	Perception and production	Computer	1,024 px	18 cm	0.3 cm	Relative estimation error
Jung, Roesch, Klein, Dacker-mann, Heller, & Moeller (2020)	Germany	Children (fifth – seventh graders)	*Bounded:* 0 – 10,000 *Unbounded:* 0 – 29	Production	Computer	1,024 px	716 px	NA	Percent absolute error & mean percent relative estimation error
Kim & Opfer (2020)	USA	4 – 12-year-old children	*Bounded:* 0 – 538 *Unbounded:* Large condition: 0 – 448 Medium condition: 0 – 132 Small condition: 0 – 58	Production	Computer	1,280 or 1,440 px	1,080 px	2 px	Beta estimates ($MaxRange_{unbounded}$)

* according to Kim & Opfer (2020)

3. A Taxonomy of Magnitude Estimation Tasks

In the previous section, I discussed methodological commonalities and differences of the 15 studies employing unbounded NLE so far. Based on this and to integrate the topic of this doctoral thesis into a broader research context, I propose an updated taxonomy of different types of number magnitude estimation tasks (see Figure 11). The recently suggested "taxonomy of paradigms of studies on magnitude estimation" (see Ebersbach et al., 2013, p. 3) served as a starting point for the classification scheme which I adapted and simplified motivated by experiments conducted and results obtained in the present dissertation.

The primary purpose of this taxonomy is to enable classification of present as well as future approaches to assess magnitude estimation distinguishing three basic categories: (1) the *estimation type* used in the experiment (i.e., non-symbolic numerosities vs. symbolic numbers), (2) the *estimation task* (i.e., numerosity estimation, bounded or unbounded NLE) and (3) the *task type* chosen (i.e., perception, production or reproduction). This systematic taxonomy developed based on above described literature review provides a hierarchical structure of reduced complexity only considering number magnitude estimation tasks that were used in past and actual research or suggested to use in future studies (see Figure 11).

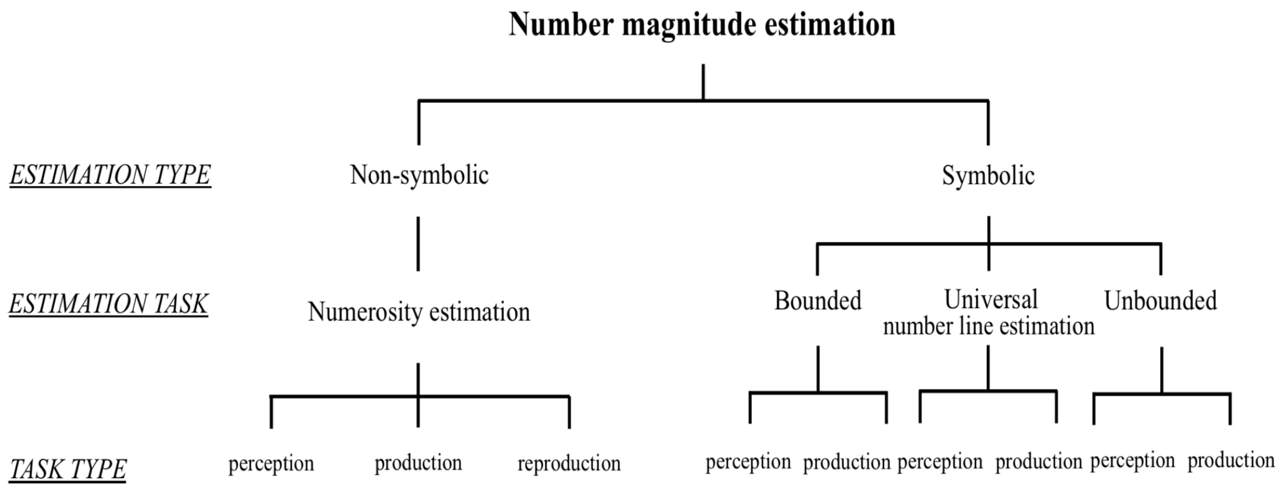

Figure 11. A taxonomy of magnitude estimation tasks.
Adapted based on Ebersbach et al., (2013, p. 3), to fit the research carried out in this dissertation.

Considering the three categories more closely, non-symbolic stimuli as an (1) estimation type can be sequences of sounds, a set of dots, or any collection of objects in a set with set size representing numerical magnitude, while symbolic stimuli are for instance spoken and/or written number words or Arabic numbers (see Crollen et al., 2011, p. 39). Furthermore, (2) estimation tasks can be differentiated according to two subcategories: first, the numerosity estimation task with non-symbolic and symbolic stimuli and, second, three different versions of the NLE task: i) the traditional bounded NLE task with a given start and end point labeled with minimum and maximum values, ii) the new unbounded task version with only a standard point given together with a segment denoting a unit (usually of value 1) as well as iii) a recently introduced hybrid of the former two, the universal NLE task, which is a generalization of both the bounded and unbounded NLE task (also called Cohen Ray number-line task). In this combined version presented by Cohen and Ray (2020), the left and right bounds can be given any value with target numbers even greater than the upper right end point. In this case, when participants for instance have to mark the position of '13' on a number line with a start point of 0 and an upper end point of 10, they can drag the response line beyond the right boundary (see Figure 12). Otherwise, for all numbers between 0 and 10 this task acts like a typical bounded NLE task and individuals can drag the response line between the left and right boundary.

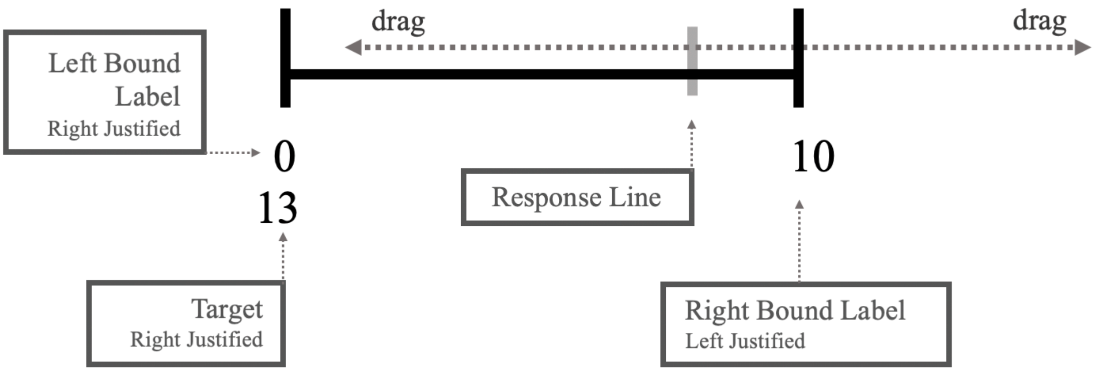

Figure 12. The structure of the universal number line (from Cohen & Ray, 2020, p. 848).

Each of these estimation tasks, in turn, can be subdivided into three distinct (3) task types: perception, production, and reproduction in the numerosity estimation task. On the one hand, in the perception variant (also called *position-to-number*), participants have to estimate the numerical value indicated by a marked position on the number line or, for instance, a collection of dots. In the alternative variant of the task, the most commonly used production version (*number-to-position*), participants are given a target number (e.g., 72) and requested to mark the spatial location of it on the number line (e.g., ranging from 0–100). Finally, in the reproduction variant (works for non-symbolic to non-symbolic conversion) can be employed as numerosity estimation task in which individuals have to reproduce, through the production of a set of dots, the numerosity of a given non-symbolic quantity (e.g., a set of dots, see especially Crollen et al., 2011, Experiment 3).

In the current dissertation, I particularly focused on the branch on the very right side of this systematic taxonomy that provides a simplified but also more comprehensive classification of paradigms used in studies employing unbounded NLE tasks. In addition, it serves as a comprehensive overview of the present state of research on magnitude estimation tasks considering crucial methodological aspects. Taking into account several of these factors, the three empirical studies conducted in this thesis substantiated that unbounded NLE indeed seems to be a valid and more pure measure for the underlying representation of number magnitude compared to traditional bounded NLE. Finally, this taxonomy suggests perspectives for future research raising and identifying unexplored research questions and may bring new insights into research of the field of numerical magnitude and unbounded NLE, which will partly be discussed in the next section.

4. Limitations and Future Perspectives

Unbounded NLE is still a fairly young research objective. Nevertheless, there is robust evidence suggesting that this new task version provides a more unbiased measure of the mental number magnitude representation. The findings of the current thesis support this claim in many respects. However, these studies are not without limitations and there are some points that have to be considered when interpreting the results obtained and/or need to be further investigated in future studies. The following paragraph reviews general limitations of the present and develops suggestions for future research.

First, a limitation identified by Cohen and Ray (2020) also applied to Study 1 of the present thesis. I might have allowed only limited room for overestimation for larger target numbers. For instance, on the unbounded number line with a length of 50 units and target numbers up to 49, it was not possible to measure overestimations of the larger target numbers properly (e.g., typically about 15-20% overestimation which corresponds to an optimal line length of about 57 to 60 units for this target number range). In case of presenting varying number lines with a length of about 54 to 66 units as implemented in Studies 2 and 3 of this thesis, this should be unproblematic. In future studies, the length of an unbounded number line should give enough space for larger target numbers to avoid that participants use this upper end point as an implicit boundary or reference point. Cohen and colleagues found that the space required to measure a positively accelerating bias is around $\beta = 1.2$ (Cohen & Ray, 2020). This would ensure that estimates are not biased by task specific constraints.

Furthermore, as also mentioned by Crollen and colleagues (2011), research on the mapping process between symbolic and non-symbolic magnitude representations is still in its infancy and should be pursued further to directly investigate different types of numerical estimation tasks as well as different mapping routes in children (see also Mundy & Gilmore, 2009). Additional studies would be desirable to take a closer look at the development of numerical mapping abilities in younger children such as kindergartners or first graders to further examine the bi-directional mapping hypothesis (Crollen et al., 2011). Such data would offer interesting insights into whether the observed biases of under- and overestimation would be similar or different in children as compared to those in adults.

In addition, Study 3 showed no significant sex differences for bounded NLE in contrast to what was observed by Bull and colleagues (2013). To investigate this inconsistency more closely, one might use bounded number lines with end points different from 0 or multiples of 10 in future studies such as 259 to 643 (see also Booth & Newton, 2012; Di Lonardo, Huebner, Newman, & LeFevre, 2020; Hurst, Leigh Monahan, Heller, & Cordes, 2014, for findings on atypical endpoints). Maybe, straight numbers as end points of the number line would increase task difficulty which might in turn increase the possibility of differential effects, in particular in adults who perform very well in bounded NLE. On the other hand, number ranges covered should be more difficult in further studies, not only in the bounded (e.g., 100,000), but also in the unbounded NLE task (e.g., ranging up to 100 or 500 at least). In both cases, error variance in general as well as sex differences in particular are supposed to increase as men were shown to be at an advantage when required to develop their own solution strategies. So far, only few studies used larger numerical ranges of up to about 400 (van der Weijden et al., 2018) or 1,000 (Kim & Opfer, 2017) in unbounded NLE tasks. However, it is important to note that also for these large ranges researchers should leave enough monitor space for overestimations to be registered properly as recommended by Cohen and Ray (2020).

Moreover, referring to the taxonomy of magnitude estimation tasks displayed in Figure 11 (see also Ebersbach et al., 2013, p. 3), there is a specific combination of categories in number magnitude estimation that has not been investigated so far. This concerns the leftmost branch in the taxonomy: "Non-symbolic numerosity estimation" using "unbounded number line estimation" as an instrument to measure number magnitude representation. I therefore recommend future studies to examine non-symbolic estimation with an adapted unbounded NLE task. In such a task version, the size of the scaling unit may be indicated by a set of points (see Figure 13 for an example). Here, as well as in a typical symbolic unbounded NLE task, the target number may be presented above the scaling unit on the left end of the number line by a dot pattern. Participants then have to mark the spatial position of the target number by a mouse click at the estimated position. Applying this modified task version in future research could contribute towards shedding light on its generalizability independently of the stimuli´s modality (i.e., non-symbolic vs. symbolic). One might speculate that the non-symbolic version of the unbounded NLE task might be an even purer measure of number magnitude representation. At least, it would allow to test even younger children not yet familiar with symbolic numbers. As such, modifying this task as proposed would allow further insights into the overall research question of this dissertation.

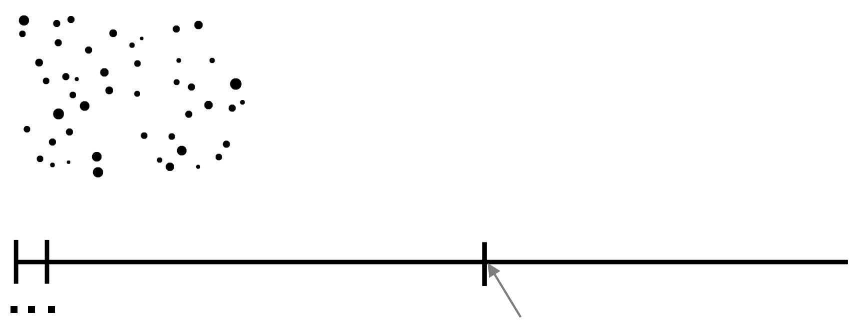

Figure 13. Non-symbolic version of an unbounded number line estimation task.

Beyond that, future research might also consider using the recently suggested universal number line (Cohen Ray number-line task) introduced by Cohen and Ray (2020). In this generalized version of a combined bounded and unbounded NLE task (see Figure 12 for its structure), the left boundary, for instance, may be '0' and the right boundary '50', so that this part of the number line range reflects bounded NLE. In contrast, for target numbers larger than 50 this constitutes unbounded NLE for which the response line must be dragged to the right beyond 50. By implementing such a combined task version in which the position of target numbers has to be indicated between the left and right bound (as in the bounded) as well as beyond the right boundary (as in the unbounded NLE task), hypotheses of both task versions may be tested at the same time.

Finally, future studies – as also recommended by Schneider and colleagues (2018a) – will have to focus more specifically on components such as training elements of NLE like for example magnitude comparison or adding numbers before participants have to indicate the position of the sum on a number line (see also Fuchs et al., 2013; Honoré & Noël, 2016; Thompson & Opfer, 2016). In particular, it remained unclear in all previous studies whether these training effects in the NLE arise as a result of the number line or rather other training components (e.g., more accurate memory recall of learners or simply an improvement of memory by receiving feedback, domain-general cognitive resources, as well as other cognitive mechanisms). Honoré and Noël (2016), for instance, recommended to investigate the specific role of specific components of the respective trainings (e.g., number line positioning, or the magnitude comparison task) by contrasting their isolated training effects. Thereby, different training effects as well as mechanisms being involved and transfer from one task to another might be evaluated.

Moreover, as suggested by Ebersbach and colleagues (2013), various methodological aspects of magnitude estimation tasks (e.g., shape, variability, and accuracy of magnitude estimations) should be systematically manipulated in further studies and could be compared thus with other features and findings of other tasks. Furthermore, it is necessary for the purpose of achieving comparability that all researchers provide exact details of the size of certain features in pixels such as for example a single unit size, the horizontal screen resolution as well as the largest possible bias (β_{max}).

In sum, there are still numerous research questions that need to be addressed in future research to answer the overarching question whether the unbounded NLE is indeed a more valid measure of number magnitude representation than the bounded task version. Notably, a larger number range as well as a modification of the stimulus modality or use of the new universal (Cohen Ray) NLE task may help to gain additional insights into this question and close some of the gaps described above and in the literature. Nevertheless, the empirical findings of the present thesis are meaningful in providing converging evidence to the claim that the unbounded NLE task captures the underlying representation of number magnitude more purely as compared to the standard bounded task version (see also Cohen & Blanc-Goldhammer, 2011).

5. Overall Conclusion

In the last few years, it has been debated controversially whether the traditional bounded NLE task in fact allows for inferences on the underlying representation of number magnitude (e.g., Barth & Paladino, 2011; Cohen & Blanc-Goldhammer, 2011; Cohen & Quinlan, 2018; Rouder & Geary, 2014; Slusser et al., 2013). Surprisingly few studies have been conducted employing the newly introduced unbounded task version – but gaining more and more popularity – although it seems to provide a more pure measure of number magnitude representation. Except the results of Kim and Opfer (2017, 2020), a total of 13 empirical studies largely substantiated this claim. Three of these studies are part of this thesis and broadened our understanding of the validity of unbounded NLE.

As an overall conclusion of the studies presented in the current dissertation, it can be noted that all results obtained here suggest that the new unbounded compared to the bounded NLE task is less influenced by the application of (proportion-judgement) strategies that do not directly reflect numerical estimation processes. As such, these findings clearly support the notion that unbounded NLE might actually be a valid and more "purely" measure of number magnitude representation. Going beyond previous investigations, the current dissertation provided converging evidence from (1) a comparison with non-symbolic numerosity estimation, (2) eye-fixation behavior as well as (3) sex differences in NLE. Considering similarities and differences with numerosity estimation revealed conceptual similarity of unbounded (but not bounded) number line and non-symbolic numerosity estimation, which is widely agreed on to be a reliable measure of number magnitude representation. The systematic patterns of underestimation of target numbers in the perception as well as overestimation in the production version in the unbounded number line and non-symbolic numerosity estimation corroborated this argument. Second, evaluating participants´ eye-fixation behavior while solving the unbounded as well as bounded NLE task confirmed that the former measures number magnitude representation more "purely". In particular, linearly decreasing numbers of fixations on the number line with increasing target number magnitude and no increase of numbers of fixations at or around reference points indicated that unbounded NLE is less influenced by proportion-based estimation strategies not directly related to numerical estimation. Finally, sex differences observed for unbounded but not bounded NLE further corroborated that no learnt solution strategies are to be applied to solve the former, but responses may specifically require numerical estimations for which an advantage for males was reported previously. All in all, the findings of this dissertation clearly suggest that the newly introduced unbounded NLE task might indeed be a purer measure of the underlying representation of number magnitude.

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