Design and Implementation of Effect Handlers for Object-Oriented Programming Languages

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To my family, who always believed in me — and my wife for her love, patience, and unwavering support throughout the journey of life and thesis.

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A PhD is challenging but you are not alone – you will always have friends and colleagues. — Paolo G. Giarrusso (2014)

My friend Paolo and I were sitting in the Sudhaus pub in Marburg, where this remark by Paolo finally convinced me to start doing a PhD. And, as it turned out, Paolo was right. In both regards. The times of my PhD studies were volatile and challenging. And, luckily, there were all these amazing people offering their friendship and support. I want to use this opportunity to express my gratitude to *all of you* who accompanied my journey – including many that are not mentioned in the following.

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Abstract

An important aspect of software development is to structure the control flow of a program. Features to structure the control flow range from simple branching between two possible execution paths, to non-local control-flow transfers by means of jumps or exceptions. Furthermore, modern programming languages offer advanced control-flow structures like async/await to avoid blocking when performing input / output actions, generators to model demand driven computation of streams, or coroutines to express control-flow transfer between different software components. Often, those advanced control-flow abstractions are built into the language and are thus not user definable. Users are thus limited to the control-flow features provided by the programming language designers and cannot create their own custom domain-specific control-flow abstractions.

Another aspect of software development is to parametrize a software component over details that might vary. Parametrizing a component with configuration, behavior, or other components, enables reuse of the same component in different contexts. Components can be configured statically, or dynamically at runtime. Manually parametrizing components can quickly become tedious, both for the component author who needs to design the component for parametrization, as well as the user of the component who needs to provide all parameters. This is especially the case for components that depend on other parametrized components. They often need to be parametrized by all transitive parameters of their dependencies.

Algebraic effects (Plotkin and Power, 2003) and their extension with handlers (Plotkin and Pretnar, 2009, 2013) offer interesting new ways to structure programs. Programs use effect operations, which, like normal functions, can receive arguments and return results. However, the implementation of effect operations is left open and depends on the context in which the program is executed. Effect handlers, much like exception handlers, handle effect operations and provide their implementation. Like throwing an exception, calling an effect operation transfers control to the corresponding handler. Unlike exceptions, the handler can resume the computation and thus transfer control back to the call site. Effect handlers support both aspects of software development concisely: they can express advanced control-flow structures as well as parametrization of software components. It has been shown in the literature that effect handlers can express many of the aforementioned control-flow structures as libraries, such as async/await, generators, and coroutines. At the same time, effect operations can conveniently be used to parametrize software components while handlers provide the configuration. Unifying both aspects also guarantees well-defined interaction between control flow and parametrization.

While algebraic effects recently gained popularity in the programming language research community, we identify two problems hindering adoption by a wider audience of programmers. Firstly, programmers are immediately confronted with the full generality of effect handlers. While effect handlers are expressive enough to model advanced control-flow structures, not all use cases require this expressivity. Secondly, algebraic effects and handlers have been conceived in the realm of functional programming languages. In consequence, most implementations of effect handlers are either standalone languages following the functional programming paradigm or are library embeddings into existing functional programming languages.

In this thesis, we propose solutions to the two aforementioned problems with the goal to facilitate adoption of effect handlers by a wider audience.

To address the first problem, we systematically present effect handlers as a combination of delimited control (that is, they allow control-flow transfers) and dynamic binding (that is, they allow parametrization). While the connection between effect handlers and delimited control has previously been established and it has been shown that effect handlers can express

dynamic binding, we discover that dynamic binding and effect handlers form a spectrum. We present a language design that embraces this spectrum. Increasing expressivity and conceptual complexity step-by-step, we introduce the three features ambient values, ambient functions, and ambient control. Ambient values coincide with dynamically bound variables and ambient control coincides with a slightly restricted form of effect handlers. The novel intermediate form of ambient functions enables abstraction similar to effect handlers, but without modifying the control flow: Ambient functions are dynamically bound, but statically evaluated. Introducing effect handlers from dynamic binding offers programmers an alternative way to approach handlers. They can incrementally learn and understand the different generalizations (ambient values, functions, and control). We hope this facilitates adoption of effect handlers by a wider audience.

To address the second problem, we present a library design, which embeds effect handlers in object-oriented programming languages. Our design embraces the object-oriented programming paradigm and we map abstractions of effect handlers to key abstractions of object-oriented programming. Combining the two paradigms not only enables programmers to use effect handlers in object-oriented programs, but also to use object-oriented programming abstractions to modularize effect handlers. Our design employs explicit capability-passing style. That is, instead of dynamically searching for a handler at runtime, we pass instances of handlers as additional arguments to methods. We present an implementation of our library design in the language Scala. It is based on a variation of a continuation monad for multi-prompt delimited control (Dybvig et al., 2007) to allow effect handlers to express advanced control-flow structures. We study the extensibility properties gained by embedding effect handlers into an object-oriented programming language. While it opens up new dimensions of extensibility, the implementation still comes with two limitations. Firstly, being based on a monad user programs have to be written in monadic style. Secondly, the implementation is not effect safe and capabilities can escape the scope of their handler. This potentially results in runtime errors. We separately address the two limitations. Firstly, we present an implementation in Java that performs a type-selective continuation-passing style transformation by rewriting of bytecode. This way, user programs in the Java implementation can be written in direct-style overcoming the limitation of the Scala implementation. Secondly, we enhance our Scala library with an embedded effect system. We index the type of effectful programs with an intersection type to track the set of used effects. Expressing the set of used effects as an intersection type allows us to reuse Scala's support for subtyping to express effect subtyping and to reuse Scala's support for type polymorphism to express effect polymorphism.

Zusammenfassung

Ein wichtiger Aspekt bei der Entwicklung von Software ist es, den Kontrollfluss eines Programmes zu strukturieren. Hierzu bieten Programmiersprachen verschiedenste Funktionalität an: Diese reichen von einfachen Verzweigungen zwischen zwei Ausführungspfaden, zu nicht-lokalem Transfer des Kontrollflusses durch Sprünge oder Ausnahmebehandlungen (engl. *Exception Handler*). Darüber hinaus lassen sich heutzutage in einigen Sprachen auch fortgeschrittene Mittel zur Kontrollflussstrukturierung finden. Beispiele hier für sind: Asynchrone Programmierung, die es zu verhindern sucht, dass Ein- und Ausgabevorgänge andere Berechnungen blockieren; Generatoren, die es vereinfachen Berechnungen erst auf Bedarf durchzuführen; und schließlich Koroutinen, die es erlauben, die Kommunikation und den Kontrollflusswechsel zwischen kooperativen Prozessen zu modellieren. Leider sind diese Kontrollflussabstraktionen häufig fest in die jeweilige Sprache eingebaut und können nicht durch Nutzer der Sprache angepasst oder neu definiert werden. Dies erschwert es Programmierern ihre eigenen, domänen-spezifischen Kontrollflussabstraktionen zu entwickeln.

Ein weiterer, wichtiger Aspekt der Softwareentwicklung ist es, Softwarekomponenten über mögliche Variationspunkte zu parametrisieren. Eine Komponente kann so in verschiedenen Kontexten wiederverwendet werden. Parametrisierung erlaubt es Konfiguration und Verhalten statisch oder zur Laufzeit anzupassen. Komponenten manuell zu parametrisieren kann schnell aufwändig werden. Dies gilt sowohl für den Autor der Komponente, welcher veränderliche Aspekte identifizieren und abstrahieren muss, als auch für den Nutzer der Komponente. Um die Komponente zu verwenden, muss jener alle nötigen Parameter bereitstellen. Eine Komponente, die von einer anderen abhängig ist, muss diese entweder konfigurieren, oder selbst über deren Variationspunkte parametrisiert werden.

Algebraische Effekte (Plotkin and Power, 2003) und ihre Erweiterung mit lokaler Effektbehandlung (Plotkin and Pretnar, 2009, 2013) (engl. *Effect Handler*) bieten interessante neue Wege, um Programme zu strukturieren. Programme nutzen Effektoperationen, die wie normale Funktionen Argumente erhalten und Ergebnisse zurückgeben können. Ein wichtiger Unterschied zu Funktionen ist jedoch, dass die Implementierung von Effektoperationen offengehalten wird und vom Kontext abhängt, in welchem das Programm ausgeführt wird. Effekthandler, vergleichbar mit Exceptionhandlern, behandeln Effekte und weisen Effektoperationen ihre Bedeutung zu. Ähnlich zu dem Werfen einer Exception, überträgt der Aufruf einer Effektoperation den Kontrollfluss zum entsprechenden Handler. Ein wichtiger Unterschied zu Exceptions ist jedoch, dass der Handler die Berechnung an der Stelle der Effektoperation fortsetzen kann und damit den Kontrollfluss zurücküberträgt. Effekthandler erleichtern beide Aspekte der Softwareentwicklung: Sie können die zuvor genannten fortgeschrittenen Kontrollflussabstraktionen als Programmbibliotheken ausdrücken, ohne dass diese in die Sprache selbst eingebaut werden müssen. Gleichzeitig bieten sie Möglichkeiten zur Parametrisierung von Komponenten. Effektoperationen können als Variationspunkte betrachtet werden, während Handler Konfigurationen darstellen. Beide Aspekte in einem Sprachkonstrukt zu vereinen garantiert, dass Interaktionen zwischen Kontrollfluss und Parametrisierung wohldefiniert sind.

Obwohl algebraische Effekte kürzlich in der Forschungsgemeinde an Popularität gewinnen konnten, lassen sich zwei Probleme identifizieren, die potentiell einer praktischen Anwendung und weiteren Verbreitung unter Softwareentwicklern im Wege stehen. Zum einen werden Entwickler häufig direkt mit der vollen Ausdrucksstärke von Effekthandlern konfrontiert. Effekthandler sind zwar ausdrucksstark genug, um wichtige Kontrollflussabstraktionen auszudrücken, jedoch ist diese Ausdrucksstärke nicht immer erforderlich. Zum anderen sind algebraische Effekte und

Handler im Rahmen der funktionalen Programmierung entwickelt worden. Infolgedessen sind die meisten Implementierungen dieses Sprachkonstrukts lediglich auf funktionale Programmierung ausgelegt. Dies gilt sowohl für eigenständige Sprachimplementierungen, als auch für eingebettete Implementierungen in Form von Programmbibliotheken.

Diese Dissertation untersucht diese zwei Probleme mit dem Ziel eine größere Verbreitung von Effekthandlern als Sprachkonstrukt zu ermöglichen.

Bezüglich des ersten Problems präsentiert diese Dissertation Effekthandler im Spannungsfeld zwischen den zwei oben genannten Aspekten der Softwareentwicklung: als Kombination von begrenzten Kontrolleffekten (engl. *Delimited Control*) mit dynamischen Gültigkeitsbereichen (engl. *Dynamic Binding*). Kontrolleffekte erlauben es den Kontrollfluss zu strukturieren, während dynamische Bindung die Parametrisierung von Komponenten ermöglicht. Existierende Arbeiten haben bereits den Zusammenhang zwischen Kontrolleffekten und Effekthandlern untersucht. Weiterhin wurde es gezeigt, dass Effekthandler dynamische Bindung ausdrücken können. Wir zeigen nun, dass dynamische Bindung und Effekthandler ein Spektrum bilden und präsentieren ein Sprachdesign, welches auf diesem Spektrum basiert. Das neuartige Sprachkonstrukt „ambiente Funktionen“ (engl. *Ambient Functions*) liegt in der Ausdrucksstärke zwischen dynamischer Bindung und Effekthandlern. Es ermöglicht Abstraktion ähnlich zu Effekthandlern, aber ohne den Kontrollfluss zu beeinflussen. Ambiente Funktionen sind dynamisch gebunden, werden aber im statischen Definitionskontext ausgewertet. Effekthandler aus der Sicht von dynamischer Bindung einzuführen, bietet Entwicklern einen neuen, alternativen Zugang. Sie können die verschiedenen Generalisierungsschritte von dynamischer Bindung, über ambiente Funktionen zu Effekthandlern separat erlernen und verstehen.

Bezüglich des zweiten Problems präsentiert diese Dissertation einen Entwurf, um Effekthandler als Softwarebibliothek in objektorientierte Programmiersprachen (OOP) einzubetten. Im Gegensatz zu existierenden Arbeiten steht objektorientierte Programmierung im Mittelpunkt des Entwurfs. Dieser bildet essentielle Abstraktionen aus der Programmierung mit Effekthandlern unmittelbar auf OOP Abstraktionen ab. Die Kombination der zwei Paradigmen ermöglicht nicht nur die Verwendung von Effekthandlern in objektorientierten Programmen, sondern erlaubt es auch OOP Modularisierungsstrategien zu verwenden, um Effekthandler zu strukturieren. Ein wichtiger Bestandteil des Entwurfs ist es, Handlerinstanzen als zusätzliche Argumente an Methoden zu übergeben, statt nach diesen zur Laufzeit zu suchen. Wir präsentieren eine Implementierung unseres Entwurfs in der Programmiersprache Scala. Um Effekthandlern Kontrollflusstransfers zu ermöglichen, basiert unsere Bibliothek auf einer monadischen Implementierung begrenzter Kontrolleffekte (Dybvig et al., 2007). Eine Evaluation unserer Bibliothek zeigt neue Dimensionen der Erweiterbarkeit auf, welche durch die Kombination von Effekthandlern und OOP erschlossen werden. Die Implementierung hat jedoch zwei Einschränkungen: Erstens gründet sie auf einer monadischen Implementierung, welches die Art Programme zu schreiben diktiert. Zweitens ist die Implementierung nicht sicher in dem Sinne, dass Handlerinstanzen ihren Gültigkeitsbereich verlassen können. Wir präsentieren mögliche Lösungen für beide Einschränkungen. Zum einen stellen wir eine weitere Implementierung in der Sprache Java vor, welche darauf basiert Bytecode zu transformieren. Auf diese Weise können Programme wie gewohnt geschrieben werden und erfordern keine monadische Schreibweise. Zum anderen erweitern wir unsere Scala Implementierung um ein eingebettetes Effektsystem. Wir indizieren den Typ effektvoller Ausdrücke mit einem Schnitttyp (engl. *Intersection Type*), welcher die Menge der genutzten Effekte repräsentiert. Die Verwendung von Schnitttypen erlaubt es uns unmittelbar Scala's Unterstützung für Subtypbeziehungen und Typpolymorphie für Effekte wiederzuverwenden.

Contents

List of Figures

Chapter 1

Introduction

Computer programming is intimately coupled to structuring control flow. From branching control flow based on conditionals, over employing a stack discipline when calling procedures, to using alternative return paths and transferring control flow with exceptions – thinking about control flow is a substantial aspect of every programmer's daily tasks.

The increasing need for distributed computing and non-blocking IO has led to the development of various advanced control-flow abstractions and programming models, such as asynchronous programming (that is, *async/await*) (Bierman et al., 2012), lightweight user-level threads and *fibers* (Dolan et al., 2013), and different forms of non-preemptive multitasking like asymmetric *coroutines* (Moura and Ierusalimschy, 2009) or *generators* (Politz et al., 2013). Each of these solutions is tailored to one particular programming problem. The way these abstractions are realized today is unsatisfactory. Often, they are either hard-coded into a language, (*i.e.*, they are not user-definable), they are not composable (*i.e.*, multiple abstractions cannot be used in one project), or implementations for one control-flow abstraction cannot be reused to implement similar other control-flow abstractions.

In contrast, algebraic effects (Plotkin and Power, 2003) in their extension with effect handlers (Plotkin and Pretnar, 2009) are a general control-flow structuring paradigm that is not tailored to a particular programming problem. Like Monads (Moggi, 1989), algebraic effects have originally been conceived as a framework to model the semantics of computational effects. Also like Monads, starting as a tool for semanticists, algebraic effects have then been rediscovered as a general program-structuring tool for programmers. However, there is a fundamental difference between the two concepts: Monads are centered on a type constructor, a semantic domain (like `Either String a`), from which effect operations (like `throwError`) emerge (Wadler, 1995). Algebraic effects instead start with the set of effect operations to then find a domain that supports the operations (Plotkin and Pretnar, 2009). Algebraic effects and handlers are thus operation centric, while monads are domain centric. Algebraic effects have their theoretical foundations in Lawvere theory (Plotkin and Power, 2003), but in this thesis we will view effect handlers more from the perspective of a software engineer. Following Kammar et al. (2013), effect handlers can be thought of as a generalization of the simpler and widely known control-flow construct *exceptions*. Similar to exceptions, programs using effect handlers are conceptually split into three parts: Effect operations, effectful functions, and effect handlers. Figure 1.1 gives pseudo code examples for each of the three components. We use Scala-like pseudo syntax for easier comparison with the exception handling mechanism.

This chapter is partially based on the contents of the following publication: Jonathan Immanuel Brachthäuser, Philipp Schuster, and Klaus Ostermann. 2018. "Effect Handlers for the Masses". *Proc. ACM Program. Lang.*, 2 (OOPSLA): 111:1–111:27. DOI: https://doi.org/10.1145/3276481.

Effect Signature:

```
effect Amb {
  def flip(): Boolean
}
```

Effectful Program:

```
def choice(n: Int): Int =
  if (n > 1 && Amb.flip() )
    choice(n - 1)
  else
    n
```

Effect Handler:

```
def collect[R](prog: () ⇒ R): List[R] =
  handle { Cons(prog(), Nil) }
  with Amb {
    def flip() = resume(true) ++
                 resume(false)
  }
```

Figure 1.1. Effect handlers generalize exception handlers. They allow resuming the computation at the call site. Examples of an effect signature, an effectful program, and an effect handler in Scala-like pseudo syntax.

Effect operations

Compared with exceptions, *effect operations* correspond to `throw` as found in languages like Java (Gosling et al., 1996). However, instead of being built in, effect operations are user defined. One can define operations like `yield` to output elements of a (push-based) stream, `getHttp` to send (asynchronous) http-requests, or `suspend` to (cooperatively) transfer control to a scheduler (Leijen, 2016). Opposed to `throw`, effect operations can potentially return results. In the example in Figure 1.1, we follow Leijen (2017c) and declare the operation `flip` which returns a boolean.

Effectful programs

Programs are *effectful* (Kammar et al., 2013; Bauer and Pretnar, 2015) if they call effect operations. This can occur either directly or indirectly via other effectful programs. Like checked exceptions in Java, some implementations of effect handlers track the effects used by an effectful program in its type. This allows the type checker to distinguish effectful programs from pure programs and to assert that all effects are eventually handled. The effectful program `choice`, adapted from Danvy and Filinski (1992, p. 154), uses the `flip` operation to nondeterministically return a number between 1 and n.

Effect handlers

Effect handlers generalize exception handlers (Plotkin and Pretnar, 2009; Bauer and Pretnar, 2013). They implement the effect operations, specifying what it means for example to yield, send http-requests or suspend. Like exception handlers delimit the scope with `try { ... }`, effect handlers delimit the *dynamic scope* in which effect operations of a particular effect signature are handled by this very handler; The example handler, adapted from Plotkin and Pretnar (2013, p. 13), handles the operation `Amb.flip` for the extent of evaluating `Cons(prog(), Nil)`.

Effect operations are typically declared (and grouped) in *effect signatures* (such as `Amb`), which act as an interface between the user of effect operations (that is, effectful programs) and the implementer of effect operations (that is, effect handlers). Programming with effect handlers encourages modularity, by separating the declaration of effect operations from their implementation in effect handlers (Kammar et al., 2013).

While conceptually related, there is one important improvement of effect handlers over exception handlers: To handle effects and to implement effect operations, handlers get access to the *delimited continuation*, which is the remainder of the program execution – from the call of

the effect operation up to the enclosing effect handler (Plotkin and Pretnar, 2013). Provided with the continuation, the effect handler can decide to *resume* the execution from the point where the effect operation was originally called. This can also be understood from an operational perspective. Following the analogy to exceptions, let us recall how traditional implementations of exceptions perform a search for the corresponding handler. Exceptions unwind the stack frame-by-frame, until a matching handler is found. The stack segment between the exception throwing site and the handler is discarded (Gosling et al., 1996).

Like exception handlers, effect handlers can be thought of as marking positions on the runtime stack. Also, the search for a corresponding handlers proceeds similar to exceptions. However, in contrast to exceptions, effect handlers do not discard the unwound stack segment. Instead, they reify the stack segment as a resumption function (Leijen, 2017c) (or continuation) and make it available to the handler implementation (Plotkin and Pretnar, 2013).

Figure 1.1 defines the handler `collect` (Plotkin and Pretnar, 2013) which takes a program `prog` potentially using the `Amb.flip` effect to compute a result of type `R`. We assume a call-by-value language and thus the program needs to be thunked to defer the effects (like `flip`) to the point of handling. To handle the `Amb.flip` operation, the `collect` handler can access the delimited continuation by using **`resume`**. Resuming with a value is similar to returning from a normal method call. Computation will proceed at the call site of the effect operation `flip`. However, like with exceptions, we can decide to never resume, or as in our example, we can even resume multiple times. We resume once with **`true`** and once with **`false`**, enumerating all possible outcomes of `flip`. Since the type of our handled program (`Cons(prog(), Nil)`) is `List[R]`, resuming will each time yield a list, which we finally concatenate to gather all results. For example, calling

```
collect(() ⇒ (choice(2), choice(3)))
```

yields the result:

```
▶ List((1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3))
```

The ability to resume the computation at the call site of the effect operation is the important ingredient that provides effect handlers with additional expressive power over exceptions. Effect handlers can express the aforementioned control-flow abstractions as libraries (Dolan et al., 2015, 2017; Wu et al., 2014; Kammar et al., 2013; Hillerström and Lindley, 2016; Leijen, 2017a). Implementing abstractions like concurrent programming (Dolan et al., 2017) and async/await (Leijen, 2017a) with effect handlers naturally allows a combined usage. This is good news for both language implementers and programmers. Implementers who want to implement multiple of the aforementioned control-flow structures now only need to support effect handlers. Interaction between the features can be considered on the user-/ library-level. Furthermore, users can use effect handlers to define custom (potentially domain specific) control abstractions.

1.1 Thesis Overview and Contributions

Effect handlers are a powerful program-structuring paradigm. However, we identify two problems that potentially hinder a widespread adoption of effect handlers.

Firstly, as in the previous section, programmers are often immediately confronted with the full generality of effect handlers. In consequence, effect handlers are reported (Abramov, 2019) to have the reputation of being difficult to understand. While effect handlers are expressive enough to model advanced control-flow structures, not all use cases require the expressiveness to the full extent. For instance, to make the current request object available in a webserver implementation does not necessarily require the ability to capture the continuation.

Secondly, algebraic effects and handlers have been conceived in the realm of functional programming languages. In consequence, many implementations of effect handlers can be found in functional programming languages. They are either built into the language, like in Eff (Bauer and Pretnar, 2015), Koka (Leijen, 2014), Frank (Lindley et al., 2017), Multicore OCaml (Dolan et al., 2017), Links (Hillerström et al., 2017), and Helium (Biernacki et al., 2019). Or they are implemented as a libraries for Haskell (Kammar et al., 2013; Kiselyov et al., 2013; Wu and Schrijvers, 2015), OCaml (Kammar et al., 2013; Kiselyov and Sivaramakrishnan, 2016), or Idris (Brady, 2013). With the notable exception of a recently presented object-oriented calculus JEff (Inostroza and van der Storm, 2018), effect handlers are not widespread in the realm of object-oriented programming (OOP).

In this thesis, we propose solutions to the two aforementioned problems with the goal to facilitate adoption of effect handlers by programmers. The thesis is structured in two parts, reflecting the two different ways in which we work towards this goal.

In the first part, we systematically present effect handlers as a combination of delimited control and dynamic binding. We follow Moreau (1998) and refer to dynamically scoped variables as *dynamic binding*. We use *late binding* (Suzuki, 1981) to refer to dynamic binding of method implementations in OOP. Step-wise generalizing dynamic binding to effect handlers, we discover that dynamic binding and effect handlers form a spectrum and present a language design that embraces this spectrum. Introducing effect handlers from dynamic binding offers programmers an alternative way to approach handlers. They can incrementally learn and understand the different generalizations.

In the second part, we present a library design that embeds effect handlers in object-oriented programming languages. Our design embraces the OOP paradigm and we map abstractions of effect handlers to key abstractions of OOP. Combining the two paradigms not only enables programmers to use effect handlers in object-oriented programs, but also to use OOP abstractions to modularize effect handlers.

We now give a more detailed overview over the two parts and summarize our contributions.

1.1.1 Effect Handlers in Perspective

In the first part of this thesis, we establish the necessary terminology and present effect handlers as a combination of two seemingly orthogonal features: delimited control *and* dynamic binding. The developments of this part of the thesis are illustrated in Figure 1.2. To introduce terminology and to discuss the abstractions provided by effect handlers, we summarize existing work in different calculi (Figure 1.2). In Chapter 2, we follow historical developments in the literature on control effects and present effect handlers in the light of delimited control (Felleisen, 1988). In Chapter 3, we then perform similar steps of generalization, but starting from dynamic binding (Moreau, 1998).

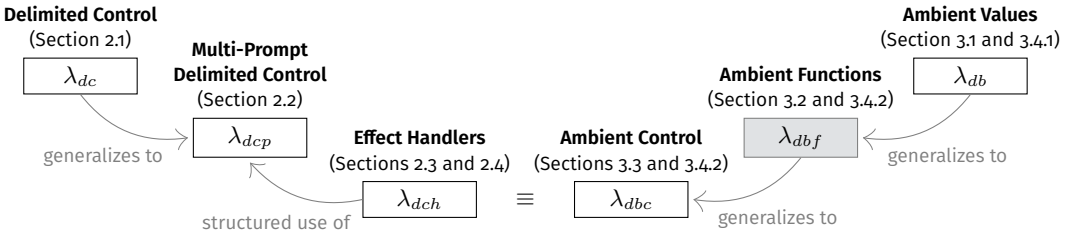

Figure 1.2. Overview of the different calculi presented in Chapters 2 and 3. The new feature of ambient values is highlighted.

Both features, delimited control and dynamic binding, have a somewhat bad reputation and naively combining them as two individual features comes with problems (Kiselyov et al., 2006). Nevertheless, as a type- and effect-safe combination of both delimited control *and* dynamic binding, effect handlers occupy a sweet spot in the design space between the two features. Both, the relation between delimited control and effect handlers (Forster et al., 2017), as well as the one between dynamic binding and effect handlers (Kammar and Pretnar, 2017) have been established previously. However, we believe that introducing effect handlers from dynamic binding offers programmers an alternative way to approach programming with effect handlers. Increasing expressiveness and conceptual complexity step-by-step, we introduce the three features ambient values, ambient functions, and ambient control. Each step highlights a different, important aspect of programming with effect handlers. Importantly, our systematic presentation helps us to discover the novel feature of *ambient functions*, which resides between dynamic binding and effect handlers. Ambient functions enable abstraction similar to effect handlers, but without modifying the control flow: They are dynamically bound, but statically evaluated.

Related publications The first part of this thesis is based on the following articles:

Jonathan Immanuel Brachthäuser, Philipp Schuster, and Klaus Ostermann. 2020. "Effekt: Capability-passing style for type- and effect-safe, extensible effect handlers in Scala". *Journal of Functional Programming, 30, E8*.

Jonathan Immanuel Brachthäuser and Daan Leijen. 2019. "Programming with Implicit Values, Functions, and Control". *Technical Report MSR-TR-2019-7*. Microsoft Research.

Contributions The first part of this thesis makes the following contributions:

- We systematically present two aspects of effect handlers. Once starting from (delimited) control operators (Chapter 2), and once starting from dynamic binding (Chapter 3). The two developments lead to two equivalent calculi (λ_{dch} and λ_{dbc}).
- While it has been previously established that effect handlers can express dynamic binding (Kammar and Pretnar, 2017), we observe that dynamic binding and effect handlers form a spectrum. We perform a step-wise generalization from ambient values (dynamic

binding) to ambient control (effect handlers). This leads us to a novel intermediate form of abstraction: ambient functions (Section 3.2).

– We present a language design that incorporates all three generalization steps (ambient values, functions, and control). We use the macro translation from ambients to effect handlers (Sections 3.5 and 3.6) to implement the language design as an extension of the programming language Koka (Leijen, 2017c).

– We present a type- and effect-system for ambient values, functions, and control (Section 3.4.2). Expressing ambient values and functions in terms of ambient control (Section 3.5) allows us to reuse existing soundness proofs (Sections 2.4 and 3.6).

1.1.2 Effect Handlers and Object-Oriented Programming

The second part of this thesis represents our main contribution. We present Effekt – a library design for embedding effect handlers in object-oriented languages. There are two important aspects of our library design.

Firstly, as opposed to many other effect libraries and language implementations, Effekt is not based on a free monad representation (Kiselyov and Ishii, 2015) but instead centers around the concept of explicit *capability-passing style* on top of a runtime system for multi-prompt delimited control (Hieb and Dybvig, 1990; Dybvig et al., 2007). That is, instead of searching for the correct handler implementation of an effect operation at runtime, we explicitly pass capabilities as additional arguments to effectful functions. Capability passing helps us to understand effects as imposing an additional requirement on the calling context, rather than a side effect that occurs in addition to computing the result. As we will see in Chapter 6, effects are thus contravariant.

Secondly, programming with effect handlers in Effekt *is* object-oriented programming. Our library design of Effekt is designed to integrate well with object-oriented programming languages and we directly map abstractions of effect handlers to abstractions in object-oriented programming. Effect signatures are *interfaces* declaring effect operations, effect handlers are *classes* implementing those interfaces and capabilities are *instances* of effect handlers. Effectful programs are written against the abstraction of effect signatures. Combining the paradigms not only enables programmers to use effect handlers in object-oriented programs, but also to use object-oriented programming abstractions to modularize effect handlers.

We evaluate our design in two implementations. A library implementation in Java (Gosling et al., 1996), which we call JavaEffekt (Section 5) and a library implementation in the programming language Scala, which we call ScalaEffekt (Sections 4 and 6). The two implementations both perform capability passing and rely on an implementation of multi-prompt delimited control. While the programming interfaces are similar, their implementations are quite different.

Our Java implementation consists of three core components: *(1)* A type selective continuation-passing style (CPS) transformation (Fischer, 1972; Reynolds, 1972) via JVM bytecode transformation, *(2)* an implementation of delimited continuations (Felleisen, 1988) on top of the bytecode transformation, and finally *(3)* a library for effect handlers in terms of delimited continuations. The CPS transformation allows us to implement the resumption behavior of effect handlers, while user programs can be written in direct style.

In contrast, our Scala implementation is based on a *monadic* implementation of delimited control (Dybvig et al., 2007). Consequently, users have to write effectful code in monadic style. Another important difference is that ScalaEffekt (as presented in Chapter 6) is effect-safe, while in JavaEffekt capabilities can leak, which might result in runtime errors.

Related publications The second part of the thesis is based on the following articles:

Jonathan Immanuel Brachthäuser and Philipp Schuster. 2017. "Effekt: Extensible Algebraic Effects in Scala (Short Paper)". In *Proceedings of the 8th ACM SIGPLAN International Symposium on Scala* (SCALA 2017). ACM, New York, NY, USA, 67-72.

Jonathan Immanuel Brachthäuser, Philipp Schuster, and Klaus Ostermann. 2020. "Effekt: Capability-passing style for type- and effect-safe, extensible effect handlers in Scala". *Journal of Functional Programming, 30, E8*.

Jonathan Immanuel Brachthäuser, Philipp Schuster, and Klaus Ostermann. 2018. "Effect Handlers for the Masses". *Proc. ACM Program. Lang.*, 2 (OOPSLA): 111:1–111:27.

Contributions The second part of this thesis makes the following contributions:

- We present the first library design of a type-safe embedding of effect handlers that revolves around object-oriented programming. The design is based on capability-passing style as an alternative to dynamic binding of effect handlers. In our library design, programming with effect handlers *is* object-oriented programming. Effect signatures correspond to interfaces; handlers correspond to implementations of the interfaces (Chapter 4).
- We implement the design in two libraries, one for Java (Chapter 5) and one for Scala (Chapters 4 and 6).
- For our implementation of `JavaEffekt`, we present an implementation of multi-prompt delimited continuations in Java. It uses trampolining and avoids the typical linear overhead of restoring the stack upon resumption common to all continuation libraries in Java that we are aware of (Section 5.2.2).
- We present a type-selective, signature preserving CPS transformation of JVM bytecode. We use closures introduced in Java 1.8 to create specialized instances of continuation frames. The general idea is applicable to any VM-based language that supports closure creation (Section 5.4).
- For our implementation of `ScalaEffekt`, we build on the operational semantics of Dybvig et al. (2007) but make it effect-safe (Section 6.1). We achieve effect safety by generalizing techniques of Launchbury and Sabry (1997) to nested regions (Kiselyov and Shan, 2008) but using intersection types of abstract type members instead of rank-2 types (Chapter 6).
- We study the extensibility gained by combining effect handlers with object orientation; in particular Scala's traits and mixin composition. We identify the *effect expression problem* as an instance of Wadler's (1998) expression problem and show how `ScalaEffekt` supports the required dimensions of extensibility (Section 4.4).
- We evaluate the performance of our Scala and Java implementations of `Effekt` (Section 5.6.2). We compare the performance of programs translated with our bytecode instrumentation with other Java bytecode instrumentations (Section 5.6).

1.2 List of Papers and Publications

The work on this thesis resulted in a number of paper submissions and publications at international conferences and workshops. Here, we briefly summarize the main contributions of each paper and discuss its relationship to the other papers and this thesis.

1.2.1 Effekt: Effect Handlers and Object-Oriented Programming

In the following articles, we presented different aspects of our library design.

Effect Handlers as a Monadic Library in Scala

> Jonathan Immanuel Brachthäuser and Philipp Schuster. 2017. "Effekt: Extensible Algebraic Effects in Scala (Short Paper)". In *Proceedings of the 8th ACM SIGPLAN International Symposium on Scala* (SCALA 2017). ACM, New York, NY, USA, 67-72.
> DOI: `https://doi.org/10.1145/3136000.3136007`

In this paper, we introduce a Scala library for programming with effect handlers. The library design is based on the concept of explicit capability-passing style. Viewing effect operations as constructors of an embedded domain specific language (Hudak, 1998), capability-passing style corresponds to a shallow embedding (Carette et al., 2007; Hofer et al., 2008). Taking this point of view, we describe the *effect expression problem* as a variant of Wadler's "Expression Problem" (Wadler, 1998). It is the first effect handler library to make extensive use of Scala's new feature of implicit function types (Odersky et al., 2017). Implicit function types allow us to reduce most of the syntactic overhead associated with capability passing. To support continuation capture, we implement our library on top of a monad for multi-prompt delimited control (Dybvig et al., 2007). Preliminary benchmarks suggest that capability passing offers significant performance improvements over an established Scala library (Torreborre, 2016), which is based on freer monads (Kiselyov and Ishii, 2015).

> Jonathan Immanuel Brachthäuser, Philipp Schuster, and Klaus Ostermann. 2020. "Effekt: Capability-passing style for type- and effect-safe, extensible effect handlers in Scala". *Journal of Functional Programming, 30, E8*. DOI: `https://doi.org/10.1017/S0956796820000027`

While passing first-class capabilities offers an interesting alternative to implement effect handlers, it does not guarantee effect safety. Effect handlers introduce capabilities, which are only valid in the dynamic scope of the effect handler. Since capabilities are first class, they can leave the scope of the handlers, which potentially results in a runtime exception. In this paper, we add an effect system to our library implementation to ensure that capabilities can only be used in the dynamic scope of the corresponding handler. The effect system is inspired by the one of Zhang and Myers (2019). In our Scala implementation of the embedded effect system, we use intersection types and path-dependent types to guarantee well scopedness of capabilities. Trying to run a program that uses a capability outside of its corresponding handler results in a Scala type error. Using the existing Scala type-system to express effect safety of our library

embedding has the advantage that we can reuse Scala's support for subtyping to express effect subtyping and Scala's support for polymorphism to express effect polymorphism.

Effect Handlers via Bytecode Transformation in Java

Jonathan Immanuel Brachthäuser, Philipp Schuster, and Klaus Ostermann. 2018. "Effect Handlers for the Masses". *Proc. ACM Program. Lang.*, 2 (OOPSLA): 111:1–111:27. DOI: `https://doi.org/10.1145/3276481`

In this paper, we evolve the original library design of ScalaEffekt to integrate better with Java. The efforts to integrate with Java result in a design, which directly maps effect signatures to interfaces, handlers to implementations, and capabilities to handler instances. While in ScalaEffekt, effectful user programs have to be written in monadic style, users of JavaEffekt write effectful programs in direct style. That is, programmers can use existing control-flow structures such as conditional branching and loops to describe effectful programs. To enable this, in the paper we introduce a type-selective, signature preserving CPS transformation of JVM bytecode. An implementation of multi-prompt delimited continuations on top of the instrumented bytecode allows capturing the continuation. The implementation performs trampolining and avoids the typical linear overhead associated with restoring the stack upon resumption. All continuation libraries in Java that we are aware of suffer from this linear overhead.

Effekt in this Thesis

The introduction of Chapter 2 is partially based on the second article (Brachthäuser et al., 2020).

Chapter 4 is based on the three articles (Brachthäuser and Schuster, 2017; Brachthäuser et al., 2018, 2020) and presents Effekt, a library design centered on capability passing. The library is designed to blend into object-oriented languages with interfaces and classes. Section 4.6.1 briefly discusses the use of implicits to remove the burden of passing capabilities (Brachthäuser and Schuster, 2017). Otherwise, in this thesis we mostly refrain from using the feature of implicit function types to make capability passing more explicit. Section 4.4 introduces the effect expression problem (Brachthäuser and Schuster, 2017) and describes different dimensions of extensibility, enabled by the design of Effekt.

Chapter 5 is closely based on the third mentioned publication (Brachthäuser et al., 2018) and presents the implementation of JavaEffekt and our CPS transformation. Compared to the publication, this thesis contains additional benchmark results. The results illustrate the linear overhead of other continuation libraries and provide further evidence for the difference in asymptotic complexity (Section 5.6)

Chapter 6 is closely based on the second mentioned article (Brachthäuser et al., 2020) and enhances the library with an embedded effect system. For a uniform presentation, and to facilitate comparison between the different stages of development, the library interfaces presented in Chapters 4 to 6 slightly differ from the original presentations in the corresponding articles.

1.2.2 From Dynamic Binding to Effect Handlers

Jonathan Immanuel Brachthäuser and Daan Leijen. 2019. "Programming with Implicit Values, Functions, and Control". *Technical Report MSR-TR-2019-7*. Microsoft Research.

It has been previously established that effect handlers can express dynamic binding (Kammar and Pretnar, 2017), that is, *dynamically scoped* variables.

In this paper, we take the opposite approach and start from dynamic binding. We step-wise generalize dynamic binding from *implicit values* to *implicit control*. While implicit values correspond to dynamic binding (Lewis et al., 2000; Kiselyov et al., 2006), implicit control closely corresponds to effect handlers (Plotkin and Pretnar, 2013).

We discover that dynamic binding and effect handlers form a spectrum and propose the novel feature of *implicit functions*, which (from an expressivity point-of-view) resides between the two other features. Like implicit values, implicit functions are bound dynamically. Like implicit control (and effect handlers), the body of implicit functions is evaluated in the lexical scope of its definition. We show how the small generalization from regular implicit values to implicit functions allows encapsulating control-effects at the definition site of the function, as opposed to leaking them (in the type and operationally) at the call site of the function.

We formalize the new feature as an extension to Moreau's calculus of dynamic binding (Moreau, 1998) and show how all three features are macro-expressible (Felleisen, 1990) in terms of effect handlers. Both, the design of a language with implicit functions, values, and control as well as the implementation on top of the Koka language (Leijen, 2017c) are the result of a PhD research internship at Microsoft Research, Redmond, USA. The formalization of the type- and effect systems in the article (Brachthäuser and Leijen, 2019) and the corresponding soundness proofs are largely due to Daan Leijen.

Ambient Functions in this Thesis

Chapter 3 is closely based on this paper. To match the most recent implementation in the Koka language (Leijen, 2017c), in Chapter 3 we renamed implicit values, functions, and control to *ambient values*, *ambient functions*, and *ambient control*, correspondingly. In the paper, we translate implicit (*i.e.*, ambient) values, functions and control to effect handlers. Instead, in Section 3.5 of this thesis we show how ambient values and functions can be macro expressed in terms of ambient control. We adjust the corresponding proofs of type- and semantics preservation. In this thesis, ambient control coincides exactly (syntactic differences notwithstanding) with the simplified effect handlers of Section 2.4. The type- and semantics preserving translation of ambient functions and values to ambient control (and effect handlers in extension) allows us to reuse the proof of semantics soundness for effect handlers (Section 2.4.1 which is adapted from Leijen (2017c)).

1.2.3 Software Modularity

Tillmann Rendel, Jonathan Immanuel Brachthäuser, and Klaus Ostermann. 2014. "From Object Algebras to Attribute Grammars". In *Proceedings of the 2014 ACM International Conference on Object-Oriented Programming Systems Languages & Applications*. (OOPSLA 2014). ACM, New York, NY, USA, 377-395.
DOI: `https://doi.org/10.1145/2660193.2660237`

In this paper, we analyze the relationship between object algebras (Oliveira and Cook, 2012) and attribute grammars (Knuth, 1968).

Many application domains require traversing tree-structured data. Amongst the most prominent examples are interpreters and compilers that traverse the abstract syntax tree to evaluate or translate the source program. There exists a multitude of techniques, based on the idea of separating the recursive traversal from the computation at each node. Examples are the visitor pattern (Gamma et al., 1995), folds (or "catamorphisms") (Meijer et al., 1991), shallow embeddings (Carette et al., 2007), and Church encodings (Barendregt, 1984, 1992). Object algebras (Oliveira and Cook, 2012; Oliveira et al., 2013) are a recent approach to structure tree traversals, similar in spirit to the aforementioned techniques. Programs written with object algebras can be structured in a pleasingly modular and extensible way. At their foundation, object algebras correspond to folds and the computation at a given node is expressed compositionally in terms of the results of the child nodes. This limits the class of programs that can be expressed in terms of object algebras.

In contrast, attribute grammars (Knuth, 1968) allow declarative specification of attributes as part of context-free grammars. Attributes on one node can be described in terms of attributes on other nodes, such as the parent node, child nodes, or sibling nodes. Attributes that can be computed bottom-up are called *synthesized*; attributes that are computed top-down are called *inherited*. Grammars with only synthesized attributes are called S-attributed (Knuth, 1968; Lewis et al., 1974).

In the paper, we show that object algebras as presented by Oliveira and Cook (2012) can be seen as a Church encoding of S-attributed grammars. We extend the expressiveness of object algebras in order to support traversals that correspond to richer classes of attribute grammars. We allow to additionally express dependencies (1) on attributes computed for child nodes, (2) on attributes computed for the current node, and (3) attributes computed on left siblings of the current node. With the latter, we are able to encode the full class of L-attributed grammars – a class that can express one-pass compilers. Implementing L-attributed grammars with object algebras has several advantages. Attributes can be defined and type-checked separately while dependencies between attributes are statically checked.

It is our prior work on this paper, which sparked the idea for shallow embeddings of effect handlers (Chapter 4). Effect handlers are often introduced as a *fold* over a computation tree (Kammar et al., 2013; Lindley, 2014). If handlers are folds, can we express them as a shallow embedding to inherit the associated modularity and extensibility benefits? The idea of shallowly embedding effect handlers resulted in explicit capability-passing style, which is an important aspect of the `Effekt` design. In Section 4.4, we study the extensibility properties that `Effekt` inherits from shallow embeddings.

Jonathan Immanuel Brachthäuser, Paolo G. Giarrusso, Tillmann Rendel, and Klaus Ostermann. "The Co-Expression Problem". Unpublished. 2016.

Programs that traverse tree-shaped data often suffer from the expression problem (Wadler, 1998), an important and well-studied problem in software engineering. The expression problem describes the difficulties of extending a recursive data type with new variants, while at the same time being able to modularly add new functions over the data type. With the notable exception of Lämmel and Rypacek (2008), the expression problem is often described only informally in the literature. Following Lämmel and Rypacek, in the paper, we formally model the expression problem using category theory. We then introduce the *co-expression problem* by applying categorical duality. While the expression problem centers on extending the least fixed point of a sum type and recursive functions consuming it — the co-expression problem is concerned with extending the greatest fixed point of a product type and co-recursive functions producing it.

In our papers on `Effekt` (Brachthäuser et al. 2017, 2018, 2020) and in Section 4.5.2 of this thesis, we use the intuition gained through the work on the (co-)expression problem to formulate the *effect expression problem* – a variant of the expression problem in the context of effect handlers.

Jonathan Immanuel Brachthäuser, Tillmann Rendel, and Klaus Ostermann. 2016. "Parsing with First-Class Derivatives". In *Proceedings of the 2016 ACM SIGPLAN International Conference on Object-Oriented Programming, Systems, Languages, and Applications* (OOPSLA 2016). ACM, New York, NY, USA, 588-606.
DOI: `https://doi.org/10.1145/2983990.2984026`

Brzozowski derivatives (Brzozowski, 1964) are a well-known concept in the context of regular expressions. Recently, they have been rediscovered to give a simplified explanation to parsers of context-free languages (Danielsson, 2010; Might et al., 2011; Adams et al., 2016). Might et al. use the derivative of a parser as part of the *implementation*. In this paper, we add derivatives as a novel first-class feature to the *interface* of a standard parser combinator language. First-class derivatives enable an inversion of the control flow. This allows modularly implementing parsers for languages that previously required separate pre-processing steps or crosscutting modifications of the involved parsers. This opens up new opportunities for reuse and supports a modular and declarative specification of layout-sensitive parsers.

While at first parsing and effect handlers might seem like two very different fields, there are interesting connections between the two. Firstly, it has been shown that effect handlers can be used to implement parsers (Leijen, 2016). In Section 4.6, we will briefly revisit this approach and see how grammars can be expressed in terms of effect operations, and how the semantics of parsers is implemented as handlers for the effect operations. In general, by varying the effect handlers, we can freely choose between different parsing strategies: Depth-first or breadth-first parsing, computing only the first result or enumerating all results. Secondly, as observed by Kiselyov (2007), delimited control (and effect handlers in extension) can be used to invert the control flow and to incrementally provide the parser with input. This way we can implement online (that is, streaming) parsers (Swierstra, 2009), or use the results of our paper to add first-class derivatives to a parser combinator language.

1.2.4 Efficient Compilation of Effect Handlers

Philipp Schuster and Jonathan Immanuel Brachthäuser. 2018. "Typing, Representing, and Abstracting Control". In *Proceedings of the 3rd ACM SIGPLAN International Workshop on Type-Driven Development* (TyDe 2018). ACM, New York, NY, USA, 14-24.
DOI: `https://doi.org/10.1145/3240719.3241788`

Despite existing optimization efforts (Pretnar et al., 2017; Leijen, 2017b; Dolan et al., 2015), to the current day, the abstraction of effect handlers still comes with a significant runtime cost that is associated with searching the correct handlers and capturing the continuation. A well-known translation strategy for delimited control in general (Danvy and Filinski, 1990), and for effect handlers in particular (Hillerström et al., 2017; Leijen, 2017c) is to compile to continuation-passing style. As a first step towards efficient compilation of effect handlers, in this paper, we combine the results of Danvy and Filinski (1990) and Danvy and Filinski (1992) and present a compilation strategy for multiple level of delimited control. We implement the translation of terms *and* of types in Idris. Our translation preserves typability by intrinsically typing both source and target language. We additionally index the type of effectful expressions by a list of *answer types*. This indexing serves two purposes: (1) it restricts the contexts in which this expression can be used and guarantees effect safety, and (2) it drives our type-directed compilation process. Each entry in the list of answer types results in one iteration of the CPS translation. We generalize standard techniques (Danvy and Filinski, 1992) to the iterated translation, in order to avoid multiple classes of administrative beta and eta-redexes.

Philipp Schuster, Jonathan Immanuel Brachthäuser, and Klaus Ostermann. 2020. "Compiling Effect Handlers in Capability-Passing Style". To appear in *Proceedings of the 25th ACM SIGPLAN International Conference on Functional Programming*.

Building on the results of this thesis, in the paper we generalize the techniques of the previous paper from delimited control to effect handlers. In particular, like in `Effekt`, we employ explicit capability-passing style. We present a calculus that guarantees full elimination of redexes associated with handler abstractions. To achieve this, we restrict capabilities to be second-class, which prevents them from being returned from functions. Passing capabilities at compile-time allows us to always specialize effectful programs to the corresponding handler implementations. While the inlining guarantees are powerful, our proposed solution comes with restrictions. Handlers are second-class, which implies that programmers cannot abstract over handlers at runtime.

1.3 Structure of the Thesis

The remainder of this thesis is structured according to the aforementioned two parts. The first part provides the necessary overview over delimited control and dynamic binding and relates both to effect handlers. The second part describes the library design of `Effekt` and discusses new modularization opportunities enabled by combing effect handlers with object-oriented programming. Chapter 7 discusses our approach, sketches potential future work and concludes.

Part I.

Effect Handlers in Perspective

Chapter 2

From Delimited Control to Effect Handlers

In this chapter, we follow historical developments in the literature and first generalize control effects to delimited control to then generalize from one control operator to a family of control operators. We finally introduce effect handlers by shifting the perspective on multi-prompt delimited continuations (Hieb et al., 1994; Dybvig et al., 2007): With effect handlers the implementation of an effect operation is localized at the handler, not at the effect operation (Sitaram, 1993).

Neither the overview, nor the generalizations bear much technical novelty and have been given in similar form by Shan (2004b) and Dybvig et al. (2007). Furthermore, the relation between effect handlers and control operators has been studied formally by Forster et al. (2017). However, presenting the different control operators in the context of effect handlers helps us to discuss modularity properties of the various control operators and effect handlers. Furthermore, this background chapter serves two purposes: firstly, it establishes the necessary vocabulary and concepts. The different calculi presented in this chapter serve as a point of reference for the rest of this thesis. In particular, we use the variant of multi-prompt delimited control to implement our effect handler libraries in the second part of this thesis. Secondly, it highlights that programming with effect handlers can be seen as structured programming with delimited control (Kammar et al., 2013, pp.150-151). This point of view is the foundation for explicit capability-passing style, established in later chapters.

Cartwright and Felleisen describe an effect as

> "a closed expression that [...] is not a value, and cannot be reduced to a value without affecting the context." — *Cartwright and Felleisen (1994, p. 248)*

As opposed to *pure* expressions, which can be reduced to a value without regarding the context (assuming well-typed expressions and neglecting non-termination), the evaluation of *effectful* expressions might depend on or even modify the context. While in general, the context can include resources like the heap, the file system, or I/O, in this work we focus on the syntactic evaluation context (Wright and Felleisen, 1994). We refer to effects that affect the evaluation context also more specifically as *control effects* and to built-in or user-defined operations that have control effects as *control operators* (Felleisen and Friedman, 1986).

Parts of this chapter are based on the following article: Jonathan Immanuel Brachthäuser, Philipp Schuster, and Klaus Ostermann. 2020. "Effekt: Capability-passing Style for Type- and Effect-safe, Extensible Effect Handlers in Scala". *Journal of Functional Programming, 30, E8*. DOI: https://doi.org/10.1017/S0956796820000027
The presentation of the type- and effect system in Section 2.4, appeared in similar form in the publication: Jonathan Immanuel Brachthäuser and Daan Leijen. "Programming with Implicit Values, Functions, and Control". *Technical Report MSR-TR-2019-7*. Microsoft Research, 2019.

For multiple decades, control operators like `call/cc` have been used to program with control effects (Friedman et al., 1984). Similar to `goto`, which can be understood as undelimited, local continuation (Landin, 1965; Kennedy, 2007), `call/cc` captures the undelimited global continuation, i.e., the entire future of the program execution. We will shortly see an example, but abstractly, invoking `call/cc`, captures the continuation, reifies it as a first-class value, and makes it available to the programmer. When the continuation is called, execution proceeds at the very point where `call/cc` was originally called (Haynes and Friedman, 1987). Such a captured continuation is undelimited, since it spans the entire future, not just some parts of it. It is global in the sense that it potentially spans across procedure boundaries.

Recently, in disguise, control effects have found their way into mainstream programming languages in the form of specialized solutions such as async/await, fibers, coroutines, generators and others. While many of these programming language constructs can be implemented in terms of other control effects (Wand, 1980; Haynes et al., 1986), programming language designers often refrained from directly offering control operators like `call/cc` to the users. And rightfully so! Like `goto`, while being very general and expressive, programs written with the control operator `call/cc` tend to be fragile, hard to understand, and difficult to maintain. In contrast, each of the programming language constructs listed above focuses on one particular problem domain. Each offers domain terminology, potentially specialized typing rules and semantics, seemingly reducing the complexity for the user of the programming language. However, the specialization to one particular solution comes with a price: Each feature is typically implemented individually in the compiler or runtime. Even worse, combining multiple features in one programming language often leads to interaction of the features, again increasing the complexity of the language for both the user and the implementer.

Also recently, the programming languages research community found new interest in control effects in the form of algebraic effects (Plotkin and Power, 2003) and their extension with handlers (Plotkin and Pretnar, 2009, 2013). Effect handlers occupy a sweet spot between the very general control operators (such as `call/cc`) and specialized programming language features (such as `async`/`await`). Like general control operators, effect handlers are powerful and can express many of the language features above as user-defined libraries (Dolan et al., 2015, 2017; Leijen, 2017a). However, unlike general control operators, effect handlers also encourage modularity by separating programs into *effect signatures* (interfaces listing the available effect operations), *effectful programs* written against those signatures, and *effect handlers* implementing the effect signatures thereby providing semantics to the effect operations (Kammar et al., 2013). This separation improves modular reasoning, simplifies type checking, and enables well-defined composition of programs using different effects.

The connection between effect handlers and delimited control is not accidental. It has been established practically (Kiselyov and Sivaramakrishnan, 2016, 2018) as well as theoretically (Forster et al., 2017; Piróg et al., 2019) that certain forms of delimited continuations can express certain forms of algebraic effect handlers. In the literature, effect handlers are sometimes introduced as a structured way to programming with delimited continuations (Kammar et al., 2013; Leijen, 2017c).

> "Effect handlers are to delimited continuations as structured programming is to goto." — *Andrej Bauer (Dagstuhl Seminar 18172, March 2018)*

Viewing programming with effect handlers as structured way of programming with delimited continuations has merits on its own.

In particular, we believe the regained interest in control operators comes from four important generalizations and improvements effect handlers make over `call/cc`:

1. Generalizing from undelimited to delimited continuations
2. Generalizing from one control operator to a family of control operators
3. Establishing answer type safety of control operators
4. Establishing effect safety of control operators

From an engineer's perspective, each of these improvements helps to write programs in a modular way, making them easier to extend, and making it easier to reason about parts of a program in isolation. While effect handlers can be seen as a revival of structured programming with delimited control operators, there exists an alternative point of view we want to emphasize: effect handlers generalize dynamic binding (Chapter 3). Hence, effect handlers encapsulate two powerful (but often frowned upon) features behind a single uniform abstraction.

Delimited control
Effect handlers can capture the (delimited) continuation when handling effect operations – effect operations are thus related to control operators and effect handlers are related to control delimiters (Kammar et al., 2013).

Dynamic binding
Effect operations are dynamically bound (or *dynamically scoped* (Moreau, 1998)) – effect operations are thus related to dynamically bound (effectful) functions and effect handlers are related to binders of such dynamically scoped (Kammar and Pretnar, 2017).

In this chapter, we follow the above generalizations in order to establish important terminology and to better understand the relation between effect handlers and delimited control. We review the design space of delimited control, explore different variants of delimited control operators, generalize from one control operator to families of control operators, to finally identify one particular variant (`spawn` / `controller`) that most closely matches the operational semantics of effect handlers in our implementations of the second part of this thesis.

2.1 Delimiting Control

The control operator `call/cc` (as present in some Scheme dialects) captures the entire continuation of the program.

Example

$$\mathsf{print}((1 \ + \ \mathsf{callcc} \ \{ \ k \Rightarrow k(10); \ k(20) \ \}) \ * \ 2)$$

In this example, reducing callcc captures the entire rest of the program execution as continuation k. The continuation corresponds to the evaluation context $\mathsf{E} = \mathsf{print}((1 \ + \ \square) \ * \ 2)$. Invoking the continuation with $k(10)$ aborts the computation and replaces the current evaluation context with E. The example thus evaluates to $\mathsf{E}[10]$ and running it prints the number 22. We can also understand callcc as a labeling instruction and calling the continuation $k(10)$ as a jump to that label with the given argument. This analogy to jumps also makes it immediate that calling the continuation *discards* the current continuation (that is, the second call $k(20)$) and replaces it with $\mathsf{print}((1 \ + \ 10) \ * \ 2)$. The call to $k(10)$ never returns.

A first step towards effect handlers is to allow users to delimit the extent to which the continuation is being captured. In variations, such a *delimited continuation* (Felleisen, 1988) is also referred to in literature as subcontinuation (Hieb et al., 1994) or partial continuation (Johnson and Duggan, 1988). Since their introduction by Felleisen, different variants of delimited control operators have been introduced – maybe the most prominent example being shift / reset (Danvy and Filinski, 1992). Using shift, we can write a variation of the above example as:

$$\mathsf{print}(\mathsf{reset}\ \{\ (1 + \ \mathsf{shift}\ \{\ k \Rightarrow k(3)\ +\ k(4)\ \}\ )\ *\ 2\ \})$$

Now the continuation (highlighted in grey) corresponds to the evaluation context $(1 + \square) * 2$. Its extent is *delimited* by reset and thus does not include the call to print. The example will print the number $((1 + 3) * 2) + ((1 + 4) * 2) = 18$. Importantly, in contrast to the example with `call/cc`, the call to the delimited continuation $k(3)$ *does* return with the value 8.

2.1.1 Four Variants of Delimited Control

Figure 2.1 formally captures the operational semantics of delimited control. It defines the syntax and semantics of a calculus for a call-by-value language with delimited control λ_{dc}. The style we present the calculi in this and the following chapter is adapted from Leijen (2017c), who in turn uses the style of Wright and Felleisen (1994). The syntax includes the usual forms for lambda abstraction ($\lambda x.e$) and application ($e(e)$), variables (x) and constants (c). In addition, it also includes the two special forms for capturing the continuation (shift) and delimiting the extent of the captured continuation (reset). In addition to the standard abbreviations for curried function definition and application, we use the following abbreviations:

$$\begin{array}{lcl} \mathsf{val}\ x\ =\ e_1;\ e_2 & \doteq & (\lambda x.\ e_2)(e_1) \\ \mathsf{def}\ f(x)\ =\ e_1;\ e_2 & \doteq & \mathsf{val}\ f\ =\ \lambda x.\ e_1;\ e_2 \\ e_1;\ e_2 & \doteq & \mathsf{val}\ \_\ =\ e_1;\ e_2 \end{array}$$

Akin to blocks in C-style languages, we sometimes use braces (instead of parenthesis) to group sequenced expressions (*i.e.*, $\{\ e_1;\ \ldots;\ e_i;\ \ldots;\ e_n\ \}$) in definitions.

To reduce an expression to a value, the expression is decomposed into an evaluation E context and a redex following the rules for forming evaluation contexts of Figure 2.1. We use $e[x \mapsto v]$ to denote capture free substitution of x by v in e. All *pure expressions* not involving the control operator shift are completely standard and can be reduced independently of the evaluation context. We use $e \longmapsto e'$ to denote reduction in an evaluation context E (rule Cong) and $e \longrightarrow^* e'$ for the reflexive, transitive closure of our reduction relation.

$$(\pm \mathit{shift} \pm) \quad \mathsf{reset}\ \{\ \mathsf{H}[\mathsf{shift}\ \{k \Rightarrow e\}]\ \} \longrightarrow \underbrace{\mathsf{reset}\ \{\ e[k \mapsto \lambda x.\ \overbrace{\mathsf{reset}\ \{\ \mathsf{H}[x]\ \}}^{\text{inner delimiter}}\ ]\ \}}_{\text{outer delimiter}}$$

The reduction rule for shift makes use of a special evaluation context H, which we call a *capture context*. Like the context E the capture context is a call-by-value context, but does not include any reset-frames. To reduce a shift, we thus capture the context delimited by the *closest dynamically surrounding* reset, reify it as a function, and bind it to k. The rule ($\pm \mathit{shift} \pm$) leaves us with interesting design choices: As summarized by Dybvig et al. (2007) and Shan (2004b),

Syntax:

Expressions	e	$::=$	$e(e)$	application
		$\mid$	$\mathsf{reset}\ \{\ e\ \}$	delimiting control effects
		$\mid$	$\mathsf{shift}\ \{\ k \Rightarrow e\ \}$	continuation capture
		$\mid$	v	value
Values	v	$::=$	$x \mid c \mid \lambda x.\ e$	

Evaluation Contexts:

$$\mathsf{E} ::= \square \mid \mathsf{E}(e) \mid v(\mathsf{E}) \mid \mathsf{reset}\ \{\ \mathsf{E}\ \}$$
$$\mathsf{H} ::= \square \mid \mathsf{H}(e) \mid v(\mathsf{H}) \mid$$

Operational Semantics:

$$\begin{array}{llll} (\delta) & c(v) & \longrightarrow \delta(c, v) & \text{if } \delta(c, v) \text{ is defined} \\ (\beta) & (\lambda x.\ e)(v) & \longrightarrow e[x \mapsto v] & \\ (\mathit{reset}) & \mathsf{reset}\ \{\ v\ \} & \longrightarrow v & \end{array} \qquad \frac{e \longrightarrow e'}{\mathsf{E}[e] \longmapsto \mathsf{E}[e']}\ [\textsc{cong}]$$

Figure 2.1. Syntax and semantics of delimited control (λ_{dc}). Reduction rule (*shift*) is defined in-text.

control operators can be characterized by whether or not they leave the (outer) delimiter behind and whether or not the body of the captured continuation is enclosed by the (inner) delimiter. That is, we can choose between four different variants of the rule ($\pm\mathit{shift}\pm$), each corresponding to a different control operator introduced in the literature (Dybvig et al., 2007):

$(+\mathit{shift}-)$ $\mathsf{reset}\ \{\ \mathsf{H}[\mathsf{shift}\ \{\ k \Rightarrow e\ \}]\ \} \longrightarrow \mathsf{reset}\ \{\ e[k \mapsto \lambda x.\ \mathsf{H}[x]\ ]\ \}$

Leave the outer delimiter behind, but do not include the inner delimiter in the continuation. This semantics corresponds to the control operator F by Felleisen (1988) also referred to as $\mathsf{control}$ in later publications.

$(-\mathit{shift}-)$ $\mathsf{reset}\ \{\ \mathsf{H}[\mathsf{shift}\ \{\ k \Rightarrow e\ \}]\ \} \longrightarrow e[k \mapsto \lambda x.\ \mathsf{H}[x]\ ]$

Neither leave the delimiter behind, nor contain it in the continuation. Shan (2004b) refers to this variant of $\mathsf{control}$ as $\mathsf{control}_0$.

$(-\mathit{shift}+)$ $\mathsf{reset}\ \{\ \mathsf{H}[\mathsf{shift}\ \{\ k \Rightarrow e\ \}]\ \} \longrightarrow e[k \mapsto \lambda x.\ \mathsf{reset}\ \{\ \mathsf{H}[x]\ \}\ ]$

Do not leave the outer delimiter behind but include the inner delimiter in the continuation. This corresponds to the control operator shift_0 by Danvy and Filinski (1989, Appendix C).

$(+\mathit{shift}+)$ $\mathsf{reset}\ \{\ \mathsf{H}[\mathsf{shift}\ \{\ k \Rightarrow e\ \}]\ \} \longrightarrow \mathsf{reset}\ \{\ e[k \mapsto \lambda x.\ \mathsf{reset}\ \{\ \mathsf{H}[x]\ \}\ ]\ \}$

Both leave the delimiter behind and contain the delimiter in the continuation. This semantics corresponds to shift as introduced by Danvy and Filinski (1989, 1992)).

We use ($\pm shift\pm$) as an abbreviation for the choice between the four different reduction rules ($+shift-$), ($-shift-$), ($-shift+$), and ($+shift+$). This classification of control operators has been presented by Dybvig et al. (2007), but we change their notation from $\pm\mathsf{F}\pm$ to $\pm shift\pm$. Since we can instantiate it with four rules for ($\pm shift\pm$), we also refer to λ_{dc} as a *calculus schema*.

Delimiters and Modular Reasoning

The choice of the evaluation semantics $\pm shift\pm$ has no influence on our particular example above. However, it makes significant difference in non-trivial programs and has implications on modular reasoning:

The outer delimiter Leaving the outer delimiter behind, rule ($+shift\pm$) guarantees encapsulation of control effects. That is, reset { e } is guaranteed to be a pure term and thus only has "a single exit point" (Felleisen, 1988, p. 186). For a terminating expression e, the code following reset { e } will be executed exactly once. This property is important to reason about resource safety in presence of control effects (Felleisen, 1988). Take the following example:

$$\mathsf{val}\ \mathit{file}\ =\ \mathit{open}(\texttt{"my-file.txt"});\ \mathsf{reset}\ \{\ \mathit{action}()\ \};\ \mathit{close}(\mathit{file})$$

No matter how *action* in the above example uses control effects; the $+shift\pm$ variants guarantee that the file will eventually be closed, given `action` terminates.

The inner delimiter Similarly, delimiting the body of the continuation ($\pm shift+$) guarantees that all subsequent calls to shift are conceptually delimited by the *same* reset. This is illustrated by the following example[2] adapted from Danvy and Filinski (1990):

$$\begin{array}{l}\mathsf{def}\ \mathit{flip}()\ =\ \mathsf{shift}\ \{\ k \Rightarrow k(\mathsf{true})\ +\!\!+\ k(\mathsf{false})\ \};\\ \mathsf{reset}\ \{\ \mathrm{List}(\mathit{flip}()\ ||\ \mathit{flip}())\ \}\end{array}$$

Here, we use the control operator shift to express an ambiguity effect. For this example, both $\pm shift+$ variants result in the list List(true, true, true, false). Delimiting the body of the continuation is essential for type-safety of the example. Both calls to k return a list and *flip* returns a boolean. This is not the case for the variant $-shift-$, which would make this example ill typed. After reducing the first shift with rule ($-shift-$) the example corresponds to:

$$\begin{array}{l}\mathsf{def}\ k(x)\ =\ \{\\ \quad \mathsf{def}\ \mathit{flip}()\ =\ \mathsf{shift}\ \{\ k \Rightarrow k(\mathsf{true})\ +\!\!+\ k(\mathsf{false})\ \};\\ \quad \mathrm{List}(x\ ||\ \mathit{flip}()\ )\\ \};\\ k(\mathsf{true})\ +\!\!+\ k(\mathsf{false})\end{array}$$

The marked call to *flip* will result in a stuck term, since the body of *flip* cannot be reduced without a corresponding delimiter. In contrast, evaluating the example with rule $+shift-$ will

[2] We use List(e_1, ..., e_n) to denote the creation of a list with n elements and the infix operator $+\!\!+$ for list concatenation. We use the infix operator $x\ ||\ y$ to express strict boolean disjunction.

diverge. Again, after one step of reducing shift with rule ($+shift-$) we obtain:

```
reset {
  def k(x) = {
    def flip() = shift { k ⇒ k(true) ++ k(false) };
    List(x || flip())
  };
  k(true) ++ k(false)
}
```

This time the second call to *flip will* be delimited. However, the captured continuation also contains the highlighted call to the first captured continuation, which in turn will call *flip* again, and again.

Multiple effects with $-shift+$ As it has already been observed by Danvy and Filinski (1989, Appendix C), the $-shift+$ variant (also called shift_0) allows to access outer contexts by shifting multiple times within the body of the shift. For example we can add a second effect operation *raise* to our example:

```
def raise(msg) = shift { k_flip ⇒ shift { k_raise ⇒ None } };
def flip()     = shift { k_flip ⇒ k_flip(true) ++ k_flip(false) };
def action()   = ...
reset { Some(reset { List(action()) }) }
```

Here *action* can make use of both *raise* and *flip*. In case of *raise* we immediately abort with None. Compare this to

```
def raise(msg) = shift { k_raise ⇒ None };
def flip()     = shift { k_raise ⇒ shift { k_flip ⇒ k_flip(k_raise(true)) ++ k_flip(k_raise(false)) } };
def action()   = ...
reset { List(reset { Some(action()) }) }
```

which enumerates all possible outcomes of *flip* in a depth-first manner. An exception terminates the search in the current execution branch and potentially continues execution in another branch. We can define *action* to model a drunk trying to flip a coin (Kammar et al., 2013):

```
def action() =
  if (!flip()) raise("dropped it")
  else if (flip()) "heads" else "tails"
```

the first interpretation results in None, while the second one yields:

- List(Some("heads"), Some("tails"), None)

By assigning a number of shifts to each effect, programs can simultaneously use different effects (exceptions and ambiguity in this example). It is this variant of delimited control, that is semantically closest to algebraic effect handlers (Kammar et al., 2013; Forster et al., 2017).

To summarize the comparison, the semantics of $-shift-$ is very general since we can easily express all other forms by manually inserting calls to reset; the $+shift\pm$ variants enable effect encapsulation and reasoning about external, linear resources; the $\pm shift+$ variants guarantee that all subsequent effects are confined to the call of the continuation. The $-shift+$ variant allows users to combine multiple effects, albeit statically fixing the ordering of effects.

	One Control Operator		Family of Control Operators	
$-shift-$	$\mathsf{control}_0$	Shan (2004b)	cupto	Gunter et al. (1995)
			pushSubCont	Dybvig et al. (2007)
$+shift-$	F	Felleisen (1988)	control	Sitaram and Felleisen (1990)
$-shift+$	shift_0	Danvy and Filinski (1989, Appx. C)	*controller*	Hieb and Dybvig (1990)
$+shift+$	shift	Danvy and Filinski (1989, 1992)	ξ_n	Danvy and Filinski (1990)

Figure 2.2. Comparison of different control operators in the literature.

2.2 Families of Delimited Control Operators

The semantics of $-shift+$ allowed us to express multiple effects, such as exceptions and ambiguity, while at the same time it guaranteed that using the effects together in the same program is well-defined. However, as we have seen in the example above, the order of effects is encoded in the number of required shifts and thus statically fixed.

To overcome this issue and to express multiple effects more robustly, Sitaram and Felleisen (1990), Danvy and Filinski (1990), and Hieb and Dybvig (1990) almost simultaneously each generalized a different variant of delimited control from one control operator to a *family of control operators*. Figure 2.2 organizes the relevant related work, grouping by the semantics in terms of $\pm shift\pm$ and by the generalization to families of control operators. While details of the approaches differ, conceptually the control operator shift and its delimiter are generalized to a family of control operators shift_p (and reset_p correspondingly) indexed by what we refer to as a *prompt* p (adopting the terminology of Felleisen (1988)). Each control operator for one particular p can be used to express a different effect without risking interference between different control operators.

Figure 2.3 follows Gunter et al. (1995) to capture this indexing formally. We extend and generalize the calculus schema λ_{dc} to λ_{dcp} (pronounced "delimited control with prompts"). In the syntax we replace the control operator shift by its indexed counterpart shift_p and reset by reset_p, accordingly. We extend values with the set of prompts $p \in \mathbb{P}$ and include an operation new_prompt (Gunter et al., 1995; Dybvig et al., 2007) to create fresh prompts. Every prompt p gives rise to a control operator shift_p and its matching delimiter reset_p. Like in λ_{dc}, the reduction rule ($\pm shift\pm$) is a rule scheme and offers four different choices for the operational semantics.

We also adjust the reduction rules accordingly. The most important change compared to the semantics in Figure 2.1 is that now the capture context H_p is also indexed (Gunter et al., 1995). Just like previously omitting reset altogether in H, the side condition ($p \neq p'$) asserts that the innermost reset delimits the extent of the captured continuation. However, now the continuation *can* contain delimiters as long as they are indexed by a different prompt. The continuation captured by shift_p represented by the capture context H_p is thus delimited by the closest reset for the very prompt p.

Four variants of delimited control As in the calculus schema λ_{dc}, we again offer four variations of reduction rule ($\pm shift\pm$). The discussion on tradeoffs of the different $\pm shift\pm$ variants carries over from the single-prompt setting to the multi-prompt setting. However, there is one important difference. Considering the $+shift\pm$ variants, a delimited expression $\mathsf{reset}_p\ \{\ e\ \}$ now is only pure with respect to one particular prompt p. In consequence, we lose the guaranty of the single-prompt setting, where $\mathsf{reset}_p\ \{\ e\ \}$ would return exactly once. This is not the case in the

Syntax:

$$
\begin{array}{llll}
\text{Expressions} & e & ::= & \ldots \\
 & & \mid & \mathsf{reset}_p\ \{\ e\ \} \qquad \text{delimiting control effects} \\
 & & \mid & \mathsf{shift}_p\ \{\ k \Rightarrow e\ \} \qquad \text{continuation capture} \\
 & & \mid & \mathsf{new\_prompt} \qquad \text{creating a fresh prompt} \\
\\
\text{Values} & v & ::= & \ldots \mid p
\end{array}
$$

Evaluation Contexts:

$$
\begin{array}{l}
\mathsf{E} \ ::= \square \mid \mathsf{E}(e) \mid v(\mathsf{E}) \mid \mathsf{reset}_p\ \{\ \mathsf{E}\ \} \\
\mathsf{H}_p ::= \square \mid \mathsf{H}_p(e) \mid v(\mathsf{H}_p) \mid \mathsf{reset}_{p'}\ \{\ \mathsf{H}_p\ \} \quad \text{if } p \neq p'
\end{array}
$$

Operational Semantics:

$$
\begin{array}{lllll}
\ldots & & & & \\
(\mathit{fresh}) & \mathsf{new\_prompt} & \longrightarrow & p\ \mathit{fresh} & \\
(\mathit{reset}) & \mathsf{reset}_p\ \{\ v\ \} & \longrightarrow & v & \\
(\pm\mathit{shift}\pm) & \mathsf{reset}_p\ \{\ \mathsf{H}_p[\mathsf{shift}_p\ \{\ k \Rightarrow e\ \}]\ \} & \longrightarrow & \mathsf{reset}_p\ \{\ e[k \mapsto \lambda x.\ \mathsf{reset}_p\ \{\ \mathsf{H}_p[x]\ \}\ ]\ \} &
\end{array}
$$

Figure 2.3. Generalization of syntax and semantics to families of control-operators (λ_{dcp}).

multi-prompt setting. Other control effects (with $p' \neq p$) can still capture $\mathsf{reset}_p\ \{\ e\ \}$ (and expressions following it) as part of the continuation. The following example illustrates that generalizing to multiple prompts invalidates any linearity assumptions:

$$\mathsf{reset}_{p'}\ \{\ \mathsf{val}\ \mathit{file}\ =\ \mathit{open}(\texttt{"my-file.txt"});\ \mathsf{reset}_p\ \{\ \mathit{action}()\ \};\ \mathit{close}(\mathit{file})\ \}$$

We can now assume *action* to be defined as $\mathsf{shift}_{p'}\ \{\ k \Rightarrow 0\ \}$, which discards the continuation k and aborts the computation up to $\mathsf{reset}_{p'}$. In consequence, $\mathsf{reset}_p\ \{\ \mathit{action}()\ \}$ aborts and the file will be opened, but not closed.

Multiple prompts give rise to multiple effects Choosing one of the variants $\pm\mathit{shift}+$, we can recast our *drunkFlip* example from the last section to use prompts to distinguish different effects:

$$
\begin{array}{l}
\mathsf{val}\ \mathit{exc}\ =\ \mathsf{new\_prompt} \\
\mathsf{val}\ \mathit{amb}\ =\ \mathsf{new\_prompt} \\
\mathsf{def}\ \mathit{raise}(\mathit{msg})\ =\ \mathsf{shift}_{\mathit{exc}}\ \{\ k \Rightarrow \mathrm{None}\ \} \\
\mathsf{def}\ \mathit{flip}()\ \ =\ \mathsf{shift}_{\mathit{amb}}\ \{\ k \Rightarrow k(\mathsf{true})\ +\!\!+\ k(\mathsf{false})\ \} \\
\mathsf{def}\ \mathit{action}()\ \ =\ \ldots \\
\mathsf{reset}_{\mathit{exc}}\ \{\ \mathrm{Some}(\mathsf{reset}_{\mathit{amb}}\ \{\ \mathrm{List}(\mathit{action}())\ \})\ \}
\end{array}
$$

To reorder the effects, with prompts, we can now keep the definitions of *flip* and *raise* unchanged. We just need to rearrange the order of the resets to:

$$\mathsf{reset}_{\mathit{amb}}\ \{\ \mathrm{List}(\mathsf{reset}_{\mathit{exc}}\ \{\ \mathrm{Some}(\ \ \mathit{action}()\ \ )\ \})\ \}$$

This demonstrates the flexibility gained from generalizing to multiple prompts. While before, we could express multiple effects with shift_0, we needed to statically assign a number of shifts to each effect and thus fix the order of effects upfront. With multiple prompts, this choice is deferred to the point of reset. In particular, the implementation of effect operations is independent of the ordering.

2.2.1 On the Choice of Prompts

So far, when generalizing to multiple prompts we have assumed that there exists a set of prompts $\mathbb{P}$ whose elements can be used to index the control operators. Different approaches in the literature not only vary in the choice of $\pm\mathit{shift}_{\pm}$ but also in their treatment of prompts. Some approaches treat prompts as first-class values, while others do not. Some include an explicit new_prompt operation, whereas others make prompt creation implicit or assume a globally static set of prompts. The exact treatment of prompts is important. This can easily be seen by removing new_prompt from the language and choosing the static singleton set $\mathbb{P} = \{ \# \}$ for the source of available prompts. In this case, the extension to multiple prompts is conservative and degenerates to λ_{dc}.

Dynamic prompts Gunter et al. (1995), Sitaram and Felleisen (1990), Hieb and Dybvig (1990), and later Dybvig et al. (2007) all allow the creation of new prompts at runtime. This way, users can dynamically create new control operators and corresponding delimiters.
Gunter et al., Sitaram and Felleisen, and Dybvig et al. additionally make prompts first class objects. Gunter et al. and Dybvig et al. introduce a separate type of prompts $\mathbb{P} =$ Prompt and offer built-in functions new_prompt (resp. newPrompt) to the user to create new prompts at runtime. In contrast, Sitaram and Felleisen use natural numbers to distinguish levels of control operators, that is $\mathbb{P} = \mathbb{N}$ and consequently do not need to offer a way to create fresh prompts. Different to the semantics presented in Figure 2.3, their prompts form a hierarchy and the comparison on prompt equality in the capture context thus needs to be replaced by $p < p'$.

Implicit prompts In contrast to approaches with first-class prompts and explicit prompt creation, Hieb and Dybvig (1990) introduce the operator spawn $\{ c \Rightarrow e \}$ that unifies creating a fresh prompt and delimiting the extent of the continuation. Conceptually, invoking spawn creates a new control operator (named c for "controller"), which is delimited by this very call to spawn. Following Dybvig et al. (2007), the semantics of spawn can be captured by

$$\mathsf{spawn}\ \{ c \Rightarrow e \} \doteq \mathsf{val}\ p = \mathsf{new\_prompt};\ \mathsf{def}\ c(\mathit{body}) = \mathsf{shift}_p\ \{ k \Rightarrow \mathit{body}(k) \};\ \mathsf{reset}_p\ \{ e \}$$

with semantics $-\mathit{shift}_{+}$ for the operator shift_p. By grouping the creation of prompts and delimiting of continuations in one feature, Hieb and Dybvig (1990) introduce a slight restriction on expressivity. Always creating a fresh prompt p prevents users from using the same prompt on multiple, potentially different reset_ps.

Static prompts The effect operation *raise* above illustrated that control effects can be used to model exceptions. Deliberately ignoring subtyping, to model the exceptional control flow of Java, we can instantiate λ_{dcp} with a globally static set of prompts, corresponding to the different types of exceptions, such as:

$$\mathbb{P} = \{ \text{ArithmeticException},\ \text{NullpointerException},\ \ldots \}$$

Conceptually, exception types are not first-class, though this view changes when also considering runtime reflection. Since exceptions discard the continuation, we can choose any of the $-shift\pm$ variants as the reduction rule.

2.2.2 Static Hierarchies

Like the generalization to multiple prompts, the *CPS-hierarchy* (Danvy and Filinski, 1990) is an alternative approach that gives rise to a family of control operators. Danvy and Filinski (1990) translate programs with multiple levels of control operators by repeatedly applying a CPS transformation. Each iteration of the CPS transformation adds an additional continuation argument to the translated program, which gives rise to a new pair of control operators $\langle\ e\ \rangle_i$ (*i.e.*, reset) and $\xi_i\ k.\ e$ (*i.e.*, shift). Since the number of CPS translations needs to be known at translation time, this way they obtain a *static* hierarchy of control operators. While in terms of delimiter treatment, their semantics corresponds to $+shift+$, there is a significant semantic difference to our description with prompts: in the CPS-hierarchy, the control operator $\xi_n\ k.\ e$ captures the first n delimited contexts and binds their composition to k in e. Likewise, $\langle\ e\ \rangle_n$ delimits the first n contexts (Danvy and Filinski, 1990). As a consequence, when implementing effects like *flip* and *raise* as above, we need to globally fix the order of effects, for instance choosing ξ_1 to express *flip* and ξ_2 to express raise. From a users point of view, programming in the CPS hierarchy is thus closer to our development of multiple effects with shift_0 ($-shift+$) in the previous section, than to programming with multiple prompts.

2.2.3 Multi-Prompt Delimited Control in Effekt

In the second part of this thesis, we base our implementations of the Effekt library on multi-prompt delimited control in the semantics $-shift+$. This holds true for both our Scala implementations in Chapters 4 and 6, as well as the Java implementation in Chapter 5. As we will see, every handler implicitly introduces a fresh prompt. Our operational semantics is thus closest to the one of spawn (Hieb and Dybvig, 1990).

2.3 From Delimited Control to Effect Handlers

In the last two sections, we have revisited two important generalizations in the history of control operators: first from undelimited control to delimited control and then from one control operator to a family of control operators. In this section, we will now draw connections between multi-prompt delimited control and effect handlers.

It was Sitaram (1993), who performed an important (though not often recognized) step on the way from delimited control to effect handlers[3]. Sitaram observed that with callcc (and likewise with other control operators)

> "the *handling* of the continuation takes place at the identical site as the creation of the continuation" — *Sitaram (1993, p. 148)*

This is fundamentally different when programming with exception handlers. With exception handlers, **`throw`** just *signals* an exception, but the *handling* is performed at the exception handler.

[3]We are grateful to Youyou Cong to bring the work of Sitaram (1993) to our attention.

Following Sitaram (1993)[4], we can apply the same idea to control effects,

and instead of

$$\begin{array}{l}\mathsf{reset}_p\ \{\\ \quad \ldots\ \mathsf{val}\ x\ =\ arg;\ \mathsf{shift}_p\ \{\ k \Rightarrow\ body\ \}\ \ldots\\ \}\end{array}$$

we now write:

$$\begin{array}{l}\mathsf{handle}_p\ \{\ (x,\ k) \Rightarrow\ body\ \}\ \mathsf{in}\ \{\\ \quad \ldots\ \mathsf{do}_p(arg)\ \ldots\\ \}\end{array}$$

That is, instead of grouping continuation capture and handling in the construct shift, we group handling with the continuation delimiter handle (called run by Sitaram). The effect operation do (called fcontrol by Sitaram) now merely signals a control transfer to the corresponding *handler*. The choice of the terminology "handler" is not accidental, as the construct was inspired by exception handling (Sitaram, 1993).

Figure 2.4 defines a calculus scheme modifying λ_{dcp} to λ_{dch} (pronounced "delimited control with handlers"). As in λ_{dcp}, prompts are first-class and can be created with new_prompt. Like in all previous presentations of calculus schemes, λ_{dch} offers the choice of four different reduction rules ${}_{\pm}do_{\pm}$. The semantics of Sitaram (1993) corresponds to ${}_{+}do_{-}$. However, as mentioned in Section 2.1, of the four different variants the reduction rule $({}_{-}shift_{+})$ is closest to effect handlers. Similarly, we will always assume rule $({}_{-}do_{+})$ in the remainder of this thesis unless otherwise noted.

Example Using prompts to distinguish different effect operations, we can express our running example as follows:

$$\begin{array}{lcl}\mathsf{val}\ amb & = & \mathsf{new\_prompt};\\ \mathsf{val}\ exc & = & \mathsf{new\_prompt};\\ \mathsf{def}\ flip() & = & \mathsf{do}_{amb}()\\ \mathsf{def}\ raise(msg) & = & \mathsf{do}_{exc}(msg)\\ \mathsf{def}\ action() & = & \ldots\end{array}$$

The effect operations *flip* and *raise* immediately forward to control operations do_{amb} and do_{exc} respectively. They do not carry any implementation details. The implementation itself is now located at the handlers:

$$\begin{array}{l}\mathsf{def}\ collect(action)\ =\ \mathsf{handle}_{amb}\ \{\ ((),\ k) \Rightarrow k(\mathsf{true})\ +\!\!+\ k(\mathsf{false})\ \}\ \mathsf{in}\ \{\ \mathrm{List}(action())\ \}\\ \mathsf{def}\ maybe(action)\ =\ \mathsf{handle}_{exc}\ \{\ (msg,\ k) \Rightarrow \mathrm{None}\ \}\ \mathsf{in}\ \{\ \mathrm{Some}(action())\ \}\end{array}$$

For more flexibility, we abstract over the handlers as functions *collect* and *maybe*. Similar to our previous examples, handling the effects gives:

$collect(\lambda().\ maybe(drunkFlip))$
▶ List(Some("heads"), Some("tails"), None)

$maybe(\lambda().\ collect(drunkFlip))$
▶ None

We can safely treat the involved prompts and definitions of effect operations as globally static. This does not prevent us from providing alternative implementations in handlers like:

$$\mathsf{def}\ report(action)\ =\ \mathsf{handle}_{exc}\ \{\ (msg,\ k) \Rightarrow log(msg)\ \}\ \mathsf{in}\ \{\ action();\ ()\ \}$$

[4]We deliberately adjusted the syntax of Sitaram (1993) to be closer to the one of effect handlers.

Syntax:

$$
\begin{array}{llll}
\text{Expressions} & e & ::= & \ldots \\
 & & \mid & \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ e\ \} \qquad \text{delimiting effects} \\
 & & \mid & \mathsf{do}_p(v) \qquad \text{effect operation call} \\
\\
\text{Handlers} & h & ::= & \{\ (x,\ k) \Rightarrow e\ \} \qquad \text{handler clause}
\end{array}
$$

Evaluation Contexts:

$$
\begin{array}{l}
\mathsf{E} \ ::= \square \mid \mathsf{E}(e) \mid v(\mathsf{E}) \mid \mathsf{handle}_p\ h \ \ \mathsf{in}\ \{\ \mathsf{E}\ \} \\
\mathsf{H}_p ::= \square \mid \mathsf{H}_p(e) \mid v(\mathsf{H}_p) \mid \mathsf{handle}_{p'}\ h \ \ \mathsf{in}\ \{\ \mathsf{H}_p\ \} \quad \text{if } p \neq p'
\end{array}
$$

Operational Semantics:

$$
\begin{array}{lll}
\ldots & & \\
(\mathit{reset}) & \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ v\ \} & \longrightarrow\ v \\
(\pm do \pm) & \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ \mathsf{H}_p[\mathsf{do}_p(v)]\ \} & \longrightarrow\ \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ e[x \mapsto v,\ k \mapsto \lambda y.\ \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ \mathsf{H}_p[y]\ \}\ ]\ \} \\
 & & \quad \mathsf{where}\ h\ =\ \{\ (x,\ k) \Rightarrow e\ \}
\end{array}
$$

Figure 2.4. Syntax and semantics of multi-prompt delimited control with handlers (λ_{dch}).

This alternative handler for exceptions assumes logging facilities to report the provided message and always return the unit value. While programs express that they use the *raise* effect, the exact semantics is determined by the handler. We can think of the handler as a *dynamic binder* for the effect operation – a point of view, which we will explore in detail in Chapter 3.

Moving the handling (*body*) from the control operation to the delimiter seems like a simple change in syntax. However, it has profound consequences on the way users can reason about effectful programs.

Localized reasoning about the continuation With shift_p each use of the control operator to capture the continuation can vary. Parts of the program might use shift_p to discard the continuation and abort with a dummy value (like *raise* above), other parts might use it as a backtracking facility (like *flip* above) for the same delimiter. Even though different occurrences of shift_p need to coordinate the way they use the continuation, they are spread all over the program. Not so with handlers. Locating *body* with the delimiter establishes an important restriction: all bodies of shift_p need to coincide. This way, a programmer can easily determine all usages of the continuation for a prompt p by analyzing the corresponding handler. The different ways a handler might capture and use the continuation is thus confined to one module. This allows syntactically localized reasoning about the continuation usage.

Localized reasoning about effects Syntactically grouping the handlers with the delimiter has another important consequence. It naturally suggests that effects used by the expression *body* are evaluated in the context of the handler, not in the context of the corresponding shift_p or fcontrol. What might seem obvious (when considering handlers) initially lead to confusion in

the context of delimited control. Danvy and Filinski (1990) describe "the desirable relation"[5]

$$\mathsf{shift}_p\ \{\ k \Rightarrow k(e)\ \}\quad \equiv\quad e$$

which as they remark, in general, does not hold for the CPS-hierarchy. Translated to our setting of multiple prompts, by simple reduction we can see that it also does not hold for any of our multi-prompt $\pm shift\pm$-variants.

$$\mathsf{reset}_p\ \{\ \mathsf{H}_p[\mathsf{shift}_p\ \{\ k \Rightarrow k(e)\ \}]\ \} \longrightarrow^* \mathsf{reset}_p\ \{\ (\lambda x.\ \mathsf{reset}_p\ \{\ \mathsf{H}_p[x]\ \}\ )(e)\ \}$$

In general, the reduction of expression e might involve a reduction of $\mathsf{shift}_{p'}$ for any other prompt $p' \neq p$. This prompt p' in turn might be delimited in the capture context H_p since the capture context only excludes delimiters for the same prompt[6]. We want to argue, that this relation is not desirable after all! As illustrated above, effects in e are evaluated in the current evaluation context, while $\mathsf{shift}_p\ \{\ k \Rightarrow k(e)\ \}$ leads to a *shift in perspective*: The body of shift_p is evaluated at the position of the corresponding reset_p. Grouping handling expressions with the delimiter clarifies this and furthermore allows syntactically localized reasoning about the effect usage. With handlers, a programmer can lexically reason about effects used by the handler since the scope of evaluation and the scope of definition is syntactically unified. In general, this is not true for languages with shift_p-like control operators.

Localized type checking Grouping delimiter and handling expressions not only helps the programmer to reason about programs, it also simplifies type checking. While type systems for various variants of control operators exist (Danvy and Filinski, 1989; Asai and Kameyama, 2007; Biernacka et al., 2011; Materzok and Biernacki, 2011) they are typically quite involved. One particular challenge is *answer type modification* (Asai and Kameyama, 2007). Answer type modification describes the situation where the body of a shift changes the resulting type (the answer type) at the corresponding reset. Take the following example:

$$\mathsf{val}\ ans\ =\ \mathsf{reset}\ \{\ 1\ +\ action()\ \}$$

Just from the type of the top-level addition, one would assume reset to return a numeric value. However, with

$$\mathsf{def}\ action()\ =\ \mathsf{shift}\ \{\ k \Rightarrow \lambda x.\ k(x)\ *\ 2\ \}$$

the body of shift changes the type of *ans* to a function between numeric values. Applying $ans(20)$ thus yields 42.
Answer type modification is powerful and can for instance express `sprintf` concisely by changing the number and the type of additional arguments based on the parsed format string (Asai and Kameyama, 2007). However, this expressiveness takes its toll on the understandability of programs and the simplicity of type checking. Handlers improve on this situation by syntactically grouping continuation usage and delimiter in one module. This simplifies type checking: Most type systems for languages with support for (algebraic) effect handlers simply require that the answer types of the handled expression and the one handling coincide (Plotkin and Pretnar, 2013; Leijen, 2017c; Lindley et al., 2017). That is, handlers do not support answer type modification. Notably, Gunter et al. (1995) achieve a similar restriction by fixing the answer type at prompt creation time.

[5] For easier comparison, we present the relation in the syntax of λ_{dcp}.
[6] As before, the optional outer and inner delimiter are highlighted in grey.

Syntax:

Expressions	e	$::=$	$e(e)$	application
		$\mid$	$\mathsf{handle}_p\ h\ \mathsf{in}\ \{\ e\ \}$	delimiting effects
		$\mid$	$\mathsf{do}_p(v)$	effect operation call
		$\mid$	v	value
Values	v	$::=$	$x \mid c \mid \lambda x.\ e$	
Handlers	h	$::=$	$\{\ (x,\ k) \Rightarrow e\ \}$	handler clause

Evaluation Contexts:

$$\begin{array}{l} \mathsf{E} \ ::= \square \mid \mathsf{E}(e) \mid v(\mathsf{E}) \mid \mathsf{handle}_p\ h\ \ \mathsf{in}\ \{\ \mathsf{E}\ \} \\ \mathsf{H}_p ::= \square \mid \mathsf{H}_p(e) \mid v(\mathsf{H}_p) \mid \mathsf{handle}_{p'}\ h\ \ \mathsf{in}\ \{\ \mathsf{H}_p\ \} \quad \text{if } p \neq p' \end{array} \qquad \frac{e \longrightarrow e'}{\mathsf{E}[e] \longmapsto \mathsf{E}[e']}\ [\textsc{cong}]$$

Operational Semantics:

$$\begin{array}{lllll} (\delta) & c(v) & \longrightarrow & \delta(c,v) & \text{if } \delta(c,v) \text{ is defined} \\ (\beta) & (\lambda x.\ e)(v) & \longrightarrow & e[x \mapsto v] & \\ (\mathit{reset}) & \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ v\ \} & \longrightarrow & v & \\ (-do+) & \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ \mathsf{H}_p[\mathsf{do}_p(v)]\ \} & \longrightarrow & e[x \mapsto v,\ k \mapsto \lambda y.\ \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ \mathsf{H}_p[y]\ \}] & \\ & & & \quad \mathsf{where}\ h\ =\ \{\ (x,\ k) \Rightarrow e\ \} & \end{array}$$

Figure 2.5. Summary of syntax and semantics for λ_{dch}, specialized to variant $-do+$ and a statically declared set of prompts (adapted from Leijen, 2017c).

2.4 A Type System for Effect Handlers

In the previous section, we argued that grouping the handling expressions with the delimiter facilitates type checking. To support this observation and as a point of reference for later chapters, in this section we present a simple monomorphic type and effect system for λ_{dch}. Figure 2.6 defines the typing rules for λ_{dch} in the variant $-do+$. Since this chapter presented the development of λ_{dch} in multiple steps, for easier reference, we also include syntax and operational semantics (Figure 2.5). For the effect system, we build on a simplified version of the effect system of Koka (Leijen, 2017c), which we presented in similar form as λ_{aeh} in previous work (Brachthäuser and Leijen, 2019). In Koka, the set of labels is statically determined by global effect declarations, such as:

```
effect amb { flip(): bool }
```

By listing effect operations as part of an effect declaration, effect operations only implicitly refer to the corresponding effect label. Hence, labels are not first-class[8]. Likewise, for the presentation in this section, we assume an arbitrary, but statically fixed set of prompts (that is, we exclude new_prompt from the language). Following Leijen (2017c), for every prompt p, we assume the presence of a unique effect declaration in Σ, that assigns a signature to the prompt (*i.e.*, $\Sigma(p) = \tau_1 \to \tau_2$). For instance, we would require $amb \in \mathbb{P}$ and $amb : unit \to bool \in \Sigma$ to express the flip operation as:

$$\mathsf{def}\ \mathit{flip}()\ \{\ \mathsf{do}_{amb}()\ \}$$

Both, the operational semantics and the effect system are adapted from Leijen (2017c). While λ_{dch} is close to the calculus presented by Leijen (2017c), there are notable differences:

- We simplify the type system of λ_{dch} to monomorphic types and effects.
- Handlers in λ_{dch} do not have return-clauses.
- Handlers in λ_{dch} are restricted to a single effect operation, whereas Leijen (2017c) allows grouping of multiple effect operations under one effect label / prompt.

Figure 2.6 introduces two typing judgments. The judgment $\Gamma \vdash_{val} v : \tau$ types values, which, by construction, cannot use any control effects. The judgment $\Gamma \vdash_{dch} e : \tau \mid \pi$ types expressions that can use control effects specified in row ϵ. Following Leijen (2017c), a row is either empty $\langle\,\rangle$ or an extension with a prompt label $\langle p|\epsilon\rangle$. We also treat rows as equivalent up to the order of different prompts (Figure 2.6) and use the following shorthands:

$$\langle p_1, \ldots, p_n|\epsilon\rangle \doteq \langle p_1|\ \ldots\ \langle p_n|\epsilon\rangle\ \ldots\ \rangle \qquad \langle p_1, \ldots, p_n\rangle \doteq \langle p_1|\ \ldots\ \langle p_n|\langle\,\rangle\rangle\ \ldots\ \rangle$$

Rule DO types control transfers do_p and introduces the prompt label in the effect row. In contrast, rule HANDLE eliminates the prompt label from the row to type a handler. Handlers in λ_{dch} do not have return clauses and thus the rule HANDLE requires e_1 and e_2 both to agree on the type τ. To type the handler body e_1, the typing environment is extended with the argument type $x : \tau_1$, and the type of the reified continuation $k : \tau_2 \to \epsilon\ \tau$. In particular, both the body of the handler and the continuation can use residual effects ϵ, which are not handled by the handler itself.

[8] Recently, Leijen (2018b) extended Koka with support for dynamic effect handlers by adding the rule $\mathsf{new\ handle}_h(v) \longrightarrow \mathsf{handle}^l_h(v(l))$ where l *fresh*, which is reminiscent of the above mentioned rule for spawn. By generating fresh labels, the set of labels is now open and by passing the label to the handled program labels effectively become first-class objects.

Syntax of Types:

Types	τ	$::=$	$\mathit{unit} \mid \mathit{bool} \mid \ldots$	set of builtin types
		$\mid$	$\tau \to \epsilon\ \tau$	effectful functions
Effect Row	ϵ	$::=$	$\langle\,\rangle$	empty row
		$\mid$	$\langle p \mid \epsilon \rangle$	row extension
Effect Labels	p	$::=$	$\mathit{exc} \mid \mathit{amb} \mid \ldots$	static set of prompts
Type Environment	Γ	$::=$	$\emptyset \mid \Gamma, x : \tau$	
Effect Signatures	Σ	$::=$	$\emptyset \mid \Sigma, p :\ \tau_1 \to \tau_2$	

Typing Rules:

$$\frac{\Gamma(x) \;=\; \tau}{\Gamma \vdash_{val}\; x \;:\; \tau}\ [\textsc{var}] \qquad \frac{\Gamma \vdash_{val}\; v \;:\; \tau}{\Gamma \vdash_{dch}\; v \;:\; \tau \mid \epsilon}\ [\textsc{val}] \qquad \frac{\Sigma(p) \;=\; \tau_1 \to \tau_2 \quad \Gamma \vdash_{val}\; v \;:\; \tau_1}{\Gamma \vdash_{dch}\; \mathsf{do}_p(v) \;:\; \tau_2 \mid \langle p \mid \epsilon \rangle}\ [\textsc{do}]$$

$$\frac{\Gamma, x : \tau_1 \vdash_{dch}\; e \;:\; \tau_2 \mid \epsilon}{\Gamma \vdash_{val}\; \lambda x.e \;:\; \tau_1 \to \epsilon\ \tau_2}\ [\textsc{lam}] \qquad \frac{\Gamma \vdash_{dch}\; e_1 \;:\; \tau_1 \to \epsilon\ \tau_2 \mid \epsilon \quad \Gamma \vdash_{dch}\; e_2 \;:\; \tau_1 \mid \epsilon}{\Gamma \vdash_{dch}\; e_1(e_2) \;:\; \tau_2 \mid \epsilon}\ [\textsc{app}]$$

$$\frac{\Sigma(p) \;=\; \tau_1 \to \tau_2 \quad \Gamma \vdash_{dch}\; e_2 \;:\; \tau \mid \langle p \mid \epsilon \rangle \quad \Gamma, x : \tau_1, k : \tau_2 \to \epsilon\ \tau \vdash_{dch}\; e_1 \;:\; \tau \mid \epsilon}{\Gamma \vdash_{dch}\; \mathsf{handle}_p\ \{\ (x,\ k) \Rightarrow e_1\ \}\ \mathsf{in}\ \{\ e_2\ \} \;:\; \tau \mid \epsilon}\ [\textsc{handle}]$$

Row Equivalence:

$$\epsilon \;\cong\; \epsilon \quad [\textsc{eq-refl}] \qquad \frac{\epsilon_1 \;\cong\; \epsilon_2}{\langle p \mid \epsilon_1 \rangle \;\cong\; \langle p \mid \epsilon_2 \rangle}\ [\textsc{eq-head}]$$

$$\frac{\epsilon_1 \;\cong\; \epsilon_2 \quad \epsilon_2 \;\cong\; \epsilon_3}{\epsilon_1 \;\cong\; \epsilon_3}\ [\textsc{eq-trans}] \qquad \frac{p_1 \;\neq\; p_2}{\langle p_1 \mid \langle p_2 \mid \epsilon \rangle\,\rangle \;\cong\; \langle p_2 \mid \langle p_1 \mid \epsilon \rangle\,\rangle}\ [\textsc{eq-comm}]$$

Figure 2.6. Syntax of types, typing rules, and equivalence of effect rows for λ_{dch} (adapted from Leijen, 2017c). Every prompt p has a uniquely defined signature in Σ.

2.4.1 Semantic Soundness

Since λ_{dch} is a restriction of the language presented by Leijen (2017c), the proofs carry over mostly unchanged. Leijen (2017c) extends a previous proof (Leijen, 2014) and uses standard techniques from Wright and Felleisen (1994) to show that well-typed effectful programs cannot go *wrong*. That is, all effect operations are eventually handled. Here, we repeat some of the necessary lemmas and proofs of Leijen (2017c). Following Leijen, we state semantic soundness:

Theorem 1. (*Semantic Soundness*)
If $\emptyset \vdash_{dch} \ e \ : \ \tau | \ \langle\rangle$ then either e diverges, or evaluates to a value $e \longmapsto^* v$ where $\emptyset \vdash_{val} \ v \ : \ \tau$.

Note, that we require the set of effects to be empty (*i.e.*, $\langle\rangle$). That is, the program should not have any control-effects, which are observable from the outside[7]. We only focus on effect safety and identify the following expressions as faulty:

Definition 1. (*Unhandled effect*)
A term has an unhandled effect p, if it has the form $\mathsf{H}_p[\mathsf{do}_p(v)]$.

Leijen (2017c) continues and proves semantic soundness using two lemmas, which we prove in the remainder of this section:

Lemma 1. (*Subject reduction*)
If $\Gamma \vdash_{dch} \ e_1 \ : \ \tau | \ \epsilon$ and $e_1 \longmapsto e_2$, then $\Gamma \vdash_{dch} \ e_2 \ : \ \tau | \ \epsilon$.

That is, types and effects are preserved by the reduction relation $\longmapsto$.

Lemma 2. (*Effects are meaningful*)
If $\Gamma \vdash_{dch} \ \mathsf{H}_p[\mathsf{do}_p(v)] \ : \ \tau | \ \epsilon$, then $p \ \in \epsilon$.

Lemma 2 states that effect types are *meaningful*. Only effect handlers can remove labels p from the effect row ϵ.

Proof of Semantic Soundness

Leijen (2017c) uses Lemma 2 (stating that effects are meaningful) together with Lemma 1 (subject reduction) to prove semantic soundness (Theorem 1).

Proof. (*of Theorem 1*) To reduce e_1 to a value (or diverge), we repeatedly apply $\longmapsto$. Every such reduction step either leads to another expression e_2 or it is stuck on a faulty expression (Definition 1), that is, an unhandled effect. Lemma 1 gives us, that the new expression e_2 has the same type τ and the same effects ϵ.

Now, let us assume that a sequence of reductions leads to an expression $\mathsf{H}_p[\mathsf{do}_p(v)]$, which is faulty. By subject reduction (Lemma 1), we know that $\Gamma \vdash_{dch} \ \mathsf{H}_p[\mathsf{do}_p(v)] \ : \ \tau | \ \epsilon$. Additionally, by Lemma 2, we know that $p \ \in \epsilon$.

However, the fact that $p \ \in \epsilon$ contradicts our assumption of Theorem 1. It requires an empty effect row. The reduction thus cannot lead to an unhandled effect. □

[7] Leijen (2017c) allows arbitray effects ϵ, which we believe is an oversight in the original formalization.

Proof of Subject Reduction

To prove subject reduction (Lemma 1), we need the following two standard lemmas (Wright and Felleisen, 1994):

Lemma 3. (*Substitution*)
a. If $\Gamma,\ x\ :\ \tau \vdash_{val}\ v'\ :\ \tau'$ and $\Gamma \vdash_{val}\ v\ :\ \tau$ then $\Gamma \vdash_{val}\ v'[x \mapsto v]\ :\ \tau'$
b. If $\Gamma,\ x\ :\ \tau \vdash_{dch}\ e\ :\ \tau' \mid \epsilon$ and $\Gamma \vdash_{val}\ v\ :\ \tau$ then $\Gamma \vdash_{dch}\ e[x \mapsto v]\ :\ \tau' \mid \epsilon$

By mutual induction on the derivations $\Gamma,\ x\ :\ \tau \vdash_{val}\ v'\ :\ \tau'$, and $\Gamma,\ x\ :\ \tau \vdash_{dch}\ e\ :\ \tau' \mid \epsilon$.

Lemma 4. (*Replacement*)
Given a typing derivation D that ends in $\Gamma \vdash_{dch}\ \mathsf{E}[e]\ :\ \tau \mid \epsilon$, and D', which is a subderivation of D ending in $\Gamma' \vdash_{dch}\ e\ :\ \tau' \mid \epsilon'$ and occurs at the hole of E, and $\Gamma' \vdash_{dch}\ e'\ :\ \tau' \mid \epsilon'$, then we have that $\Gamma \vdash_{dch}\ \mathsf{E}[e]\ :\ \tau \mid \epsilon$.

Proof by induction on the derivation tree and case analysis of the evaluation context E.

Since effect operations allow to transfer control flow and send values "up the stack", we also need the following non-standard lemma:

Lemma 5. (*Context inversion*)
If we have a typing derivation D that ends in $\Gamma \vdash_{dch}\ \mathsf{E}[e]\ :\ \tau \mid \epsilon$, and D' is a subderivation of D ending in $\Gamma' \vdash_{dch}\ e\ :\ \tau' \mid \epsilon'$. Then Γ' can be weakened to Γ.

That is, the evaluation context E has no influence on the type environment Γ'. Proof by induction on the derivation tree and case analysis of the evaluation context E. Informally, only the rules App and Handle are applicable. Neither changes the typing context.

Proof. (*Of Lemma 1*) We are now ready to prove Lemma 1. The proof proceeds by induction over reductions $\longrightarrow$. We assume that the induction hypothesis holds for subterms. The interesting cases are rules (*reset*) and ($_{-}do_{+}$):

case ($_{-}do_{+}$)
We need to construct a derivation for: $\Gamma\ \vdash\ e[x \mapsto v,\ k \mapsto \lambda y.\ \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ \mathsf{H}_p[y]\ \}]\ :\ \tau \mid \epsilon$. From rule Handle we have $\Sigma(p)\ =\ \tau_1 \to \tau_2$ **(1)**, $\Gamma \vdash_{dch}\ \mathsf{H}_p[\mathsf{do}_p(v)] :\ \tau \mid \langle p|\epsilon\rangle$ **(2)**, and $\Gamma,\ x\ :\ \tau_1,\ k :\ \tau_1 \to \epsilon \vdash_{dch}\ e\ :\ \tau \mid \epsilon$ **(3)**.
The derivation (2) includes a subderivation for $\Gamma' \vdash\ \mathsf{do}_p(v)\ :\ \tau_2 \mid \epsilon'$ **(4)** with the premise $\Gamma' \vdash_{val}\ v : \tau_1$ **(5)**. From (5) and Lemma 5, we can obtain $\Gamma \vdash_{val}\ v\ :\ \tau_1$. With (2), (4), assuming $\emptyset \vdash_{dch}\ y\ :\ \tau_2 \mid \epsilon'$, and Lemma 4, we can derive $\Gamma \vdash_{dch}\ \mathsf{H}_p[y]\ :\ \tau \mid \langle p|\epsilon\rangle$ **(6)**. Using Handle, (1), (6), and (3) we have $\Gamma \vdash_{dch}\ \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ \mathsf{H}_p[y]\ \}\ :\ \tau \mid \epsilon$. By Lam, we obtain $\Gamma \vdash_{val}\ \lambda y.\ \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ \mathsf{H}_p[y]\ \}\ :\ \tau_2 \to \epsilon\ \tau$ **(7)**. Assuming $x, k\ \notin\ \mathsf{fv}(\mathsf{H}_p[y])$, and using Lemma 3 twice (once for x and once for k), we finally construct the desired typing derivation.

case (*reset*)
We need to construct a typing derivation for $\Gamma \vdash_{dch}\ v\ :\ \tau \mid \epsilon$. From rule Handle we have the premise $\Gamma \vdash_{dch}\ v\ :\ \tau \mid \langle p|\epsilon\rangle$ **(1)**. Since v is a value, the derivation has to end in an application of Val with the premise $\Gamma \vdash_{val}\ v\ :\ \tau$ **(2)**. From (2) we can again apply Val choosing a different effect ϵ (instead of $\langle p|\epsilon\rangle$). This gives us the desired derivation. Intuitively, values do not have effects and we can thus choose arbitrarily.

□

Proof that Effects are Meaningful

After having shown subject reduction (Lemma 1), we continue and show that faulty expressions are not typeable. To show that faulty expressions are not typeable, we need to prove Lemma 2.

Proof. (*of Lemma 2*) We proceed by induction over the structure of the capture context H_p.

case $\square$
By rule Do, we immediately have $\epsilon = \langle p|\epsilon'\rangle$ and thus $p \in \epsilon$.

case $\mathsf{H}'_p(e)$
We can apply rule App to obtain the premise $\Gamma \vdash_{dch} \mathsf{H}'_p[\mathsf{do}_p(v)] : \tau_1 \to \epsilon\ \tau_2 \mid \epsilon$ **(1)**. By **(1)** and the induction hypothesis, we have $p \in \epsilon$.

case $v(\mathsf{H}'_p)$
Similar to the previous case.

case $\mathsf{handle}_{p'}\ h\ \mathsf{in}\ \{\ \mathsf{H}'_p\ \}\ \mathsf{if}\ p \neq p'$
By the grammar for capture contexts, we have $p \neq p'$ **(1)**. Applying rule Handle, we obtain $\Gamma \vdash_{dcp} \mathsf{H}'_p[\mathsf{do}_p(v)] : \tau|\ \langle p'|\epsilon\rangle$ **(2)**. By the induction hypothesis, (2) we have $p \in \langle p'|\epsilon\rangle$. By side condition (1) we finally get $p \in \epsilon$.

$\square$

2.5 Related Work and Chapter Conclusion

In this chapter, we introduced effect handlers from the perspective of control effects and discussed the relation between effect handlers and multi-prompt delimited control. In a natural progression, starting from undelimited control, via delimited control and multiple prompts, we finally arrived at effect handlers (λ_{dch}). Since this chapter summarized existing work, most related work has already been discussed throughout the text. Here, we want to briefly put λ_{dch} into perspective of other calculi and implementations of delimited control and effect handlers.

Delimited Control and Effect Handlers

It has previously been established that algebraic effect handlers can be expressed with delimited control. Forster et al. (2017) formally relate the expressive power of effect handlers with the expressive power of single-prompt delimited control in variant $-shift_+$ (that is, shift_0). Piróg et al. (2019) extend the work by Forster et al. (2017) to a typed setting and show that it requires type polymorphism. They provide a typed correspondence between deep handlers (Kammar et al., 2013) and shift_0. They also give a typed correspondence between shallow handlers, as they can be found in the language Frank (Lindley et al., 2017), and the control operator $\mathsf{control}_0$ (that is, $-shift-$). Kiselyov and Sivaramakrishnan (2016) present a practical embedding of the language Eff into OCaml in terms of multi-prompt delimited control. They build on semantics $-shift-$ but manually delimit the body of the continuation.

Dybvig et al. (2007) give a comprehensive overview over the different control operators and show how they can be expressed in terms of withSubCont (that is, semantics $-shift-$). We adopt their notation to classify control operators by delimiter treatment, but write $\pm shift\pm$ instead of $\pm\mathsf{F}\pm$. While they present the operational semantics of control operator withSubCont in terms

of an abstract machine, we adapt the style of Wright and Felleisen (1994) and use the capture context H (respectively, H_p). This is also how Leijen (2017c) defines the operational semantics of Koka, which inspired the presentation of the calculi in this and the following chapter.

Comparison with Effect Handlers in Koka

As established by Forster et al. (2017), of the four different variants the reduction rule ($-shift_+$) is closest to effect handlers. One can also see this relation between delimited control and algebraic effect handlers clearly when comparing the operational semantics of λ_{dch} with the one of Koka (Leijen, 2017c, 2018b), a language with support for algebraic effect handlers. We repeat the relevant reduction rules of Leijen (2017c) where necessary. In Koka, handlers can use the delimited continuation to provide semantics to an effect operation. The continuation capture is expressed by the following evaluation context,

$$\mathsf{H}^l \;=\; \Box \mid \mathsf{H}^l(e) \mid v(\mathsf{H}^l) \mid \mathsf{handle}_h^{l'}(\mathsf{H}^l) \quad \text{if } l \neq l'$$

and the following reduction rule (*handle*):

$$\mathsf{handle}_h^l(\mathsf{H}^l[op^l(v)]) \longrightarrow e[x \mapsto v,\ resume \mapsto \lambda y.\ \mathsf{handle}_h^l(\mathsf{H}^l[y])] \quad \text{where } (op^l(x) \to e) \in h$$

The reduction rule shows how the call to the effect operation op is handled by the handler h. Comparing this to λ_{dch} and mapping effect labels l to prompts p, we can see that the delimited context H^l directly corresponds to th capture context H_p and that the *handle*-rule corresponds to rule ($-do_+$), repeated here for easier comparison:

$$\mathsf{handle}_p\ h\ \mathsf{in}\ \{\ \mathsf{H}_p[\mathsf{do}_p(v)]\ \} \longrightarrow e[x \mapsto v,\ resume \mapsto \lambda y.\ \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ \mathsf{H}_p[y]\ \}] \quad \text{where } h \;=\; \{\ (x,\ resume) \Rightarrow e\ \}$$

Like in rule ($-do_+$), the continuation bound to *resume* is again delimited by handle_h^l and no outer delimiter is left behind. Also, both Koka and λ_{dch} perform lookup for the handler body.

However, there are also a few differences. As mentioned above, effect declarations in Koka are not limited to just one effect operation. If we also want to express multiple, different effects in λ_{dch} by using one prompt p, we can do so by tagging the operations (Sitaram, 1993). For instance, we can express the state effect as:

$$\begin{array}{l}\mathsf{val}\ state \;=\; \mathsf{new\_prompt}\\ \mathsf{def}\ get()\ \ \mathsf{do}_{state}(\texttt{"get"})\\ \mathsf{def}\ set(value)\ \ \mathsf{do}_{state}(\texttt{"set"},\ value)\end{array}$$

Assigning static types to operations tagged this way would require some form of GADTs (Peyton Jones et al., 2006).

In addition to the implementation of effect operations, effect handlers in Koka also provide a "return clause". For comparison, the collect handler would be expressed as:

```
val collect = handler {
  return x → Cons(x, Nil)
  flip() → append(resume(True), resume(False))
}
```

In our examples, we inlined the return clauses. This naive solution is not always possible since the body of the return might refer to an outer handler of the same effect. Leijen (2018b) shows how to express return clauses as effect operations, which circumvents this problem.

2.5.1 Chapter Conclusion

In this background chapter, we presented effect handlers from the perspective of delimited control to illustrate that there are three important aspects to effect handlers:

1. One of the most important features of effect handlers is the ability to capture the continuation. We summarized existing work in λ_{dc} and λ_{dcp} to illustrate the design space of delimited control.
2. Effect handlers group delimiters and delimiter usage (control operator) in one module. This grouping allows localized reasoning about the usage of continuations and other control effects by effect operations. We highlight this difference by presenting both λ_{dcp} and λ_{dch}, where the only difference is the syntactic position of the handling body.
3. With effect handlers, effect operations are dynamically bound and looked up in the evaluation context. They are thus related to dynamically scoped variables. Chapter 3 explores this connection in more detail.

With the presented calculi we not only provided necessary preliminaries for the implementations of effect handlers presented in the second part of this thesis; we also hope that our presentation puts Andrej Bauer's statement that "effect handlers are to delimited continuations as structured programming is to goto" (at Dagstuhl Seminar 18172, March 2018) into a fresh perspective.

Chapter 3

From Dynamic Binding to Effect Handlers

In this chapter, we offer a second, alternative perspective on effect handlers: as a generalization of dynamic binding. To illustrate the relationship between effect handlers and dynamic binding, we gradually generalize dynamic binding in two steps. From *ambient values*, over *ambient functions*, to *ambient control*.

Ambient values are a typed implementation of dynamic binding (Moreau, 1998; Kiselyov et al., 2006). Ambient functions are *bound dynamically but evaluated in the lexical scope of their binding*. Ambient control further generalizes ambient functions by adding the ability modify the control flow. It coincides exactly with effect handlers as presented in the previous chapter. Each generalization steps reveals a certain aspect of effect handlers. Effect operations are dynamically bound, evaluated in the lexical scope of their binding, and can capture the continuation to resume to the call site.

We formalize all three features, ambient values, functions, and control as an extension to Moreau's calculus of dynamic binding (1998). Unifying all language features in one framework guarantees that the interaction between ambient values, functions, and control is well defined. We also show ambient values and ambient functions can be macro expressed by ambient control. They can thus be seen as a restriction of effect handlers.

While the restricted features help us to understand effect handlers, they also serve as better programming abstraction to the user. In particular, ambient values and functions are easier to reason about than full effect handlers, while offering a similar form of effect encapsulation.

An important aspect of software development is to parametrize a software component over aspects that might vary (Parnas, 1972). Parametrizing a component with configuration, behaviour, or other components enables reuse of the same component in different contexts. Manually parametrizing components, however, can quickly become tedious, both for the component author who needs to design the component for parametrization, as well as the user of the component who needs to provide all parameters. This is especially the case for components that depend on other parametrized components. They often need to be parametrized by all transitive parameters of their dependencies. Instead of explicitly passing around parameters, we could use dynamically scoped variables.

> "A *dynamic variable* is a variable whose association with a value exists only within the dynamic extent of an expression, that is, while control remains within that expression. If several associations exist for the same variable at the same time, the latest one takes effect. Such association is called a *dynamic binding*."
>
> — *Kiselyov et al. (2006, p. 26)*

This chapter is closely based on the following publication: Jonathan Immanuel Brachthäuser and Daan Leijen. "Programming with Implicit Values, Functions, and Control". *Technical Report MSR-TR-2019-7*. Microsoft Research, 2019.

In contrast to lexically scoped variables, the meaning of a dynamically bound variable is not determined in its lexical scope, but in its *dynamic scope*. In particular, lambda abstractions do not close over dynamically bound variables. In this thesis, we use *dynamic binding* (Moreau, 1998) exclusively to describe dynamically scoped variables. This is not to be confused with dynamic binding of method implementations in OOP, for which we use *late binding* (Suzuki, 1981). Due to the overloaded meaning of both *dynamic* and *variable*, in this chapter, we prefer to use the term *ambient value* to refer to a typed discipline of (immutable) dynamic variables.

Ambient values are a lightweight language feature that can be used to parametrize software components. Instead of manually parametrizing a component with additional arguments, component authors directly refer to the ambient value. Similarly, component users do not have to explicitly provide the parameters. Instead, conceptually, those parameters are bound and passed *implicitly*. This is useful in many practical situations, ranging from passing a type environment in a compiler, maintaining context information like input-positions in parsers, to associating the current request object in a web server implementation (Kiselyov et al., 2006).

Dynamic binding has a somewhat bad reputation – and not without a reason. In an untyped, unrestricted setting (as in the original Lisp (McCarthy, 1960)) one might bind dynamic variables accidentally. One might even forget to provide a binding, at all. We therefore follow the discipline of Lewis et al. (2000) who introduce *implicit parameters* as "dynamically bound values with explicit typing". Likewise, we track ambient values in the type of a function to inform callers that they might (accidentally or not) bind them. It also helps us guarantee that all ambients are eventually bound.

The structure of this chapter follows the generalization from ambient values to ambient control, that is, effect handlers. We gradually generalize ambient values in three steps, offering new programming abstractions and insight into effect handlers:

Ambient values
: We introduce ambient values and give examples in Section 3.1. Restricting us to ambient values highlights the aspect of effect handlers that effect operations are dynamically bound.

Ambient functions
: The main contribution of this chapter is ambient functions: these are *bound dynamically but evaluated in the lexical scope of their binding* (Section 3.2). Ambient functions highlight the aspect of effect handlers that effect operations are evaluated at the handler (the binding site) not at the call site. We show how this small change from ambient values improves abstraction. In particular, we show how ambient functions allow users to provide precise interfaces without leaking the (side) effects of any particular implementation into the types.

Ambient control
: We then generalize the notion of ambient functions to *ambient control*. Just like effect handlers, ambient control allows modifying the control flow. This is essential to implement control operations like exceptions and backtracking search (Section 3.3). Ambient control highlights the aspect of effect handlers that effect operations can capture the continuation to express advanced control structures – an aspect that has been the focus of the previous chapter.

We use the calculus of dynamic binding λ_{db} as defined by Moreau (1998) in its revised form by Kiselyov, Shan, and Sabry (2006) to formalize the semantics of ambient values (which coincides exactly). We then extend it, with ambient functions as calculus λ_{dbf}, and with ambient control as λ_{dbc}. After having introduced the different language features, we present a type system, extended with *ambient rows*, that guarantees that all ambients are eventually bound

(Section 3.4). We give a type- and semantics preserving *macro* translation (Felleisen, 1990) of ambient values and functions to ambient control (Section 3.5). Ambient control (λ_{dbc}) coincides exactly with effect handlers of the previous chapter (λ_{dch}). The two calculi only differ in syntax (Section 3.6). Nevertheless, we chose to present them a separately to emphasize the respective aspect we started with: delimited control and dynamic binding.

Even though we can translate ambient values and functions to ambient control (and effect handlers), we argue that each individual concept merits study. Following the *principle of least power* (Berners-Lee, 2005), each feature gradually adds expressiveness to ambient values. Explicitly distinguishing between ambient values, functions, and control makes it *(a)* easier to reason about programs – for example, a program only using ambient values does not alter the control flow, *(b)* easier to efficiently implement – compilers can use the additional information to generate code specialized to the more limited control flow, *(c)* easier to learn the new concepts – users can incrementally understand the generalizations from ambient values, via ambient functions, to ambient control. We thus believe that the incremental generalization from ambient values to control offers a novel and more approachable way of introducing effect handlers.

We implemented the language design presented in this chapter in the Koka language (Leijen, 2017c), which we use to provide code examples. The implementation is available at:

```
https://github.com/koka-lang/koka
```

Koka is a strict functional language that tracks (side) effects in the types (including exceptions and divergence). The reason to use Koka is two-fold: it already has a type system based on row-types, which we adapted to track ambient bindings, and the run-time system supports effect handlers, which we use to implement ambient control. However, the ideas described in this chapter also apply to other programming languages and are not tied to Koka.

3.1 Ambient Values

To illustrate the use ambient values, we repeat an example of the canonical paper on implicit parameters by Lewis, Launchbury, Meijer, and Shields (2000). For the example, we assume that we are writing a pretty printing library that produces pretty strings from documents. Our library has the following top-level entry-point:

```
fun pretty(d: doc): string
```

Unfortunately, deep inside the code, it has a hard-coded maximum display width (*i.e.*, `40`), which we would like to make configurable to use our pretty printer in different contexts with different display width.

```
... if (line.length ≤ 40) then ...
```

We have two choices to abstract over the maximum display width. We can either turn the width into a global (mutable) variable, or add an extra explicit width parameter and thread it around manually. Both options come with limitations. Global mutable state is difficult to encapsulate and test, passing extra parameters requires significant change to the code base.

However, with an *ambient value* we can solve this cleanly. First, we *declare* the type signature of an ambient value at the module level:

```
ambient val width: int
```

This way, we can simply refer to the ambient value `width` inside our library implementation:

```
... if (line.length ≤ width) then ...
```

The type system of Koka tracks the use of such ambient values in row types denoted between angle brackets. In particular, the new inferred type of the pretty printing function is:

```
fun pretty(d: doc): <width> string
```

The type expresses that `pretty` can only be used in a context, which provides a binding for the ambient value `width`. We can provide such a binding as follows:

```
fun pretty-thin(d: doc ): <> string {
  with val width = 40 in pretty(d)
}
```

Here, for the dynamic extent of evaluating the body expression `pretty(d)`, the ambient value `width` is dynamically bound to `40`. Importantly, the scope of the **`with`**-binder is not limited lexically. Note, how the inferred type of `pretty-thin` (that is, `<> string`) now reflects that there are no more unbound ambient values.

In our implementation, we also offer a statement form of the **`with`**-expression that scopes over the rest of the current block. Using this form we can write the previous example as function `pretty-thin` on the left:

```
fun pretty-thin(d) {
  with val width = 40;
  pretty(d)
}
```

```
fun pretty-wide(d: doc): <width> string {
  with val width = width * 2;
  pretty(d)
}
```

Of course, we can also re-bind (that is, *shadow*) ambient values. For example, function `pretty-wide` (on the right) pretty prints part of a document doubling the current display width. The type of `pretty-wide` reflects that, even though it binds `width`, it still depends on a `width` binding in its own context.

3.1.1 Ambient Values Operationally

To formalize the calculus of ambient values, Figure 3.1 repeats the calculus of dynamic binding (λ_{db}) by Moreau (1998) as presented by Kiselyov et al. (2006), slightly adapted to our setting. There is one difference to the presentation of Kiselyov et al. (2006): To ease comparison with calculi presented in the previous chapter, the operational semantics of ambient values uses a special evaluation context H_p (Gunter et al., 1995) instead of a side condition $p \notin \mathsf{bp}(\mathsf{E})$ on rule (*dval*) (Kiselyov et al., 2006). Otherwise, our presentation mostly differs in formatting. In this section, we present the operational semantics. Section 3.4 then presents the type-system.

Syntax:

Expressions	e	$::=$	$e(e)$	application
		$\mid$	$\mathsf{with}\ b_p\ \mathsf{in}\ \{\ e\ \}$	dynamic binding
		$\mid$	p	ambient name
		$\mid$	v	value
Binding	b_p	$::=$	$\mathsf{val}\ p\ =\ v$	ambient value binding
Values	v	$::=$	$x \mid c \mid \lambda x.\ e$	

Evaluation Contexts:

$$\mathsf{E} ::= \square \mid \mathsf{E}(e) \mid v(\mathsf{E}) \mid \mathsf{with}\ b_p\ \mathsf{in}\ \{\ \mathsf{E}\ \}$$
$$\mathsf{H}_p ::= \square \mid \mathsf{H}_p(e) \mid v(\mathsf{H}_p) \mid \mathsf{with}\ b_{p'}\ \mathsf{in}\ \{\ \mathsf{H}_p\} \quad \text{if } p \neq p'$$

Operational Semantics:

(δ)	$c(v)$	$\longrightarrow$	$\delta(c, v)$	if $\delta(c, v)$ is defined
(β)	$(\lambda x.\ e)(v)$	$\longrightarrow$	$e[x \mapsto v]$	
$(dret)$	$\mathsf{with}\ b_p\ \mathsf{in}\ \{\ v\ \}$	$\longrightarrow$	v	
$(dval)$	$\mathsf{with}\ \mathsf{val}\ p\ =\ v\ \mathsf{in}\ \{\ \mathsf{H}_p[p]\ \}$	$\longrightarrow$	$\mathsf{with}\ \mathsf{val}\ p\ =\ v\ \mathsf{in}\ \{\ \mathsf{H}_p[\ v\ ]\ \}$	

$$\frac{e \longrightarrow e'}{\mathsf{E}[e] \longmapsto \mathsf{E}[e']}\ [\textsc{cong}]$$

Figure 3.1. Language of ambient values, λ_{db} (adapted from Kiselyov et al., 2006).

Following Kiselyov et al. (2006), we assume two disjoint sets of variables: lexical variables denoted by x, and ambient names (dynamic variables) denoted by p. Note that ambient names are not values, they are effectful expressions. Meta variables b_p, ranging over binders, are indexed by the name of the ambient p they bind. The evaluation context E contains the clause $\mathsf{with}\ \mathsf{val}\ p\ =\ v\ \mathsf{in}\ \{\ \mathsf{E}\ \}$. This illustrates that dynamic bindings are part of the evaluation context (that is, the stack). The right-hand-side of an ambient value binding is syntactically restricted to be a value. This way, the bound expression is guaranteed to be fully evaluated at the definition site before it is bound.

The operational semantics is given by the four rules for the relation $\longrightarrow$. Like in the previous chapter, the relation $\longmapsto$ lets us evaluate according to the evaluation context. The most important reduction rule $(dval)$ captures the semantics of ambient values. It dynamically resolves an ambient binding p to its bound value v by searching in the evaluation context E. Similar to capture contexts in Section 2.2, the special evaluation context H_p is a restriction on contexts E and excludes other binders for p (Gunter et al., 1995). Ambient values thus resolve to the closest with-binding that *dynamically* surrounds the ambient value. In the reduction rule $(dval)$, we highlight the unmodified context of the *call site* to the ambient value p. This emphasize will become important in the following sections when discussion the generalizations of ambient values and ambient control.

3.1.2 Ambient Values Summarized

Let us summarize some key aspects of ambient values

Declared on the module level
Ambient values have to be *declared*, prior to their use. The declaration **`ambient val`** p: τ brings both the term name p and an equally named type into scope. We use the latter to track ambients as part of the type of a function (Section 3.4).

Resolved by name
Ambient values are resolved by name, not by type. This is similar to implicit parameters as proposed by Lewis et al. (2000) and to their implementation in Haskell. It is different to implicits in Scala (Odersky et al., 2017), which are resolved by type (see Section 3.7).

Dynamically scoped
Ambient values are dynamically scoped. Thus, lambdas do *not* close over bindings. This is illustrated by the following example:

```
with val width = 40;                with val width = 60;
val x = 1;                          val x = 2;
val f = fun() { width + x }         f()
```

The lambda `f` is created in a context on the left where `x` is bound to `1` and `width` is bound to `40`. Nevertheless, it only closes over the value of `x`, not over `width`. This also becomes visible in its inferred type `() → <width> int`. Hence, running the example results in `61`.

Type- and effect checked
The usage of ambients is tracked in the type of functions. The type and effect system guarantees, that all ambients are eventually bound. Section 3.4 presents a type and effect system.

Ambient types are inferred
While the ambient value `width` could be used in some deeply nested helper function of the pretty-printing library, we neither have to change the call to that helper function, nor to any other transitive caller. Also, by inferring the types, we do not need to change any of the function's type signatures. Being able to infer types is important for maintainability.

The following example again illustrates scoping of ambient bindings.

```
with val width = 40;
val g = fun() { with val width = 80 in width + 1 };
val h = with val width = 80 in (fun() { width + 1 });
println(g());
println(h())
```

Here we bind two functions `g` and `h`, and invoke them. The first `println(g())` prints `81`, since during the evaluation of `g()` the ambient value `width` is bound to its innermost binding of `80`. However, `println(h())` prints `41` instead: during evaluation of `h()`, the `width` is bound dynamically to `40` – the binding to `80` was only present during the evaluation of the function value as such. This illustrates that functions do not close over ambient values.

3.2 Ambient Functions

Of course we can bind function values as ambient values. We refer to those dynamically bound function values as *ambient lambdas*. For example, we can declare the ambient value

```
ambient val emit-naive: (s: string) → <> ()
```

and use the ambient lambda somewhere in our pretty-printing library to emit partial output:

```
...
match(doc) {
  Text(s) → emit-naive(s)
  ...
}
```

Unfortunately, the ambient value declaration has a type signature that severely limits the possible implementations. For example, an implementation might want to use the display width:

```
with val width = 80;
// type error: emit-naive cannot use ambient value 'width'
with val emit-naive = (fun(s: string){ println(s.truncate(width)) });
...
```

This leads to a type error, since the type signature of `emit-naive` promises to return unit without using any ambient values (`<> ()`). To work around this restriction, we can of course change the type signature of the value declaration to:

```
ambient val emit-naive : ((s : string) → <width> ())
```

But now the signature exposes accidental details of one particular implementation. Even worse, it might not be the desired semantics, since the width would be dynamically bound at the *call site* of `emit-naive`, while we often want to bind it at the *definition site* and encapsulate this implementation detail in the definition of `emit-naive`.

Ambient Values and Closure In this particular example, we could lookup the ambient value once, bind it to an explicit value and close over that value. In fact, the desired semantics is related to closure: the binding should be resolved at the definition site of the function, not the call site. However, not only the ambient bindings, which an ambient lambda uses, are resolved at the call site – the same also extends to other (control) effects. For example, `emit-naive` might append the output to a file as:

```
with val emit-naive = (fun(s){ ... width ... append(file, s) });
...
```

The function `append` uses the `io` effect. In Koka (Leijen, 2014), this is reflected in the type of the defined function (*i.e.*, `(string) → <width,io> ()`). Hence, just as before, this definition of the ambient value `emit-naive` does not coincide with the declaration and is rejected by the type checker.

3.2.1 Ambient Functions: Dynamic Binding with Lexical Scoping

As motivated above, we would like to encapsulate implementation effects at the definition site of a function. Using the traditional closure semantics suffices for simple values, but does not scale to more general (control) effects. What we need instead is our new feature of *ambient functions*, which we summarize by:

> Ambient functions are *dynamically bound*, but *statically evaluated* in the lexical context of their binding.

Similar to ambient values, we first declare an ambient function as:

```
ambient fun emit(s: string ): ()
```

Like with ambient values, the use of ambient functions is reflected in the type of an effectful program. For example, if we call `emit(...)` somewhere in our pretty-printing library, the inferred type of `pretty` becomes:

```
fun pretty(d: doc): <width, emit> string
```

The type now reflects that the function depends on ambient bindings for both the display width and an emitter function. Ambient functions are bound similar to ambient values. We can change our implementation of `pretty-thin` to also provide a binding for `emit`:

```
fun pretty-thin(d) {
  with val width = 40;
  with fun emit(s) { println(s.truncate(width)) };
  pretty(d)
}
```

This definition of `emit` is very different from the previous binding as an ambient value `emit-naive`. Like before, `emit` is dynamically bound and can be used for the extent of evaluating `pretty(d)`. However, when the ambient function `emit` is called, its body now executes in the lexical context of its definition and not in its calling context. In particular, the ambient value `width` in the body of `emit` now always resolves to `40` even if `width` is dynamically rebound inside `pretty(d)`.

Importantly, the fact that the implementation of `emit` uses the ambient value `width` is encapsulated at the *call site*. It neither leaks into the type of `emit` nor into its callers. Since emit additionally uses `println` (*i.e.*, the `console` effect), the inferred return type of `pretty-thin` is `<console> string`.

3.2.2 Ambient Functions Operationally

Figure 3.2 extends the syntax and operational semantics of λ_{db} to λ_{dbf} (pronounced "dynamic binding with functions") to add support for ambient functions. In contrast to the ambient value rule (*dval*), in rule (*dfun*) we first evaluate the function body e *outside* the calling evaluation context H_p. That is, the function body is evaluated in the *scope of its definition*. Again, we highlight the original call-site context to emphasize this. After evaluating the body, we resume in the original calling context with the result bound to a fresh variable y. While in the calculus we distinguish between calls to ambient lambdas (*i.e.*, $(p)(e)$) and ambient function calls (*i.e.*, $p(e)$), we make no such difference in the surface syntax of our Koka implementation. However,

Extended Syntax:

Expressions	e	$::=$	$\ldots$	
		$\mid$	$p(v)$	ambient call
Binding	b_p	$::=$	$\ldots$	
		$\mid$	$\mathsf{fun}\ p(x)\ \{\ e\ \}$	ambient function binding

Extended Evaluation Contexts:

$$\begin{array}{lll} \mathsf{E} & ::= \ldots \mid & p(\mathsf{E}) \\ \mathsf{H}_p & ::= \ldots \mid & p'(\mathsf{H}_p) \end{array}$$

Extended Operational Semantics:

$$\ldots$$
$$(\mathit{dfun}) \quad \mathsf{with\ fun}\ p(x)\ \{\ e\ \}\ \mathsf{in}\ \{\ \mathsf{H}_p[p(v)]\ \} \longrightarrow (\lambda y.\ \mathsf{with\ fun}\ p(x)\ \{\ e\ \}\ \mathsf{in}\ \{\ \mathsf{H}_p[\ y\ ]\ \})(e[x \mapsto v])$$

Figure 3.2. Extension of λ_{db} with ambient functions (λ_{dbf}).

we require the ambient signatures (declared at the top level of a module) to be globally unique. This way, the compiler (and users) *can statically* determine whether a call refers to an ambient function or a lambda bound by an ambient value by inspecting the corresponding signatures.

It is important to highlight the difference between the call to an ambient function and an ambient lambda. To see this difference, let us compare rule (*dfun*) with the following reduction using (*dval*) followed by (β):

$$\begin{array}{rcl} \mathsf{with\ val}\ p\ =\ \lambda x.\ e\ \mathsf{in}\ \{\ \mathsf{H}_p[(p)(v)]\ \} & \longrightarrow & \mathsf{with\ val}\ p\ =\ \lambda x.\ e\ \mathsf{in}\ \{\ \mathsf{H}_p[(\lambda x.\ e)(v)]\ \} \\ & \longmapsto & \mathsf{with\ val}\ p\ =\ \lambda x.\ e\ \mathsf{in}\ \{\ \mathsf{H}_p[e[x \mapsto v]]\ \} \end{array}$$

The only difference between the above reduction of an ambient lambda and rule (*dfun*) is the context in which the expression e is evaluated. For ambient lambdas it is context of the call site (that is, $\mathsf{with\ val}\ p\ =\ \lambda x.\ e\ \mathsf{in}\ \{\ \mathsf{H}_p[\ \square\ ]\ \}$). In contrast, the body of ambient functions is evaluated in is context of the definition site (that is, $(\lambda y.\ \ldots)(\square)$). This small difference in operational semantics makes a big difference in terms of encapsulation.

3.2.3 A Novel Abstraction Mechanism

Changing the evaluation context to the defining scope seems like a minor extension, but it has profound implications on abstraction. In particular, we are now able to confine ambient bindings and other used effects to the lexical context of the ambient function definition. Continuing with our example, we may want to collect all generated output using local mutable state:

```
fun emit-collect(action) {
  var out := "";
  with fun emit(s) { out := out + s + "\n" } in action();
  out
}
```

The dynamically bound function `emit` can access the locally scoped mutable variable `out` from its body and update it. As we will see in Section 3.3.4, our local mutable variables are stack allocated and thus should not be accessed outside of their dynamic scope. In general, variable access can escape the scope through closures (Osvald et al., 2016). Similarly, if we would bind the `emit-naive` function as an ambient value as in

```
with val emit-naive = (fun(s) { out := out + s + "\n" }) in action()
```

a reference to the `out` variable might indirectly escape through `emit-naive`. To prevent this, Koka not only tracks ambients but also the use of mutable state and other effects as part of the type. Thus, the Koka type checker rejects this example due to the restrictive ambient declaration of `emit-naive`, which does not admit mutable state.

In contrast, the body of ambient functions executes in the lexical context of their definition and our example type-checks. Any implementation related effects, such as the use of `out`, are encapsulated in the scope of `emit-collect`. At the same time, in `action`, the ambient function `emit` can be used as a function of type `(string) → <emit> ()`. For example, the expression

```
emit-collect(fun() { emit("hello"); emit("world") })
```

just returns the string `"hello\nworld\n"` without any side effects observable from the outside.

Closures and mutable variables Readers with a background in C#, JavaScript, or Scala would expect this very behavior. Those languages inhabit a middle ground: lambda expressions can capture local variables by reference and would thus behave like in our example. While they encapsulate the state-effect, they still leak other effects, like throwing an exception, to the calling context.

Example: Depth-First Traversal

To illustrate the abstraction power of ambient functions, we adapt the example of Lewis, Launchbury, Meijer, and Shields (2000). The authors describe a depth-first traversal of a graph (King and Launchbury, 1995), where the auxiliary function `dfs-loop` is implicitly parameterized by three functions: one to mark vertices, one to query if a vertex is marked, and one to get the children of a vertex in the graph. In the original example, the authors then use `runST` (Peyton Jones and Launchbury, 1995) to efficiently implement the marking with isolated mutable state. Figure 3.3 translates the example to Koka, using ambient functions.

The `dfs` function completely encapsulates the use of (scoped) mutable state to efficiently implement the marking of the visited vertices. The inferred type of `dfs-loop` reflects that it only depends on the declared ambient functions and has no other effects:

```
fun dfs-loop(vs : list<vertex>) : <children, marked, mark> list<rose>
```

In contrast, Lewis et al. (2000) bind functions as *values* and thus leak the effects (that is, mutable state) of one particular implementation into the type of the `dfs-loop` function. The loop needs to be written in a monadic style and has the type:

```
dfsLoop :: (?children :: Graph → [Vertex],
            ?marked :: Vertex → ST s Bool,
            ?mark :: Vertex → ST s ()) ⇒ [Vertex] → ST s [Rose]
```

```
alias vertex = int
type graph { ... }
type rose  { Rose(v: vertex, sub: list<rose> ) }

ambient fun marked(v: vertex): bool
ambient fun mark(v: vertex ): ()
ambient fun children(v: vertex ): list<vertex>

fun dfs(g: graph, vs: list<vertex>): list<rose> {
  var visited := vector(g.bound, False);
  with fun children(v) { g.gchildren(v) };
  with fun marked(v)   { visited[v] };
  with fun mark(v)     { visited[v] := True };
  dfs-loop(vs)
}

fun dfs-loop(vs: list<vertex>) { match(vs) {
  Nil → Nil
  Cons(v, rest) →
    if (marked(v)) then { dfs-loop(rest) }
    else { mark(v); Cons(Rose(v, dfs-loop(children(v))), dfs-loop(rest)) }
}}
```

Figure 3.3. Implementation of a depth-first traversal (adapted from Lewis et al., 2000). Call sites of ambient functions highlighted.

Here the `ST` effect, used in the implementation of the operations, leaks into the definition of `dfsLoop`. Note also how the type of `dfsLoop` is quite verbose. With implicit parameters, the names of implicits (like `?children`) are not declared on the top level. This is more flexible but also leads to larger type signatures, as it is the case for `dfsLoop`. With explicit type declarations for ambient values, the types are more concise and allow simplified type checking at the use site of an ambient function.

Ambient functions encapsulate used effects as implementation details. Encapsulation is particularly relevant when combining ambient functions with general control effects, like exceptions. To express control effects, in the next section, we thus perform a last step of generalization.

3.3 Ambient Control

On our journey to effect handlers, we started with ambient values, which are dynamically scoped. We continued with the generalization of ambient functions, which encapsulate effects at the definition site. We have seen the interaction between stack-allocated mutable state and ambient functions and alluded to the fact that ambient functions integrate well with other control effects, like exceptions. However, so far our language does not support control effects.

To also express general control effects in our unified framework, we perform one further step of generalization: from ambient functions to *ambient control*. Ambient functions are evaluated in their defining lexical scope. But like regular functions they return to the calling context. Ambient control additionally *returns* to the defining lexical scope – similar to how exceptions "return" to their innermost **`try`** block.

3.3.1 Aborting Control

To motivate ambient control, we follow Leijen (2016) and use ambient functions and ambient control to implement parsers. First, we declare the two ambients `next` and `fail`:

```
ambient fun next(): char
ambient control fail(msg: string): a
```

Using the two ambients, we can readily write a parser that expects a certain character in the input stream or fails otherwise:

```
fun expect(e: char): <next, fail> char {
  val c = next();
  if (c == e) then c else fail("Expected " + e + " but got " + c)
}
```

Similarly, the parser that exactly accepts the word `"hello"` can be expressed by:

```
fun hello(): <next, fail> () {
  expect('h'); expect('e'); expect('l'); expect('l'); expect('o'); ()
}
```

As we will see, syntactic differences notwithstanding, ambient control is identical to effect handlers from Section 2.3. It is thus not surprising, that ambient control also subsumes exception handling. In our case, `fail` corresponds to **`throw`** and an ambient control binding corresponds to a **`try`** handler. For example, here is a function that transforms an exception throwing action to a `maybe<a>` result:

```
fun to-maybe(action: () → <fail|e> a): e maybe<a> {
  with control fail(msg) { Nothing };
  Just(action())
}
```

The syntax `maybe<a>` denotes type constructor application to the universally quantified type parameter `a`.

A note on effect polymorphism Koka implements effect (and ambient) polymorphism in terms of row polymorphism (Leijen, 2005, 2014, 2017c). In fact, the use of ambients is just one particular class of effects that we track in the types. In the above example, `action` can thus use arbitrary other effects `e` besides the `fail` effect. The binder `to-maybe` binds and removes the ambient `fail` from the list of effects, while the resulting program still uses effects `e`. The syntax `<p|e>` denotes row extension with the abbreviations defined in Section 2.4. Additionally, `e` is short for the empty row extension `<|e>`. All one character type- and row-variables implicitly introduce a universal quantification. The full type signature of `to-maybe` thus is: **`forall`**`<a, e>. (action: () → <fail|e> a) → <|e> maybe<a>`.

To run the parser, we also need to define a binding for the ambient function `next`:

```
fun reader(input: string, action: () → <next, fail|e> a): <fail|e> (a, string) {
  var cs := input.list;
  val res = with fun next() {
    match(cs) {
      Nil        → fail("Unexpected EOS")
      Cons(c,cc) → { cs := cc; c }
    }
  } in action();
  (res, cs.string)
}
```

Similar to `emit-collect`, the function `reader` uses scoped mutable state to keep track of the remaining characters. It binds `next` in the dynamic extent of `action` to return the next character or otherwise uses the ambient control `fail` if no such character is available. Note, that this time, we purposefully do not encapsulate the use of `fail`. The program `action` itself can also use `fail` and potentially other effects `e`.

Finally, we can use the two binder functions `to-maybe` and `reader` to check whether a given input has the string `"hello"` as a prefix.

```
fun parse-hello(input: string): maybe<((), string)> {
  to-maybe(fun() { reader(input, fun() { hello() }) })
}
```

If `fail(...)` is called, it will directly return to its definition point with the value `Nothing` as the result of `parse-hello`. This is the case when invoking `parse-hello("help")`. In contrast, calling `parse-hello("hello world")` will successfully parse the prefix and yield `Just(((), " world"))`.

Remark Higher-order functions like `to-maybe` and `reader(input)` can be thought of as first-class binders for ambients – for `fail` respectively `next` in this case. In Koka, we treat those binder functions uniformly like **`val`**, **`fun`**, or **`control`** binders and use the following extension of the **`with`** statement syntax:

```
fun parse-hello(input: string): maybe<((), string)> {
  with to-maybe;
  with reader(input);
  hello()
}
```

That is, if **`with`** is followed by an expression, i.e. not a keyword **`val`**, **`fun`**, or **`control`**, we treat the expression as a higher-order binder function and pass the rest of the block as a function argument (Leijen, 2018b, Sec. 4.4). Also, note the curried application of `reader`.

3.3.2 Resuming Control

In the previous parsing example, we defined a new ambient control `fail` to abort the parsing in case of an error condition. Using `fail` together with `next` allowed us to describe expected sequences of characters, like `"hello"`. However, one important feature of parsers is still missing:

how can we express alternative productions in a grammar? To this end, we declare a second ambient control function `choice`:

```
ambient control choice(): bool
```

We can use `choice` to recognize the language `AsB ::= 'a' AsB | b`, multiple characters `a` followed by a single `b`, with the following parser:

```
fun as-b() : <fail, next, choice> int {
  if (choice()) then { expect('a'); as-b() + 1 }
  else { expect('b'); 0 }
}
```

As it turns out, ambient control functions like `fail` and `choice` not only *return* to their defining context, but also allow us to *resume* at the call site. To enable this, a control binding is passed an extra (implicit) argument `resume` that can be used to return to the calling context. Like in the previous chapter, `resume` is a first-class function and represents the continuation delimited by the corresponding control binder.

Using `resume`, we can bind `fail` and `choice` to collect all possible parse results in a list:

```
fun collect(action: () → <fail, choice|e> a): e list<a> {
  with control choice() { append(resume(True), resume(False)) };
  with control fail()   { Nil };
  Cons(action(), Nil)
}
```

The binder for `choice` immediately returns to the call site with the value `True`. Resuming will eventually produce a value of type `a`, which is wrapped in a singleton list. Since the extent of the continuation `resume` is delimited by the control binder for `choice`, both calls to `resume` will return a list of results. Similarly, the extent of the continuation captured by calling `fail` is delimited by the `fail` binder. The order of binders is important here: since the binder for `choice` is the outermost one, capturing (and discarding) the continuation at the binder for `fail` only aborts the *current* alternative with an empty list.

Finally, we can run the parser with

```
fun parse-as-b(input: string): list<int> {
  with collect;
  with reader(input);
  as-b()
}
```

giving `Cons(3, Nil)` for `parse-as-b("aaab")` and `Nil` for `parse-as-b("aaac")`. Importantly, we can only parse a string like `"b"` since local variables in `reader` are stack-allocated, not heap-allocated. Conceptually, the captured continuation corresponds to a segment of the runtime stack and also includes the values of mutable, stack-local variables at the time of capturing. This enables the necessary backtracking behavior, when resuming a second time with `False`. If we would use global (heap-allocated) state instead, the call to `resume(False)` would not backtrack the position of the input stream, but would continue where the (potentially failed) alternative `resume(True)` last left off. The same also holds when swapping the order of **`with`** `collect` and **`with`** `reader(input)`. State changes are then persisted across different alternatives.

Extended Syntax:

$$\begin{array}{lrll} \text{Binding} & b_p & ::= & \ldots \\ & & \mid & \mathsf{control}\ p(x,\ k)\ \{\ e\ \} \qquad \text{ambient control binding} \end{array}$$

Extended Operational Semantics:

$$\begin{array}{l} \ldots \\ (dctl)\quad \mathsf{with\ control}\ p(x,\ k)\ \{\ e\ \}\ \mathsf{in}\ \{\ \mathsf{H}_p[p(v)]\ \} \longrightarrow e[x{\mapsto}v,\ k{\mapsto}\lambda y.\ \mathsf{with\ control}\ p(x,\ k)\ \{\ e\ \}\ \mathsf{in}\ \{\ \mathsf{H}_p[\ y\ ]\ \}] \end{array}$$

Figure 3.4. Extension of λ_{dbf} with ambient control (λ_{dbc}).

3.3.3 Ambient Control Operationally

Figure 3.4 defines λ_{dbc} (pronounced "dynamic binding with control") as an extension of λ_{dbf} with ambient control. In contrast to the surface syntax, in the calculus we explicitly bind the continuation argument k. For *abortive ambient control* binders, which discard the continuation k (like `to-maybe` for `fail`), we can simplify the reduction rule (*dctl*) to:

$$\begin{array}{lll} (abort) & \mathsf{with\ control}\ p(x,\ k)\ \{\ e\ \}\ \mathsf{in}\ \{\ \mathsf{H}_p[p(v)]\ \}\ \text{ where }\ k\ \notin e & \longrightarrow (\lambda x.\ e)(v) \\ & & \longrightarrow e[x{\mapsto}v] \end{array}$$

This derived rule is similar to the rule (*dfun*) for ambient functions except that we do not continue evaluation in the original context.

3.3.4 Mutable Variables as Ambient Control

The previous example illustrated that the interaction between local mutable state and ambient control is subtle due to the first-class (delimited) continuation captured by `resume`. This is already remarked upon by Moreau (1998, p. 276), who calls for "a single framework integrating continuations, side effects, and dynamic binding.". Kiselyov et al. (2006) studied the in the context of delimited continuations.

However, it turns out we can view local mutable state in terms of ambient control and thus do not need a special semantic treatment. In particular, we can use the translation by Kammar and Pretnar (2017, Fig. 7) who show how to express mutable dynamic variables in terms of algebraic effect handlers. We reuse their translation, except that we use ambient control instead of effect handlers. First, we α-rename such that local variables are uniquely named. For every local variable `s`, we define ambient declarations (on the left) and translate lexically bound occurrences of `s` either to $\texttt{get}_\texttt{s}$ or $\texttt{put}_\texttt{s}$ operations (on the right):

```
ambient control get_s(): τ                s          ⇝  get_s()
ambient control put_s(x: τ): ()           s := expr  ⇝  put_s(expr)
```

Every binding of a local variable `s` of some type τ is then translated to a function application of a $\texttt{local}_\texttt{s}$ function:

```
var s: τ := init;  ⇝  local_s(init, fun() { ... })
...
```

Finally, the binder function $\texttt{local}_\text{s}$ provides semantics to the operations $\texttt{get}_\text{s}$ and $\texttt{put}_\text{s}$.

```
fun local_s(init: τ, action: () → <get_s, put_s|e> a): e a {
  val f = { with control get_s()  { (fun(st) { resume(st)(st) }) };
            with control put_s(x) { (fun(st) { resume(())(x) }) };
            val x = action();
            (fun(st){ x })
          }
  f(init)
}
```

The main difference to the translation in Kammar and Pretnar (2017) is that we use two separate ambient control functions, while they group the two operations under a single handler. Otherwise, both translations express mutable state by returning a function that gets the current state as input. This as also essentially the way Kiselyov et al. (2006) express dynamic binding in terms of delimited control. To implement the lookup of a binding $\texttt{p}$, they shift to the binder with $\mathsf{shift}_p\ \{\ f \Rightarrow \lambda y.\ f(y)(y)\ \}$ (Kiselyov et al., 2006, Fig. 3). This corresponds to the definition of $\texttt{get}_\text{s}$, mapping f to our `resume` and y to our `st` parameter.

Of course, this translation is useful from a semantic perspective, but in a practical implementation, we can use more efficient mechanisms. The Koka implementation directly uses mutation of variables that are (handler) stack allocated. It also ensures that the state is properly captured as part of the stack and restored on resume.

3.4 Type-Safety of Ambient Values

While our examples in Koka already used the type- and effect-system, so far, we only presented the dynamic semantics of our calculi. In this section, we start again with λ_{db} and repeat the type system of Kiselyov et al. (2006). We then extend the type system with ambient-rows to assert that all ambient values are eventually bound. We show that our type system is a conservative extension of the original type system by Kiselyov et al. (2006). Finally, we extend the row-based effect system with support for ambient functions and control.

Figure 3.5 repeats the standard typing rules for λ_{db}. Like with the operational semantics of the base calculus (Figure 3.1), except for formatting, the typing rules are the same as the ones of Kiselyov et al. (2006). Like Kiselyov et al., we assume a global mapping Σ from dynamic variables p to signatures $\mathsf{val}\ \tau$. We denote signature extension by $\Sigma, p : \mathsf{val}\ \tau$ and require that the mapping is globally unique.

The evaluation can get stuck if an ambient value is not bound, and Kiselyov et al. (2006) define such terms as *bp*-stuck:

Definition 2. (*bp-stuck*)
A term is *bp*-stuck if it has the form $\mathsf{E}[p]$ where $p \notin \mathsf{bp}(\mathsf{E})$.

The set of bound ambient variables in a context E is denoted as $\mathsf{bp}(\mathsf{E})$ and defined by:

$$\begin{array}{lll} \mathsf{bp}(\square) & = \emptyset & \mathsf{bp}(v(\mathsf{E})) = \mathsf{bp}(\mathsf{E}) \\ \mathsf{bp}(\mathsf{E}(e)) & = \mathsf{bp}(\mathsf{E}) & \mathsf{bp}(\mathsf{with}\ b_p\ \mathsf{in}\ \{\ \mathsf{E}\ \}) = \{p\} \cup \mathsf{bp}(\mathsf{E}) \end{array}$$

Kiselyov et al. use a side condition on rule (*dval*) instead of the special context H_p. However, by induction over the evaluation context, it can be seen that $\mathsf{E} = \mathsf{H}_p$ if and only if $p \notin \mathsf{bp}(\mathsf{E})$.

Syntax of Types:

Types	τ	$::=$	$\mathit{unit} \mid \mathit{bool} \mid \ldots$	builtin types
		$\mid$	$\tau \to \tau$	functions
Ambient Names	p	$::=$	$\mathit{width} \mid \mathit{next} \mid \mathit{fail} \mid \ldots$	
Type Environment	Γ	$::=$	$\emptyset \mid \Gamma, x : \tau$	
Ambient Signatures	Σ	$::=$	$\emptyset \mid \Sigma, p : \mathsf{val}\ \tau$	

Type Rules:

$$\frac{\Gamma(x) \;=\; \tau}{\Gamma \vdash_{db} \; x \;:\; \tau}\ [\textsc{var}] \qquad \frac{\Sigma(p) \;=\; \mathsf{val}\ \tau}{\Gamma \vdash_{db} \; p \;:\; \tau}\ [\textsc{dval}]$$

$$\frac{\Gamma, x : \tau_1 \vdash_{db} \; e \;:\; \tau_2}{\Gamma \vdash_{db} \; \lambda x.e \;:\; \tau_1 \to \tau_2}\ [\textsc{lam}] \qquad \frac{\Gamma \vdash_{db} \; e_1 \;:\; \tau_1 \to \tau_2 \qquad \Gamma \vdash_{db} \; e_2 \;:\; \tau_2}{\Gamma \vdash_{db} \; e_1(e_2) \;:\; \tau_2}\ [\textsc{app}]$$

$$\frac{\Sigma(p) \;=\; \mathsf{val}\ \tau_1 \qquad \Gamma \vdash_{db} \; v \;:\; \tau_1 \qquad \Gamma \vdash_{db} \; e \;:\; \tau_2}{\Gamma \vdash_{db} \; \mathsf{with\ val}\ p \;=\; v\ \mathsf{in}\ \{\ e\ \} \;:\; \tau_2}\ [\textsc{wval}]$$

Figure 3.5. Standard typing rules for λ_{db} (adapted from Kiselyov et al., 2006, Figure 2).

That is, we can equivalently define a term to be *bp*-stuck, if it has the form $\mathsf{H}_p[p]$. The definition of *bp*-stuck terms thus closely resembles Definition 1 of unhandled effects (Section 2.4).

Unbound ambient values notwithstanding, the type system is sound and Kiselyov et al. (2006) prove progress and preservation:

Theorem 2. (*Preservation*)
If $\Gamma \vdash_{db} \; e \;:\; \tau$ and $e \longmapsto e'$, then $\Gamma \vdash_{db} \; e' \;:\; \tau$.

Theorem 3. (*Progress*)
If $\emptyset \vdash_{db} \; e \;:\; \tau$ and e is not a value and not *bp*-stuck, then $e \longmapsto e'$ for some term e'.

3.4.1 Effect Safety for Ambient Values

While the type system by Kiselyov et al. (2006) is pleasantly simple, it does not prevent *bp*-stuck terms, which is left to future work by the authors. To address this issue, Figure 3.6a defines more precise type rules for λ_{db} that additionally track the use of ambient values. Following the original type-system of Koka (Leijen, 2014, 2017c), we annotate every function arrow with an *ambient row* π. Like in Section 2.4, we introduce two typing judgments. The judgment $\Gamma \vdash_{val} \; v \;:\; \tau$ to type values, and the judgment $\Gamma \vdash_{exp} \; e \;:\; \tau \mid \pi$ to type expressions, which can use the ambients specified in row π.

To focus on the aspect of dynamic binding, and to allow a better comparison with the type system of Kiselyov et al., our type system is monomorphic. Leijen (2005) shows how rows can

be naturally extended with polymorphism while allowing full unification, making them well suited to combine with Hindley-Milner (Milner, 1978; Hindley, 1969) style type inference. In our implementation, we macro express ambient values, functions, and control in terms of effect handlers (see Section 3.6). This way, we are able to reuse Koka's support for type- and effect polymorphism (Leijen, 2017c).

Tracking ambients in effect rows Following Leijen (2005), the ambient rows can contain multiple occurrences of a name and are considered equal up to the order of the ambient names in the row (see Section 2.4, Figure 2.6). As Leijen points out, allowing duplicates in rows is important for typing ambient bindings that refer themselves to the same ambient name. For example, consider typing $(\lambda x.\ \mathsf{with\ val}\ p\ =\ x\ \mathsf{in}\ \{\ e\ \})(p)$:

$$\frac{\dfrac{\dots}{\Gamma \vdash_{exp}\ \lambda x.\mathsf{with\ val}\ p\ =\ x\ \mathsf{in}\ \{\ e\ \}\ :\ \tau_1 \to \langle p|\pi\rangle\ \tau|\ \langle p|\pi\rangle}\ [\textsc{lam}] \quad \dfrac{\Sigma(p)\ =\ \mathsf{val}\ \tau_1}{\Gamma \vdash_{exp}\ p\ :\ \tau_1\ |\ \langle p|\pi\rangle}\ [\textsc{dval}]}{\Gamma \vdash_{exp}\ (\lambda x.\mathsf{with\ val}\ p\ =\ x\ \mathsf{in}\ \{\ e\ \})(p)\ :\ \tau|\ \langle p|\pi\rangle}\ [\textsc{app}]$$

which means that the first premise is typed as:

$$\frac{\Sigma(p)\ =\ \mathsf{val}\ \tau_1 \quad \Gamma, x:\tau_1 \vdash_{exp}\ x\ :\ \tau_1\ |\ \langle p|\pi\rangle \quad \Gamma \vdash_{exp}\ e\ :\ \tau|\ \langle p|\langle p|\pi\rangle\rangle}{\Gamma, x:\tau_1 \vdash_{exp}\ \mathsf{with\ val}\ p\ =\ x\ \mathsf{in}\ \{\ e\ \}\ :\ \tau|\ \langle p|\pi\rangle}\ [\textsc{wval}]$$

This leads to typing e with two occurrences of p in the ambient row. Having duplicates keeps the system simple and avoids the need for row constraints (Rémy, 1994).

Conservative Extension

Our type system is a conservative extension of the original type system by Kiselyov et al. (2006). If we define a simple erasure function $\hat{\cdot}\ :\ \tau_{imp} \to \tau_{db}$ as

$$\begin{aligned} \hat{c} \qquad\qquad &= c \\ (\tau_1 \widehat{\to\ \pi}\ \tau_2) &= \hat{\tau_1} \to \hat{\tau_2} \end{aligned}$$

and extend it over type environments, we can then state the following lemma:

Lemma 6. (*Conservative Extension*)
If $\Gamma \vdash_{exp}\ e\ :\ \tau|\ \pi$ then $\hat{\Gamma} \vdash_{db}\ e\ :\ \hat{\tau}$.

Immediate by erasing ambient rows from the derivation and removing identity (VAL) derivations.

3.4.2 Effect Safety for Ambient Functions and Control

Figure 3.6b extends the type system of the base language λ_{db} (Figure 3.6a) with rules for ambient functions and ambient control. The ambient signatures Σ are extended with new forms $\mathsf{fun}\ \tau_1 \to \tau_2$ and $\mathsf{control}\ \tau_1 \to \tau_2$ for ambient function declarations and ambient control declarations, correspondingly. As mentioned above, we require that bindings are globally unique. That is, for a given ambient name p, from the signature environment we can uniquely determine whether it is an ambient value, function, or control.

Syntax of Types:

Types	τ	$::=$	$\mathit{unit} \mid \mathit{bool} \mid \ldots$	builtin types
		$\mid$	$\tau \to \pi\ \tau$	effectful functions
Ambient Row	π	$::=$	$\langle\,\rangle$	empty row
		$\mid$	$\langle p \mid \pi \rangle$	row extension
Ambient Names	p	$::=$	$\mathit{width} \mid \mathit{next} \mid \mathit{fail} \mid \ldots$	
Type Environment	Γ	$::=$	$\emptyset \mid \Gamma, x : \tau$	
Ambient Signatures	Σ	$::=$	$\emptyset \mid \Sigma, p : \mathsf{val}\ \tau$	

Type Rules:

$$\frac{\Gamma(x) \;=\; \tau}{\Gamma \vdash_{val} \; x \;:\; \tau}\ [\textsc{var}] \qquad \frac{\Gamma \vdash_{val} \; v \;:\; \tau}{\Gamma \vdash_{exp} \; v \;:\; \tau \mid \pi}\ [\textsc{val}] \qquad \frac{\Sigma(p) \;=\; \mathsf{val}\ \tau}{\Gamma \vdash_{exp} \; p \;:\; \tau \mid \langle p \mid \pi \rangle}\ [\textsc{dval}]$$

$$\frac{\Gamma, x : \tau_1 \vdash_{exp} \; e \;:\; \tau_2 \mid \pi}{\Gamma \vdash_{val} \; \lambda x.e \;:\; \tau_1 \to \pi\ \tau_2}\ [\textsc{lam}] \qquad \frac{\Gamma \vdash_{exp} \; e_1 \;:\; \tau_1 \to \pi\ \tau_2 \mid \pi \quad \Gamma \vdash_{exp} \; e_2 \;:\; \tau_2 \mid \pi}{\Gamma \vdash_{exp} \; e_1(e_2) \;:\; \tau_2 \mid \pi}\ [\textsc{app}]$$

$$\frac{\Sigma(p) \;=\; \mathsf{val}\ \tau_1 \quad \Gamma \vdash_{val} \; v \;:\; \tau_1 \quad \Gamma \vdash_{exp} \; e \;:\; \tau_2 \mid \langle p \mid \pi \rangle}{\Gamma \vdash_{exp} \; \mathsf{with\ val}\ p \;=\; v\ \mathsf{in}\ \{\ e\ \} \;:\; \tau_2 \mid \pi}\ [\textsc{wval}]$$

(a) Improved type rules for λ_{db}. We add *ambient rows* to ensure all ambient values are bound.

Extended Syntax of Types:

Ambient Signature	Σ	$::=$	$\ldots \mid \Sigma, p : \mathsf{fun}\ \tau_1 \to \tau_2 \mid \Sigma, p : \mathsf{control}\ \tau_1 \to \tau_2$	

Extended Type Rules:

$$\frac{\Sigma(p) \;=\; \mathsf{fun}\ \tau_1 \to \tau_2 \quad \Gamma \vdash_{val} \; v \;:\; \tau_1}{\Gamma \vdash_{exp} \; p(v) \;:\; \tau_2 \mid \langle p \mid \pi \rangle}\ [\textsc{dfun}] \qquad \frac{\Sigma(p) \;=\; \mathsf{control}\ \tau_1 \to \tau_2 \quad \Gamma \vdash_{val} \; v \;:\; \tau_1}{\Gamma \vdash_{exp} \; p(v) \;:\; \tau_2 \mid \langle p \mid \pi \rangle}\ [\textsc{dctl}]$$

$$\frac{\Sigma(p) \;=\; \mathsf{fun}\ \tau_1 \to \tau_2 \quad \Gamma, x : \tau_1 \vdash_{exp} \; e_1 \;:\; \tau_2 \mid \pi \quad \Gamma \vdash_{exp} \; e_2 \;:\; \tau \mid \langle p \mid \pi \rangle}{\Gamma \vdash_{exp} \; \mathsf{with\ fun}\ p(x)\ \{\ e_1\ \}\ \mathsf{in}\ \{\ e_2\ \} \;:\; \tau \mid \pi}\ [\textsc{wfun}]$$

$$\frac{\Sigma(p) \;=\; \mathsf{control}\ \tau_1 \to \tau_2 \quad \Gamma,\ x :\ \tau_1,\ k :\ \tau_2 \to \pi\ \tau \vdash_{exp} \; e_1 \;:\; \tau \mid \pi \quad \Gamma \vdash_{exp} \; e_2 \;:\; \tau \mid \langle p \mid \pi \rangle}{\Gamma \vdash_{exp} \; \mathsf{with\ control}\ p(x,\ k)\ \{\ e_1\ \}\ \mathsf{in}\ \{\ e_2\ \} \;:\; \tau \mid \pi}\ [\textsc{wctl}]$$

(b) Extending the typing rules of Figure 3.6a to ambient functions and control.

Figure 3.6. Effect safety for ambient values, functions, and control. Ambient rows π are considered equivalent up to the order of the names in the row.

Translation of Terms:

$$
\begin{array}{lcll}
[\![\ x\ ]\!] & = & x & \\
[\![\ \lambda x.e\ ]\!] & = & \lambda x.\ [\![\ e\ ]\!] & \\
\\
[\![\ e(e')\ ]\!] & = & [\![\ e\ ]\!]([\![\ e'\ ]\!]) & \\
[\![\ p\ ]\!] & = & p(()) & \text{with } p\ :\ \mathsf{val}\ \tau \in \Sigma \\
[\![\ p(v)\ ]\!] & = & p([\![\ v\ ]\!]) & \text{with } p\ :\ \mathsf{fun}\ \tau \to \tau'\ \in \Sigma \text{ or } p\ :\ \mathsf{control}\ \tau \to \tau'\ \in \Sigma \\
[\![\ \mathsf{with}\ b_p\ \ \mathsf{in}\ \{\ e\ \}\ ]\!] & = & \mathsf{with}\ [\![\ b_p\ ]\!]\ \ \mathsf{in}\ \{\ [\![\ e\ ]\!]\ \} &
\end{array}
$$

Translation of Binders:

$$
\begin{array}{lcll}
[\![\ \mathsf{val}\ p\ =\ v\ ]\!] & = & \mathsf{control}\ p((),\ k)\ \{\ k([\![\ v\ ]\!])\ \} & \text{with } k\ \notin \mathsf{fv}(v) \\
[\![\ \mathsf{fun}\ p(x)\ \{\ e\ \}\ ]\!] & = & \mathsf{control}\ p(x,\ k)\ \ \{\ k([\![\ e\ ]\!])\ \} & \text{with } k\ \notin \mathsf{fv}(e) \text{ and } k \neq x \\
[\![\ \mathsf{control}\ p(x,\ k)\ \{\ e\ \}\ ]\!] & = & \mathsf{control}\ p(x,\ k)\ \ \{\ [\![\ e\ ]\!]\ \}\ ] & \\
\end{array}
$$

Translation of Evaluation Contexts:

$$
\begin{array}{lcl}
[\![\ \Box]\!] & = & \Box \\
[\![\ \mathsf{E}(e)\ ]\!] & = & [\![\ \mathsf{E}\ ]\!]([\![\ e\ ]\!]) \\
[\![\ v(\mathsf{E})\ ]\!] & = & [\![\ v\ ]\!]([\![\ \mathsf{E}\ ]\!]) \\
[\![\ \mathsf{with}\ b_p\ \ \mathsf{in}\ \{\ \mathsf{E}\ \}\ ]\!] & = & \mathsf{with}\ [\![\ b_p\ ]\!]\ \ \mathsf{in}\ \{\ [\![\ \mathsf{E}\ ]\!]\ \}
\end{array}
$$

Translation of Signatures:

$$
\begin{array}{lcl}
[\![\ \emptyset]\!] & = & \emptyset \\
[\![\ \Sigma, p\ :\ \mathsf{val}\ \tau]\!] & = & [\![\ \Sigma\ ]\!],\ p:\ \mathsf{control}\ \mathit{unit} \to \tau \\
[\![\ \Sigma, p\ :\ \mathsf{fun}\ \tau \to \tau'\ ]\!] & = & [\![\ \Sigma\ ]\!],\ p:\ \mathsf{control}\ \tau \to \tau' \\
[\![\ \Sigma, p\ :\ \mathsf{control}\ \tau \to \tau'\ ]\!] & = & [\![\ \Sigma\ ]\!],\ p:\ \mathsf{control}\ \tau \to \tau'
\end{array}
$$

Figure 3.7. Translating ambient values and ambient functions to ambient control. The translation is homomorphic except for the highlighted cases.

We also extend the type rules with two new rules for checking ambient functions (WFUN) and control (WCTL) binders. The rules are similar to the WVAL rule in Figure 3.6a where the ambient name p is discharged from the ambient row. The type of the resumption function k in the type rule for control is interesting: it gets an argument of type τ_2 (the result type of the ambient p) and returns a value of τ, (the result type of the body e_2). Since the body of the continuation is evaluated under a with binding (see (*dctl*)), the ambient row is just π without p.

To show semantic soundness of the (extended) system, we perform two steps. Firstly, we show that ambient values and ambient functions are expressible in terms of ambient control. The translation is type and semantics preserving. Some minor syntactic differences notwithstanding, the target language with ambient control λ_{dbc} and its type system is identical to λ_{dch} (Section 2.4). Semantic soundness of λ_{dbc} and its type system follows as a corollary.

3.5 Ambient Values and Ambient Functions as Ambient Control

In this chapter, we performed two steps of generalization. First from ambient values to ambient functions, second from ambient functions to ambient control. Ambient control truly generalizes the two other features: ambient values and ambient functions are expressible (in the sense of Felleisen (1990)) in terms of ambient control. Additionally, as we will see in the next section, our language of ambient control is identical to our typed language with effect handlers λ_{dch} (Section 2.4). Maybe unsurprisingly, the generalization from ambient values to ambient control has lead us to effect handlers – a fact that we use as an implementation strategy for ambients in Koka. The strong theoretical foundation and expressiveness of algebraic effect handlers make this an excellent target – and not without precedent, as Kammar and Pretnar (2017) already show how to translate mutable dynamic binding to algebraic effects.

3.5.1 Translation to Ambient Control

Figure 3.7 defines a translation function $[\![\cdot]\!]$ from λ_{dbc} (a language with ambient values, functions, and control) to a subset that only uses ambient control. We translate values, expressions, evaluation contexts, and signatures. We do not translate types and ambient rows, since they do not distinguish between values, functions, and control.

Translating ambient values As can be seen from the translation of signatures, we translate ambient values to ambient control operations taking an argument of type *unit*. Binders for ambient values immediately resume with the bound value:

$$[\![\ \mathsf{with\ val}\ p\ =\ v\ \mathsf{in}\ \{\ e\ \}\ ]\!]\ =\ \mathsf{with\ control}\ p((),\ k)\ \{\ k([\![\ v\ ]\!])\ \}\ \mathsf{in}\ \{\ [\![\ e\ ]\!]\ \}$$

Accordingly, the lookup of an ambient value p is translated to a ambient call $p(())$. A very similar translation has previously been given by Kiselyov et al. (2006, Fig. 3), who show how to express dynamic binding in terms of (multi-prompt) delimited control, and by Kammar and Pretnar (2017, Fig. 7) who show that effect handlers can express dynamically bound state.

Translating ambient functions Similar to ambient values, ambient function binders translate to control binders that resume after evaluating the function body:

$$[\![\ \mathsf{with\ fun}\ p(x)\ \{\ e_1\ \}\ \mathsf{in}\ \{\ e_2\ \}\ ]\!]\ =\ \mathsf{with\ control}\ p(x,\ k)\ \{\ k([\![\ e_1\ ]\!])\ \}\ \mathsf{in}\ \{\ [\![\ e_2\ ]\!]\ \}$$

Ambient function calls translate to ambient control calls. All other language constructs, including ambient control are translated unchanged.

3.5.2 Type Preservation

The translation preserves types:

Theorem 4. (*Type Preservation*)
If $\Gamma \vdash_{exp} e : \tau | \pi$, then $[\![\Gamma]\!] \vdash_{exp} [\![e]\!] : [\![\tau]\!] \mid [\![\pi]\!]$.

Proof. Induction over the type rules of λ_{dbc}. We only list the two relevant cases of translating ambient values and functions.

case WFUN

We type $\Gamma \vdash_{exp} \mathsf{with\ fun}\ p(x)\ \{\ e_1\ \}\ \mathsf{in}\ \{\ e_2\ \} : \tau \mid \pi$. By induction, we can assume

- $[\![\ \Sigma\ ]\!](p) = \mathsf{control}\ \tau_1 \to \tau_2$ **(1)**,
- $\Gamma,\ x : \tau_1 \vdash_{exp} [\![\ e_1\ ]\!] : \tau_2 \mid \pi$ **(2)**, and
- $\Gamma \vdash_{exp} [\![\ e_2\ ]\!] : \tau \mid \langle p|\pi \rangle$ **(3)**.

We need to verify now that we can check $\Gamma \vdash_{exp} [\![\mathsf{with\ fun}\ p(x)\ \{\ e_1\ \}\ \mathsf{in}\ \{\ e_2\ \}\ ]\!] : \tau \mid \pi$ which equals $\Gamma \vdash_{exp} \mathsf{with\ control}\ p(x,\ k)\ \{\ k([\![\ e_1\ ]\!])\ \}\ \mathsf{in}\ \{\ [\![\ e_2\ ]\!]\ \} : \tau \mid \pi$, with $k \notin \mathsf{fv}(e_1)$ and $k \neq x$. To use rule WCTL, we can immediately satisfy two of its premises with (1) and (3). That leaves us to derive $\Gamma,\ x : \tau_1,\ k : \tau_2 \to \pi\ \tau \vdash_{exp} k([\![\ e_1\ ]\!]) : \tau \mid \pi$. We can use rule APP to check $\Gamma,\ x : \tau_1,\ k : \tau_2 \to \pi\ \tau \vdash_{exp} [\![\ e_1\ ]\!] : \tau_2 \mid \pi$. Since $k \notin \mathsf{fv}(e_1)$, we can apply weakening and it suffices to show $\Gamma,\ x : \tau_1 \vdash_{exp} [\![\ e_1\ ]\!] : \tau_2 \mid \pi$, which holds by (2).

case WVAL

Similar to the previous case. We type $\Gamma \vdash_{exp} \mathsf{with\ val}\ p = v\ \mathsf{in}\ \{\ e\ \} : \tau_2 \mid \pi$.
Again, by induction, we can assume
$[\![\ \Sigma\ ]\!](p) = \mathsf{control}\ \mathit{unit} \to \tau_1$ **(1)**, $\Gamma \vdash_{val} [\![\ v\ ]\!] : \tau_1$ **(2)**, and $\Gamma \vdash_{exp} [\![\ e\ ]\!] : \tau_2 \mid \langle p|\pi \rangle$ **(3)**.
We need to derive $\Gamma \vdash_{exp} \mathsf{with\ control}\ p((),\ k)\ \{\ k([\![\ v\ ]\!])\ \}\ \mathsf{in}\ \{\ [\![\ e\ ]\!]\ \} : \tau_2 \mid \pi$, with $k \notin \mathsf{fv}(v)$
We can again satisfy two premises of WCTL with (1) and (3). This leaves us to show $\Gamma,\ k : \tau_1 \to \pi\ \tau_2 \vdash_{exp} k([\![\ v\ ]\!]) : \tau_2 \mid \pi$. Using rule APP, leaves us to check the type of the argument to the continuation $\Gamma,\ k : \tau_1 \to \pi\ \tau_2 \vdash_{exp} [\![\ v\ ]\!] : \tau_1 \mid \pi$. Again, since $k \notin \mathsf{fv}(v)$, we can check $\Gamma \vdash_{exp} [\![\ v\ ]\!] : \tau_1 \mid \pi$, which holds by rule VAL and (2).

The cases for DVAL and DFUN follow immediately from the induction hypothesis. □

3.5.3 Preservation of Semantics

This brings us to our theorem that the translation preserves semantics:

Theorem 5. (*Semantic Soundness*)
If $e \longmapsto e'$ then $[\![\ e\ ]\!] \longmapsto^* [\![\ e'\ ]\!]$

To prove this, we need the following lemmas about the translation:

Lemma 7. (*Translation preserves free variables*)
$\mathsf{fv}(e) = \mathsf{fv}([\![\ e\ ]\!])$

Proof by induction over the structure of expressions. Translating ambient values and function binders introduces a variable k. The side conditions $k \notin \mathsf{fv}(v)$ and $k \notin \mathsf{fv}(e)$ prevent capture.

Lemma 8. (*Translation preserves contexts*)
$[\![\ \mathsf{E}[e]\ ]\!] = [\![\ \mathsf{E}\ ]\!][\![\ e\ ]\!]$

Proof by induction over evaluation contexts.

Lemma 9. (*Translation preserves bound ambients*)
$[\![\ \mathsf{H}_p[e]\ ]\!] = [\![\ \mathsf{H}_p\ ]\!][\![\ e\ ]\!]$

Proof by induction over the grammar of capture contexts H_p. Informally, only the kind of binder changes, not the name of the ambient.

Lemma 10. (*Translation is substitution safe*)
$[\![\ e[x{\mapsto}v]\ ]\!] \ = \ [\![\ e\ ]\!][x{\mapsto}[\![\ v\ ]\!]\ ]$

Proof by induction over the size of the expression.

Proof. (*Of Theorem 5*) The proof proceeds by induction over the reduction rules of λ_{dbc}.

case (*dfun*) – $\mathsf{with\ fun}\ p(x)\ \{\ e\ \}\ \mathsf{in}\ \{\ \mathsf{H}_p[p(v)]\ \} \longrightarrow (\lambda y.\ \mathsf{with\ fun}\ p(x)\ \{\ e\ \}\ \mathsf{in}\ \{\ \mathsf{H}_p[y]\ \})(e[x{\mapsto}v])$

Applying the translation, we derive:

$[\![\ \mathsf{with\ fun}\ p(x)\ \{\ e\ \}\ \mathsf{in}\ \{\ \mathsf{H}_p[p(v)]\ \}\ ]\!]$
$\quad = \ \{$ definition of $[\![\ \cdot\ ]\!]$ with $k \notin \mathsf{fv}(e)$ $(\mathbf{1})\ \}$
$\mathsf{with\ control}\ p(x,\ k)\ \{\ k([\![\ e\ ]\!])\ \}\ \mathsf{in}\ \{\ [\![\ \mathsf{H}_p[p(v)]\ ]\!]\ \}$
$\quad = \ \{$ Lemma 8, Lemma 9 $\}$
$\mathsf{with\ control}\ p(x,\ k)\ \{\ k([\![\ e\ ]\!])\ \}\ \mathsf{in}\ \{\ [\![\ \mathsf{H}_p\ ]\!][\ [\![\ p(v)\ ]\!]\ ]\ \}$
$\quad \longrightarrow \ \{$ definition of $[\![\ \cdot\ ]\!]\ \}$
$\mathsf{with\ control}\ p(x,\ k)\ \{\ k([\![\ e\ ]\!])\ \}\ \mathsf{in}\ \{\ [\![\ \mathsf{H}_p\ ]\!][p([\![\ v\ ]\!])]\ \}$
$\quad \longrightarrow \ \{$ definition of $(dctl)\ \}$
$(k([\![\ e\ ]\!])[x \mapsto [\![\ v\ ]\!],\ k \mapsto \lambda y.\ \mathsf{with\ control}\ p(x,\ k)\ \{\ k([\![\ e\ ]\!])\ \}\ \mathsf{in}\ \{\ [\![\ \mathsf{H}_p\ ]\!][y]\ \}]$
$\quad = \ \{$ substitute $\}$
$(\lambda y.\ \mathsf{with\ control}\ p(x,\ k)\ \{\ k([\![\ e\ ]\!])\ \}\ \mathsf{in}\ \{\ [\![\ \mathsf{H}_p\ ]\!][y]\ \})([\![\ e\ ]\!][x \mapsto [\![\ v\ ]\!],\ k{\mapsto}\ \ldots])$
$\quad = \ \{$ (1) and Lemma 7 $\}$
$(\lambda y.\ \mathsf{with\ control}\ p(x,\ k)\ \{\ k([\![\ e\ ]\!])\ \}\ \mathsf{in}\ \{\ [\![\ \mathsf{H}_p\ ]\!][y]\ \})([\![\ e\ ]\!][x \mapsto [\![\ v\ ]\!]\ ])$
$\quad = \ \{$ Lemma 10 $\}$
$(\lambda y.\ \mathsf{with\ control}\ p(x,\ k)\ \{\ k([\![\ e\ ]\!])\ \}\ \mathsf{in}\ \{\ [\![\ \mathsf{H}_p\ ]\!][y]\ \})([\![\ e[x \mapsto v]\ ]\!])$
$\quad = \ \{$ (1) and definition of $[\![\ \cdot\ ]\!]\ \}$
$[\![\ (\lambda y.\ \mathsf{with\ fun}\ p(x)\ \{\ e\ \}\ \mathsf{in}\ \{\ \mathsf{H}_p[y]\ \})(e[x \mapsto v])\ ]\!]$

case (*dval*) – $\mathsf{with\ val}\ x\ =\ v\ \mathsf{in}\ \{\ \mathsf{H}_p[p]\ \} \longrightarrow \mathsf{with\ val}\ x\ =\ v\ \mathsf{in}\ \{\ \mathsf{H}_p[v]\ \}$

Structurally similar to the previous case, but requires an additional application of rule (β).

$[\![\ \mathsf{with\ val}\ x\ =\ v\ \mathsf{in}\ \{\ \mathsf{H}_p[p]\ \}\ ]\!]$
$\quad \ldots$
$(\lambda y.\mathsf{with\ control}\ p(x,\ k)\ \{\ k([\![\ v\ ]\!])\ \}\ \mathsf{in}\ \{\ [\![\ \mathsf{H}_p\ ]\!][y]\ \})([\![\ v\ ]\!])$
$\quad = \ \{$ definition of (β) and substitution $\}$
$\mathsf{with\ control}\ p(x,\ k)\ \{\ k([\![\ v\ ]\!])\ \}\ \mathsf{in}\ \{\ [\![\ \mathsf{H}_p\ ]\!][\ [\![\ v\ ]\!]\ ]\ \}$
$\quad = \ \{$ definition of $[\![\ \cdot\ ]\!]\ \}$
$[\![\ \mathsf{with\ val}\ p\ =\ v\ \mathsf{in}\ \{\ \mathsf{H}_p[v]\ \}\ ]\!]$

□

The proof for case (*dval*) explains why Theorem 5 maps one step in the source program to potentially multiple reduction steps in the translated program. In particular, we could phrase the theorem more precisely: When the source program takes one reduction step, the translated program takes either one or two reduction steps.

$$
\begin{array}{lcl}
\lceil\cdot\rceil \ : \ \lambda_{dbc} \to \lambda_{dch} & & \\
\lceil x \rceil & = & x \\
\lceil \lambda x.e \rceil & = & \lambda x.\lceil e \rceil \\
 & & \\
\lceil e(e') \rceil & = & \lceil e \rceil(\lceil e' \rceil) \\
\lceil p(v) \rceil & = & \mathsf{do}_p(\lceil v \rceil) \\
\lceil \mathsf{with\ control}\ p(x,\ k)\ \{\ e_1\ \}\ \mathsf{in}\ \{\ e_2\ \} \rceil & = & \mathsf{handle}_p\ (x,\ k) \Rightarrow \lceil e_1 \rceil\ \mathsf{in}\ \{\ \lceil e_2 \rceil\ \} \\
 & & \\
\lceil\cdot\rceil \ : \ \Sigma_{dbc} \to \Sigma_{dch} & & \\
\lceil \emptyset \rceil & = & \emptyset \\
\lceil \Sigma, p \ : \ \mathsf{control}\ \tau \to \tau' \rceil & = & \lceil \Sigma \rceil,\ p : \ \tau \to \tau' \\
 & & \\
\lceil\cdot\rceil \ : \ \mathsf{E}_{dbc} \to \mathsf{E}_{dch} & & \\
\lceil \Box \rceil & = & \Box \\
\lceil \mathsf{E}(e) \rceil & = & \lceil \mathsf{E} \rceil(\lceil e \rceil) \\
\lceil v(\mathsf{E}) \rceil & = & \lceil v \rceil(\lceil \mathsf{E} \rceil) \\
\lceil \mathsf{control}\ p(x,\ k)\ \{\ \lfloor e \rfloor\ \mathsf{in}\ \{\ \mathsf{E}\ \}\ \rceil & = & \mathsf{handle}_p\ \{\ (x,\ k) \Rightarrow e\ \}\ \mathsf{in}\ \{\ \lceil \mathsf{E} \rceil\ \}
\end{array}
$$

(a) Translating ambient control (λ_{dbc}) to delimited control with handlers (λ_{dch}).

$$
\begin{array}{lcl}
\lfloor\cdot\rfloor \ : \ \lambda_{dch} \to \lambda_{dbc} & & \\
\lfloor x \rfloor & = & x \\
\lfloor \lambda x.e \rfloor & = & \lambda x.\lfloor e \rfloor \\
 & & \\
\lfloor e(e') \rfloor & = & \lfloor e \rfloor(\lfloor e' \rfloor) \\
\lfloor \mathsf{do}_p(v) \rfloor & = & p(\lfloor v \rfloor) \\
\lfloor \mathsf{handle}_p\ \{\ (x,\ k) \Rightarrow e_1\ \}\ \mathsf{in}\ \{\ e_2\ \} \rfloor & = & \mathsf{with\ control}\ p(x,\ k)\ \{\ \lfloor e_1 \rfloor\ \}\ \mathsf{in}\ \{\ \lfloor e_2 \rfloor\ \} \\
 & & \\
\lfloor\cdot\rfloor \ : \ \Sigma_{dch} \to \Sigma_{dbc} & & \\
\lfloor \emptyset \rfloor & = & \emptyset \\
\lfloor \Sigma, p \ : \ \tau \to \tau' \rfloor & = & \lfloor \Sigma \rfloor,\ p : \ \mathsf{control}\ \tau \to \tau' \\
 & & \\
\lfloor\cdot\rfloor \ : \ \mathsf{E}_{dch} \to \mathsf{E}_{dbc} & & \\
\lfloor \Box \rfloor & = & \Box \\
\lfloor \mathsf{E}(e) \rfloor & = & \lfloor \mathsf{E} \rfloor(\lfloor e \rfloor) \\
\lfloor v(\mathsf{E}) \rfloor & = & \lfloor v \rfloor(\lfloor \mathsf{E} \rfloor) \\
\lfloor \mathsf{handle}_p\ \{\ (x,\ k) \Rightarrow e\ \}\ \mathsf{in}\ \{\ \mathsf{E}\ \} \rfloor & = & \mathsf{with\ control}\ p(x,\ k)\ \{\ \lfloor e \rfloor\ \}\ \mathsf{in}\ \{\ \lfloor \mathsf{E} \rfloor\ \}
\end{array}
$$

(b) Translating delimited control with handlers (λ_{dch}) to ambient control (λ_{dbc}).

Figure 3.8. Translating the language of ambient control (λ_{dbc}) to delimited control with handlers (λ_{dch}) and back. Both translations are fully homomorphic.

3.6 Translating to Effect Handlers and Back

We have seen that ambient values and ambient functions can be macro expressed in terms of ambient control. The translation is type- and semantics preserving. Comparing the language with only ambient control (λ_{dbc}) with the language of effect handlers in Section 2.4, we find that they only differ in surface syntax. To recall, for the type system of λ_{dbc} we assumed semantics $-do+$, a globally static set of prompts, and corresponding signatures. Both, operational semantics and the type system are equivalent up to syntax.

Theorem 6. (*Macro Equivalence*)
The effect-safe calculus λ_{dbc} (with ambient control) and effect-safe λ_{dch} (in variant $-do+$) are macro equivalent.

To capture the relation between the two calculi formally, Figure 3.8 defines translations from ambient control to effect handlers (Figure 3.8a) and back (Figure 3.8b). Both translations are fully homomorphic and only map effect handlers to ambient control, and back. The translations are inverse to each other, as can be easily seen from the translation rules. Every ambient binder for p corresponds to a handler for a given prompt. Since prompts (p) coincide with ambient names, and effect rows ϵ with ambient rows π – we do not need to translate types besides renaming meta-variables for effect rows.

The translation is type and semantics preserving as can be verified by comparing the respective rules. For easier comparison, here we repeat the relevant reduction rules:

$$
\begin{array}{lll}
(dctl) & \mathsf{with}\ b_p\ \mathsf{in}\ \{\ \mathsf{H}_p[p(v)]\ \} & \longrightarrow\ e[x \mapsto v,\ k \mapsto \lambda y.\ \mathsf{with\ control}\ p(x,\ k)\ \{\ e\ \}\ \mathsf{in}\ \{\ \mathsf{H}_p[y]\ \}] \\
 & & \quad \mathsf{where}\ b_p\ =\ \mathsf{control}\ p(x,\ k)\ \{\ e\ \} \\
(-do+) & \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ \mathsf{H}_p[\mathsf{do}_p(v)]\ \} & \longrightarrow\ e[x \mapsto v,\ k \mapsto \lambda y.\ \mathsf{handle}_p\ h\ \mathsf{in}\ \{\ \mathsf{H}_p[y]\ \}] \\
 & & \quad \mathsf{where}\ h\ =\ \{\ (x,\ k) \Rightarrow e\ \}
\end{array}
$$

We also repeat the typing judgements for ambient control and effect handlers:

$$
\frac{\Sigma(p)\ =\ \mathsf{control}\ \tau_1 \to \tau_2 \quad \Gamma \vdash_{exp}\ e_2\ :\ \tau |\ \langle p|\pi\rangle \quad \Gamma,\ x:\ \tau_1,\ k:\ \tau_2 \to \pi\ \tau \vdash_{exp}\ e_1\ :\ \tau |\ \pi}{\Gamma \vdash_{exp}\ \mathsf{with\ control}\ p(x,\ k)\ \{\ e_1\ \}\ \mathsf{in}\ \{\ e_2\ \}\ :\ \tau |\ \pi}\ [\textsc{wctl}]
$$

$$
\frac{\Sigma(p)\ =\ \tau_1 \to \tau_2 \quad \Gamma \vdash_{dch}\ e_2\ :\ \tau |\ \langle p|\epsilon\rangle \quad \Gamma,\ x:\ \tau_1,\ k:\ \tau_2 \to \epsilon\ \tau \vdash_{dch}\ e_1\ :\ \tau |\ \epsilon}{\Gamma \vdash_{dch}\ \mathsf{handle}_p\ \{\ (x,\ k) \Rightarrow e_1\ \}\ \mathsf{in}\ \{\ e_2\ \}\ :\ \tau |\ \epsilon}\ [\textsc{handle}]
$$

Comparing the rules, it becomes evident that they only differ in surface syntax. Having established the equivalence between the calculi, semantics soundness of λ_{dbc} follows as corollary.

Corollary. (*Semantic Soundness*)
If $\emptyset \vdash_{exp}\ e\ :\ \tau |\ \langle\,\rangle$ then either e diverges, or evaluates to a value $e \longmapsto^*\ v$ where $\emptyset \vdash_{val}\ v\ :\ \tau$.

This immediately follows from the fact that typed λ_{dbc} is equivalent to typed λ_{dch} (Theorem 6) together with semantic soundness of λ_{dch} (Theorem 1).

3.7 Related Work and Chapter Conclusion

We briefly review work related to dynamic binding and effect handlers.

Implicit parameters Ambient values are perhaps most closely related to *implicit parameters* as described by Lewis, Launchbury, Meijer, and Shields (2000). In particular, implicit parameters are immutable, named, and statically typed. In contrast to our approach, implicit parameters do not need to be declared and can be used and bound at any type. This is flexible, but can lead to large types (as shown in Section 3.2.3). It also delays possible type errors to the binding site. Lewis et al. (2000) show how implicit parameters can be elegantly implemented using regular parameter passing. This is similar to the dictionary passing translation of type classes (Wadler and Blott, 1989; Jones, 1992). Such translation could possibly be applied to ambient values as well, turning every member p in an ambient row into an additional parameter. In the second part of this thesis, we use the similar technique of explicit capability passing to implement effect handlers in object-oriented programming languages (Chapters 4 to 6).

Dynamic binding and delimited control In an untyped setting, dynamic binding first appeared in McCarthy Lisp (as a bug) (McCarthy, 1960). Modern dialects have lexical scoping but still provide dynamic binding: in Common Lisp one can use the `special` declaration (Steele Jr., 1990), and MIT Scheme has `fluid-let` bindings (Hanson, 1991). The semantics of dynamic binding was formalized by Moreau (1998). Later, Kiselyov et al. (2006) extend upon that work by giving a translation into delimited control, providing a unified framework for control effects and dynamic binding. Later, Kammar and Pretnar (2017) perform a similar translation and show how (mutable) dynamic variables can be expressed in terms of algebraic effect handlers – our translation of local mutable variables in Section 3.3.4 is based on this. Forster, Kammar, Lindley, and Pretnar (2017) show that in a untyped setting, algebraic effect handlers, delimited control, and monads, can all express each other through a local macro-translation (Felleisen, 1990) and thus all can express dynamic binding. Piróg et al. (2019) extend the results to the typed setting, which requires some form of type-polymorphism.

Resolution by type Instead of binding ambient values by name, an alternative is to resolve dynamic bindings implicitly based on their *type*. Implicit parameters in Scala (Oliveira and Gibbons, 2010; Odersky, 2019a; Odersky et al., 2017) are probably the most common example. Implicit parameters are declared on a method signature but provided automatically at the call site based on their type. Siek and Lumsdaine (2005) introduce system F^G, which uses type based resolution for concept-based generic programming. Haskell type classes (Wadler and Blott, 1989; Jones, 1992; Kiselyov and Shan, 2004) are another instance where dictionaries are passed implicitly, based on their type. Oliveira et al. (2012) formalize a notion of implicit parameters, which are resolved based on their type, in the *implicit calculus*. The implicit calculus is interesting since implicit values are not only resolved by their type, but also referred to by their type. For example,

$$\mathsf{implicit}\ 1\ \mathsf{in}\ \mathsf{implicit}\ \mathit{True}\ \mathsf{in}\ (\mathit{even}(?\mathit{int})\ \&\&\ ?\mathit{bool})$$

evaluates to *False* where $?\tau$ is used to implicitly look up a value of type τ. Resolving by type leads to issues with coherence, and stability under type substitution, as well as problems when combined with polymorphism (Schrijvers et al., 2019). Ambient values in contrast, are resolved

by name, globally declared, and have an untyped operational semantics. This way, ambient values are naturally stable under type substitution.

3.7.1 Discussion: Ambient Values and Ambient Functions

In this chapter, we introduced effect handlers by step-wise generalizing dynamic binding (that is, ambient values). We discovered ambient functions as a new language feature that resides between ambient values and ambient control. Ambient functions are a small extension to ambient values, but create new opportunities for abstraction, while avoiding the need for full generality of continuation capture. Both features, ambient values and ambient functions, are macro expressible in terms of ambient control. So, why should we consider them separately? As mentioned in the beginning of this chapter, distinguishing between ambient values, function and control has several advantages.

Learnability of concepts Users can incrementally understand the generalizations from ambient values, via ambient functions, to ambient control. In this chapter we followed these developments to open up a new perspective on effect handlers as generalization of dynamic binding. Studying the difference between lambdas bound to ambient values and ambient functions shows how effect handlers can serve as abstraction barrier, even if they do not use delimited control themselves.

Reasoning about programs A program *only* using ambient values and ambient functions does not alter the control flow. From the type of our example function `pretty` (Section 3.1), we can immediately see that it only uses ambient values. The following code example uses this:

```
val f = open("output.txt");
val res = pretty(doc);
try-write(f, res);
close(f)
```

The effect row of `pretty` only mentions the ambient `width`. Since `pretty` only uses ambient values, it cannot alter the control flow and thus will exactly return once with a string value. Assuming that the function `try-write` also does not have control effects, cannot raise exceptions, and does not diverge[9], we can be sure that the file handle `f` will be closed. This would still be true if `pretty` would use ambient functions, as long as the binders of those ambient functions do not use control-flow altering effects themselves.

Efficient implementation Compilers can use the additional information to generate code specialized to the more limited control flow. For ambient values and ambient functions we know that the continuation will be used in a tail-position (also called *tail resumptive* by Leijen (2017b)). In this case, we do not need to capture the continuation at all, which can lead to performance improvements (Leijen, 2017b). Koka previously already performed similar optimizations (Leijen, 2018a, Sec. 5) but needed to infer this information from the handler definitions. With ambient values and functions this information is readily present in the ambient declarations. The optimization thus applies naturally to our implementation of ambient values and ambient functions in Koka, as well.

[9] Divergence is conservatively approximated by the Koka effect system (Leijen, 2014). If it cannot determine that a recursive function is structurally recursive, it adds the `div` effect to the function's effect row.

3.7.2 Conclusion

In this chapter, we step-wise generalized ambient values. Each generalization illustrated an important aspect of effect handlers. Starting from ambient values, we revisited existing work on dynamic binding. The operational semantics of λ_{db} (Figure 3.1) and its simple type system (Figure 3.5), are based on the *DB* calculus by Kiselyov et al. (2006). Ambient values emphasize the fact that the implementation of an *effect operation is dynamically bound*.

In a second step, we generalized ambient values to the novel feature of ambient functions, which in turn distils yet another aspect of effect handlers: the implementation of an effect operation is dynamically bound, but *statically evaluated* in the context of its binding. The expressiveness of ambient functions resides between ambient values and ambient control (that is, effect handlers). Unlike ambient control, ambient functions do not obtain access to the continuation. In consequence, they cannot alter the control flow. However, like effect handlers, ambient functions encapsulate implementation effects at the definition site. Ambient functions thus support abstraction similar to effect handlers, without the conceptually challenging aspect of delimited control.

The third and last generalization, ambient control, is equivalent to effect handlers as presented in the previous chapter (Section 3.6). We gave a translation of ambient values and ambient functions to ambient control (Section 3.5). By macro expressing ambient values and ambient functions in terms of ambient control, we were able to reuse the soundness proof of Section 2.4.1, which in turn is an adaption of the proof for Koka by Leijen (2017c). The fact, that delimited control can express dynamic binding is not new (Kiselyov et al., 2006), the same holds for effect handlers (Kammar et al., 2013). To implement ambients in the Koka language, we use a similar translation from ambient values to effect handlers.

In the second part of this thesis, we explore an alternative to dynamic binding of effect operations: we employ *explicit capability passing* to embed effect handlers in object-oriented programming languages. That is, instead of dynamically searching for the respective handler in the evaluation context, user programs are required to explicitly pass handler instances.

Part II.

Effect Handlers and Object-Oriented Programming

Chapter 4

Effekt – A Library Design

Effect handlers are a program-structuring paradigm with rising popularity in the functional programming language community and can express many advanced control flow abstractions. In this chapter, we present Effekt: the first design of effect handlers that revolves around object-oriented programming. Our design maps effect handler abstractions to key abstraction of object-oriented programming languages.

Different to most other existing languages with effect handlers, Effekt is centered on (explicit) capability-passing style as an alternative to dynamic binding. Capability-passing style integrates well with the paradigm of object-oriented programming.

We explore the combination of effect handlers and object-oriented programming and describe new potential to structure effectful programs and effect handlers, enabled by reusing existing abstraction. We capture the different dimensions of extensibility in a variant of Wadler's expression problem (Wadler, 1998), which we call the *effect expression problem*.

Effect handlers, as introduced in the first part of this thesis, have been conceived in the setting of functional programming languages. Given this heritage, it is not surprising that effect handlers have been implemented as libraries for functional programming languages like Haskell (Kammar et al., 2013; Kiselyov et al., 2013; Wu and Schrijvers, 2015), Idris (Brady, 2013), or OCaml (Kammar et al., 2013; Kiselyov and Sivaramakrishnan, 2016). Similarly, most standalone languages that support effect handlers are following the functional programming paradigm. Examples include Eff (Bauer and Pretnar, 2015), Koka (Leijen, 2014), Frank (Lindley et al., 2017), Links (Hillerström et al., 2017), and Helium (Biernacki et al., 2019).

Implementations in the realm of object-oriented programming (OOP) are much more difficult to find. No implementation, that we are aware of, explicitly uses OOP features in its API (see discussion of related work in Section 4.7.3). Instead, library implementations like Eff in Scala (Torreborre, 2016) often re-implement the work by Kiselyov and Ishii (2015) and neglect the object-oriented nature of their host language. In this chapter, we present Effekt – the first design for effect handlers that revolves around object-oriented programming. In particular, we directly

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This chapter is based on an extended version that appeared in the Journal of Functional Programming: Jonathan Immanuel Brachthäuser, Philipp Schuster, and Klaus Ostermann. 2020. "Effekt: Capability-passing Style for Type- and Effect-safe, Extensible Effect Handlers in Scala". *Journal of Functional Programming, 30, E8*. DOI: https://doi.org/10.1017/S0956796820000027

It is also contains contents of the following publication: Jonathan Immanuel Brachthäuser, Philipp Schuster, and Klaus Ostermann. 2018. "Effect Handlers for the Masses". *Proc. ACM Program. Lang.*, 2 (OOPSLA): 111:1–111:27. DOI: https://doi.org/10.1145/3276481.

map the abstractions of effect handlers to key abstractions in object-oriented programming, establishing the mantra of our design:

> Programming with effect handlers in Effekt *is* object-oriented programming.

In particular, effect signatures are *interfaces* declaring the effect operations, effect handlers are *classes* implementing those interfaces and effectful programs are *methods* written against the abstraction of the interface (Figure 4.1). While in this chapter we present Effekt as a Scala library, the high-level concepts are independent of Scala and applicable to every object-oriented programming language with interfaces, classes, and simple generics. In the next chapter, we present an implementation of Effekt in the language Java.

Directly mapping effect handlers to OOP features has several advantages:

Economy of concepts
Reusing existing concepts to structure effect handlers and effectful programs lets users focus on what is new with effect handlers. At the same time users can immediately apply their knowledge and intuition about interfaces and classes to structure effect signatures and handlers, correspondingly.

Reuse of abstractions
As we will see, reusing existing abstractions from OOP allows users to combine the advantages of those abstractions with the abstraction of effect handlers. The combination of effect handlers with object-oriented features enables novel modularization strategies, both for effectful programs and for effect handler implementations. For instance, with our design, users can use standard OOP techniques like inheritance, subtyping, private state, dynamic dispatch, and others to structure and implement effect handlers. By embedding handlers in a language like Scala, users can additionally use more advanced features (like mixin composition or abstract type members) to modularily describe effects and handlers.

Ease of implementation
Language implementers benefit from reusing existing abstractions since this removes the burden of re-implementing very similar abstractions all over again. Having fewer features also potentially reduces the risk of bad feature interaction. For library implementers, reuse is even more essential since it simply might not be possible to add new abstractions to a language without modifying its syntax and semantics. Our design allowed us to implement effect handlers as libraries for Scala (this chapter and Chapter 6) and Java (Chapter 5) without modifications to the languages.

To get a first impression of Effekt, consider the following piece of code written in Scala and using our library. As before in Section 2.1.1, we use the effect operation `flip` to nondeterministically determine whether the gambler is too drunk to catch the coin (Kammar et al., 2013). In this case, we use the effect operation `raise` to raise an exception. Otherwise, we return the result of a second coin toss as a string.

```
                    Capability-Passing Style   Monad for Delimited Control
def drunkFlip(amb: Amb, exc: Exc): Control[String] = for {
  caught ← amb.flip()
  heads  ← if (caught) amb.flip() else exc.raise("We dropped the coin.")
} yield if (heads) "Heads" else "Tails"
```

The two effect operations `flip` and `raise` are methods declared in corresponding interfaces

Effect Handlers	Object-Oriented Programming
Effect Signatures	Interfaces
Effect Operation	Method
Effect Handlers	Implementations (Classes)
Effectful Programs	Interface Users
Effect Capability	Instances / Objects

Figure 4.1. Mapping concepts from effect handlers to object-oriented programming.

modeling effect signatures `Amb` (for ambiguity) and `Exc` (for exceptions). We will see the declarations of the effect signatures in the next section.

In the first part of this thesis, we revisited two important aspects of effect handlers: dynamic binding and delimited control. Both aspects can already be identified in the type signature of the method `drunkFlip`. In particular, our library design incorporates these two aspects in the following ways:

Dynamic binding
In contrast to most other implementations (and our presentation in Chapters 2 and 3), we do not perform a search for the implementation of effect operations at runtime. Instead, effect operations are methods on effect instances `amb` and `exc`, which `drunkFlip` receives as arguments. We refer to this alternative to dynamic binding as *capability-passing style*. The terminology of *capability* (Dennis and Van Horn, 1966) suggests that they entitle the holder to use the effect.

Delimited control
The result type of the program suggests that it does not simply compute a string, but uses control effects to do so (hence the name of the type constructor `Control`). We base our Scala library on a monadic implementation of *multi-prompt delimited control* (Dybvig et al., 2007) with semantics $-shift+$ (Section 2.1.1). This is also reflected in the use of Scala's `for`-comprehensions (Odersky et al., 2006) to sequence effectful expressions. While having to write programs in monadic style might not seem very object-oriented, it is not essential to our library design and does not prevent us from using OOP features for other aspects of the library. Chapter 5 explores an alternative implementation strategy of delimited control that lifts this limitation and enables writing programs in direct style.

Our library design revolves around the concept of *capability passing*. As we will see in the remainder of this chapter, capabilities in `Effekt` encapsulate three different aspects:

Capabilities *contain* effect implementations They give semantics to effect operations (Section 4.1). In the example program `drunkFlip`, we call effect operations as methods on the capabilities `amb` and `exc`.

Capabilities *contain* prompt markers Effect operations can capture the continuation, which is delimited by the corresponding handler (Chapter 2). Programs written with our library have type `Control` (Section 4.2), a monadic implementation of delimited control with first-class prompts (Dybvig et al., 2007). Our capabilities contain such a prompt marker as a value member.

Capabilities *grant* privileges Capabilities can be seen as constructive legitimization that the holder is entitled to use a particular effect (Dennis and Van Horn, 1966; Miller, 2006). While passing capabilities encourages developers to mention effects in the method signature, this is not enough to statically guarantee effect safety. In particular, capabilities can leak, potentially resulting in runtime errors (Osvald et al., 2016). Chapter 6 shows how we can use Scala's advanced type system features of path-dependent types and intersection types to encode regions (Moggi and Sabry, 2001) and obtain effect safety.

In summary, the contributions of this chapter are:

- We present a library design to embed effect handlers in object-oriented programming languages. While we use Scala for our presentation, the idea of mapping effect handlers to OOP concepts is independent of Scala as a host language (Section 4.1).
- We introduce *capability-passing style* as an alternative to dynamically binding effect handlers. While the translation of dynamic binding to dictionary passing is not new (Lewis et al., 2000; Kammar et al., 2013), we embrace *explicit* passing of capabilities as part of the user facing syntax. As we will see in Chapter 6, capability passing is essential to our approach of establishing effect safety.
- We study the modularization potential newly gained by combining effect handlers and object-oriented programming. We show how reusing existing abstractions opens up new and interesting ways to modularize effect handlers. Section 4.4 relates effect handlers to the expression problem, explores several dimensions of extensibility, and shows how `Effekt` supports them.
- We present a novel design for stateful handlers to guarantee well-behaved interaction with continuation capture (Section 4.3). Our design is an OOP alternative to parameterized handlers that can be found in functional programming languages like Koka (Leijen, 2017c).
- We hide most of the syntactic overhead of capability passing (Section 4.6.1) by making it implicit using the Scala features of implicit parameters (Odersky, 2019a) and implicit function types (Odersky et al., 2017).

While the general design of `Effekt` is independent of a concrete object-oriented language, for the presentation in this thesis, we choose the programming language Dotty (in version 0.19) – a version of the Scala language that will eventually become Scala 3 (Odersky, 2018). We explain necessary Scala specifics on first encounter.

4.1 Programming with Effect Handlers in Effekt

To get a first impression of how to program with effect handlers in `Effekt`, and to illustrate how effect handler concepts map to object-oriented programming, let's again look at our running example of `drunkFlip`.

4.1.1 Effect Signatures are Interfaces

The implementation of `drunkFlip` uses capabilities `amb` and `exc`, which are instances of effect signatures `Amb` and `Exc`, correspondingly. We sometimes interchangeably use the terminology

```
trait Control[+A] {
  def map[B](f: A ⇒ B): Control[B]                    def pure[A](a: ⇒ A): Control[A]
  def flatMap[B](f: A ⇒ Control[B]): Control[B]       def run[A](ca: Control[A]): A
  def andThen[B](c: Control[B]): Control[B]
}
```

Figure 4.2. The control interface – a monad for multi-prompt delimited control.

capability and *handler instance*. While "capability" (Dennis and Van Horn, 1966) puts a focus on the concept of entitling the holder to use an effect, "handler instance" highlights the fact that handlers are implementations of effect signatures. Following our mapping of concepts from the introduction, we define effect signatures as interfaces:

```
trait Exc {                                    trait Amb {
  def raise[A](msg: String): Control[A]          def flip(): Control[Boolean]
}                                              }
```

For the purpose of this section, it is enough to understand Scala traits as the equivalent to interfaces in languages like Java. The methods `raise` and `flip` model *effect operations*. Their return type `Control[A]` (and `Control[Boolean]`, correspondingly) suggests that they are *effectful*, that is, they can have control effects.

The concrete implementation of `Control` is not essential to the design of Effekt. For now, it suffices to understand it as some monadic type constructor with the operations summarized in Figure 4.2. In the following Section 4.2, we will go into more detail on our implementation of multi-prompt delimited control and relate it to our presentation of Chapter 2.

The methods to sequence programs without control effects (*i.e.* `map`) and with control effects (*i.e.* `flatMap`) enable us to write effectful programs in an imperative style via Scala's for-comprehensions[10]. The function `pure` corresponds to monadic **`return`** (Wadler, 1995) and lifts a Scala expression without control effects into a computation. While the expression is free of control effects, it can have other side effects such as I/O. Since Scala is call-by-value, we mark the parameter (*i.e.*, `⇒ A`) as being by-name (Odersky et al., 2006) to defer those side effects to the point of running the computation by means of `run`. Using `run`, we can execute a program with control effects that computes a value of type `A` to eventually obtain that value. While we can already understand `map` as the constructive proof that `Control` is covariant in the type parameter `A` – we additionally prefix the type parameter with `+` to inform the type checker about the covariance and establish subtyping of `Control[A] <: Control[B]`, whenever `A <: B` (Odersky et al., 2006).

4.1.2 Effect Handlers are Implementations

The method `drunkFlip` does not rely on any concrete implementation of the effect operations `raise` and `flip`. It merely uses the interfaces `Amb` and `Exc` in its signature. The caller is free to pick any implementations of `Amb` and `Exc`, determining the semantics of the effect operations. For

[10] Similar to Haskell's **`do`**-notation, **`for`**-comprehensions in Scala syntactically simplify writing monadic code. In general, **`for`** { $x_1 \leftarrow e_1$; $x_2 \leftarrow e_2$; ... xn ← en } **`yield`** e desugars to e_1.flatMap { x_1 ⇒ e_2.flatMap { x_2 ⇒ ... en.map { xn ⇒ e }}} (Odersky et al., 2006).

example, we can ignore control effects altogether and use native side effects to implement `Amb` and `Exc`. For `flip` we use a random number generator and for `raise` we use native exceptions[11].

```
class NativeExceptions extends Exc {
  def raise[A](msg: String): Control[A] = pure { throw new RuntimeException(msg) }
}
class RandomFlip extends Amb {
  def flip(): Control[Boolean] = pure { Math.random() < 0.5 }
}
```

To run our example program, we simply pass instances of `NativeExceptions` and `RandomFlip` to `drunkFlip`, which will randomly result in either a runtime exception or one of the two possible string values.

```
run { drunkFlip(new RandomFlip(), new NativeExceptions()) }
```

Following our mapping of concepts, the classes `NativeExceptions` and `RandomFlip` are *effect handlers*: they provide semantics to the effect operations by implementing the effect signatures. However, to do so, they immediately return a *pure* expression without any control effects. Again, for our purposes, pure expressions do not have control-effects – they can have other side effects.

In the remainder of this section, we will pick up our handler implementations of Chapter 2 that did use control effects. We will explore an interpretation of programs, which use `Exc` and compute some result of type `R`, into programs that return an `Option[R]`. Calling `raise` will immediately abort the program with `None`. We will also explore an interpretation of programs, which use `flip` to compute a result of type `R`, into programs that return a `List[R]` by enumerating and collecting all possible outcomes.

4.1.3 Handling with Control Effects

We now show how to freely mix handlers if we implement them using our Effekt library. This is an example of the modularity benefits provided by effect handlers. The implementation of our handlers will be given shortly. For now, let us assume the following definitions:

```
class Maybe[R]   extends Exc with Handler[R, Option[R]] { ... }
class Collect[R] extends Amb with Handler[R, List[R]] { ... }
```

Like our first interpretation of `Amb` and `Exc` above, the classes `Maybe` and `Collect` again implement the corresponding effect signatures. This time, however, the effect handlers also extend our library trait `Handler[R, E]` (defined in Figure 4.3b). In general, every handler for an effect gives semantics to the corresponding effect operations. To do so, it interprets an effectful program, which would compute a result of type `R` (mnemonic for *return type*) into a new semantic domain of type `E`, the *effect domain*. When implementing effect handlers, programmers choose the effect domain to have enough structure to implement the effect operations.

[11] In Scala, methods with one argument can be called with braces (`pure { expr }`) instead of parenthesis (`pure(expr)`). This is purely syntactical and does not lead to thunking of the expression (Odersky et al., 2006).

Handlers introduce capabilities Before looking at the details of how the handlers are implemented, it is instructive to understand how they can be used. Recall our notion of effect handlers in Section 2.3:

$$
\begin{array}{l}
\mathsf{handle}_{amb}\ \{\ \ldots\ \}\ \mathsf{in}\ \{ \\
\quad \mathsf{handle}_{exc}\ \{\ \ldots\ \}\ \mathsf{in}\ \{ \\
\quad\quad \mathit{drunkFlip}() \\
\quad \} \\
\}
\end{array}
$$

In comparison, using Effekt, handling effects `Amb` and `Exc` looks like[12]:

```
val res1: Control[List[Option[String]]] =
  new Collect handle { amb ⇒
    new Maybe handle { exc ⇒
      drunkFlip(amb, exc)
    }
  }
```

The lambda, which is passed as body to the handler `new Collect().handle(amb ⇒ ...)`, represents the dynamic scope in which the capability `amb` can be used. Besides delimiting the dynamic scope, `handle` also passes the handler instance unmodified as argument to the body. Thus, in our example the variable `amb` will be bound to the instance of `Collect` created on the very same line. This is also reflected by our usage of the singleton type **`this.type`** (Odersky and Zenger, 2005b) as type of the argument (see Figure 4.3b). We only use the singleton type for better type inference, such that `amb` can be used as a capability of type `Amb` without requiring manual type annotations.

Having handled both effects, running `drunkFlip` now yields for `run { res1 }`:

```
▶ List(Some("Heads"), Some("Tails"), None)
```

As in Section 2.2, we can easily swap the two handlers to obtain a different semantics where an exception leads to a termination of the search.

```
val res2: Control[Option[List[String]]] =
  new Maybe handle { exc ⇒
    new Collect handle { amb ⇒
      drunkFlip(amb, exc)
    }
  }
```

Since at least one of the control paths raises an exception, running `drunkFlip` with the effects handled in the different order yields for `run { res2 }`:

```
▶ None
```

[12] Scala methods with only one argument can be used as an infix operators. Also type parameters can be omitted, if they can be inferred. Hence, handling with the `Maybe` handler could also be written more explicitly as `new Maybe[String]().handle(exc ⇒ ... )`.

```
class Maybe[R] extends Exc with Handler[R, Option[R]] {
  def unit(result: R) = Some(result)
  def raise(msg: String) = use { resume ⇒ pure(None) }
}
class Collect[R] extends Amb with Handler[R, List[R]] {
  def unit(result: R) = List(result)
  def flip() = use { resume ⇒ for {
    ts ← resume(true)
    fs ← resume(false)
  } yield ts ++ fs }
}
```

(a) The two effect handlers `Maybe` and `AmbList` utilizing `use` to capture the continuation.

```
trait Handler[R, E] {
  // Subclasses need to specify how to convert a result into the effect domain
  protected def unit(result: R): Control[E]

  // Handler implementations can use this method to capture the continuation
  protected def use[A](body: (A ⇒ Control[E]) ⇒ Control[E]): Control[A] = ...

  // Handler users can use this method to delimit the scope of the continuation
  def handle(prog: this.type ⇒ Control[R]): Control[E] = ...
}
```

(b) The handler interface – the essence of the effect handler library. Handlers interpret programs of type `R` into an effect domain of type `E`. Implementations of `use` and `handle` given in Figure 4.8.

Figure 4.3. Implementation of effect handlers for `Exc` and `Amb` using the library class `Handler`.

This shows the power of effect handlers: effectful programs can be written fully agnostic of both the semantic domain into which the effects will be interpreted, as well as the order in which the effects will be handled. Importantly, changing the order of handlers changes of the result type. A program that uses `Exc` returns an optional and a program that uses `Amb` returns a list. We did not have to change the effectful program that uses the effects. Notably, both, its type and its implementation can remain unchanged.

We could also implement the example without support for effect handlers. However, to write programs that use both effects (like `drunkFlip`), we would need to decide and fix upfront whether raising an exception terminates the search for possible outcomes and thus returns `Option[List[R]]`, or whether it only terminates one branch in our search and thus returns `List[Option[R]]`. Revising decisions like this in retrospect can be costly, since they potentially affect the whole codebase. Additionally, if we want to use programs, which use only one of `Exc` and `Amb`, together with programs that use both, we have to manually *lift* (Liang et al., 1995) the programs that use only one of the two effects. Effect handlers address these problems.

4.1.4 Implementing Effect Handlers

Having seen how effects can be handled, we will now take a closer look at how handlers are implemented. In Figure 4.3a, we use the Effekt library to give the implementation of the handlers `Maybe` and `Collect`. The handlers extend the corresponding effect signatures (`Exc` and `Amb`), as well as the library trait `Handler` (Figure 4.3b).

Every handler needs to implement the abstract method `unit` to specify how handled programs, which do not use the corresponding effect, are lifted from `R` into the effect domain `E`. The method `unit` is also known as the "return-clause" in the literature on effect handlers (Bauer and Pretnar, 2015; Kammar et al., 2013). Additionally, a handler needs to implement all the effect operations, which are specified in the effect signature. To implement the effect operations, handlers are able to utilize the instance method `use`, as provided by the library trait `Handler`. Specialized to the handler `Maybe`, the method `use` has type:

```
def use[A](body: (A ⇒ Control[Option[R]]) ⇒ Control[Option[R]]): Control[A]
//               ^^^ the continuation ^^^
```

In our implementation of `raise`, calling `use { resume ⇒ ... }` captures the continuation and binds it to the identifier `resume` in the provided body. To implement the expected semantics of exceptions unwinding the stack, we discard the continuation and immediately return `None`.

The handler for ambiguity, in turn, captures the continuation and invokes it twice. Once with **`true`** and once with **`false`**, each time yielding a list of possible results. Finally, it concatenates the two lists. It is important to stress again that this is only possible, because the continuation captured by `use` is delimited by the corresponding call to `handle` and the handler `Collect` defines the effect domain to have the type `List[R]`.

The methods `unit` and `use` of the handler interface are marked as protected to signal that they are implementation details of the handler and should not be called from the outside.

4.1.5 Design of the Effekt Library

We designed Effekt as a library for object-oriented languages around object-oriented idioms. In summary, the Effekt library is based on the following design decisions.

Effect signatures are interfaces and handlers are classes implementing the interfaces. We establish a capability-passing style where users explicitly pass handler instances to programs that use effect operations. As we will discuss in Section 4.4, this corresponds to a shallow embedding of effect operations and is similar to the use of type classes by Kammar et al. (2013).

Handler implementations capture the continuation by explicitly calling `use`. In Effekt, *tail resumptive* handler implementations, which exclusively invoke the continuation in tail position, can be expressed as simple methods that do not capture the continuation at all.

Effekt is a library and neither changes the language nor the type system. Handlers and the operations `use` and `handle` are designed to not require an advanced type system. In particular, different to approaches based on monad transformers (Liang et al., 1995; Jones, 1995), Effekt does not require type constructor polymorphism. The usage of the singleton type **`this.type`** in the signature of `handle` is not essential and can be removed at the cost of type inference or some more boilerplate.

The design of Effekt, as presented in this chapter, does not establish an effect typing discipline. Users need to make sure that an effect handler is only used in the dynamic extent of a corresponding call to `handle`. Invoking `use` outside of the corresponding handler scope will result in a runtime error. We address this issue in Chapter 6.

```
trait Prompt[Result] {}
def reset[R](prog: Prompt[R] ⇒ Control[R]): Control[R]
def shift[A, R](prompt: Prompt[R])(body: (A ⇒ Control[R]) ⇒ Control[R]): Control[A]
```

Figure 4.4. The prompt interface – control operators `shift` / `reset` and the marker trait `Prompt`.

4.2 Delimited Control

Our Scala implementation of Effekt implements effect handlers in terms of a monad for multi-prompt delimited control. In this section, we present a simplified version of our implementation of this monad as a specialization of the one presented by Dybvig et al. (2007). The monad `Control[+A]` is very close to the monad for multi-prompt delimited control by Dybvig et al. (2007), but we specialize the exposed interface to better fit effect handlers. While Dybvig et al. present a very general framework with semantics $-shift-$ that allows for the implementation of different control operators, we build on $-shift+$ as our control operator of choice. This means that the body of the captured continuation is always delimited by a reset. As highlighted by Kammar et al. (2013), $-shift+$ matches the semantics of *deep handlers*, where the same effect is already handled in the continuation. It additionally avoids the problem of accumulating delimiters, as observed by Dybvig et al. (2007). Section 4.7 offers a closer comparison of our implementation with the one by Dybvig et al. (2007). The version of our monad for delimited control that we introduce in this section is answer-type-safe, but it is not effect-safe. In Chapter 6, we show how to establish effect safety.

We briefly repeat the developments of Chapter 2, this time in Scala and using our monad `Control`. Consequently, the structure of the following sections is very similar: we first recapitulate delimited control, generalize to families of control operators with multiple prompts, and finally introduce the abstraction of effect handlers.

4.2.1 Delimiting Control

To recall, the following example program, which we adapt from Danvy and Filinski (1990), uses the control operator shift and delimiter reset of Section 2.1.1 in variant $-shift+$:

$$1 \;+\; \mathsf{reset}\;\{\; 10 \;+\; \mathsf{shift}\;\{\; k \Rightarrow k(k(100))\;\}\;\}$$

The control operator shift captures the continuation and binds it to `k`. The continuation is delimited: only the evaluation context up to the enclosing reset is captured and thus the continuation corresponds to $10 \;+\; \square$. This example reduces in the following steps:

$$\begin{array}{lll}
1 \;+\; \mathsf{reset}\;\{\; 10 \;+\; \mathsf{shift}\;\{\; k \Rightarrow k(k(100))\;\}\;\} & & \\
1 \;+ & k(k(100)) & \textbf{where} \quad k \;=\; x \Rightarrow \mathsf{reset}\;\{\; 10 \;+\; x\;\} \\
1 \;+ & k(\mathsf{reset}\;\{\; 10 \;+\; 100\;\}) & \\
1 \;+ & k(110) & \\
1 \;+ & \mathsf{reset}\;\{\; 10 \;+\; 110\;\} & \\
1 \;+ & 120 & \\
121 & &
\end{array}$$

The continuation k does *not* contain the frame $1 \;+\; \square$, which is outside of the delimiting reset. From the reduction steps, we can recognize the semantics $-shift+$: The body of the continuation k is again delimited by reset, while the body of shift (*i.e.* $k(k(100))$) is not.

Example 1

Using the `Control` monad, we can translate the above example to Scala:

```
val ex: Control[Int] = reset { p ⇒
  shift(p) { k ⇒ k(100) flatMap { x ⇒ k(x) } } map { y ⇒ 10 + y }
} map { z ⇒ 1 + z }
```

Figure 4.2 already introduced the first part of the interface of the monad for delimited control. To actually capture continuations and delimit their scope, Figure 4.4 now additionally introduces the type `Prompt[Result]` and two functions `shift` and `reset`. The delimiter function `reset { p ⇒ PROG }` introduces a *fresh prompt* `p` and delimits the control effects for `p` in the provided program `PROG`. Our control operator `shift` takes a prompt as a parameter. This allows us to select which `reset` we want to shift to. It captures the current continuation up to the corresponding `reset` and passes the continuation to the body. The current example only uses one prompt `p` but we will shortly see how to utilize this additional expressivity.

4.2.2 Families of Delimited Control Operators

Every call to `reset` introduces a fresh prompt `p`, or in other words, each prompt `p` labels the corresponding `reset`. This gives rise to a dynamic number (that is, a *family*) of control operators `shift(p)`, one for each prompt created by a `reset`. The following example illustrates the use of multiple resets:

Example 2

We use `reset` twice, introducing two different prompts $\texttt{p}_1$ and $\texttt{p}_2$.

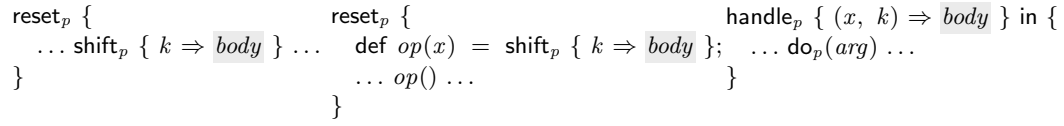
```
val ex₂: Control[Int] = reset { p₁ ⇒
  reset { p₂ ⇒
    shift(p₁) { k ⇒ pure(21) }
  } map { if (_) 1 else 2 }
} map { 2 * _ }
```

The captured continuation `k` contains the program segment delimited by prompt $\texttt{p}_1$. It corresponds to the evaluation context $\mathsf{if}\ (\mathsf{reset}_{p_2}\ \{\ \square\ \})\ 1\ \mathsf{else}\ 2$. In this example the body of `shift` discards the continuation `k` and immediately returns `21`. Hence, `run { ex₂ }` evaluates to `42`.

4.2.3 Answer Type Safety

Operationally, `shift(p)(k ⇒ PROG)` replaces the corresponding reset by the body `PROG`. To be type safe, the return type of the body thus has to match the answer type at the `reset`. In our setting of multiple first-class prompts, we guarantee this by following Dybvig et al. (2007) (respectively Gunter et al. (1995)) and parameterize prompts over the answer type `R`. In Example 2, the two prompts have type $\texttt{p}_1$`: Prompt[Int]` and $\texttt{p}_2$`: Prompt[Boolean]`.

Furthermore, the type of `reset[R]` requires three types to be `R`: the answer type of the created prompt (*i.e.*, `Prompt[R]`), the result of the given program (*i.e.*, `Control[R]`), and the return type of `reset`. Similarly, the type of `shift` uses the answer type of the given prompt and requires that *(a)* the return type of the continuation and *(b)* the return type of the given body agree



(a) Programming with delimited control. (b) Structured programming with delimited control. (c) Programming with effect handlers.

Figure 4.5. Programming with effect handlers as structured programming with delimited control.

with *(c)* the answer type expected at the `reset` that introduced the prompt. Answer type safety is especially important in the presence of multiple prompts. Each reset potentially introduces a prompt with a different answer type. Shifting to a reset with the wrong type should be statically rejected. For instance, shifting to p_2 would render the example type incorrect, since this would require the body of shift to return a computation of type `Boolean`, not `Int`.

4.2.4 Structured Programming with Delimited Control

Multi-prompt delimited continuations are implementation internals that Effekt does not expose to the user. However, to highlight the point of Chapter 2, that programming with effect handlers is structured programming with delimited control (Kammar et al., 2013), we express our running example in terms of the operators `shift` and `reset`. Figure 4.5 provides an overview of the developments, where Figure 4.5c corresponds to programming with effect handlers and Figure 4.5b depicts the style of structured programming, which we will employ in the rest of this section. It is instructive to compare the different ways of implementing effects, since structured programming with delimited control immediately gives rise to capability-passing style.

Example 3

Figure 4.6 translates the handlers of our running example to directly use control effects. Let us assume the type aliases for effectful programs with the signatures of `flip` and `raise` (Figure 4.6a). The user program from the introduction then carries over almost unchanged.

```
def drunkFlip(raise: Exc, flip: Amb): Control[String] = for {
  caught ← flip()
  heads  ← if (caught) flip() else raise("We dropped the coin.")
} yield if (heads) "Heads" else "Tails"
```

The program `drunkFlip` now takes effectful functions `raise` and `flip` directly as parameters. The effectful functions directly correspond to effect operations. While the program was previously written in capability-passing style, it is now written in a very similar *operation-passing style*.

We implement what could be viewed as handlers for `raise` (Figure 4.6b) and `flip` (Figure 4.6c) as higher order functions that construct implementations of effect operations and pass them to the given program. We refer to those functions as *handler functions*. They also change the result type `R` to the effect domain `Option[R]` and `List[R]` respectively, just as we have previously seen. The pattern to directly implement effect handlers in terms of delimited control shows that handler functions encapsulate three aspects of effect handling in one module:

```
type Amb = () ⇒ Control[Boolean]
type Exc = String ⇒ Control[Nothing]
```

(a) Effect signatures for exception and ambiguity as type aliases for effectful functions.

```
def maybe[R](prog: Exc ⇒ Control[R]): Control[Option[R]] = reset { p ⇒
  val raise: Exc = msg ⇒ shift(p) { resume ⇒ pure(None) }
  prog(raise) map { x ⇒ Some(x) }
}
```

(b) Handler function for the exception effect.

```
def collect[R](prog: Amb ⇒ Control[R]): Control[List[R]] = reset { p ⇒
  val flip: Amb  = () ⇒ shift(p) { resume ⇒ for {
    xs ← resume(true)
    ys ← resume(false)
  } yield xs ++ ys }
  prog(flip) map { x ⇒ List(x) }
}
```

(c) Handler function for the ambiguity effect.

Figure 4.6. Using answer-type-safe delimited control to declare and handle exception and ambiguity effects.

1. The handler function uses `reset` to delimit the scope of the captured continuation.
2. It *locally* uses the fresh prompt introduced by `reset` to implement the effect operations in terms of `shift`. The effect operations close over the prompt and are thus the only way to capture the continuation.
3. It finally lifts the return type of the handled function `R` into the effect domain, which makes it the answer type of the `reset`.

Grouping these aspects of effect handling in one module, it is possible to locally reason about type safety. The implementation of `raise` is only safe because we statically know from the type of `p: Prompt[Option[R]]` that the answer type expected at the `reset` is `Option[R]`. Likewise, in `collect` we use the fact that we statically know that the answer type in the body of `shift(p)` is `List[R]` to safely concatenate the results of the two calls of the continuation `resume`.

As in the introduction, we can use both handler functions in a different order to run the program, getting different results of different type.

```
val res1: List[Option[String]] = run {
  collect { flip ⇒ maybe { raise ⇒ drunkFlip(raise, flip) }}
}
val res2: Option[List[String]] = run {
  maybe { raise ⇒ collect { flip ⇒ drunkFlip(raise, flip) }}
}
```

The operations `raise` and `flip` are first-class functions and close over the fresh prompts `p` that we introduced with `reset`. Writing effect handlers in this style, we can think of `reset` as introducing a fresh `shift` that is only available in the *lexical scope*.

```
new Collect handle { amb ⇒
  var x = 0
  amb.flip() map { b ⇒
    if (b) { x = 2 } else {}
    x
  }
}
▶ List(2, 2)
```

```
new Collect handle { amb ⇒ region { state ⇒
  val x = state.Field(0)
   amb.flip() flatMap { b ⇒
     if (b) x.put(2) else pure(())
  } andThen x.get()
}}

▶ List(2, 0)
```

(a) Example illustrating the difference between global (left) and local, backtrackable state (right). Access to the state is highlighted.

```
trait State {
  def Field[T](init: T): Field[T]

  trait Field[T] {
    def get(): Control[T]
    def put(value: T): Control[Unit]
    def update(f: T ⇒ T): Control[Unit] = get().map(f).flatMap(put)
  }
}
def region[R](prog: (s: State) ⇒ Control[R]): Control[R] = ...
```

(b) Effect signature of the state effect and its built-in handler `region`.

Figure 4.7. The `State` effect: mutable state that interacts well with multiple resumptions.

4.3 Ambient State

Many practical handler implementations require some form of mutable state. Our host-language Scala already supports mutable state and effect handlers can readily use it. However, combining mutable state and delimited control can interact in unforeseen ways (Kiselyov et al., 2006; Leijen, 2018b). This is illustrated by the example in Figure 4.7a. In the program on the left, we introduce a local mutable variable `x`. Only if the `flip` operation returns true we modify it. Still, running the example outputs `List(2, 2)`. Surprisingly, the change to `x` is also visible in the branch where `flip` returns **`false`**. Built-in mutable state is *global* and does not backtrack across different resumptions.

However, for some effect handlers, we want *local*, backtrackable state. Running the example program should yield `List(2, 0)`, backtracking the local state when resuming for the second time. Leijen (2017c) calls this form of state "ambient". There are multiple ways to obtain local, backtrackable state. As in Section 3.3.4, we could define a state effect in terms of control effects. This technique has been presented by Kammar et al. (2013) for effect handlers and by Kiselyov et al. (2006) for delimited control. Alternatively, we could also offer a generalized form of effect handlers that support local state. For example, Koka chooses this solution and supports "parameterized handlers", which manually perform state-passing (Leijen, 2017c).

For our design of Effekt, we decided to offer a built-in `State` effect that exhibits the correct backtracking behavior when combined with our implementation of delimited control. Figure 4.7b defines the interface of the effect signature `State` and the corresponding built-in handler `region`. The effect signature `State` contains a nested type `Field` with the necessary operations to retrieve (*i.e.*, `get`) and update (*i.e.*, `put` and `update`) the state. The use of the state effect is illustrated in the right column of Figure 4.7a. Given a capability `state` we create a new field `x` with **`val`** `x = state.Field(...)`. To read and update the state, we use the effect operations `get` and `put` on the field. In our design, state access is effectful and thus operations `get` and `put` return a result in the `Control` monad.

Our implementation of `Control` is specialized to properly save and restore the fields for each `State` effect. The `region` handler introduces a new mutable frame, which holds the allocated fields on the stack that the implementation of `Control` is based on. Capturing the continuation with `shift` will capture parts of the stack, shallowly copying the current values of the fields. Calling the continuation restores the values. This implements dynamically scoped state (Kiselyov et al., 2006) and allows constant time access and modification at the cost of copying on continuation capture. If we would switch the order of the effect handlers to

```
region { state ⇒ new Collect handle { amb ⇒ ... }}
```

running the example would again yield `List(2, 2)`. In this order, changes to the state are persistent across multiple resumptions. In the next section, we will encounter a few examples that use this state effect.

Note Our design for ambient state is only necessary, since continuations in Effekt can be resumed multiple times. Continuations in Effekt are thus *multi-shot* (Hieb et al., 1990), while for instance continuations in Multicore OCaml (Dolan et al., 2014) are *one-shot*[13] (Dolan et al., 2015). Restricting Effekt to one-shot continuations, it would be safe to use native mutable state, since we cannot observe the difference between local and global state anymore.

[13] Multicore OCaml offers support to manually clone continuations. This way, continuations can also be resumed multiple times (Kiselyov and Sivaramakrishnan, 2018).

4.4 Composing Effect Signatures

In the previous Section 4.2.4, we have seen how to program directly with multi-prompt delimited control. Comparing the implementations of handler functions `maybe` and `collect` with the handlers `Maybe` and `Collect`, we find that there is only a small difference between programming with multi-prompt delimited control (Section 4.2) and programming with effect handlers (Section 4.1). In the following sections, we introduce the missing interfaces to program with effect handlers.

We also give additional examples to evaluate the different dimensions of extensibility gained by embedding Effekt into Scala. Mapping effect signatures and handlers to existing features of object-oriented programming allows us to reuse the abstractions those features offer. In particular, as we will see, mapping signatures to Scala's traits opens up interesting new modularity benefits. With the advent of default methods (Gosling et al., 2015), many of those benefits also apply to Java's interfaces. Here, we revisit some of the abstractions and highlight the modularity benefits newly gained by implementing effect handlers this way.

We will see how to compose effect signatures, effect handlers, and effectful programs.

4.4.1 Extending Effect Signatures

Since signatures are traits (Odersky et al., 2006), we can extend them and add new operations. Here we extend the `Amb` trait (Section 4.1) with an additional effect operation `choose`.

```
trait Choose extends Amb {
  def choose[A](first: A, second: A): Control[A]
}
```

This introduces a subtyping relationship between `Amb` and `Choose` capabilities. A `Choose` handler thus can also be used to handle `Amb`. This cannot be expressed in Koka, for example, where a handler handles precisely the effects of a single given effect signature (Leijen, 2017c).

4.4.2 Default Methods: Primitive vs. Derived Effect Operations

Traits in Scala cannot only contain abstract method declarations, but also *concrete* method implementations. Similarly, our effect signatures cannot only contain abstract operations, but also concrete effect operation implementations, as illustrated in the following example.

```
trait Fiber {
  // primitive effect operations
  def suspend(): Control[Unit]
  def fork(): Control[Boolean]
  def exit(): Control[Nothing]

  // derived effect operations
  def forked(p: Control[Unit]): Control[Unit] = for {
    b ← fork()
    r ← if (b) p andThen exit() else pure(())
  } yield r
}
```

Here, the `Fiber` effect (Dolan et al., 2015) for cooperative multitasking has abstract effect operations `suspend`, `fork`, and `exit` that need to be implemented by handlers. It additionally contains a concrete effect operation `forked`, which is implemented in terms of `fork` and `exit`. We refer to operations like `suspend` as *primitive effect operation* and to operations like `forked` as *derived effect operations*. Of course, in other languages with effect handlers like Koka, `forked` could be defined as a simple effectful program using the `Fiber` effect. In Effekt, however, handlers that implement the `Fiber` effect can choose to overwrite the `forked` implementation – for instance for efficiency purposes.

The derived operation `forked` does not make any assumptions about the handler implementation. In particular, it does not explicitly capture the continuation with `shift` but only uses the other effect operations of `Fiber`. This illustrates an important difference to languages like Koka, where every effect operation always automatically captures the continuation (Leijen, 2017c). As a result, in Koka, the effect operation call `fork()` would *not* refer to the same handler, but an *outer* one. In our example, however, `forked` does not explicitly capture the continuation and it is hence safe to call `fork` while referring to the same handler. For similar reasons, the following implementation of `forked` could potentially result in a runtime error:

```
def forked[E](p: Control[Unit]) = use { resume ⇒ fork() flatMap resume } // BAD
```

In Section 6, we will see how to embed an effect system in Scala to statically reject implementations like the above.

4.4.3 Abstract Type Members: Effect Signatures as Module Interfaces

Another particularly interesting example of abstraction reuse are Scala's abstract type members (Odersky and Zenger, 2005b). Mapping effect signatures to Scala traits, signatures cannot only describe effect operations, but also have (abstract) type members. This opens up interesting ways to structure effect signatures, illustrated by the signature for the async effect (Dolan et al., 2017):

```
trait Async {
  type Promise[T]
  def async[T](prog: Control[T]): Control[Promise[T]]
  def await[T](p: Promise[T]): Control[T]
}
```

The abstract type member `Promise` allows handler implementations to choose the representation of promises. The effect signature declares two effect operations (`async` and `await`) that refer to the abstract type. In a concrete capability of type `Async`, this choice of representation is hidden existentially. Odersky and Zenger (2005b) show how to use abstract type members and self-type annotations to express *family polymorphism* (Ernst, 2001): multiple (potentially mutually recursive) types form a family. Family polymorphism requires that the involved types can be covariantly refined together as a family, similar to open recursion on the level of types. With Effekt, the implementation strategy of using abstract type members to express family polymorphism now can also be applied to effect signatures and effect handlers. In our example, hiding the representation of promises behind an abstract type member ensures that promises can only be awaited by the capability that instantiated them.

Example: Asynchronous Programming

Having declared the effect signatures for `Async` and `Fiber` we can write effectful programs using these effects:

```
def asyncExample(f: Fiber, a: Async) = for {
    p ← a.async { for {
        _ ← log("Async 1")
        _ ← f.suspend()
        _ ← log("Async 2")
        _ ← f.suspend()
      } yield 42 }
    _ ← log("Main")
    r ← a.await(p)
    _ ← log("Main with result " + r)
  } yield ()
```

This example illustrates how two advanced control-flow structures, one for asynchronous programming and one for cooperative multitasking, can naturally be used together. We will see how to run this example after having defined the handlers for `Fiber` and `Async`.

4.4.4 Nested Traits: Families of Effectful Types

Abstract type members like `Promise` are not the only way to express a family of effectful types. Traits in Scala can also be nested, and we can refactor the effect signature `Async` to:

```
trait AsyncNested {
  trait Promise[T] { def await: Control[T] }
  def async[T](prog: Control[T]): Control[Promise[T]]
}
```

This is reminiscent of the `State` effect (Figure 4.7b), where the trait `Field` is expressed as nested type.

4.4.5 Mixing Effect Signatures

Scala supports mixin composition on traits (Odersky et al., 2006). This way, we can mix independently declared effect signatures:

```
trait Nondet extends Amb with Exc
```

Furthermore, since traits can contain both abstract and concrete definitions, effect signatures can be mixed to mutually implement primitive (*i.e.*, abstract) effect operations in one signature by derived (*i.e.*, concrete) effect operations in another. This mutually recursive matching of definitions works for both, abstract types and abstract methods (Odersky and Zenger, 2005b).

4.5 Composing Effect Handlers

In this section, we introduce the missing interfaces to program with effect handlers.

4.5.1 From Delimited Control to Effect Handlers

We purposefully presented programming with delimited control close to programming with effect handlers to highlight one small, but important difference:

> "the *handling* of the continuation takes place at the identical site as the creation of the continuation" — *Sitaram (1993, p. 148)*

For the purpose of this section, we like to rephrase this quote to:

> Effect handlers syntactically encapsulate the introduction of a prompt and its use.

Specifically, users do not manually pass *prompts* around, to then use them at arbitrary points in the program and to capture the continuation. Instead, effect operations that use a prompt are lexically related to the `reset` that introduced the prompt. As pointed out in Chapter 2, we believe that this is one of the most important aspects that make programming with effect handlers more approachable than programming with (multi-prompt) delimited control. Of course, we can voluntarily restrict ourselves to this mode of use, as we did in Section 4.2.4. While explicitly using `reset` and closing over prompts is a good way to understand the details of delimited control, it surfaces too much technical detail. In the following, we focus on effect operations and the high-level abstraction of handlers. In particular, each handler instance contains exactly one prompt marker, which it only uses internally. It would be possible to use the handler abstraction together with `reset` and have handlers close over prompts, as in:

```
reset { p ⇒ new Handler(p).handle(...) }
```

However, we want to fully hide prompts as a concept from users of Effekt. To ease the instantiation of handler, we thus include the following library-internal function, which corresponds to *pushPrompt* by Dybvig et al. (2007).

```
def resetWith[R](prompt: Prompt[R])(prog: Control[R]): Control[R]
```

Of course, we can express `reset` in terms of `resetWith`:

```
def reset[R](prog: Prompt[R] ⇒ Control[R]): Control[R] = {
  val p = new Prompt[R] {}; resetWith(p)(prog(p))
}
```

We are now finally ready to fully implement the Handler interface of Figure 4.3b. The expressive power of effect handlers comes from the two operations `handle` and `use`, which are encapsulated in the library trait `Handler`. Figure 4.8 shows the implementation of these two operations in terms of delimited control. Handlers internally create a prompt marker of type `Prompt[E]` with the effect domain `E`. The answer type of delimited continuations thus will be the effect domain `E`. In the implementation of `handle`, we install the prompt marker as a delimiter before resuming with `prog`. The handler thus delimits the extent of captured continuations. By running `unit`

```
trait Handler[R, E] {
  protected def unit(result: R): Control[E]

  // Each capability contains a unique prompt
  private val prompt = new Prompt[E] {}

  // Handlers encapsulate capturing ...
  protected def use[A](body: (A ⇒ Control[E]) ⇒ Control[E]): Control[A] =
    shift(prompt)(body)

  // ... and delimiting the continuation in one interface
  def handle(prog: this.type ⇒ Control[R]): Control[E] =
    resetWith(prompt)(prog(this) flatMap unit)
}
```

Figure 4.8. The handler implementation – handlers contain a prompt marker, `use` captures the continuation and `handle` delimits the scope.

after `prog`, programs that use no further effects will be lifted from `R` to `E`. The method `use` calls `shift` with `prompt` as a prompt marker to capture the continuation up to the most recent call to `handle` on this very handler instance. The method `unit` is still left abstract, as it needs to be implemented by concrete handlers like `Collect` and `Maybe`.

Effect handlers in Effekt are *deep handlers* (Kammar et al., 2013). That is, all effect operations are recursively handled by the same handler. To implement deep handlers, we choose $-shift+$ as underlying semantics of `Control`. This asserts that all subsequent calls to `use` on this handler are again delimited by `prompt`. Our operations `handle` and `use` are thus conceptually very similar to *spawn* and the corresponding *controller* by Hieb and Dybvig (1990).

Defining handlers as traits allows us to use mixin composition and thereby discover new opportunities for extensible handler definitions, which we explore in the remainder of this section.

4.5.2 The Effect Expression Problem

Many implementations of libraries and languages for (algebraic) effects and handlers are based on a *deep embedding* (Boulton et al., 1992) of effect operations (Kiselyov and Ishii, 2015). They reify effect operations as alternatives in a sum type and represent effectful computations as a command-response tree. For instance, the `flip` effect operation would be reified as a constructor of an algebraic data type `Amb`. Handlers fold over the tree of computation and use pattern matching to interpret the reified effect operations (Leijen, 2017c; Hillerström et al., 2017; Bauer and Pretnar, 2015; Kiselyov and Ishii, 2015; Kiselyov and Sivaramakrishnan, 2016). To mix programs with different effects means to extend an open union type of reified effect operations.

In contrast, by performing capability passing and representing effect signatures as traits, Effekt builds on a *shallow embedding* (Hudak, 1998; Carette et al., 2007) of effect operations. Instead of folding over the tree of computation, user programs directly call effect operations on the handler. In a language with mixin composition, shallow embeddings can be structured in a pleasingly extensible way (Oliveira and Cook, 2012). Thus, Effekt has a solution to the *expression problem* (Wadler, 1998) at its foundation, a property it shares with many other effect handler

implementations. For instance, languages like Koka (Leijen, 2014), Frank (Lindley et al., 2017), and Links (Hillerström et al., 2017) are based on row polymorphism (Gaster and Jones, 1996) and Extensible Effects (Kiselyov et al., 2013; Kiselyov and Ishii, 2015) are based on open unions (Swierstra, 2008).

Viewing the tree of computation as a recursive data type, we can define the *effect expression problem* as modularly and type safe being able

a. to implement new handlers for an effect operation – this corresponds to adding a new function definition over the recursive data type in the original expression problem;
b. to add new effect operations – this corresponds to adding a new variant to the recursive data type in the original expression problem.

The analogy to the expression problem (Wadler, 1998), however, is not perfect: Most descriptions of the expression problem only consider a single algebra, whereas with effect handlers we typically have more than one effect signature and the order of handling / folding over the operations affects the semantics.

4.5.3 Dimensions of Extensibility

We can relate extensibility dimensions discussed in the literature on the expression problem to the effect handler setting and show how Effekt supports them. Importantly, by embedding effect handlers into a general purpose programming language like Scala, the modularity features of the host language become available to structure effectful programs and handlers.

Adding new Handlers for an Effect

The first dimension of the effect expression problem. A central feature of every implementation of effects and handlers is the ability to define a new handler for an existing effect. Effekt supports this feature: users can define a new trait or class that implements an existing effect signature.

Adding new Operations to an Effect

The second dimension of the effect expression problem. We can distinguish between adding an operation to an existing effect signature, and adding a new effect signature. Effekt supports modular extension of effect signatures as illustrated by the example trait `Choose` of Section 4.4. Many other languages, like Koka, cannot extend or compose effect signatures. In those languages, it is therefore also not necessary to extend or compose handler implementations. In contrast, Effekt allows the programmer to extend handler implementations modularly.

```
trait CollectChoose[R] extends Collect[R] with Choose {
  def choose[A](first: A, second: A): Control[A] = for {
    b ← flip()
  } yield if (b) first else second
}
```

In this example, the handler for the extended effect signature `Choose` extends the existing `Collect` handler and only implements the missing effect operation `choose`. The example also illustrates that we can reuse the implementation of `flip` to implement `choose`.

```
trait Poll extends Async {
  val state: State; val fiber: Fiber
  type Promise[T] = state.Field[Option[T]]

  def async[T](prog: Control[T]) = for {
    p ← pure(state.Field[Option[T]](None))
    _ ← fiber.forked { prog flatMap { r ⇒ p.put(Some(r)) }
  } yield p

  def await[T](p: Promise[T]) = p.get() flatMap {
    case Some(r) ⇒ pure(r)
    case None ⇒ fiber.suspend() andThen await(p)
  }
}
```

Figure 4.9. Handler for the `Async` effect – using two effects `State` and `Fiber`.

4.5.4 Mixing Handlers – Horizontal Composition of Handlers

The description of the expression problem has seen many extensions and additional requirements. One additional requirement, described by Odersky and Zenger (2005a), is that the programmer should be able to combine independently developed extensions. For effect handlers this translates to compose two existing effect handlers. This feature might seem unnecessary in the context of effect handlers, where handler composition can already be expressed by nesting handlers (that is, by function composition). However, by using trait mixin composition to combine two handlers, the handler implementations can share implementation details like private methods, private state, and dependencies on other internally used effects. As an example, we define another handler for ambiguity that performs backtracking to compute only the first successful result:

```
trait Backtrack[R] extends Amb with Handler[R, Option[R]] {
  def flip() = use { resume ⇒ for {
    attempt ← resume(true)
    res ← if (attempt.isDefined) pure(attempt) else resume(false)
  } yield res }
}
```

We can implement the `Nondet` effect simply by mixing the handlers `Backtrack` and `Maybe`:

```
class FirstResult[R] extends Nondet with Maybe[R] with Backtrack[R]
```

The use of mixin composition is legal, since the two handlers assume the same effect domain. In general, handler implementations should be defined as traits in order to enable this form of composition. In languages like Java, using interfaces, a similar style of composition is possible but comes with a list of restrictions. In Dotty however, most of those restrictions have been lifted and traits can contain method implementations, fields, and receive constructor arguments. Using the composed handler `FirstResult`, we can handle `Exc` and `Amb` simultaneously:

```
val res3: Option[String] = run { new FirstResult handle { n ⇒ drunkFlip(n, n) } }
▶ Some("Heads")
```

The example illustrates how handlers can be composed *horizontally* with mixin composition under the condition that they interpret the effects into the same effect domain. Operationally, they share the same prompt marker. By subtyping, the combined handler can be used to handle both effects. In $\texttt{res}_3$, the capability is passed down twice, once to handle the `Amb` effect and once to handle the `Exc` effect. Every handler trait only implements one aspect of the overall handler component, which is then created by mixin composition. Based on traits and mixin composition, many OOP modularization strategies can now be applied to implement handlers:

- Programmers can use method overriding, super calls, and dynamic dispatch to model effectful *extension points* as methods that can be overridden in subclassing handler traits.
- Programmers can use abstract methods to describe the required interfaces of a handler trait and concrete method implementations to describe the provided interface. Mixin composition then matches the abstract and concrete method implementations based on the method signatures. This also allows expressing mutually recursive dependencies between handler components (Odersky and Zenger, 2005b).
- Programmers can use visibility modifiers to control how effect operations can be accessed. This way, helper effect operations can be restricted to be only locally accessible from the current handler implementation.

4.5.5 Composition over Inheritance – Vertical Composition of Handlers

Effect handlers allow us to locally handle a subset of effects, used by a program. To do so, handlers can again use effects in their implementation, which are then handled by other handlers. That is, we can compose handlers *vertically*.

Different to most other formulations of effect handlers, handlers in Effekt do not have to capture and use the continuation and consequently do not have to inherit from the `Handler` trait. It is up to the handler implementation to decide. Figure 4.9 presents an example of such a handler that does *not* explicitly capture the continuation. Instead, it uses the effects `State` and `Fiber` and therefore requires the capabilities `state` and `fiber` as abstract value members. We define `Promise` to be type `state.Field`. That is, we store the result of the asynchronous computation in a field provided by the `state` capability. The handler function `poll` takes the two required capabilities to construct an instance of the handler trait `Poll`:

```
def poll[R](s: State, f: Fiber)(prog: Async ⇒ Control[R]): Control[R]
  = prog(new Poll { val state = s; val fiber = f })
```

Compared to the expression problem literature, this forwarding to other handlers is reminiscent of family self references (Oliveira et al., 2013) or base algebras (Hofer et al., 2008).

4.5.6 Vertical Composition and Continuation Capture

The `Poll` handler (Figure 4.9) uses two effects `State` and `Fiber` in its implementation, but does not capture the continuation. All other effect handlers we have seen so far did inherit from `Handler` to capture the continuation but did not use any other effects. However, there is another way to use effects that we have not seen so far: First capture the continuation and *then* use another effect. Figure 4.10 and uses this technique and adapts the handler implementation by Dolan et al. (2015) to implement the `Fiber` effect as a round robin scheduler. In addition

```
trait Scheduler[R] extends Fiber with Handler[R, Unit] {
  val state: State

  def unit(r: R) = pure(())

  type Queue = List[Control[Unit]]
  lazy val queue = state.Field[Queue](Nil)

  def exit() = use { resume ⇒ pure(()) }
  def fork() = use { resume ⇒
    queue.update { resume(true) :: resume(false) :: _ } andThen run
  }
  def suspend() = use { resume ⇒
    queue.update { _ appended resume(()) } andThen run
  }

  private def run: Control[Unit] = queue.get() flatMap {
    case Nil ⇒ pure(())
    case p :: rest ⇒ queue.put(rest) andThen p andThen run
  }
}
```

Figure 4.10. Handler for the `Fiber` effect – using the `State` effect *after* capturing the continuation.

to inheriting from `Handler`, it also requires the `state` capability. The difference now is in the combined usage of continuation capture (via `use`) and the `state` capability. The implementation of the effect operation `exit` is not interesting, as it simply discards the continuation. However, the two remaining operations `fork` and `suspend` both capture the continuation to then use the `state` within the body passed to `use`. As seen in Section 4.3, the interaction between state and continuation capture can be subtle. For the `Scheduler` handler, the state should be persisted across different fibers, forked by resuming once with **`true`** and once with **`false`**. That is, the state effect is required to be the outer handler. This also becomes visible in the handler function, which requires the state capability:

```
def scheduler(st: State)(prog: Fiber ⇒ Control[Unit]) =
  new Scheduler { val state = st } handle { fiber ⇒ prog(fiber) }
```

Capturing the continuation to then use some other effect can easily lead to runtime error. This is, for example, the case if the captured continuation contains the delimiter of the other effect. Using the other effect, after capturing the delimiter in the continuation results in a runtime error. In Section 6.1, we will see how statically reject those programs by embedding an effect system. We then revisit the scheduler example, using the effect-safe variant of ScalaEffekt (*cf.* Figure 6.6b).

Example: Running Asynchronous Programs

Having defined handlers for the `Fiber` and `Async` effects, we can finally run the program `asyncExample`, which results in the output on the right:

```
region { s ⇒
  scheduler(s) { f ⇒
    poll(s, f) { a ⇒
      asyncExample(f, a)
    }
  }
}
```

```
Async 1
Main
Async 2
Main with result 42
```

4.6 Composing Effectful Programs

By passing capabilities explicitly, in the presence of multiple instances of the same effect, we are able to explicitly select which instance of an effect to use. For example, we can use multiple instances of `Fiber` in one program to model different thread pools. At the same time, passing capabilities explicitly can be a burden since it introduces manual boilerplate. We illustrate this by reimplementing an example from Section 3.3: an effectful program that uses effects to express a parser (Leijen, 2016):

```
// AB ::= a AB | b
def parseAsB(amb: Amb, exc: Exc, in: Input): Control[Int] = alternative(
  accept('a')(in, exc) andThen parseAsB(amb, exc, in) map { _ + 1 },
  accept('b')(in, exc) map { x ⇒ 0 })(amb)

def accept(exp: Char)(in: Input, exc: Exc) = in.read() flatMap { t ⇒
  if (t == exp) pure(()) else exc.raise("Expected " + exp) }

def alternative[A](fst: Control[A], snd: Control[A])(amb: Amb) =
  amb.flip() flatMap { b ⇒ if (b) fst else snd }
```

In the example, we use an `Input` effect that allows reading from an input stream.

```
trait Input extends Eff { def read(): Control[Char] }
```

Effect handlers allow us to choose between many different semantics without having to change concrete parsers, like `parseAsB`. To implement breadth-first (Swierstra, 2009) parsing, we can view alternatives in a grammar as cooperative (parsing) processes (using the `Fiber` effect) and use effect handlers like `Scheduler` to schedule these processes (Dolan et al., 2017). Similarly, an alternative handler implementation of `Input` could capture the continuation at the call to the effect operation `read()`. This way, to implement online (or streaming) parsers (Swierstra, 2009), we can then store the continuation until further input is available. The continuation of `read()` can also be seen as the *derivative* of the parser (Kiselyov, 2007). Offering this derivative as part of a parser combinator library to the grammar author, it is possible to express parsers for two-dimensional layout in a modular way (Brachthäuser et al., 2016).

Composing effectful programs that use different effects, the programmer needs to manually pass the capabilities to the respective function calls. In particular, a program (like `parseAsB`) that uses other effectful programs (like `accept` and `alternative`) needs to take the *union* of all capabilities, required by its dependencies. To call them, the programmer needs to select the correct subset of capabilities and provide them along other arguments.

4.6.1 Implicits for Capability-Passing Style

While the overall design of Effekt is largely independent of Scala, there are certain features that ease the use of the library. One such feature is implicit parameters (Odersky, 2019a). Implicit parameters (called "given-clauses" or "contextual parameters" in Scala 3 (Odersky, 2019c)) can help to automatically pass function arguments based on their type (Odersky et al., 2017). This makes implicits a perfect fit for the Effekt library.

Implicit Parameters for Effectful Functions

To implicitly lookup capabilities, for every effect signature we define functions like:

```
def Amb given (a: Amb) = a
```

Calling the nullary function `Amb` implicitly searches for a value of the equally named type in the current scope and returns it. Using these helpers, we can now rewrite the above example to:

```
def parseAsB given Amb given Exc given Input: Control[Int] = alternative(
  accept('a') andThen parseAsB map { _ + 1 },
  accept('b') map { x ⇒ 0 })

def accept(exp: Char) given Input given Exc = Input.read() flatMap { t ⇒
  if (t == exp) pure(()) else Exc.raise("Expected " + exp) }

def alternative[A](fst: Control[A], snd: Control[A]) given Amb =
  Amb.flip() flatMap { b ⇒ if (b) fst else snd }
```

The signature of `accept` informs us that it relies on an instance of `Input` and an instance of `Exc` being available in scope at the call site[14]. At the same time, it brings these two instances in scope for the method body, so `Input.read` and `Exc.raise` will resolve to method calls on the corresponding implicit argument. Note that `Input` is the nullary method call to a boilerplate function as defined above. For the purpose of this chapter, it is enough to understand that implicit search is performed at compile time, is lexically scoped, and type directed. Implicit resolution results in a program very similar to the explicit capability passing variant above. We can also choose to bind implicit parameters to explicit names.

```
def parseAsB given (a: Amb) given (e: Exc) given (i: Input): Control[Int]
```

Binding capabilities to names also enables us to fall back to passing them explicitly (*e.g.*, `accept('a') given a given e`). This is important to resolve conflicts in case of ambiguous implicits, which result in a compile time error.

[14] Since Scala 3, naming implicitly bound variables is optional (Odersky, 2019c). The signature thus roughly corresponds to `def accept(exp: Char)(implicit $1: Input, $2: Exc)` in Scala 2.

4.6.2 Binding Implicit Parameters

Similarly, we can modify the signature of the `handle` method to accept an implicit function type, or "contextual function" **given** `A ⇒ B` (Odersky et al., 2017). That is, a lambda that takes an implicit parameter of type `A` to return a `B`:

```
def handle(prog: given this.type ⇒ Control[R]): Control[E]
```

Handlers thus introduce bindings for implicit parameters. Since the parameter to the lambda is implicit, the compiler will perform automatic eta-expansion, based on the expected function type. This allows us to rewrite the handler applications to hide capability passing altogether. Assuming adapted versions of our previous examples, we can see how implicits

```
run { new Collect handle { new Maybe handle { drunkFlip } } }
```

further simplify the process of rearranging the handlers:

```
run { new Maybe handle { new Collect handle { drunkFlip } } }
```

Implicit parameters are thus a perfect fit for capability passing, removing most its syntactic overhead.

4.6.3 Reducing the Overhead by Composition

Another strategy to reduce the burden of passing capabilities is by composition. We can define a trait that contains the necessary capabilities as members:

```
class Parser(val amb: Amb, val exc: Exc, val input: Input)
```

Now all three methods can be refactored to only take one (potentially implicit) argument of type `Parser`, manually projecting the fields where necessary.

Not only functions can obtain their arguments implicitly by type – the same also holds for classes and their constructor arguments. We can rewrite the composed parser as follows:

```
class Parser given (val amb: Amb, val exc: Exc, val input: Input)
```

Like with implicit parameters, when creating an instance of `Parser`, the constructor arguments can be omitted and will be filled in by the Scala compiler. Using the same technique, handlers can mark the effects they are dependent on as implicit.

4.7 Related Work and Chapter Conclusion

In this section, we discuss closely related work. In particular, we compare our approach of explicit capability passing to other implementations of effect handlers, put our implementation of `Control` in perspective with freer monads (Kiselyov and Ishii, 2015), and relate our effect handlers design for object-oriented languages to others.

4.7.1 Capability-Passing Style

We start by discussing some properties of explicit capability-passing style.

Shallow embedding of effect operations Many implementations of libraries and languages for effect handlers are based on a *deep embedding* (Boulton et al., 1992) of effect operations. In contrast, by performing capability passing, Effekt builds on a *shallow embedding* (Hudak, 1998; Carette et al., 2007) of effect operations. Interpretation of effect operations is moved from (external) pattern matching to (internal) dynamic dispatch, which makes the shallow embedding of effect operations a good fit for object-oriented programming languages. Similarly, Kammar et al. (2013) base their library implementation of algebraic effect handlers on Haskell type classes, effectively performing a shallow embedding. Using type classes and the associated dictionary passing helps Kammar et al. (2013) to achieve good performance results since it prevents the materialization of constructors for effect operations. It also avoids any search for the matching handler implementation in some kind of handler stack, as it is done in Koka (Leijen, 2017c, 2017b).

Simplified typing Another advantage of capability passing is that it simplifies typing. Combining object-oriented programming with effect handlers, we define `use` as a method on `Handler`. As a method, it naturally shares the type of the effect domain with its implementing class. We thus use dynamic dispatch instead of implementing a pattern-matching interpreter. This helps us to avoid advanced typing features, such as type constructor polymorphism (Kiselyov et al., 2013) or generalized algebraic data types typically associated with interpreter based solutions (Kiselyov and Ishii, 2015). In a previous presentation of Effekt (Brachthäuser and Schuster, 2017), we represented capabilities as a pair of a prompt marker and the handler implementation. To hide answer types existentially in the user program, this required us to use type members and path-dependent types. In contrast, Effekt as presented in this chapter is designed to remove requirements on the type system and to blend in with OOP paradigms. We immediately represent capabilities as instances of type `Exc` and hide implementation details like answer types by means of simple subtyping.

Manually selecting effects By performing capability passing, we require the user to explicitly select the handler to use. In other libraries and languages for effect handlers, an effect operation always resolves to the dynamically closest handler implementation. Explicitly calling methods on handlers has the advantage that no confusion arises when multiple handlers for the same effect are present. Other languages added features post hoc in order to allow programmers to select effects more explicitly. Leijen (2018b) added `inject` to the Koka language, and Convent et al. (2020) added `mask` and adaptors to Frank. Both `inject` and `mask` are conceptually similar to the *lift* operation proposed by Biernacki et al. (2017), which allows to select effect handlers in terms of the *distance* to the effect operation. For example, in Koka, `flip()` will be handled

by the closest handler for ambiguity, whereas `inject<amb>(fun{flip()})` will be handled by the second closest handler, and so forth. This is reminiscent of programming with De Bruijn indices (de Bruijn, 1972), where variables also refer to their binders by distance. In contrast, with explicit capability passing different effects can be referred to by name.

Effect polymorphism Since capabilities are first-class, functions (and objects) can close over them. This allows users to express some form of effect polymorphism (Osvald et al., 2016). Take the following effectful map function, adapted from Rytz et al. (2012), as an example:

```
def mapM[A, B](lst: List[A], f: A ⇒ Control[B]): Control[List[B]]
```

The signature of `mapM` only informs us that `f` has control effects, but is silent on the concrete effects. Still, we can invoke map passing a lambda that uses an effect like `Amb`.

```
new Collect handle { amb ⇒ map(List(1,2,3), a ⇒ amb.flip()) }
```

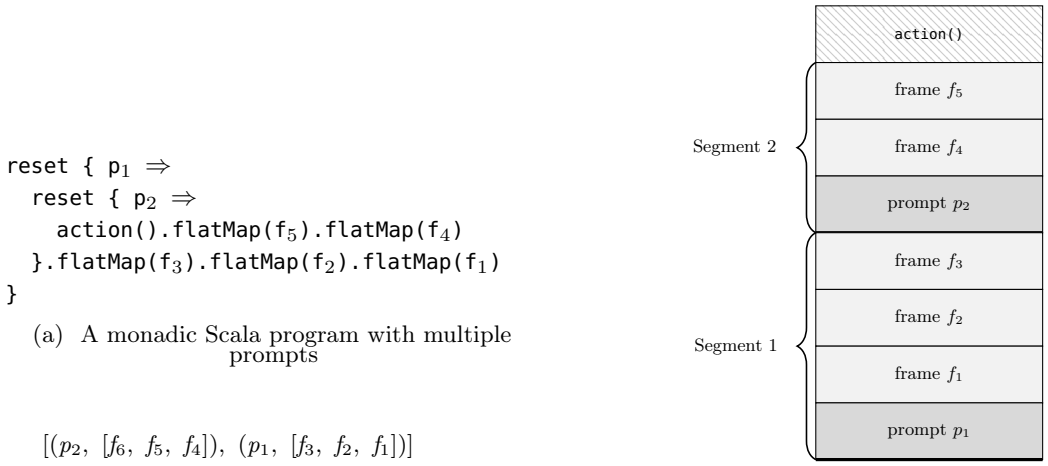
The function passed as argument `f` simply *closes* over the capability. The use of the `Amb` effect does not become visible in its type, which is `Int ⇒ Control[Boolean]`. The function `mapM` is thus effect polymorphic.

Effect safety Passing capabilities as evidence is not new. Osvald et al. (2016) also perform capability passing in Scala. They present capability passing as an alternative approach to traditional type and effect systems. A capability serves as a constructive proof, that the holder is entitled to use the actions associated with the capability. To statically prevent the leaking of capabilities, Osvald et al. introduce a new type system feature: arguments to functions can be marked as second class (or "local"). Similar to region based memory management (Kiselyov and Shan, 2008), the type checker then guarantees that the capability cannot leave the dynamic scope of the function call. They show how second-class values can be used to model checked exceptions, as in the following example, adopted from Osvald et al. (2016):

```
def Try[T](fn: (@local (Exception ⇒ Nothing)) ⇒ T): Option[T]
Try { throw ⇒ throw(new Exception) } // Safe usage of 'throw'
```

The function **`throw`** is introduced by the exception handler and marked as `@local`. This guarantees that **`throw`** cannot escape the dynamic scope of the function `fn`. Much like effect handlers, **`try`** thus introduces a capability. Liu (2016) presents a different approach to capability based effect safety, by distinguishing between functions that can capture capabilities and others that cannot (called "stoic"). In our design of Effekt, we adopt the capability passing of Osvald et al. (2016) and use it together with a monad for delimited control to generalize exception handlers to effect handlers. While second-class values might be an interesting way to achieve effect safety, in Chapter 6 we explore a different approach based on path-dependent types and intersection types.

Performance To implement important performance optimizations, Leijen (2017b) explicitly tags each effect operation in a handler with information about how the continuation is used. Similarly explicit, in Effekt, handlers capture the continuation with `use`. *Tail resumptive* handlers (Leijen, 2017b), *i.e.* handlers that only call the continuation in tail position, do not need to capture the continuation and do not call `use`. Not having to capture the continuation is only possible due to our combination of capability-passing style and multi-prompt delimited



(a) A monadic Scala program with multiple prompts

(b) Representation of the stack as a prompt-separated cons-list of frames.

(c) Visualization of the stack at the point of evaluating `action()`.

Figure 4.11. Representing stacks as prompt-separated list of frames.

continuations. As already mentioned in Section 2.3 and what is the essence of ambient functions (Chapter 3), capturing the continuation also leads to a "shift in perspective". That is, effects are dynamically resolved at the handler site – as opposed to at the call site of the effect operation. In Effekt, we can avoid capturing the continuation altogether since we replace dynamic lookup of effect handlers by explicit capability passing. Effect operations explicitly refer to capabilities at handler site by closing over them.
Building on the insights of this thesis, in related work (Schuster and Brachthäuser, 2018; Schuster et al., 2020), we started to explore strategies to pass capabilities *at compile time*. This way, programs are always fully specialized to the handlers under which they are executed.

Deep handlers and shallow embeddings Due to our design decision of a shallow embedding of effect operations, handlers in Effekt are *deep handlers* (Kammar et al., 2013). That is, all effect operations in the continuation captured by `use` will automatically be handled recursively by the very same handler. However, if a *shallow handler* semantics is required, it can be achieved by reifying the command-response trees of selected effect operations. A reifying effect handler interprets a program of type `R` into a free-structure `Free<R>` (Kiselyov et al., 2013), which then can be interpreted step-by-step. While this encoding is possible, it can lead to performance problems and memory leaks. Hillerström and Lindley (2018) present an encoding of shallow handlers in terms of deep handlers that does not suffer from these problems.

4.7.2 Implementing Monadic Delimited Control

Kiselyov and Sivaramakrishnan (2016) embed effect handlers into OCaml. They also build on an implementation of multi-prompt delimited control (Kiselyov, 2012), which makes use of the fact that OCaml includes bytecode instructions that support manipulation of the runtime stack. Just like Kiselyov (2012), our implementation of delimited control presented in this chapter is

based on the work by Dybvig et al. (2007). Since the JVM does not allow for stack manipulation, however, we build on their monad `CC` for multi-prompt delimited control (Dybvig et al., 2007). In particular, we choose to represent "continuations as sequences of frames" (Dybvig et al., 2007, Section 7.3) – but make the following adjustments:

- Dybvig et al. choose the semantics $-shift-$ for their control operator `withSubCont`. We instead choose $-shift+$ as it is closer to deep effect handlers (Kammar et al., 2013) and allows for a more uniform representation of stack segments, which are always delimited by a prompt.
- Dybvig et al. (Section 7.3) represent continuations as a type-aligned sequence of frames. As the authors remark, capturing the delimited continuation and searching for the prompt is thus linear in the number of individual frames. We slightly refine the representation and additionally group frames, which are not separated by prompts in a nested list (Figure 4.11). Our continuation is thus a prompt separated list of frames. This representation and the choice of $-shift+$ allows us to guarantee that prompt search, continuation capture, and resumption is always at most linear in the number of prompts.
- Dybvig et al. index the `CC` monad by the final answer type and then use rank-2 types to prevent mixing prompts between different "runs". We choose to reduce the required type system features at the cost of additional unsafety. However, much like our capabilities, which can escape, Dybvig et al. (2007) also do not guarantee that every continuation capture for a particular prompt will succeed. The continuation might not be delimited, which results in a runtime error that the prompt cannot be found. Considering this, mixing prompts between different runs seems like a minor additional unsafety compared to this existing source of runtime errors. In Chapter 6, we develop an effect-safe variant of multi-prompt delimited control that rules out both sources of unsafety.
- Dybvig et al. eventually perform all computation in a monad `P`, which is essentially a state monad used to generate fresh prompts. Since the setting of our embedding is the JVM, which does not support tail call elimination, but mutable state, we use object identity as the source for prompts and instead instantiate `P` as a trampoline (Ganz et al., 1999).

4.7.3 Effect Handlers and Object-Oriented Programming

As highlighted in Section 4.4, effect safe programming with effect handlers in a language with objects comes with new opportunities for modularization. As we will see in Chapter 6, it also comes with new challenges – mediating encapsulation and flexible use of objects. Not much prior work exists that combines effect handlers and object-oriented programming.

Eff (Torreborre, 2016) is a translation of the work by Kiselyov and Ishii (2015) to Scala. It almost exclusively targets the functional aspects of Scala and thus integrates well with other popular libraries for functional programming in Scala.

A notable exception is JEff (Inostroza and van der Storm, 2018), which appeared after our first presentation of `Effekt` and simultaneously with our publication on `JavaEffekt` (Brachthäuser et al., 2018). Inostroza and van der Storm (2018) also combine effect handlers and object-orientated programming in their language JEff. Similar to `Effekt`, JEff maps effect signatures to interfaces and handlers to classes implementing the interfaces. The following example, adapted from Inostroza and van der Storm (2018), gives the effect signature for an exception effect and the corresponding handler.

```
interface Exc { eff Nothing raise(String s) }
class Maybe<T>() implements Exc, Handler<Option<T>, T> {
  Option<T> return(T t) = new Some(t)
  eff Nothing raise(String s) = new None()
}
```

We take the striking similarities to Effekt as empirical support for our design. Despite the similarities in the combination of object-oriented programming and effect handlers, there are also a few notable differences. JEff, as a calculus, performs dynamic lookup and continuation capture similar to λ_{dch}. In contrast, as a library embedding in Scala, in Effekt we perform capability passing instead of dynamic binding and build on a monadic implementation of delimited control. In JEff, the continuation takes an updated copy of the effect handler as additional argument. This allows both to model stateful handlers and even to change the handler implementation for the rest of the computation, similar to shallow handlers. In Effekt, we added special support for ambient state as a separate effect and handlers are deep. The design of Effekt, presented in this chapter is not effect-safe. In contrast, Inostroza and van der Storm (2018) describe an effect system for their calculus. Their effect system does not feature effect polymorphism, which rules out many important examples. In Chapter 6, we present an effect-safe variant of ScalaEffekt that also supports effect polymorphism.

4.7.4 Conclusion

In this chapter, we presented the design of Effekt, a library integrating effect handlers with object-oriented programming. We followed the mantra that programming with effect handlers in Effekt *is* object-oriented programming and mapped effect signatures to interfaces and handlers to classes. We highlighted several dimensions of extensibility, which are supported by Effekt and showed modularization opportunities gained by combining effect handlers with object-oriented programming. We explored capability passing as an alternative to the traditional dynamic binding of effect handlers (*cf.* Chapter 3). While capability passing imposes some syntactic overhead on the user, it offers new interesting perspectives on performance optimizations (such as tail resumptions) and effect polymorphism. Most of the syntactic overhead associated with capability passing can be reduced by using Scala's implicits.

Despites the benefits, our implementation of Effekt in Scala suffers from two drawbacks: user programs have to be written in monadic style and capabilities can leave the dynamic scope of the handler. We separately address the two issues in the following two chapters. Chapter 5 presents JavaEffekt, a direct-style implementation of effect handlers for Java. Chapter 6 reiterates the design of ScalaEffekt to add an effect system that statically prevents the usage leaked capabilities.

Chapter 5

Java Effekt – Effectful Programming in Direct Style

In this chapter, we further evaluate our library design of Effekt by implementing it in another object-oriented language: Java.

The implementation of ScalaEffekt is based on a monad for delimited control. While Scala offers for-comprehensions to facilitate writing monadic code, there is no such support in Java. Writing monadic code in Java is non-idiomatic and verbose. In this chapter, we overcome this limitation and allow user programs to be written in direct style. To achieve this, we present a framework that consists of three core components: A type selective continuation-passing-style (CPS) transformation on the level of Java Virtual Machine (JVM) bytecode, an implementation of delimited continuations on top of the bytecode transformation, and finally a library for effect handlers in terms of delimited continuations.

We evaluate and compare the performance of our bytecode-transformation-based implementation against other bytecode transformations that allow continuation capture. The measurements indicate that the performance of our library is competitive with other existing transformations.

The programming language Java lacks a general mechanism to express advanced control flow. In consequence, many control-flow abstractions like generators[16], asynchronous programming with async/await [17], the coroutine programming model[18], and fibers [19], that is lightweight user-level threads, are currently implemented by custom source-to-source or bytecode transformations. Since each extension makes different assumptions about the generated code, combining the different concepts in a single project ranges from non-trivial to impossible. As has been shown in the literature, effect handlers can express many of these control-flow abstractions as simple libraries (Dolan et al., 2015, 2017; Leijen, 2017a). Having support for effect handlers thus would naturally allow a combined usage of those features. We will revisit some examples of those control-flow abstractions in the context of Java in Section 5.3.

In this chapter, we present an implementation of the Effekt design in Java, which we call JavaEffekt. As in the previous chapter, we follow our mantra that programming with effect handlers in JavaEffekt is object-oriented programming. Hence, effect signatures are Java interfaces, effect

This chapter is closely based on the following publication: Jonathan Immanuel Brachthäuser, Philipp Schuster, and Klaus Ostermann. 2018. "Effect Handlers for the Masses". *Proc. ACM Program. Lang.*, 2 (OOPSLA): 111:1–111:27. DOI: https://doi.org/10.1145/3276481

[16] https://github.com/peichhorn/lombok-pg/wiki/Yield

[17] https://github.com/electronicarts/ea-async

[18] https://github.com/offbynull/coroutines

[19] https://github.com/puniverse/quasar

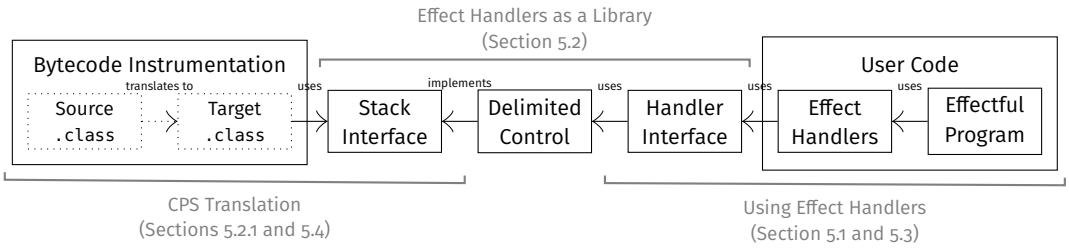


Figure 5.1. Structure of the JavaEffekt framework. Directed, solid arrows express dependencies.

handlers are Java classes that implement those interfaces, capabilities are instances of handlers, and effectful functions are Java functions that use capabilities.

In contrast to the previous chapter, user programs in this chapter can be written in direct-style. Importantly, this way programmers can use effects and handlers together with existing control-flow mechanisms like branching (*i.e.*, **`if`** and **`switch`**), loops (*i.e.*, **`while`**), and exceptions. Also, in contrast to our implementation in Scala, in JavaEffekt, local variables (but not heap-allocated fields) show the correct backtracking behavior. To enable this, our implementation consists of three components: A type-selective CPS transformation of bytecode, an implementation of delimited continuations on top of the bytecode transformation, and a library for effect handlers in terms of delimited continuations. While all three components are designed in concert to implement the effect handler library, they can be used and understood individually. The bytecode transformation is performed independent of Java as the source language and could potentially be reused with other JVM languages such as Scala, Kotlin, JRuby, Clojure, and others.

In short, the contributions of this chapter are:

- The first library design for programming with effect handlers in Java.
- An implementation of multi-prompt delimited continuations in Java. It uses trampolining and avoids the typical linear overhead of restoring the stack upon resumption common to all continuation libraries in Java that we are aware of.
- A type-selective, signature preserving CPS transformation of JVM bytecode. We use closures introduced in Java 1.8 (Gosling et al., 2015) to create specialized instances of continuation frames. The general idea is applicable to any VM-based language that supports closure creation.
- A performance evaluation, comparing JavaEffekt with other libraries that perform bytecode instrumentation to allow continuation capture. The measurements confirm the asymptotic improvement that we obtain by trampolining. We also compare the performance of JavaEffekt with our implementation ScalaEffekt and another monadic library in Scala.

Overview

The chapter structure follows the structure of our framework which is presented in Figure 5.1.

User code We show how to program with effect handlers in Java using our library. Section 5.1 briefly presents the library design of JavaEffekt by means of our standard running example; Section 5.3 illustrates the expressiveness of JavaEffekt with several more complex examples.

```
String drunkFlip(Amb amb, Exc exc) throws Effects {
  boolean caught = amb.flip();
  if (!caught) {
    return exc.raise("We dropped the coin.");
  } else {
    return amb.flip() ? "Heads" : "Tails";
  }
}
```

(a) Our running example, translated to JavaEffekt.

```
interface Exc {
  <A> A raise(String msg) throws Effects;
}

interface Amb {
  boolean flip() throws Effects;
}
```

(b) Effect signatures for `Exc` and `Amb`.

Figure 5.2. Example of using two effects in an effectful program.

Bytecode instrumentation Section 5.2.1 illustrates the CPS translation by applying it to the running example and Section 5.4 describes the implementation of the translation in more detail.

Delimited control Section 5.2.2 describes the implementation of multi-prompt delimited continuations (Dybvig et al., 2007) in Java. It interfaces with the instrumented bytecode by implementing the `Stack` interface that the instrumented bytecode uses. This means that the bytecode instrumentation component can potentially be used as a backend for applications other than our implementation of delimited continuations. At the same time, our instrumentation could be exchanged by another backend that implements the `Stack` interface.

Effect handlers as a library Section 5.2.3 shows the implementation of our effect handler library in terms of multi-prompt delimited continuations (analogous to Section 4.4). The effect handler library is independent of our concrete implementation of `DelimitedControl`. It could for instance be used together with a modified JVM runtime that directly supports delimited continuations.

After presenting our framework, Section 5.5 discusses our approach in the context of related work. We evaluate the performance overhead induced by our translation and compare the performance of JavaEffekt to other continuation libraries in Java (Section 5.6.1), as well as to our implementation of ScalaEffekt and an existing functional effect library in Scala (Section 5.6.2).

5.1 Programming with Effect Handlers in JavaEffekt

The design of JavaEffekt follows the design principles of Effekt. In this Section, we briefly review our running example to highlight similarities and point out differences.

Figure 5.2a uses JavaEffekt to express our running example. As before, the two effect operations `flip` and `raise` are declared in corresponding effect signatures `Amb` (for ambiguity) and `Exc` (for exceptions) in Figure 5.2b. The biggest difference compared to the implementation in Scala (Chapter 4) is that the program now is written in direct style. This is also visible in the type signature of `drunkFlip`: Previously, we used the monadic type constructor `Control` to signal that the program might use control effects and used for-comprehensions to sequence effectful programs. In contrast, effectful programs now are declared as such by adding the

checked exception `Effects` to their `throws` clause. Like other bytecode instrumentation such as "Quasar" (Parallel Universe Software Co., 2013), we use checked exceptions as a course grained effect system (Section 5.5 provides a comparison of JavaEffekt with Quasar). This allows us to distinguish pure programs (that do not use control-effects) from effectful programs. This information is important to guide our bytecode instrumentation. For our CPS translation to be sound, the `Effects` exception should never be handled in user code.

5.1.1 Handling Effects

Analogous to the previous chapter, we first review how effects are handled before we give the full implementation of the involved handlers.

```
class Maybe<R>   extends Handler<R, Optional<R>> implements Exc { ... }
class Collect<R> extends Handler<R, List<R>> implements Amb { ... }
```

As before, the handlers extend our library interface `Handler<R, E>` to express that they represent effect handlers[20]. In JavaEffekt, using the handlers to handle effects `Amb` and `Exc` looks like:

```
List<Optional<String>> res1 =
  Handler.handle(new Collect<Optional<String>>(), amb →
    Handler.handle(new Maybe<String>(), exc → drunkFlip(amb, exc)));
```

Due to the lack of singleton types in Java, the method `handle(h, body)` is now a static method on `Handler`. Compared to the Scala implementation of Effekt, this is the only superficial difference. Running `drunkFlip` with both effects handled yields for $\texttt{res}_1$:

```
▶ [Optional["Heads"], Optional["Tails"], Optional.empty]
```

5.1.2 Implementing Effect Handlers

We will now revisit how handlers are implemented. Figure 5.3b shows the Handler interface, which is relevant for implementing effect handlers. It also shows the type of effectful functions `Eff<S, T>`, which is just like the Java function interface `Function<S, T>`, but with its single abstract method being marked as throwing `Effects`. The interface `Eff<S, T>` is thus the Java equivalent of a function `S ⇒ Control[T]` in Scala. Similarly, interface `CPS<A, E>` corresponds to the nested effectful function type `Eff<Eff<A, E>, E>` (that is, `(A ⇒ Control[E]) ⇒ Control[E]`).

Like in our Scala implementation, both handlers `Maybe` and `Collect` implement their effect operations in terms of `use` (Figure 5.3a). To implement the `raise` effect, the handler `Maybe` captures the continuation and deliberately discards it. The handler for ambiguity, in turn, captures the continuation and invokes it twice, each time yielding a list of possible results. Specialized to the `Collect` handler, the type of the captured continuation is `Eff<Boolean, List<R>>`. Hence, it is safe to finally concatenate the two resulting lists[21]. Similarly, in `raise` it is safe to discard the continuation and immediately return `Optional.empty` because the caller of `handle(new Maybe<String,Optional<String>>(), ...)` expects a value of type `Optional<String>`.

[20] `Optional<A>` is an interface for optional values of type `A`, introduced in Java 1.8.
[21] For this example, we assume lists to be immutable and only be constructed by `singleton` and `concat`.

```
class Maybe<R> extends Handler<R, Optional<R>> implements Exc {
  Optional<R> unit(R r) { return Optional.of(r); }
  <A> A raise(String msg) throws Effects {
    return use(k → Optional.empty());
  }
}
class Collect<R> extends Handler<R, List<R>> implements Amb {
  List<R> unit(R r) { return Lists.singleton(r); }
  boolean flip() throws Effects {
    return use(k → Lists.concat(k.resume(true), k.resume(false)));
  }
}
```

(a) The two effect handlers `Maybe` and `Collect` utilizing `use` to capture the continuation.

```
abstract class Handler<R, E> {
  E unit(R r) throws Effects;
  <A> A use(CPS<A, E> body) throws Effects { ... }
  static <R, E, H extends Handler<R, E>> E handle(H h, Eff<H, R> p) throws Effects {
    ...
  }
}
interface Eff<S, T> { T resume(S value) throws Effects; }
interface CPS<A, E> { E apply(Eff<A, E> k) throws Effects; }
```

(b) Interface of the library class `Handler` and the necessary functional interfaces.

Figure 5.3. Implementation of effect handlers for `Exc` and `Amb` using the library class `Handler`.

5.2 Implementing Effect Handlers for Java in three Steps

As can be seen from our running example, programming with effect handlers in JavaEffekt is almost just standard Java programming. Only the control operator `use` and its counterpart `handle` make the difference in expressivity. This section describes how these control operators can be implemented, bottom up. We start with a CPS translation, that rewrites all methods annotated with `throws Effects`, build a library for delimited continuations upon the translation, and finally implement effect handlers in terms of delimited continuations.

5.2.1 Step 1: Type Selective CPS Transformation by Example

To support accessing the continuation with `use`, the JavaEffekt framework performs a type selective CPS transformation by instrumenting (that is, rewriting) JVM bytecode. This can either be achieved by hooking into the class loading mechanisms of Java and transforming a classfile at runtime when it is loaded, or "ahead of time" by a separate pre-processing phase that rewrites the class files once (Binder et al., 2007); our implementation supports both. Implementing the transformation on the level of JVM bytecode opens up the opportunity of reuse for other JVM languages. While the implementation of JavaEffekt rewrites JVM bytecode, for easier accessibility this section presents the CPS transformation as a semantically equivalent[22] source-to-source rewriting of the example program `drunkFlip`. This section provides an overview, Section 5.4 formally describes the bytecode transformation and explains how we treat control flow and exceptions.

Effect calls and entrypoints Figures 5.4b and 5.4c show the result of transforming the method `drunkFlip`. We instrument only *effectful methods* and identify those by means of the checked exception `Effects`. Using Reynolds (1972) terminology we only consider methods marked with `throws Effects` to be "serious". All other functions are "trivial" and do not require any instrumentation. Consequently, we also only instrument call sites of effectful methods (*effect calls*). In `drunkFlip`, there are three such effect calls, two to `flip` and one to `raise`. We exclude *tail effect calls* from the translation, that is, effect calls immediately followed by a return. For `drunkFlip`, this means that we instrument the two `flip` calls, since the call to `raise` is in tail position. We call the code immediately following an effect call an *entrypoint*. We also treat the initial entrypoint of a function as an entrypoint in this sense. Similar to Prokopec and Liu (2018), for each entrypoint in an effectful method, we generate a separate *entrypoint method*. For our example, these are the methods $\texttt{drunkFlip}_0$ (the initial entrypoint method), $\texttt{drunkFlip}_1$ and $\texttt{drunkFlip}_2$ (corresponding to the two invocations of `flip`). Importantly, entrypoint methods take the *function local state* as arguments. That is, all values on the operand stack and all local variables that are needed to resume the function execution after the effect call would return.

The stack interface and continuation frames Similar to how the JVM pushes a stack frame before it enters a method call (Lindholm et al., 2015), we rewrite every effect call to push a *continuation frame*. By calling `Effekt.push`, the continuation frame is pushed to a global, user-level stack. Class `Effekt` has a global, static field `Effekt.stack` that implements the interface `Stack` shown in Figure 5.5. The interface `Stack` contains all necessary methods used by the instrumented

[22] For the example in this section, we manually verified that the bytecode of the source-to-source transformation is equivalent to the result of the bytecode transformation (modulo some superfluous register stores/loads).

```
String drunkFlip(Amb amb, Exc exc) throws Effects {
  boolean caught = amb.flip();
  if (!caught) {
    return exc.raise("We dropped the coin.");
  } else {
    return amb.flip() ? "Heads" : "Tails";
  }
}
```

(a) Source method with highlighted effect calls.

```
String drunkFlip(Amb amb, Exc exc) throws Effects {
  Effekt.push(() → drunkFlip0(amb, exc));
  return null;
}
```

(b) Method stub, only pushing the initial entrypoint.

```
static void drunkFlip0(Amb amb, Exc exc) {
  Effekt.push(() → drunkFlip1(amb, exc));
  amb.flip();
}
static void drunkFlip1(Amb amb, Exc exc) {
  boolean caught = Effekt.result();
  if (!caught) {
    exc.raise("We dropped the coin.");
  } else {
    Effekt.push(() →
      drunkFlip2(amb, exc, caught));
    amb.flip();
  }
}
static void drunkFlip2(Amb amb, Exc exc,
    boolean caught) {
  boolean res1 = Effekt.result();
  Effekt.returnWith(res1?"Heads":"Tails");
}
```

(c) Entrypoints as separate, static methods.

Figure 5.4. CPS translation of the example in Figure 5.4a, presented as a source-to-source transformation.

bytecode. We assume that all methods on `Stack` are available as static methods on `Effekt` and write `Effekt.push` instead of `Effekt.stack.push`. In the presence of multiple threads, we would store the current stack in a thread local variable, such that each thread maintains its own stack, but refrain from doing so for simplicity. A continuation frame is an instance of the `Frame` interface, also shown in Figure 5.5. We use Java 8 lambdas (Gosling et al., 2015) to create instances of the `Frame` interface. The lambdas close over the function local state. When invoked with `enter`, they pass the state to the entrypoint methods. Conceptually, we represent continuation frames as instances of classes that have one field for each local variable they store. After pushing the continuation frame, the entrypoint methods call the effectful method and immediately return. Thus, all effect calls become tail calls in the translated program.

Custom calling convention Instrumented effectful methods use a special calling convention: We rewrite all returns to `Effekt.returnWith` calls. Correspondingly, entrypoint methods use `Effekt.result` to obtain the result of the previous effect call. We transform the original method `drunkFlip` to a stub (Figure 5.4b) that pushes a continuation frame for the *initial entrypoint* and immediately returns a dummy value. Callers of `drunkFlip` have to use `Effekt.result` to eventually get the actual return value. Our entrypoints follow the same calling convention for consistency, as we can observe in $\texttt{drunkFlip}_2$. The calling convention is motivated by the goal to maximize interoperability with other Java features with as little specialization of the transformation as possible. The most important consequence of this design goal is that our translation preserves method signatures. It thus does only translate terms, not types or signatures. A standard CPS translation changes the type of a computation that returns `A` to a function type `(A → R) → R` for some answer type `R` (Meyer and Wand, 1985). However, this

```
interface Stack {
  // special calling convention
  void returnWith(Object r);
  void unwindWith(Throwable t);
  <A> A result() throws Throwable;

  // stack of frames
  void push(Frame frame);
  Frame pop();
  boolean isEmpty();

  void trampoline();
}

interface Frame {
  void enter();
}
```

```
abstract class RTStack implements Stack {
  Object res; Throwable exc;

  void returnWith(Object r) { res = r; exc = null; }
  void unwindWith(Throwable t) { res = null; exc = t; }

  <A> A result() throws Throwable {
    if (exc != null) throw exc;
    return (A) res;
  }

  void trampoline() {
    while (!isEmpty())
      try { pop().enter(); }
      catch (Throwable t) { unwindWith(t); }
  }
}
```

Figure 5.5. Interface and example implementation of the user-level stack. Implementations of stack operations (`push`, `pop` and `isEmpty`) are left abstract.

change is precluded by our decision of not changing method signatures. Instead, the continuation `A → R` is obtained via the global instance of `Stack`, as we will see in the next subsection. Still, the return type of an effectful computation changes from `A` to `R`. To accommodate for the change in return type, we make effectful methods use our own custom calling convention.

Implementing the stack interface Instead of using the JVM stack for continuation frames, we use a separate user-level stack. A canonical implementation of `Stack` recovering the expected runtime behavior of the JVM stack is sketched as class `RTStack` in Figure 5.5. Only the methods implementing our calling convention are provided. The straightforward stack operations `push`, `pop`, and `isEmpty` are left abstract. Our stack implementation performs trampolining (Ganz et al., 1999). To run an instrumented program[23], we first invoke it to push a frame that corresponds to its initial entrypoint onto `Effekt.stack`:

```
static <A> A run(Eff<Void, A> prog) {
  prog.resume();
  Effekt.trampoline();
  return Effekt.result();
}
```

```
Effekt.run(() →
  Handler.handle(new AmbList<>(), amb →
    Handler.handle(new Maybe<>(), exc →
      drunkFlip(amb, exc))));
```

To actually start execution, we call `Effekt.trampoline`, which will continue to pop and enter frames until the stack is empty. If an exception is raised, the trampoline will unwind the user-level stack frame by frame. Each frame starts with a call to `result`, re-raising the exception after restoring the method state. Section 5.4 shows more details on how we deal with exceptions.

[23] When effectful programs do not require an argument we will use `Eff<Void, A>`. To avoid materializing instances of `Void` and binding them, we write `f.resume()` as a short hand for `f.resume(null)` and `(() → ...)` instead of `(unusedVoid → ...)`. Effectful programs of type `Eff<Void, A>` corresponds to values of type `Control[A]` in our Scala implementation.

```
interface Prompt<E> {}
```

(a) Interface `Prompt`, used to mark positions on the stack.

```
class SeqStack extends RTStack {
  Seq<Frame> s = Seq.empty();
  void push(Frame frame) { s = s.push(frame); }
  Frame pop() { Frame f = s.head(); s = s.tail(); return f; }
  boolean isEmpty() { return s.isEmpty(); }
}
```

(b) Implementation of `Stack`, forwarding to an immutable stack `Seq`.

```
class DelimCC extends SeqStack implements DelimitedControl {
  <E> E resetWith(Prompt<E> p, Eff<Void, E> prog)
      throws Effects {
    s = s.mark(p); return prog.resume();
  }

  <A, E> A shift(Prompt<E> p, CPS<A, E> body)
      throws Effects {
    Seq<Frame> init = s.before(p); s = s.after(p);
    Eff<A, E> k = (A value) → {
      s = init.prependTo(s); return (E) value; }
    return (A) body.apply(k);
  }
}
```

(c) Implementation of delimited control in terms of `Seq`.

```
interface Seq<A> {
  boolean isEmpty();
  A       head();
  Seq<A>  tail();
  Seq<A>  before(PromptEx p);
  Seq<A>  after(PromptEx p);

  Seq<A>  push(A element);
  Seq<A>  mark(PromptEx p);
  Seq<A>  prependTo(Seq<A> init);

  static <A> Seq<A> empty() {...}
}
```

(d) Immutable, splittable stack.

Figure 5.6. Implementation of control operators. Usage of the splittable stack implementation `Seq` is highlighted.

5.2.2 Step 2: Delimited Continuations

A program that has been transformed with our CPS translation still uses the JVM stack for non-effectful calls but uses our user-level stack for effectful method calls. Different implementations of the interface `Stack` give rise to different additional operations that exploit the corresponding stack representation. In consequence, effectful programs that are executed against a particular stack implementation can make use of those additional operations. In this section, we will develop one particular implementation of `Stack` that implements additional operations to capture delimited continuations. All code in the rest of this chapter is subject to the CPS bytecode transformation and all methods annotated with `throws Effects` will be instrumented.

The Interface of Delimited Control

As already shown in Section 4.2, the effect handler library can be implemented as a very thin layer on top of multi-prompt delimited continuations. Again, our implementation of multi-prompt delimited continuations is similar to the one presented in Chapter 4. However, we translate it to Java and adapt it to our setting of bytecode instrumentation. We extend the above `Stack` implementation with the following interface `DelimitedControl`.

```
interface DelimitedControl {
  <E>         E resetWith(Prompt<E> p, Eff<Void, E> prog) throws Effects;
  <A, E>      A shift(Prompt<E> p, CPS<A, E> body)        throws Effects;
  default <E> E reset(Eff<Prompt<E>, E> prog)             throws Effects { ... }
}
```

Instances of `Prompt<E>` (Figure 5.6a) are used to mark positions on the stack. The type of an effectful function `Eff` and of effectful programs that use an effectful continuation `CPS`, have been defined in Figure 5.3b. The type parameter `E` of `Prompt` unifies with the type of the computation that we delimit with `resetWith`. Capturing a continuation with `shift`, the return type of the continuation and of the body have to match the type of the prompt `E`. The implementation of `reset` in terms of `resetWith` directly follows the one from Section 4.4.

Using Delimited Continuations

Assuming the global stack instance supports the methods from `DelimitedControl`, we can reimplement the examples of Section 4.2, this time in direct style.

```
1 + Effekt.reset(p1 → 10 + Effekt.shift(p1, k → k.resume(k.resume(100))));
▶ 1 + (10 + (10 + 100)) = 121
```

The second example illustrates the use of multiple prompts.

```
2 * Effekt.reset(p1 → Effekt.reset(p2 → Effekt.shift(p1, k → 21)) ? 1 : 2);
▶ 2 * 21 = 42
```

The captured continuation `k` contains the program segment marked by prompt p_1. We discard the continuation and replace it by the value `21`.

A Splittable Stack Implementation

Figure 5.6b implements `Stack` by storing a list of frames in a field `s`. It implements all abstract operations of `Stack` by forwarding to this list `s`. For now, let's assume `Seq` is an immutable implementation of a stack data structure with elements of type `A`. While very simple, running a program with this stack implementation already has the benefit that it performs trampolining and thus reduces JVM stack usage, which might avoid stack overflows. However, the real power of the translation comes from the fact that `Stack` implementations can add new methods, which expose additional (control) operators. Following Dybvig et al. (2007), to implement the additional control operators `resetWith` and `shift`, we need to mark positions on the runtime stack (`mark`), slice the stack at given positions (`before`, `after`), and prepend whole stack segments (`prependTo`). The stack data structure `Seq` (Figure 5.6d) that we already used above offers exactly these operations. As presented in Figure 4.11, `Seq<A>` is a prompt-separated cons-list of elements of type `A`. Calling `s.before(p)` returns the initial segment up to (and including) the first occurrence of the marker `p`. This segment contains all the recently pushed elements (frames and prompts) after `p` has been pushed. Calling `s.after(p)` returns the remainder of the stack. This segment contains all elements, which have been pushed before marker `p` has been pushed.

As mentioned before, effect handlers in Effekt are *deep handlers* (Kammar et al., 2013) and our implementation of delimited control has semantics $-shift+$. Consequently, the stack segment obtained by seq.before(p) is delimited by the prompt marker `p`, while seq.after(p) is not.

Pushing a Prompt

Figure 5.6c shows the implementation of the `DelimitedControl` interface, using `Seq`. To implement `resetWith`, we mark the stack using the provided prompt `p` and update the mutable reference `s` with the now marked stack. We then resume with the effectful program `prog`. Being effectful, the program `prog` pushes additional frames onto the stack. We can capture those frames later by slicing the stack at the position of the installed marker `p`.

Capturing a Continuation

The control operator `shift(p, body)` captures the continuation `k` up to the next dynamically enclosing `resetWith(p, ...)` and conceptually replaces the call to `resetWith` with a call to `body.apply(k)`. Its implementation in Figure 5.6c stores all frames that have been pushed after `p` in a local variable `init`. This segment corresponds to the delimited continuation from type `A` to type `E`. That is, the top most frame expects `Effekt.result` to return a value of type `A`. The initial segment and the prompt marker are then removed from the stack by mutating it with `s = s.after(p)`. This leaves a segment on the stack, which expects a value of type `E` to continue program execution. The continuation `k` implements the functional interface `Eff` with method `resume() throws Effects`. It closes over the initial stack segment `init` and, when invoked, prepends it to the stack `s`. This implements the desired semantics of resuming the delimited continuation: The runtime system will first run the initial stack segment `init` before it eventually continues at the call site of `resume` within `body`.

There are two casts involved that require some explanation. Both `shift` and the continuation `k` are effectful. Thus, the respective caller will be instrumented. However, by mutating field `s` and modifying the stack, we change the execution context. In `shift` we remove the initial segment of the stack and thus the new caller expects a value of type `E` not `A`. In the continuation we prepend the initial segment and thus the caller now expects an `A` not `E`. The Java type checker is unaware of our transformation and the modifications to our own call stack. Hence the casts[24].

5.2.3 Step 3: Implementation of the JavaEffekt Library

In the previous subsections, we have seen how programs, which contain `throws Effects` annotations, are CPS translated. To support multi-prompt delimited continuations, we extended the runtime environment in which those translated programs are executed. Equipped with multi-prompt delimited continuations, we are now ready to implement the effect handler library.

The expressive power of effect handlers comes from the two operations `handle` and `use`, which are encapsulated in the library class `Handler`. Figure 5.7 shows the implementation of these two operations in terms of delimited continuations. As in the Scala implementation, handlers contain an instance of a prompt marker `Prompt<E>` as a field. The answer type of delimited continuations thus will be the effect domain `E`. In the implementation of `handle`, we push the prompt before resuming with `prog`. The pushed prompt will delimit the extent of continuations captured by that handler. By calling `h.unit` after resuming, programs that use no effects will be lifted from `R` to `E`. The method `use` simply forwards to `shift`. It again uses the prompt to capture the continuation up to the most recent call to `handle` for this handler instance.

[24]Since `A` and `E` are generic type parameters, they will be erased and the program can safely be executed.

```
abstract class Handler<R, E> {
  E unit(R r) throws Effects;

  private Prompt<E> prompt = new Prompt<E>() {};

  <A> A use(CPS<A, E> body) throws Effects {
    return Effekt.shift(prompt, body);
  }
  static <R, E, H extends Handler<R,E>> E handle(H h, Eff<H,R> prog) throws Effects {
    return Effekt.resetWith(h.prompt, () → h.unit(prog.resume(h)));
  }
}
```

Figure 5.7. The essence of the effect handler library: The `Handler` class.

5.3 Use Cases

Having implemented effect handlers as a library for Java, programmers can now freely combine object-oriented Java programming with effect handlers. We revisit selected examples of Section 4.4 to show how they can be expressed in JavaEffekt. Two differences to ScalaEffekt are particularly interesting. Firstly, in JavaEffekt, control-flow structures like conditionals and **`while`** loops can immediately be used in direct style. In ScalaEffekt, control flow needs to be lifted into the `Control` monad. Secondly, function-local (mutable) state (*i.e.*, local variables) can be used and has the correct backtracking behavior in presence of multiple resumptions.

5.3.1 Handling Multiple Effects in one Handler

All handlers we have seen in this chapter so far only implemented a single effect signature. However, sometimes it is necessary to group the implementation of multiple effect signatures in a single handler. Since effect signatures are interfaces and handlers are classes implementing those interfaces, this is straightforward. Like with ScalaEffekt, effect implementations grouped in a single handler share the same effect domain `E`, share the private state of the handler and they can be implemented in terms of each other. In particular, sharing the effect domain is important if the handler wants to express interaction between different effect operations. Two examples combining `Amb` and `Exc` in one handler to share the effect domain are:

```
class Nondet<R> extends Collect<R> implements Exc {
  <A> A raise(String msg) throws Effects {
    return use(k → Lists.empty());
  }
}
```

```
class Backtrack<R> extends Maybe<R> implements Amb {
  boolean flip() throws Effects {
    return use(k → {
      Optional<R> res = k.resume(true);
      return res.isPresent() ? res : k.resume(false); });
  }
}
```

Handler `Nondet` extends `AmbList` and only provides the definition for `raise`. It shares the effect domain `List<R>` with `AmbList`. Similarly, handler `Backtrack` extends `Maybe` and adds the implementation for `flip`. It shares the effect domain `Optional<R>` with `Maybe`. By subtyping, the combined handlers can still be used as handlers for the individual effects `Amb` or `Exc`:

```
Handler.handle(new Nondet, nd → drunkFlip(nd, nd))
▶ ["Heads", "Tails"]

Handler.handle(new Backtrack, bt → drunkFlip(bt, bt))
▶ Optional["Heads"]
```

This also illustrates reuse of handler implementations by inheritance. We only needed to provide the missing definitions; all other implementations of effect operations are inherited.

5.3.2 Alternatives to Capability Passing

In previous examples, effectful methods were written in capability-passing style. They thus expressed their use of effects by expecting handler instances (capabilities) as arguments. In an object-oriented programming language like Java, it is natural to explore other means to get access to a handler instance. Capabilities can be passed to constructors and stored in fields. Take the following implementation of a reader effect (specialized to characters) as an example:

```
interface Input { char read() throws Effects; }
```

The following handler `StringInput` uses an instance of an `Exc`-capability to raise the end-of-stream exception. The capability is passed on construction and stored in the field `exc`.

```
class StringInput<R> extends Handler<R, R> implements Input {
    final Exc exc; final String input; int pos = 0;
    StringInput(String s, Exc e) { this.input = s; this.exc = e; }
    public char read() throws Effects {
        return if (pos ≥ input.length()) exc.raise("EOS");
               else input.charAt(pos++);
    }
}
```

Methods of `StringInput`, which use the `Exc` effect, can only be safely executed in the corresponding dynamic scope of the `Exc`-handler. In JavaEffekt this is up to the user of `StringInput` to ensure. The next chapter revises our Scala library to statically guarantee effect safety.

5.3.3 Ambient State and Parametrized Handlers

Our bytecode transformation only saves and restores function local state, that is, mutable local variables. It does not deep-copy heap-allocated state and nothing prevents the user from using mutable fields in the implementation of handlers. The handler `StringInput` is an example since it mutates the field `pos`. However, as demonstrated in Section 4.3, heap-allocated state and delimited continuations can interact in unforeseen ways. In JavaEffekt, we support ambient state as follows: Handler implementations, like `StringInput`, can just use mutable fields. To turn the handler state into ambient state, a handler needs to implement the `Stateful<S>` interface below.

```
interface Stateful<S> { S exportState(); void importState(S state); }
class StringInput2<R> extends StringInput<R> implements Stateful<Integer> {
  Integer exportState() { return pos; }
  void importState(Integer state) { pos = state; }
}
```

Our delimited control implementation is extended to export the handler state when the continuation is captured and to restore it on resumption. It is up to the user to identify and potentially deep-copy all relevant aspects of the handler state.

5.3.4 Case Study: Parsing

Equipped with nondeterministic choice, exceptions and reader we can implement parsers (Leijen, 2016). For convenience, we group `Amb`, `Exc`, and `Input` into one effect signature for parsers.

```
interface Parser extends Amb, Exc, Input {}
int digit(Parser p) throws Effects {
  char t = p.read();
  return isDigit(t)
    ? getNumericValue(t)
    : p.raise("Not a digit: " + t);
}
```

```
int number(Parser p) throws Effects {
  int res = digit(p);
  while (true)
    if (p.flip()) {
      res = res * 10 + digit(p);
    } else { return res; }
}
```

While we already presented a similar example in Section 4.6, implementing it again in JavaEffekt is interesting for two reasons. Firstly, it demonstrates that programs, now written in direct style, can also use Java's control-flow constructs like **`while`** and conditionals. In the monadic counterpart of Effekt, those control-flow constructs need to be lifted into the monad. Secondly, it also illustrates that effectful programs can use mutable, function local state. To parse a number, every time after reading a digit, we mutate the local variable `res` to update the result. Our byte code transformation makes sure that this function local state will be saved on continuation capture, allowing a well-defined interaction with multiple resumptions and backtracking.

To parse the string `"123"` with the `number` parser, we use the `ParserForward` handler. The handler stores capabilities for `Amb`, `Exc` in `Input` in fields and implements the corresponding effect operations by forwarding. We can handle effect operations `flip` and `raise` with either the `Nondet` handler to get a list of all possible parses

```
Handler.handle(new Nondet<>(), nd →
  Handler.handle(new StringInput2<>("123", nd), r →
    number(new ParserForward(nd, nd, r))));
▶ ["123", "12", "1"]
```

or with the `Backtrack` handler to only obtain the first successful parse, if it exists:

```
Handler.handle(new Backtrack<>(), bt →
  Handler.handle(new StringInput2<>("123", bt), r →
    number(new ParserForward(bt, bt, r))));
▶ Optional["123"]
```

Note, that the parsers can now be written in direct style. At the same time, handlers (like `Nondet`) can still transparently access the continuation in their implementation of effect operations.

5.3.5 Case Study: Generators

In the programming language Python, the built-in control operation **`yield`** can be used to describe a stream of values also known as generators (Politz et al., 2013). While generators are built into Python, with effect handlers we can implement them as a library.

```
interface Writer<A> { void write(A value) throws Effects; }
void numbers(int to, Writer<Integer> w) throws Effects {
  int n = 0; while (n ≤ to) { w.write(n++); }
}
```

The method `numbers` describes a generator that yields integers up to a given value using the `Writer` effect. Again, it uses a mutable variable `n` together with native control-flow structures (*i.e.*, **`while`**). We can handle the writer effect with the `Iterate` handler. To suspend the generator until the next value is requested, the hander captures and stores the continuation on every write.

```
class Iterate<A, R> extends Handler<R, IteratorEff<A>> implements Writer<A> {...}
interface IteratorEff<A> {
  boolean hasNext() throws Effects;
  A next() throws Effects;
}
IteratorEff<Integer> it = Handler.handle(new Iterate()<>, w → numbers(10, w));
while (it.hasNext()) { println(it.next()); }
```

Since the iterator is effectful, we cannot reuse the Java interface `Iterator`. Instead, the interface `IteratorEff` duplicates the interface and adds the `throws Effects` annotations. As a consequence, this prevents users from using Java's **`for`** `(A a : iterator)` syntax (Gosling et al., 2015), since it requires `iterator` to implement the "pure" iterator interface.

5.3.6 Case Study: Cooperative Multitasking

Like generators, cooperative multitasking and async/await can be implemented as a library (Dolan et al., 2017; Leijen, 2017a). Programs can use the `Fiber` effect (*cf.* Section 4.4) to fork and suspend processes. A process is an effectful program `Eff<Void, Void>`. Like in our implementation in Section 4.4, the `Fiber` effect makes use of Java's default methods to implement the derived effect operation `forked`. The `Scheduler` handler in Figure 5.8 implements a round-robin scheduler that keeps a queue of all running processes in its mutable handler state. Suspending is implemented by enqueueing the continuation of the process and immediately returning. The handler will then pick the next process to execute.

```
interface Fiber {
  void suspend() throws Effects;
  boolean fork() throws Effects;
  void exit() throws Effects;
  default void forked(Eff<Void, Void> p) throws Effects {
    if (fork()) { p.resume(); exit(); }
  }
}
```

(a) The `Fiber` effect signature.

```
Handler.handle(new Scheduler(), f → {
  f.fork(() → {
    println("world");
  });
  println("hello");
  f.suspend();
});
```

(b) User program using the `Fiber` effect.

```
class Scheduler extends Handler<Void, Void> implements Fiber {
  void suspend() throws Effects { use(k → { queue.add(k); run(); }); }
  void exit() throws Effects { use(k → null); }
  boolean fork() throws Effects { return use(k → {
    queue.add(() → k.resume(true));
    queue.add(() → k.resume(false));
    run();
  })}

  private Queue<Eff<Void, Void>> queue = new LinkedList<>();
  private void run() throws Effects {
    while (!queue.isEmpty()) queue.remove().resume();
  }
}
```

(c) A round robin handler for the `Fiber` effect.

Figure 5.8. Effect signature `Fiber` with operations for cooperative multitasking and a round-robin scheduler implemented as handler `Scheduler`.

5.4 Implementation of the Type Selective CPS Transformation

Section 5.2.1 illustrated the type-selective CPS translation by example as a source-to-source transformation. In the current section, we now present the implementation of the translation on the level of bytecode and show how we handle control-flow constructs like jumps and exceptions. The translation is interesting in that it uses Java 8 closures to create continuations. This is different to other bytecode translations, which we will compare with in Section 5.5.1. We implemented the described transformation described using the OPAL framework by Eichberg and Hermann (2014) for static analysis and synthesis of JVM bytecode. Our implementation of the CPS transformation as well as all other components of JavaEffekt can be found online:

`http://github.com/b-studios/java-effekt`

We minimize the presentation to a relevant core set of language features and leave out details that distract from the essence of the transformation: Storing all necessary function-local state in closures and inserting calls to the API as described in Section 5.2. We model the JVM (Lindholm et al., 2015) and assume an abstract machine with registers (also referred to as *locals*, since they are function local), an operand stack (also referred to as *operands*, again function-local) and a frame stack (commonly referred to as *stack*). We use the term *function-local state* to refer to the values stored in locals and operands at a given time in the execution of a method. We adopt the JVM calling convention: function arguments are pushed on the operand stack by the caller, but accessible as locals by the callee, starting from register index 0. We assume every bytecode instruction is labeled, but omit labels that we never refer to. In our description of bytecode, labels are drawn from the set $\mathbb{L}$. We use sans-serif font to refer to object-language labels (*e.g.*, init, or loop) and italic font for meta-language variables ranging over labels (*e.g.*, *label*, or *eff*). Similarly, we use serif font to refer to method names, types etc. (*e.g.*, doLoop, or MyExc) and italic font for meta-language variables ranging over names (*e.g.*, *name*, or *exc*).

5.4.1 An Example with Jumps and Exceptions

We explain the transformation on an example method doLoop with the following Java source:

```
boolean doLoop() throws Effects {
  Input r = Inputs.getInput(); // static method
  loop: try { while ('\n' != r.read()) {} }
        catch (MyExc e) { return false; }
  exit: return true;
}
```

The bytecode of the method doLoop is shown in Figure 5.9a. The method doLoop performs effect calls to `r.read()` until the result is either a newline or `r.read()` raises a native exception. The example includes exception handling to illustrate, in detail, how the bytecode instrumentation interacts with native exceptions. As in the JVM, exception handling is modeled external to the list of bytecode instructions of a method and exception handlers are given in the form of regions:

excregion *tryStartLabel tryEndLabel catchLabel exceptionType*

In our example, let us assume an exception region **excregion** loop break catch MyExc is in place. That is, if an exception with a type name MyExc is raised in the dynamic region between the labels loop and break execution will be continued at label catch.

```
method doLoop 1 throws Effects {
  init :   invoke Inputs.getInput 0
           store 1
  loop :   const '\n'
           load 1    // load Input from local 1
  op :     invoke Input.read 1
           ifeq exit
  break :  goto loop
  catch :  const false
           return
  exit :   const true
           return
}
excregion loop break catch MyExc
```

(a) Bytecode of method doLoop.

$$
\begin{aligned}
\mathcal{S}[\![\ \text{doLoop}\ ]\!] &= \textbf{method}\ \text{doLoop}\ 1\ \textbf{throws}\ \text{Effects}\ \{ \\
&\quad \textbf{load}\ 0 \quad \textit{// load 'this'} \\
&\quad \textbf{closure}\ \text{Frame.enter doLoop}_{\text{init}}\ 1 \\
&\quad \textbf{const false} \quad \textit{// load dummy value} \\
&\quad \textbf{return} \\
&\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{E}[\![\ \text{doLoop}\ ]\!]_{\text{init}} &= \textbf{method}\ \text{doLoop}_{\text{init}}\ 1\ \{ \\
&\quad \textbf{goto}\ \text{init} \\
&\quad \ldots \\
&\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{E}[\![\ \text{doLoop}\ ]\!]_{\text{op}} &= \textbf{method}\ \text{doLoop}_{\text{op}}\ 2\ \{ \\
&\quad \textbf{goto}\ \text{ep\$op} \\
&\quad \ldots \\
&\}
\end{aligned}
$$

(b) Generated methods – method bodies in Fig. 5.9d.

$$
\begin{array}{llll}
\mathit{effCalls}(\text{doLoop}) = [\text{init}, \text{op}] & \mathit{operands}(\text{init}) = 0 & \mathit{locals}(\text{init}) = [0] & \mathit{ep}(\text{init}) = \text{init} \\
\mathit{tmpLocal}(\text{doLoop}) = 2 & \mathit{operands}(\text{op}) = 1 & \mathit{locals}(\text{op}) = [1] & \mathit{ep}(\text{op}) = \text{ep\$op}
\end{array}
$$

(c) Meta information as obtained by static analysis.

```
init :   invoke Inputs.getInput 0
         store 1
loop :   const '\n'
         load 1
op :     store 2                          // save call operands
         load 1                           // load live locals
         closure Frame.enter doLoop_op 2  // close over two values
         invoke Effekt.push 1             // push closure to stack
         load 2                           // restore call operands
         invoke Input.read 1
         return_void
ep$op :  load 0                           // load arguments
         load 1
         store 1                          // restore locals
         invoke Effekt.result 0           // get results
         ifeq exit
break :  goto loop
catch :  const false
         invoke Effekt.returnWith 1       // store results
         return_void
exit :   const true
         invoke Effekt.returnWith 1       // store results
         return_void
```

(d) Result of translating the instructions of method doLoop.

Figure 5.9. Example of translating a method doLoop

The syntax of the term-language is summarized in Figure 5.10a. For simplicity of the presentation, we do not concern ourselves with types and thus choose a uni-typed term-language. The translation does not distinguish instance methods and static methods. Hence, we only include a single method definition that consists of a name, a list of potentially raised exceptions and a list of labeled instructions. We omit the list of exception handler regions (the *exception table*) since it does not require any modification. The syntax of bytecode instructions *Instr* in Figure 5.10a includes instructions to load constant values (**const** v) to the operand stack, instructions to load from and store into function-local registers (**load** *index*, **store** *index*) and control-flow instructions (**return**, **throws**, **ifeq**[25] and **goto**). Finally, as with method declarations, we only include a single form of method invocation (**invoke** *name arity*) that subsumes static and virtual method calls. Virtual method calls take the receiver as the first argument and thus always have an arity greater than or equal to one. The syntax of instructions also includes an instruction **closure**, which we discuss in Section 5.4.3.

5.4.2 Translation of Methods

Only effectful methods that are marked as throwing Effects exceptions are translated. All other methods of a class are copied unchanged.

As can be seen in the source-to-source transformation in Figure 5.4, for one effectful method m, we generate multiple methods: a single method stub, using the translation function $\mathcal{S}[\![\cdot]\!]$ and one entrypoint method for each effect call at label *eff* using the translation function $\mathcal{E}[\![\cdot]\!]_{eff}$ (both translation functions are defined in Figure 5.10b). Every effect call (*eff* : **invoke** *name arity*) inside a given method m gives rise to an entrypoint uniquely identified by the label *eff*. The entrypoint itself is labeled $ep(eff)$ and represents the continuation of the method m after the effect call returned. For consistency, we also treat the initial entrypoint at label init as effect call. The special entrypoint $ep(\mathsf{init}) = \mathsf{init}$ refers to the label of the first original instruction of the method. By explicitly pushing the initial entrypoint, we perform trampolining for each effect call. For the example method doLoop, we thus generate three methods: the method stub doLoop and the two entrypoint methods $\text{doLoop}_{\mathsf{init}}$ and $\text{doLoop}_{\mathsf{op}}$ (Figure 5.9b).

Rule T-Stub generates the method stub that first saves the local state and then immediately returns a dummy value. At that point, the local state only consists of the arguments supplied to the function call. As we will see shortly, *saveState* thus pushes a closure that closes over the call arguments. When invoked, it resumes with the method corresponding to the initial entrypoint. For our example, this initial entrypoint method is $\text{doLoop}_{\mathsf{init}}$. The returned result of the stub method will never be used; hence *loadDummyResult* can load any constant value.

Rule T-Entrypoint takes a method m and a label *eff* to generate a new method corresponding to the entrypoint at *eff*. For our example, we generate two static entrypoint methods $\text{doLoop}_{\mathsf{init}}$ and $\text{doLoop}_{\mathsf{op}}$. The entrypoint methods use the same exception table as the original method doLoop. After the initial **goto** instruction, the bodies of the generated methods are identical. For our example, the common part is given in Figure 5.9d. The only difference between the two generated methods is the initial jump. Since generating almost identical methods for each entrypoint leads to unnecessary growth of the class file, in our implementation of JavaEffekt, we perform dead code elimination after generating the bytecode. Using closures to save state, only saving live variables, and performing dead code elimination ultimately results in code, which (in spirit) is very close to handwritten code in continuation-passing style (as in Figure 5.4c).

[25] For our example, we assume **ifeq** *label* pops two values and jumps to the given *label* if the two values are equal.

$$
\begin{array}{rcl}
\mathit{instr} \in \mathit{Instr} ::= & \textbf{const}\ \mathit{Value} \mid \textbf{load}\ \mathbb{N} \mid \textbf{store}\ \mathbb{N} \\
\mid & \textbf{return} \mid \textbf{throw} \mid \textbf{goto}\ \mathbb{L} \\
\mid & \textbf{invoke}\ \mathit{Name}\ \mathbb{N} \\
\mid & \textbf{closure}\ \mathit{Name}\ \mathit{Name}\ \mathbb{N}
\end{array}
\qquad
\begin{array}{rcl}
m \in \mathit{Method} ::= & & \\
& \textbf{method}\ \mathit{Name}\ \mathbb{N}\ \textbf{throws}\ \overline{\mathit{Name}}\ \{\ \overline{\mathbb{L}:\ \mathit{Instr}}\ \} \\
\mathit{label},\ \mathit{eff},\ \ldots \in \mathbb{L} & := & \mathsf{init} \mid \mathsf{loop} \mid \mathsf{op} \mid \ldots \\
\mathit{name},\ \mathit{exc},\ \ldots \in \mathit{Name} & := & \mathrm{doLoop} \mid \mathrm{Effekt.push} \mid \ldots
\end{array}
$$

(a) Syntax of methods and bytecode instructions.

(T-Stub)

$$
\begin{array}{l}
\mathcal{S}[\![\cdot]\!] : \mathit{Method} \to \mathit{Method} \\
\mathcal{S}[\![\ \textbf{method}\ \mathit{name}\ \mathit{arity}\ \textbf{throws}\ \overline{\mathit{exc}}\ \{\ \overline{\mathit{instr}}\ \}\ ]\!] = \\
\quad \textbf{method}\ \mathit{name}\ \mathit{arity}\ \textbf{throws}\ \overline{\mathit{exc}}\ \{ \\
\qquad \mathit{saveState}(\mathit{name}, \mathsf{init}) \\
\qquad \mathit{loadDummyResult} \\
\qquad \textbf{return} \\
\}
\end{array}
$$

(T-Invoke-Eff)

$$
\begin{array}{l}
\mathcal{I}[\![\cdot]\!]_{(\ \cdot\ )} : (\mathbb{L}:\ \mathit{Instr}) \to \mathit{Method} \to \overline{\mathbb{L}:\ \mathit{Instr}} \\
\mathcal{I}[\![\ \mathit{eff}: \quad \textbf{invoke}\ \mathit{name}\ \mathit{arity}\ ]\!]_m\ \textbf{if}\ \mathit{effectful}\ (\mathit{name}) = \\
\quad \mathit{eff}: \quad \mathit{saveCallOperands}(\mathit{tmpLocal}(m),\ \mathit{arity}) \\
\qquad\qquad\ \ \mathit{saveState}(\mathit{methodName}(m),\ \mathit{eff}) \\
\qquad\qquad\ \ \mathit{restoreCallOperands}(\mathit{tmpLocal}(m),\ \mathit{arity}) \\
\qquad\qquad\ \ \textbf{invoke}\ \mathit{name}\ \mathit{arity} \\
\qquad\qquad\ \ \textbf{return}_{\textbf{void}} \\
\quad \mathit{ep}(\mathit{eff}): \ \mathit{restoreState}(\mathit{eff}) \\
\qquad\qquad\ \ \textbf{invoke}\ \mathrm{Effekt.result}\ 0
\end{array}
$$

(T-Entrypoint)

$$
\begin{array}{l}
\mathcal{E}[\![\cdot]\!]_{(\ \cdot\ )} : \mathit{Method} \to \mathbb{L} \to \mathit{Method} \\
\mathcal{E}[\![\ \textbf{method}\ \mathit{name}\ \mathit{arity}\ \textbf{throws}\ \overline{\mathit{exc}}\ \{\ \overline{\mathit{instr}}\ \}\ ]\!]_{\mathit{eff}} = \\
\quad \textbf{method}\ \mathit{ep}(\mathit{name},\ \mathit{eff})\ \mathit{epArity}(\mathit{eff})\ \textbf{throws}\ \emptyset\ \{ \\
\qquad \textbf{goto}\ \mathit{ep}(\mathit{eff}) \\
\qquad \overline{\mathcal{I}[\![\ \mathit{instr}\ ]\!]_m} \\
\quad \}
\end{array}
$$

(b) Transformation of effectful methods.

(T-Return)

$$
\begin{array}{l}
\mathcal{I}[\![\ \mathit{label}: \quad \textbf{return}]\!]_m = \\
\quad \mathit{label}: \quad \textbf{invoke}\ \mathrm{Effekt.returnWith}\ 1 \\
\qquad\qquad\ \ \textbf{return}_{\textbf{void}}
\end{array}
$$

(T-Other)

$$
\mathcal{I}[\![\ \mathit{label}: \quad \mathit{instr}]\!]_m = \mathit{label}: \ \mathit{instr}
$$

(c) Transformation of bytecode instructions.

$$
\begin{array}{l}
\mathit{saveCallOperands}: \mathbb{N} \times \mathbb{N} \to \overline{\mathbb{L}:\ \mathit{Instr}} \\
\mathit{saveCallOperands}(\mathit{first},\ n) = \\
\quad \mathit{storeLocals}(\mathit{first}\ ..\ (\mathit{first} + n - 1))
\end{array}
$$

$$
\begin{array}{l}
\mathit{restoreCallOperands}: \mathbb{N} \times \mathbb{N} \to \overline{\mathbb{L}:\ \mathit{Instr}} \\
\mathit{restoreCallOperands}(\mathit{first},\ n) = \\
\quad \mathit{loadLocals}((\mathit{first} + n - 1)\ ..\ \mathit{first})
\end{array}
$$

$$
\begin{array}{l}
\mathit{saveState}: \mathit{Name} \times \mathbb{L} \to \overline{\mathbb{L}:\ \mathit{Instr}} \\
\mathit{saveState}(\mathit{name},\ \mathit{eff}) = \\
\quad \mathit{loadLocals}(\mathit{locals}(\mathit{eff})) \\
\quad \textbf{closure}\ \mathrm{Frame.enter}\ \mathit{ep}(\mathit{name},\ \mathit{eff})\ \mathit{epArity}(\mathit{eff}) \\
\quad \textbf{invoke}\ \mathrm{Effekt.push}\ 1
\end{array}
$$

$$
\begin{array}{l}
\mathit{restoreState}: \mathbb{L} \to \overline{\mathbb{L}:\ \mathit{Instr}} \\
\mathit{restoreState}(\mathit{eff}) = \\
\quad \mathit{loadLocals}(0\ ..\ (\mathit{epArity}(\mathit{eff}) - 1)) \\
\quad \mathit{storeLocals}(\mathit{reverse}(\mathit{locals}(\mathit{eff})))
\end{array}
$$

(d) Helper functions used to manage function local state.

Figure 5.10. Type selective CPS translation via bytecode transformation.

5.4.3 Saving Function Local State

To generate state saving and restoring code, we use the following information about a method m, which we obtain by static analysis of the bytecode:

$effCalls : Method \to \overline{\mathbb{L}}$

the set of labels that mark effect calls, including the label $ep_{\mathsf{init}} = \mathsf{init}$ but no effect tail calls,

$tmpLocal : Method \to \mathbb{N}$

the index of the first free register, not used by the original instructions of the given method.

Likewise, for each effect call $eff \in \mathbb{L}$ in a method m, the transformation uses the following information, which we again obtain by static analysis:

$operands : \mathbb{L} \to \mathbb{N}$

the number of operands on the operand stack after the effect call (not including the result),

$locals : \mathbb{L} \to \overline{\mathbb{N}}$

the list of indices of local registers, which are live after the effect call,

$ep : \mathbb{L} \to \mathbb{L}$

a unique label representing the entrypoint to jump to on resumption,

$ep : Name \times \mathbb{L} \to Name$

for each pair of original method name and label – a unique method name for the corresponding entrypoint method.

We also define $epArity(eff)$ to equal $operands(eff) + |locals(eff)|$, referring to the total number of values that need to be stored in the closure. That is, all operands and the number of locals, which are live *after* the effect call. The static information for doLoop is given in Figure 5.9c.

To actually save the state, the meta-function *saveState* generates code that stores those parts of the function-local state, which are necessary to resume the execution of the function. This includes all operands (after the effect call) and the contents of all registers, which correspond to live local variables. In the code generated by *saveState*(doLoop, op),

```
load 1                                  // load live locals
closure Frame.enter doLoop_op 2         // close over two values
invoke Effekt.push 1                    // push closure to stack
```

we can see that saving the state is achieved in three steps:

1. all live local variables are loaded to the operand stack; the operands do not need to be loaded since they are already on the operand stack
2. a new instance of a Frame is created as a lambda; the lambda uses the given method $ep(name, eff)$ to implement the interface Frame.enter and closes over $epArity(eff)$-many values on the operand stack;
3. finally, the newly created frame is pushed using Effekt.push.

In JVM bytecode, closures are created by issuing a specific **invokedynamic** call to a *lambda metafactory*[26]. We refer to this call only in its specialized form as

closure *interface name arity*

The call to **closure** is provided with name of a *functional interface* (Gosling et al., 2015) $interface \in Name$[27] (*i.e.*, an interface with a single abstract method), a method $name \in Name$, which serves as the implementation of the interface, and an $arity \in \mathbb{N}$ that specifies the number of values the lambda should close over. The JVM runtime passes the closed-over values as additional arguments to the implementing method when the closure is applied.

5.4.4 Translation of Instructions

To generate the bodies of entrypoint methods, Figure 5.10c defines the translation function $\mathcal{I}[\![\ label\ :\ instr\ ]\!]_m$. It specifies how a single labeled instruction is translated within a method m. The result of translating the body of doLoop can be found in Figure 5.9d.

Figure 5.10d gives the implementation of some of the helper functions that expand at translation time to generated code. Given a list of register indices, the unlisted functions *loadLocals* and *storeLocals* generate bytecode that loads from (respectively stores to) all given locals. We use the notation $x\ ..\ y$ to denote a range of indexes from x to y, both ends inclusive.

The translation of bytecode instructions behaves as identity (Rule T-Other) except for effect calls (Rule T-Invoke-Eff) and returns (Rule T-Return). To stress, non-effectful calls require no modification. To translate effect calls, rule T-Invoke-Eff saves the function-local state, performs the effect call, and suspends the method by returning to the trampoline (**return**$_{\textbf{void}}$). In the translated program, all jumps to the effect call within m should point to the instrumented call instead. Therefore, we change label *eff* to point to the first instruction of the state saving code. This also affects exception regions that mention *eff*, which do not require further modification. As in our doLoop example, the effect call might require arguments that reside on the operand stack at the time of state saving. To account for this, we use temporary locals (which will not be stored in the closure) to set the call operands temporarily aside.

The remainder of the function after the effect call is labeled with $ep(eff)$. Since it immediately follows a return, this part of the code is *only* reachable by the **goto** $ep(eff)$ in the corresponding entrypoint method. At that time, all function state necessary for resumption has been passed as arguments and is thus stored in first $epArity(eff)$-many registers. For our example of doLoop, the code at label $ep(\texttt{op}) = \texttt{ep\$op}$ assumes that one operand (the constant `'\backslash{}n'`) and one local (an instance of type Input) have been passed as arguments and are thus available via registers 0 and 1. Before the function can be resumed, its state needs to be restored to where it has been left off. The meta-function *restoreState* loads all saved operands and locals (in this order) to the operand stack. It then writes the locals to the original registers (in reverse order). The result of the previous effect call is obtained by Effekt.result.

If the previous effect call exited abnormally by throwing an exception, Effekt.result as implemented in Figure 5.5, will re-raise this exception. Since we already restored all operands and locals, the exception will be raised in the correct context and trigger the correct exception handlers. Being defined in terms of labels, our translation does not need to modify the exception table. The rule T-Return replaces every return with a call to Effekt.returnWith to install the second half of the special calling convention.

[26] `https://docs.oracle.com/javase/8/docs/api/java/lang/invoke/LambdaMetafactory.html`
[27] For the translation, *interface* will always be Frame.enter.

5.5 Discussion and Related Work

We discuss design decisions and related work. Existing implementations of libraries and languages for (algebraic) effect handlers are either translations to a high-level language or involve a custom runtime implementation. High-level implementations translate effect handlers into delimited continuations (Kammar et al., 2013; Kiselyov and Sivaramakrishnan, 2016), free monads (Kiselyov and Ishii, 2015) or perform a CPS translation into another high-level language (Leijen, 2017c; Hillerström et al., 2017). Other implementations require a custom runtime that supports stack manipulation (Bauer and Pretnar, 2015; Dolan et al., 2017) or setjump / longjump (Leijen, 2017b). In this chapter, we explored a new implementation technique for effect handlers in terms of a CPS transformation of bytecode. Similarly, the discussion focuses on other bytecode transformation.

5.5.1 Continuations on the Java Virtual Machine

We review related work on (delimited) continuations and CPS transformations in the context of the Java virtual machine. Experimental implementations that modify the JVM exist (Dragos et al., 2007; Stadler et al., 2009) or are under development (Pressler, 2017). While those specialized runtimes could potentially be used as backend for our effect handler library, here we will focus on library solutions. We compare our CPS transformation with three other Java projects that perform bytecode instrumentation. A library for fibers "Quasar" (Parallel Universe Software Co., 2013), a library for one-shot continuations "JavaFlow"[28] (Silaev, 2015), and a library for coroutines "Coroutines" (Faghihi, 2015).

Continuation instantiation Approaches to capture the continuation can be characterized by the point-in-time the continuation is materialized. CPS transformations create the continuation *before* the execution of an effectful call. This is how we implemented JavaEffekt. It is also the case for implementations of effect handlers that rely on CPS (Hillerström et al., 2017) and corresponding monadic implementations in eager languages like ScalaEffekt. Quasar also explicitly stores all function state, before entering an effect call. Another approach is to instantiate the continuation only when it is needed. Effectful functions can also use a special exception at runtime to signal that the continuation needs to be captured (Sekiguchi et al., 2001; Pettyjohn et al., 2005; Loitsch, 2007). Other alternatives to signal continuation capture are sum types (Kiselyov and Sivaramakrishnan, 2016) or global flags (JavaFlow, Coroutines).

Stack restoration All bytecode continuation libraries in Java that we are aware of resume a continuation by replaying all function calls. This way, the JVM stack is restored before the execution of the program continues. Similarly, Koppel et al. (2018) showed recently how delimited continuations can be expressed with exceptions and state by replaying all effectful function calls on resumption. This implementation technique simplifies integration with exceptions, stack traces, and debuggers. It also is a technical consequence of not having a first class representation of continuation frames. However, restoring the stack is linear in the depth of the stack since all function calls need to be replayed.[29] In contrast, in JavaEffekt we explicitly reify each continuation frame as a closure. Upon resumption, we just enter the first frame without restoring the full

[28] We compare JavaEffekt with a recently maintained fork of the equally named Apache Commons project. The original project is located at: `https://commons.apache.org/sandbox/commons-javaflow`

[29] For Quasar, this observation has been made by Aleksandar Prokopec (Oracle Labs) – personal communication.

Java stack. While this helps to reduce the asymptotic complexity from quadratic to linear, stack traces in JavaEffekt only show very few frames, which can impede debugging.

Function state representation The state, necessary to later resume a suspended function, consists of the function-local state and an entrypoint label. It can be represented and stored in different ways. The entrypoint label can be encoded as a number that will be dispatched upon with a switch statement at the beginning of the method. This is commonly combined with storing the function local data in a stack like data structure (Quasar, JavaFlow, Sekiguchi et al. (2001), Bierman et al. (2012)). An alternative is to replace the switch by dynamic dispatch and to store the function data in a closure (Pettyjohn et al., 2005). This is how JavaEffekt is implemented.

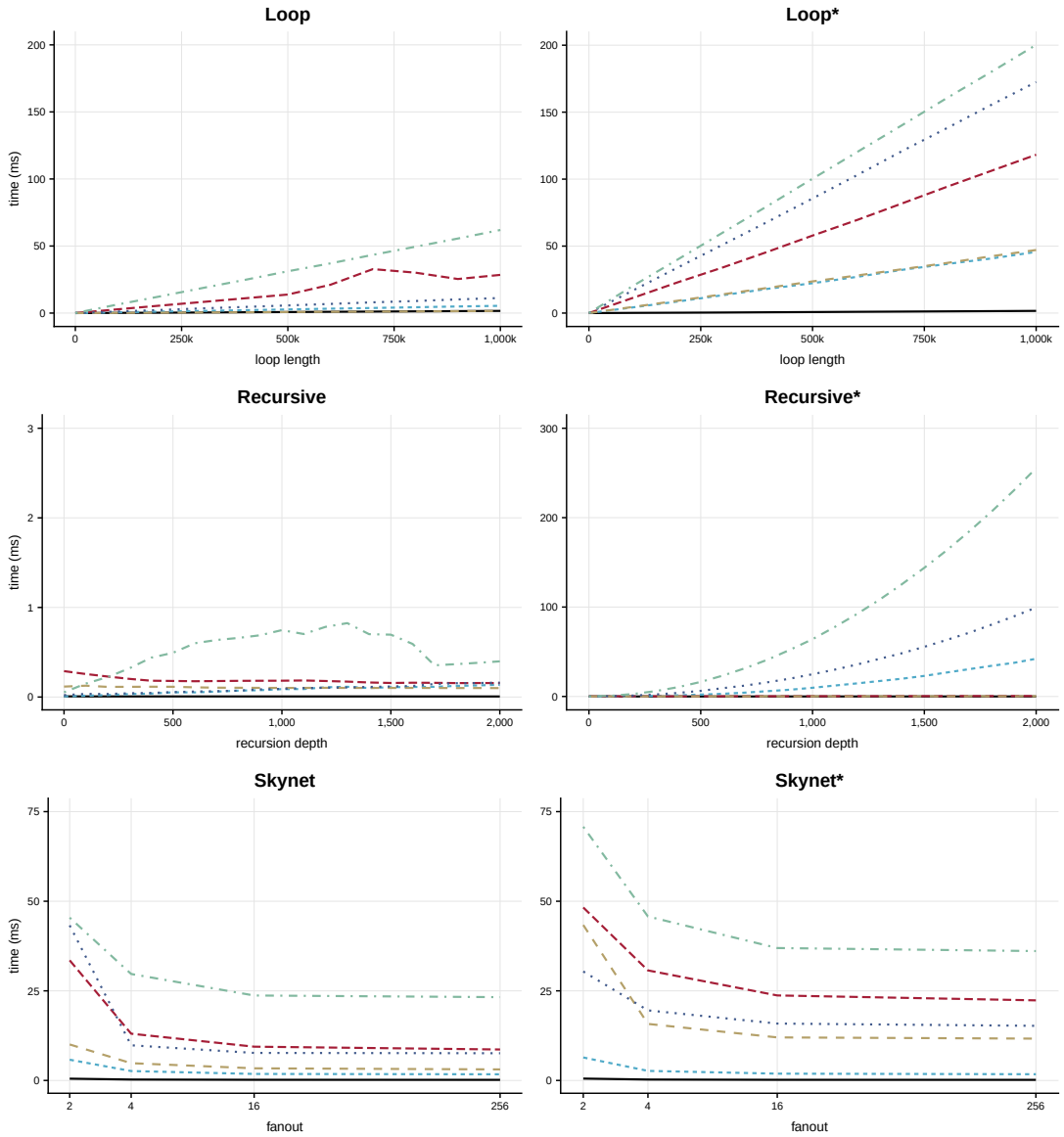
Multiple resumptions We designed JavaEffekt in a way that continuations can naturally be resumed multiple times. In implementations supporting only *one-shot* continuations, state update can be destructive, which makes it easier to implement continuations efficiently (Dolan et al., 2015). While JavaEffekt maintains one global immutable runtime stack (as in Figure 5.5), Quasar and JavaFlow maintain one mutable stack per delimited continuation / fiber. In such a setting, multiple resumptions are implemented by deeply cloning the corresponding stack and all nested stacks before resuming. In JavaEffekt, function-local state is copied into immutable frames. In addition, stack segments are immutable and can be shared across multiple resumptions.

Marking effectful functions Effekt uses Java's checked exception mechanism as a (coarse grained) effect type system. Thus, all effect operations and consequently all effectful functions that transitively use effect operations are required to be marked as throwing a special `Effects` exception. The exception is only used as a marker to distinguish pure from effectful methods. It will never be thrown at runtime. JavaFlow requires the user to annotate methods with a `@continuable` annotation. Java Coroutines only treats those methods as effectful that take a value of type `Continuation` as an argument. Quasar offers the user the choice between checked exceptions and Java annotations.

5.6 Performance of Effekt

We report on some preliminary performance results. The evaluation of performance is split into two parts: A part on the CPS transformation and a part on Effekt as a library for programming with effect handlers. All benchmarks were executed on a 2.5 GHz Intel Core i7 with 16GB of memory using ScalaMeter (Prokopec, 2012), a state of the art JVM benchmarking library. Each benchmark consisted of 50 runs, executed in a warmed up JVM instance (JDK version $1.8.0_{181}$).

The implementation of JavaEffekt slightly deviates from the formal presentation in Section 5.4. To avoid push-pop-enter cycles, the initial entrypoint is not explicitly pushed but inlined. We also show the performance results for a variant of JavaEffekt that implements several optimizations: Similar to the approach by Pettyjohn et al. (2005), our alternative implementation does not materialize continuation frames upfront, but only when needed. In addition, only methods that contain at least one non-tail effect call are instrumented. Like in our formal presentation, the alternative implementation also materializes continuation frames by creating closures. We refer to this variant as $\text{JavaEffekt}_{\text{opt}}$. The user programs using JavaEffekt do not need to be changed.



	Time in ms (Confidence Interval)					
Benchmark	**Baseline**	**JavaEffekt**	**JavaEffekt$_{opt}$**	**Coroutines**	**Quasar**	**JavaFlow**
Loop (1M)	1.6 ±0.1	28.5 ±0.7	1.6 ±0.0	5.4 ±0.3	62.0 ±1.2	11.2 ±0.3
Loop*(1M)	1.6 ±0.1	118.2 ±3.1	47.1 ±1.5	45.7 ±1.6	200.2 ±2.8	172.5 ±2.2
Recursive (1K)	0.0 ±0.0	0.2 ±0.0	0.1 ±0.0	0.1 ±0.1	0.7 ±0.5	0.1 ±0.0
Recursive*(1K)	0.0 ±0.0	0.3 ±0.0	0.2 ±0.0	9.7 ±0.9	64.0 ±0.6	24.9 ±0.9
Skynet (2)	0.5 ±0.0	33.5 ±10.5	10.0 ±3.4	5.8 ±2.1	45.4 ±1.0	43.3 ±9.3
Skynet*(2)	0.5 ±0.1	48.3 ±1.1	43.4 ±13.2	6.4 ±2.4	70.8 ±1.3	30.4 ±0.7

* Variants that perform regular suspend and resume.

Figure 5.11. Performance of bytecode instrumentation libraries. Runtime in ms, lower is better.

5.6.1 Performance of the Bytecode Instrumentation

We evaluate the performance of our CPS transformation comparing with Quasar in version 0.7.9 (Parallel Universe Software Co., 2013), JavaFlow in version 2.6.0 (Silaev, 2015), and Coroutines in version 1.4.2 (Faghihi, 2015). We also measure the overhead compared to a baseline that does not capture continuations. All libraries perform some sort of bytecode transformation to support capturing the continuation. Since each of the libraries targets a particular domain (coroutines / fibers), capturing the continuation also involves additional overhead specific to the target application. Where possible, we reduced this overhead by disabling features – focusing on the continuation capturing aspect, only. As an example, the measurements for the Quasar library where executed using a trivial single threaded scheduler. The results of the measurements can be found in Figure 5.11.

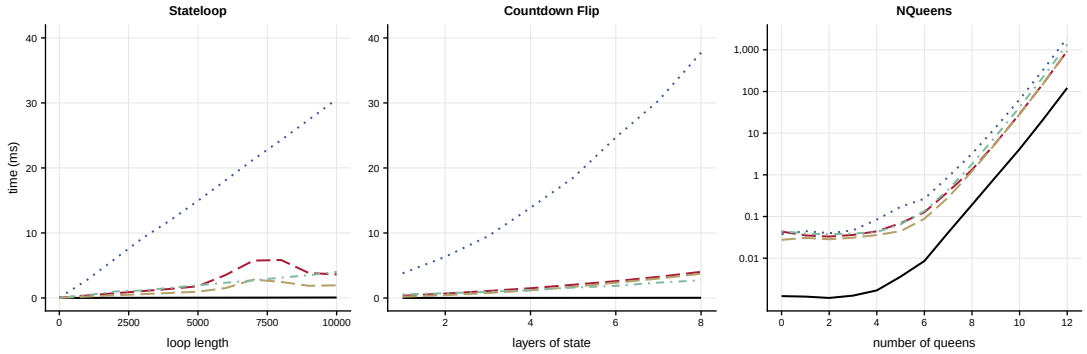
To assess the instrumentation overhead, the Loop benchmark counts down from a given number to zero, performing some computation work at each step but not capturing the continuation. We can see that most of the overhead of creating continuation frames is eliminated in the alternative JavaEffekt$_{\text{opt}}$. Java Coroutines save the function local state in arrays before entering a potentially suspending function call. This is unnecessary for the Loop benchmark, which does not suspend. The Loop* variant captures and immediately resumes the continuation at each step of counting down. Here the cost of storing the function state pays off for Java Coroutines, which now aligns with JavaEffekt$_{\text{opt}}$. The baseline for Loop* does not suspend and is thus the same as for Loop.

To measure performance of capturing the continuation, the Recursive and Recursive* benchmarks also count down, but as recursive functions. For Recursive, we suspend the computation once before returning the result (at stack-depth n); correspondingly, for Recursive* we suspend once at every recursive call. Resuming continuations is linear in stack depth for all implementations but the two JavaEffekt implementations. In consequence, for the other implementations, Recursive* has a running time that is quadratic in n while it is linear for JavaEffekt and JavaEffekt$_{\text{opt}}$. Resuming in our implementations is linear in the number intermediate prompts, while the other implementations are linear in the number of individual frames.

To measure the overhead of delimiting continuations, the Skynet benchmark[30] recursively spawns n fibers until $m = 2^{16}$ fibers are created at the leafs. In total, $\frac{n^{\log_n(m)\ +\ 1}\ -\ 1}{n\ -\ 1}$ fibers are created. We also refer to n as the fanout. Each fiber recursively spawns fibers, performs some computation, and finally aggregates the results. One fiber corresponds to one delimited continuation. The Skynet variant never suspends, but just creates the fibers which immediately return. The Skynet* variant in contrast suspends each fiber once before returning, resulting in a continuation capture and resumption per fiber. Quasar and JavaFlow maintain one stack per delimited continuation / fiber, each pre-allocating memory to store the function state. JavaFlow additionally maintains one stack per primitive type and copies the stack on every resumption. This leads to hundred thousands of arrays copies[31]. The Coroutines library is optimized for one-shot continuations and large parts of the library are inlined in the generated bytecode. It also does not suffer from its linear stack restoration in the Skynet benchmark since the stack size of each fiber on suspension is at most one.

[30] `https://github.com/atemerev/skynet`

[31] In reaction to the first publication of our results (Brachthäuser et al., 2018), the authors of JavaFlow significantly improved the performance of their library in general, and one-shot continuations in particular. We report the updated results here.



	Time in ms (Confidence Interval)				
Benchmark	**Baseline**	**JavaEffekt**	**JavaEffekt$_{opt}$**	**ScalaEffekt**	**Scala Eff**
Stateloop (10k)	0.07 ±0.0	3.62 ±0.24	1.94 ±0.08	4.0 ±0.27	30.56 ±0.9
CountdownFlip (8)	0.03 ±0.0	4.02 ±0.1	3.73 ±0.17	2.72 ±0.14	37.73 ±1.0
NQueens (12)	121.83 ±0.98	936.17 ±13.44	935.81 ±29.65	1350.58 ±211.64	1933.68 ±32.18

Figure 5.12. Performance of effect libraries. Runtime in ms, lower is better.

5.6.2 Performance of the Effect Library

To evaluate the performance of the overall framework, we compare our Java implementation of JavaEffekt with our implementation in Scala (ScalaEffekt, Chapter 4), and the effect library "Eff" (Torreborre, 2016), which we refer to as "Scala Eff" to avoid confusion with the language Eff by Bauer and Pretnar (2015). Scala Eff is a library for functional programming with effects, based on extensible effects and freer monads (Kiselyov and Ishii, 2015). The results of the measurements can be found in Figure 5.12.

The Stateloop Benchmark uses a state effect to count down from a given number n. To allow a better comparison with Scala Eff, both of our library implementations use the functional state translation of Kammar et al. (2013) instead of our `Stateful` interface. The baseline implementation directly uses mutable state.

The CountdownFlip benchmark layers one ambiguity effect over eight state effects (Kiselyov and Ishii, 2015). It uses the state to count down from $1,000$ and flips once before returning. The baseline does not model ambiguity but always returns true for `flip`.

NQueens is an effect library benchmark from the literature (Kammar et al., 2013). For the baseline, we adopted a Java implementation found online[32]. The baseline makes use of mutable state and arrays, whereas the other four implementations use control effects and immutable lists.

In both state benchmarks, Stateloop and CountdownFlip, our implementations use the functional state translation. Of course we could also use our specialized state interface of Section 5.3.3 instead. Using the `Stateful` interface in the CountdownFlip benchmark, for JavaEffekt and eight layers, the measured run time goes down to 0.22ms ± 0.02ms. Similarly, for JavaEffekt$_{opt}$, we measured 0.08ms ± 0.01ms – a speed up of about 46x compared to using functional state.

[32] `https://rosettacode.org/wiki/N-queens_problem`

The benchmarks indicate that the Java implementations of JavaEffekt are on par with ScalaEffekt. Compared to Scala Eff, our implementations offer speed ups of 1.4-16x using functional state and of 470x using our specialized state interface. Besides specializing state, we account the biggest performance over Scala Eff, that tail resumptive operations can be dynamic method calls in Effekt. A similar finding has been reported by Leijen (2017b). Compared to ScalaEffekt, performance improvements might be related to inlining the monadic Scala code by bytecode instrumentation and only capturing the continuation on demand ($\text{JavaEffekt}_{\text{opt}}$).

5.7 Chapter Conclusion

We presented the first library for programming with effect handlers in direct style in Java. We showed how such a library can be implemented in terms of a continuation-passing style transformation and multi-prompt delimited continuations. Our continuation-passing style transformation allows trampolining, multiple resumptions and is competitive in its performance.

Chapter 6

Scala Effekt – Effect Safety through Regions

The implementations of Effekt, as presented in the previous chapters, do not guarantee effect safety. Capabilities can leak, which in turn leads to runtime errors.

In this chapter, we improve the involved (effect) types and present a variant of ScalaEffekt that supports effect safety and effect polymorphism. Our effect system guarantees that all effects are eventually handled and runtime errors caused by leaked capabilities are statically ruled out.

To the best of our knowledge, Effekt, as presented in this chapter, is the first library implementation of effect handlers that supports effect safety and effect polymorphism, without resorting to type-level programming. We describe a novel way of achieving effect safety in a library embedding by using intersection types and path-dependent types. The effect system of our library design fits fits well into the programming paradigm of capability passing and is inspired by the effect system of Zhang and Myers (2019). Capabilities carry an abstract type member, which represents an individual effect type and reflects the use of the capability on the type level. Handlers introduce capabilities and remove components of the intersection type. Reusing the existing type system of Scala, we get effect subtyping and effect polymorphism for free.

Let us recall our running example using our ScalaEffekt library, presented in Chapter 4:

```
def drunkFlip(amb: Amb, exc: Exc) = for {
  caught ← amb.flip()
  heads  ← if (caught) amb.flip() else exc.raise("Too drunk")
} yield if (heads) "Heads" else "Tails"
```

As before, effectful methods mention the effects they use as additional arguments (*i.e.*, *capabilities*). Handlers introduce a dynamic scope in which the corresponding capability can be used – the *handler region*. However, it is not guaranteed that methods *only* acquire capabilities by means of capability passing. For example, capabilities can be stored in references to be used

The contents of this chapter first appeared in: Jonathan Immanuel Brachthäuser and Philipp Schuster. 2017. "Effekt: Extensible Algebraic Effects in Scala (Short Paper)". In *Proceedings of the 8th ACM SIGPLAN International Symposium on Scala* (SCALA 2017). ACM, New York, NY, USA, 67-72. DOI: https://doi.org/10.1145/3136000.3136007

This chapter is closely based on an extended version that appeared in the Journal of Functional Programming: Jonathan Immanuel Brachthäuser, Philipp Schuster, and Klaus Ostermann. 2020. "Effekt: Capability-passing Style for Type- and Effect-safe, Extensible Effect Handlers in Scala". *Journal of Functional Programming, 30, E8*. DOI: https://doi.org/10.1017/S0956796820000027

later – even outside of the handler region. This can lead to runtime exceptions and is a source of unsafety.

In this chapter, we solve the problem of effect safety and present a variant of ScalaEffekt that statically prevents programs from using leaked capabilities outside of the handler region. We repeat user programs of Chapter 4 and change their types, which are more precise now. This also becomes visible in the inferred type of `drunkFlip`:

```
                                                    Result Type      Effect Typing
def drunkFlip(amb: Amb, exc: Exc): Control[String, amb.effect & exc.effect]
```

We describe a novel way of achieving effect safety in a library embedding by using intersection types and path-dependent types (Odersky et al., 2006). The effect system of our library design fits well into the programming paradigm of capability passing and is inspired by the effect system of the $\lambda_{\Downarrow\Uparrow}$-calculus by Zhang and Myers (2019). As we will see in Section 6.1, we represent effect rows as the contravariant intersection of effect types, where an individual effect of a capability (like `amb`) is represented by the abstract type member `amb.effect`[33]. Handlers remove components of the intersection type. By reusing the existing type system of Scala, we get effect subtyping and effect polymorphism for free.

By the nature of an embedding into a practical, but unsound host language (Amin and Tate, 2016), type- and effect-safety of our library can only be guaranteed up to soundness of the host language. Furthermore, we do not present formal proofs of soundness and safety, but the design for a library implementation as an embedding into an existing, mainstream programming language. Our effect system is based on $\lambda_{\Downarrow\Uparrow}$ – but we leave a formal treatment and corresponding soundness proofs to future work. To enable a more direct comparison, Section 6.6.1 presents a library embedding of the $\lambda_{\Downarrow\Uparrow}$-calculus (Zhang and Myers, 2019) into Scala. While we do not present formal proofs, from our experience of working with the library embedding, we are confident it inherits the following properties from $\lambda_{\Downarrow\Uparrow}$:

Effect safety We embed an effect system, which makes sure that attempting to run a program with unhandled effects lead to Scala type error. For example, we can only call `run` on a program when all effects are handled and we reject programs like `run { amb.flip() }`. Like with systems based on monadic regions (Launchbury and Sabry, 1997; Moggi and Sabry, 2001; Kiselyov and Shan, 2008), capabilities can *leave* their defining handler scope. Our effect system only ensures that they cannot be *used* outside their defining handler scope.

Effect subtyping We use Scala's support for subtyping of intersection types to implement effect subtyping. A program with type `Control[Int, exc.effect]` can be used where a program of type `Control[Int, exc.effect & amb.effect]` is expected.

Effect polymorphism We use Scala's support for polymorphism to express effect polymorphic functions like, such as the following function adapted from Rytz et al. (2012):

```
def mapM[A, B, E](ls: List[A], f: A ⇒ Control[B, E]): Control[List[B], E]
```

Here, `mapM` is polymorphic in the effects `E` used by function `f`.

[33] Despite being written lowercase, **`effect`** is not different from other type members. We just render it as a keyword to highlight its importance for our approach.

Existing implementations of languages with effect handlers either completely lack a static effect system – this includes Multicore OCaml (Dolan et al., 2014), Eff (Bauer and Pretnar, 2015), embeddings of Eff in OCaml (Kiselyov and Sivaramakrishnan, 2016), and previous versions of `Effekt` in Scala (Chapter 4) and Java (Chapter 5) – or they do not have sufficient support for effect polymorphism (Kammar et al., 2013; Inostroza and van der Storm, 2018). Languages and libraries with effect systems like Extensible Effects (Kiselyov et al., 2013), Koka (Leijen, 2017c), Links (Hillerström et al., 2017), Frank (Lindley et al., 2017), and Helium (Biernacki et al., 2019) require explicit lifting annotations to *encapsulate effects* in effectful higher order functions, like the function `mapM` above. Without such manual liftings, the implementation detail of effects used within `mapM` would leak into its type signature. In contrast, our effect safe version of `ScalaEffekt` requires no such manual lifting.

In summary, the contributions of this chapter are:

- For our implementation, we build on the operational semantics of Dybvig et al. (2007). We achieve effect safety by generalizing techniques of Launchbury and Sabry (1997) to nested regions (Kiselyov and Shan, 2008), and we use intersection types of abstract type members (Parreaux et al., 2017) to express nesting of regions. Our effect system rules out some use cases of multi-prompt delimited control (Kobori et al., 2016), but can express many interesting use cases of effect handlers.
- We implement `ScalaEffekt` as a thin layer on top of multi-prompt delimited continuations. Importantly, we demonstrate how effect safety for effects and handlers follows from the newly gained effect safety of our implementation of multi-prompt delimited continuations.
- We evaluate desirable properties like effect safety and effect parametricity of our library embedding. We discuss limitations and potential solutions.
- To evaluate the expressive power of our effect system, we revisit examples of Chapter 4. We show how they can safely be expressed. The programs do not have to change, only the (effect) types are more precise now.
- We discuss interesting opportunities to explore type- *and effect-safe* modularization of effectful programs, opened up by embedding `Effekt` into Scala.

The remainder of this chapter is structured as follows. Section 6.1 identifies multiple sources of unsafety. It then shows how to add effect safety to the implementation of delimited control as presented in Section 4.2. Section 6.2 presents an effect-safe variant of our interface for ambient state. In Section 6.3, we repeat the developments of previous chapters and express effect handlers as a thin library on top of (now effect-safe) delimited control. We revisit examples from previous chapters and show how to make them effect-safe. We discuss desirable properties of our effect-system and potential threats in Section 6.5. Section 6.4 discusses the combination of object-oriented programming and effect-safe handlers. Effect safety surfaces interesting new encapsulation challenges. Section 6.6 discusses related work and concludes.

6.1 Effect-Safe Delimited Control

In Section 4.2, we have seen a version of `Control` that has one type parameter `Result`. By also indexing the type `Prompt` with a type parameter `Result`, we statically track answer types and guarantee that capturing and calling the continuation is type-safe (Gunter et al., 1995). However, this version of `Control` is not effect-safe: capabilities can leave their handler region. In particular, capabilities can close over prompts. Using a prompt outside of the dynamic scope of the corresponding `reset` leads to a runtime error. We identify two ways to leave the scope of `reset`.

1) Leaving the scope by returning We can leave the scope of `reset` by returning from it Osvald et al. (2016). In this case it is possible to leak the prompt either through the heap

```
var p: Prompt[Unit] = null
val problem1 = run {
  for {
    _ ← reset { prompt ⇒ p = prompt; pure(()) }
    _ ← shift(p) { resume ⇒ pure(()) } // Exception: Prompt not found
  } yield ()
}
```

or by simply returning it as a result:

```
val problem2 = run {
  for {
    p ← reset { p ⇒ pure(p) }
    _ ← shift(p) { resume ⇒ pure(()) } // Exception: Prompt not found
  } yield ()
}
```

As observed by Osvald et al. (2016), both sources of leakage can also occur indirectly through functions or objects that close over the prompt. Prompts might even leave the scope of the enclosing `run` to then be used in the scope of a different `run`. Dybvig et al. (2007) use rank-2 types to prevent this particular source of error, but leave others to future work.

2) Leaving the scope by shifting We can also leave the scope of `reset` by means of control effects.

```
val problem3 = run {
  reset { p ⇒
    shift(p) { resume ⇒
      shift(p) { resume ⇒ pure(()) } // Exception: Prompt not found
    }
  }
}
```

Due to our choice of $-shift_{+}$, shifting to a prompt removes the enclosing `reset`. Consequently, shifting a second time inside the body of `shift` results in a runtime error. The evaluation context of the second call to `shift` is `run` $\{\ \square\}$ and does not contain any delimiters. Danvy

and Filinski (1990) operationally prevent this kind of runtime error by leaving the outer `reset` behind. However, in the setting of multiple prompts this is not sufficient. The delimiter can also be removed by shifting to a different prompt:

```
val problem4 = run {
  reset { p1 ⇒ reset { p2 ⇒
    shift(p1) { resume ⇒
      shift(p2) { resume ⇒ pure(()) } // Exception: Prompt not found
    }
  }}
}
```

We now introduce our implementation of an effect system that rules out the above four problematic programs, prevents the use of escaped capabilities, and guarantees effect safety. The underlying problem our effect system solves is a general one: we need to restrict the lifetime of a resource (capabilities in our case) to a certain dynamic region (the call to `reset` in our case). This problem also occurs in the domain of region based resource management (Kiselyov and Shan, 2008), object capabilities (Haller and Loiko, 2016), delimited control (Dybvig et al., 2007), scope safety in type-safe meta programming (Parreaux et al., 2017), as well as with prompt based implementations of effect handlers (as in Chapters 4 and 5). Our effect system is inspired by the $\lambda_{\Downarrow\Uparrow}$ calculus (Zhang and Myers, 2019), which uses dependent types to track the set of used labels (*i.e.*, prompts and capabilities) in the effect type. In Section 6.5, we discuss some properties of our effect system embedding, but leave formal proofs of safety and soundness to future work. For a better comparison, Section 6.6.1 gives a more immediate embedding of $\lambda_{\Downarrow\Uparrow}$ into Scala.

6.1.1 Tracking and Delimiting Prompt Usage

Following Zhang and Myers (2019), our effect system builds on the idea of tracking the set of prompts that a program uses in its type. We represent prompts on the type level by their *singleton type* (`p.type`, for some prompt `p`).

In Scala, a singleton type is only inhabited by that very value it refers to. Two (path-dependent) singleton types (`p1.type` and `p2.type`) are equal, if and only if, their prefix paths (`p1` and `p2`) are stable and can be unified (Odersky et al., 2003; Odersky and Zenger, 2005b). That is, the type checker treats `p1.type` and `p2.type` as equal types, if and only if, it can show that `p1` and `p2` refer to the same immutable binding. This way, we prevent leakage via mutable references as in `problem1`.

We use Scala's intersection types to describe a set of prompts. For example, we would model the term level set of prompts

`val ps = Set(p1, p2, p3)` by the type `type Ps = p1.type & p2.type & p3.type`

Term-level set union is represented as type-level intersection. We call programs that use prompts *effectful* and consequently refer to types like `Ps` as "effect types" or simply as "effects". As we will see, effectful programs are contravariant in their effect type. The top of Scala's subtyping lattice (`Any`) describes the empty set. To emphasize this, we define the type alias

```
type Pure = Any
```

```
trait Control[+A, -Effects] {
  def map[B](f: A ⇒ B): Control[B, Effects]
  def flatMap[B, FX](f: A ⇒ Control[B, FX]): Control[B, FX & Effects]
  def andThen[B, FX](c: Control[B, FX]: Control[B, FX & Effects]
}

def pure[A](value: ⇒ A): Control[A, Pure]
def run[A](c: Control[A, Pure]): A
```

Figure 6.1. The effect-safe control interface – changes compared to Figure 4.2 are highlighted. Type alias `Pure` defined in text.

and pure programs thus have an effect type `Pure`. Again, it is important to point out that the purity only refers to delimited control. Programs with a pure effect row still can use side effects like writing to files or accessing the network. The intersection of singleton types might not be inhabited, but this is irrelevant for our use case. We only use the intersection type as a phantom type to track used effects.

The Control Interface

To enable tracking of prompts (and effects in extension), Figure 6.1 revises the type `Control` and adds a second type parameter `Effects`. As before, the type `Control[+A, -Effects]` is marked as covariant in its first type parameter `A`. It is also marked as *contravariant* (Odersky et al., 2006) in its second type parameter `Effects` to obtain the correct subtyping relation between effectful programs. This also aligns well with our view that capabilities are *obligations* to the caller. By contravariance, pure programs (that do not use any prompts) are a subtype of effectful programs (that do use a non-empty intersection of prompt types). As an example, assuming prompts p_1, p_2, and p_3, we write the type of the effectful program that uses prompts p_1 and p_2 to compute an integer as:

```
val prog1: Control[Int, p1.type & p2.type]
```

By (effect) subtyping, the effect type can be weakened

```
p1.type & p2.type & p3.type <: p1.type & p2.type
```

and thus the following assignment is valid:

```
val prog2: Control[Int, p1.type & p2.type & p3.type] = prog1
```

The type of `Control.map` shows that mapping pure functions does not add any new effects to the set of effects. In contrast, the function passed to `Control.flatMap` can have additional effects `FX` to produce a value of type `B`. The effects are aggregated using the intersection type `FX & Effects`. Lifting computation into the control monad with `pure` has no control effects. Dually, only programs without any unhandled control effects can be executed using `run`.

```
trait Prompt[Result, Effects] {}
def reset[R, FX](prog: (p: Prompt[R, FX]) ⇒ Control[R, p.type & FX]): Control[R, FX]
def shift[A, R, FX](p: Prompt[R, FX])(
  body: (A ⇒ Control[R, FX]) ⇒ Control[R, FX]
): Control[A, p.type]
```

Figure 6.2. The effect-safe prompt interface – changes compared to Figure 4.4 are highlighted.

6.1.2 The Prompt Interface – From Answer Type Safety to Effect Safety

In the previous variant of `Control` (Chapter 4), prompts carried the answer type `Result` to ensure that using control effects is type-safe (Gunter et al., 1995). To also make them effect-safe and prevent programs like $\texttt{problem}_4$ from type checking, the type `Prompt` (Figure 6.2) now additionally contains a type parameter `Effects`.

```
trait Prompt[Result, Effects]
```

As before, prompts are introduced by `reset`.

```
def reset[R, FX](prog: (p: Prompt[R, FX]) ⇒ Control[R, p.type & FX]): Control[R, FX]
```

There are a few important aspects of our embedded effect system that become visible in this type signature:

Ambient effects
: The type parameter `FX` describes the set of ambient effects, which are available at the call site to `reset`. Not surprisingly, the return type of the delimited program `prog` also signals that the program can use these effects. Prompts now track both, the expected return type `Result` and the set of ambient effects `Effects` available at the corresponding reset.

Prompt usage and intersection types
: The type of `prog` is a *dependent function type* (Odersky, 2019b). As before in Section 4.2, the program receives the fresh prompt as argument, which it may use in its body. However, this fact is now also expressed in the program's return type `Control[R, p.type & FX]`.

Path dependency and safety
: To guarantee safety, we have to make sure that the only way to remove a prompt type from the intersection type is by delimiting the program with `reset`. To achieve this, the set of effects of the program `prog` refers to `p.type`, a *path-dependent singleton type* (Odersky and Zenger, 2005b). Different calls to `reset` lead to different singleton types that cannot be unified on the type level (even if it would be the same object at runtime). Hence, only the very call to `reset` that introduced a prompt can remove the prompt's singleton type from the effect type.

The return type of the control operator `shift` expresses the use of the prompt on the type level:

```
shift(p)(...): Control[A, p.type]
```

Tainting the set of effects with the singleton type rules out problematic programs $\texttt{program}_1$ and $\texttt{program}_2$. In case of the first program, the prompt is stored in a mutable reference and used

outside of the corresponding `reset`. In Scala, two (path-dependent) types are equal if and only if their prefix paths are stable and they can be unified (Odersky and Zenger, 2005b). Informally, a path is stable if it does not contain a mutable component. This way we prevent leakage via mutable references as in $\texttt{problem}_1$. The second program is also statically ruled out, since `run` requires the set of effects to be empty. Furthermore, all attempts to delimit a program that uses the prompt will fail: The singleton type of the prompt will not unify with the new delimiter.

As mentioned in Section 2.3, the body of a `shift` is conceptually evaluated at the position of the corresponding `reset`. This is now reflected in its type, which is:

```
body: (A ⇒ Control[R, FX]) ⇒ Control[R, FX]
```

Both, the answer type `R` and the effects `FX` have to match with the ones at the corresponding `reset[R, FX]`. Thus, capability-passing style is not only essential for operationally delimiting control effects but also necessary to carry both the expected answer type as well as the available effects from the `reset` to the `shift` that uses the prompt. Since the body of `shift` has to return `Control[R, FX]`, it cannot shift to the same prompt (as in $\texttt{problem}_3$). This would require a type of `Control[R, p.type & FX]`. The problematic program $\texttt{problem}_4$ is ruled out too, since $\texttt{p}_1$ has type `Prompt[Int, Pure]`. The body of the first shift needs to be pure and cannot use $\texttt{p}_2$.

To emphasize, we do not prevent leakage of prompts. Instead, we taint the effect type whenever we perform `shift` on a prompt. This is reflected in the return type of `shift` (Figure 6.2). The type `Control[A, p.type]` indicates the use of the prompt `p` on the type level. While we do not prevent leakage, the type of `run` asserts that only pure programs can be executed. That is, all prompts have to be delimited and the intersection has to be empty (`Pure`). The way we achieve safety is similar to how rank-2 types can be used to enable type-safe monadic regions in Haskell (Launchbury and Sabry, 1997; Moggi and Sabry, 2001; Kiselyov and Shan, 2008). In those approaches, resources are indexed by a type parameter. To handle programs that use resources, the state handler `runST` then requires that the type index can be quantified over universally. Every user of a resource is "infected with the type of that state thread" (Launchbury and Sabry, 1997, p. 229). Similarly, using a prompt `p` taints the effect row with the prompt's singleton type. By building on path-dependent types, we move the universal quantification from the type level to the term level.

6.1.3 Structured Programming with Effect-Safe Delimited Control

Equipped with our enhanced implementation of `Control`, we are now ready to revisit the examples from Section 4.2 and assign effect types.

Example 1 - Effect Typed

The first example (Danvy and Filinski, 1990) does not need to change. Only the type is a bit more precise:

```
val ex: Control[Int, Pure] = reset { p: Prompt[Int, Pure] ⇒
  shift(p) { k ⇒ k(100) flatMap { x ⇒ k(x) } } map { y ⇒ 10 + y }
} map { z ⇒ 1 + z }
```

It is now clear from the effect type that, after resetting, there are no more control effects left to delimit. We can thus safely run `ex`.

Example 2 - Effect Typed

The second example illustrates how each `reset` removes its prompt from the set of effects.

```
val ex2: Control[Int, Pure] = reset { p1: Prompt[Int, Pure] ⇒
  reset { p2: Prompt[Boolean, p1.type] ⇒
    shift(p1) { k ⇒ pure(21) }       // Control[Int, p2.type & p1.type]
  } map { x ⇒ if (x) 1 else 2 }      // Control[Int, p1.type]
} map { y ⇒ 2 * y }                  // Control[Int, Pure]
```

The body of the second `reset` has type `Control[Int, p1.type & p2.type]`. By effect subtyping, we can use `shift(p1) { ... }`, which has type `Control[Int, p1.type]`.

This example also highlights another important aspect of our effect-safe control operator: prompts track the available effects at the *definition site*, not the *use site*. For instance, the type of p_2 informs us that within a body of `shift(s2) { ... }` the s_1 prompt could be used. In this case, the return type of `shift(s2)` would only mention `s2.type`, not `s1.type`. The potential usage of s_1 within the body is an implementation detail that is encapsulated at the definition site. It does not leak to the use site of `shift(s2)`.

Example 3 - Effect Typed

Adding effect types to the third example is a bit more involved. In Section 4.2, we defined the following type aliases (Figure 4.6a):

```
type Amb = () ⇒ Control[Boolean]
type Exc = String ⇒ Control[Nothing]
```

Without effect safety, it was sufficient to say that `flip` and `raise` use *any* control effects by making them return for example `Control[Boolean]`. To establish effect safety, we now have to be more specific. Figure 6.3a defines the effect signatures `Amb` and `Exc` as traits. Comparing the traits to the equally named type aliases, we see a few differences:

1. Effect signatures inherit from the library trait `Eff`.
2. Effect operations are now named (that is, `flip` and `raise` are explicitly named methods) whereas earlier we used the `apply` method that Scala generates for function types.
3. Effect operations refer to the type member **`effect`**, declared in `Eff`, in their return type. Note that the reference to **`effect`** is short for **`this.effect`** (Odersky and Zenger, 2005b).

The last difference is especially important. Guided by the previous implementation, we know that a handler will eventually use *some effect* to implement the effect operations of the effect signature. The concrete effects used, however, are considered an implementation detail of the handler. We thus hide this detail existentially (Mitchell and Plotkin, 1988; Rossberg et al., 2010) behind a type member **`effect`**.

The use of these effects is now exactly as in the introductory example:

```
def drunkFlip(exc: Exc, amb: Amb): Control[String, amb.effect & exc.effect] =
  for {
    caught ← amb.flip()
    heads  ← if (caught) amb.flip() else exc.raise("Too drunk")
  } yield if (heads) "Heads" else "Tails"
```

```scala
trait Eff { type effect }
trait Exc extends Eff { def raise(msg: String): Control[Nothing, effect] }
trait Amb extends Eff { def flip(): Control[Boolean, effect] }
```

(a) Effect Signatures for exception and ambiguity.

```scala
def maybe[R, FX](prog: (e: Exc) ⇒ Control[R, e.effect & FX]): Control[Option[R], FX] =
  reset { (p: Prompt[Option[R], FX]) ⇒
    val exc = new Exc {
      type effect = p.type
      def raise(msg: String) = shift(p) { resume ⇒ pure(None) }
    }
    prog(exc) map { x ⇒ Some(x) }
  }
```

(b) Handler function for the exception effect. Implementation of `raise` like in Figure 4.6b.

```scala
def collect[R, FX](prog: (a: Amb) ⇒ Control[R, a.effect & FX]): Control[List[R], FX] =
  reset { (p: Prompt[List[R], FX]) ⇒
    val amb = new Amb {
      type effect = p.type
      def flip() = shift(p) { resume ⇒ for {
        xs ← resume(true)
        ys ← resume(false)
      } yield xs ++ ys }
    }
    prog(amb) map { x ⇒ List(x) }
  }
```

(c) Handler function for the ambiguity effect. Implementation of `flip` like in Figure 4.6c.

Figure 6.3. Using effect-safe delimited control to declare and handle exception and ambiguity effects. Effect type related changes compared to Figure 4.6 highlighted.

The implementation of handler functions is given in Figures 6.3b and 6.3c. The implementations of `raise` and `flip` in the handler functions are exactly as in Figure 4.6, but we repeat them for easier reference. However, we now assign types that are more precise! Handlers have to state that they implement the effect signatures in terms of delimited control. This is achieved by the type assignment **`type effect = p.type`**, which is necessary to unify **`amb.effect`** with **`p.type`**. This way `reset`, which removes the **`p.type`** from the set of effects, also removes **`amb.effect`**.

To emphasize, the simple library trait `Eff` plays an essential role in abstracting over the (control) effects used by a handler. This implementation detail is hidden from the handled program. Its type only refers to the abstract type member **`effect`**, the concrete implementation in terms of delimited control is hidden existentially.

With effect types assigned to `maybe` and `collect`, we are ready to handle our running example:

```
val res1 = run {
  collect { amb ⇒
    maybe { exc ⇒
      drunkFlip(amb, exc)   // Control[String, exc.effect & amb.effect]
    }                       // Control[Option[String], amb.effect]
  }                         // Control[List[Option[String]], Pure]
}                           // List[Option[String]]
```

6.2 Effect-Safe Ambient State

As seen in Section 4.3, and pointed out by Kiselyov et al. (2006), the interaction between delimited control and mutable state is subtle. The functional state translation of Kammar et al. (2013) allows us to guarantee the correct backtracking behavior, even in presence of handlers that call the resumption multiple times. However, the benchmarks of Section 5.6.2 suggest that using our specialized state interface offers significant speedups over using functional state. Figure 6.4 defines the effect signature of our built-in state effect. Only the types are more precise, compared to Figure 4.7b. As before, the interface `Field` is nested in the state effect. However, it now refers to the outer effect's type member **`effect`**. This way, all fields created by calling the method `Field` share the same effect type (Leijen (2018b) also refers to this parent effect as "umbrella effect"). Simply by nesting the types, we are able to express scoped resources, whose lifetime is conceptually coupled to the outer effect. This allows creating a dynamic number of fields, while maintaining the invariant that fields cannot escape the region of the corresponding state handler.

Given the built-in handler `region` (Figure 6.4), we can again assign more precise types to the equivalent example of Section 4.3:

```
collect { amb ⇒
  region { state ⇒
    val x = state.Field(0)
     amb.flip() flatMap { b ⇒
       if (b) x.put(2) else pure(())
    } andThen x.get() // Control[Int, state.effect & amb.effect]
  }                   // Control[Int, amb.effect]
}                     // Control[List[Int], Pure]
```

```
trait State extends Eff {
  def Field[T](init: T): Field[T]

  trait Field[T] {
    def get(): Control[T, effect]
    def put(value: T): Control[Unit, effect]
    def update(f: T ⇒ T): Control[Unit, effect]
  }
}
def region[R, FX](prog: (s: State) ⇒ Control[R, s.effect & FX]): Control[R, FX] = ...
```

Figure 6.4. Effect-safe state effect – effect signature of the built-in state effect and its handler `region`.

Like the delimiter for continuation capture (*i.e.*, `reset`) creates a fresh prompt, the delimiter for ambient state (*i.e.*, `region`) creates a fresh instance of `State`. This instance can only be used in the scope delimited by `region`, which is guaranteed by the types.

6.3 From Effect-Safe Delimited Control to Effect Handlers

In Section 6.1, we have seen how to make delimited control effect-safe. This allowed us to safely re-implement our running example in the style of structured programming with delimited continuations (Kammar et al., 2013). The handlers of the previous section were implemented as anonymous inner classes. This style of implementing handlers precludes certain forms of reuse.

What is left, is the small step from structured programming with delimited control to effect handlers. A step we have already taken twice in Sections 4.4 and 5.2.2 for our Scala and Java implementations, correspondingly. Analogous to the previous chapters, we can express the handler interface of Effekt in terms of effect-safe multi-prompt delimited control. As explored in Section 4.4, implementing handlers as traits opens up interesting new opportunities for modularity and extensibility.

In this section, we define the missing `Handler` interface, revisit a selection of examples of Chapter 4, and show how to implement them in our effect-safe variant of ScalaEffekt. Not only is it possible to implement all the examples of Chapter 4 in an effect-safe way, adding effect types also makes important details visible that were previously hidden.

6.3.1 Programming with Effect-Safe Effect Handlers

Many handler implementations need to capture the delimited continuation and use delimited control as an implementation effect. Figure 6.5b abstracts over this common mode of use in the effect-safe definition of the `Handler` trait.

```
class Maybe[R, FX] extends Exc with Handler[R, Option[R]] { type Effects = FX; ... }
class Collect[R, FX] extends Amb with Handler[R, List[R]] { type Effects = FX; ... }
```

(a) The two effect handlers are parametric in the effects `FX`.

```
trait Handler[R, E] extends Eff {
  // Abstract members that need to be specified by implementing classes
  type Effects
  protected def unit(result: R): Control[E, Effects]

  protected def use[A](
    body: (A ⇒ Control[E, Effects]) ⇒ Control[E, Effects]
  ): Control[A, effect] = ...

  def handle(
    prog: this.type ⇒ Control[R, effect & Effects]
  ): Control[E, Effects] = ...
}
```

(b) The effect-safe handler interface. The abstract type member `Effects` describes the effects the handler may use.

Figure 6.5. Implementation of effect handlers for `Exc` and `Amb` using the effect-safe library class `Handler` – changes compared to Figure 4.3 are highlighted.

The Effect-Safe Handler Interface

Most importantly, the library trait `Handler` declares the type member `Effects`, which represents the *implementation effects* a handler uses *besides* delimited control. As we will see, handler implementations override the type member to specify concrete effects. Inspecting the type signatures of the three methods (*i.e.*, `unit`, `use`, and `handle`) sheds some light on how a handler can use the implementation effects.

The type of the method `unit` signals that the return clause can use the specified implementation effects, but not the handled effect itself (**`effect`**).

The type of the method `use` shows that the body, which is passed to it, can use the implementation effects. Since the implementation effects are not handled by this handler, the type of the continuation (`A ⇒ Control[E, Effects]`) also mentions `Effects`. As already pointed out in Chapter 2, effect encapsulation is one of the most important aspects of effect handlers. The effects at the handler site (`Effects`) are encapsulated by `use` whose return type only mentions the effect of the handler itself (**`effect`**). This effect encapsulation is the only difference between ambiently bound lambdas and ambient functions in Chapter 3.

Analogous to `reset`, the type of the method `handle` states that the current effect `h.effect` is removed from the set of the effects used by `prog`. The implementation effects are not affected by calling `handle`.

Figure 6.5 uses the effect-safe handler interface to, once more, implement the handlers `Maybe` and `Collect`. The implementation of the methods stays the same. Only the types change. The example handlers `Maybe` and `Collect` do not use any other effects. They are parametric in the ambient effects `FX`, much like they are parametric in the answer type `R`. While the result type is

provided as a type parameter to the handler, for the effects we use the type member `Effects`. As we will see, this offers us some more flexibility, since type members can refer to other type members of the same class, while type parameters cannot.

The Effect-Safe Handler Implementation

As in the previous chapters, in our implementation of ScalaEffekt, we implement the effect handler interface by adding a field that stores a unique prompt marker:

```
val prompt: Prompt[E, Effects]
```

Unlike before, however, the type of the prompt marker now carries the additional information about the effects at the handler site (`Effects`). For convenient handling, we again extend our implementation of multi-prompt delimited continuations with the method `resetWith`. The method bodies of `use` and `handle` are thus syntactically equivalent to the ones of Figure 4.8. The `Handler` trait uses control-effects. In our effect-safe implementation, this also needs to be declared on the type level:

```
type effect = prompt.type
```

Using this type equality, the implementations of `use` and `handle` can be type checked without any modifications.

While with the `Handler` trait we aim to abstract over the implementation detail of multi-prompt delimited control, the type member **`effect`** *and* its definition are part of the public interface. Once fully implemented, the proposed Scala feature of *opaque types* will help us to fully encapsulate this implementation detail (Osheim and Cantero, 2017).

6.3.2 Case Study: Effect-Safe Cooperative Multitasking

Using our effect-safe handler interface, we can implement all the effects and handlers from the Chapter 4, but most implementations are straightforward and do not bare much new technical insight. However, two handler implementations surface interesting properties of our Effekt design and our effect system: the `Poll` handler (for the `Fiber` effect) and `Scheduler` handler (for the `Async` effect). To recall, the handler for the `Fiber` effect does not explicitly capture the continuation, but only uses other effects in its implementation. In contrast, the handler for the `Async` effect does capture the continuation and *additionally* uses another effect (*i.e.*, `State`).

We now revisit these two effects (*cf.* Section 4.4) and handlers and add effect types. As we will see, adding effect types makes important details visible that were previously hidden.

The Scheduler Handler – Storing effectful computation

Figure 6.6 defines the effect-safe variants of the `Fiber` effect and its handler `Scheduler`. All term-level implementations are left unchanged but are repeated for convenience.

Adding effect types to the `Fiber` effect signature (Figure 6.6a) exhibits an interesting detail. The derived effect operation `forked` is a higher-order effect operation (Wu et al., 2014): it takes a *computation* as argument. While it is higher-order, the effect types inform us that it is not a handler. The computation passed as argument can have arbitrary effects `FX`. However, these effects are left unchanged. They still appear in the return type of `forked`.

```
trait Fiber extends Eff {
  def suspend(): Control[Unit, effect]
  def fork(): Control[Boolean, effect]
  def exit(): Control[Nothing, effect]
  def forked[FX](p: Control[Unit, FX]): Control[Unit, effect & FX] = for {
    b ← fork()
    r ← if (b) p andThen exit() else pure(())
  } yield r
}
```

(a) The `Fiber` effect signature – the derived operation `forked` is polymorphic in ambient effects `FX`.

```
trait Scheduler[R, FX] extends Fiber with Handler[R, Unit] {
  val state: State
  type Effects = state.effect & FX

  def unit(r: R) = pure(())

  type Queue = List[Control[Unit, Effects]]
  lazy val queue = state.Field[Queue](Nil)

  def exit() = use { resume ⇒ pure(()) }
  def fork() = use { resume ⇒
    queue.update { resume(true) :: resume(false) :: _ } andThen run
  }
  def suspend() = use { resume ⇒
    queue.update { _ appended resume(()) } andThen run
  }
  private def run: Control[Unit, Effects] = queue.get() flatMap {
    case Nil ⇒ pure(())
    case p :: rest ⇒ queue.put(rest) andThen p andThen run
  }
}
```

(b) Handler for the `Fiber` effect – enqueued processes now have more precise types.

Figure 6.6. Effect-safe versions of the `Fiber` effect signature and the `Scheduler` handler. All term-level implementations are unchanged.

Also the types of the `Poll` handler (Figure 6.6b) are more precise now. By defining

```
type Effects = state.effect & FX
```

the handler implementation declares that it uses the `state` capability. Specializing the type of `use[A]` to this handler implementation, we obtain for the argument `body`:

```
body: (A ⇒ Control[Unit, state.effect & FX]) ⇒ Control[Unit, state.effect & FX]
```

This shows two things. Firstly, the handler is allowed to use the `state` effect within the body passed to `use`. Secondly, continuations captured with `use` also still have the `state` effect and other ambient effects `FX`. One can easily make mistakes when implementing the scheduler example *without* effect types. As before, the scheduler stores continuations in a queue to later run them in a potentially different context (Dolan et al., 2017). The previous type of stored computations (*i.e.*, `Control[Unit]`) did not provide much information about when it is safe to run. In the effect-safe implementation, the type of `Queue` needs to be more specific. It now contains computations of type `Control[Unit, state.effect & FX]`. Importantly, since the computation is indexed by the involved effect types, it cannot be used outside of the handler for `state`. The handler function for the scheduler leaves the state effect open:

```
def scheduler[R, FX](st: State)(prog: (f: Fiber) ⇒ Control[R, f.effect & FX]) =
  new Scheduler[R, E] { val state: st.type = st } handle { fiber ⇒ prog(fiber) }
```

The type refinement **`val state: st.type`** is necessary for a precise return type, which is inferred to be `Control[Unit, st.effect & FX]`. Without the type refinement, the state handler could not remove the state effect from the set of effects.

The Async Handler – Forwarding without Capturing

Also the improved types of the `Async` effect (Figure 6.7a) and its handler implementation (Figure 6.7b) reveal some interesting details. Like the derived effect operation `forked`, the effect operation `async` is higher-order and takes a computation. Its return type also mentions the effect parameter `FX`, which suggests that promises contain effectful computations. Those computations are conceptually evaluated at the original call site to `async`. Correspondingly, the signature of `await` mentions no effects beside **`this.effect`**.

As mentioned before, handlers in Effekt do not have to capture and use the continuation and consequently do not have to inherit from the `Handler` trait. It is up to the handler implementation to decide. Figure 6.7b repeats the implementation of the `Poll` effect handler but adds effect types. The only necessary change is the definition of the type member **`effect`**:

```
type effect = state.effect & fiber.effect
```

Instead of extending the library trait `Handler`, the `Poll` handler directly implements the `Async` effect in terms of the two effects `State` and `Fiber`. This is not possible in languages like Koka, where every effect handler always has to capture the continuation. As before, the handler function `poll` takes the two required capabilities to construct an instance of the handler `Poll`:

```
def poll[R, FX](s: State, f: Fiber)(prog: (a: Async) ⇒ Control[R, a.effect & FX])
  = prog(new Poll { val state: s.type = s; val fiber: f.type = f })
```

```
trait Async extends Eff {
  type Promise[T]
  def async[T, FX](prog: Control[T, FX]): Control[Promise[T], effect & FX]
  def await[T](p: Promise[T]): Control[T, effect]
}
```

(a) The `Async` effect signature – promises contain effectful computations.

```
trait Poll extends Async {
  val state: State; val fiber: Fiber

  type effect = state.effect & fiber.effect
  type Promise[T] = state.Field[Option[T]]

  def async[T, FX](prog: Control[T, FX]) = for {
    p ← pure(state.Field[Option[T]](None))
    _ ← fiber.forked { prog flatMap { r ⇒ p.put(Some(r)) }
  } yield p

  def await[T](p: Promise[T]) = p.get() flatMap {
    case Some(r) ⇒ pure(r)
    case None ⇒ fiber.suspend() andThen await(p)
  }
}
```

(b) Handler for the `Async` effect – using two effects `State` and `Fiber`.

Figure 6.7. Effect-safe versions of the `Async` effect signature and the `Poll` handler. All term-level implementations are unchanged.

By refining the types of `state` and `fiber` to singleton types, the inferred return type of `poll` is `Control[R, s.effect & f.effect & FX]`. It thus communicates precisely that we implement the `Async` effect in terms of the given state and fiber capabilities.

We are now ready to safely handle the example program `asyncExample` from Section 4.4. The user program does not require any modifications.

```
region { s ⇒
  scheduler[Unit, s.effect](s) { f ⇒
    poll(s, f) { a ⇒
      asyncExample(f, a) // Control[Unit, f.effect & a.effect]
    }                    // Control[f.effect & s.effect]
  }                      // Control[s.effect]
}                        // Control[Pure]
```

The effect types illustrate that the `poll` handler function removes the async effect (`a.effect`), but adds the state effect (`s.effect`) to the set of effects. The `scheduler` handler function uses the same state effect but in turn removes the fiber effect (`f.effect`). Finally, the `region` handler handles the state effect. Type inference is not proficient enough to infer the removal of effects and we need to annotate the effect type `s.effect` at the call to `scheduler`.

This concludes our presentation of effect-safe programming with effect handlers. In this section, we have introduced the missing `Handler` interface, revisited some non-trivial example usages, and showed how they can be expressed in effect-safe ScalaEffekt.

6.4 Discussion: Effect Handlers and Object Orientation

Effekt is an embedding of effect handlers in a language with support for object-oriented programming. Naturally, the question arises how these two features interact. We will now revisit this combination in the light of our effect-safe implementation.

Object-oriented programming has a strong focus on encapsulation. In particular, the concrete implementation of an object and its internal state is often hidden behind an interface (Canning et al., 1989). That is, the implementation can differ with the granularity of a single object. Another important feature is that objects are first-class and are typically stored on the heap (Lindholm et al., 2015). In contrast, effects and handlers are tied to a stack discipline. Effect handler implementations can capture parts of the stack as a continuation, handlers delimit segments of the stack and effect typing asserts that these stack operations are safe.

Which effects are used by an object's implementation can be seen either as part of the public interface or as a private implementation detail. It is a design decision the programmer should make. However, if the effects used by an object are hidden behind an interface, how can we assert effect safety? For instance, if an object closes over a capability, the object's lifetime needs to be restricted to the capability's lifetime (Osvald et al., 2016). Otherwise, the use of the capability within the object might not be effect-safe. In this section, we will discuss possible design choices when combining effect handlers with object-oriented programming, while maintaining effect safety. The following interface will serve as a running example:

```
trait Person {
  def greet(other: String): Unit
}
```

Alternative 1. Effects as Part of the Public Interface

An implementation of this interface might want to use the following effect to print the greeting on the console.

```
trait Console extends Eff { def print(msg: String): Control[Unit, effect] }
```

However, the method `greet` as defined above does not mention the `Console` effect. Of course, we can change the interface accordingly.

```
trait Person {
  def greet(other: String)(out: Console): Control[Unit, out.effect]
}
```

Now, the `Console` effect is part of the public interface and all implementations of `Person` can make use of it to implement method `greet`. The effect has to be handled by the caller of `greet`. In this variant, it is possible to have multiple implementations of `Person` and store the instances in data structures on the heap.

```
var p1: Person = new Person { ... }
var p2: Person = new Person { ... }
val ps = List(p1, p2)
```

Alternative 2. Hiding Effects behind an Interface

Changing the interface of `Person` to mention the effects, which are used by a particular implementation, leaks implementation details. This problem also occurs with checked exceptions in Java. We can think of `Console` as a checked exception that is not mentioned in the interface of `greet`. Java programmers often resort to wrapping checked exceptions in unchecked ones to work around this problem (Zhang et al., 2016). Sometimes, the exceptions our implementation throws are considered implementation details that we might want to encapsulate. Similarly, in Effekt, we can hide the effects behind an abstract type member **`effect`**.

```
trait Person {
  type effect
  def greet(other: String): Control[Unit, effect]
}
```

An implementation of `Person` closes over the effect capabilities, just like the handlers from the previous section.

```
class MyPerson extends Person {
  val out: Console
  type effect = out.effect
  def greet(other: String) = out.print("Hello " + other)
}
```

This way, the lifetime of an object of type `MyPerson` is tightly coupled to the lifetime of the capability `out`. Let us assume some handler `withConsole` for the `Console` effect:

```
withConsole { o ⇒
  ...
  val p = new MyPerson { val out: o.type = o }
  ...
}
```

The instance `p` must not be used outside the scope of the handler `withConsole`, which is ensured by our effect system: `out.effect` is an abstract type that only unifies with this one particular call to `withConsole`. As before, to eventually be able to handle the effects used by the implementation, users thus always need to have stable paths to an object. In the following example p_1 and p_2 are arguments of method `user`. They have stable paths that can be used in the return type.

```
def user(p₁: Person, p₂: Person): Control[Unit, p₁.effect & p₂.effect] = for {
  _ ← p₁.greet("Alice")
  _ ← p₂.greet("Bob")
} yield ()
```

In general, the requirement of path stability (Odersky and Zenger, 2005b) excludes objects to be stored in mutable references or in containers like lists. While we can store p_1 in a reference, the effect system will prevent us from running a program that calls an effectful methods on it.

Alternative 3. Grouping Objects by their Effect Implementations

The first alternative requires all objects to use the same effects in their implementation and the second alternative allows each object to individually differ in their effect implementation. Both solutions also have drawbacks: the former constrains the implementer while the latter imposes restrictions on the user. As a compromise between the two extremes, we can generalize over the effect implementation and thereby group objects by their effect implementations.

```
trait Person[FX] {
  def greet(other: String): Control[Unit, FX]
}
```

Like with abstract type members, implementing classes can instantiate `FX` to the desired implementation effects. Like with the first alternative, objects of type `Person[out.effect]` leak the implementation detail that they use the `Console` effect in their implementation. Users can be parametric in the effect type of the particular implementation:

```
def user[FX](p₁: Person[FX], p₂: Person[FX]): Control[Unit, FX] = for {
  _ ← p₁.greet("Alice")
  _ ← p₂.greet("Bob")
} yield ()
```

While we now can store objects of type `Person[FX]` in mutable references or lists of type `List[Person[FX]]`, this requires all instances to have the same effect implementation. Like in the second alternative, instances of type `Person[out.effect]` are coupled to the lifetime of the corresponding capability `out`. If an implementation depends on more than one effect, it can only be used in the intersection of the corresponding handler regions. Using path-dependent types, our effect-system allows programmers to express this dependency on the type level.

6.5 Discussion: Properties of the Effect System

Our effect system is based on the $\lambda_{\Downarrow\Uparrow}$ calculus by Zhang and Myers (2019). We embed their calculus into the practical programming language Scala that has no full formal specification. It is therefore not in the scope of this thesis to formally prove properties of our library embedding. Nevertheless, in this section we discuss some properties of our embedded effect system and explain under which assumptions we believe them to hold.

6.5.1 Effect Safety

Effect safety is the absence of runtime errors, caused by capabilities being used outside of their defining handlers. Assuming a sound subset of Scala, such as the pDot calculus (Rapoport and Lhoták, 2019), we are confident that our effect system establishes effect safety – though we do not give formal proofs. Adding mutable variables and fields should also not affect our effect system which relies on *stable*, that is, immutable paths (Odersky and Zenger, 2005b). The same also holds for native exceptions, though unlike JavaEffekt, our library as presented in this chapter is not prepared to interact with native exceptions. A formal treatment is left to future work. In our experience, adding effect types to existing advanced case studies (such as the `Scheduler` handler in Section 4.10) helped us to discover subtle bugs. The effect system also guided us in the design of the interface for ambient state as presented in Section 6.2.

6.5.2 Effect Subtyping

By marking the set of capabilities in `Control` as contravariant, we use Scala's support for subtyping of intersection types to express effect subtyping. This is an important advantage over effect systems that encode effect rows using type-level lists. In those systems, effect subtyping typically has to be implemented manually by performing type-level computation (Kiselyov and Ishii, 2015). In contrast, using intersection types to express the set of effects integrates well with other Scala features like variance annotations and type bounds. Type inference for monotonically growing intersection types is well supported and in consequence, most return types of effectful functions (like `drunkFlip`) and effect handlers (like `maybe`) can be omitted.

6.5.3 Effect Polymorphism

We also reuse Scala's support for type polymorphism to express effect polymorphic functions. Rytz et al. (2012) give an example of an effect polymorphic, higher-order function:

```
def mapM[A, B, E](lst: List[A], f: A ⇒ Control[B, E]): Control[B, E]
```

The function `mapM` is effect polymorphic in the effects `E` used by function `f`. The return type of `mapM` indicates that it potentially calls `f` in its implementation and so has the same effects as `f`. The effects `E` still need to be handled by the caller of `mapM`. Handler functions like `collect` and `maybe` (Section 6.1) are other examples for effect polymorphic functions. Type inference for calling higher-order functions (like `mapM`) that do not alter the set of effects is well supported. In contrast, nesting multiple handler applications (like `collect` and `maybe`) often requires explicit effect annotations.

6.5.4 Effect Parametricity

The example function `mapM` is polymorphic in the effects `E`. Following Zhang and Myers (2019), we claim that it *should not be possible* for the implementation of `mapM` to (accidentally) handle any concrete effect in `E`; no matter what `E` will be instantiated to at the call site. That is, in the following user program, we should be able to determine *statically* that `flip` is handled by `collect`. No implementation of `mapM` should be able to violate this assumption.

```
collect { amb ⇒ mapM(List(1,2,3), n ⇒ amb.flip()) }
```

Zhang and Myers (2019) refer to this *parametricity* of effect polymorphism as *abstraction safety*. Generally speaking, given a type of a function, parametricity of type polymorphism allows us to infer properties of the function's implementation (Reynolds, 1983; Wadler, 1989). In the case of effect parametricity, we want to statically determine that a given higher-order function *cannot* handle a particular effect. Otherwise, we speak of *accidental handling*. But does effect parametricity hold for our implementation of Effekt? The answer is subtler than in most other implementations of effects and handlers because Effekt is based on capability passing style and multi-prompt delimited continuations. We need to distinguish two aspects of accidental handling.

Implementation abstraction Accidental handling of effects can occur in languages without static effect systems like Eff (Bauer and Pretnar, 2015) and Multicore OCaml (Dolan et al., 2014). Those languages dynamically search handlers at runtime. In those systems, `mapM` could (accidentally or purposefully) handle `Amb` and, for example, change the semantics of `flip` to always return **`true`**. Similarly, the presentations of Effekt of Chapters 4 and 5 do not have static effect systems. Still, they already support this aspect of effect parametricity. Capabilities are passed down to their use site and not looked up at runtime. In our example, the function passed to `mapM` closes over the capability `amb`, which fixes the implementation of `flip`.

Control-flow abstraction Because Effekt uses an implementation of multi-prompt delimited continuations, there is a second aspect of effect parametricity to consider. Looking at the example call to `mapM` again, we would also like to be sure that the continuation captured by `flip` will always be delimited by the corresponding call to `collect` and nowhere else. Because `mapM` does not know about `amb`, it should not be possible for `mapM` to delimit the continuation captured by `amb.flip()`. However, as we will see next, in Effekt we can construct examples that violate this property[34].

Capturing the Delimiter by Capturing the Continuation

The following example uses the untyped λ_{dcp} calculus (Section 2.2) to illustrate (accidental) delimiting of continuations.

$$\mathsf{def}\ \mathit{example}() \ =\ \mathsf{reset}_{\mathit{exc}}\ \{\ \mathit{delimit}\ (\lambda().\ \mathsf{shift}_{\mathit{exc}}\ \{\ k \Rightarrow \texttt{"aborted"}\ \}\ );\ \texttt{"resumed"}\ \}$$

In the example, we use $\mathsf{reset}_{\mathit{exc}}$ to delimit a program with a prompt *exc*. Within the range of $\mathsf{reset}_{\mathit{exc}}$, we then use $\mathsf{shift}_{\mathit{exc}}$ to abort the current computation and return the string `"aborted"` as the overall result. Assuming that *delimit* has no access to the prompt *exc*, we might want

[34] The observation of the loss of parametricity is due to an anonymous reviewer, who we are grateful for. The reviewer also presented a counter-example, which, in adapted form, our analysis in this section builds on.

to reason statically, that the continuation captured by shift_{exc} is delimited by the surrounding call to reset_{exc}. Independent of the implementation of *delimit*, running the program should always return the string `"aborted"`. This neither holds in λ_{dcp}, nor in any of the versions of Effekt presented in this thesis. The following code illustrates how to violate this assumption. Given some prompt r, we can define the function *delimit*:

$$\mathsf{def}\ \mathit{delimit}(\mathit{prog})\ =\ (\mathsf{shift}_r\ \{\ k \Rightarrow k(\lambda().\ k(\mathit{prog}))\ \})();$$
$$\mathsf{reset}_r\ \{\ \mathit{example}()\ \}$$

The call to shift_r returns a *computation* (highlighted in grey). In particular, it returns a computation that, when forced, calls the continuation again with the provided program *prog*. We finally force this computation *outside* of the call to shift. Operationally, passing the continuation to the continuation itself duplicates the evaluation context between the call to *delimit* and (including) the call to reset_r. Let us recall the operational semantics of $lambda_{dcp}$ (Section 2.2). In particular, rule $\mathit{-shift_+}$:

$$(\mathit{-shift_+})\quad \mathsf{reset}_p\ \{\ \mathsf{H}_p[\ \mathsf{shift}_p\ \{\ k \Rightarrow e\ \}\ ]\ \}\ \longrightarrow\ e[k \to \lambda x.\ \mathsf{reset}_p\ \{\ \mathsf{H}_p[\ x\ ]\ \}]$$

Evaluating the call to shift_p reifies the capture context H_p and binds it to k. Importantly, the capture context H_p can contain *arbitrary other* delimiter frames $\mathsf{reset}_{p'}\ \{\ \square\}$ where $p \neq p'$. Calling the continuation reinstalls those delimiters. Given the above implementation of *delimit*, we have the following reduction:

$$\mathsf{reset}_r\ \{\ \mathsf{H}_r\ [\ \mathit{delimit}\ \{\ e\ \}\ ]\ \}\quad \longrightarrow^*\quad \mathsf{reset}_r\ \{\ \mathsf{H}_r\ [\ \mathsf{reset}_r\ \{\ \mathsf{H}_r\ [\ e\ ]\ \}\ ]\ \}$$

That is, in our example, the capture context contains the delimiter with prompt *exc*, which is

$$\mathsf{H}_r\ =\ \mathsf{reset}_{exc}\ \{\ \square\ ;\ \text{"resumed"}\ \}\ ]$$

If *body* now captures and discards the continuation, only the innermost copy of the context H_r is removed. Hence, our example returns `"resumed"`. To emphasize again, delimiting the extent of the continuation captured by shift_{exc} is possible *without* direct access to the prompt marker *exc*.

Violating Control-Flow Abstraction in ScalaEffekt

We can translate the above example to the effect-safe handler setting of ScalaEffekt.

```
trait Delimit[R, E] extends Eff {
  def delimited[FX](prog: Control[R, FX]): Control[Control[R, FX & E], effect]
  def delimit[FX](prog: Control[R, FX]): Control[R, FX & E & effect] =
    delimited(prog) flatMap { c ⇒ c }
}
```

The `Delimit` effect has a primitive effect operation `delimited` and a derived effect `delimit`. The latter simply calls the former and forces the returned computation. The type parameter `E` describes the set of effects at the handler – `FX` describes the set of effects at the call to `delimit`. It is an effect polymorphic effect operation. We implement `Delimit` like before:

```
class Reset[R, E] extends Delimit[R, E] with Handler[R, R] {
  def unit(result: R) = pure { result }
  def delimited[FX](prog: Control[R, FX]) = use { k ⇒ k(k(prog)) }
}
```

We do not need to thunk the result of `k(prog)` since it already is a computation of type `Control[R, FX & E]`. Intuitively, all handlers of effects present in `FX`, but not present in `E`, are duplicated by calling `delimited`. In the following example, we use the effect `Delimit` together with the exception effect.

```
new Reset[Option[String], Pure] handle { r ⇒
  new Maybe handle { exc ⇒
    r.delimit {
      exc.raise("Abort")
    } map { _ ⇒ "Resumed" }
  }
}
```

Like the above program in λ_{dcp}, the example returns `Some("Resumed")`. Importantly, the type of the effect operation `delimited` does *not* mention the exception effect. Instead, it is parametric in the effects `FX`. So why can it handle (or, to be more precise *delimit*) the exception effect? In our embedding of `Effekt`, the effect type of the continuation `k` does not represent the fact that it captures delimiters. It only mentions the effects it *uses* not the ones it *delimits*[35]. However, the loss of parametricity is not a threat to effect safety. Handlers are duplicated, not removed.

6.5.5 Effect Encapsulation

Another property, *effect encapsulation*, is related to effect parametricity and can be observed in languages featuring an ML-like type system with row-polymorphism for effect types like Koka and Frank (Lindley, 2018; Leijen, 2018b). The following program is adapted from Leijen (2018b) and written in the Koka language. It shows how an effect used by f_1 leaks into its type – it is not encapsulated.

```
fun f1(action: () → <exc|e> a): e option<a> { // types inferred
  maybe { if (...) { raise("abort") }; action() }
}
```

Here, f_1 is a higher-order function taking an effectful function `action` as argument. The function f_1 uses exceptions in its implementation but locally handles them with the `maybe` handler. This implementation detail *leaks* as part of the inferred type, which states the fact that `exc` effects of `action` will be handled by f_1. Koka implements effect subtyping via row polymorphism (Leijen, 2014), so the effect row of `action()` is unified with those of other statements under `maybe`.

Manually Encapsulating Effects

Operationally, Koka will handle any exception effect used in `action` with the `maybe` handler in f_1. We cannot hide this fact by annotating the parameter `action` of f_1 with the type `() → e a`. This does not type check. However, if we do not want any exceptions thrown by `action` to be handled by `maybe`, languages like Koka and Frank offer some form of manual lifting operation (Biernacki et al., 2017; Convent et al., 2020).

[35] This has been brought to our attention by the aforementioned, anonymous reviewer.

```
fun f2(action: () → e a): e option<a> { // types inferred
  maybe {
    if (...) { raise("abort") }
    inject<exc> { action() }
  }
}
```

Manually injecting the `exc` effect into the effect row also has operational content as described by Leijen (2018b): the runtime search for the exception handler will skip the next handler for `exc`. Now the type of f_2 truthfully states that it does not handle any effects in `e` – including any exception effects.

In Effekt, we can also express the two variants of the function `f` with different types:

```
def f1[A, E](action: (exc: Exc) ⇒ Control[A, exc.effect & E]): Control[Option[A], E]
def f2[A, E](action: () ⇒ Control[A, E]):  Control[Option[A], E]
```

The type of f_1 makes it clear that `action` has an unhandled exception effect, handled by f_1.

6.6 Related Work and Chapter Conclusion

In this section, we discuss closely related work. In particular, we compare our approach of achieving effect safety with others.

6.6.1 Abstraction-Safe Effect Handlers via Tunneling

Both, the dynamic and static semantics of Effekt is closely related to $\lambda_{\Downarrow\Uparrow}$, presented by Zhang and Myers (2019). Handling an effect with a handler `h` introduces a fresh label. Like prompts in Effekt, the label is used on the term level to delimit the scope of captured continuations. As in the first presented versions of ScalaEffekt (Brachthäuser and Schuster, 2017), capabilities in $\lambda_{\Downarrow\Uparrow}$ are tuples of a label and the handler implementation. To ensure effect safety, Zhang and Myers use a simple form of dependent types: Using an effect handler `h` introduces `h.lbl` in the effect type, which is effectively a set of labels. The dependent label corresponds to the abstract type member `h.effect` in ScalaEffekt. Like in ScalaEffekt, this dependent effect type can only be discharged by the very same delimiter (denoted by $\Downarrow^{\ell}\ e$) that introduced the label. Due to the embedding of Effekt in Scala, prompts are first class while labels in $\lambda_{\Downarrow\Uparrow}$ are not first class. Instead, the binding of a label and its use in a handler implementation is statically scoped.

Zhang and Myers formalize $\lambda_{\Downarrow\Uparrow}$ and show effect safety. However, they do not provide an implementation of their calculus. We use intersection types and path-dependent types to encode the ideas of the $\lambda_{\Downarrow\Uparrow}$ effect system and thereby make Effekt effect-safe.

To facilitate comparison of $\lambda_{\Downarrow\Uparrow}$ with Effekt, Figure 6.8 presents an embedding of the $\lambda_{\Downarrow\Uparrow}$-calculus (Zhang and Myers, 2019) into Scala. We use the `Control` monad to express the operational semantics. We present a practical embedding into Scala and leave a formal translation and corresponding soundness proofs to future work. However, assuming a sound subset of Scala that corresponds to $\lambda_{\Downarrow\Uparrow}$, we conjecture that our embedding faithfully models the calculus by Zhang and Myers. In particular, the restriction to a subset of Scala excludes the use of mutable state, recursive function definitions, recursive data-types, and exceptions. In addition, effect signatures have to be declared on the top level and should neither use mixin composition, type

```
type ℓ[T, E] = Prompt[T, E]

trait 𝔽[A, B] { type lbl; def apply(arg: A): Control[B, this.lbl] }

class Handler[A, B, T, E, L <: ℓ[T, E]](
  val label: L,
  val impl: A ⇒ (B ⇒ Control[T, E]) ⇒ Control[T, E]
) extends 𝔽[A, B] {
  type lbl = label.type
  def apply(arg: A) = shift(label) { k ⇒ impl(arg)(k) }
}

def ⇩[R, E](prog: (l: ℓ[R, E]) ⇒ Control[R, l.type & E]): Control[R, E] = reset { prog }
def ⇧[A, B](f: 𝔽[A, B]): A ⇒ Control[B, f.lbl] = a ⇒ f.apply(a)
```

Figure 6.8. Using `Control` to embed the $\lambda_{⇩⇧}$ calculus into Scala.

members, type-bounds, subtyping, or any other advanced Scala feature.

Example To ease comparison with the original calculus, we use ℓ as the type of labels and $\mathbb{F}$ as the type of effect signatures. Effect signatures `𝔽[A, B]` only declare a single effect operation with argument type `A` and return type `B`. For example, we can express the signatures of the ambiguity and exception effect as:

```
trait Amb extends 𝔽[Unit, Boolean]
trait Exc extends 𝔽[String, Nothing]
```

Handlers, modeled by the class `Handler`, are pairs of labels and effect implementations[36]. In this style, the handler for expressions can be expressed as:

```
def maybe[R, E](prog: (exc: Exc) ⇒ Control[R, exc.effect & E]) = ⇩ { l ⇒
  val exc = new Handler(l, msg ⇒ k ⇒ pure(None)) with Exc
  prog(exc) map { r ⇒ Some(r) }
}
```

Calling an effect operation amounts to calling ⇧ on the handler:

```
maybe { h ⇒ ... ⇧(h)("Failed!") ... }
```

While in the $\lambda_{⇩⇧}$ calculus, the operation ⇧ performs the continuation capture, in the embedding we perform the capturing in the implementation of `Handler.apply` by means of `shift(l)`. This is necessary to have the available answer types (`T` and `E`) in scope. We use subtyping (*i.e.*, `Handler <: 𝔽`) to existentially hide the answer types and other implementation details of `Handler` when passing capabilities of type $\mathbb{F}$ (or `Exc` and `Amb` to be precise).

[36] Like with the handlers of ScalaEffekt, the field `label` needs to be refined to the singleton type `l.type`. That is, the constructor calls requires annotation: `new Handler[String, Nothing, Option[R], E, l.type](...)`.

6.6.2 Effect Safety by Region Safety

In ScalaEffekt, we establish effect safety by ensuring that a capability cannot be used outside of its handler region. The problem of ensuring region safety is a general one and a wide range of solutions exist.

Safety for multi-prompt delimited control As in our presentation in Section 4.2, Dybvig et al. (2007) guarantee answer type safety by indexing prompts with the expected answer type. Furthermore, they use rank-2 types to prevent prompts from being used across different instances of `run`. But, as they observe, this is not enough to achieve effect safety, which they explicitly leave to future work. We generalize the idea of region safety and guarantee that capabilities cannot be used outside of the scope that they are created in. Instead of rank-2 types, we use abstract type members. This lets us easily nest scopes using intersection types. As of now, Dotty has better support for path-dependent function types than for rank-2 types. For instance, we can use the lambda syntax (*e.g.*, `amb ⇒ amb.flip()`), which is currently not possible with rank-2 types.

Safety for effects Many languages with effect handlers base their effect system on some form of row polymorphism. Prominent examples are Koka (Leijen, 2014), Frank (Lindley et al., 2017; Convent et al., 2020), and Links (Hillerström and Lindley, 2016; Hillerström et al., 2017). In contrast, effect-safe library embeddings like Extensible Effects (Kiselyov et al., 2013; Kiselyov and Ishii, 2015) often use various forms of open union types (Swierstra, 2008) to track the list of unhandled effects. In Effekt, we index the monad for effectful computations with an intersection of all capabilities effects used by the computation. This way, capabilities cannot be used outside of their handler region.

Safety for resources To achieve resource safety, Kiselyov and Shan (2008) generalize from a single region (Launchbury and Sabry, 1997; Moggi and Sabry, 2001) to multiple nested regions. They achieve region polymorphism and region subtyping together with good type inference for their library in Haskell. On the type level, they represent nested regions as a nesting of monad transformers. In contrast, we represent nested delimiters by an intersection of abstract type members. To achieve region polymorphism, they reuse Haskell's polymorphism and to achieve region subtyping they use Haskell's type class instance search. To achieve effect polymorphism, we reuse Scala's polymorphism and to achieve subtyping we reuse subtyping for intersection types built into Scala.

Safety for variable scopes Parreaux et al. (2017) apply a strategy very similar to ours in order to implement scope safety in the context of type-safe meta programming. The type parameter `Ctx` of their type `Code[+Typ, -Ctx]` is used to track the set of free variables. Addtionally, variables have a type member that describes the scope the variable can be used in.

```
class Variable[A] {
 type Ctx;
 def substitute[T,C](pgrm: Code[T, Ctx & C], v: Code[A, C]): Code[T,C]
}
```

As can be seen from the type of `substitute`, substitution of free variables removes `Ctx` from the intersection type and thus corresponds to handling of effects.

Safety for capabilities Like in this thesis, Osvald et al. (2016) perform capability passing in Scala: A capability serves as a constructive proof, that the holder is entitled to use the actions associated with the capability. To prevent leaking of capabilities, Osvald et al. (2016) introduce a type-based escape analysis as an alternative approach to traditional effect systems: arguments to functions can be marked as second class (or "local"). The type checker then guarantees that the capability cannot leave the dynamic scope of the function call. Liu (2016) presents a different approach to capability based effect safety, by distinguishing between functions that can capture capabilities and others that cannot (called "stoic"). In our design of Effekt, we adopt the capability passing of Osvald et al. (2016). Their approach of second class values might be an interesting alternative to the effect system presented in this thesis. However, they are not available in Scala, while our library can readily be used.

6.6.3 Effect Parametricity

Our implementation of ScalaEffekt does not guarantee effect parametricity due to accidental delimiter capture (Section 6.5). Effect polymorphic effect operations like `delimited` may seem contrived. However, effect polymorphic effect operations are essential to express examples like `Async.async` (Figure 6.7a). Languages without support for effect polymorphic operations, like $\lambda_{\Downarrow\Uparrow}$ or Koka cannot directly express this signature. To work around this limitation in Koka, Leijen (2017a); fixes the set of effects used by asynchronous programs to `io`. We embed Effekt into Scala and use intersection types as sets of effects. This allows us to reuse Scala's type system for effects. However, as shown in Section 6.5, it also allows to violate control-flow abstraction. We leave it to future work to estimate the impact of lost parametricity and to implement strategies to restore it. We see two ways to restore parametricity in ScalaEffekt – both come with significant trade-offs.

Restricting effect signatures We conjecture that control-flow abstraction can be restored by restricting the expressivity of effect signatures. Languages like Koka or the $\lambda_{\Downarrow\Uparrow}$ d oo't support effect polymorphic effect operations and thus cannot express operations like `delimited` (Section 6.6.1). In Effekt, effect signatures are arbitrary traits and effect operations can use Scala's polymorphism to express effect polymorphism (*e.g.*, type parameter `FX` in the signature of `delimited`). This is not expressible in our embedding of $\lambda_{\Downarrow\Uparrow}$ (Figure 6.8). There, one effect signature $\mathbb{F}$`[A, B]` always only describes a single effect operation from type `A` to `B`, which excludes effect polymorphic operations. While this may restore effect parametricity, this might exclude many of the newly supported extensibility scenarios of Section 4.4.

Resuming continuations as effect To duplicate parts of the evaluation context and the delimiters contained therein, the implementation of the effect operation `delimit` calls the continuation with the continuation. It thus relies on the fact that the continuation can escape the scope of the `shift` that captured it! We can prevent this by treating continuation resumption itself as an effect:

```
trait Cont[A, R, FX] extends Eff { def resume(a: A): Control[R, effect & FX] }
def shift[A, R, FX](p: Prompt[R, FX])(
  body: (k: Cont[A, R, FX]) ⇒ Control[R, k.effect & FX]
): Control[A, p.type]
```

This way, the continuation can only be used within the scope of the body provided to `shift`. While we believe that this solution restores parametricity, it also rules out handlers (*e.g.*, `Scheduler`, Figure 4.9), which need to store the continuation to resume it later.

6.6.4 Effect Handlers and Object-Oriented Programming

In Chapters 4 and 5, we started to explore the combination of effect handlers and object orientation. However, those versions of Effekt did not guarantee effect safety. The present chapter shows how to add effect safety to Effekt and support effect polymorphism. As highlighted in Section 4.7.3, effect-safe programming with effect handlers in a language with objects comes with new challenges – mediating encapsulation and flexible use of objects. Inostroza and van der Storm (2018) also combine effect handlers and object orientation in the language JEff. In JEff, the continuation takes an updated copy of the effect handler as additional argument, which models dynamically scoped state (Kiselyov et al., 2006). It also allows to change the handler implementation for the rest of the computation, similar to shallow handlers. The effect system of JEff does not feature effect polymorphism and hence problems with effect encapsulation do not arise. Inostroza and van der Storm (2018) present a static effect system for the language JEff. However, in contrast to our embedded effect system, JEff does not support effect polymorphism. In the simpler setting without effect polymorphic functions, violations of effect parametricity are not a concern.

6.6.5 Conclusion

In this chapter, we presented Effekt, a monadic library for programming with effect handlers in Scala that features effect polymorphism, effect subtyping and effect safety. We use intersection types and path-dependent types to track the set of effects a program might use. This allows us to directly reuse Scala's support for polymorphism for effect polymorphism and Scala's support for subtyping for effect subtyping. Combining effect-safe delimited control with object-oriented programming both offers new ways to modularize effectful programs but also comes with new challenges.

Chapter 7

Discussion and Conclusion

Effect handlers enable control-flow abstractions that are user-definable, modular, and compositional. In this thesis, we explored the modularity and extensibility opportunities gained by combining effect handlers with object-oriented programming. First, we revised two important aspects of effect handlers, delimited control (Chapter 2) and dynamic binding (Chapter 3) to elaborate the operational and conceptual essence of effect handlers. Building on this foundation, we presented a design to integrate effect handlers with object-oriented programming (Chapter 4). We showed how to enable user programs to be written in direct-style (Chapter 5) and how to establish effect safety (Chapter 6). In this chapter, we summarize the conclusions of the previous chapters, highlight the most important aspects of each chapter, discuss some limitations, and sketch potential future work.

Delimited control In the first part of this thesis, we revisited several different forms of delimited control in the literature. We presented existing generalizations of delimited control in two steps to arrive at effect handlers. These two steps helped us to highlight important characteristics of effect handlers: First, the generalization from one control operator to a family of control operators enables the safe use of multiple different effects in one program. With multi-prompt delimited control, prompt markers can be used to uniquely describe the connection between a control operator (shift_p) and its corresponding delimiter (reset_p) without having to risk interference with other operators and delimiters. Second, syntactically moving the implementation of a control effect from the control operator to the delimiter simplifies reasoning about the continuation usage. Maybe most importantly, it emphasizes that handlers perform effect encapsulation: Handlers can use other control effects to implement effect operations. However, these control effects are not evaluated at the call site of the implemented effect operation – they are evaluated at the definition site of the handler. Effect handlers thus offer an interesting combination of dynamic and static scoping. Handlers dynamically bind effect operations, but execute their implementation in the static scope of their definition.

Dynamic binding Starting from dynamic binding (*e.g.*, ambient values), we again performed two steps of generalization to arrive at effect handlers. While the generalizations from delimited control carried some insight, but no technical novelty – this is different for the generalization from dynamic binding. In particular, the novel feature of *ambient functions* provides the same aforementioned effect encapsulation that effect handlers provide, without the complexity of delimited continuations. Avoiding this complexity bears fruit for both language implementors and users. On the one hand, it is easier to efficiently implement ambient functions than it is to implement ambient control. This is the case since ambient functions are always tail-resumptive, which allows important optimizations (Leijen, 2017b). On the other hand, users can readily apply their intuition about closure and lexical scoping to reason about effect encapsulation. In the future, we hope to see more languages that will support this feature.

Effekt In the second part of this thesis, we presented Effekt, a library design for effect handlers that integrates well with object-oriented programming languages. Building on the insights from the first part, we designed Effekt around the concept of explicit capability-passing style as an alternative to dynamic binding. Explicit capability-passing style has several advantages. Since it only requires basic features, it is well supported by existing programming languages. Modern implementations of the Java Virtual Machine offer a multitude of optimization strategies for dynamic dispatch and virtual method calls such as polymorphic inline caches (Hölzle et al., 1991) and inlining of monomorphic calls (Kotzmann et al., 2008). Explicit capability passing directly benefits from these optimizations. Combining capability passing with multi-prompt delimited control also gives rise to an alternative of expressing effect polymorphism (Osvald et al., 2016). Lastly, capability passing helps us to understand effects as imposing an additional requirement on the calling context, rather than a side effect that occurs in addition to computing the result. In our effect-safe implementation, we used this insight and marked the type parameter `Effect` on `Control` as contravariant.
The combination of effect handlers and object-oriented programming proved to be a fruitful one. On the one hand, embedding effect handlers in an object-oriented programming offers new ways to modularize effect handlers. On the other hand, having control effects and handlers available in a language allows users to express user-defined control-flow constructs. Nevertheless, the combination also comes with new challenges. If no special measures are taken, allocation of objects and modification of fields has to be understood as *global* effects. In contrast, effect handlers express control flow *locally*. They can use the captured continuation in non-linear ways, potentially violating assumptions about global resources, such as heap-allocated objects. In Section 4.3 we presented a specialized state handler to implement ambient state, which interacts well with non-linear control flow. In particular, it implements the desired backtracking behavior, when resuming a continuation multiple times. The solution of handlers with ambient state is only a partial one as it only backtracks the handler state shallowly. In future work, it would be interesting to develop a more holistic approach to integrate heap-state with effect handlers – potentially following Leijen (2018b).

Direct-style Effekt in Java We presented the first library design for programming with effect handlers in Java. In contrast to our Scala implementations, programs in JavaEffekt can be written in direct style. We enable the direct style by applying a CPS transformation on the level of bytecode. Our transformation reduces stack utilization by trampolining, performs resumptions in constant time (in the stack-depth), supports multiple resumptions and is competitive in its performance. In the future, once delimited continuations are natively supported by the JVM (Pressler, 2017), the bytecode transformation backend could be switched to an implementation in terms of native continuations. As of this writing, switching to the development version of the native implementation still comes with performance penalties. More importantly, it does not (yet) allow multiple resumptions.

Effect-safe Effekt in Scala To the best of our knowledge, we are the first to present an effect-safe embedding of monadic, multi-prompt delimited control. Our effect-system is conservative and excludes some programs that could be expressed safely without effect-types (Kobori et al., 2016). Building on monadic delimited control, we then showed how to make our Scala implementation of Effekt effect safe. We use intersection types and path-dependent types to track the set of effects a program might use. This allows us to readily reuse Scala's support for polymorphism to express effect polymorphism and Scala's support for subtyping to express effect subtyping. Our

implementation supports effect encapsulation without manual lifting annotations, by building on explicit capability passing and multi-prompt delimited control. Using singleton types and intersection types also has limitations. Handling effects, that is, removing elements from the intersection type, often requires explicit type and effect annotations. Forwarding of effects, that is, handlers using other effects in their implementation, requires manual type refinements to the corresponding singleton type. As we have seen in Chapter 6, this is necessary to unify the path-dependent types. In the future, it would be interesting to investigate whether type inference can be improved by either modifying Scala's type system or our embedding of the effect system. Furthermore, reusing type polymorphism in Scala to express effect polymorphism allows users to express effect polymorphic effect operations. However, using those operations, control-flow abstraction and effect parametricity can be violated.

7.1 Future Work: Effect-Safe and Direct-Style Effekt

We identified two major limitations of our Effekt implementation in Chapter 4: user programs have to be written in monadic style and the implementation does not guarantee effect safety. We separately addressed the two issues in Chapters 5 and 6.

In the future, it would be interesting to combine the two solutions and obtain an effect-safe embedding where user programs can be written in direct-style. However, Java's type system is not expressive enough to embed an effect system in the style of Chapter 6. In particular, it does not support path-dependent types. We conjecture, the most promising way is to use the effect system of ScalaEffekt together with a CPS translation like the one presented in Chapter 5.

Since Scala does not support checked exceptions, one could instead use annotations to guide the transformation (Rompf et al., 2009; Parallel Universe Software Co., 2013). Rompf et al. (2009) implement delimited continuations as a Scala compiler plugin. They annotate effectful programs with `A @cps[B, C]` denoting a computation that computes a result of type `A` and changes the answer type from `B` to `C` (Rompf et al., 2009). Since annotations are not automatically propagated (as opposed to checked exceptions in Java) Rompf et al. need to manually implement annotation inference and checking of annotations. We could employ a similar strategy and instead of the monadic type constructor `Control[+A, -Effects]` use an annotation `A @effects[-Effects]`. Using such an annotation based approach, our running example could be expressed as

```
def drunkFlip(amb: Amb, exc: Exc): String @effects[amb.effect & exc.effect] =
  if (amb.flip()) {
    if (amb.flip()) { "Heads" } else { "Tails" }
  } else {
    exc.raise("Too drunk")
  }
```

Like the type constructor `Control`, the effects annotation could be indexed by a contravariant intersection type to track the set of used effects.

7.2 Future Work: Efficient Compilation of Effect Handlers

Effect handlers allow high-level, user-definable, and composable control abstractions. However, a significant runtime cost is associated with searching the correct handlers and capturing the continuation. By means of capability passing, our implementations of `Effekt` reduce the handler search to a simple dynamic dispatch. Still, capturing the continuation involves a runtime search through the (user level) stack. This becomes visible in our performance evaluation of Section 5.6.2. Comparing the baseline, not using effects, with our optimized version of `JavaEffekt` suggests room for improvement of around 27x for the Stateloop benchmark and 7x for the NQueens benchmark. Future work could investigate how to further improve the performance of our effect handler implementations. There exist two lines of closely related work to minimize the performance overhead of the abstraction of effect handlers: runtime optimizations and compile-time optimizations.

Runtime optimizations
Some languages limit the expressivity of effect handlers to allow more efficient implementation strategies in the language runtime. For example, Multicore OCaml originally only allowed continuations to be called once to support efficient (and destructive) stack switching (Dolan et al., 2014, 2013). Other languages detect usage patterns of the continuation, which then can be supported more efficiently. In the Koka implementation, Leijen (2017b) syntactically recognizes that the continuation is only used once in tail position to optimize capture / resume sequences.

Compile-time optimizations
Leijen (2017c) uses effect types to distinguish pure from effectful computations and only applies a CPS transformation to the latter. Pretnar et al. (2017) explore compile time source-to-source transformation rules to implement an optimizing compiler for the language Eff. The idea is to repeatedly apply rewrite rules in order to specialize effectful programs to their handlers (Pretnar et al., 2017).

In previous work (Schuster and Brachthäuser, 2018), we explored an alternative strategy to efficiently compile control effects. We started to adapt the approach as a compilation strategy for effect handlers (Schuster et al., 2020), which we outline in the remainder of this subsection. Our compilation strategy rests on the following observations: As pointed out in Chapter 2, the semantics of a program with control-effects depends on the evaluation context (the stack). The concrete stack can only be known at runtime. But, what if certain information about the stack can be determined statically at compile time? In languages like Koka (Leijen, 2017c), the stack carries both, the effect handler implementations as well as markers delimiting the captured continuations. Both are part of the evaluation context, as discussed in Chapter 2. If some of this information would be available at compile-time, can we use it to partially evaluate (that is, specialize) the program? We distinguish the following two classes of potentially static information:

Stack shape.
We might know in which *order* effect handlers will be present on the stack, when evaluating a part of the program. We refer to this information as the *shape* of the stack. Statically knowing the order in which effect handlers appear would allow us to specialize the control flow and continuation capture. For instance, we could use this information to drive an iterated CPS translation (Danvy and Filinski, 1990), that is, perform one CPS translation for every

handler in the stack shape. This is similar to the translation by Hillerström et al. (2017), however, we propose to additionally also take the static ordering of effect handlers into account. Like in the CPS hierarchy by Danvy and Filinski (1990), dynamic runtime search for a delimiter could be replaced by direct composition of explicit continuation fragments.

Handler implementation.
We might know the concrete handler implementation that will be present on the stack when evaluating a particular part of the program. This would allow inlining of the handler implementations at the call site of the effect operation. Inlining the handlers not only removes the runtime search for the handler implementation, but also potentially opens up further local optimizations.

We believe that capability-passing style, as performed by Effekt, can be an important step to achieve handler inlining. Use staging annotations (Taha and Sheard, 1997) capabilities can be marked as stage-time information. This is similar to how Kammar et al. (2013) use Haskell's support for inlining type-class dictionaries. In future work, it would be interesting to fully develop the ideas into a compiler for a language with effect handlers like Koka. This will come with interesting challenges, since in Koka, both the stack shape and handlers are not always statically known. However, we believe that it is beneficial to start from the restricting assumption that handlers and the stack shape are always statically known and guarantee full elimination of the associated runtime costs. We conjecture that having to lift some of the restrictions that come with this requirement will make the associated runtime costs explicit.

7.3 Future Work: Effectful Traversals and Modular Interpreters

Often, interpreters for programming languages are structured as recursive functions over the abstract syntax tree of the object language. If the recursion structure of the interpreter follows the recursion structure of the object language, the traversal itself can be defined separately as a *fold*. Maybe most importantly, separating the recursion structure from the computation immediately gives rise to fusion of traversals (Meijer et al., 1991).

Often, interpreters for programming languages use computational effects of the host language to express effects of the object language. Another way to model effects of the object language is by monads (Wadler, 1995). Expressing effects with monads allows implementing interpreters in host languages that do not support the effects of the object language.

The two approaches can be naturally combined to implement interpreters using effectful (or monadic) folds (Meijer and Jeuring, 1995). This allows describing traversals that can be fused, while being able to use effects not available in the host language. Object algebras (Oliveira and Cook, 2012; Oliveira et al., 2013) are a recent way to describe folds in a modular and extensible way. Different aspects of a traversal can be implemented in separate modules, while still allowing some dependencies. Computations can access attributes computed on the current node and on immediate child nodes. The following example, adopted from Oliveira et al. (2013), shows a simple algebra for arithmetic expressions with numeric literals and addition:

```
trait ExpAlg[E] {
  def lit(n: Int): E
  def add(e1: E, e2: E): E
}
```

Instead of constructing the recursive data structure, terms are represented by their church encoding (*i.e.*, $\forall r.\ (ExpAlg[r] \Rightarrow r) \Rightarrow r$) and are constructed by immediately calling into the algebra (Barendregt, 1992; Oliveira and Cook, 2012).

In future work, it would be interesting to extend object algebras to support *modular effectful traversals*. We believe that combining object algebras with effect handlers as presented in this thesis is especially promising. Different components of effectful traversals can use different monads with different semantic domains, which might be difficult to compose. In contrast, using Effekt the only necessary monad is `Control` and effectful algebras can easily be specialized to it:

```
type ExpEff[E] = ExpAlg[Control[E]]
```

Inostroza and Storm (2015) extend the object algebra approach to modularly propagate context information (like a type-environment) through the traversal. Each traversal component can depend on different aspects of the context. The framework of Inostroza and Storm (2015) hides the necessary context composition and projections. The same approach could potentially be used to propagate capabilities.

In prior work (Rendel et al., 2014), we extended the expressiveness of object algebras to also allow dependencies on left siblings, encoding the full class of L-attributed grammars. This way, object algebras can be used to modularly implement one-pass compilers. Using the results of this thesis, traversals expressed with object algebras can also be *effectful*. That is, attribute specifications can use control effects. Using the encoding techniques of Rendel et al. (2014) together with the design for effect handlers, as presented in this thesis, it might be possible to implement effectful one-pass compilers.

Combining tree traversals with *delimited control* effects is particularly interesting, since it allows expressing non-local tree rewritings, such as let-insertion (Yallop, 2016, 2017) or transformations to A-normal form (Thiemann, 1996). Similarly, it has recently been shown by Wang et al. (2019) that delimited control operators can be used to express reverse-mode automatic differentiation. Using effectful tree traversals, it might be possible to describe such transformations as separate, reusable modules.

7.4 Future Work: Naturalistic DSLs and Effectful Syntax

It is the goal of domain specific languages (DSLs) to bridge the conceptual gap between languages used in the particular domain and the computer language domain experts use to solve the domain problems (Hudak, 1996). Domain language is often close to natural (*i.e.*, spoken) language, making the task of DSL design particularly challenging (Lopes et al., 2003). Natural language has the reputation of being lexically and syntactically ambiguous, having complicated and context dependent binding structures, and often has non-trivial semantics, which rarely is compositional.

In consequence, often DSLs are still far away from being close to natural language. This is in particular the case for DSLs, which are *embedded* (Hudak, 1996) into a general-purpose language. With embedded DSLs, the host language additionally imposes its own syntactical restrictions and typing discipline on the DSL designer. Many limitations have been addressed in their own line of work. Examples include syntax extensions as libraries (Erdweg et al., 2011; Biboudis et al., 2016) and domain specific type system extensions (Lorenzen and Erdweg, 2016)). However, non-context-free linguistic constructs are often neglected.

In about the last decade, many developments in modeling the semantics of natural languages

have been inspired by computer science and the theory of abstract machines and control operators in particular. Importantly, Shan (2005) describes "noncompositional phenomena in natural languages" as *linguistic side effects*. Delimited continuations have successfully been used to model linguistic side effects, such as quantification, focus, and polymorphic coordination (Shan, 2004a, 2005; Barker and Shan, 2004). Maršík and Amblard (2016) recently used algebraic effects with handlers to give a compositional semantics to deixis (*"John loves* me*"*), quantification with scope islands (*"John loves* every woman*"*), and implicature (*"John,* my best friend*, loves me"*).

Implementors of (domain specific) programming languages often reside to effects to describe the *semantics* of the language (Wadler, 1995). Inspired by the recent developments in natural language semantics, we propose to follow Maršík and Amblard (2016) and use effects and handlers to describe the *syntax* of a DSL. In particular, we propose to group the different syntactic constructs of a DSL according to the following aspects: *(1) pure syntax*, that can be understood as compositional construction of the abstract syntax tree. *(2) effectful syntax*, that, like linguistic side effects (Shan, 2005), (often) requires context for interpretation and results in some form of non-local rewriting of the syntax tree. Effect operations can be used to express effectful syntax. *(3) binding syntax*, which provides the necessary context. Binding syntax can be expressed as effect handlers for effectful syntax.

7.4.1 Effectful Syntax in Scala

Scala comes equipped with many features that allow designing elegant embedded DSLs (Moors et al., 2012). Examples include infix notation of method application, implicit coercions, implicit resolution, and techniques to extend classes retroactively with new methods (Odersky et al., 2006). Using the results of this thesis, we can now additionally use effect handlers to implement effectful DSLs. To illustrate the gained expressivity, we implement examples from Maršík and Amblard (2016) as an embedded domain specific language in Scala. Maršík and Amblard (2016) already use a calculus of effects and handlers to express the semantics. We simply translate the examples to Scala, using ScalaEffekt as presented in Chapter 6. We also use Scala implicits to hide the details of capability passing (Section 4.6.1) and to focus on the application domain of natural language.

The Speaker Effect. We begin with a simple sentence that uses the speaker effect to refer to the contextual speaker of the sentence:

```
def s1 given Speaker = John said { Mary loves me }
```

The declaration of sentence `s1` requires an implicit parameter of type `Speaker`, which tells us that the sentence uses the speaker effect:

```
trait Speaker extends Eff { def me(): Control[NominalPhrase, effect] }
```

Here, the type `NominalPhrase` is part of the abstract syntax tree of our DSL. Making capability passing and other DSL implementation techniques explicit, the above example corresponds to:

```
def s1(s: Speaker) = s.me().map { p ⇒ Said(John, Loves(Mary, p)) }
```

The syntax tree describing the sentence can only be fully constructed, once the speaker is known. The speaker effect can be handled locally by using the handler `saidQuote`:

```
def s2 = John saidQuote { Mary loves me }
```

Again, making capability passing and other DSL implementation techniques explicit, the sentence corresponds to:

```
def s2: Control[Sentence, Pure] = saidQuote(John) { s ⇒
  s.me().map { p ⇒ Said(John, Loves(Mary, p)) }
}
```

The type of the sentence now suggests that no effect is left to be handled, and we can run the sentence to obtain `Said(John, Loves(Mary, John))`.

The `Scope` effect. Passing down context information, as we did with the speaker effect, does not require full handlers. In Koka we could use the feature of ambient values as presented in Chapter 3. Likewise, in Scala, implicit parameters suffice to express this effect. Things become more interesting when we consider the scope effect, which can be used to model universal quantification (Maršík and Amblard, 2016).

```
def s3 = scoped { John saidQuote { every(Woman) loves me } }
```

The effect operation `every` takes a predicate (*i.e.*, `Woman`) and introduces a universal quantification at the position of the handler `scoped`. It is declared in the signature of the `Scope` effect.

```
trait Scope extends Eff {
  def every(pred: NominalPhrase ⇒ Sentence): Control[NominalPhrase, effect]
}
```

Running `s3`, we see that this leads to a systematic "rewrite" of the syntax tree, moving the introduced binder and the predicate up to the handler:

```
▶ Forall(x ⇒ Implies(Woman(x), Said(John, Loves(x, John))))
```

Every invocation of the effect operation `every` introduces an additional quantifier. This non-local rewriting of the syntax tree to introduce a binder is very similar to let-insertion (Yallop, 2016, 2017). Yallop (2017) show how to use effect handlers to perform let-insertion.

With effectful syntax, we propose to revisit linguistic side effects (Shan, 2005) in the context of domain specific languages. In particular, we propose to apply Maršík and Amblard's (2016) idea of using effect handlers to model natural language semantics to embedded domain specific languages. The library presented in this thesis enabled us to combine the expressivity of Scala with the expressivity of effect handlers to modularly implement effectful syntax. We conjecture that systematically using effect handlers and effectful syntax leads to user programs that communicate the usage of linguistic features in their types, opens up new modularization strategies for DSL implementations, and potentially offers improved error reporting and better IDE support. We believe that effectful syntax opens up a new interesting perspective on the design and implementation of naturalistic DSLs. In future work, it would be interesting to explore the idea of effectful syntax on application domains like (financial) contracts (Peyton Jones et al., 2000) or test specifications.

7.5 Conclusion

In this thesis, we addressed our goal to make the powerful program structuring technique of effect handlers available to a wider audience in two ways.

Firstly, we provided a fresh perspective on effect handlers, viewing them as a combination of delimited control and dynamic binding. Approaching effect handlers from dynamic binding, the new feature of ambient functions emerged. Ambient functions are easier to understand and to reason about than effect handlers since they do not manipulate the control flow. At the same time, they show the same powerful effect encapsulation properties of effect handlers. The way to approach handlers from dynamic binding and ambient functions as an intermediate form of abstraction can make a significant difference in a wider adoption of effect handlers.

Secondly, we explored the design space of embedding effect handlers in existing mainstream object-oriented programming languages. We have demonstrated that effect handlers can be implemented efficiently on top of existing object-oriented languages. Our approach finally enables programmers of object-oriented languages to also use effect handlers to structure their programs. At the same time, it allows to use object-oriented programming techniques to structure effect handlers. This way, our language design opens up previously unexplored dimensions of extensibility. We presented a novel bytecode transformation, which enables writing user programs of effect handlers in direct-style. The transformation makes significant use of closure creation and is competitive in performance. Finally, we demonstrated a new way to achieve effect safety in a library embedding of effect handlers. Using intersection types and path-dependent types allowed us to model regions and establish that capabilities cannot be used outside of their region. We are confident that our library design can guide the research on effect handlers towards more modular and extensible language designs, regardless of the programming paradigm.

References

Dan Abramov. Algebraic effects for the rest of us, 2019. URL `https://overreacted.io/algebraic-effects-for-the-rest-of-us/`. [Last access: 09-30-2019].

Michael Adams, Celeste Hollenbeck, and Matthew Might. On the complexity and performance of parsing with derivatives. In *Proceedings of the Conference on Programming Language Design and Implementation*, pages 224–236, New York, NY, USA, 2016. ACM.

Nada Amin and Ross Tate. Java and Scala's type systems are unsound: The existential crisis of null pointers. In *Proceedings of the Conference on Object-Oriented Programming, Systems, Languages and Applications*, pages 838–848, New York, NY, USA, 2016. ACM.

Kenichi Asai and Yukiyoshi Kameyama. Polymorphic delimited continuations. In *Proceedings of the Asian Symposium on Programming Languages and Systems*, pages 239–254, Berlin, Heidelberg, 2007. Springer.

Henk P. Barendregt. *The Lambda Calculus: Its Syntax and Semantics. Revised Edition.* North-Holland, Amsterdam, The Netherlands, 1984.

Henk P. Barendregt. Lambda calculi with types. In *Handbook of Logic in Computer Science (vol. 2): Background: Computational Structures*, pages 117–309. Oxford University Press, New York, NY, USA, 1992.

Chris Barker and Chung-chieh Shan. Continuations in natural language. Technical report CSR-04-1, School of Computer Science, University of Birmingham, Birmingham, UK, 2004.

Andrej Bauer and Matija Pretnar. An effect system for algebraic effects and handlers. In *International Conference on Algebra and Coalgebra in Computer Science*, pages 1–16, Berlin, Heidelberg, 2013. Springer.

Andrej Bauer and Matija Pretnar. Programming with algebraic effects and handlers. *Journal of Logical and Algebraic Methods in Programming*, 84 (1): 108–123, 2015.

Tim Berners-Lee. The principle of least power, 2005. URL `https://www.w3.org/2001/tag/doc/leastPower-2005-12-19.html`. [Last access: 10-09-2019].

Aggelos Biboudis, Pablo Inostroza, and Tijs van der Storm. Recaf: Java dialects as libraries. In *Proceedings of the Conference on Generative Programming and Component Engineering*, pages 2–13, New York, NY, USA, 2016. ACM.

Gavin Bierman, Claudio Russo, Geoffrey Mainland, Erik Meijer, and Mads Torgersen. Pause'n'play: Formalizing asynchronous C#. In *Proceedings of the European Conference on Object-Oriented Programming*, pages 233–257, Berlin, Heidelberg, 2012. Springer.

Malgorzata Biernacka, Dariusz Biernacki, and Sergueï Lenglet. Typing control operators in the CPS hierarchy. In *Proceedings of the Conference on Principles and Practice of Declarative Programming*, pages 149–160, New York, NY, USA, 2011. ACM.

References

Dariusz Biernacki, Maciej Piróg, Piotr Polesiuk, and Filip Sieczkowski. Handle with care: Relational interpretation of algebraic effects and handlers. *Proc. ACM Program. Lang.*, 2 (POPL): 8:1–8:30, December 2017. ISSN 2475-1421.

Dariusz Biernacki, Maciej Piróg, Piotr Polesiuk, and Filip Sieczkowski. Abstracting algebraic effects. *Proc. ACM Program. Lang.*, 3 (POPL): 6:1–6:28, January 2019. ISSN 2475-1421.

Walter Binder, Jarle Hulaas, and Philippe Moret. Advanced Java bytecode instrumentation. In *Proceedings of the International Conference on Principles and Practice of Programming in Java*, pages 135–144, New York, NY, USA, 2007. ACM.

Richard J. Boulton, Andrew Gordon, Michael J. C. Gordon, John Harrison, John Herbert, and John Van Tassel. Experience with embedding hardware description languages in HOL. In *Proceedings of the International Conference on Theorem Provers in Circuit Design: Theory, Practice and Experience*, pages 129–156, Amsterdam, The Netherlands, 1992. North-Holland Publishing Co.

Jonathan Immanuel Brachthäuser and Philipp Schuster. Effekt: Extensible algebraic effects in Scala (short paper). In *Proceedings of the International Symposium on Scala*, New York, NY, USA, 2017. ACM. doi:10.1145/3136000.3136007.

Jonathan Immanuel Brachthäuser, Philipp Schuster, and Klaus Ostermann. Effect handlers for the masses. *Proc. ACM Program. Lang.*, 2 (OOPSLA): 111:1–111:27, October 2018. ISSN 2475-1421. doi:10.1145/3276481.

Jonathan Immanuel Brachthäuser, Philipp Schuster, and Klaus Ostermann. Effekt: Capability-passing style for type- and effect-safe, extensible effect handlers in Scala. *Journal of Functional Programming*, 2020. doi:10.1017/S0956796820000027.

Jonathan Immanuel Brachthäuser and Daan Leijen. Programming with implicit values, functions, and control. Technical Report MSR-TR-2019-7, Microsoft Research, 2019.

Jonathan Immanuel Brachthäuser, Tillmann Rendel, and Klaus Ostermann. Parsing with first-class derivatives. In *Proceedings of the Conference on Object-Oriented Programming, Systems, Languages and Applications*, New York, NY, USA, 2016. ACM. doi:10.1145/2983990.2984026.

Edwin Brady. Programming and reasoning with algebraic effects and dependent types. In *Proceedings of the International Conference on Functional Programming*, pages 133–144, New York, NY, USA, 2013. ACM.

Janusz A Brzozowski. Derivatives of regular expressions. *Journal of the ACM (JACM)*, 11 (4): 481–494, 1964.

P. S. Canning, W. R. Cook, W. L. Hill, and W. G. Olthoff. Interfaces for strongly-typed object-oriented programming. In *Proceedings of the Conference on Object-Oriented Programming, Systems, Languages and Applications*, pages 457–467, New York, NY, USA, 1989. ACM.

Jacques Carette, Oleg Kiselyov, and Chung-Chieh Shan. Finally tagless, partially evaluated. In *Proceedings of the Asian Symposium on Programming Languages and Systems*, pages 222–238, Berlin, Heidelberg, 2007. Springer LNCS 4807.

Robert Cartwright and Matthias Felleisen. Extensible denotational language specifications. In Masami Hagiya and John C. Mitchell, editors, *Theoretical Aspects of Computer Software*, pages 244–272, Berlin, Heidelberg, 1994. Springer Berlin Heidelberg.

Lukas Convent, Sam Lindley, Conor McBride, and Craig McLaughlin. Doo bee doo bee doo. *Journal of Functional Programming*, 30: e9, 2020. doi:10.1017/S0956796820000039.

Nils Anders Danielsson. Total parser combinators. In *Proceedings of the International Conference on Functional Programming*, pages 285–296, New York, NY, USA, 2010. ACM.

Oliver Danvy and Andrzej Filinski. Representing control: A study of the CPS transformation. *Mathematical Structures in Computer Science*, 2 (4): 361–391, 1992.

Olivier Danvy and Andrzej Filinski. A functional abstraction of typed contexts. *DIKU Rapport 89/12, DIKU, University of Copenhagen*, 1989.

Olivier Danvy and Andrzej Filinski. Abstracting control. In *Proceedings of the Conference on LISP and Functional Programming*, pages 151–160, New York, NY, USA, 1990. ACM.

Nicolaas Govert de Bruijn. Lambda calculus notation with nameless dummies. a tool for automatic formula manipulation with application to the Church-Rosser theorem. *Indagationes Mathematicae*, 34: 381–392, 1972.

Jack B. Dennis and Earl C. Van Horn. Programming semantics for multiprogrammed computations. *Commun. ACM*, 9 (3): 143–155, March 1966. ISSN 0001-0782.

Stephen Dolan, Servesh Muralidharan, and David Gregg. Compiler support for lightweight context switching. *ACM Transactions on Architecture and Code Optimization (TACO)*, 9 (4): 36:1–36:25, 2013. ISSN 1544-3566.

Stephen Dolan, Leo White, and Anil Madhavapeddy. Multicore OCaml. In *OCaml Workshop*, 2014.

Stephen Dolan, Leo White, KC Sivaramakrishnan, Jeremy Yallop, and Anil Madhavapeddy. Effective concurrency through algebraic effects. In *OCaml Workshop*, 2015.

Stephen Dolan, Spiros Eliopoulos, Daniel Hillerström, Anil Madhavapeddy, KC Sivaramakrishnan, and Leo White. Concurrent system programming with effect handlers. In *Proceedings of the Symposium on Trends in Functional Programming*. Springer LNCS 10788, 2017.

Iulian Dragos, Antonio Cunei, and Jan Vitek. Continuations in the Java virtual machine. In *Second ECOOP Workshop on Implementation, Compilation, Optimization of Object-Oriented Languages, Programs and Systems (ICOOOLPS 2007)*, Berlin, Germany, 2007. Technische Universität Berlin.

R. Kent Dybvig, Simon L. Peyton Jones, and Amr Sabry. A monadic framework for delimited continuations. *Journal of Functional Programming*, 17 (6): 687–730, 2007.

Michael Eichberg and Ben Hermann. A software product line for static analyses: The OPAL framework. In *Proceedings of the 3rd ACM SIGPLAN International Workshop on the State of the Art in Java Program Analysis*, SOAP '14, pages 1–6, New York, NY, USA, 2014. ACM.

Sebastian Erdweg, Tillmann Rendel, Christian Kästner, and Klaus Ostermann. Sugarj: Library-based syntactic language extensibility. In *Proceedings of the Conference on Object-Oriented Programming, Systems, Languages and Applications*, pages 391–406, New York, NY, USA, 2011. ACM.

Erik Ernst. Family polymorphism. In *Proceedings of the European Conference on Object-Oriented Programming*, pages 303–326, London, United Kingdom, 2001. Springer LNCS 2072.

Kasra Faghihi. Coroutines – Java toolkit that allows you to write coroutines, 2015. URL `https://github.com/vsilaev/tascalate-javaflow`. [Last access: 10-01-2019].

Matthias Felleisen. The theory and practice of first-class prompts. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 180–190, New York, NY, USA, 1988. ACM.

Matthias Felleisen. On the expressive power of programming languages. In *Proceedings of the European Symposium on Programming*, pages 134–151. Springer, Berlin, Heidelberg, 1990.

References

Matthias Felleisen and Daniel P. Friedman. Control operators, the SECD-machine, and the λ-calculus. In *Formal Description of Programming Concepts III*, pages 193–217. Elsevier (North-Holland), Amsterdam, 1986.

Michael J. Fischer. Lambda calculus schemata. In *Proceedings of ACM Conference on Proving Assertions About Programs*, pages 104–109, New York, NY, USA, 1972. ACM.

Yannick Forster, Ohad Kammar, Sam Lindley, and Matija Pretnar. On the expressive power of user-defined effects: Effect handlers, monadic reflection, delimited control. *Proc. ACM Program. Lang.*, 1 (ICFP): 13:1–13:29, August 2017. ISSN 2475-1421.

Daniel P. Friedman, Christopher T. Haynes, and Eugene Kohlbecker. Programming with continuations. In Peter Pepper, editor, *Program Transformation and Programming Environments*, Berlin, Heidelberg, 1984. Springer-Verlag.

Erich Gamma, Richard Helm, Ralph Johnson, and John Vlissides. *Design Patterns: Elements of Reusable Object-Oriented Software*. Addison-Wesley Publishing Co., Boston, Massachusetts, USA, 1995.

Steven E. Ganz, Daniel P. Friedman, and Mitchell Wand. Trampolined style. In *Proceedings of the International Conference on Functional Programming*, pages 18–27, New York, NY, USA, 1999. ACM.

Benedict R. Gaster and Mark P. Jones. A polymorphic type system for extensible records and variants. *Technical Report NOTTCS-TR-96-3*, 1996.

James Gosling, Bill Joy, and Guy L. Steele. *The Java Language Specification*. Addison-Wesley Publishing Co., Boston, MA, USA, 1996. ISBN 0201634511.

James Gosling, Bill Joy, Guy Steele, Gilad Bracha, and Alex Buckley. *The Java Language Specification – Java SE8 Edition*. Oracle America, Inc., Redwood City, CA, USA, 2015.

Carl A. Gunter, Didier Rémy, and Jon G. Riecke. A generalization of exceptions and control in ML-like languages. In *Proceedings of the Conference on Functional Programming Languages and Computer Architecture*, pages 12–23, New York, NY, USA, 1995. ACM.

Philipp Haller and Alex Loiko. LaCasa: Lightweight affinity and object capabilities in Scala. In *Proceedings of the Conference on Object-Oriented Programming, Systems, Languages and Applications*, pages 272–291, New York, NY, USA, 2016. ACM.

Chris Hanson. MIT Scheme reference manual. Massachusetts Institute of Technology, January 1991.

Christopher T. Haynes and Daniel P. Friedman. Embedding continuations in procedural objects. *ACM Trans. Program. Lang. Syst.*, 9 (4): 582–598, October 1987. ISSN 0164-0925.

Christopher T. Haynes, Daniel P. Friedman, and Mitchell Wand. Obtaining coroutines from continuations. *Computer languages*, 11 (3-4): 143–153, 1986.

Robert Hieb and R. Kent Dybvig. Continuations and concurrency. In *Proceedings of the Second ACM SIGPLAN Symposium on Principles & Practice of Parallel Programming*, PPOPP '90, pages 128–136, New York, NY, USA, 1990. ACM.

Robert Hieb, R. Kent Dybvig, and Carl Bruggeman. Representing control in the presence of first-class continuations. In *Proceedings of the Conference on Programming Language Design and Implementation*, pages 66–77, New York, NY, USA, 1990. ACM.

Robert Hieb, R. Kent Dybvig, and Claude W. Anderson, III. Subcontinuations. *Lisp Symb. Comput.*, 7 (1): 83–110, January 1994. ISSN 0892-4635.

Daniel Hillerström and Sam Lindley. Liberating effects with rows and handlers. In *Proceedings of the Workshop on Type-Driven Development*, New York, NY, USA, 2016. ACM.

Daniel Hillerström and Sam Lindley. Shallow effect handlers. In Sukyoung Ryu, editor, *Proceedings of the Asian Symposium on Programming Languages and Systems*, pages 415–435, Cham, 2018. Springer International Publishing.

Daniel Hillerström, Sam Lindley, Bob Atkey, and KC Sivaramakrishnan. Continuation passing style for effect handlers. In *Formal Structures for Computation and Deduction*, volume 84 of *LIPIcs*. Schloss Dagstuhl–Leibniz-Zentrum für Informatik, 2017.

J.R. Hindley. The principal type scheme of an object in combinatory logic. *Trans. of the American Mathematical Society*, 146: 29–60, Dec. 1969.

Christian Hofer, Klaus Ostermann, Tillmann Rendel, and Adriaan Moors. Polymorphic embedding of DSLs. In *Proceedings of the Conference on Generative Programming and Component Engineering*, pages 137–148, New York, NY, USA, 2008. ACM.

Urs Hölzle, Craig Chambers, and David Ungar. Optimizing dynamically-typed object-oriented languages with polymorphic inline caches. In *Proceedings of the European Conference on Object-Oriented Programming*, pages 21–38. Springer, 1991.

Paul Hudak. Building domain-specific embedded languages. *ACM Computing Surveys*, 28 (4es), December 1996. ISSN 0360-0300.

Paul Hudak. Modular domain specific languages and tools. In *Proceedings of the Conference on Software Reuse*, pages 134–142. IEEE Computer Society Press, June 1998.

Pablo Inostroza and Tijs van der Storm. Modular interpreters for the masses: Implicit context propagation using object algebras. In *Proceedings of the Conference on Generative Programming and Component Engineering*, pages 171–180, New York, NY, USA, 2015. ACM.

Pablo Inostroza and Tijs van der Storm. JEff: Objects for effect. In *Proceedings of the 2018 ACM SIGPLAN International Symposium on New Ideas, New Paradigms, and Reflections on Programming and Software*, Onward! 2018, New York, NY, USA, 2018. ACM.

G. F. Johnson and D. Duggan. Stores and partial continuations as first-class objects in a language and its environment. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 158–168, New York, NY, USA, 1988. ACM.

Mark P. Jones. A theory of qualified types. In *Proceedings of the European Symposium on Programming*, volume 582 of *Lecture Notes in Computer Science*, pages 287–306. Springer-Verlag, February 1992.

Mark P. Jones. Functional programming with overloading and higher-order polymorphism. In Johan Jeuring and Erik Meijer, editors, *Advanced Functional Programming*, pages 97–136, Berlin, Heidelberg, 1995. Springer Berlin Heidelberg.

Ohad Kammar and Matija Pretnar. No value restriction is needed for algebraic effects and handlers. *Journal of Functional Programming*, 27 (1), January 2017.

Ohad Kammar, Sam Lindley, and Nicolas Oury. Handlers in action. In *Proceedings of the International Conference on Functional Programming*, pages 145–158, New York, NY, USA, 2013. ACM.

Andrew Kennedy. Compiling with continuations, continued. In *Proceedings of the International Conference on Functional Programming*, pages 177–190, New York, NY, USA, 2007. ACM.

References

David J. King and John Launchbury. Structuring depth-first search algorithms in Haskell. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 344–354, New York, NY, USA, 1995. ACM.

Oleg Kiselyov. Incremental, undoable parsing in OCaml as the general parser inversion. Posted on the OCaml mailing list, July 2007.

Oleg Kiselyov. Delimited control in OCaml, abstractly and concretely. *Theoretical Computer Science*, 435: 56–76, 2012.

Oleg Kiselyov and Hiromi Ishii. Freer monads, more extensible effects. In *Proceedings of the Haskell Symposium*, pages 94–105, New York, NY, USA, 2015. ACM.

Oleg Kiselyov and Chung-chieh Shan. Functional pearl: Implicit configurations–or, type classes reflect the values of types. In *Proceedings of the Haskell Symposium*, pages 33–44, New York, NY, USA, 2004. ACM.

Oleg Kiselyov and Chung-chieh Shan. Lightweight monadic regions. In *Proceedings of the Haskell Symposium*, Haskell '08, New York, NY, USA, 2008. ACM.

Oleg Kiselyov and KC Sivaramakrishnan. Eff directly in OCaml. In *ML Workshop*, 2016.

Oleg Kiselyov and KC Sivaramakrishnan. Eff directly in OCaml. In Kenichi Asai and Mark Shinwell, editors, *Proceedings of the ML Family Workshop / OCaml Users and Developers workshops*, volume 285 of *Electronic Proceedings in Theoretical Computer Science*, pages 23–58. Open Publishing Association, 2018. doi:10.4204/EPTCS.285.2.

Oleg Kiselyov, Chung-chieh Shan, and Amr Sabry. Delimited dynamic binding. In *Proceedings of the International Conference on Functional Programming*, pages 26–37, New York, NY, USA, 2006. ACM.

Oleg Kiselyov, Amr Sabry, and Cameron Swords. Extensible effects: An alternative to monad transformers. In *Proceedings of the Haskell Symposium*, pages 59–70, New York, NY, USA, 2013. ACM.

Donald E. Knuth. Semantics of context-free languages. *Mathematical Systems Theory*, 2 (2): 127–145, 1968.

Ikuo Kobori, Yukiyoshi Kameyama, and Oleg Kiselyov. Answer-type modification without tears: Prompt-passing style translation for typed delimited-control operators. *arXiv preprint arXiv:1606.06379*, 2016.

James Koppel, Gabriel Scherer, and Armando Solar-Lezama. Capturing the future by replaying the past (functional pearl). *Proc. ACM Program. Lang.*, 2 (ICFP): 76:1–76:29, July 2018. ISSN 2475-1421.

Thomas Kotzmann, Christian Wimmer, Hanspeter Mössenböck, Thomas Rodriguez, Kenneth Russell, and David Cox. Design of the Java HotSpot™ client compiler for Java 6. *ACM Transactions on Architecture and Code Optimization (TACO)*, 5 (1): 7:1–7:32, May 2008.

Ralf Lämmel and Ondrej Rypacek. The Expression Lemma. In *Proceedings of the Conference on Mathematics of Program Construction*. Springer LNCS 5133, July 2008.

Peter J Landin. A generalization of jumps and labels. In *Report, UNIVAC Systems Programming Research*, 1965.

John Launchbury and Amr Sabry. Monadic state: Axiomatization and type safety. In *Proceedings of the International Conference on Functional Programming*, ICFP '97, pages 227–238, New York, NY, USA, 1997. ACM.

Daan Leijen. Extensible records with scoped labels. In *Proceedings of the Symposium on Trends in Functional Programming*, pages 297–312, 2005.

Daan Leijen. Koka: Programming with row polymorphic effect types. In *Proceedings of the Workshop on Mathematically Structured Functional Programming*, 2014.

Daan Leijen. Algebraic effects for functional programming. Technical report, MSR-TR-2016-29. Microsoft Research technical report, 2016.

Daan Leijen. Structured asynchrony with algebraic effects. In *Proceedings of the Workshop on Type-Driven Development*, pages 16–29, New York, NY, USA, 2017a. ACM.

Daan Leijen. Implementing algebraic effects in C. In *Proceedings of the Asian Symposium on Programming Languages and Systems*, pages 339–363, Cham, Switzerland, 2017b. Springer International Publishing.

Daan Leijen. Type directed compilation of row-typed algebraic effects. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 486–499, New York, NY, USA, 2017c. ACM.

Daan Leijen. Algebraic effect handlers with resources and deep finalization. Technical Report MSR-TR-2018-10, Microsoft Research, April 2018a.

Daan Leijen. First class dynamic effect handlers: Or, polymorphic heaps with dynamic effect handlers. In *Proceedings of the Workshop on Type-Driven Development*, pages 51–64, New York, NY, USA, 2018b. ACM.

Jeffrey R. Lewis, John Launchbury, Erik Meijer, and Mark B. Shields. Implicit parameters: Dynamic scoping with static types. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 108–118. ACM, 2000.

P. M. Lewis, D. J. Rosenkrantz, and R. E. Stearns. Attributed translations. *Journal of Computer and System Sciences*, 9 (3): 279–307, December 1974.

Sheng Liang, Paul Hudak, and Mark Jones. Monad transformers and modular interpreters. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 333–343, New York, NY, USA, 1995. ACM.

Tim Lindholm, Frank Yellin, Gilad Bracha, and Alex Buckley. *The Java Virtual Machine Specification – Java SE8 Edition*. Oracle America, Inc., Redwood City, CA, USA, 2015.

Sam Lindley. Algebraic effects and effect handlers for idioms and arrows. In *Proceedings of the Workshop on Generic Programming*, pages 47–58, New York, NY, USA, 2014. ACM.

Sam Lindley. Encapsulating effects. *Dagstuhl Reports*, 8 (4), 2018.

Sam Lindley, Conor McBride, and Craig McLaughlin. Do be do be do. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 500–514, New York, NY, USA, 2017. ACM.

Fengyun Liu. A study of capability-based effect systems. Master's thesis, École Polytechnique Fédérale de Lausanne, Switzerland, 2016.

Florian Loitsch. Exceptional continuations in JavaScript. In *Workshop on Scheme and Functional Programming*, 2007.

Cristina Videira Lopes, Paul Dourish, David H. Lorenz, and Karl Lieberherr. Beyond AOP: Toward naturalistic programming. In *Proceedings of the Conference on Object-Oriented Programming, Systems, Languages and Applications (Onward! track)*, Anaheim, 2003. ACM.

Florian Lorenzen and Sebastian Erdweg. Sound type-dependent syntactic language extension. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 204–216, New York, NY, USA, 2016. ACM.

References

Jirka Maršík and Maxime Amblard. Introducing a calculus of effects and handlers for natural language semantics. In *International Conference on Formal Grammar*, pages 257–272. Springer LNCS 9804, 2016.

Marek Materzok and Dariusz Biernacki. Subtyping delimited continuations. In *Proceedings of the International Conference on Functional Programming*, pages 81–93, New York, NY, USA, 2011. ACM.

John McCarthy. Recursive functions of symbolic expressions and their computation by machine, Part I. *Communications of the ACM*, 3 (4): 184–195, 1960. doi:10.1145/367177.367199.

Erik Meijer and Johan Jeuring. Merging monads and folds for functional programming. In *International School on Advanced Functional Programming*, pages 228–266. Springer LNCS 925, 1995.

Erik Meijer, Maarten Fokkinga, and Ross Paterson. Functional programming with bananas, lenses, envelopes and barbed wire. In *Proceedings of the Conference on Functional Programming Languages and Computer Architecture*, pages 124–144. Springer-Verlag, 1991.

Albert R. Meyer and Mitchell Wand. Continuation semantics in typed lambda-calculi. In *Workshop on Logic of Programs*, pages 219–224. Springer LNCS 173, 1985.

Matthew Might, David Darais, and Daniel Spiewak. Parsing with derivatives: A functional pearl. In *Proceedings of the International Conference on Functional Programming*, pages 189–195, New York, NY, USA, 2011. ACM.

Mark Samuel Miller. *Robust Composition: Towards a Unified Approach to Access Control and Concurrency Control*. PhD thesis, Johns Hopkins University, Baltimore, Maryland, USA, 2006. AAI3245526.

Robin Milner. A theory of type polymorphism in programming. *Journal of Computer and System Sciences*, 17: 248–375, 1978.

John C. Mitchell and Gordon D. Plotkin. Abstract types have existential type. *ACM Trans. Program. Lang. Syst.*, 10 (3): 470–502, July 1988. ISSN 0164-0925.

Eugenio Moggi. Computational lambda-calculus and monads. In *Proceedings of the Symposium on Logic in Computer Science*, pages 14–23. IEEE, 1989.

Eugenio Moggi and Amr Sabry. Monadic encapsulation of effects: a revised approach (extended version). *Journal of Functional Programming*, 11 (6): 591–627, November 2001.

Adriaan Moors, Tiark Rompf, Philipp Haller, and Martin Odersky. Scala-virtualized. In *Proceedings of the Workshop on Partial Evaluation and Program Manipulation*, pages 117–120, New York, NY, USA, 2012. ACM. ISBN 978-1-4503-1118-2.

Luc Moreau. A syntactic theory of dynamic binding. *Higher-Order and Symbolic Computation*, 11 (3): 233–279, Sep 1998. ISSN 1573-0557.

Ana Lúcia De Moura and Roberto Ierusalimschy. Revisiting coroutines. *ACM Trans. Program. Lang. Syst.*, 31 (2): 6:1–6:31, February 2009. ISSN 0164-0925.

Martin Odersky. Towards Scala 3, 2018. URL `https://www.scala-lang.org/blog/2018/04/19/scala-3.html`. [Last access: 09-30-2019].

Martin Odersky. The Scala language specification, 2.8, 2019a. URL `https://docs.scala-lang.org/tour/implicit-parameters.html`. [Last access: 09-30-2019].

Martin Odersky. Dotty documentation – dependent function types, 2019b. URL `https://dotty.epfl.ch/docs/reference/new-types/dependent-function-types.html`. [Last access: 09-15-2019].

Martin Odersky. Dotty documentation – given parameters, 2019c. URL https://dotty.epfl.ch/docs/reference/contextual/given-clauses.html. [Last access: 09-15-2019].

Martin Odersky and Matthias Zenger. Independently extensible solutions to the expression problem. In *Proceedings of the Workshop on Foundations of Object-Oriented Languages*, 2005a.

Martin Odersky and Matthias Zenger. Scalable component abstractions. In *Proceedings of the Conference on Object-Oriented Programming, Systems, Languages and Applications*, pages 41–57, New York, NY, USA, 2005b. ACM.

Martin Odersky, Vincent Cremet, Christine Röckl, and Matthias Zenger. A nominal theory of objects with dependent types. In Luca Cardelli, editor, *Proceedings of the European Conference on Object-Oriented Programming*, pages 201–224. Springer LNCS 2743, 2003. ISBN 978-3-540-40531-3.

Martin Odersky, Philippe Altherr, Vincent Cremet, Gilles Dubochet, Burak Emir, Philipp Haller, Stéphane Micheloud, Nikolay Mihaylov, Adriaan Moors, Lukas Rytz, Michel Schinz, Erik Stenman, and Matthias Zenger. The Scala language specification, 2.13, 2006. URL https://scala-lang.org/files/archive/spec/2.13. [Last access: 09-30-2019].

Martin Odersky, Olivier Blanvillain, Fengyun Liu, Aggelos Biboudis, Heather Miller, and Sandro Stucki. Simplicitly: Foundations and applications of implicit function types. *Proc. ACM Program. Lang.*, 2 (POPL): 42:1–42:29, December 2017. ISSN 2475-1421.

Bruno C. d. S. Oliveira and William R. Cook. Extensibility for the masses: Practical extensibility with object algebras. In *Proceedings of the European Conference on Object-Oriented Programming*, pages 2–27. Springer LNCS 7313, 2012.

Bruno C. d. S. Oliveira and Jeremy Gibbons. Scala for generic programmers. *Journal of Functional Programming*, 20 (3–4): 303–352, October 2010.

Bruno C. d. S. Oliveira, Tijs van der Storm, Alex Loh, and William R. Cook. Feature-oriented programming with object algebras. In *Proceedings of the European Conference on Object-Oriented Programming*, pages 27–51. Springer LNCS 7920, 2013.

Bruno C.d.S. Oliveira, Tom Schrijvers, Wontae Choi, Wonchan Lee, and Kwangkeun Yi. The implicit calculus: A new foundation for generic programming. In *Proceedings of the Conference on Programming Language Design and Implementation*, PLDI '12, pages 35–44. ACM, 2012.

Erik Osheim and Jorge Vicente Cantero. Scala improvement proces (SIP-35) - opaque types, 2017. URL https://docs.scala-lang.org/sips/opaque-types.html. [Last access: 09-30-2019].

Leo Osvald, Grégory Essertel, Xilun Wu, Lilliam I González Alayón, and Tiark Rompf. Gentrification gone too far? affordable 2nd-class values for fun and (co-) effect. In *Proceedings of the Conference on Object-Oriented Programming, Systems, Languages and Applications*, pages 234–251, New York, NY, USA, 2016. ACM.

Parallel Universe Software Co. Quasar, 2013. URL http://docs.paralleluniverse.co/quasar. [Last access: 10-01-2019].

David. L. Parnas. On the criteria to be used in decomposing systems into modules. *Communications of the ACM*, 15 (12): 1053–1058, 1972.

Lionel Parreaux, Antoine Voizard, Amir Shaikhha, and Christoph E. Koch. Unifying analytic and statically-typed quasiquotes. *Proc. ACM Program. Lang.*, 2 (POPL): 13:1–13:33, December 2017. ISSN 2475-1421.

References

Greg Pettyjohn, John Clements, Joe Marshall, Shriram Krishnamurthi, and Matthias Felleisen. Continuations from generalized stack inspection. In *Proceedings of the International Conference on Functional Programming*, pages 216–227, New York, NY, USA, 2005. ACM.

Simon Peyton Jones, Jean-Marc Eber, and Julian Seward. Composing contracts: An adventure in financial engineering (functional pearl). In *Proceedings of the International Conference on Functional Programming*, pages 280–292, New York, NY, USA, 2000. ACM.

Simon L. Peyton Jones and John Launchbury. State in Haskell. *Lisp and Symbolic Comp.*, 8 (4): 293–341, 1995.

Simon L. Peyton Jones, Dimitrios Vytiniotis, Stephanie Weirich, and Geoffrey Washburn. Simple unification-based type inference for GADTs. In *Proceedings of the International Conference on Functional Programming*, pages 50–61, New York, NY, USA, 2006. ACM.

Maciej Piróg, Piotr Polesiuk, and Filip Sieczkowski. Typed equivalence of effect handlers and delimited control. In *Formal Structures for Computation and Deduction*, LIPIcs, pages 30:1–30:16. Schloss Dagstuhl–Leibniz-Zentrum für Informatik, 2019.

Gordon Plotkin and John Power. Algebraic operations and generic effects. *Applied Categorical Structures*, 11 (1): 69–94, 2003.

Gordon Plotkin and Matija Pretnar. Handlers of algebraic effects. In *European Symposium on Programming*, pages 80–94. Springer-Verlag, 2009.

Gordon D. Plotkin and Matija Pretnar. Handling algebraic effects. *Logical Methods in Computer Science*, 9 (4), 2013.

Joe Gibbs Politz, Alejandro Martinez, Matthew Milano, Sumner Warren, Daniel Patterson, Junsong Li, Anand Chitipothu, and Shriram Krishnamurthi. Python: The full monty. In *Proceedings of the Conference on Object-Oriented Programming, Systems, Languages and Applications*, pages 217–232, New York, NY, USA, 2013. ACM. ISBN 978-1-4503-2374-1.

Ron Pressler. Loom Project: Fibers and Continuations for the Java Virtual Machine. OpenJDK Project, HotSpot Group, September 2017. URL `http://mail.openjdk.java.net/pipermail/discuss/2017-September/004390.html`.

Matija Pretnar, Amr Hany Shehata Saleh, Axel Faes, and Tom Schrijvers. Efficient compilation of algebraic effects and handlers. Technical report, Department of Computer Science, KU Leuven; Leuven, Belgium, 2017.

Aleksandar Prokopec. ScalaMeter, 2012. URL `https://scalameter.github.io`. [Last access: 10-01-2019].

Aleksandar Prokopec and Fengyun Liu. Theory and practice of coroutines with snapshots. In *Proceedings of the European Conference on Object-Oriented Programming*, volume 109, pages 3:1–3:32. Schloss Dagstuhl–Leibniz-Zentrum für Informatik, 2018.

Marianna Rapoport and Ondrej Lhoták. A path to DOT: formalizing fully-path-dependent types. *CoRR*, abs/1904.07298, 2019. URL `http://arxiv.org/abs/1904.07298`.

Didier Rémy. Type inference for records in natural extension of ML. In *Theoretical Aspects of Object-oriented Programming*, pages 67–95, Cambridge, MA, USA, 1994. MIT Press. doi:10.1.1.48.5873.

Tillmann Rendel, Jonathan Brachthäuser, and Klaus Ostermann. From object algebras to attribute grammars. In *Proceedings of the Conference on Object-Oriented Programming, Systems, Languages and Applications*, pages 377–395, New York, NY, USA, 2014. ACM. doi:10.1145/2714064.2660237.

John C. Reynolds. Definitional interpreters for higher-order programming languages. In *Proceedings of the ACM annual conference*, pages 717–740, New York, NY, USA, 1972. ACM.

John C. Reynolds. Types, abstraction and parametric polymorphism. In *Proceedings of the IFIP World Computer Congress*, pages 513–523, Amsterdam, The Netherlands, 1983. Elsevier (North-Holland).

Tiark Rompf, Ingo Maier, and Martin Odersky. Implementing first-class polymorphic delimited continuations by a type-directed selective CPS-transform. In *Proceedings of the International Conference on Functional Programming*, pages 317–328, New York, NY, USA, 2009. ACM. ISBN 978-1-60558-332-7.

Andreas Rossberg, Claudio V. Russo, and Derek Dreyer. F-ing modules. In *Proceedings of the 5th ACM SIGPLAN Workshop on Types in Language Design and Implementation*, pages 89–102, New York, NY, USA, 2010. ACM.

Lukas Rytz, Martin Odersky, and Philipp Haller. Lightweight polymorphic effects. In James Noble, editor, *Proceedings of the European Conference on Object-Oriented Programming*, pages 258–282, Berlin, Heidelberg, 2012. Springer Berlin Heidelberg.

Tom Schrijvers, Bruno C.D.S. Oliveira, Philip Wadler, and Koar Marntirosian. Cochis: Stable and coherent implicits. *Journal of Functional Programming*, 29, 2019.

Philipp Schuster and Jonathan Immanuel Brachthäuser. Typing, representing, and abstracting control. In *Proceedings of the Workshop on Type-Driven Development*, pages 14–24, New York, NY, USA, 2018. ACM. doi:10.1145/3240719.3241788.

Philipp Schuster, Jonathan Immanuel Brachthäuser, and Klaus Ostermann. Compiling effect handlers in capability-passing style. To appear in Proceedings of the International Conference on Functional Programming, 2020.

Tatsurou Sekiguchi, Takahiro Sakamoto, and Akinori Yonezawa. Portable implementation of continuation operators in imperative languages by exception handling. In Alexander Romanovsky, Christophe Dony, Jørgen Lindskov Knudsen, and Anand Tripathi, editors, *Advances in Exception Handling Techniques*, pages 217–233. Springer-Verlag, Heidelberg, Berlin, Germany, 2001.

Chung-chieh Shan. Delimited continuations in natural language. In *Continuation Workshop*, 2004a.

Chung-chieh Shan. Shift to control. In *Proceedings of the 5th workshop on Scheme and Functional Programming*, pages 99–107, 2004b.

Chung-chieh Shan. Linguistic side effects. In *Proceedings of the Symposium on Logic in Computer Science*. University Press, 2005.

Jeremy G. Siek and Andrew Lumsdaine. Essential language support for generic programming. In *Proceedings of the Conference on Programming Language Design and Implementation*, PLDI '05, pages 73–84. ACM, 2005.

Valery Silaev. Tascalate JavaFlow – continuations / coroutines for Java 1.5 - 11, 2015. URL `https://github.com/vsilaev/tascalate-javaflow`. [Last access: 10-01-2019].

Dorai Sitaram. Handling control. In *Proceedings of the Conference on Programming Language Design and Implementation*, pages 147–155, New York, NY, USA, 1993. ACM.

Dorai Sitaram and Matthias Felleisen. Control delimiters and their hierarchies. *LISP and Symbolic Computation*, 3 (1): 67–99, Jan 1990.

Lukas Stadler, Christian Wimmer, Thomas Würthinger, Hanspeter Mössenböck, and John Rose. Lazy continuations for Java virtual machines. In *Proceedings of the International Conference on Principles and Practice of Programming in Java*, pages 143–152, New York, NY, USA, 2009. ACM.

References

Guy Lewis Steele Jr. *Common Lisp. The Language, Second Edition*. Digital Press, 1990.

Norihisa Suzuki. Inferring types in smalltalk. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 187–199, New York, NY, USA, 1981. ACM.

S. Doaitse Swierstra. Combinator parsing: A short tutorial. In *Language Engineering and Rigorous Software Development*, pages 252–300. Springer, 2009.

Wouter Swierstra. Data types à la carte. *Journal of Functional Programming*, 18 (4): 423–436, July 2008.

Walid Taha and Tim Sheard. Multi-stage programming with explicit annotations. In *Proceedings of the Workshop on Partial Evaluation and Program Manipulation*, pages 203–217, New York, NY, USA, 1997. ACM.

Peter J. Thiemann. Cogen in six lines. In *Proceedings of the International Conference on Functional Programming*, pages 180–189, New York, NY, USA, 1996. ACM.

Eric Torreborre. Eff monad for cats, 2016. URL `https://github.com/atnos-org/eff`. [Last access: 09-24-2019].

Philip Wadler. Theorems for free! In *Proceedings of the Conference on Functional Programming Languages and Computer Architecture*, pages 347–359, New York, NY, USA, 1989. ACM.

Philip Wadler. Monads for functional programming. In *International School on Advanced Functional Programming*, pages 24–52. Springer, 1995.

Philip Wadler. The expression problem. Note to Java Genericity mailing list, November 1998.

Philip Wadler and Stephen Blott. How to make ad-hoc polymorphism less ad hoc. In *Proceedings of the Symposium on Principles of Programming Languages*, pages 60–76, New York, NY, USA, 1989. ACM.

Mitchell Wand. Continuation-based multiprocessing. In *Proceedings of the Conference on LISP and Functional Programming*, LFP '80, pages 19–28, New York, NY, USA, 1980. ACM.

Fei Wang, Daniel Zheng, James Decker, Xilun Wu, Grégory M. Essertel, and Tiark Rompf. Demystifying differentiable programming: Shift/reset the penultimate backpropagator. *Proc. ACM Program. Lang.*, 3 (ICFP): 96:1–96:31, July 2019. ISSN 2475-1421.

Andrew K. Wright and Matthias Felleisen. A syntactic approach to type soundness. *Inf. Comput.*, 115 (1): 38–94, November 1994.

Nicolas Wu and Tom Schrijvers. Fusion for free - efficient algebraic effect handlers. In *Proceedings of the Conference on Mathematics of Program Construction*. Springer LNCS 9129, 2015.

Nicolas Wu, Tom Schrijvers, and Ralf Hinze. Effect handlers in scope. In *Proceedings of the Haskell Symposium*, Haskell '14, pages 1–12, New York, NY, USA, 2014. ACM.

Jeremy Yallop. Staging generic programming. In *Proceedings of the Workshop on Partial Evaluation and Program Manipulation*, pages 85–96, New York, NY, USA, 2016. ACM.

Jeremy Yallop. Staged generic programming. *Proc. ACM Program. Lang.*, 1 (ICFP): 29:1–29:29, August 2017. ISSN 2475-1421.

Yizhou Zhang and Andrew C. Myers. Abstraction-safe effect handlers via tunneling. *Proc. ACM Program. Lang.*, 3 (POPL): 5:1–5:29, January 2019. ISSN 2475-1421.

Yizhou Zhang, Guido Salvaneschi, Quinn Beightol, Barbara Liskov, and Andrew C. Myers. Accepting blame for safe tunneled exceptions. In *Proceedings of the Conference on Programming Language Design and Implementation*, pages 281–295, New York, NY, USA, 2016. ACM.